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A Dynamic North-South Model of Demand-Induced Product Cycles∗

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Abstract

This paper presents a dynamic North-South general equilibrium model with non-homothetic preferences. Innovation takes place in the rich North while firms in the poor South at random imitate products manufactured in the North. The model is able to generate endogenous product cycles as described by Vernon (1966) where the different stages of the product cycle are not only determined by supply side factors but also by the distribution of income between North and South. We simulate comparative statics results of changes in Southern labor productivity, changes in inequality across regions, and changes in the savings rate. We further provide suggestive evidence for the product cycle stages.

JEL classification: F1, O3

Keywords: Product cycles, Inequality, International trade

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1 Introduction

In his seminal article Vernon (1966) explained international trade patterns with product cycles. He hypothesized that new goods would be introduced in countries with high per capita incomes (catering to the needs of such a market), after a while demand for these goods emerges abroad (as incomes grow) and they are exported. Later on, goods are imitated by less advanced countries, which have a relative cost advantage, such that the production moves there. Hence, goods that were once exported by rich countries are eventually imported by them.

In a follow-up paper, Vernon (1979), page 260, explicitly emphasized the role of the demand side in shaping the typical product cycle: "In the early part of the post-war period, the US economy was the repository of a storehouse of innovations not yet exploited abroad, innovations that responded to the labour-scarce high-income conditions of the US market. As the years went on, other countries eventually achieved the income levels and acquired the relative labour costs that had prevailed earlier in the United States. As these countries tracked the terrain already traversed by the US economy, they developed an increasing demand for the products that had previously been generated in response to US needs. That circumstance provided the consequences characteristically associated with the product cycle sequence [...]"

In this paper we develop a formal model that is able to generate a product cycle like the one Vernon (1966) described. There are two regions, a wealthy North, and a poor South. Households have non-homothetic preferences over indivisible and differentiated products such that consumption patterns differ across regions. In particular, households in the North can afford to consume more and newer products than households in the South. Monopolistic firms in the North innovate new products (horizontal innovations) whereas competitive firms in the South randomly target Northern products for imitation. Trading products across regions is costless. In the steady state, products go on average through the following cycle. A new prod-
uct is developed and introduced in the North. Only after a certain time have households in
the South become rich enough to afford a "new" product that is produced in the North. This
demand lag\textsuperscript{1} is endogenously determined and depends \textit{ceteris paribus} positively on the degree
of inequality across regions and negatively on the innovation rate. In other words, if Southern
households, relative to Northern households, are poor the demand lag is long, similarly, if
incomes grow at a low rate the demand lag is long too. At this stage of the product cycle
the North is exporting the product. As time elapses further the South eventually masters the
technology to manufacture the product itself. Southern firms chose at random Northern prod-
ucts to imitate that have not yet been copied. They must invest resources in order to reverse
engineer the production process of the chosen product. If they have invested the necessary
resources they enter into a price competition with the innovating firm in the North. Because
they have a cost advantage due to lower wages they can underbid the Northern innovator and
capture the whole market. Hence, the South becomes an exporter of that product. The average
time span a product is manufactured in the North is determined endogenously... In sum, we
get on average a product cycle as described by Vernon (1966): At the stage of a new product
they are manufactured and consumed in the North, at the mature stage they are exported to
the South, and eventually, at the stage of the standardized product they are manufactured in
the South and exported to the North.

The remainder of the paper is structured as follows. In Section 2, we distinguish our
contribution from the existing literature. Section 3 presents suggestive evidence for the product
cycle described in the introductory Section 1 (in particular, we look at six major consumer

\textsuperscript{1}Our use of the term "demand lag" differs from Posner's (1961). He thinks of the demand lag as the delay
in the acceptance of foreign goods in the domestic market, i.e. foreign goods might not be considered perfect
substitutes for home-produced goods until some time elapses. We think of the demand lag as the time it takes
in the poor South for incomes to grow sufficiently such that households there can afford to buy goods produced
in the North abstracting from differences in tastes.
durables). In Section 4, we introduce the model and solve for the steady state. Comparative statics results of changes in Southern productivity, changes in inequality across regions and changes in the savings rate are discussed in Section 5. Eventually, Section 6 concludes.

2 Related Literature

Vernon’s (1966) theory grew out of dissatisfaction with classical trade theories, which explain trade between countries with differences in relative factor endowments (Heckscher-Ohlin) or differences in relative productivities (Ricardo). On the one hand, these theories missed characteristics like countries per capita incomes (Burenstam-Linder, 1961) that are thought to be important determinants of international trade (Markusen, 1986; Bernasconi, 2011), and on the other hand, seemed to be unable to explain observed trade flows (Leontief, 1953, 1956).

It has been almost 50 years since Vernon put forward his verbal theory of the product cycle. Since then a number of economists have both formalized the product cycle theory in theoretical models as well as put it to empirical tests.

One of the first to study product cycles in a theoretical model was Krugman (1979). In his model, an advanced North introduces new products at a constant exogenous rate (i.e. the product space expands), and a less advanced South copies these goods also at a constant exogenous rate. Higher per capita income in the North depends on quasi rents from Northern monopoly of new goods (i.e. North must continually innovate to maintain its relative and absolute position). Later, Grossman and Helpman (1991) extended Krugman’s model, and endogenized innovation and imitation rates. In their model, long-run growth is faster the larger the resource base of the South and the more productive its resources (in learning the production process). The reason is that profits during the monopoly phase are higher when a smaller number of Northern firms compete for resources in the manufacturing sector, which outweighs the effect of a higher risk-adjusted interest rate since profits accrue on average for a shorter
period of time. Both models focus on the supply side part of the product cycle theory, i.e.
how the diffusion of technology and the determination of relative wages depend on technology
(and preference parameters of the model). However, in both models, demand patterns in
the North and South are the same because agents have homothetic constant-elasticity-of-
substitution (CES) preferences. In other words, the consumption basket demanded in the
North is simply a scaled up clone of the one in the South. "This is clearly at odds with the fact,
stressed by Vernon, that new goods are not typically consumed in the South until later in the
with vertically differentiated goods the North manufactures high-quality products whereas the
South manufactures low-quality products. Since agents have non-homothetic preferences all
of the products manufactured in the North are consumed domestically but only the lower-
quality products are exported. Stokey (1991) is interested in the effect of population size and
productivity changes on relative wages, production respectively trade patterns, and the terms
of trade. Kugler and Zweimüller (2005) build a dynamic North-South model where households
have non-homothetic preferences. Their model is closest to our setup. However, the model
in Kugler and Zweimüller (2005) is not a general-equilibrium model because interest rates are
exogenously determined. Furthermore, the focus of their analysis is on the composition of
aggregate demand rather than on product cycles.

Our model differs from the existing literature in the following ways. In contrast to e.g.
Stokey (1991) or Flam and Helpman (1987) we focus on horizontal instead of vertical in-
novations. In addition, we differ from Stokey (1991) and Matsuyama (2000) who build a
static Ricardian trade model where agents have (hierarchic) non-homothetic preferences by
developing a dynamic general equilibrium model. We model the demand side as Foellmi and
Zweimüller (2006). On the supply side, we borrow from Grossman and Helpman (1991). In-
corporating non-homothetic preferences into these types of models enables us to formalize the
product cycle hypothesis and analyze the effects of the demand side on the product cycle.

3 Motivation

There have been many attempts to test the product cycle hypothesis empirically. Among the first who found evidence for the product cycle theory were Wells (1969) for consumer durables and Hirsch (1967) for electronic products. Mullor (1983) finds evidence that industrial product groups behave according to the product cycle theory. Feenstra and Rose (2000) find evidence for product cycles by ranking goods (and countries) according to the year they are first exported (export) to the United States.

Besides empirical evidence there is anecdotal evidence for the product cycle hypothesis, e.g. products like color T.V.s, computer games, or electric can openers seem to follow or have followed Vernon’s product cycle.

3.1 Suggestive evidence on 6 major consumer durables

Instead of attempting to empirically test the product cycle hypothesis (a conclusive analysis would require time series data on production and consumption at the product level across a large sample of countries - the gathering and analysis of such data is beyond the scope of this paper), we take a different route and provide suggestive evidence by looking at three distinct features characterizing the product cycle described by Vernon (1966) that should be observed in the data. First, new products are not introduced in all countries simultaneously, with the lag of introduction varying negatively with GDP per capita. Second, as the production of goods migrates from North to South, the North should start out as a net exporter of a given product, and over time become a net importer of that good. Third, production of a given good should be high in developed relative to developing countries early in the product cycle, and low later on.
In particular, we take a closer look at 6 major consumer durables (dishwasher, dryer, freezer, microwave oven, VCR, and washing machine) for which we know the year of introduction in 16 European countries (Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and United Kingdom), and which we can identify in the trade data at the 5 digit SITC level. The date of introduction for these consumer durables were kindly provided by Tellis et al. (2003)\(^2\), and the trade data are U.S. import and export data from 1972-2006, which are provided by The Center for International Data at UC Davis. Own research shows that all of these products were first introduced in the United States where the electric dishwasher was introduced in 1950 (Hobart Corp.), the automatic electric clothes dryer in 1949 (Hamilton Manufacturing Corp., GE), the domestic deep freezer in 1949 (GE), the countertop microwave oven in 1967 (Amana Corp.), the VCR in 1965 (Sony, Ampex, RCA), and the automatic electric washing machine in 1947 (Bendix, GE).

3.2 Suggestive evidence on demand lags

Table 1 shows the year of introduction (defined as the first year commercial sales for the corresponding product were registered) of the 6 consumer durables across the 16 European countries. Green numbers indicate the first year of introduction whereas red numbers indicate the last year of introduction. For example, the countertop microwave oven was first introduced in the US in 1967, and last introduced in Greece and Portugal in 1982. We observe that the year of introduction varies across countries. On average, products were first introduced in countries with a high GDP per capita like the US and UK, and last introduced in countries with a low GDP per capita like Greece, Portugal and Spain.

\(^2\)Unfortunately, we don't have data on the diffusion of the 6 consumer durables
Let us look closer at the microwave oven, which we consider a typical household appliance. In 1946, Percy Spencer, an American engineer, while working on radar technology for the U.S. defense company Raytheon Corporation accidentally discovered that microwaves are capable of heating food very quickly (the story goes that a candy bar in Spencer’s pocket melted during an experiment). Spencer realized the commercial potential (especially for a high-income market like the US) of his discovery, and the Raytheon Corporation filed for patents. In 1947, Raytheon produced the first commercial microwave oven named “Radarange” (which was sold to restaurants etc.). Twenty years later, in 1967, Amana, a division of Raytheon, introduced the first domestic countertop microwave oven, marking the beginning of the use of microwave ovens in American kitchens.

Figure 1 below shows the relationship between the demand lag in years of the countertop microwave oven relative to the US across the 16 European countries, and their GDP per capita.

<table>
<thead>
<tr>
<th>Country</th>
<th>Dishwasher</th>
<th>Dryer</th>
<th>Freezer</th>
<th>Microwave Oven</th>
<th>VCR</th>
<th>Washing Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>na</td>
<td>1966</td>
<td>na</td>
<td>1973</td>
<td>1978</td>
<td>na</td>
</tr>
</tbody>
</table>

Source: Tellis et al. (2003)
relative to the United States in 1967 (PWT 7.0 data, PPP adjusted, in 2005 USD), the year the microwave oven was introduced in the US. We observe that on average the lower a countries’ GDP per capita relative to the US in 1967, the longer the time span until households in a country start purchasing the microwave oven. For example, the Netherlands had a GDP per capita in 1967 that was about 20 percent lower than the US and households started consuming microwave ovens 4 years later than the US whereas Portugal’s GDP per capita in 1967 was about 30 percent of that in the US and households began purchasing microwave ovens 15 years later. Note that countries below the line of fit have a higher average growth rate between 1950-1990 than countries above the line of fit (calculations based on PWT 7.0). Switzerland is an extreme outlier in the sense that even though its GDP per capita 1967 was higher relative to the US, households first purchased 6 years later than households in the US.

![Lag in Introduction of Microwave Ovens](image)

**Figure 1: Relationship between GDP per capita and demand lag for the microwave oven**

In Appendix A.8, Figures 10-14 show the same graphs for the other 5 consumer durables. We observe that for all 6 consumer durables the pictures show a negative relationship between
GDP p.c. and the demand lag. Figure 2 plots for each country the average of GDP per capita relative to the US in the year of introduction across all 6 consumer durables against the average lag in years in the introduction of these consumer durables. We conclude from Figure 2 that there is a negative correlation, suggesting that on average, in countries where households enjoy a high income, these consumer durables are purchased sooner\footnote{We expected GDP per capita to be more important for product take-off (i.e. when a certain threshold of sales has been reached) than for the time of introduction. However, it seems that managers base their decisions to launch new products (and form their expectations for future performance) as much on the general level of development in a country (e.g. high average income level) as on the existence of a small group of rich people.}. Again, we notice that Switzerland is an extreme outlier that might be explained by its lacking integration into the European market (at that time) and its relatively highly regulated domestic market.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{mean_lag_of_introduction_selected_consumer_durables.png}
\caption{Mean Lag of Introduction of Selected Consumer Durables}
\end{figure}

\textit{Source: Tellis et al. (2003) and PWT 7.0}
3.3 Suggestive evidence on export performance

We now turn to the export performance of the United States in these product categories. In particular, we look at the value of US exports in product category $i$ at time $t$ relative to the value of US exports plus imports in category $i$ at time $t$.

\[
\frac{\text{exports}_{it}}{\text{exports}_{it} + \text{imports}_{it}} \in \begin{cases} [0, 0.5) & \text{US is a net importer in product category } i \text{ at time } t \\ (0.5, 1] & \text{US is a net exporter in product category } i \text{ at time } t \end{cases}
\]  

(1)

The product cycle hypothesis offers an explanation for a falling export ratio (1) in the data as the US starts out as a net exporter and becomes a net importer over time in a given product category. Again, in Figure 3 we first take a look at the export performance of the United States in the product category of microwave ovens.

![Figure 3: US export ratio in microwave ovens across 16 European countries](Source: U.S. Import and Export Data 1978-2006 http://cid.econ.ucdavis.edu)

We observe that the US starts out as a net exporter of microwave ovens at the beginning...
of the sample period in 1978 and ends up as a net importer at the end of the sample period in 2006, switching around 1983/84. A possible interpretation for the decline in the export ratio is that firms in the 16 European countries mastered the technology to produce microwave ovens, and due to lower production costs are able to compete with US firms in their home markets as well as in the US market. In other words, US firms became less competitive in their export markets and/or European firms became more competitive in the US market, such that US exports relative to U.S. imports decrease.

In Figure 4 we plot the value of U.S. exports across all 16 European countries in the 6 product categories dishwasher, dryer, freezer, microwave oven, VCR, and washing machine relative to the value of exports plus the value of imports in those product categories from the 16 European countries over the period 1978-2006.

![Figure 4: US export ratio across all 6 consumer durables and 16 European countries](http://cid.econ.ucdavis.edu/)

We see that the value of US exports relative to its imports also declines across all 6 consumer durables implying a fall in the export ratio.
3.4 Suggestive Evidence on production patterns

Eventually, for the microwave oven we have production data for the United Kingdom, the United States, South Korea, Brazil, Russia, and Argentina during the time period of 1982-2008 from the United Nations Industry Statistics. The United Nations Commodity Statistics Yearbook collects data on production of industrial commodities by country. The data is collected through annual questionnaires sent to national statistical authorities. The data reported by the United Nations Commodity Statistics Yearbook reflect volume (and value) of production sold during the survey period, which is defined as the production carried out at some time, which has been sold (invoiced) during the reference period. Unfortunately, we don’t have data on the production in China and India. However, Figure 5 plots the number of units (in thousands) produced in the countries listed above over the time period of 1982 to 2008. We note that in the developed countries like the US, the UK and the "Asian Tiger" South Korea the production was on average relatively high at the beginning of the sample period and has decreased towards the end (from 2000 onwards) whereas the production pattern in todays emerging countries like Brazil, Argentina, and Russia was relatively low at the beginning and high at the end of the sample period. Again, this is consistent with the product cycle hypothesis, which suggests that the production of microwave ovens should move from developed countries to developing countries as firms in these countries acquire the technology to produce microwave ovens and have lower production costs. In other words, we would expect to see the inverse U-shape of production in each country as observed in the data.
Figure 5: Production of microwave ovens across a sample of countries

We interpret the patterns observed in the data of these 6 consumer durables as suggestive evidence for the type of product cycles Vernon (1966) had in mind.
4 Model

4.1 Distribution and Endowments

The economy consists of two regions, an industrialized region $N$ (North), and a less developed region $S$ (South). The population size of the economy is $L$, a fraction $\beta$ lives in the South and a fraction $1 - \beta$ in the North. We assume that each household regardless of its residence inelastically supplies one unit of labor on the local labor market. This implies that aggregate labor supply in the South is given by $\beta L$, and by $(1 - \beta)L$ in the North. Furthermore, suppose that each households holds only domestic assets. Hence, income inequality is endogenously determined and originates from differences in labor and capital incomes across countries.

In order to study *ceteris paribus* effects of income inequality across countries we introduce a transfer system (e.g. foreign aid) between the North and the South so that each household in the North pays/receives a lump-sum tax/benefit $T_N(t)$, respectively $T_S(t)$. We assume that the transfer system runs a balanced budget in each period so that $(1 - \beta)L T_N(t) = \beta L T_S(t)$. We will take $T_S(t)$ as the exogenous variable so that through the balanced budget condition $T_N(t)$ is endogenously determined.

4.2 Preferences

There is a continuum of differentiated products in the economy indexed by $j \in [0, \infty)$, where only a subset $N(t)$ is available on the market at each point in time. We assume differentiated products to be indivisible, and model consumption as a binary decision. Hence, households consume either 1 unit of product $j$ at time $t$, or they don’t consume that product. Instantaneous utility is non-homothetic and takes the following form

$$u \left( \{c(j, t)\}_{j=0}^{N(t)} \right) = \int_{0}^{N(t)} c(j, t) \, dj$$  \hspace{1cm} (2)
where \( c(j, t) \) is an indicator function that takes the value one if product \( j \) is consumed at time \( t \), and zero otherwise. Note that this indicator function will be specific to the income group, i.e. the region. The specification of the instantaneous utility function contrasts with the constant-elasticity-of-substitution (CES) form. Here, households can only choose consumption along the extensive margin, i.e. choose how many different products they want to purchase, whereas with CES preferences they can only choose consumption along the intensive margin, i.e. how many units of each product they want to buy. In that sense, our preferences are no less special or general than CES preferences. For the sake of illustration, suppose that the whole product set available to households consists of the 6 consumer durables in Section 3. With the preferences specified in (2) wealthy households in the North would consume one unit of all 6 consumer durables available whereas poor households in the South could not afford to consume all goods available, and for example could only purchase one washing machine and one freezer (some of the "older" goods available). With CES preferences Northern and Southern households would both consume all 6 consumer durables available. However, Northern households would purchase e.g. 10 units of each good whereas Southern households could only buy 1 unit each. Furthermore, note that preferences (2) are symmetric, i.e. no product is intrinsically better or worse than any other product. In other words, there is no explicit consumption hierarchy. This allows us to order products in ascending order from old to new, such that product \( k \) is developed before product \( k' \), where \( k' > k \). Note that the same ordering would emerge if we assumed instantaneous utility to take the following form \( u(c(j, t)) = \int_0^{N(t)} j^{-\eta} c(j, t) dj \). The power function \( j^{-\eta} \) implies that (instantaneous) marginal utility is falling in the index \( j \), i.e. higher indexed goods yield lower marginal utility than lower indexed goods. The parameter \( \eta \in (0, 1) \) determines the “steepness” of the hierarchy, i.e. how fast marginal utility falls in index \( j \). With these preferences households start consuming low-indexed goods (as they yield higher marginal utility) and expand consumption towards high-indexed goods until their income is
used up. To keep the model simple, we will assume that such an hierarchy in consumption latently exists rather than explicitly modeling it.

The household’s intertemporal objective function is given by

$$U(0) = \int_{0}^{\infty} \exp(-\rho t) \log u \left( \left\{ c(j,t) \right\}_{j=0}^{N(t)} \right) dt$$

(3)

where $\rho > 0$ denotes the time preference rate. Note that the intertemporal preferences given by (3) are homothetic. Households maximize their lifetime utility (3) subject to the non-negativity constraint $c(j,t) \geq 0$, and to their lifetime budget constraint

$$\int_{0}^{\infty} \int_{0}^{N(t)} p(j,t)c(j,t) dj \exp(-R(t)) dt \leq a(0) + \int_{0}^{\infty} w(t) \exp(-R(t)) dt$$

where $R(t) = \int_{0}^{t} r(s) ds$ denotes the cumulative interest rate and $a(0)$ initial wealth. The solution of the household problem has been relegated to Appendix A.1. From the maximum principle conditions we derive the individual Marshallian demand functions for product $j$ as follows

$$c(j,t) = \begin{cases} 
1 & p(j,t) \leq z(j,t) \\
0 & p(j,t) > z(j,t) 
\end{cases}$$

(4)

where $z(j,t) \equiv u(\cdot)^{-1}/\lambda(t)$ denotes the willingness to pay. The costate variable, which can be interpreted as marginal utility of wealth at time $t$, is denoted by $\lambda(t)$. Households purchase one unit of a product if the price of that product does not exceed their willingness to pay. Since preferences are symmetric over all products the willingness to pay is identical for all products $j$. However, the willingness to pay depends on the costate variable $\lambda(t)$, i.e. on the shadow price of (lifetime) income. Hence, consumption patterns differ across regions since by our distributional assumptions (lifetime) incomes are different in the two regions. In particular, for wealthy households $c(j,t) = 1$ holds for more $j$’s in (4) than for poor households. In other words, wealthy households consume a larger set of products than poor households.
4.3 Technology

4.3.1 Innovation technology in the North

According to Grossman and Helpman (1991) it is a stylized fact that most new products are designed and developed in high-income countries.\textsuperscript{4} There is data available from the World Bank on research and development spending of low/middle and high income countries in percent of GDP from 2000-2007. These data confirm that high income countries on average spend about 2.5 times as much on R\&D in percent of their GDP than low and middle income countries.

Each firm in the North is a single-product firm, which has access to the same innovation technology. The creation of a new product requires $F^N(t) = F^N/N(t)$ units of labor, with $F^N > 0$. The production of 1 unit of the product requires $b^N(t) = b^N/N(t)$ units of labor, where $b^N > 0$. Note that firms in the North benefit in R\&D and production from the stock of knowledge $N(t)$ previous innovators in the North have created. The technology implies decreasing average costs and constant marginal costs in the steady state as wages will grow at the innovation rate. New products are protected by infinite patents but face a positive probability of being copied by a Southern firm (patent infringement). We assume that firms in the North cannot license technology to Southern firms, or set up manufacturing plants themselves in the South (foreign direct investment).

4.3.2 Imitation technology in the South

As in Grossman and Helpman (1991) we assume that each new product, which has been developed in the North at time $t$ faces the same positive probability of being imitated by a Southern firm at some time $T > t$. At the time the product is developed date $T$ is unknown, \textsuperscript{4}In principle, one could think that both North and South have access to the innovation technology but that the South is sufficiently unproductive at developing new products compared to the North such that in equilibrium no innovation takes place in the South.
i.e. $T$ is a random variable that represents the age of a product at the time of imitation. A Southern firm selects at random one of the existing products, which has not yet been copied, in the North for imitation. Reverse engineering\footnote{Suppose, less realistically, that reverse engineering is costless and the technology of imitated goods does become public knowledge. Furthermore, firms in the South operate under conditions of perfect competition. Then price competition between imitator and innovator would drive prices down to marginal cost of innovators $w^{N}(t)b^{N}(t)$. At that point, the innovator would drop out of the market. Price competition among Southern firms would drive prices further down to marginal costs of Southern firms. However, as long as $w^{N}(t)b^{N}(t) > w^{S}(t)b^{S}(t)$ Southern firms enter the market until, in the limit, marginal cost in the South have increased to $w^{N}(t)b^{N}(t)$. However, all results discussed in the main text go through.} of the production process of the selected product requires $F^{S}(t) = F^{S}/N(t)$ units of labor, with $F^{S} > 0$. Reverse engineering is not an uncertain activity, i.e. a Southern firm has success in learning the production process of the randomly chosen product with probability one. The production of 1 unit of a copied good requires $b^{S}(t) = b^{S}/N(t)$ units of labor. Note that firms in the South benefit in reverse engineering and production from the total stock of knowledge (i.e. there are international knowledge spillovers).

4.4 Equilibrium

Two (decentralized) equilibria can emerge, (i) households in the South are too poor to afford any Northern products or (ii) they can afford at least some Northern products. In case (i) no trade equilibrium exists. Hence, we focus on case (ii) and assume in the following that households in the South can afford some Northern products.

Let us denote the set of all products available in the economy as $N(t) = N^{N}(t) + N^{S}(t)$, where $N^{N}(t)$ denotes the subset of products that have not yet been imitated by the South, and $N^{S}(t)$ the subset of products that have been copied by the South.

We further assume that no trade costs occur if products are shipped between regions.
4.4.1 Aggregate demand

In the equilibrium we consider, households in the North consume all products available in the market $N_N(t) = N(t)$, whereas households in the South consume only a subset of all products $N_S(t) \subset N(t)$, which includes all products manufactured in the South and some but not all Northern products.

Aggregate demand for product $j$ can be derived by horizontally aggregating individual demand (4) across regions. It is given by

$$C(j,t) = \begin{cases} 0, & p(j,t) > z_N(j,t) \\ (1 - \beta) L, & z_S(j,t) < p(j,t) \leq z_N(j,t) \\ L, & p(j,t) \leq z_S(j,t) \end{cases}$$

(5)

where $z_i(j,t)$, where $i \in \{N,S\}$, denotes the willingness to pay of households in the North, respectively, the South. Since the willingness to pay is the same for all products $j$, aggregate demand is the same for all products. Aggregate demand is also depicted in Figure 6 below.

![Figure 6: Aggregate Demand](image-url)
We observe that if the price of a product exceeds the willingness to pay of Northern households there is no demand for that product. If the price lies between the willingness to pay of Southern and Northern households only the latter purchase the product, whereas if the price does not exceed the willingness to pay of households in the South everyone purchases the product. Note that in Figure 6 we assumed that the willingness to pay of Southern households does not fall short of marginal costs $b^N(t)w^N(t)$, otherwise no firm in the North would ever export to the South. We further assume that the marginal cost in the North exceed those in the South. This imposes the restriction that the endogenously determined relative wage rate $w^N(t)/w^S(t)$ cannot fall short of relative labor productivities $b^S/b^N$ in the production sector.\(^6\)

### 4.4.2 Aggregate supply

Let us first consider the problem of a monopolistic firm $j$ located in the North. Firm $j$ maximizes operating profits

$$\pi^N(j,t) = [p(j,t) - w^N(t)b^N(t)] C(j,t)$$

subject to aggregate demand (5) by choosing a price $p(j,t)$ such that marginal revenue equals marginal cost. From Figure 6 and the discussion in the previous section it follows that there are two candidates for the price that maximizes profits (6). Firm $j$ could either choose to set a high price equal to the willingness to pay of Northern households $z^N(j,t)$ and sell exclusively to domestic households, or it could decide to set a low price equal to the willingness to pay of Southern households $z^S(j,t)$ and serve the domestic market as well as export to the South.

---

\(^6\)Note that if $b^N = b^S$ this implies that $w^N(t) > w^S(t)$ must hold. In the equilibrium we consider this will hold endogenously for the following reason. Relative wages between North and South are determined by relative labor demand. Due to the fixed labor requirement to develop a new product the per-unit labor input in a Northern good exceeds the per-unit labor input in a Southern good. Northern labor is *ceteris paribus* scarce relative to Southern labor and therefore remunerated at a higher rate.
We assume that firms cannot price discriminate across regions. In other words, export firms cannot set a low price \( z_S(j, t) \) in the South and a high price \( z_N(j, t) \) in the North, else there would be arbitrageurs taking advantage of the price differential. That restricts export firms to set the same price in both regions.

In equilibrium, not all Northern firms will export. Suppose that at every point in time all firms in the North would set prices equal to the willingness to pay of Southern households and sell to everyone. In that case, households in the North would not exhaust their budgets, i.e. the shadow price of their (lifetime) income would become zero. That would imply an infinitely large willingness to pay for an additional product. The firm developing that additional product could earn infinite profits and therefore had an incentive to deviate from selling to everyone and sell only in the North. Hence, a situation where all Northern firms serve all households cannot be an equilibrium. Also, by the same argument, a situation where all Northern firms sell exclusively to Northern households cannot be an equilibrium as the willingness to pay of Southern households for a Northern product is infinitely large.

In an equilibrium where some Northern firms serve all households in both regions, and others serve exclusively the domestic region, firms must be indifferent between selling only to Northern households and selling to all households at any point in time. Hence, the following arbitrage condition must hold

\[
\left[ z_N(j, t) - w^N(t)b^N(t) \right] (1 - \beta) L = \left[ z_S(j, t) - w^N(t)b^N(t) \right] L. 
\]

On the aggregate level there is at any point in time a set of firms that sells in the North and South whereas the complement of firms sells only in the North. However, the behavior of a single firm is indeterminate. We think of following the firm behavior at the micro level that generates the described outcome at the macro level. Suppose, after developing a new product each firm starts to market its product in the domestic market and only after a certain period
of time has elapsed (i.e. the time it takes for incomes in the South have grown sufficiently) begins exporting. In that case, there are at any point in time, new products that are sold exclusively in the domestic market, and older products that are exported as well.

Let us denote the time period until incomes of Southern households have increased so much that they can afford to buy Northern products by \( \Delta \), and refer to it from now on as the demand lag. Remember that we are free to order products from oldest to newest, in particular, we index products in such a way that a low index corresponds to an old product (which has been developed some time ago), and a high index to a new product, such that \( N(t) \) denotes the most recently developed product. The demand lag is then endogenously determined by \( N(t) = N_S(t + \Delta) \), where \( N_S(t) \) denotes the latest Northern good consumed by households in the South.

The Northern firm which develops product \( j \) at time \( t \) faces a positive probability that its product will be copied by a Southern firm at some time \( T > t \). Until time \( T \), which is unknown to the Northern firm, product \( j \) is supplied by the Northern firm. From time \( t \) up to time \( t + \Delta < T \) the firm supplies its product exclusively to the domestic market at the profit-maximizing price \( z_N(j, t) \). and from time \( t + \Delta \) until time \( T \) it continues to sell in the domestic market and additionally starts to export to the South at the profit maximizing price \( z_S(j, t) \).

After time \( T \) product \( j \) is copied a Southern firm. This firm maximizes operating profits

\[
\pi^S(j, t) = [p(j, t) - w^S(t)b^S(t)] C(j, t)
\]

where \( C(j, t) = L \) is given by (5). After the firm in the South has copied the Northern product \( j \) it enters into a price competition with the Northern firm currently producing \( j \) (the innovating firm). This forces the Southern firm to set a limit price equal to the marginal costs of the competing firm in the North. Hence, optimal prices of Southern products are
equal to \( w^N(t) b^N(t) \). The wide-gap case discussed in Grossman and Helpman (1991) where Southern firms can set the monopoly price can not occur here since we assume that \( z^S(t) > w^N(t) b^N(t) > w^S(t) b^S(t) \). Otherwise, no firm in the North would export to the South, and no trade equilibrium exists. Our case is similar to their narrow-gap case where Southern firms charge prices marginally below the marginal cost of Northern firms.

To sum up

\[
p(j, t) = \begin{cases} 
  z^N(j, t) = 1/\lambda^N(t)N(t) & N^S(t) < j \leq N(t) \\
  z^S(j, t) = 1/\lambda^S(t)N^S(t) & N^S(t) < j \leq N^S(t) \\
  w^N(t) b^N(t) & 0 < j \leq N^S(t) 
\end{cases}
\] (9)

Prices evolve over time as follows

\[
\frac{\dot{p}(j, t)}{p(j, t)} = \begin{cases} 
  r^N(t) - \rho - \dot{N}(t)/N(t) & N^S(t) < j \leq N(t) \\
  r^S(t) - \rho - \dot{N}^S(t)/N^S(t) & N^S(t) < j \leq N^S(t) \\
  w^N(t)/w^N(t) - \dot{N}(t)/N(t) & 0 < j \leq N^S(t) 
\end{cases}
\] (10)

4.4.3 Labor markets

Labor is immobile across regions but regional labor markets are assumed to be perfect, in particular, in the North labor is completely mobile between production and R&D.

Labor market clearing in the North demands that

\[
(1 - \beta) L = \dot{N}(t) F^N(t) + \int_{N^S(t)}^{N^S(t)} b^N(t) C(j, t) \, dj + \int_{N^S(t)}^{N(t)} b^N(t) C(j, t) \, dj \\
(1 - \beta) L = \frac{\dot{N}(t)}{N(t)} F^N(t) + \frac{b^N L}{N(t)} [N^S(t) - N^S(t)] + \frac{(1 - \beta) b^N L}{N(t)} [N(t) - N^S(t)] 
\] (11)

The first term in (11) on the right-hand side denotes labor demand from the R&D sector, the second term labor demand from the production of older Northern products consumed by all households in both regions, and the third term labor demand from the production of newer Northern products only Northern households consume.
Labor market clearing in the South requires

\[ \beta L = \dot{N}^S(t)F^S(t) + \int_0^{N^S(t)} b^S(t)C(j, t) \, dj \]

\[ \beta L = \frac{\dot{N}^S(t)}{N(t)} F^S + \frac{b^S L}{N(t)} N^S(t) \]  

(12)

The right-hand side in (12) denotes labor demand from reverse engineering and production of imitated products, which are consumed by all households in both regions.

### 4.4.4 Capital markets

We assume that regional capital markets are perfect but capital is immobile across regions (i.e. Southern households cannot buy shares in Northern firms, and vice versa). In particular, the North has perfect capital markets (including a stock market) such that households can diversify away the idiosyncratic risk of a single firm in the North of being copied by holding a portfolio of shares in all Northern firms (i.e. there is no aggregate risk). Since households can save by either buying riskless (consumption) bonds or shares in monopolistic firms, no arbitrage ensures that the riskless return on bonds equals the return to a portfolio of Northern firms.

At time \( t \) when product \( j \) is developed time \( T > t \) when it is imitated is unknown. The instantaneous rate of imitation (hazard rate) is given by \( \mu(t) \equiv \dot{N}^S(t)/N^N(t) \), i.e. the rate at which product \( j \) is imitated at time \( t \), given it has not been imitated before time \( t \). For a detailed definition of the hazard rate the reader is referred to Appendix A.2.

We can calculate the present discounted value of product \( j \), which has been developed at time \( t \), and is imitated at time \( T \) (at which point in time the profit flow stops) as follows

\[ \Pi^N(j, t, T) = \int_t^T \exp \left( -\left( R^N(s) - R^N(t) \right) \right) \pi^N(j, s) \, ds \]

We can write the expected present discounted value of profits of product \( j \) that was intro-
duced at time $t$ as

$$v^N(j,t) = \int_t^\infty \Pi^N(j,t,\tau) f_T(\tau) \, d\tau$$

where $f_T(\tau)$ denotes the probability density function (pdf) of $T$.

Differentiating $v^N(j,t)$ with respect to time $t$, and rearranging terms yields the following standard no arbitrage condition in the capital market of the North

$$\frac{\pi^N(j,t)}{v^N(j,t)} + \frac{\dot{v}^N(j,t)}{v^N(j,t)} = r^N(t) + \mu(t)$$

(13)

stating that the inverse of the price earning ratio, i.e. $\pi^N(j,t)/v^N(j,t)$, plus the rate of capital gains (or losses) on share holdings, i.e. $\dot{v}^N(j,t)/v^N(j,t)$, equals the risk-adjusted interest rate, i.e. $r^N(t) + \mu(t)$ (the risk premium is equal to the rate of imitation).

We further assume free entry into product development in the North. Hence, the expected value of product $j$ must equal its development costs in equilibrium

$$v^N(j,t) = w^N(t) F(t)$$

(14)

Note that since reverse engineering in the South is not an uncertain activity, the present discounted value of profits at time $t$ is given by

$$v^S(j,t) = \int_t^\infty \exp \left( - \left( R^N(s) - R^N(t) \right) \right) \pi^S(j,\tau) \, d\tau$$

(15)

The no arbitrage condition in the South’s capital market is given by

$$\frac{\pi^S(j,t)}{v^S(j,t)} + \frac{\dot{v}^S(j,t)}{v^S(j,t)} = r^S(t)$$

(16)

Free entry into reverse engineering in the South implies that the present discounted value of profits must equal the cost of reverse engineering in equilibrium

$$v^S(j,t) = w^S(t) F^S(t)$$

(17)
4.4.5 Balance of payments

Suppose that the balance of payments is balanced period by period. Since we assume that capital is immobile, the current account must also be balanced in every period. In other words, we require that the sum of the balance of trade plus net transfer payments is zero.

\[
\left\{ [N_S(t) - N^S(t)] z_S(t) \beta L - N^S(t) w^N(t) b^N(t) (1 - \beta) L \right\} - \beta T_S(t) = 0
\]  

(18)

where the first term in brackets on the left-hand side denotes the balance of trade and the second term net transfer payments. Note that if \( T_S(t) > 0 \) for all \( t \), the South runs a permanent trade deficit, i.e. the value of its exports falls short of the value of its imports.

4.5 Steady state

We define the steady state as follows

**Definition 1.** A steady state is a state where Northern firms introduce new products at a constant rate \( g \), and Southern firms imitate at a constant rate \( \mu \). Shares of resources devoted to R&D and production are constant, and the fraction of Northern products that have not yet been imitated, i.e. \( \zeta^N(t) \equiv N^N(t)/N(t) \), is constant. Furthermore, prices of Northern and Southern products, and therefore, profits of Northern firms are constant.

Let us choose the marginal costs of production of Northern firms as the numeraire (in other words, we measure everything in terms of Northern production costs), i.e. set \( w^N(t) b^N(t) = 1 \) for all \( t \).

First, we turn to the equilibrium in the labor markets. The resource constraint in the South (12) becomes

\[
\beta L = g \zeta^S F^S + \zeta^S b^S L
\]  

(19)

where \( \zeta^S \equiv N^S(t)/N(t) \) denotes the fraction of products that have been imitated. A higher \( \zeta^S \) implies *ceteris paribus* that there is more imitation activity in the South so that on average
Northern products are copied sooner. This tends to depress innovation activity in the North implying a lower $g$. Note that if firms in the South would benefit only from intertemporal knowledge spillovers in the stock of knowledge $N^S(t)$ that has been created by past imitators the innovation rate $g = (\beta - b^S) \frac{L}{F^S}$ would be determined by the South’s resource constraint.

The resource constraint of the North (11) can be written as follows in the steady state

\[
(1 - \beta) L = gF^N + Lb^N (n - \zeta) + (1 - \beta) Lb^N (1 - n)
\]

(20)

where $n(t) \equiv N_S(t)/N(t)$ denotes the "consumption gap" between South and North. Note that ceteris paribus a higher share of the South in total production $\zeta$ releases resources from the production sector in the North that can be reallocated to the R&D sector. This allows the North to introduce new products at a higher rate $g$. Furthermore, a higher consumption share of the South $n$ induces ceteris paribus a reallocation from the R&D sector to the production sector in the North (since $\beta < 1$) to satisfy the additional demand for existing Northern products by the South, thereby depressing innovation in the North.

Next, a fixed inter-sectoral allocation of labor implies that prices of Northern goods must be constant in the steady state. We denote the price of a new product that is sold exclusively to Northern households as $z_N$, which is identical for all new products $j \in (N_S(t), N(t)]$ since all firms face the same demand curve and have the same cost structure. From the arbitrage condition (7) for monopolistic firms follows that prices for all old Northern goods $j \in (N^S(t), N_S(t)]$, which are sold to all households, are also constant and determined by $z_S = \beta + (1 - \beta) z_N$. Moreover, this implies that profits are constant over time.

From the evolution of prices (10) it follows that $g = r^N(t) - \rho$, and $g = r^S(t) - \rho$, which implies that interest rates are constant in the steady state and equalize across regions, i.e. $r = r^N = r^S$. Hence, the optimal evolution of consumption of Northern and Southern households,
i.e. the Euler equation, in steady state is given by

\[ g = r - \rho \]  

(21)

Prices of Southern products \( w^N(t)b^N(t) \) are equal to 1 due to our choice of numeraire and therefore constant in steady state as well (else the demand for Southern labor would change over time.

Let us consider the average life cycle in the steady state of some product \( j \), which is introduced at time \( t \). At the time of introduction product \( j \) is sold at price \( z_N \) exclusively to Northern households. At time \( t + \Delta \) the Northern firm manufacturing product \( j \) lowers the price to \( \beta + (1 - \beta)z_N \) and exports it to the South. At time \( T > t + \Delta \) product \( j \) is imitated by a Southern firm. Due to lower production costs in the South, Southern firms can set a price marginally below 1, the marginal costs of Northern firms. Hence, the Northern firm stops producing product \( j \) and the product is now exported to the North.

For the average product cycle described above we look at the case where the average lifetime \( \mu^{-1} \) of product \( j \) (see Appendix A.3 for a derivation), i.e. the time until a Northern product is copied, exceeds the demand lag \( \Delta \). In the steady state, the demand lag \( \Delta \) is determined by \( N_S(t) \exp(g\Delta) = N(t) \), remember \( N(t) \) denotes the latest good developed in the North and \( N_S(t) \) the newest Northern good consumed by Southern households. It follows that \( \Delta = -\log n/g > 0 \), where \( n \equiv N_S(t)/N(t) < 1 \) denotes the consumption share of Southern households. The demand lag is decreasing in the consumption share \( n \) and the innovation rate \( g \). However, there will be some products that are imitated before households in the South become rich enough to afford them; they will skip the export stage. The average life cycle of product \( j \) in terms of sales volume is depicted below in Figure 7.

\[ \text{Note that in the other case(s) where } \mu^{-1} \leq \Delta \text{ goods would on average skip the export stage. This implies that in the steady state trade would not be balanced, i.e. the North would run a permanent trade deficit. This cannot be an equilibrium outcome in this model.} \]
From the definition of the hazard rate $\mu = \dot{N}^S(t)/N^N(t)$ we can express the production share of the South in the total number of differentiated products as

$$\zeta^S = \frac{\mu}{g + \mu}$$

(22)

which must be constant in the steady state.

Next, the no arbitrage condition in capital markets (13) and the zero-profit condition (14) in the North imply that in the steady state the value of a firm is equal to the expected present discounted value of its future profits. Together with the no arbitrage condition (7) and the choice of our numeraire $w^N(t)b^N(t) = 1$ we obtain

$$\frac{[z_N - 1](1 - \beta)L}{r + \mu} = \frac{F^N}{b^N}$$

(23)

In the South, the no arbitrage condition (16) together with the zero-profit condition (17) yield

$$\frac{[1 - \omega^S b^S]L}{r} = \omega^S F^S$$

(24)
where \( \omega^S(t) \equiv w^S(t)/N(t) \) is constant since wages in the South grow at rate \( g \).

Last, in steady state, the balance of payments (18) becomes

\[
(n - \zeta^S) \left[ \beta + (1 - \beta)z_N \right] \beta = \zeta^S(1 - \beta) + \beta T
\]  

(25)

where \( T \equiv T_S(t)/N(t) \). For a derivation of the balance of payments in the steady state see Appendix A.6. Note that due to Walras’ law the balance of payments is implied by the budget constraints, the zero-profit conditions and the resource constraints.

The 7 equations (19)-(25) in the 7 unknowns \( g, \mu, n, \zeta^S, r, z_N, \) and \( \omega^S \) fully characterize the steady state.

We can reduce the 7 equations to 2 equations in 2 unknowns \( g \) and \( \zeta^S \). The first equation describes steady state relationships between \( g \) and \( \zeta^S \) that are consistent with labor market clearing in the South:

\[
g = \frac{\beta - \zeta^S b^S}{\zeta^S F^S / L}
\]  

(26)

The second equation describes steady state relationship between \( g \) and \( \zeta^S \) that are consistent with labor market clearing in the North, balance of payments and the no arbitrage condition

\[
\beta T + \zeta^S(1 - \beta) = \frac{1 - \frac{F_N}{L}g - \beta + b^N(1 - \beta)(1 - \zeta^S)(1 - \zeta^S) + \frac{F_N}{L}(g + \rho(1 - \zeta^S)))}{(b^N)^2(1 - \zeta^S)}
\]  

(27)
5 Discussion

In this section we discuss the effect of the demand side on the product cycle as well as the incentives to innovate and imitate in detail. Furthermore, we explore the implications of the South becoming more productive in the production of imitated products. Last, we look into the channel of a lower savings rate. Despite the simplicity of the model we will see that in some cases the comparative static results are ambiguous. It turns out that it is difficult to sign the effects analytically. Hence, instead of proving our results we will appeal to simulations. We choose the following parameter configuration for our baseline simulation: $L = 1$, $F_N = F_S = 5$, $b^N = b^S = 0.75$, $\beta = 0.5$, $\rho = 0.04$, and $T = 0$. Note that the figures depicting the simulation results have been relegated to Appendix A.9.
5.1 Increase in Southern labor productivity

Result 1. In a steady state with high Southern labor productivity the innovation rate $g$ and the consumption share of Southern households $n$ are high relative to a steady state with low Southern labor productivity. Relative wages $w^N(t)/w^S(t)$ are lower whereas the terms of trade for the North $[\beta + (1 - \beta)z_N] / w^N(t)b^N(t)$ are higher. The first (exclusive) and second (North’s export) stage of the product cycle are shorter whereas the third (North’s import) stage is longer.

Suppose that labor in production or reverse engineering in the South becomes more productive at every point in time $t$, i.e. $b^S$ or $F^P$ falls.

This implies that ceteris paribus reverse engineering or the production of imitated goods becomes cheaper. Hence, the value of an imitation relative to an innovation increases. This implies that relatively more imitation takes place, i.e. $\mu$ increases, which leads to a higher risk-adjusted interest rate and hence to a lower present discounted value of profits earned from innovation. This tends to diminish the incentives to innovate. At the same time, households in the South become relatively richer and can afford to purchase more (and newer) products so that $n$ rises. Hence, the (marginal) willingness to pay of Southern households increases. From the arbitrage condition of Northern firms follows that the (marginal) willingness to pay of Northern households must increase as well in equilibrium. This implies that profits from developing new products, and therefore the incentives to innovate increase. In our simulations the positive effect on innovation dominates the negative effect so that the innovation rate $g$ rises. It turns out that the the fraction of imitated goods $\zeta^S$ increases, i.e. the imitation rate $\mu$ increases more than the innovation rate $g$.

Since imitation activity in the South increases relative to innovation activity in the North, the demand for Southern labor increases relative to the demand for Northern labor. Hence, the relative wage rate $w^N(t)/w^S(t)$ decreases. Because the willingness to pay of Southern
households increases for a given product, the North’s export prices, \( z_S = \beta + (1 - \beta) z_N \), increase whereas the North’s import prices, \( w^N(t)b^N(t) = 1 \), are constant (due to our choice of the numeraire). Therefore, the terms of trade move in favor of the North.

The first stage of the product cycle where products are exclusively sold in the North becomes shorter. In other words, the demand lag \( \Delta \) becomes shorter because first, households in the South are relatively richer (effect of higher \( n \)), and second, the income of Southern firms grows at a higher rate (effect through higher \( g \)), which both induces the firm in the North that produces the latest good to export sooner to the South. Since the imitation rate \( \mu \) increases, the average time span a product in manufactured in the North \( 1/\mu \) becomes shorter, which increases the third stage of the product cycle where goods are imported by the North. In our simulations, the second stage where products are exported by the North \( (1/\mu - \Delta) \) becomes shorter, implying that the average time span of production in the North \( 1/\mu \) falls more than the demand lag \( \Delta \).

5.2 Changes in inequality across regions

**Result 2.** Consider a transfer from poor South to rich North. In a steady state with high inequality relative to one with low inequality the growth rate \( g \) is high and the income share of the South \( n \) low. The wage rate in the North relative to the South and the North’s terms of trade are higher. The first (exclusive) stage and the third (North’s import) stage of the product cycle are longer whereas the second (North’s export) stage is shorter.

The intuition behind the result above is the following. A regressive transfer leads to higher incomes in the North and lower incomes in the South, *ceteris paribus*. Inequality measured by the Gini coefficient increases.\(^8\) Higher incomes in the North imply that the (marginal) willingness to pay for a new product is higher, holding the number of products consumed by

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\(^8\)For a derivation of the Gini coefficient see Appendix A.5.
Northern households \( N(t) \) constant. Hence, the incentives to develop a new product in the North rise, which increases the innovation rate \( g \). At the same time, lower incomes in the South translate \textit{ceteris paribus} into a lower (marginal) willingness to pay for older products produced in the North. This implies that profits of innovating firms in the North from selling only to Northern households exceed profits from selling to all households creating a disequilibrium in the North (arbitrage condition is violated). This induces some Northern firms (in particular, the ones with newer products) to stop exporting. As Southern households consume less and less products, i.e. \( N_S(t) \) falls, their marginal willingness to pay \textit{ceteris paribus} increases until the equilibrium in the North is restored. In the new equilibrium, households in the South consume a lower fraction of all products \( n \), and their (marginal) willingness to pay is higher.

Hence, the North’s export prices \( \beta + (1 - \beta)z_N \) increase, and as the North’s import prices \( w^N(t)b^N(t) = 1 \) are constant (due to our choice of numeraire), the terms of trade move in favor of the North.

The same argument as above can also be made as follows. A higher innovation rate \( g \) \textit{ceteris paribus} absorbs more resources in the R&D sector of the North. This implies that labor demand from the production sector in the North must be lower. On the one hand, lower incomes in the South imply that the consumption share of Southern households \( n \) lower, which tends to lower the labor demand from the production sector in the North. On the other hand, a higher innovation rate implies a lower fraction of imitated goods \( \zeta^S \), holding the imitation rate \( \mu \) constant for the moment. This indicates that the labor demand from the production sector in the North is higher. However, at the same time, a lower \( \zeta^S \) \textit{ceteris paribus} suggests that in the South more resources are available for reverse engineering. Hence, the imitation rate \( \mu \) tends to be higher, which implies that \( \zeta^S \) increases. It turns out that in the new steady state, the innovation rate \( g \) increases more than the imitation rate \( \mu \) so that \( \zeta^S \) is lower.

Higher innovation activity in the North implies that Northern wages \( w^N(t) \) increase rel-
ative to Southern wages $w^S(t)$, *ceteris paribus*. This makes reverse engineering in the South (relatively) cheaper, which tends to increase the imitation activity. At the same time, a higher innovation rate $g$ implies a higher interest rate, and therefore a lower present discounted value of profits accruing from imitation. This tends to depress imitation activity in the South, i.e. lower the copy rate $\mu$ (holding everything else constant). In equilibrium, the first effect dominates so that the imitation rate $\mu$ is higher in the new steady state.

The first stage of the product cycle where new products are exclusively sold in the North becomes longer, i.e. $\Delta$ is longer. There are two opposing effects on the demand lag. On the one hand, households in the South are poorer so that the Northern firm producing the latest product would like to export later (effect of higher $n$). On the other hand, even though the level of income for Southern households is lower, their income grows at a higher rate. This induces the Northern producer of the latest product to export sooner (effect of higher $g$). In our simulations, the first effect dominates so that the first product cycle stage becomes longer. As the imitation rate $\mu$ becomes higher, the average time span a product is manufactured in the North $1/\mu$ becomes shorter so that the third stage during which the North imports a product increases. Since the first stage becomes longer, the time period during which the North exports a product $(1/\mu - \Delta)$ becomes shorter.

5.3 An increase in the time preference rate

**Result 3.** In a steady state with a low time preference rate $\rho$ compared to one with a high time preference rate the innovation rate $g$ and the consumption share of the South $n$ are low. Relative wages North to South and the terms of trade for the North are higher. The first (exclusive) and third (North’s import) stage of the product cycle are longer whereas the second (North’s export) stage is shorter.

Consider the following change in the time preference rate. Suppose that households in both
regions become more impatient, i.e. $\rho$ increases.

There are two effects on the value of an innovation. First, a higher $\rho$ implies that *ceteris paribus* the interest rate must increase to equate savings with investments. This means that future profits are more heavily discounted so that the present discounted value of profits decreases. This is a standard channel. Second, if households are more impatient they have a higher willingness to pay (today relative to tomorrow) for new products. Hence, prices and markups rise *ceteris paribus*, increasing the value of an innovation. This channel does not exist in the standard model since markups are constant. In our simulations the latter effect dominates so that the innovation rate $g$ increases. As $\rho$ increases the value of an innovation increases relative to the value of an imitation (i.e. increases inequality across regions). Hence, households in the South can afford to buy less products so that $n$ decreases.

Relatively more innovation activity in the North implies that labor demand North relative to South has increased (note that therefore $\zeta^S$ decreases). Hence, relative wages $w^N(t)/w^S(t)$ are higher. As households in the South cannot afford to consume as many products as before their (marginal) willingness to pay for a given good is now higher. Hence, the terms of trade move in favor of the North.

The intuition behind the changes in the duration of the different stages of the product cycle is the same as in the previous section.

6 Conclusion

Vernon’s (1966) celebrated product cycle theory hypothesizes that new products go through the following stages. In the first stage, new products are developed and first introduced in high-income countries. Later in the cycle, incomes in the poorer countries have grown sufficiently such that demand for these products appears there, and the products, which were only consumed in high-income countries before, are exported to these countries. In the third
stage, production moves from high-income countries to low-income countries because they have learned the technology to produce these goods (or there is foreign direct investment) and are able to produce at lower costs.

In this paper we provide suggestive evidence for the different stages of the product cycle hypothesis. We show that 6 major consumer durables seem to have gone (or still go) through a "typical" product cycle. In particular, we show that new products are not introduced simultaneously across countries, and that the lag in introduction depends negatively on relative GDP per capita (i.e., relative to the first country where a product is introduced). In other words, new products are introduced in wealthy countries before they are introduced in less prosperous countries.

Our contribution to the literature is to build a dynamic general-equilibrium model that is able to generate the product cycle described by Vernon (1966). In this model, a wealthy North develops new products which a poor South randomly attempts to copy. Besides technology, the incentives to innovate (and imitate) are determined by the distribution of income across regions. In other words, the demand side is an important determinant of the product cycle stages. Aside from simulating changes in Southern labor productivity and changes in the time preference rate, we simulate the effects of a redistribution of income between North and South such that inequality across regions decreases. We show that a decrease in inequality across regions leads to a decline in the innovation rate and hence a slowdown of imitation activity in the South (for a given share of the South in total production). Since Southern households are wealthier after the redistribution of income they can afford to purchase a higher share of goods available in the world market (in particular, they can afford more newer goods produced in the North). The first stage of the cycle becomes shorter since Southern households are wealthier (even though their incomes grow at a lower rate) firms in the North want to export their products sooner. At the same time the average duration new products are manufactured
in the North increases because imitation activity in the South has slowed down. Hence, the second stage of the product cycle where new goods are exported by the North to the South becomes longer. The increase in the average duration of manufacturing in the North implies that firms in the South master the technology to produce a good later so that it takes longer for the production to move there (because of the cost advantage). Therefore, the third stage of the cycle where the production moves to the South and is standardized becomes shorter.

Future work could for example incorporate foreign direct investment by Northern firms or inequality within the North and/or South.
References


A Appendix

A.1 Household problem

Households maximize logarithmic intertemporal utility, where consumption \( c(j,t) \) is its control, and asset holdings \( a(t) \) its (endogenous) state variable

\[
\max_{\{c(j,t)\}_{t=0}^{\infty}} U(0) = \int_{0}^{\infty} \exp(-\rho t) \log u \left( \{ c(j,t) \}_{j=0}^{N(t)} \right) dt
\]

where \( \rho > 0 \) denotes the time preference rate, subject to the non-negativity constraint \( c(j,t) \geq 0 \), and the flow budget constraint

\[
\dot{a}(t) = r(t)a(t) + w(t) - e(t)
\]

with \( a(0) \geq 0 \), \( c(j,t) \in \{0,1\} \), and \( e(t) = \int_{0}^{N(t)} p(j,t)c(j,t) dj \). Furthermore, households face a no-Ponzi game condition of the following form

\[
\lim_{t \to \infty} \exp(-R(t)) a(t) = 0
\]

where \( R(t) = \int_{0}^{t} r(s) ds \) denotes the cumulative interest rate. Households take the time paths of the interest rate, the wage rate, prices for all goods \( j \), as well as the set of differentiated products in the economy \( \{ r(t), w(t), p(j,t), N(t) \}_{t=0}^{\infty} \) as given.

The current value Hamiltonian is given by

\[
H(t, c(j), a, \lambda, \mu) = \log u(\cdot) + \lambda(t) [r(t)a(t) + w(t) - e(t)] + \mu(t)c(j,t)
\]

where \( \lambda(t) \) denotes the costate variable on the flow budget constraint and \( \mu(t) \) the one on the non-negativity constraint. The maximum principle conditions are

\[
\max_{\{c(j,t)\}_{j=0}^{N(t)}} H(t, c(j), a, \lambda, \mu) \text{ for all } t \in [0, \infty], j \in [0, N(t)] :
\]

\[
u(\cdot)^{-1} - \lambda(t)p(j,t) = \begin{cases} 0, & c(j,t) = 1 \\ 0, & c(j,t) = 0 \end{cases}
\]

\[
u(\cdot)^{-1} - \lambda(t)p(j,t) \leq 0, \quad c(j,t) = 0
\]
\[
\lambda(t) r(t) = -\dot{\lambda}(t) + \rho \lambda(t)
\]
\[
\dot{a}(t) = r(t) a(t) + w(t) - e(t)
\]
\[
\lim_{t \to \infty} \exp(-\rho t) \lambda(t) a(t) = 0
\]

### A.2 Hazard rate

The instantaneous rate of imitation (hazard rate) is given by \( \mu(t) \equiv \dot{N}^S(t)/N^N(t) \), i.e. the rate at which product \( j \) is imitated at time \( t \), given it has not been imitated before time \( t \).

The imitation rate in period \( t \) is equal to the number of products copied by Southern firms in period \( t \), \( \dot{N}^S(t) \), relative to the number of products that are not imitated in period \( t \), \( N^N(t) \).

By the definition of the hazard rate (the instantaneous rate of imitation for firms, that have not been imitated until time \( t \), during the next instant of time \( dt \)) follows

\[
\mu(t) \equiv \lim_{dt \to 0} \frac{\text{Prob}(t \leq T < t + dt | T > t)}{dt} = \frac{1}{\text{Prob}(T > t)} \lim_{dt \to 0} \frac{\text{Prob}(t \leq T < t + dt)}{dt} = \frac{f_T(t)}{S_T(t)}
\]

where \( S_T(t) = \text{Prob}(T > t) = 1 - F_T(t) \) denotes the survival rate. Hence, the hazard rate can be written as

\[
\mu(t) = -\frac{d \log S_T(t)}{dt}
\]

Integrating on both sides from 0 to \( t \) yields

\[
\int_0^t \mu(u) du = -\int_0^t \frac{d \log S_T(u)}{du} du = -\log S_T(t) + \log S_T(0) \Leftrightarrow \\
\exp\left(-\int_0^t \mu(u) du\right) = S_T(t)
\]

since \( S_T(0) = 1 - F_T(0) = 1 \). Hence, we get

\[
F_T(t) = \text{Prob}(T \leq t) = 1 - S_T(t) = 1 - \exp\left(-\int_0^t \mu(u) du\right).
\]
In the text, date $T$, representing imitation of product $j$ at age $T$, is a random variable with support $[t, \infty]$ it follows that the probability of product $j$, which was introduced at time $t$, and is imitated at some time $t < T \leq \tau$ is given by

$$F_T(\tau) = \text{Prob}(T \leq \tau) = 1 - S_T(\tau) = 1 - \exp \left( - \int_t^\tau \mu(u) du \right).$$

Note that we assume that immediate imitation is not possible, i.e. survival in period $t$, when the product is introduced, is certain. Hence, $S_T(t) = \text{Prob}(T > t) = 1$, which is a standard assumption in survival analysis. The probability density function of $T$ is given by

$$f_T(\tau) = \frac{dF_T(\tau)}{d\tau} = \mu(\tau)S_T(\tau) = \mu(\tau) \exp \left( - \int_t^\tau \mu(u) du \right).$$

We observe that the hazard rate $\mu(\tau)$ fully characterizes the distribution of $T$.

### A.3 Average lifetime of a Northern product

The average lifetime of product $j$ that is introduced at time 0 can be calculated as

$$= \frac{1}{1 - F_T(0)} \int_0^\infty t f_T(t) \, dt$$

$$= \frac{1}{S_T(0)} \int_0^\infty t \mu S_T(t) \, dt$$

$$= \frac{1}{S_T(0)} \int_0^\infty t \mu \exp(-\mu t) \, dt$$

$$= \frac{1}{\mu}$$

We note that in the steady state the age of a product, $T$, follows an exponential distribution according to the cumulative distribution function $F_T(\tau) = 1 - \exp(-\mu(\tau - t))$. Hence, we recognize $\mu^{-1}$ as the mean of the exponential distribution, which implies that the higher the rate of imitation $\mu$ (hazard rate) the shorter the expected lifetime of a particular product. The assumption that the expected lifetime of a product exceeds the duration until the South starts consuming it becomes $\mu^{-1} > -\log n/g$. 

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A.4 Budget constraints

The intertemporal budget constraint of households in the North is in the steady state given by

\[ N(t) \left\{ \zeta^N + (n - \zeta^N) [\beta + (1 - \beta) z_N] + (1 - n) z_N \right\} = (r - g)a_N(t) + w^N(t) - T_N(t) \]

where \( y_N(t) = a_N(t) + w^N(t)/(r - g) - T_N(t)/(r - g) \) denotes the lifetime income of a Northern household. We observe that Northern households save only out of their capital income (note that \( r - g = \rho \)), and consume all their labor income (and possible transfer income). In other words, the marginal propensity to consume out of labor and transfer income is one.

Similarly, in the steady state the intertemporal budget constraint of households in the South becomes

\[ N(t) \left\{ \zeta^S + (n - \zeta^S) [\beta + (1 - \beta) z_N] \right\} = (r - g)a_S(t) + w^S(t) + T_S(t) \]

where \( y_S(t) = a_S(t) + w^S(t)/(r - g) + T_S(t)/(r - g) \) denotes the lifetime income of a household in the South. Similarly to Northern households, Southern households save only out of capital income and consume all labor income.

Hence, relative lifetime incomes per capita in the steady state are (endogenously) determined by

\[
\begin{align*}
\frac{y_S(t)}{y_N(t)} &= \frac{\rho a_S(t) + w^S(t) + T_S(t)}{\rho a_N(t) + w^N(t) - T_N(t)} \\
\frac{y_S(t)}{y_N(t)} &= \frac{\zeta^S \omega^S F^S / \beta L + \omega^S + T}{\rho (1 - \zeta^S) F^N / b^N (1 - \beta) L + (1/b^N) - \beta T/(1 - \beta)}
\end{align*}
\]

where \( T \equiv T_S(t)/N(t) \) denotes (exogenous) transfers.

A.5 Measuring income inequality across regions

We will measure income inequality across regions in the steady state with the Gini coefficient. We construct the Gini coefficient from the Lorenz curve shown in Figure 9 in the following way (see Ray, 1998). The Gini coefficient \( G \) is the ratio of the area between the Lorenz curve
and the line of perfect equality (shaded area ABD) to the area of the triangle below the 45 degree line (area ABC), i.e. \( G = \text{Area ABD}/\text{Area ABC} \). In Figure 9, we denote the share of Southern aggregate lifetime income in world lifetime income

\[
\frac{Y_S(t)}{Y(t)} = \frac{1}{1 + \left( \frac{1-\beta}{\beta} \right) \frac{y_N(t)}{y_S(t)}}
\]

where \( y_N(t)/y_S(t) \) denote relative lifetime incomes per capita in steady state given by

\[
\frac{y_N(t)}{y_S(t)} = \frac{\rho a_N(t) + w^N(t) - T_N(t)}{\rho a_S(t) + w^S(t) + T_S(t)}.
\]

Obviously, if the South’s income share \( Y_S(t)/Y(t) \) equals its population share \( \beta \) the Lorenz curve lies on the 45 degree line of perfect equality, and the Gini coefficient is zero.

A.6 Balance of Payments

The intertemporal budget constraint of households in the South, the resource constraint in the South, and the zero-profit condition in the South imply the balance of payments as stated in
Due to Walras’ law, the intertemporal budget constraint of the North is redundant.

First, rewrite the intertemporal budget constraint of a household in the South as follows:

\[ \zeta^S \beta L + (n - \zeta^S) [\beta + (1 - \beta) z_N] \beta L = \rho \frac{a_S(t)\beta L}{N(t)} + \frac{w^S(t)\beta L}{N(t)} + \frac{\beta LT_S(t)}{N(t)} \]

\[ \zeta^S \beta L + (n - \zeta^S) [\beta + (1 - \beta) z_N] \beta L = \rho \zeta^S \omega^S F^S + \omega^S \beta L + \beta LT \]

Second, we rewrite the resource constraint in the South:

\[ g \zeta^S \omega^S F^S + \zeta^S \omega^S b^S L = \omega^S \beta L \]

Next, the zero-profit condition in the South can be written as:

\[ \rho \zeta^S \omega^S F^S = \zeta^S L - (g \zeta^S \omega^S F^S + \zeta^S \omega^S b^S L) \]

Finally, substituting the resource constraint and the zero-profit condition into the intertemporal budget constraint yields the balance of payments:

\[ \zeta^S \beta L + (n - \zeta^S) [\beta + (1 - \beta) z_N] \beta L = \zeta^S L - \omega^S \beta L + \omega^S \beta L + \beta LT \]

\[ (n - \zeta^S) [\beta + (1 - \beta) z_N] \beta L = \zeta^S (1 - \beta) L + \beta LT \]

### A.7 Consumer durables

Figures 10-14 show the relationship between the lag in introduction and the GDP per capita across the 16 European countries relative to the US for the dishwasher, dryer, freezer, VCR, and the washing machine.
Figure 10: Introduction of the dishwasher across European countries

Figure 11: Introduction of the dryer across European countries
Figure 12: Introduction of the freezer across European countries

Figure 13: Introduction of the VCR across European countries
Figures 14-19 show the export performance for the dishwasher, dryer, freezer, microwave oven, VCR, and the washing machine of the U.S. across the 16 European countries. We note that except for the domestic deep freezer the export ratio decreases from a value close to one to a low value close to zero.
Figure 16: U.S. export performance in dryers across 16 European countries

Figure 17: U.S. export performance in freezers across 16 European countries
Figure 18: U.S. export performance in VCRs across 16 European countries

Figure 19: U.S. export performance in washing machines across 16 European countries

A.8 Simulations

Figures 20-22 show the comparative statics results of a change in labor productivity in production in the South.
Figure 20: Effect on innovation rate and consumption share of the South

Figure 21: Effect on relative wages
Figures 23-25 show the comparative statics results of a change in labor productivity in R&D in the South.

Figure 22: Effect on the stages of the product cycle

Figure 23: Effect on innovation rate and consumption share of the South
Figures 24-26 depict the effects of an increase in inequality across regions due to a regressive transfer, i.e. a transfer from poor South to rich North.
Figure 26: Effect on innovation rate and consumption share of the South

Figure 27: Effect on relative wage rate and terms of trade
Figures 29-31 show the effects of a decrease in the savings rate implied by a decrease in the 
time preference rate (households become more impatient in both regions).

Figure 29: Effect on innovation rate and consumption share of the South
Figure 30: Effect on relative wage rate and terms of trade

Figure 31: Effect on the stages of the product cycle