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# Forward trading and collusion of firms in volatile markets

*Aichele, Markus F.*

## **Abstract**

Assuming deterministic demand Liski and Montero (2006) show that forward trading is able to facilitate collusion. We present a more concise model incorporating the main reason for forward trading: Uncertainty.

In general, fluctuations make collusion harder to sustain (Rotemberg and Saloner, 1986). However, using forward contracts, firms are able to decrease the incentives to deviate from a collusive agreement even in very volatile markets. This makes collusive strategies more sustainable and decreases social welfare.

## **1 Introduction**

Liski and Montero (2006) model an infinitely repeated oligopoly game where firms are allowed to act on the spot as well as on the forward market. They show under deterministic demand that forward trading helps to sustain collusive profits in cases where otherwise collusive profits could not be achieved. Commodities like gas, power and oil seem to match the described market structure. However, stochastic influences play a crucial role in these markets and are one of the main reasonings for trading forward. In order to gain a deeper insight into forward trading and collusion a volatile market context is added to the existing economic literature.

As has been known since Rotemberg and Saloner (1986), when firms solely interact on the spot market stochastic market conditions make collusive agreements harder to sustain. This paper aims to connect the findings of Liski and Montero (2006) and Rotemberg and Saloner (1986)

in order to analyze the effects of forward trading in volatile markets on collusive agreements. The intuition behind the effect of forward trading on collusion is as follows: Firms fix a certain quantity at a certain price via forward trading. This induces two effects: On one hand this decreases demand available for a deviating firm. Here, the consequence of forward trading is pro-collusive. On the other hand, forward trading decreases demand that is available for collusive price-setting. Here, the consequence of forward trading is contra-collusive. Destabilizing effects of fluctuations can be reduced, using a certain structure of forward contracts. We will show that trading with short-term forward contracts strictly promotes collusion, since forward trading sharply decreases the range of parameters in which collusive agreements fail.

In table 1, volumes for different commodities traded at the Leipzig European-Energy-Exchange in 2009 and 2010 are presented. The data come from the annual report of European-Energy-Exchange (2010). The spot, the forward, the total market volume and the ratio of forward traded volume and total market volume are given for power, emission rights and natural gas. In table 1 the volume of power is given in terrawatt-hours (TWh), the volume of emissions in thousand tons (1000 t.) and the volumes of gas in gigawatt-hours (GWh). The column Forwards m.share shows the ratio of forward contracted volume and total market volume (spot and forward market volume). For all three commodities, most of the trading takes place on the forward market since market share of forwards exceeds 0.65 for all commodities and years. The markets for power, emissions and gas are volatile. Hence the assumptions in the presented model match with key properties of these commodities. Surely there are important other reasons than collusive behavior for forward trading in these markets, e.g. risk sharing. However, it will be shown that (for whatever reasons traded) forward contracts help to stabilize collusive agreements in volatile markets.

Commodity	Spot m.		Forward m.		Total m.		Forwards m.share	
	2009	2010	2009	2010	2009	2010	2009	2010
Power (TWh)	203	279	1025	1208	1228	1487	0,83	0,81
Emissions (1000 t.)	9709	25184	23642	127197	33351	152381	0,71	0,83
Gas (GwH)	3516	15026	11361	31863	14877	46889	0,76	0,68

Tab. 1: Commodity volumes traded at European-Energy-Exchange (2010)

## 2 The model

### 2.1 Assumptions and general remarks

Collusive behavior of firms can occur if and only if there is no incentive for any firm to deviate from the collusive agreement unilaterally. There exists no incentive for any firm to break the collusive agreement unilaterally if the net present value of profits gained by collusion is greater than or equal to the net present value of profits gained by ending collusion. The highest profit that can be earned in every period is the monopoly profit. The profit is shared equally by both firms.

The exact outcome of prices, quantities and profits is stochastic and depends on the difference between the reservation price ( $a$ ) and marginal costs ( $c$ ). We do not distinguish between demand and supply shocks. The difference between the reservation price and marginal costs ( $\gamma = a - c$ ) will be called “spread” in the analysis. The first two moments of this “spread” are given by  $E[\gamma]$  and  $V[\gamma]$ . In order to give comparative static results, the König-Huygens theorem is used later in this paper to decompose the expectation of the squared “spread” into its variance and its squared expectation ( $E[\gamma^2] = V[\gamma] + E[\gamma]^2$ ).

Whenever we use monopoly prices, quantities and profits for the argumentation, we refer to monopoly prices, quantities and profits for a given realization of the stochastic difference between reservation price and marginal costs. Using a linear demand function we denote the price, quantity and profit associated to the one-period monopoly solution by  $p^m = \frac{a+c}{2}, q^m = \frac{a-c}{2}$

$$\text{and } \Pi_m = (p^m - c)q^m = \frac{(a-c)^2}{4}.$$

The spot and the forward market are connected as follows: In the first period, both firms choose simultaneously the amount of forward contracts they want to trade (forward market period). In the second period, contracts are settled and firms choose the amount they want to sell additionally on the spot market (spot market period). The forward market opens in the even periods ( $t = 0, 2, \dots$ ) and the spot market in the odd periods ( $t = 1, 3, \dots$ ). For comparability with pure-spot market games the per period discount factor is given by  $\sqrt{\delta}$ . Alternatively the spot market opens a marginal unit of time right after the forward market and the discount factor is given by  $\delta$ . The important fact is that the only discounting is between two spot markets, two forward markets or the forward market in  $t$  and the spot market in  $t + 1$ . Hence no discounting takes place between consecutive forward and spot markets. The structure of trading initially on the forward market and settling contracts afterwards as well as meeting residual demand on spot market is infinitely repeated. One can think of firms deciding around Christmas each year about forward contracts delivered in the following year.

## 2.2 Profits of Collusion and Deviation

Firms compete in prices. Whenever firm  $i$  sets a price lower than competing firm  $j$  firm  $i$  meets the whole spot market demand. When prices are equal they equally split the market. Consider following trigger strategy:

In the first forward market round (period 0), firm  $i$  sells  $f_i^{0,1}$  and  $f_i^{0,l} = 0$  for all  $l > 1$ . Hence firms only sell forward contracts that will be settled in the following spot market. In this following spot market period firm  $i$  sets the monopoly price ( $p_i^t = p^m$ ) if and only if in every period preceding  $t$  both firms have set monopoly prices in the spot market and have contracted in the forward market the collusive amount  $f_i^{0,1} = f_j^{0,1} = f$  one period ahead. Whenever firm

$j$  deviates from this agreement, firm  $i$  sets price at marginal cost in the spot market and sells any arbitrary amount of forward contracts forever.

Liski and Montero (2006) do not allow forward contracts exceeding monopoly quantity in their model of forward trading and collusion in a deterministic market structure. However, in a volatile market, firms do not know in any forward market period the demand and cost structure they will face in the following spot market period. Hence, firms might have traded forward more than the quantity they can sell with monopoly prices on spot market. This may happen e.g. for a relatively small realization of the difference of reservation price and marginal costs. Therefore the critical discount factor will be derived for the forward traded amount being less than monopoly quantity as well as for the forward traded amount being larger than monopoly quantity.

In general two possibilities of deviation are possible. Firstly, to set the price lower than the collusive price in the spot market. Secondly, to increase forward sales in the forward market. The latter is never profitable since speculators, which are taking the counterpart, immediately realize any deviation from collusion in the forward market and are not willing to pay any higher price than the next period's stock market price, which is given by marginal costs. Hence profitable deviation is restricted to the spot market and a firm trying to deviate knows the actual state of the economy.

The demand that can be achieved on the spot market for a firm deviating is restricted by already sold future contracts. Each firm has a already secured supply of  $f_i$ . The secured supply of both firms is given by  $F$  ( $f_i + f_j = F$ ). Total traded amount decreases accessible demand ( $a - F$  instead of  $a$ ). This gives (residual) demand function on the spot market:

$$D_i^R = \begin{cases} (a - F - p_i) & \text{if } p_i < p_j, \\ \frac{1}{2}(a - F - p_i) & \text{if } p_i = p_j, \\ 0 & \text{if } p_i > p_j \end{cases} \quad (1)$$

A firm deviating from collusion maximizes its profit over its (deviation) price. This leads to following optimal deviation price and quantity:

$$\begin{aligned} \max_p \quad \Pi_i &= (p_i - c)(a - F - p_i) \\ p^d &= \frac{1}{2}[a + c - F], q^d = \frac{1}{2}(a - F - c), \Pi^d = \frac{1}{4}[a - c - F]^2 \end{aligned} \quad (2)$$

Deviation price, quantity and profit are quite similar to price, quantity and profit in a deviation from collusion without forward trading. However, the already contracted amount decreases the demand that is reachable on the spot market ( $\frac{\partial D_i^d}{\partial F} < 0$ ). Hence deviation price, quantity and profit become smaller.

When the total contracted amount exceeds or equals the Bertrand quantity ( $q^B$ ), which is given by twice monopoly quantity ( $F \geq q^B = 2q^m = a - c$ ), no positive deviation profit can be earned since any deviation would require a price that is lower than the Bertrand price on the spot market, which is given by marginal costs.

After a deviation, profits can neither be earned on forward nor on spot markets (see collusive strategy in 2.1). Consequently, the net present value of deviation is given solely by the deterministic deviation profit of this single period:

$$E_{NPV}[Deviation] = \begin{cases} \frac{1}{4}[a - c - F]^2 & \text{if } F < 2q^m \\ 0 & \text{if } F \geq 2q^m, \end{cases} \quad (3)$$

The demand that can be reached by collusive behavior in this period is restricted by already sold forward contracts too. Colluding firms set monopoly prices as if no forward trading has occurred before ( $p^m = \frac{a-c}{2}$  instead of  $p^m = \frac{a-F-c}{2}$ ). When not doing so firms would not be able to sell collusive forward contracts at expected (monopoly) prices since speculators would anticipate the (expected) price discount on spot market. When firms set this collusive price they split residual demand that is given by  $D^R = a - F - p^m$  and earn a per-unit-profit of

$\pi^C = p^m - c$ . Each firms' collusive profit on spot market can be stated as:

$$\begin{aligned}\Pi^C &= \frac{1}{2}D^R\pi^C = \frac{1}{2}(a - F - p^m)(p^m - c) \\ &= \frac{1}{2}\left[\frac{1}{4}(a - c - F)^2 - \frac{1}{4}F^2\right] = \frac{1}{2}\Pi^d - \frac{1}{8}F^2\end{aligned}\quad (4)$$

When the total forward traded amount does not exceed or equal monopoly quantity ( $F < q^m$ ), collusive behavior leads to collusive profits in this period (given in equation 4). Additionally collusive profits given by the half of expected monopoly profit are expected in all upcoming periods.

When the total forward traded amount exceeds or equals monopoly quantity ( $F \geq q^m$ ) no collusive profits can be earned in this period, since the total demand for the monopoly price is already satisfied. However, not deviating from collusion promises half of the expected monopoly profit in all upcoming periods. This gives net present value of collusion as a piecewise defined function:

$$E_{NPV}[Collusion] = \begin{cases} \frac{1}{2}\Pi^d - \frac{1}{8}F^2 + \frac{1}{2}\frac{\delta}{1-\delta}E[\Pi^m] & \text{if } F < q^m \\ \frac{1}{2}\frac{\delta}{1-\delta}E[\Pi^m] & \text{if } q^m \leq F < 2q^m \end{cases} \quad (5)$$

The different collusion profits in the period of (possible) deviation lead to one of three different cases. In the first case, the total forward traded amount is less than the monopoly quantity of the economic state ( $F < q^m$ ). In the second case, the total forward traded amount exceeds monopoly quantity but is less than Bertrand quantity ( $q^m < F < q^B = 2q^m$ ). In the third case, the total traded amount exceeds Bertrand quantity ( $q^B = 2q^m < F$ ). However, the third case is not analyzed deeper since neither collusive nor deviation profits can be earned in the corresponding period and a firm is not facing the trade-off between collusion or deviation in the corresponding period. Hence both firms will stay in the collusive agreement since it is the only way that promises future (collusive or deviation) profits.

## 2.3 To cheat or not to cheat: Deviation vs. Collusion

A firm facing an collusive agreement with its competitor has two alternative strategies. Firstly, it can collude and gain a profit in the corresponding period and in future periods. Secondly, it can deviate and gain an additional profit in the corresponding period but forgo all collusive profits in future periods. A firm chooses the strategy which gives higher expected net present value of profits.

Comparison of this net present values leads to an inequality, which represents the trade-off between collusion and deviation. This inequality is called the no deviation constraint. This no deviation constraint will be used to find the critical discount factor.

**For total traded amount less than monopoly quantity ( $F < q^m$ )**

For a stable collusive agreement, the net present value of collusion must be larger than the net present value of deviation. Hence for the forward traded amount less than monopoly quantity, following inequality has to be fulfilled for a stable collusive agreement:

$$\begin{aligned}
 E_{NPV}[Deviation] &\leq E_{NPV}[Collusion] \\
 \Pi^d &\leq \frac{1}{2}\Pi^d - \frac{1}{8}F^2 + \frac{1}{2}\frac{\delta}{1-\delta}E[\Pi^m] \\
 \frac{1}{4}\gamma^2 + \frac{1}{2}F^2 - \frac{1}{2}F\gamma &\leq \frac{\delta}{1-\delta}\frac{1}{4}[E[\gamma]^2 + V[\gamma]]
 \end{aligned} \tag{6}$$

This inequality will be called the no deviation constraint and can be used to find a semi-collusive strategy. This semi-collusive strategy becomes necessary when firms have discount factors for which monopoly behavior would lead to deviation whereas lower (still collusive) prices and profits would be stable.

Rearranging the no deviation constraint gives the critical discount factor for collusive behavior as:

$$\begin{aligned}
 \frac{\delta}{1-\delta} &\geq \frac{\gamma^2 + 2F^2 - 2F\gamma}{E[\gamma]^2 + V[\gamma]} \\
 \delta &\geq \delta^* = 1 - \frac{E[\gamma]^2 + V[\gamma]}{E[\gamma]^2 + V[\gamma] + \gamma^2 - 2F\gamma + 2F^2}
 \end{aligned} \tag{7}$$

**For total traded amount higher than monopoly quantity but less than Bertrand quantity ( $q^m < F < 2q^m$ )**

For a stable collusive agreement, the net present value of collusion must be larger than the net present value of deviation. No collusive profits are earned on the spot market since total forward traded amount exceed monopoly quantity ( $q^m < F$ ). Hence the net present value of collusion is restricted to half of the future expected monopoly profits. For the forward traded amount exceeding monopoly quantity this gives following inequality for a stable collusion:

$$\begin{aligned}
 E_{NPV}[Deviation] &\leq E_{NPV}[Collusion] \\
 \Pi^d &\leq \frac{1}{2} \frac{\delta}{1-\delta} E[\Pi^m] \\
 \frac{1}{2}\gamma^2 - F\gamma + \frac{1}{2}F^2 &\leq \frac{\delta}{1-\delta} \frac{1}{4} [E[\gamma]^2 + V[\gamma]]
 \end{aligned} \tag{8}$$

Rearranging the no deviation constraint gives the critical discount factor for collusive behavior as:

$$\begin{aligned}
 \frac{\delta}{1-\delta} &\geq \frac{2\gamma^2 - 4\gamma + 2F^2}{E[\gamma]^2 + V[\gamma]} \\
 \delta &\geq \delta^* = 1 - \frac{E[\gamma]^2 + V[\gamma]}{E[\gamma]^2 + V[\gamma] + 2\gamma^2 - 4F\gamma + 2F^2}
 \end{aligned} \tag{9}$$

## 2.4 Effects of forward trading on the critical discount factor

The critical discount factor for any forward traded amount under full-collusion is given by:

$$\delta^* = \begin{cases} 1 - \frac{E[\gamma]^2 + V[\gamma]}{E[\gamma]^2 + V[\gamma] + \gamma^2 - 2F\gamma + 2F^2} & \text{if } F < q^m \\ 1 - \frac{E[\gamma]^2 + V[\gamma]}{E[\gamma]^2 + V[\gamma] + 2\gamma^2 - 4F\gamma + 2F^2} & \text{if } q^m \leq F < 2q^m \end{cases} \tag{10}$$

How is the critical discount factor influenced by the realization of the random difference between reservation price and marginal costs ( $\gamma$ ), by the amount of forward contracts ( $F$ ), by the expected difference between reservation price and marginal cost ( $E[\gamma]$ ) and by the variance of the difference between reservation price and marginal cost ( $V[\gamma]$ )?

The partial derivative of the critical discount factor due to the difference between reservation price and marginal costs is given by:

$$\frac{\partial \delta^*}{\partial \gamma} = \begin{cases} 2 \frac{[\gamma-F][E[\gamma]+V[\gamma]]}{[E[\gamma]^2+V[\gamma]+\gamma^2-2F\gamma+2F^2]^2} \geq 0 & \text{if } F < q^m \\ 4 \frac{[\gamma-F][E[\gamma]^2+V[\gamma]]}{[E[\gamma]^2+V[\gamma]+2\gamma^2-4F\gamma+2F^2]^2} \geq 0 & \text{if } q^m \leq F < 2q^m \end{cases} \quad (11)$$

A higher difference of reservation price and marginal costs leads to a higher profit which leads to an higher critical discount factor, since deviation becomes more attractive.

The partial derivative of the critical discount factor due to the amount of contracts is given by:

$$\frac{\partial \delta^*}{\partial F} = \begin{cases} -2 \frac{[\gamma-2F][E[\gamma]^2+V[\gamma]]}{[E[\gamma]^2+V[\gamma]+\gamma^2-2F\gamma+2F^2]^2} \leq 0 & \text{if } F < q^m \\ -4 \frac{[\gamma-F][E[\gamma]^2+V[\gamma]]}{[E[\gamma]^2+V[\gamma]+2\gamma^2-4F\gamma+2F^2]^2} \leq 0 & \text{if } q^m \leq F < 2q^m \end{cases} \quad (12)$$

A higher forward contracted amount strictly reduces the critical discount factor, since for forward traded amount less than monopoly quantity ( $0 \leq F < q^m = \frac{1}{2}\gamma$ ) deviation profit is cut more sharply than collusive profit in the corresponding period. This is derived analytically in the appendix (equation 22 and 23). If the forward traded amount is larger than monopoly quantity ( $\frac{1}{2}\gamma = q^m \leq F < q^m = \gamma$ ), the deviation profit is cut solely since no collusive profit can be earned in the corresponding period. Hence forward contracts strictly promote collusion. The partial derivative of the critical discount factor with respect to the expected difference between reservation price and marginal costs is given by:

$$\frac{\partial \delta^*}{\partial E[\gamma]} = \begin{cases} -2 \frac{[\gamma^2-2F\gamma+2F^2]E[\gamma]}{[E[\gamma]^2+V[\gamma]+\gamma^2-2F\gamma+2F^2]^2} \leq 0 & \text{if } F < q^m \\ -2 \frac{[2\gamma^2-4F\gamma+2F^2]E[\gamma]}{[E[\gamma]^2+V[\gamma]+2\gamma^2-4F\gamma+2F^2]^2} \leq 0 & \text{if } q^m \leq F < 2q^m \end{cases} \quad (13)$$

A higher expected difference of reservation price and marginal costs decreases the critical discount factor. Deviation from collusion becomes less attractive. A higher expected difference

increases future collusive profits which cannot be earned after a deviation. Hence the additional profits earned by a deviation become smaller in relative terms.

The partial derivative of the critical discount factor with respect to the variance of the difference between reservation price and marginal costs is given by:

$$\frac{\partial \delta^*}{\partial V[\gamma]} = \begin{cases} -\frac{\gamma^2 - 2F\gamma + 2F^2}{[E[\gamma]^2 + V[\gamma] + \gamma^2 - 2F\gamma + 2F^2]^2} & \leq 0 \quad \text{if } F < q^m \\ -\frac{2\gamma^2 - 4F\gamma + 2F^2}{[E[\gamma]^2 + V[\gamma] + 2\gamma^2 - 4F\gamma + 2F^2]^2} & \leq 0 \quad \text{if } q^m \leq F < 2q^m \end{cases} \quad (14)$$

A higher variance of the difference of reservation price and marginal costs decreases the critical discount factor. At a first glance this seems to be counter-intuitive since fluctuations are said to threaten collusions. One should keep in mind the relationship between variance squared, expectation and expectation squared used above ( $E[\gamma^2] = E[\gamma]^2 + V[\gamma]$ ). As can be seen, expected profit given by  $\frac{1}{4}E[\gamma^2]$  is *ceteris paribus* increased by an increasing variance. As presented above, a higher expected profit increases the stability of collusion. It is not the variance itself that decreases the stability of an collusive agreement. It is more precisely the appearance of a high realization of the random difference between reservation price and marginal costs. For a higher variance, this high realization of the random variable is more likely to be drawn. However, for a given realization of the random variable, a higher variance decreases the critical discount factor.

Table 2 summarizes partial effects on the critical discount factor. Expected difference between reservation price and marginal costs and its variance have *ceteris paribus* a stabilizing effect on a collusive agreement, as well as the total forward traded amount. High realizations of the difference between reservation price and marginal costs have a destabilizing affect on a collusive agreement.

Figure 1 shows the evolution of the critical discount factor due to forward contracts and due to the ratio of boom and expected profits. In the dimension of sold contracts figure 1 starts

Variable	Partial Effect	I Contracts inferior monopoly quantity	II Contracts superior monopoly quantity
“Spread”	$\frac{\partial \delta^*}{\partial \gamma}$	$\geq 0$	$\geq 0$
Forwards	$\frac{\partial \delta^*}{\partial F}$	$\leq 0$	$\leq 0$
Expected “spread”	$\frac{\partial \delta^*}{\partial E[\gamma]}$	$\leq 0$	$\leq 0$
Variance of “spread”	$\frac{\partial \delta^*}{\partial V[\gamma]}$	$\leq 0$	$\leq 0$

Tab. 2: Summary of partial effects on critical discount factor

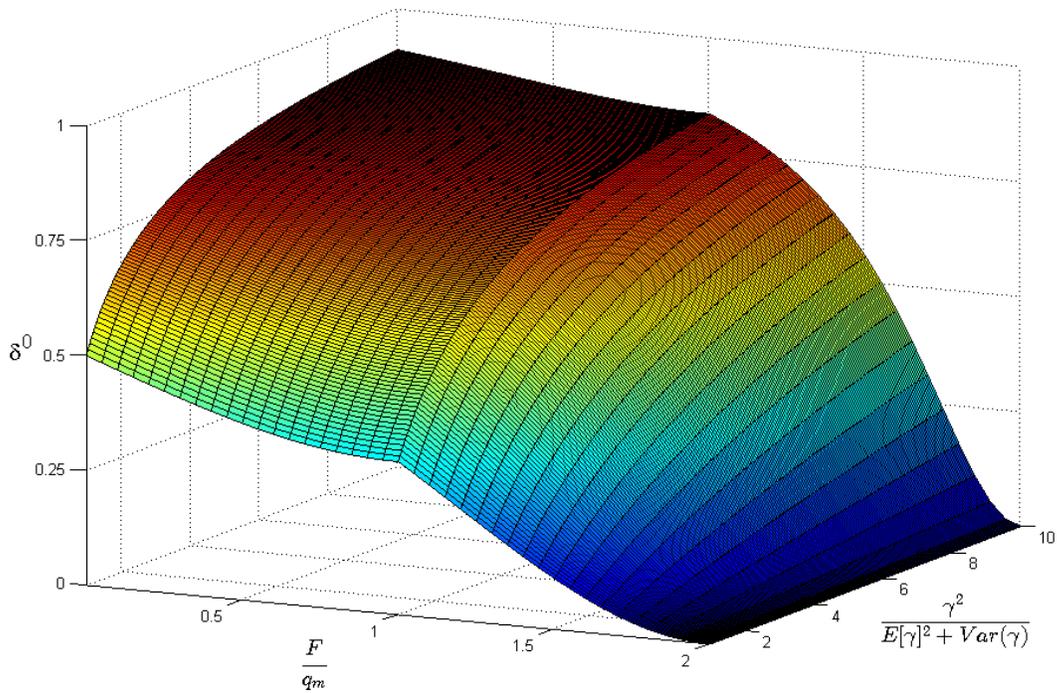


Fig. 1: Effects of forward trading and ratio of boom and expected profit on discount factor

at 0. The discount factor is plotted for positive ratios of contracted amount and monopoly quantity. Neither collusive nor deviation profits can be earned for a higher amount of contracts than the Bertrand quantity and the critical discount factors becomes zero. Hence the graph is stopped at a ratio of the forward traded amount and monopoly quantity of two.

It is known from Rotemberg and Saloner (1986) that deviation from collusion is more profitable in booms. Figure 1 starts in the dimension of profits at 1, since in booms per definition profits are higher than the expected ones. It ends in this dimension at a profit that is ten times the expected one.

The horizontal front-line of figure 1 shows the evolution of the discount factor for expected profit equal to actual profit ( $\frac{\gamma^2}{E[\gamma^2]+V[\gamma]} = 1$ ). This represents the case of certainty described by Liski and Montero (2006), since without any forward contracts and without any volatility the critical discount factor is one half and when total monopoly quantity is traded forward the discount factor is one-third. For forward contracts between these both extreme cases ( $0 \leq \frac{F}{q^m} < 1$ ), the critical discount factor strictly decreases in forward contracts.

When firms have contracted more than the monopoly quantity of the corresponding state ( $1 \leq \frac{F}{q^m} < 2$ ), the critical discount factor still decreases in forward contracts. However, it seems to decrease more rapidly. One reason might be that for forward contracts less than monopoly quantity ( $0 \leq \frac{F}{q^m} < 1$ ), additional contracts decrease deviation as well as collusive profits. This leads to an altogether pro-collusive effect. For forward contracts exceeding monopoly quantity ( $1 \leq \frac{F}{q^m} < 2$ ) deviation profit is solely decreased by forward contracts. Hence additional forward contracts decrease the discount factor more sharply.

For total traded amount equal to zero and without volatility ( $F = 0 \wedge \frac{\gamma^2}{E[\gamma^2]+V[\gamma]} = 1$ ) the critical discount factor equals one half. Introducing a volatile market creates an incentive to deviate from collusion during booms. Without forward contracts ( $F = 0$ ) the critical discount factor strictly increases and converges to one for boom profit increasing to infinity. The functional form of the critical discount factor depends on the ratio of boom and expected profit

and is given by  $\frac{\gamma^2}{E[\gamma^2]+V[\gamma]+\gamma^2} = \delta^0 \leq \delta$ . This functional form is equivalent to that derived by Rotemberg and Saloner (1986).

When contracts are traded forward and at the same time boom profits are larger than expected profits, the above described evolution of the critical discount factor is not changed fundamentally. Other things being equal, a higher amount of contracts decreases the critical discount factor, whereas boom profits exceeding expected profit increase the critical discount factor. This can be seen graphically in figure 1 by the evolution of the plane between the above described front-lines.

When firms contract a sufficiently high quantity, stable collusion becomes possible for any discount factor. However, since the collusive profits decrease when firms deliver via the forward or spot market quantities that are higher than monopoly quantities, excessive contracting is not a (good) option.

Table 3 summarizes the previous analysis of collusive behavior in a volatile market when firms are allowed to trade forward. The first column represents case I when forward trades do not exceed monopoly quantity ( $F < q^m$ ), whereas the second column represents case II when forward trades exceed the monopoly quantity ( $q^m < F < q^B$ ). For both cases deviation and collusive profits, the net present value of collusive behavior as well as the no deviation constraint and the critical discount factor are given.

## 2.5 A two-state economy as a special case

We specify the distribution of the “spread” as a two-state random variable with a low and a high realization called recession and boom. A recession happens with probability  $\mu$  and a boom happens with the complementary probability  $1 - \mu$ . Firms split the monopoly profit in each state equally ( $\Pi_H^C = \frac{1}{2}\Pi_H^M, \Pi_L^C = \frac{1}{2}\Pi_L^M$ ).

For an amount of forward contracts smaller than the monopoly quantity ( $F < q^m$ ), equation

Case	I Contracts inferior monopoly quantity	II Contracts superior monopoly quantity
Deviation profit (t=0)	$\Pi^d = \frac{1}{4} (a - c - F)^2$	$\Pi^d = \frac{1}{4} (a - c - F)^2$
Collusive profit (t=0)	$\Pi^c = \frac{1}{8} (a - c - F)^2 - \frac{1}{8} F^2$	0
NPV of collusion	$\frac{1}{2} \Pi^d - \frac{1}{8} F^2 + \frac{1}{2} \frac{\delta}{1-\delta} E[\Pi^m]$	$\frac{1}{2} \frac{\delta}{1-\delta} E[\Pi^m]$
No dev. constraint	$\Pi^m + \frac{1}{2} F^2 \leq \frac{\delta}{1-\delta} E[\Pi^m] + Fq_m$	$\Pi^m + \frac{1}{4} F^2 \leq \frac{1}{2} \frac{\delta}{1-\delta} E[\Pi^m] + Fq_m$
Critical disc. factor	$\delta^* = 1 - \frac{E[\gamma]^2 + V[\gamma]}{E[\gamma]^2 + V[\gamma] + \gamma^2 - 2F\gamma + 2F^2}$	$\delta^* = 1 - \frac{E[\gamma]^2 + V[\gamma]}{E[\gamma]^2 + V[\gamma] + 2\gamma^2 - 4F\gamma + 2F^2}$

Tab. 3: Summary of the effect of total contracted amount on collusive firms

6 can be rearranged to:

$$\begin{aligned} \Pi^m &\leq \frac{\delta}{1-\delta} E[\Pi^m] + Fq_m - \frac{1}{2} F^2 \\ \Pi_H^C &\leq \frac{\mu\delta}{1-\delta(2-\mu)} \Pi_L^C + \frac{1}{2} \frac{1-\delta}{1-\delta(2-\mu)} \left[ Fq_m - \frac{1}{2} F^2 \right] \end{aligned} \quad (15)$$

For an amount of forward contracts larger than the monopoly quantity ( $F > q^m$ ) equation 8 can be brought to:

$$\begin{aligned} \Pi^m &\leq \frac{1}{2} \frac{\delta}{1-\delta} E[\Pi^m] + Fq_m - \frac{1}{2} F^2 \\ \Pi_H^C &\leq \frac{\mu\delta}{2-3\delta+\mu} \Pi_L^C + \frac{1-\delta}{2-3\delta+\mu} \left[ Fq_m - \frac{1}{4} F^2 \right] \end{aligned} \quad (16)$$

When firms do not have a sufficient high discount factor to sustain monopoly profits in a boom, equation 15 or 16 can be used to derive a semi-collusive spot market strategy. For a given discount factor, recession probability and total forward traded amount, firms choose the highest collusive boom profit such that equation 15 or 16 is fulfilled. However, to derive the “best semi-collusive strategy” one should model firms’ *a priori* trade-off between sustaining collusion via forward or spot market actions. For this trade-off, firms anticipate the implications that a certain degree of forward trading has on the collusion sustaining spot market profit. This is not dealt with in this paper because its scope is collusive agreements at monopoly

prices.

Inserting the recession probability of  $\mu = \frac{1}{2}$  in the no deviation constraint of the two state model (equation 15) gives:

$$\Pi_H^C \leq \frac{\delta}{2-3\delta} \Pi_L^C + \frac{1-\delta}{2-3\delta} \left[ Fq_m - \frac{1}{2}F^2 \right] \quad (17)$$

When firms do not trade any contracts on the forward market ( $F = 0$ ), the discount factor derived in equation 17 is equivalent to that derived by Rotemberg and Saloner (1986) for a two state economy (Tirole, 1988, p.249). For a discount factor higher than  $\delta^0 = \frac{2}{3}$ , full-collusion becomes possible, even without forward trading. The term that is added is strictly positive in the relevant range of parameters. Hence the necessary semi-collusive adjustment of spot market prices and profits is always lower than when there is no forward trading. This points out the pro-collusive effects of trading forward.

## 2.6 Negligible uncertainty as a special case

Under certainty, firms never trade more than the monopoly quantity in a full collusive agreement, since trading forward more than (a priori known) monopoly quantity would decrease profits. Total traded amount is given by summing up the single (symmetrically) traded amount where  $x$  gives the proportion of monopoly quantity that is traded forward ( $F = f_i + f_j = 2f = xq^m = \frac{1}{2}\gamma x$ ). Under certainty, the “spread” equals its expectation and the variance of the “spread” is equal to zero. Then the critical discount factor (equation 7) can be brought to:

$$\begin{aligned} \delta \geq \delta^* &= 1 - \frac{E[\gamma]^2 + V[\gamma]}{E[\gamma]^2 + V[\gamma] + \gamma^2 - 2F\gamma + 2F^2} \\ &= 1 - \frac{\gamma^2}{2\gamma^2 - x\gamma^2 + \frac{1}{2}x^2\gamma^2} \\ &= 1 - \frac{2}{(2-x)^2 + 2x} \end{aligned} \quad (18)$$

The partial derivative of the critical discount factor due to proportion of monopoly quantity traded forward is given by:

$$\frac{\partial \delta^*}{\partial x} = \frac{-4[1-x]}{(2-x)^2 + 2x} \leq 0 \quad (19)$$

The partial derivative of the critical discount factor due to proportion of monopoly quantity traded forward is strictly negative. Hence, in a deterministic market structure, trading forward is able to stabilize collusive agreements as well. The critical discount factor neglecting uncertainty (equation 18) is equivalent to the factor found by Liski and Montero (2006, p.219).

### 3 Conclusion

Liski and Montero (2006) model the collusive strategy of firms facing a spot as well as a forward market. They use deterministic market conditions. Uncertainty, volatility and fluctuations are the most frequent reasons given for forward trading (see e.g. Hull (2000)). The contribution of this paper is the simultaneous analysis of fluctuations and forward contracts on collusive agreements. The incorporation of stochastic market conditions leads to a more precise understanding of the effects of forward trading and collusion. In terms of economic literature, the gap between Rotemberg and Saloner (1986) and Liski and Montero (2006) has been closed.

As a measure for the stability of collusive agreements, we used the critical discount factor as concept. In order to gain an insight into the consequences of stochastic market conditions and forward trading on a collusive agreement, we took partial derivatives of the critical discount factor. We found that high realizations of the random difference between reservation price and marginal costs (“spread”) have a destabilizing effect, whereas a higher expectation of the “spread” has a stabilizing effect on collusive agreements. This is totally in line with the analysis of Rotemberg and Saloner (1986). However, decomposition of the expectation of the squared

“spread” into its squared expectation and variance led to an interesting insight. For a given positive fluctuation (boom), a higher variance increases the stability of collusion, since a higher variance makes a boom common. Hence it is not the variance itself that decreases the stability of a collusive agreement in volatile markets, but rather the appearance of high realizations of the “spread” that destabilizes collusive agreements. However, extraordinary booms only occur if the distribution of the spread is characterized by a sufficient degree of dispersion. In our analysis, short term forward contracts can be used by firms to strictly stabilize collusion. This is in line with the analysis of Liski and Montero (2006).

Thinking about forward trading and collusion in volatile markets still leads to several unsolved questions. In our analysis, only short term forward contracts were traded by collusive firms. Green and Coq (2010) analyzed the effects of the length of contracts on collusive agreements in a deterministic market structure. In order to account for the effects of longer lasting contracts in volatile markets, long term contracts could be added.

It has been known since Rotemberg and Saloner (1986) that when the discount factor of firms is not sufficiently high enough to sustain monopoly price and profit in booms, firms can set prices lower than monopoly prices in order to sustain collusion. The best semi-collusive strategy would be an interesting expansion of the presented model. Equation 6 and 8 could be used as a starting point for this expansion since, whether adapting forward prices, adapting spot prices or adapting both prices, promises highest semi-collusive profits cannot be said without further analysis.

Firms need capacities to produce or exploit commodities traded on forward markets. It has been known since Kreps and Scheinkman (1983) that quantity competition gives a way to incorporate questions of capacity planning. Liski and Montero (2006) analyzed collusive strategy of firms facing quantity competition based on the models of Allaz (1992) and Allaz and Villa (1993). Adding volatility in a market with quantity competition and forward trading would help to analyze collusive strategies of firms building up capacities.

In our analysis, uncertainty exists only about the market conditions of upcoming spot and forward markets. One also can introduce imperfect observability of spot and forward market conditions. Imperfect observability of spot and forward positions leads to totally different strategic implications since a firm facing a decrease in spot market demand cannot be sure whether this happens due to a economic recession or a deviation of its opponent. For pure spot market games, this analysis has been done by Green and Porter (1984). The strategic implications of imperfect observability of spot and/or forward market positions seem to be another exciting direction for further research.

## 4 Appendix

### 4.1 Why does forward trading make collusion more stable?

Deviation profit (equation 2) can be rearranged to

$$\begin{aligned}
 \Pi^d &= \frac{1}{4} [a - c - F]^2 \\
 &= \frac{1}{4} [(a - c)^2 - 2F(a - c) + F^2] \\
 &= \Pi^m \left[ 1 - \frac{F}{\frac{1}{2}(a - c)} + \frac{1}{4} \frac{F^2}{\frac{1}{4}(a - c)^2} \right] \\
 &= \Pi^m \left[ 1 - \frac{1}{2} \frac{F}{q^m} \right]^2
 \end{aligned} \tag{20}$$

Collusive profit in a spot market period (equation 4) can be brought to:

Remember: Collusive profit in a spot market period can be earned if and only if  $F < q_m$

$$\begin{aligned}
 \Pi^C &= \frac{1}{2} \left[ \frac{1}{4} (a - c)^2 - \frac{1}{2} F(a - c) \right] \\
 &= \frac{1}{2} \left[ \Pi^m - \frac{2}{4} (a - c)^2 \frac{F}{a - c} \right] \\
 &= \frac{1}{2} \Pi^m \left[ 1 - \frac{F}{q^m} \right]
 \end{aligned} \tag{21}$$

As can be seen easily, deviation profit as well as collusive profit in a spot market period is decreased by forward contracts. However, as long as the total amount of forward contracts is

less than the monopoly quantity, the decreasing effect is stronger on deviation profit. This is due to the fact that forward trading influences deviation profit squared ( $\Pi^D = \Pi^m \left[1 - \frac{1}{2} \frac{F}{q^m}\right]^2$ ) whereas collusive profit is influenced linearly ( $\Pi^C = \frac{1}{2} \Pi^m \left[1 - \frac{F}{q^m}\right]$ ).

**Proof:**

Partial derivatives of collusion and deviation profit in a spot market period are given by:

$$\begin{aligned}\frac{\partial \Pi^C}{\partial F} &= -\frac{1}{2} \frac{\Pi^m}{q^m} \\ \frac{\partial \Pi^D}{\partial F} &= -\frac{\Pi^m}{q^m} \left[1 - \frac{1}{2} \frac{F}{q^m}\right]\end{aligned}\tag{22}$$

Comparing both partial derivatives leads to

$$\begin{aligned}-\frac{1}{2} \frac{\Pi^m}{q^m} &\geq -\frac{\Pi^m}{q^m} \left[1 - \frac{1}{2} \frac{F}{q^m}\right] \\ \frac{1}{2} &\geq \frac{1}{2} \frac{F}{q^m} \\ q^m &\geq F\end{aligned}\tag{23}$$

If the forward traded amount is less than the respecting monopoly quantity ( $F < q^m$ ), additional forward contracts decrease deviation profit more sharply than collusive profit.

If the forward traded amount is greater than the respective monopoly quantity ( $F > q^m$ ), no collusive profits in the corresponding period can be earned. Additional forward contracts decrease deviation profit. Hence the effect of additional forward contracts on the critical discount factor increases.

Representation of the critical discount factor used for plotting in figure 1:

$$\begin{aligned}\delta^* &= 1 - \frac{E[\gamma] + V[\gamma]}{E[\gamma] + V[\gamma] + \gamma^2 - 2F\gamma + 2F^2} \\ &= 1 - \frac{2}{2 + \frac{\gamma^2}{E[\gamma] + V[\gamma]} \left[2 - 2\frac{F}{q^m} + \frac{F^2}{q^{m2}}\right]}\end{aligned}\tag{24}$$

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