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Expectation Damages and Bilateral Cooperative Investments *

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Abstract

We examine the efficiency of the standard breach remedy expectation damages in a setting of bilateral cooperative investment by a buyer and a seller. Contracts may specify a required quality level and an upper bound to the cost of production. We find that it is optimal to write an augmented Cadillac contract that sets one threshold such that it cannot be met with positive probability together with an extreme price. Then, one of the parties becomes a residual claimant of the trade relationship. The other threshold can be used to balance the incentives of the other party.

Keywords: breach remedies, expectation damages, Cadillac contracts, incomplete contracts, cooperative investments.

JEL-Classification: K12, C70, D86, L14, J41.

1 Introduction

In this paper, we consider a situation where a buyer and a seller, both risk neutral, contract for the future delivery of a good. Between the signing of the contract and production of the good both parties may invest cooperatively to stochastically increase the benefit of their trading partner. After uncertainty is resolved and investments are sunk but before trade is finalized, the parties may renegotiate to reach an ex-post optimal trade decision. Contracts are important to solve the standard hold-up problem that results in

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underinvestment and arises if trading parties solely rely on ex-post negotiations to induce investment incentives (see e.g. Williamson, 1985; Hart and Moore, 1988).

It is important to emphasize that the present paper is situated in the economic analysis of contract law and not in the literature on incomplete contracts originating from Hard and Moore (1988). The latter assumes that contracts are insufficiently contingent, i.e. cannot condition directly or indirectly on the state of the world. The main research question thus is whether the first best can be implemented. The former, in contrast, usually imposes less strict informational assumptions, i.e. cost and quality are verifiable.

This branch of the literature is interested to analyze whether contracts governed by real-world institutions such as standard legal breach remedies can implement the first best. The earlier literature on the economic analysis of contract law has extensively dealt with and offered solutions for the unilateral investment problem. For selfish investment, Edlin (1996) demonstrates that the classic overreliance result for the standard breach remedy of expectation damages (see e.g. Shavell, 1980, 1984; Rogerson, 1984) vanishes if the parties write a so-called Cadillac contract that specifies the highest possible quantity and/or quality combined with a sufficiently low price.

The idea behind this commonly-applied legal remedy is that the victim of breach receives a payment that makes him or her as well off as performance would have. In a setting of cooperative investment, Stremitzer (2010) confirms the optimality of Cadillac contracts. In contrast to selfish investment, an optimal Cadillac contract specifies, in his setting, the highest possible quantity and quality.

There do not exist many results for the bilateral investment problem. Edlin and Reichelstein (1996) analyze selfish investment and consider contracts that specify price and a quantity to be traded. Their main finding is that if the trade decision is continuous the socially optimal solution can be attained under the specific performance remedy.

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1 In most situations the court must only be able to form an unbiased estimate of damages, see Edlin (1996 p. 114).
2 An alternative solution is offered by Edlin and Reichelstein (1996) who find that expectation damages and another standard breach remedy, specific performance, can induce the first best if quantity is continuous and can therefore be used as an instrument to balance the overinvestment effect of Shavell (1980) against the hold-up effect that arises in absence of contractual protection.
3 Several alternative solutions to the cooperative investment problem have been proposed in the literature. For example, Che and Chung (1999) and Schweizer (2006) allow the parties to specify a required investment level. They show that the first best can be attained by a contract that is governed by reliance damages and expectation damages, respectively.
whereas expectation damages perform poorly. Ohlendorf (2009), however, points out that the poor performance of expectation damages can be attributed to Edlin and Reichelstein’s (1996) assumption of a deterministic and linear cost function: If it is sufficiently concave, expectation damages can indeed induce the first best in Edlin and Reichelstein’s (1996) setting. So far, the literature on the economic analysis of contract law has not dealt with bilateral cooperative investment. This may be due to Che and Hausch’s (1999) pessimistic view about the value of contracting in such an environment. In their article, situated in the incomplete contracts literature, they demonstrate that contracting becomes irrelevant if investment is of a sufficiently cooperative nature. Even though the economic analysis of contract law generally works under less strict assumptions than the literature on incomplete contracts, Che and Hausch’s (1999) result suggests that it may be difficult to balance both parties’ investment incentives in a setting of purely cooperative investment. Bilateral cooperative investment is, however, highly relevant in practise. Consider, for example, a situation where a manufacturer with a fixed train of machines invests to custom tailor his product to the wishes of his buyer. The buyer, in turn, may exert effort to improve coordination between both parties. Dyer and Ouchi (1993) report that Japanese automakers sent consultants to their suppliers to help decrease cost of production. Burt (1989) mentions Xerox that incorporates many supplier-designed components into its products and thus has to adapt its production lines and procedures to individual suppliers. Examples of cooperative investment by the seller include suppliers that make relationship-specific investments to customize parts for their buyers (Asanuma, 1989 p.14). Moreover, according to Nishiguchi (1994 p. 138) suppliers “send engineers to work with [automakers] in design and production. They play innovative roles in ... gathering information about [the automakers’] long-term product strategies.”

The aim of this paper is twofold. First, we demonstrate that the bilateral holdup problem that often arises under cooperative investment can be solved by a contract

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Che and Hausch (1999) give many many more real-world examples of cooperative investment: Cooperative investment plays a crucial role in the principal-agent literature where the agent’s effort directly benefits the principal. It is also present in quality-enhancing R & D efforts where suppliers and workers can spend effort to do a better job. Cooperative investment is also critically important in modern manufacturing where the adoption of quick-response inventory systems and flexible manufacturing approaches has increased the need for coordination across different production stages. Such coordination often requires investments of time and resources that have cooperative elements.
governed by the standard breach remedy expectation damages. The optimal contract turns out to be a combination between a Cadillac and a balancing contract and is thus henceforth called an augmented Cadillac contract. To obtain this optimality result, we do not impose stronger informational requirements than Edlin and Reichelstein (1996). All that is needed is that the court is able to unbiasedly estimate the buyer’s benefit from trade and the seller’s cost of production. Second, we derive our optimality result using a generally applicable machinery that does not require differentiability. The contracts we consider are simple contracts that may specify, besides price and quantity, a required quality level. Similar to Edlin (1996) and Stremitzer (2010), this threshold serves as a baseline for calculating damages should the seller deliver one or more goods of non-conforming quality. Quality thresholds are highly relevant in practice. For example, it is inconceivable that a car manufacturer and an engineering firm who contract for the future delivery of a motor write a contract that stays silent about fuel consumption and performance. Similarly, we allow the contracting parties to stipulate an upper bound for the seller’s cost of production such that all cost above this threshold are born by the buyer. Because investment is of a purely cooperative nature, not the seller’s but only the buyer’s investment decreases cost of production. A standard Cadillac contract specifies a single threshold and sets it at an extreme level such that it cannot be met with positive probability. Together with either a very low or a very high price this ensures that one of the parties accepts/delivers the quantity specified in contract whereas the other party breaches to the socially efficient quantity. Then, the breaching party is a residual claimant of the trade relationship and thus has efficient incentives to invest. What is crucial here is that the parties may freely specify a second threshold without disturbing the incentives of the residual claimant. Thus, this threshold can be used to balance the incentives of the non-breaching party. Our result challenges the viewpoint expressed in Edlin (1996) and Edlin and Reichelstein (1996) that expectation damages are poorly suited to solve problems of bilateral investment. Moreover, in contrast to Ohlendorf (2009), this paper also suggests that it may not be necessary that both parties face the risk of ex-post breach.\footnote{If, similar to Ohlendorf (2009), both parties make a purely selfish investment, one can readily show}
The paper is organized as follows. Section 2 introduces the model before we discuss the legal consequences of breach in section 3. In Sections 4, 5, and 6 we consider binary trade. Section 4 assumes that the buyer takes the performance decision whereas the seller takes it in Section 5. We demonstrate that in both cases a standard Cadillac contract can be augmented such that the first best is induced. Section 6 considers the ex-post breach game induced by expectation damages. We show that the first-best contracts of Sections 4 and 5 induce the parties to behave as if the buyer or as if the seller takes the performance decision, respectively. In Section 7 we extend, under the assumption that contracts are divisible, our first best result to non-binary trade. Section 8 concludes.

2 The model

Consider a situation where a buyer and a seller, both risk neutral, have to incur relationship-specific investments before they potentially trade a quantity \( q \) of some good. At date 1, the parties sign a contract to induce an efficient outcome (see Figure 1). The contract, which is governed by expectation damages, specifies a fixed price \( p \) and a quantity \( \bar{q} \) to be traded. Furthermore, it may stipulate an upper bound for the seller’s cost of production \( \bar{c} \), a required quality \( \bar{v} \) and an upfront transfer \( t \), where the latter is used to divide the expected gains from trade after price and thresholds have been chosen to maximize joint surplus. The legal implications of this type of contract are discussed in Section 3. At date 2, the buyer invests to decrease the seller’s expected cost of production whereas the seller invests to increase the buyer’s expected benefit from trade. We denote the cost of that the first best can be achieved by a contract that specifies an extreme price together with an intermediate cost or quality threshold. Similar to a standard Cadillac contract, this contract determines the breaching party. The result is available upon request.
their investments by $\beta \in [0, \beta_{\text{max}}]$ and $\sigma \in [0, \sigma_{\text{max}}]$, respectively, and assume that the investment decisions are not contractible. Then, at date 3, the state of the world $\omega \in \Omega$ is realized and hence the potential level of the seller’s cost of production and the buyer’s benefit from trade become commonly known.

At date 4, the parties play an extensive form breach game, which is described in detail in Section 3 and results in a quantity $q \in Q$ to be traded at date 5. In Sections 3 to 6 the trade choice is binary. W.l.o.g., we consider $Q = \{0, 1\}$. If it is non-binary, as in Section 7 we consider $Q = \{0, 1, 2, ..., q_{\text{max}}\}$. The payoffs determined by expectation damages serve as a disagreement point in subsequent renegotiations, which are costless and can take place anywhere between date 4 and date 5. The parties split the potential renegotiation surplus at an exogenously given ratio with the seller receiving a share of $\alpha \in [0, 1]$.

Let us denote the seller’s cost of producing $q$ units by $C(\beta, \omega, q)$ and the buyer’s benefit of obtaining $q$ units by $V(\sigma, \omega, q)$.

To identify cost and benefit associated with each single unit, we denote them, for the $i$th unit, by $C_i(\beta, \omega)$ and $V_i(\sigma, \omega)$, respectively. Let us assume that they are additive separable: For any unit $i \in \{1, 2, ..., N\}$, the seller’s cost of production and the buyer’s benefit do not depend on whether the remaining units are produced or not. Finally, the following technical assumptions apply throughout:

**Assumption 1** For any $\beta \in [0, \beta_{\text{max}}]$, $\sigma \in [0, \sigma_{\text{max}}]$ and $\omega \in \Omega$, $C(\beta, \omega, q)$ and $V(\sigma, \omega, q)$ are monotonically increasing in $q$ and $C(\beta, \omega, 0) = V(\sigma, \omega, 0) = 0$.

**Assumption 2** For any $\omega \in \Omega$ and $q > 0$, $V(\sigma, \omega, q)$ is monotonically increasing in $\sigma$ and $C(\beta, \omega, q)$ is monotonically decreasing in $\beta$.

**Assumption 3** There exist $c_l$ and $c_h$ s.t. for any unit $i \in \{1, 2, ..., N\}$ and for all $\beta \in [0, \beta_{\text{max}}]$ and $\omega \in \Omega$, $0 \leq c_l \leq C_i(\beta, \omega) \leq c_h < \infty$. Moreover, there exist $v_l$ and $v_h$ s.t. for any unit $i \in \{1, 2, ..., N\}$ and for all $\sigma \in [0, \sigma_{\text{max}}]$ and $\omega \in \Omega$, $0 \leq v_l \leq V_i(\sigma, \omega) \leq v_h < \infty$.

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6 This is, for example, the case if the parties play a bargaining game that results in the generalized Nash bargaining solution.

7 For simplicity, we do not distinguish between the buyer’s benefit of and the quality of a good. For our results to go through it is only important that the court’s perception of quality is an unbiased estimate of the buyer’s true benefit from trade. See Edlin (1996) for a detailed discussion on imperfectly informed courts.
Assumption 1 establishes that, ex-post, the total cost of production and the buyer’s benefit are increasing in the quantity to be traded. Assumption 2 represents that investment is beneficial and of a purely cooperative nature. It directly follows that the expected value of the buyer’s benefit is increasing and of the seller’s cost is decreasing in the other parties’ investment. Assumption 3 establishes that for any unit to be potentially traded the cost of production and the buyer’s benefit lie within a bounded interval. The ex-post social surplus of the transaction,

\[ W(\beta, \sigma, \omega, q) = V(\sigma, \omega, q) - C(\beta, \omega, q), \]

is maximized at the socially efficient quantity

\[ Q^*(\beta, \sigma, \omega) \in \arg \max_{q \in [0, q_{\text{max}}]} W(\beta, \sigma, \omega, q). \]

The efficient investment levels, denoted by \((\beta^*, \sigma^*)\), maximize the expected welfare gains from trade

\[ E[W(\beta, \sigma, \omega, Q^*(\beta, \sigma, \omega))] - \beta - \sigma \]

in \([0, \beta_{\text{max}}] \times [0, \sigma_{\text{max}}]\) contingent on an efficient trade decision. We assume that \(\beta^*\) and \(\sigma^*\) are unique maximizers.

3 Ex-post payoff

In this section, we work out the parties’ ex-post payoffs induced by expectation damages when the buyer’s benefit and the seller’s cost of production are observable by the contracting parties and the court. For clarity of exposition, we first tackle the case where trade is binary, \(Q \equiv \{0, 1\}\). To simplify notation, let us denote cost of production and quality of the good by \(c := C(\beta, \omega, 1)\) and \(v := V(\sigma, \omega, 1)\), respectively. To stay in line with the literature, the contract breach game we are going to consider is similar to the one in Edlin and Reichelstein (1996) and Ohlendorf (2009).

Suppose that before production and trade takes place at date 5 (see the timeline), first the seller and then the buyer announce their anticipatory breach decisions at date

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8 This assumption is not required to establish any of our results. It makes, however, the proofs considerably less tedious.

9 Alternatively, we consider non-binary trade in Section 7.
These decisions are anticipatory because they are announced before the potentially unwanted good is produced. Between date 4 where the decisions are announced and date 5 where delivery is contracted to take place, the parties may enter renegotiations. In these, the outcome determined by expectation damages serves as a disagreement point. The parties split any additional renegotiation surplus at an exogenously given ratio with the seller receiving a share of $\alpha \in [0, 1]$.

Let us now consider the parties' payoffs in absence of breach. In that case, the buyer has to pay the contracted price and receives the good whereas the seller receives the price but has to incur production costs. These payoffs are given by $v - p$ and $p - c$, respectively.

In our setting there exist two types of breach. First, either the buyer or the seller may refuse to trade the good. Second, the stipulated cost- and/or quality thresholds may be violated. In any case, the expectation damages rule must ensure that the victim of breach is compensated such that he or she is in as good a position as if the contract had been fulfilled. This rule is, however, not applied literally if breach is advantageous for the breached-against party. In that case damage payments are zero.

Recall that the quality threshold $\bar{v}$ serves as a baseline for calculating damages if the seller delivers a non-conforming good. If the seller does so and the buyer accepts delivery, she may claim compensation amounting to specified minus actual quality, $\bar{v} - v$. Similarly, the parties may specify a cost threshold $\bar{c}$ which is defined such that any additional cost

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Figure 2. Breachgame induced by expectation damages.

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10The order of announcements does not affect any of our results.
above $\bar{c}$ are borne by the buyer.

Thus, if the seller announces to deliver (D) and the buyer accepts (A), the latter’s ex-post payoff amounts to

$$B^{D,A}(\beta, \sigma, \omega, q) = \max[v, \bar{v}] - p - \max[c - \bar{c}, 0] + (1 - \alpha) \max[c - v, 0]$$

where $(1 - \alpha) \max[c - v, 0]$\(^{11}\) represents the Buyer’s potential bargaining surplus. Likewise, the Seller’s ex-post payoff corresponds to

$$S^{D,A}(\beta, \sigma, \omega, q) = p - \min[c, \bar{c}] - \max[\bar{v} - v, 0] + \alpha \max[c - v, 0].$$

Suppose the seller announces to deliver (D) but the buyer refuses to accept $(\bar{A})$. If the good is conforming to the contract $v \geq \bar{v}$, the seller receives, as compensation, his contractually assured trade surplus $p - \bar{c}$. However, if quality is inferior $v < \bar{v}$, we assume, as Edlin (1996), that the buyer has a broad duty to mitigate damages. She is obliged to accept performance whether the good is conforming to the contract or not and may only collect $\max[\bar{v} - v, 0]$ in damages. This prevents her from threatening to reject performance unless the seller agrees to pay larger damages. If she decides, despite of her obligation, to refuse performance, the seller may claim damages. Total damages amount to the seller’s contractually assured surplus $p - \bar{c}$ minus the damages the seller would have to pay to the buyer had she accepted, $\max[\bar{v} - v, 0]$\(^{12}\). Consequently, the buyer’s ex-post payoff amounts to

$$B^{D,\bar{A}}(\beta, \sigma, \omega, q) = -\max[p - \bar{c} - \max[\bar{v} - v, 0], 0] + (1 - \alpha) \max[v - c, 0]$$

whereas the seller receives

$$S^{D,\bar{A}}(\beta, \sigma, \omega, q) = \max[p - \bar{c} - \max[\bar{v} - v, 0], 0] + \alpha \max[v - c, 0].$$

If the seller announces not to deliver $(\bar{D})$ the buyer may sue for damages (ED). Then, her payoff is given by

$$B^{D,ED}(\beta, \sigma, \omega, q) = \max[\bar{v} - p - \max[c - \bar{c}, 0], 0] + (1 - \alpha) \max[v - c, 0]$$

\(^{11}\)Recall that in the case under consideration the parties only renegotiate if production and delivery is inefficient, i.e. if $v < c$.

\(^{12}\)If total damages were equivalent to the seller’s contractually assured trade surplus $p - \bar{c}$, he would be, in contrast to the aim of expectation damages, in a better position as if the contract had been fulfilled. Also recall that the buyer cannot sue for a reward if total damages are negative. The case where the seller does not claim damages is equivalent to the one where total damages are zero. In both situations, the parties’ ex-post payoff is identical to their respective bargaining shares.
whereas the seller receives

$$S^{\bar{D},ED}(\beta, \sigma, \omega, q) = -\max[\bar{v} - p - \max[c - \bar{c}, 0], 0] + \alpha \max[v - c, 0].$$ \hspace{1cm} (6)$$

The term $\max[c - \bar{c}, 0]$ represents the damages the buyer had to pay to the seller had the seller delivered the good. Similar to before, this term must be subtracted from the buyer’s contractually assured trade surplus $\bar{v} - p$. Only then, the buyer is in as good a position as if the seller had delivered the good. Suppose the buyer does not sue for damages (0). In that case, neither party has to pay damages and both parties’ ex-post payoffs

$$B^{\bar{D},0}(\beta, \sigma, \omega, q) = (1 - \alpha) \max[v - c, 0]$$ \hspace{1cm} (7)$$

and

$$S^{\bar{D},0}(\beta, \sigma, \omega, q) = \alpha \max[v - c, 0]$$ \hspace{1cm} (8)$$

are identical to their respective bargaining shares.\footnote{Note that the buyer’s decision is trivial in the sense that she is always weakly better off if she claims damages.} As we will see next section, the efficient breach property of expectation damages ensures that there is no scope for renegotiation. Thus the renegotiation terms in equations (1) - (8) turn out to be zero.

## 4 Buyer takes the performance decision

In the economic analysis of contract law it is frequently assumed that only one party takes the performance decision, see e.g. Che and Chung (1999) or Schweizer (2006). Let us for now assume that the buyer takes the performance decision, i.e. the seller always announces to deliver.\footnote{Even though we assume that the buyer takes the performance decision, the parties may renegotiate her decision.} We show that it is optimal for the parties to write an augmented Cadillac contract that combines aspects from Cadillac and balancing contracts. Edlin (1996) defines a Cadillac contract as a contract that stipulates the highest possible quantity and/or quality. A balancing contract, in contrast, sets the contractual parameters at an intermediate level. For example, Chung (1991) and Edlin and Reichelstein (1996) consider contracts where the parties stipulate an intermediate quantity such that the
investors sometimes receive more and sometimes less than the marginal social return of their investment. The contracts we consider in this section stipulate the lowest possible cost of production $\bar{c} = c_l$, some intermediate quality threshold $\bar{v} \in [v_l, v_h]$, a high price $p \geq v_h + \max[c_l - v_l, 0]$, and that the good has to be traded $\bar{q} = 1$. In Section 6, the parties play the ex-post breach game described in the previous section. We demonstrate that the combination of a low cost threshold and a high price induces the parties to behave as if the buyer takes the performance decision.

In Lemma 1, we show that the buyer performs whenever it is socially desirable to do so. Thus, the efficient breach property of expectation damages continues to hold. This property is desirable because it ensures that there is no scope for renegotiation. Given that a contract is optimal in absence of renegotiation, it must also be optimal if costless renegotiation is possible. After having derived the buyer’s performance decision, we summarize the parties’ expected payoffs. Let us denote the buyer’s expected payoff, at date 1, by $B^{\bar{c}\bar{v}}(\beta, \sigma)$ and the seller’s by $S^{\bar{c}\bar{v}}(\beta, \sigma)$, respectively. Here, $\bar{c} \in [c_l, c_h]$ and $\bar{v} \in [v_l, v_h]$ indicate which cost and quality thresholds are specified in contract.

**Lemma 1** Any contract of the form $(\bar{c} = c_l, \bar{v} \in [v_l, v_h], p \geq v_h + \max[c_l - v_l, 0], \bar{q} = 1)$ induces the buyer to accept the good whenever $v \geq c$ and to reject otherwise. The buyer’s expected payoff is given by

$$B^{\bar{c}\bar{v}}(\beta, \sigma) = E[W(\beta, \sigma, \omega, Q^*(\beta, \sigma, \omega))] + E[\max[\bar{v} - v, 0]] - (p - c_l) - \beta$$

whereas the seller receives

$$S^{\bar{c}\bar{v}}(\beta, \sigma) = p - c_l - E[\max[\bar{v} - v, 0]] - \sigma.$$

**PROOF:** See the Appendix.

Let us define the buyer’s best response to investment of the seller as

$$\beta_B(\bar{c}, \bar{v}, p, \bar{q}) := \arg \max_{\beta} B^{\bar{c}\bar{v}}(\beta, \sigma)$$

15It is a well known standard result of the literature that expectation damages induce an efficient ex-post trade decision (see e.g. Posner, 1977; Shavell, 1980; Kornhauser, 1986; Craswell, 1988). However, Göller and Stremitzer (2009) explain that a contract that specifies a quality threshold may induce expectation damages to lose its efficient breach property. Even though in the present paper the parties may specify a quality threshold, this problem does not occur because the contracts we consider specify a sufficiently low or high price.
and the seller’s best response to investment of the buyer as

$$\sigma_S(\bar{c}, \bar{v}, p, \bar{q}) := \arg \max_\sigma S^{*\bar{q}}(\beta, \sigma).$$

Having derived the parties’ expected payoffs in Lemma 1, the following proposition describes their investment incentives for different levels of the quality threshold.

**Proposition 1** For any \( p \geq v_h + \max[c_l - v_l, 0] \), \( \bar{c} = c_l \) and \( \bar{q} = 1 \), it holds for any \( \sigma \in [0, \sigma^{\max}] \) that

$$\arg \max_\beta B^{c_l\bar{v}}(\beta, \sigma) = \arg \max_\beta E[W(\beta, \sigma, Q^*(\beta, \omega))] - \beta$$

and for any \( \beta \in [0, \beta^{\max}] \) that

$$\sigma_S(c_l, v_l, p, 1) = 0.$$

Moreover, for \( \beta = \beta^* \), any

$$\sigma \in \sigma_S(c_l, v_h, p, 1) \geq \sigma^*.$$

**Proof.** The first statement is true because for all quality thresholds \( \bar{v} \in [v_l, v_h] \) the buyer’s expected payoff

$$B^{c_l\bar{v}}(\beta, \sigma) = E[W(\beta, \sigma, Q^*(\beta, \omega))] + E[\max[\bar{v} - v, 0] - (p - c_l) - \beta]$$

is equivalent to expected social surplus plus or minus a term that is constant with respect to the buyer’s investment. Moreover, the second statement is true because the seller’s expected payoff

$$S^{c_l\bar{v}}(\beta, \sigma) = p - c_l - \sigma$$

is strictly decreasing in \( \sigma \). To prove statement three observe that, given that the buyer invests efficiently, the difference between the seller’s expected surplus and expected social surplus

$$S^{c_l\bar{v}}(\beta^*, \sigma) - [E[W(\beta^*, \sigma, Q^*(\beta^*, \omega))] - \beta^* - \sigma]$$

$$= p - v_h - c_l + E[\min[C(\beta^*, \omega, 1), V(\sigma, \omega, 1)] + \beta^*$$
is monotonically increasing in $\sigma$ due to Assumption 2. Therefore it must hold for any $\sigma < \sigma^*$ that

$$S_{clvh}(\beta^*, \sigma) - \left[E[W(\beta^*, \sigma, Q^*(\beta^*, \sigma, \omega))]-\beta^* - \sigma\right] \leq S_{clvh}(\beta^*, \sigma^*) - \left[E[W(\beta^*, \sigma^*, Q^*(\beta^*, \sigma, \omega))]-\beta^* - \sigma^*\right]\]$$

or equivalently

$$\left[E[W(\beta^*, \sigma^*, Q^*(\beta^*, \sigma, \omega))]-\beta^* - \sigma^*\right] - \left[E[W(\beta^*, \sigma, Q^*(\beta^*, \sigma, \omega))]-\beta^* - \sigma\right] \leq S_{clvh}(\beta^*, \sigma^*) - S_{clvh}(\beta^*, \sigma).$$

(9)

Because social welfare is uniquely maximized by $(\beta^*, \sigma^*)$, the term in the first line of (9) is positive and consequently $S_{clvh}(\beta^*, \sigma^*) < S_{clvh}(\beta^*, \sigma)$ for all $\sigma \in [0, \sigma^*)$. ■

From Proposition 1 we can deduce how different levels of the quality threshold influence the parties’ best responses to efficient investment of the other party. The first statement tells us that for a low cost threshold $\bar{c} = c_l$ and a sufficiently high price $p \geq v_h + \max[c_l - v_l, 0]$, it holds for any quality threshold $\bar{v} \in [v_l, v_h]$ that the buyer’s best response is equivalent to the socially best response. Thus, if the seller invests efficiently, it is a best response for the buyer to invest efficiently herself. Under these contractual parameters, the seller’s best response to efficient investment of the buyer is to not invest at all for $\bar{v} = v_l$ and to overinvest for $\bar{v} = v_h$. Figure 3 illustrates that continuity of the buyer’s and the seller’s best response is a sufficient condition for Theorem 1 to hold.

We can now sum up our main result.

**Theorem 1** If the parties’ best responses are continuous and the contract specifies a sufficiently high price $p \geq v_h + \max[c_l - v_l, 0]$, $\bar{c} = c_l$ and $\bar{q} = 1$, there exists a quality threshold $\bar{v}^* \in (v_l, v_h]$ such that the first-best investment levels $(\beta^*, \sigma^*)$ constitute a subgame-perfect equilibrium of the induced game.

As mentioned, the first-best contract of Theorem 1 combines aspects from Cadillac- and balancing contracts. It is a Cadillac contract because, as in Edlin (1996), it stipulates a combination of an extreme price and an extreme threshold. The buyer has efficient incentives to invest for two reasons. First, because the efficient breach property of expectation damages ensures that she accepts the good whenever it is socially desirable to
Figure 3 shows, for any \( p \geq v_h + max[c_l - v_l, 0] \), contracts with the property that the best response to efficient investment is equal to efficient investment.

do so. Second, because the low cost threshold ensures that she has to pay damages in any state of the world. Thus, she internalizes the full benefit of her investment indirectly through the damages she has to pay to the seller. Because the buyer is a residual claimant of the trade relationship for any level of specified quality \( \bar{v} \in [v_l, v_h] \), the quality threshold can be used as an instrument to balance the seller’s incentives.

To be sure that the first best can be implemented, the seller’s best response must be continuous. Yet, what does that mean in a model that does not assume differentiability? Because the buyer is a residual claimant, she plays a social best response to the seller’s investment. Thus if the seller invests \( \sigma^* \), it is a best response for the buyer to invest \( \beta^* \). What is crucial only is that for some fixed \( p \geq v_h + max[c_l - v_l, 0] \) and \( \bar{c} = c_l \) the seller’s investment depends continuously on the level of the quality threshold \( \bar{v} \). Thus, our optimal contract essentially transforms the two-sided investment problem into a one-sided one as, for example, in Stremitzer (2010). The assumptions made in Stremitzer (2010) on the distribution of the buyer’s benefit from trade \( v \) are therefore an example in which the aforementioned relationship is continuous.\(^{16}\)

5 Seller takes the performance decision

In contrast to the previous section, let us assume that the seller takes the performance decision. As before, we find that it is optimal for the parties to write an augmented Cadillac contract. The contracts we consider in this section stipulate an intermediate cost threshold \( \bar{c} \in [c_l, c_h] \), the highest possible quality \( \bar{v} = v_h \), a low price \( p \leq c_l - \max[c_h - v_h, 0] \) and that the good has to be traded \( \bar{q} = 1 \). Next section, where we consider the ex-post breach game induced by expectation damages, we demonstrate that the combination of a high quality threshold and a low price induces the parties to behave as if the seller takes the performance decision.

In the following lemma, we show that the seller’s performance decision is efficient and summarize the parties’ expected payoffs.

**Lemma 2** Any contract of the form \((\bar{c} \in [c_l, c_h], \bar{v} = v_h, p \leq c_l - \max[c_h - v_h, 0], \bar{q} = 1)\) induces the seller to deliver the good whenever \( v \geq c \). The buyer’s expected payoff is given by

\[
B^{v_h}(\beta, \sigma) = v_h - p - E[\max[c - \bar{c}, 0] - \beta]
\]

whereas the seller receives

\[
S^{v_h}(\beta, \sigma) = E[W(\beta, \sigma, \omega, Q^*(\beta, \sigma, \omega))] + E[\max[c - \bar{c}, 0] - (v_h - p) - \sigma].
\]

**PROOF:** See the Appendix.

Having derived the parties’ expected payoffs in Lemma 2, the following proposition describes their investment incentives for different levels of the cost threshold.

**Proposition 2** For any \( p \leq c_l - \max[c_h - v_h, 0] \), \( \bar{v} = v_h \) and \( \bar{q} = 1 \), it holds for any \( \beta \in [0, \beta_{\text{max}}] \) that

\[
\arg\max_{\sigma} S^{v_h}(\beta, \sigma) = \arg\max_{\sigma} E[W(\beta, \sigma, \omega, Q^*(\beta, \sigma, \omega))] - \sigma
\]

and for any \( \sigma \in [0, \sigma_{\text{max}}] \) that

\[
\beta_B(c_h, v_h, p, 1) = 0.
\]
Moreover, for $\sigma = \sigma^*$, any

$$\beta \in \beta_B(c_l,v_h,p,1) \geq \beta^*.$$  

**Proof.** The first statement is true, because, for all cost thresholds $\bar{c}$, the seller’s expected payoff

$$S^{v_h}(\beta,\sigma) = E[W(\beta,\sigma,\omega,Q^*(\beta,\sigma,\omega))] + E[max[c - \bar{c},0] - (v_h - p) - \sigma$$

is equivalent to expected social surplus plus or minus a term that is constant with respect to the buyer’s investment. Moreover, the second statement is true because the buyer’s expected payoff

$$B^{c_h,v_h}(\beta,\sigma) = v_h - p - \beta$$

is strictly decreasing in $\beta$.

To prove statement three observe that, given that the seller invests efficiently, the difference between the buyer’s expected surplus and expected social surplus

$$B^{v_h}(\beta,\sigma^*) - [E[W(\beta,\sigma^*,\omega,Q^*(\beta,\sigma^*,\omega))]-\beta - \sigma^*]$$

$$= v_h - p + c_l - E[min[C(\beta,\omega,1),V(\sigma^*,\omega,1)] + \sigma^*$$

is monotonically increasing in $\beta$ due to Assumption 2. Therefore it must hold for any $\beta < \beta^*$ that

$$B^{v_h}(\beta^*,\sigma^*) - [E[W(\beta^*,\sigma^*,\omega,Q^*(\beta^*,\sigma^*,\omega))]-\beta^* - \sigma^*] \leq$$

$$B^{v_h}(\beta^*,\sigma^*) - [E[W(\beta^*,\sigma^*,\omega,Q^*(\beta^*,\sigma^*,\omega))]-\beta^* - \sigma^*]$$

or equivalently

$$[E[W(\beta^*,\sigma^*,\omega,Q^*(\beta^*,\sigma^*,\omega))]-\beta^* - \sigma^*] - [E[W(\beta,\sigma^*,\omega,Q^*(\beta,\sigma^*,\omega))]-\beta - \sigma^*]$$

$$\leq B^{v_h}(\beta^*,\sigma^*) - B^{v_h}(\beta,\sigma^*).$$

(10)

Because social welfare is uniquely maximized by $(\beta^*,\sigma^*)$, the term in the first line of [10] is positive and consequently $B^{v_h}(\beta,\sigma^*) < B^{v_h}(\beta^*,\sigma^*)$ for all $\beta \in [0,\beta^*)$.  

If the seller takes the performance decision, the combination of a high quality threshold $\bar{v} = v_h$ and a sufficiently low price $p \leq c_l - max[c_h - v_h,0]$ ensures that, for any level of
the cost threshold $\bar{c} \in [c_l, c_h]$, the seller’s best response is equivalent to the socially best response. The buyer’s best response to efficient investment of the seller is to not invest at all for $\bar{c} = c_h$ and to overinvest for $\bar{c} = c_l$. As before, continuity of the buyer’s and the seller’s best response is a sufficient condition for the following theorem to hold.

**Theorem 2** If the parties’ best responses are continuous and the contract specifies a sufficiently low price $p \leq c_l - \max[c_h - v_h, 0]$, $\bar{v} = v_h$ and $\bar{q} = 1$, there exists a cost threshold $\bar{c}^* \in [c_l, c_h)$ such that the first-best investment levels $(\beta^*, \sigma^*)$ constitute a subgame-perfect equilibrium of the induced game.

The intuition behind Theorem 2 and Theorem 1 is closely related. The combination of an extremely low price and a sufficiently high quality threshold ensures that the seller is a residual claimant of the trade relationship. Because this is the case for any cost threshold $\bar{c} \in [c_l, c_h]$, it can be used as an instrument to fine tune the buyer’s incentives. This residual claimant argument also is the driving force behind the efficiency result of Stremitzer (2010). He considers a situation where only the seller makes a purely cooperative investment and finds that a contract, that is governed by expectation damages and specifies $\bar{v} = v_h$, induces the first best. We can, however, deduce from Proposition 2 that such a pure cadillac contract performs poorly if both parties invest. The buyer receives, in all states of the world, a fixed payoff amounting to promised quality minus price and therefore has no incentive to invest.\[17\]

6 Ex-post breach game

So far, we assumed that either the buyer (Section 4) or the seller (Section 5) takes the performance decision. In this section, both parties may breach the contract. We consider the ex-post breach game described in Section 3 and show that a contract of the form $(\bar{c} = c_l, \bar{v} \in [v_l, v_h], p \geq v_h + \max[c_l - v_l, 0], \bar{q} = 1)$ induces the parties to behave as if the buyer takes the performance decision whereas a contract of the form $(\bar{c} \in [c_l, c_h], \bar{v} = v_h, p \leq c_l - \max[c_h - v_h, 0], \bar{q} = 1)$ induces them to behave as if the seller takes the performance decision. We prove the following lemma.

\[17\] The first best contract of Stremitzer (2010) does not specify any cost threshold. Technically, this situation is similar to a contract that specifies a cost threshold that is never violated, $\bar{c} = c_h$.\[17\]
Lemma 3  (i) Any contract of the form \((\bar{c} = c_1, \bar{v} \in [v_1, v_h], p \geq v_h + \max[c_1 - v_1, 0], \bar{q} = 1)\) induces the following behavior on the equilibrium path. The seller always announces to deliver the good. The buyer accepts whenever \(v \geq c\) and rejects otherwise. (ii) In contrast, under any contract of the form \((\bar{c} \in [c_l, c_h], \bar{v} = v_h, p \leq c_l - \max[c_h - v_h, 0], \bar{q} = 1)\) the behavior on the equilibrium path is as follows. The seller announces to deliver if and only if \(v \geq c\) whereas the buyer always accepts delivery.

PROOF: See the Appendix.

From Lemma 3 we can deduce that the combination of an an extreme price and an extreme threshold determines the breaching party. Next section, we use this finding to extend our first-best result to the case of non-binary trade.

7 Divisible contracts

As a matter of real world contracting, parties are often interested to trade several units of the same good, \(Q = \{0, 1, 2, \ldots, q^{max}\}\). In this section, we demonstrate that our first-best results continue to hold if contracts are divisible and specify maximum quantity \(\bar{q} = q^{max}\). Divisible contracts are specified such that each unit together with the per-unit price can be breached independently. Cost and quality threshold then serve as a baseline for calculating damages for every single unit. The buyer’s benefit of consuming \(q\) units is given by \(V(\sigma, \omega, q) - pq\) and the seller’s profit by \(pq - C(\beta, \omega, q)\). It may frequently occur that some but not all units violate a threshold. Let us denote the quantity of units that is non-conforming to stipulated cost or quality threshold by \(q^\circ\) or \(q^\bar{v}\), respectively.\(^{18}\) The seller’s cost of producing \(q^\circ\) units can then be written as \(C(\beta, \omega, q^\circ)\) whereas the buyer’s benefit of consuming \(q^\bar{v}\) units is denoted by \(V(\sigma, \omega, q^\bar{v})\).

As before, to establish the first best, it is crucial that the parties have a broad duty to mitigate damages. Each party is obliged to accept inferior and/or partial performance and may only collect the difference between contractually assured and actual performance as damages. Since contracts are divisible, the parties may breach each single unit independently. The legal implications of breach are then, for each unit, the same as discussed\(^{18}\)These quantities depend on investments and the state of the world. For our analysis, it is only important that \(q^\circ\) does not depend on the seller’s and that \(q^\bar{v}\) does not depend on the buyer’s investment. Hence, we write \(q^\circ\) instead of \(q^\circ(\beta, \omega)\).
Lemma 4  
(i) Any contract of the form \((\bar{c} = c_l, \bar{v} \in [v_l, v_h], p \geq v_h + \max[c_l - v_l, 0], \bar{q} = q^{max})\) induces the following behavior on the equilibrium path. The seller announces to deliver \(q^{max}\) units whereas the buyer breaches to the socially efficient quantity \(Q^*(\beta, \sigma, \omega)\).

(ii) Under any contract of the form \((\bar{c} \in [c_l, c_h], \bar{v} = v_h, p \leq c_l - \max[c_h - v_h, 0], \bar{q} = q^{max})\) the behavior on the equilibrium path is as follows. The seller announces to deliver the socially efficient quantity \(Q^*(\beta, \sigma, \omega)\) which is then accepted by the buyer.

Proof. Because cost of production and quality are additive separable, the parties have a brought duty to mitigate damages and contracts are divisible, we can consider each single unit separately to check if the parties trade it or not. (i) From Lemma 3, we can deduce that the combination of a high price \(p \geq v_h + \max[c_l - v_l, 0]\) and a low cost threshold \(\bar{c} = c_l\) induces the seller to deliver all units whereas the buyer accepts all units that yield a positive net joint benefit.

(ii) Moreover, from Lemma 3 we can also deduce that the combination of a low price \(p \leq c_l - \max[c_h - v_h, 0]\) and a high quality threshold \(\bar{v} = v_h\) induces the seller to deliver all units that are socially desirable to trade which are then accepted by the buyer.

Hence, in both cases, the efficient breach property of expectation damages continues to hold and the parties trade the socially efficient quantity \(Q^*(\beta, \sigma, \omega)\).

Let us summarize the parties’ expected payoffs in the following lemma.

Lemma 5  
(i) For any \(p \geq v_h + \max[c_l - v_l, 0]\) and \(q = q^{max}\), the expected payoffs under subgame-perfect equilibrium amount to

\[
B^{cg}(\beta, \sigma) = E[W(\beta, \sigma, \omega, Q^*(\beta, \sigma, \omega))] + E[q^v \bar{v} - V(\sigma, \omega, q^v)] - q^{max}(p - c_l) - \beta
\]

and

\[
S^{cg}(\beta, \sigma) = q^{max}(p - c_l) - E[q^v \bar{v} - V(\sigma, \omega, q^v)] - \sigma.
\]

---

19As an example, suppose the parties write a contract that stipulates some required quality and a quantity of 10 units. The seller announces to deliver 9 units from which one is non-conforming to the contract. Due to her broad duty to mitigate damages, the buyer is obliged to accept all 9 units. She may, however, claim damages. Suppose the buyer refuses to accept all 9 units. Then, the seller may claim compensation amounting to price minus cost for all 9 units he was willing to deliver minus the damages he would have to pay if the buyer had accepted the ninth unit.
Specifically,

\[ B^{cvh}(\beta, \sigma) = E[W(\beta, \sigma, Q^*(\beta, \sigma, \omega))] + E[q^{max} v_h - V(\sigma, \omega, q^{max})] - q^{max} (p - c_l) - \beta \]

and

\[ S^{cvh}(\beta, \sigma) = q^{max} (p - c_l) - E[q^{max} v_h - V(\sigma, \omega, q^{max})] - \sigma. \]

(ii) For any price \( p \leq c_l - \max[c_h - v_h, 0] \) and \( q = q^{max} \), the expected payoffs under subgame-perfect equilibrium amount to

\[ B^{cvh}(\beta, \sigma) = q^{max} (v_h - p) - E[C(\beta, \omega, q^{\beta}) - q^{\beta \bar{c}}] - \beta \]

and

\[ S^{cvh}(\beta, \sigma) = E[W(\beta, \sigma, Q^*(\beta, \sigma, \omega))] + E[C(\beta, \omega, q^{\beta}) - q^{\beta \bar{c}}] - q^{max} (v_h - p) - \sigma. \]

Specifically,

\[ B^{cvh}(\beta, \sigma) = q^{max} (v_h - p) - E[C(\beta, \omega, q^{max})] - q^{max} c_l] - \beta. \]

and

\[ S^{cvh}(\beta, \sigma) = E[W(\beta, \sigma, Q^*(\beta, \sigma, \omega))] + E[C(\beta, \omega, q^{max})] - q^{max} c_l] - q^{max} (v_h - p) - \sigma. \]

PROOF: See the Appendix.

Having derived the parties’ expected payoffs, we analyze in the following proposition their investment incentives for different levels of the cost and quality thresholds.

**Proposition 3**

(i) For any \( p \geq v_h + \max[c_l - v_l, 0] \), \( \bar{c} = c_l \) and \( q = q^{max} \), it holds for any \( \sigma \in [0, \sigma^{max}] \) that

\[ \arg \max_{\beta} B^{cvh}(\beta, \sigma) = \arg \max_{\beta} E[W(\beta, \sigma, Q^*(\beta, \sigma, \omega))] - \beta \]

and for any \( \beta \in [0, \beta^{max}] \) that

\[ \sigma_S(c_l, v_l, p, q^{max}) = 0. \]

Moreover, for \( \beta = \beta^* \), any

\[ \sigma \in \sigma_S(c_l, v_h, p, q^{max}) \geq \sigma^*. \]
(ii) For any $p \leq c_l - \max[c_h - v_h, 0]$, $\bar{v} = v_h$ and $q = q^{\max}$, it holds for any $\beta \in [0, \beta^{\max}]$ that

$$\arg \max \sigma S^{\text{opt}}(\beta, \sigma) = \arg \max \sigma E[W(\beta, \sigma, \omega, Q^{\ast}(\beta, \sigma, \omega))] - \sigma$$

and for any $\sigma \in [0, \sigma^{\max}]$ that

$$\beta^B(c_h, v_h, p, q^{\max}) = 0.$$

Moreover, for $\sigma = \sigma^{\ast}$, any

$$\beta \in \beta^B(c_l, v_h, p, q^{\max}) \geq \beta^{\ast}.$$

PROOF: See the Appendix.

The intuition behind Proposition 3 is closely related to Proposition 1 and 2. The combination of an extreme price and an extreme threshold ensures that one of the parties is a residual claimant of the trade relationship. Because this holds true regardless how the remaining threshold is set, it can be used to balance the incentives of the other party. Assuming that the best response correspondences are continuous, we can establish the following theorem.

**Theorem 3** If the parties’ best responses are continuous and the contract specifies a sufficiently high price $p \geq v_h + \max[c_l - v_l, 0]$, $\bar{c} = c_l$ and $\bar{q} = q^{\max}$, there exists a quality threshold $\bar{v}^{\ast} \in (v_l, v_h]$ such that the first-best investment levels $(\beta^{\ast}, \sigma^{\ast})$ constitute a subgame-perfect equilibrium of the induced game. Moreover, if the parties’ best responses are continuous and the contract specifies a sufficiently low price $p \leq c_l - \max[c_h - v_h, 0]$, $\bar{v} = v_h$ and $\bar{q} = q^{\max}$, there exists a cost threshold $\bar{c}^{\ast} \in [c_l, c_h)$ such that the first-best investment levels $(\beta^{\ast}, \sigma^{\ast})$ constitute a subgame-perfect equilibrium of the induced game.

If the parties trade several units from the same good and contracts are divisible, they have an additional instrument, the quantity, at hand. This cannot destroy the optimality result obtained under binary trade. Our first-best contracts induce that one party is interested to trade all units stipulated in contract. The other party then breaches to the socially efficient quantity. To obtain the first best it is not necessary to specify $\bar{q} = q^{\max}$ in contract. In theory, it is sufficient to set quantity such that it is at least as high as the highest possible realization of the socially efficient quantity.
8 Conclusion

We have shown that expectation damages, the default remedy of common law, can achieve the first best in an environment of bilateral cooperative investment. In contrast to selfish investment, where Ohlendorf (2009) has demonstrated that it is sufficient to specify price and quantity to achieve the first best, cooperative investment requires better contractual protection. We found that it is optimal to write a contract that protects the contracting parties by stipulating a cost and a quality threshold that guarantee both parties a fixed payoff in bad states of the world. The parties set one threshold at an extreme level such that one of them becomes a residual claimant of the trade relationship whereas the incentives of the remaining party can be balanced by setting the remaining threshold at an intermediate level.

Similarly to a pure Cadillac contract, our first-best contracts determine which party may have an incentive to ex-post breach the contract. Hence, our optimality result stands in contrast to the opinion expressed in Ohlendorf (2009) that in a setting of bilateral investment it is important that both parties face the risk of ex-post breach. For the breaching party to sign the contract, it must receive a substantial transfer up-front. If one believes, however, that the parties are truly sophisticated, they could theoretically divide the ex-ante expected gains from trade by stipulating a lottery between both types of optimal contracts.

From a legal perspective, the assumption that damages in case of non-delivery are equivalent to specified quality minus price may seem questionable. Indeed, courts may be inclined to use true quality instead of specified quality as a baseline for calculating damages. Our optimality result may suggest that this is not the right way to measure expectation damages. Also, this paper supports the viewpoint that courts should be ready to enforce contracts that request a broader view to mitigate damages. If courts do not, expectation damages may lose the efficient breach property in our setting.

This paper also contributes to the viewpoint that to judge the performance of a breach remedy it is important to consider the interplay between breach remedy and contract. As an example, consider Che and Chung (1999) who argue that expectation damages perform
poorly in an environment of unilateral cooperative investment. This viewpoint has been refuted by Schweizer (2006) and Stremitzer (2010) who have shown that it is possible to augment the contracts considered in Che and Chung (1999) such that expectation damages can indeed induce the first best.
9 Appendix

Proof of Lemma 1. The buyer announces to accept the good whenever \( B_{D,A}(\beta, \sigma, \omega, q) \geq B_{D,\bar{A}}(\beta, \sigma, \omega, q) \) or equivalently

\[
\max[v, \bar{v}] - p - (c - c_l) + (1 - \alpha)\max[c - v, 0] \geq \\
-\max[p - c_l - \max[\bar{v} - v, 0], 0] + (1 - \alpha)\max[v - c, 0].
\] (11)

Note that the first max operator on the second line of (11) can be dropped because \( p - c_l - \max[\bar{v} - v, 0] \geq 0 \) \( \forall (v, \bar{v}) \in [v_l, v_h]^2 \)\(^{20}\). Consequently, we can simplify (11) and conclude that the buyer announces to accept \((A)\) whenever \( v \geq c \).

To obtain the seller’s expected payoff, we first have to derive his ex-post payoff. Using that the buyer performs whenever \( v \geq c \) and inserting the contract terms into (2) yields the seller’s ex-post payoff for the case that the buyer performs. It amounts to

\[
p - c_l - \max[\bar{v} - v, 0].
\]

Likewise, we get the seller’s ex-post payoff for the case that the buyer does not perform by inserting the contract terms into (4). In that case, the seller also receives

\[
p - c_l - \max[\bar{v} - v, 0].
\]

Note that all renegotiation terms are equivalent to zero because the buyer’s performance decision is efficient. We get the seller’s expected payoff by merging both cases, subtracting the seller’s investment level and taking expectations. The buyer’s expected payoff is equivalent to expected social surplus minus the seller’s expected payoff. \( \blacksquare \)

Proof of Lemma 2. The seller announces to deliver the good whenever \( S_{D,A}(\beta, \sigma, \omega, q) \geq S_{D,\bar{D}}(\beta, \sigma, \omega, q) \)\(^{21}\) or equivalently

\[
p - \min[c, \bar{c}] - (v_h - v) + \alpha \max[c - v, 0] \geq \\
-\max[v_h - p - \max[c - \bar{c}, 0], 0] + \alpha \max[v - c, 0]
\] (12)

\(^{20}\)Recall that \( p \geq v_h + \max[c_l - v_l, 0] \).

\(^{21}\)Recall that, after the seller announced not to deliver, the buyer is weakly better off claiming damages than staying passive.
Note that the first max operator on the second line of (12) can be dropped because \( v_h - p - \max[c - \bar{c}, 0] \geq 0 \ \forall \ (c, \bar{c}) \in [c_l, c_h] \). Consequently, we can simplify (12) and conclude that the seller announces to deliver (D) whenever \( v \geq c \).

To obtain the buyer’s expected payoff, we first have to derive her ex-post payoff. Using that the seller performs whenever \( v \geq c \) and inserting the contract terms into (1) yields the buyer’s ex-post payoff for the case that the seller performs. It amounts to

\[
v_h - p - \max[c - \bar{c}, 0].
\]

Likewise, we get the buyer’s ex-post payoff for the case that the seller does not perform by inserting the contract terms into (5). In that case, the buyer also receives

\[
v_h - p - \max[c - \bar{c}, 0].
\]

Note that all renegotiation terms are equivalent to zero because the seller’s performance decision is efficient. We get the buyer’s expected payoff by merging both cases, subtracting the buyer’s investment level and taking expectations. The seller’s expected payoff is equivalent to expected social surplus minus the buyer’s expected payoff.

Proof of Lemma 3. We prove the first part only. The second part can be proven in a similar way. (i) To prove Lemma 3, we solve the game depicted in Figure 2 by backwards induction. If the seller announces not to deliver (\( \bar{D} \)), the buyer is indifferent between claiming damages (\( ED \)) and staying passive (0). In both cases she receives an ex-post payoff of

\[
B^{D,ED}(\beta, \sigma, \omega, q) = (1 - \alpha) \max[v - c, 0].
\]

If the seller announces to deliver (D), the buyer’s best response is to accept (A) if

\[
B^{D,A}(\beta, \sigma, \omega, q) \geq B^{D,\bar{A}}(\beta, \sigma, \omega, q)
\]

or equivalently

\[
\max[v, \bar{v}] - p - (c - c_l) + (1 - \alpha)\max[c - v, 0] \geq
\]

\[
-\max[p - c_l - \max[\bar{v} - v, 0], 0] + (1 - \alpha)\max[v - c, 0].
\]

(13)

Recall that \( p \leq c_l - \max[c_h - v_h, 0] \).

The buyer receives no damages if she opts for (ED). To see this, insert the contract terms into equation (5).
Note that the first max operator on the right-hand side of (13) can be dropped because \( p - c_l - \max[\bar{v} - v, 0] \geq 0 \) \( \forall (v, \bar{v}) \in [v_l, v_h]^2 \). Consequently, we can simplify (13) and conclude that the buyer announces to accept \((A)\) whenever \( v \geq c \). Anticipating the buyer’s decision, the seller always delivers \((D)\). To see this, first consider the case where \( v \geq c \). The seller announces to deliver if \( S^{D,A}(\beta, \sigma, \omega, q) \geq S^{D,ED}(\beta, \sigma, \omega, q) \) or equivalently
\[
 p \geq \max[\bar{v} - v, 0] + c_l + \alpha(v - c). 
\]

Note that
\[
 p \geq v_h + c_l - (1 - \alpha)v - \alpha c \geq \max[\bar{v} - v, 0] + c_l + \alpha(v - c)
\]
for any \( \bar{v} \in [v_l, v_h] \). Consequently, the seller delivers. Finally, consider the case where the buyer refuses to accept, \( v < c \). Here, the seller announces to deliver \((D)\) because \( S^{D,A}(\beta, \sigma, \omega, q) \geq S^{D,ED}(\beta, \sigma, \omega, q) \) or equivalently
\[
 \max[p - c_l - \max[\bar{v} - v, 0], 0] \geq 0.
\]

Proof of Lemma 5. (i) Recall that divisibility allows us to treat each unit as a separate contract. We know from Lemma 1 that the seller’s payoff per unit does not depend on whether the buyer accepts the unit or not. Thus, ex-post, the seller receives a payoff amounting to \( p - c_l - \max[\bar{v} - V_i(\sigma, \omega)] \) for each of the \( q^{\max} \) units the parties stipulated in contract. Because \( q^{\bar{v}} \) of the \( q^{\max} \) units are non-conforming to the contract, the seller’s ex-post payoff amounts to \( q^{\max}(p - c_l) - [q^{\bar{v}} - V(\sigma, \omega, q^{\bar{v}})] \). Taking expectations and subtracting the seller’s investment level then yields the seller’s expected payoff. The buyer’s expected payoff is equivalent to total social surplus minus the seller’s expected payoff. If the contract specifies \( \bar{v} = v_h \), all units are non-conforming to the contract. Thus, the seller’s ex-post payoff is given by \( q^{\max}(p - c_l) - [q^{\max}v_h - V(\sigma, \omega, q^{\max})] \). The parties’ expected payoff can be derived as before.

(ii) The second part of the proof can be performed in a similar way and is left to the reader.

Proof of Proposition 3. (i) The first statement is true, because, for all quality thresholds \( \bar{v} \), the buyer’s expected payoff is equivalent to expected social surplus plus or
minus a constant term. Moreover, the second statement must also be true because the seller’s expected payoff $S_{clv}(\beta, \sigma) = q^{\text{max}}(p - c_l) - \sigma$ is strictly decreasing in $\sigma$. To prove statement three, we first show that given the buyer invests efficiently the difference of the seller’s expected surplus and expected social surplus

$$S^{\text{equiv}}(\beta^*, \sigma) - \left[ E[W(\beta^*, \sigma, \omega, Q^*(\beta^*, \sigma, \omega))] - \beta^* - \sigma \right]$$

$$= q^{\text{max}}(p - c_l) - E[q^{\text{max}}v_h - V(\sigma, \omega, q^{\text{max}})] - E[W(\beta^*, \sigma, \omega, Q^*(\beta^*, \sigma, \omega))] + \beta^*$$

$$= q^{\text{max}}(p - c_l - v_h) + E[V(\sigma, \omega, q^{\text{max}})] - E[C(\beta^*, \omega, q^{\text{max}})] + \beta^*$$

$$= q^{\text{max}}(p - c_l - v_h) + E[W(\beta^*, \sigma, \omega, q^{\text{max}})] - E[W(\beta^*, \sigma, \omega, Q^*(\beta^*, \sigma, \omega))] + \beta^*$$

is monotonically increasing in $\sigma$. To see this, take any pair $(\sigma', \sigma'')$ with $\sigma' < \sigma''$ and note that

$$E[W(\beta^*, \sigma', \omega, q^{\text{max}} - Q^*(\beta^*, \sigma', \omega))]$$

$$\leq E[W(\beta^*, \sigma', \omega, q^{\text{max}} - Q^*(\beta^*, \sigma'', \omega))]$$

$$\leq E[W(\beta^*, \sigma'', \omega, q^{\text{max}} - Q^*(\beta^*, \sigma'', \omega))] .$$

The first inequality follows because, for $\beta^*$, $\sigma'$ and $\omega$ the quantity $q^{\text{max}} - Q^*(\beta^*, \sigma', \omega)$ represents all units that yield a negative net joint profit. Thus, this quantity minimizes ex-post social welfare. The second inequality follows from Assumption 2. To see this, recall that Assumption 2 implies that expected welfare is increasing in $\sigma$ for any fixed quantity $q > 0$. Consider some state $\omega' \in \Omega$. Ex-post, together with $\beta^*$ and $\sigma'$ this state determines how many units are associated with a negative net joint profit. Therefore, due to Assumption 2, for quantity $q^{\text{max}} - Q^*(\beta^*, \sigma'', \omega')$ it must hold that

$$E[W(\beta^*, \sigma', \omega', q^{\text{max}} - Q^*(\beta^*, \sigma'', \omega'))] \leq E[W(\beta^*, \sigma'', \omega', q^{\text{max}} - Q^*(\beta^*, \sigma'', \omega'))] . \ (14)$$

Because (14) holds for any state, it must also hold for the expected value over all states.

We have established that the difference between the seller’s expected surplus and
expected social welfare is increasing, hence it must hold for any \( \sigma < \sigma^* \) that

\[
S_{c_{\text{svh}}}^q(\beta^*, \sigma) - [E[W(\beta^*, \sigma, \omega, Q^*(\beta^*, \sigma, \omega))] - \beta^* - \sigma] \leq \\
S_{c_{\text{svh}}}^q(\beta^*, \sigma^*) - [E[W(\beta^*, \sigma^*, \omega, Q^*(\beta^*, \sigma^*, \omega))] - \beta^* - \sigma^*]
\]

or equivalently

\[
[E[W(\beta^*, \sigma^*, \omega, Q^*(\beta^*, \sigma^*, \omega))] - \beta^* - \sigma^*] - [E[W(\beta^*, \sigma, \omega, Q^*(\beta^*, \sigma, \omega))] - \beta^* - \sigma] \\
\leq S_{c_{\text{svh}}}^q(\beta^*, \sigma^*) - S_{c_{\text{svh}}}^q(\beta^*, \sigma).
\] (15)

Because social welfare is uniquely maximized by \((\beta^*, \sigma^*)\), the term in the first line of (15) is positive and consequently \(S_{c_{\text{svh}}}^q(\beta^*, \sigma) < S_{c_{\text{svh}}}^q(\beta^*, \sigma^*)\) for all \(\sigma \in [0, \sigma^*]\). (ii) The second part of the proof can be performed in a similar way and is left to the reader. ■

References


