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Futures Premium Regressions and a Latent Factor Day by Day

Kerstin Bernoth† Jürgen von Hagen‡ Casper G. de Vries§

10 June 2011

Abstract Futures instead of forwards are used to study the complete maturity spectrum of the correlations between the spot returns and premium from two days up to six months. //I took out ”puzzle” on purpose, cdv// The correlation decreases with increasing maturity. We hypothesize this maturity effect is part of a latent factor. Futures data allow us to control for the influence of an unobserved factor that can be decomposed into this time-to-maturity effect and contract-specific risk drivers. Doing this, we find that the coefficient on the forward premium is significantly positive. The contract-specific part is shown to be related to conventional proxies of risk.

JEL classification: F31, F37, G13

Keywords: forward premium puzzle, futures rates, latent factor

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1 Introduction

According to the Expectations Hypothesis (EH), forward exchange rates should be efficient predictors of future spot exchange rates. The hypothesis, which assumes rational expectations and risk neutral speculators, is an important building block of models of international macro and finance. Its empirical support, however, is weak at best. Fama (1984) first reported that, in a regression of monthly foreign exchange returns on monthly forward premiums, the estimated slope coefficient is negative instead of being one. This result has become known as the ‘forward premium puzzle’ (FPP) and is the subject of a large body of empirical research; see e.g. Froot and Frankel (1990) and Engel (1996) for overviews. Froot and Frankel (1990) found that the average estimate of the slope coefficient in over 75 published empirical studies was (−0.88). This suggests that market participants may not even be able to predict the direction of exchange-rate changes correctly.

Various explanations of the FPP have been offered. One branch of the literature argues that the forward premium contains a time-varying risk premium which is negatively correlated with the expected change in the exchange rate (e.g. Fama, 1984; Hodrick and Srivastava, 1987; and Hsieh, 1984). Another branch argues that the forward premium contains a systematic forecast error due to learning about regime shifts, Peso problems, or irrational information processing, see e.g. Bilson, (1981); Mark and Wu, (1998); Krasker, (1980); Evans and Lewis, (1995); Lewis, (1995); Gourinchas and Tornell, (2004). Following McCallum (1994), Meredith and Ma (2002) argue that the estimate of the slope coefficient is biased due to the response of monetary policy to output and inflation which in turn are correlated with the exchange rate. Recently, Bacchetta and van Wincoop (2007, 2010) have argued that infrequent portfolio revisions and incomplete information processing can explain the FPP.

In this paper, we first investigate the entire spectrum of the premium correlations with the spot returns by using exchange rate futures data rather than forward contracts. While data for forward contracts are available only for fixed maturity horizons, futures contracts have fixed delivery dates. Since futures contracts are traded in secondary markets, we can take futures rates from the first to the last trading day of a given contract and construct the full maturity spectrum of futures premiums in daily units. While the differences in trading mechanisms, default, or liquidity premiums between forward and futures contracts might cloud the comparability of the
two, empirical studies suggest that this is not the case.\textsuperscript{1} Moreover, at the weekly and monthly maturities, our futures data generate the same results as the forward data.

To our knowledge, no other study has used futures data for this purpose.\textsuperscript{2} The nature of our data allows us to vary the time horizon over which expectations are formed from two days all the way up to six months. We find that the slope coefficient is decreasing in the length of the maturity horizon, though point estimates are surrounded by large confidence bands.\textsuperscript{3} Hence, the puzzle gradually seem to become stronger as the time horizon over which expectations are formed increases. //note I took out the two sentences on the EH test and positive coefficients at first, cdv// But we also infer from the data that ultimately this decrease will be reversed. This is consistent with recent findings in the empirical literature on UIP;\textsuperscript{4} see e.g. Chinn and Meredith (2004) and Chinn (2006) that the premium puzzle disappears for very long maturity horizons.

We hypothesize that the maturity effect is part of a latent factor that is correlated with the futures premium. In line with the above discussed first strand of literature that tried to explain the FPP,

\textsuperscript{1}See e.g. Cornell and Reinganum, 1981; Hodrick and Srivastava, 1987; Polakoff and Grier, 1991; Chang and Chang, 1990.

\textsuperscript{2}Hodrick and Srivastava (1987) use data from three-months futures contracts to test a hypothesis related to the EH, i.e., that the futures rate at time $t$ from a contract expiring in $t + k$ is an efficient predictor of the futures rate at time $t + 1$ from the same contract. They present evidence rejecting this hypothesis.

\textsuperscript{3}Using weekly and monthly forward rates, Yang and Shintani (2006) and Ding and To (2010) also obtain a declining pattern, though Ding (2006) adds that this finding is not very robust and depends significantly on the time period observed. Chaboud and Wright (2005) use the interest differential around the time of the daily ex-interest rate moment and observe uncovered interest rate parity (UIP), while the UIP already fails at the 12 hour horizon. Since we use futures rates, the ex-dividend effect is not in our data. Moreover, futures rates reflect market views about the specific future spot rate. Therfore, these rates may incorporate other information than short interest rates (which are driven by Central Bank rates), and hence may easily reflect motives for e.g. carry trade.

\textsuperscript{4}UIP holds that the difference between the interest rate on two otherwise identical assets denominated in different currencies equals the expected relative change in the exchange rate over the holding period. Under UIP, the slope of a regression of foreign exchange returns on interest rate differentials should also be one. Like the EH, UIP has been strongly rejected. Several studies testing UIP for assets with multi-year maturities confirm that the slope coefficient depends on the length of the holding period of the asset, see Alexius (2001), Chinn and Meredith (2004), Fama (2006) and Fama and Bliss (1987).
we further argue that this latent factor is also driven by a contract specific risk factor due to the specific (macro) economic circumstances at the time of the contract. A multiplicative composition of the two effects contributing to the latent factor produces a risk premium that is shown to be consistent with a standard utility function and pricing kernel specification. Of course, this is not to say that the theoretical model is unique, given the observed data patterns.

The great advantage of futures data is that these permit us to control for the influence of the unobserved factor. To this end we employ Pesaran’s (2006) common correlated effects (CCE) estimator. The multiplicative structure of the latent factor is conducive to Pesaran’s technique. The technique does not estimate the latent factor directly, but through ‘prior adjustment’ corrects for possible omitted-variable bias. Once one indirectly controls for the presence of the unobserved factor, the slope coefficients on the futures premiums turn out to be significantly positive across the maturity spectrum. The results support Fama’s explanation of the FPP as an omitted-variable bias.

Even though this approach does not allow one to identify the level of the latent factor, one can identify the contract specific part up to an unknown scale factor. To explore the nature of this latent common factor, we extract a time-series approximation of the latent factor associated with each futures contract in our sample. We find that it is significantly correlated with measures and determinants of currency risk that have been used elsewhere in the literature. These include sovereign credit ratings, the general risk attitude of global investors, and some monetary variables, which is in line with recent results by Burnside, Eichenbaum, Kleshchelski and Rebelo (2006). Together, these results are consistent with the notion that at least part of the the latent factor is a risk premium.

The remainder of this paper proceeds as follows. Section 2 investigates the relationship between the premium and the spot returns over the entire time spectrum using futures data. Section 3 develops a pricing kernel for the latent factor. Section 4 introduces the CCE estimator and gives the estimates of the premium coefficient in the presence of the common factor; the nature of the unobserved factor is analyzed in section 5. The section 6 concludes. Some results are delegated Appendices.
the next section; if you agree with this approach, the rest of the sections have
to be made consistent with the above, cdv //

2 Spot Returns and Futures Premia

Let \( s_t \) denote the log of the spot exchange rate at time \( t \) and \( f_{t-m}^t \) the log of the futures exchange rate at time \( t-m \) with delivery at time \( t \) and maturity \( m \). We begin by investigating the relationship between \( s_t \) and \( f_{t-m}^t \) for \( m \) ranging over the entire maturity spectrum of the available data. This is motivated by the the expectations hypothesis (EH), which holds that \( f_{t-m}^t \) should be an efficient predictor for the spot exchange rate \( s_t \), and the empirical results reported in previous studies. This could be investigated empirically by regressing \( s_t \) on \( f_{t-m}^t \). Since exchange rates are known to be nonstationary, the EH is usually not investigated by regressing \( s_t \) on \( f_{t-m}^t \), but instead the lagged spot exchange rate is subtracted from both sides to obtain stationary data:

\[
 s_t - s_{t-m} = \alpha + \beta(f_{t-m}^t - s_{t-m}) + \varepsilon_t. \tag{1}
\]

Under the EH, \( \alpha = 0 \) and \( \beta = 1 \).

Our empirical work uses daily closing spot exchange rates and daily closing prices for three-months futures contracts for US$/DM, US$/GBP, and US$/Yen exchange rates. After the introduction of the Euro, we use US$/EUR rates instead of US$/DM rates. The delivery dates for the futures contracts are the third Wednesdays of March, June, September, and December. For all three currencies our data set contains 59 futures contracts with settlement days between 11 June 1993 and 26 June 2007. For each contract we consider the futures prices with maturities running from two days to six months or 126 business days.\(^5\) Let \( k = 1, ..., 59 \) be the index for the delivery dates of the individual futures contracts and \( m \in \{2, ..., 126\} \) the length of time to maturity. Each day in our sample is defined by a tuple \((m, k)\) which denotes \( m \) days before the delivery date \( k \). Define the log spot return \( y_{m,k} = s_{0,k} - s_{m,k} \) and the futures premium \( x_{m,k} = f_{m,k}^k - s_{m,k} \). With these

\(^5\)Dickey-Fuller tests show that the futures premium and the differenced exchange rates are stationary.
definitions, we rewrite regression equation (1) as follows:

\[ y_{m,k} = \alpha_m + \beta_m x_{m,k} + \varepsilon_{m,k}. \]  

(2)

We refer to specification (2) as the *Fama Regression*.

One way to estimate equation (2) is to apply an OLS estimator separately for every \( m \in \{2, \ldots, 126\} \). In this case we would end up with 125 individual regressions each based on 59 observations. However, since futures and spot prices are correlated across maturities, these regressions would not be independent of each other and the error vectors \( \varepsilon_{m,k} \) would be correlated across \( m \). Taking this dependence into account leads to more accurate estimates of the standard errors. To do that, we treat equation (2) as a panel with maturity length \( m \) as the cross-sectional and the maturity dates of the individual contracts as the time dimension and estimate it applying the Beck and Katz (1995) OLS estimator with panel-corrected standard errors. The estimator corrects for heteroskedasticity, correlation across maturities and, if necessary, serial correlation. Figures 1 - 3 in the appendix plot the estimates for the slope coefficients together with their 95 percent confidence intervals against the maturity length, \( m \).

For all three currencies we observe that the slope coefficients decrease with the length of the maturity \( m \). For maturities shorter than one month (22 working days) the estimated slope coefficients are generally positive. Of these, four are significantly different from zero for US$/DM or US$/Yen futures contracts, respectively, and two for US$/Pound futures contracts. For maturities ranging between one and slightly under two months, the slope coefficients are close to zero and ambiguous in sign. //Repeating the Fama regression for one-week forward contracts covering the same currencies and time period as with our futures data set, we confirm the result of Ding (2006). As shown in Table 5, the sign of the coefficients at the short maturity horizon is rather inconclusive and mostly insignificant at this maturity length. Furthermore repeating the analysis for forward rates for the one month maturity horizon, we see that the coefficients are lower compared to the one week coefficients. So the same declining pattern is observed for the forward rates based regressions. But the futures date provide more details, as this pattern is shown for the entire spectrum. TO BE CONFIRMED BY Kerstin// However, for maturities exceeding 60 days, the slope coefficients for US$/DM and US$/Yen futures rates are with two exceptions all negative. The same result holds for US$/Pound futures rates for
maturities exceeding 70 days. In 37 regressions based on US$/DM contracts, 25 regressions based on US$/Yen contracts and ten regressions based on US$/Pound contracts, the coefficients are statistically significantly negative at the 5% level.

Table 1: Relationship between ‘Fama coefficients’ and maturity length

<table>
<thead>
<tr>
<th>$\beta_m$</th>
<th>US$/DM</th>
<th>US$/Yen</th>
<th>US$/Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept</td>
<td>1.359</td>
<td>1.494</td>
<td>1.065</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>maturity</td>
<td>-0.062</td>
<td>-0.053</td>
<td>-0.052</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>maturity$^2$</td>
<td>0.0003</td>
<td>0.0002</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.03)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.68</td>
<td>0.62</td>
<td>0.49</td>
</tr>
<tr>
<td>N</td>
<td>125</td>
<td>125</td>
<td>125</td>
</tr>
<tr>
<td>$H_0$: cons=1 $^a$</td>
<td>0.05</td>
<td>0.11</td>
<td>0.76</td>
</tr>
<tr>
<td>Link test $^a$</td>
<td>0.51</td>
<td>0.79</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Notes: $p$-values in parentheses.

$^a$ Test results represented in p-values.

To summarize the relationship between the estimated slope coefficients and the maturity length of the futures contracts, we regress the estimated $\beta_m$’s on an intercept and the maturity length $m$ in levels and squared values. Table 1 has the results. The $R^2$ values suggest that maturity length explains roughly two thirds of the variation in the estimated slope coefficients. The estimated intercepts are positive and highly significant, and, as shown by a $t$-test, not significantly different from one. The linear maturity terms have negative and significant coefficients, while the squared maturity terms have significantly positive coefficients that are much smaller in absolute value. Thus, with increasing maturity, $m$, the slope coefficients of the ‘Fama regression’ first decline and become negative, but eventually increase and become positive again. For example, the results for the DM/Euro futures contracts indicate that the slope coefficient of the ‘Fama regression’

\[ \text{We test for the validity of this functional form by performing a link test. This test takes the fitted values of the residuals from the original regression and squares them, then reinserts these into the model as an additional regressor. The latter is never significant at the five percent level, indicating that the quadratic form is adequate.} \]
turns negative for maturity horizons longer than 26 days, reaches its minimum value of $-2.00$ at a maturity length of 116 days, and thereafter slowly increases again. Although this is not observed in our sample, the results suggest that the slope coefficient would eventually become positive again with maturities larger than 194 days.\footnote{\footnotetext{This is in line with the results of Ding and To (2010), who focus on forward data and find that the forward premium puzzle appears to be most significant for medium maturities, while disappearing for both very short and long maturities. Similarly, Alexius (2001), Chinn and Meredith (2004) and Chinn (2006) find focussing on UIP at the multi-year level that the rejection of the UIP becomes less decisive if the maturity horizon increases. As an alternative to the regressions in Table 1, we also estimate a panel of equations covering each maturity $m = 2, 3\ldots$ imposition the linear quadratic relationship as a cross-equation restriction on the coefficients $\beta_m$. The results are similar to those reported in Table 1, confirming the negative linear and positive quadratic term.}}

One might suggest that the result that the slope coefficients in the ‘Fama regression’ depend on the number of days left to maturity of a contract is an artefact due to a winding down of trading activity as the expiration day of a contract comes closer. To assure that this is not the case, we analyse the trading volume, the average daily return and the annualized volatility of the futures prices with respect to maturity. The results are shown in Figures 4, 5 and 6 in the appendix. We find that the liquidity of futures markets seems to be high also for futures contracts close to expiry. For very short maturities of less than five days, the trading volume is somewhat lower, but it increases steadily in maturity and reaches its maximum seven days prior to expiry. For maturities up to 77 days the liquidity stays high. However, thereafter the trading volume decreases rapidly, which confirms the usual finding that investors tend to invest mostly in the nearest-maturity futures contract. However, the pattern of the trading volume seems to have no significant effect on the daily returns nor on the volatility of futures prices, see Figures 5 and 6.\footnote{We have also repeated all our estimations dropping all observations for maturities longer than 77 days and all our results were unaffected.}

To summarize, the slope coefficients appear to be declining with the maturity horizon of the futures contracts. For small maturity horizons, the rejection of the EH is less decisive. Thus, the FPP emerges only emerges gradually with increasing maturity.
3 A latent factor model of foreign exchange returns

One common interpretation of the FPP is that foreign exchange returns contain an unobserved variable such as a risk premium in addition to the rational expectation of the exchange rate and the associated expectation error. Thus, let $\Pi_{m,k}$ be an unobserved variable such that:

$$y_{m,k} = \beta'_m x_{m,k} + \Pi_{m,k} + \varepsilon'_{m,k}. \quad (3)$$

The relationship between the foreign exchange return and the futures premium is now clouded by the unobservable variable. It is well known that the OLS estimate of the slope coefficient $\beta'_m$ is downwards biased, if the unobservable variable $\Pi_{m,k}$ and the futures premium are negatively correlated. From this perspective, the observation that the slope coefficient is a decreasing function of the maturity length suggests that the negative correlation between the unobservable variable and the futures premium grows stronger as the time to maturity becomes longer.

This suggestion is supported by the results of Hodrick and Srivastava (1987), who show that the dynamic relationship between futures premiums with different maturities and the same delivery date can be approximated by the following autoregressive structure:

$$(f^t_{t-m+n} - s_{t-m}) = \tau_n (f^t_{t-m} - s_{t-m}) + \psi_{t-m+n}, \quad (4)$$

where $\tau_n \simeq (\tau_1)^n < 1$. Hodrick and Srivastava conclude that daily risk premiums must be highly positively autocorrelated. Expressing equation (4) in expectations based on the information available at time $t-m$, we obtain:

$$E_{t-m}(f^t_{t-m+n} - s_{t-m}) = \tau_n (f^t_{t-m} - s_{t-m}), \quad (5)$$

Setting $m = n$ and using the boundary condition $f^t_t - s_t = 0$, it follows:

$$E_{t-m}(s_t - s_{t-m}) = \tau_m (f^t_{t-m} - s_{t-m}). \quad (6)$$

Taking expectations of the latent factor model (3) and subtracting (6) gives the following expression for the expected unobserved factor $\Pi_{m,k}$:

$$E_{t-m}(\Pi_{m,k}) = - (\beta'_m - \tau_m) x_{m,k}. \quad (7)$$

$^9$Note, that for simplicity we drop the intercept term from our notation. However, in our regressions we control for the possible presence of an intercept.
Thus, if $\beta'_m > \tau_m$, which would be true if $\beta'_m = 1$, the latent variable $\Pi_{m,k}$ associated with an $m$-period foreign exchange return is indeed negatively correlated with the $m$-period futures premium.

Furthermore, we observe that the futures premiums of a given contract with different maturities are highly correlated. On average we estimate a correlation coefficient of 0.90 between the futures premiums $x_{mk}$ and $x_{m-1,k}$. Combining the correlation structure between futures premia of a given contract $k$ with equation (7), we get:

$$E_{t-M+n}(\Pi_{M-n,k}) = -\rho^n \left( \beta'_{M-n} - \tau_{M-n} \right) x_{Mk},$$

(8)

where $M$ denotes the maturity of the first trading day of a futures contract $k$, and where $\rho$ measures the correlation between two consecutive forward premia of a given contract. Thus, the unobserved variables associated with the same contract $k$ and different maturities are all linked in expectation to the same unexpected variable on the first trading day of the contract.

This, together with our empirical results from section 2, suggests the following factor structure of the unobserved variable:

$$\Pi_{m,k} = \gamma_m g_k + \psi_{m,k},$$

(9)

such that the first part of the first term, $\gamma_m$, depends only on maturity, $m$, while the second part, $g_k$, is specific to each futures contract but independent of maturity, and $\psi_{m,k}$ is the stochastic part of the latent variable $\Pi_{m,k}$. We can now rewrite equation (3) as:

$$y_{m,k} = \beta'_m x_{m,k} + \gamma_m g_k + \eta_{m,k},$$

(10)

where $\eta_{m,k} = \varepsilon_{m,k} + \psi_{m,k}$. In Appendix A we show how the factor structure described by equation (9) can be derived from expected utility maximization and a pricing kernel.

To estimate this model, we regard equation (10) again as a panel data model with maturity $m$ as the cross-section and maturity date $k$ as the time-series dimension. We apply Pesaran’s (2006) common correlated effects (CCE) estimator, which allows the unobserved factor $g_k$ to be correlated with the regressor $x_{m,k}$ and the random variable $\eta_{m,k}$ to be serially correlated and heteroscedastic. Furthermore, the CCE estimator does not require the maturity-specific regressors to be identically and/or independently distributed over the cross-section units, which is particularly relevant for the
analysis of our data set, since the futures premium and foreign exchange returns for different maturities are not independent of each other.

Since, for a given contract, $g_k$ is fixed, we can average foreign exchange returns across maturities:

$$
\bar{y}_k = \frac{\gamma \bar{y}_k}{\gamma} + \frac{\gamma \bar{y}_k}{\gamma} + \eta_k, \quad (11)
$$

where the bars indicate averages. Assuming that the errors are i.i.d., the law of large numbers implies that $\bar{y}_k \approx 0$ for sufficiently large $M$. This allows us to approximate the unobserved factor for each contract $k$ by:

$$
g_k \approx 1 \frac{\gamma \bar{y}_k}{\gamma} - \frac{\gamma \bar{x}_k}{\gamma} \quad (12)
$$

Substituting $g_k$ into the equation (12) gives:

$$
y_{mk} = \beta'_m x_{mk} + a_m \bar{y}_k + d_m \bar{x}_k + \eta'_m \quad (13)
$$

with $a_m = \frac{\gamma m}{\gamma}$, $d_m = -\frac{\gamma m \beta'_m}{\gamma}$ and $\eta'_m = \varepsilon_{mk} - a_m \bar{y}_k$. Pesaran (2006) shows that by running a multiple regression of $y_{mk}$ on $(x_{mk}, \bar{y}_k, \bar{x}_k)$ for a given $m$, one obtains a consistent and unbiased estimate of $\beta'_m$. A simplified proof is provided in Appendix B.11

Estimating equation (13) also yields consistent estimates of the coefficients $a_m$ and $d_m$. Thus, given that $a_m = \gamma m / \gamma$, we are able to identify the individual factor loadings $\gamma_m$ up to a scaling factor $\gamma$. Further, using the consistent estimates of $\beta'_m$ for $m = 2, \ldots, M$, we can recover $\gamma g_k$ from equation (12) for every contract $k = 1, \ldots, T$. Therefore, we are also able to identify the unobserved factor $g_k$ up to the scaling factor $\gamma$.

We implement the CCE estimator in two ways. First, we allow the intercept and the coefficient $\beta'_m$ to vary with maturities. The resulting estimates of $\beta'_m$ are plotted in Figures 7 to 9 together with their 95% confidence interval. The figures show that the downward trend in the slope coefficient has

---

10The CCE estimator is based on the assumption that the slope coefficient $\beta'_m$ follows a random coefficient model, thus $\beta'_m = \beta' + v_m$, where the random deviations $v_m$ are distributed independently from the model coefficients and other error terms. Therefore, it holds that $\beta'_m x_k = \beta'_m x_k$.

11In appendix C we show that the approximation of the unobserved factor by a linear combination of the cross-maturity averages of the futures premium and the forex returns is closely related to the model for the term premium proposed by Cochrane and Piazzesi (2005).
has vanished for all three currencies. The slope coefficients are now generally positive and very often significantly larger than zero. Regressions of the slope coefficient on the maturity and the squared maturity of the same type as in table 1 confirm the visual impression, i.e., that neither regressor has a significant coefficient.

Table 2 reports the average intercept and $\beta'_{m}$ obtained from these estimates together with the standard deviations of the mean. Pesaran (2006) calls this the CCE mean group estimator. For all three currencies, the average intercept is not significantly different from zero. The slope coefficients are all significantly positive, but smaller in magnitude than one. However, only for the US$/Yen contracts, the Null Hypothesis that the mean group estimator equals one cannot be rejected at standard significance levels, although even here the $p$-value is 0.09.

Next, we estimate the CCE pooled estimator, which restricts the slope coefficients $\beta'_{m}$ to be the same for all maturities $m$ (see Pesaran (2006)). The lower panel of Table 2 reports these estimates. Again, we find that the slope coefficient is significantly positive, but it is also still significantly different from one. We investigate the robustness of this latter conclusion further at the end of this section.

The relevance of adding $\left( y_{k}, x_{k} \right)$ to the standard ‘Fama specification’ (2) can be tested by means of the Likelihood ratio test. Table 2 reports the $p$-values of testing (13) against (2) on basis of the mean group estimator and the pooled estimator. It appears that in none of the cases one can omit the regressors $\left( y_{k}, x_{k} \right)$ on basis of their statistical significance. Moreover, as already noted, adding these regressors to the estimation equation substantially increases the value of the futures premium coefficient, indicating that the coefficient estimates from the standard specification (2) are severely downward biased.

Figures 10 to 12 in the appendix plot the factor loadings $\gamma_{m}$ against the maturity $m$ resulting from the mean group CCE estimator.\(^{12}\) The figures show that the factor loadings increase with increasing maturity. This confirms our suggestion from above: OLS estimates of the slope coefficients decline with increasing maturity, because the importance of the latent factor for the foreign exchange return increases. The figures also suggest that we can approximate the factor loadings by a linear-quadratic function of maturity.

\(^{12}\)The factors and factor loadings estimated by the mean group CCE, pooled CCE and the error-in-variables CCE regressions look almost identical.
Table 2: Slope Coefficients from CCE Estimator

<table>
<thead>
<tr>
<th></th>
<th>US$/DM</th>
<th>US$/Yen</th>
<th>US$/Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Group Estimator</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average intercept</td>
<td>0.002</td>
<td>0.019</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.90)</td>
<td>(0.76)</td>
<td>(0.92)</td>
</tr>
<tr>
<td>average $\beta'$</td>
<td>0.688</td>
<td>0.749</td>
<td>0.822</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>LR test$^a$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$H_0 : \beta' = 1$ $^a$</td>
<td>0.00</td>
<td>0.09</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>Pooled Estimator</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average intercept</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.99)</td>
<td>(1.00)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>$\beta'$</td>
<td>0.664</td>
<td>0.656</td>
<td>0.780</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>LR test$^a$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$H_0 : \beta' = 1$ $^a$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Notes: p-values shown in paranthesis.

$^a$) Test results represented in p-values.

\[ \gamma_m = \delta_0 + \delta_1 m + \delta_2 m^2 + u_m, \]  

with $\delta_2 < 0 < \delta_1$.

Table 3 reports the results of OLS estimates of equation (14) for all three currencies. It shows that the linear-quadratic function indeed fits the factor loadings extremely well. The coefficients on the linear and the quadratic maturity are almost identical for all three currencies. Note that the linear quadratic functions imply that the factor loadings eventually decrease in maturity. Based on our estimates, they would reach zero again after approximately 248 working days, or one year. Although this is not in our sample, it is consistent with the results from the previous section.

To summarize, the main results from this section are three-fold. First, we have shown that we can characterize the term structure of daily foreign exchange returns by the corresponding futures premium and a latent, futures-contract specific factor. Second, the factor loadings can be characterized by a simple, linear-quadratic function, a result which is reminiscent of Cochrane and Piazzesi’s (2005) findings for the term structure of interest rates. Third, taking into account the latent factor and the bias it causes in OLS estimates,
the slope coefficients on the futures premium are positive and close to but significantly below one.

4 Robustness

We investigate the robustness of our estimates against two possible problems. First, as noted above, the trading volume for very short and for longer maturities decreases, see Figure 4 in the Appendix. Although Figures 5 and 6 suggest that the trading volume has no significant impact on the daily returns nor on the volatility of futures prices, we re-estimate the CCE regressions by dropping all observations for maturities shorter than 15 and larger than 77 days. The estimates are reported in Table 6 in the appendix. In case of the US$/DM futures contracts, the CCE mean group estimator and the CCE pooled estimator hardly change in comparison with the estimates that are based on the complete maturity spectrum. For the US$/Yen and the US$/Pound futures contracts the slope coefficients are lower. However, for all three currencies, the slope coefficients remain significantly positive.

A second potential problem is the timing mismatch between when the futures exchange rate $f_{t-m}^t$ and the spot exchange rate $s_{t-m}$ are recorded.\footnote{The spot exchange rates are recorded by JP Morgan at 4 p.m. London time, while the closing prices of the futures exchange rates are recorded at 2 p.m. at the Chicago Mercantile Exchange. Thus, there is a timing mismatch of four hours between the futures and the spot prices.} If this mismatch has an impact at all, it has more impact at short maturi-

<table>
<thead>
<tr>
<th></th>
<th>US$/DM</th>
<th>US$/Yen</th>
<th>US$/Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_0$</td>
<td>0.048</td>
<td>0.012</td>
<td>0.106</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.024</td>
<td>0.023</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>-0.0001</td>
<td>-0.0001</td>
<td>-0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.98</td>
<td>0.99</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Notes: p-values represented in parentheses.
ties than at long maturities, since the variance of the futures premium $\sigma_{xm}$ increases with the maturity $m$. Therefore, we re-estimate our estimations by deleting this time only the futures prices with maturities shorter than 15 days from the sample. The estimates are shown in Table 7 in the appendix. For both, the CCE mean group estimator and the CCE pooled estimator, dropping the 15 maturities at the short end hardly changes the estimates in comparison with the results that are based on the complete maturity spectrum.

5 The nature of the unobserved factor

Figures 13 to 15 show the latent factors $\hat{g}_k$ estimated for the three currencies using the CCE estimator. Remember that these latent factors are identified up to a scaling factor. However, since we are interested in analysing the driving forces of the unobserved factors and less so in their levels, this identification problem can be ignored. There are some large movements in the factors in the mid-1990s, around the turn of the millenium and between the third quarter of 2001 and the second quarter of 2002.

The correlation between the US$/Yen factor and the US$/Pound factor is 0.34, the correlation between the US$/Yen factor and the US$/DM factor is 0.44, while the correlation between the US$/Pound and the US$/DM factors is 0.73. Market fluctuations seem to be more interdependent in European markets than between European and Asian markets.

To explore the nature of these factors further, we analyze the correlation between the contract-specific factors and several conventional measures of financial market risk. The first is the spread between BBB US corporate bonds and US Treasury bonds, a commonly used indicator of risk aversion in international financial markets. In addition, we use historical (20 days rolling window) volatilities of stock indices, 10-year government bonds, and currency indices in the US, Germany, Japan, and the UK. This is motivated by dynamic international asset pricing model that suggests that the risk premium in forward prices is closely related to the realized volatility of exchange rates. Further, we add some macro economic variables that are commonly regarded as determinants of currency risk and country risk; see e.g. Burnside

---

14The increase in $\sigma_{xm}$ reduces the attenuation bias due to the measurement error as the timing mismatch does not increase with the maturity $m$. Estimates of $\sigma_{xm}$ are available from the authors on request.
et al. (2006). Specifically, we use the three countries’ debt service to export ratios, their current account balances relative to GDP, and the growth rates of broad money (M2).\textsuperscript{15} Burnside et al (2006) show that primarily monetary variables have a significant impact on risk factors in currency markets. Additionally, we use the growth rates of real GDP to capture business cycle effects and economic dynamics and a variable measuring the size of central bank foreign exchange interventions. Finally, we include a measure for the recession probability in the USA \textsuperscript{16} to measure external shocks and changes in investors’ risk attitudes.

To condense the information contained in these time series, we use factor analysis. While the scree plot suggest that the optimal number of factors to describe our set of variables is four, the Kaiser-Guttman rule\textsuperscript{17} suggest that the optimal number is three. We decided to retain three factors, since they explain around 78\% of the variation of our explanatory variables, and the fourth factors does not add a lot. We extract the factors loadings by applying the principle factor method and redefine these further by oblique rotation. The estimates for the factor loadings are listed in the lower panel of Table 4.

The first and third factor focus entirely on US variables. The first factor captures to a large extent information on US M2 growth, the US debt service ratio and the historical volatility of US stock indices. The third factor is related to the variables capturing the real economic position of the USA, i.e. the US recession probability measure and US GDP growth and also two variables measuring financial market risk, i.e. the interest spread between low graded corporate bonds and US government bonds and the historical volatility of US interest rate index. The second factor is closely related to domestic determinants, namely the domestic debt service ratio, domestic monetary variables, i.e. M2 growth and current account balance, and the historical volatilities of the 10-government bond and forex index.

We then regress the latent variables from our CCE model on these three factors and a constant. The Durbin Watson statistic gives no indication for

\textsuperscript{15}We also added the inflation rate, the debt to GDP ratio and the reserves to import ratio, but these variables had not much additional explanatory power so that we dropped these from the model.

\textsuperscript{16}Source: \url{http://www.uoregon.edu/~jpiger/us_recession_probs.htm}

\textsuperscript{17}The Kaiser-Guttman rule says that the optimal choice of factors is determined by the number of factors with eigenvalue greater than 1.0.
Table 4: Determinants of Unobserved Factor

<table>
<thead>
<tr>
<th></th>
<th>US$/DM</th>
<th>US$/Yen</th>
<th>US$/Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor 1</td>
<td>-1.46</td>
<td>-1.39</td>
<td>-1.07</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.08)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Factor 2</td>
<td>1.65</td>
<td>3.06</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.00)</td>
<td>(0.33)</td>
</tr>
<tr>
<td>Factor 3</td>
<td>1.08</td>
<td>-0.35</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.55)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.29</td>
<td>-3.20</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
<td>(0.00)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>DW</td>
<td>1.95</td>
<td>1.72</td>
<td>1.81</td>
</tr>
<tr>
<td>R2</td>
<td>0.29</td>
<td>0.16</td>
<td>0.19</td>
</tr>
<tr>
<td>N</td>
<td>59</td>
<td>59</td>
<td>59</td>
</tr>
</tbody>
</table>

Factor loadings

<table>
<thead>
<tr>
<th>Variable</th>
<th>Factor1</th>
<th>Factor2</th>
<th>Factor3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob.of US Recession</td>
<td>0.241</td>
<td>-0.091</td>
<td>0.631</td>
</tr>
<tr>
<td>M2 Growth</td>
<td>0.025</td>
<td>-0.562</td>
<td>-0.009</td>
</tr>
<tr>
<td>M2 Growth US</td>
<td>0.636</td>
<td>0.007</td>
<td>0.577</td>
</tr>
<tr>
<td>CB intervention</td>
<td>-0.106</td>
<td>0.366</td>
<td>0.091</td>
</tr>
<tr>
<td>Fed intervention</td>
<td>-0.355</td>
<td>0.133</td>
<td>-0.166</td>
</tr>
<tr>
<td>Debt service</td>
<td>-0.333</td>
<td>0.717</td>
<td>-0.080</td>
</tr>
<tr>
<td>Debt service US</td>
<td>-0.904</td>
<td>0.042</td>
<td>0.043</td>
</tr>
<tr>
<td>CA Balance</td>
<td>-0.121</td>
<td>0.519</td>
<td>0.074</td>
</tr>
<tr>
<td>CA Balance US</td>
<td>-0.108</td>
<td>0.119</td>
<td>-0.384</td>
</tr>
<tr>
<td>GDP growth</td>
<td>0.113</td>
<td>-0.450</td>
<td>-0.286</td>
</tr>
<tr>
<td>GDP growth US</td>
<td>0.264</td>
<td>0.161</td>
<td>-0.711</td>
</tr>
<tr>
<td>Spread</td>
<td>0.212</td>
<td>0.089</td>
<td>0.882</td>
</tr>
<tr>
<td>HV stock index</td>
<td>0.373</td>
<td>0.377</td>
<td>0.500</td>
</tr>
<tr>
<td>HV stock index US</td>
<td>0.663</td>
<td>0.123</td>
<td>0.512</td>
</tr>
<tr>
<td>HV interest</td>
<td>0.165</td>
<td>0.715</td>
<td>0.169</td>
</tr>
<tr>
<td>HV interest US</td>
<td>-0.021</td>
<td>0.186</td>
<td>0.702</td>
</tr>
<tr>
<td>HV forex</td>
<td>0.287</td>
<td>0.687</td>
<td>-0.018</td>
</tr>
<tr>
<td>HV forex US</td>
<td>-0.025</td>
<td>0.221</td>
<td>0.216</td>
</tr>
</tbody>
</table>

DW= Durbin-Watson statistic, Prob. of US recession=Recession probability in the USA; see http://www.uoregon.edu/~jpiger/us_recession_probs.htm, CB and Fed interventions=purchases of Million USD, CA=current account; Spread=interest rate spread between low graded US corp.bonds and govern.bonds; HV=historical volatility (20 day window)
autocorrelation in the error terms.\footnote{As a robustness check we have repeated the regression with an additional lagged dependent variable to control for possible autocorrelation, but this variable turned out to be highly insignificant and the regression results did not differ much.} The results are shown in the upper panel of Table 4. In the case of US$/DM futures, the fit of the regression is quite good, the $R^2$ being 0.29. For US$/Pound and US$/Yen futures, the explanatory power is somewhat less.

For all three currencies, the coefficient on the first factor, which reflects to a large extent US monetary variables and the US debt service ratio, turns out to be significant. The second factor, which is closely related to domestic monetary variables and domestic financial risk indicators, has significant coefficients in the regressions for US$/DM and US$/Yen futures. Finally, the third factor, which reflects the condition of the US economy, has significant coefficients in the US$/DM and the US$/Pound regressions.

These results are consistent with the notion that the latent variables extracted from the ‘Fama regressions’ contain a risk premium as suggested by Fama (1984). They are also in line with the results of Burnside et al. (2008).

\section{Conclusion}

In this paper we investigated the relation between the spot return and the futures premium for the US dollar exchange rates against the Yen, the British Pound, and the German DM (the Euro after 1999). Futures rates allow us to analyze the relationship over the complete maturity spectrum, ranging from the first until the last trading day of a futures contract. This creates a panel structure of our data with time to maturity as the cross sectional dimension and the maturity dates of individual contracts as the time series dimension.

Our first result is that the slope coefficient of the conventional ‘Fama regression’ (1984) is a U-shaped function of the time to maturity, declining over the range from one day to two month. Subsequently, we hypothesized that there is a latent factor composed of this maturity effect and contract specific risk factors. Such a multiplicative latent factor was shown to be consistent with a standard utility function and pricing kernel. This multiplicative latent
factor structure permits one to exploit the panel structure of the futures data set to control for the omitted variable bias that this latent factor might cause. To this end we used Pesaran’s (2006) common correlated effects estimator. The latent factor is contract specific with factor loadings that have a linear-quadratic structure and which increase with declining increments in the time to maturity. Once we account for this factor, the slope coefficients between the realized foreign exchange returns and the futures premiums are consistently positive and no longer depend on the time to maturity. This result is reminiscent of Cochrane’s and Piazzesi’s (2005) findings concerning the Expectations Hypothesis for the term structure of interest rates. Our third result is that the scale of the latent variable extracted from the data is correlated with a number of observables that are conventionally regarded as variables which are linked to currency risk. This suggests that the latent factor at least partially has the nature of a risk premium.
References


A Risk premium structure - A Pricing Kernel Approach

In this section we show that formally obtain the somewhat intuitive derivation of the factor specification described in equation (9) from a pricing kernel cum utility analysis. Consider the following consumer problem at time $t - m + n$ for consumption at $t - m + n$, $n \leq m$, and future consumption at time $t$ with discount factor $\delta$:

$$\max_{C_{t-m+n},C_t} U(C_{t-m+n}) + \delta^{m-n} E_{t-m+n}[U(C_t)].$$

The following two budget constraints apply

$$W_{t-m+n} = C_{t-m+n} + D$$

and

$$0 = C_t - R D - F^t_{t-m+n} A + S_t A.$$

Here $W_{t-m+n}$ is wealth at time $t - m + n$ and $D$ is the amount invested in a riskless domestic bond that pays gross interest $R$ at time $t$. At time $t$ the consumer can also buy currency futures contracts $A$ that mature at time $t$ and promise delivery of foreign currency at price $F^t_{t-m+n}$. The spot price of foreign currency at time $t$ is denoted by $S_t$. For simplicity, we assume that there are no margin payments to be made up front. All costs for the futures contract are in the agreed contract price. This explains why the currency transactions only appear in the second budget constraint. Per contrast, in case of spot speculation one buys foreign currency (bonds) in the first period and sells the proceeds the second period.

Substituting the budget constraints into the utility function, we get:

$$\max_{D,A} \delta^{m-n} E_{t-m+n}[U(RD + F^t_{t-m+n} A - S_t A)] + [U(W_{t-m+n} - D)].$$

From the first order conditions we have a.o.

$$\delta^{m-n} E_{t-m+n}[\frac{\partial U(C_t)}{\partial C_t}(F^t_{t-m+n} - S_t)] = 0.$$

\footnote{Note that the additive expected utility structure easily permits an extension to incorporation of a multi-period decision problem.}
Since the forward rate \( F_{t-m+n}^t \) is known at time \( t \), we can write:

\[
1 = \frac{E_{t-m+n}[\vartheta U(C_t) S_t]}{E_{t-m+n}[\vartheta U(C_t)]}.
\]

Furthermore we can re-express this as

\[
1 = E_{t-m+n}[\vartheta U(C_t) S_{t-m+n} F_{t-m+n}^t] / E_{t-m+n}[\vartheta U(C_t)].
\]

Suppose that as a simplification of (3) and (10), omitting superfluous indices and assuming \( \beta_m' = 1 \),

\[
y_{m-n} = x_{m-n} + \gamma g + \varepsilon.
\]

Hence, (15) can be written as

\[
1 = E_{t-m+n}[\vartheta U(C_t) e^{y_{m-n,k} - x_{m-n,k}}] / E_{t-m+n}[\vartheta U(C_t)].
\]

Postulate the following specification for the marginal utility

\[
\frac{\partial U(C_t)}{\partial C_t} = \exp \left( -\frac{1}{2} \lambda^2 \sigma^2 - \lambda \varepsilon \right)
\]

and where \( \varepsilon \) follows a mean zero normal distribution with variance \( \sigma^2 \). Thus \( \partial U(C_t)/\partial C_t \) follows a lognormal distribution and has expected value

\[
E_{t-m}[\vartheta U(C_t)] = \exp \left( -\frac{1}{2} \lambda^2 \sigma^2 + \frac{1}{2} \lambda \varepsilon \right) = 1.
\]

Combining expressions and using (16) gives

\[
\frac{\partial U(C_t)}{\partial C_t} e^{y_{m-n} - x_{m-n}} = \exp \left( -\frac{1}{2} \lambda^2 \sigma^2 - \lambda \varepsilon + \gamma g + \varepsilon \right)
\]

\[
= \exp \left( -\frac{1}{2} \lambda^2 \sigma^2 + (1 - \lambda) \varepsilon + \gamma g \right).
\]
Hence,

\[ E_{t-m+n} \left[ \frac{\partial U(C_t)}{\partial C_t} e^{y_{m-n}x_{m-n}} \right] = E_{t-m} \left[ \exp \left( -\frac{1}{2} \lambda^2 \sigma^2 + (1 - \lambda) \varepsilon + \gamma g \right) \right] \]

\[ = \exp \left( -\frac{1}{2} \lambda^2 \sigma^2 + \gamma g + \frac{1}{2} (1 - \lambda)^2 \sigma^2 \right) \]

\[ = \exp \left( \gamma g - \lambda \sigma^2 + \frac{1}{2} \sigma^2 \right). \]

Substitute all this into the first order condition (16) to get

\[ 1 = e^{\gamma g - \lambda \sigma^2 + \frac{1}{2} \sigma^2}. \]

Take logarithms on both sides to solve for \( \lambda \)

\[ \lambda = \gamma g / \sigma^2 + \frac{1}{2}. \]

To conclude, we have given a form of the utility function and pricing kernel consistent with the linear specification (10) that is adopted for the regression analysis.

**B Consistency and unbiasedness of the common correlated effects estimator**

Consider (10) from the main text

\[ y_{m,k} = \beta_m x_{m,k} + \gamma_m g_k + \eta_{m,k}, \tag{17} \]

Take averages across maturities \( m \) to get the equivalent of (11)

\[ \bar{y}_k = \beta'_m \bar{x}_k + \gamma_k + \eta_k. \]

Hence, one can express the unobserved factor as

\[ g_k = \frac{1}{\bar{y}_k} \bar{g}_k - \frac{1}{\bar{y}_k} \beta'_m \bar{x}_k - \frac{1}{\bar{y}_k} \eta_k. \tag{18} \]

Substitute this expression for \( g_k \) into (17), to get

\[ y_{m,k} = \beta'_m x_{m,k} + \frac{\gamma_m}{\bar{y}_k} \bar{g}_k - \frac{\gamma_m}{\bar{y}_k} \beta'_m \bar{x}_k - \frac{\gamma_m}{\bar{y}_k} \eta_k + \eta_{m,k}. \tag{19} \]
Under the assumption that $\beta'_m$ follows a random coefficient model, i.e. $\beta'_m = \beta' + v_m$, we can write

$$\beta'_m x_k = \beta' x_k + v_m x_k,$$

where $v_m$ is an IID error term. Subsequently, we can restate (19) as follows

$$y_{m,k} = \beta'_m x_{m,k} + \gamma m y_k - \beta' \gamma m x_k - \gamma m v_m x_k - \gamma m \eta_k + \eta_{m,k}.$$

Using matrix notation, this can further be written as

$$y_m = X_1 \beta'_m + X_2 c + \Upsilon_m \quad (20)$$

where $c^T = (\gamma m / \gamma, -\beta' \gamma m / \gamma)$, $X_1 = x_m$ and $X_2 = (y_k, x_k)$ and

$$\Upsilon_m = -\frac{\gamma m}{\gamma} v_m x_k - \frac{\gamma m}{\gamma} \eta_k + \eta_{m,k}.$$

Note that $X_2$ is observable, i.e. can be constructed from the data.

An OLS regression across $k$ contracts and maturities $m$ using only $X_1$ as regressor, as is the standard procedure in the premium regressions, gives

$$\hat{\beta}'_m = (X_1^T X_1)^{-1} X_1^T y_m$$

$$= (X_1^T X_1)^{-1} X_1^T [X_1 \beta'_m + X_2 c + \Upsilon_m]$$

$$= \beta'_m + (X_1^T X_1)^{-1} X_1^T X_2 c + (X_1^T X_1)^{-1} X_1^T \Upsilon_m.$$

and where

$$(X_1^T X_1)^{-1} X_1^T \Upsilon_m$$

is assumed to be small (see below). We see that if $X_1$ and $X_2$ are correlated, then the second term $(X_1^T X_1)^{-1} X_1^T X_2 c$ will be non-zero. In this case, ignoring the presence of the unobserved factor in equation (17), which corresponds to the ‘Fama regression’, gives a biased estimate of $\beta'_m$.

With prior adjustment, see Maddala (1977, p. 462), one uses the idempotent matrix

$$Q = I - X_2 (X_2^T X_2)^{-1} X_2^T$$

to transform the dependent variable and regressor by respectively $Q y_m$ and $Q X_1$ prior to running the regression. With prior adjustment the OLS esti-
mator becomes
\[
\hat{\beta}'_m = (X_1^T Q^2 X_1)^{-1} X_1^T Q^2 y_m
\]
\[
= (X_1^T Q X_1)^{-1} X_1^T Q [X_1 \beta'_m + X_2 c + \Upsilon_m]
\]
\[
= \beta'_m + (X_1^T Q X_1)^{-1} X_1^T Q X_2 c + (X_1^T Q X_1)^{-1} X_1^T Q \Upsilon_m
\]
\[
= \beta'_m + (X_1^T Q X_1)^{-1} X_1^T Q \Upsilon_m,
\]
where we used the two features of an idempotent matrix that $Q^2 = Q$ and
\[
Q X_2 = X_2 - X_2 (X_2^T X_2)^{-1} X_2^T X_2
\]
\[
= X_2 - X_2 = 0.
\]
Thus beta is now consistently estimated by $\hat{\beta}'_m$ if $(X_1^T Q X_1)^{-1} X_1^T Q \Upsilon_m$ is small. Pesaran (2006) gives sufficient conditions for this latter requirement to hold.\(^\text{20}\)

\section{Combining Pesaran’s CCE estimator (2006) with Cochrane and Piazzesi (2005)}

Cochrane and Piazzesi (2005) show in their seminal paper that a single factor, described by a linear combination of forward rates, has a high predictive power for the excess returns on one- to five-year maturity bonds. In this section we transfer this idea to our set-up and show that Cochrane and Piazzesi’s finding is closely related to Pesaran’s CCE estimator applied in our empirical section.

In the related literature on the forward premium puzzle, the difference between the realized spot exchange rate innovation, $y_{m,k}$, and the forward premium, $x_{m,k}$, is very often denoted as the ‘excess return’. Similar to Cochrane and Piazzesi (2005) we relate this ‘excess return’, which is expressed in our model by the unobserved factor $\gamma_m g_k$, to a set of the forward premia. Suppose that $g_k$ is linear in all the forward premia concerning contract $k$.

\begin{equation}
\begin{aligned}
g_k &= \sum_{i=1}^{M} \lambda_i x_{i,k}.
\end{aligned}
\end{equation}

\text{E.g.} either by assumptions on the Euclidean norm, if beta is non-stochastic, or if $\beta'_m - \beta = v_m$ are mean zero iid across maturities.
Thus, conditional on the information available at maturity $t - m$, the excess return contained in futures contracts at time of contract $k$ is described by:

$$y_{m,k} - x_{m,k} = \gamma_m \{ \lambda_M x_{M,k} + \ldots + \lambda_m x_{m,k} + E_m [\lambda_{m-1} x_{m-1,k} + \ldots + \lambda_1 x_{1,k}] \} + \eta_{m,k}.$$

Suppose that

$$E_m [\lambda_{m-1} x_{m-1,k} + \ldots + \lambda_1 x_{1,k}]$$

can be estimated by linear projection on the realized ex-post premia

$$\lambda_{m-1} x_{m-1,k} + \ldots + \lambda_1 x_{1,k}.$$

Then the regression model becomes

$$y_{m,k} = x_{m,k} + \gamma_m \sum_{i=1}^{M} \lambda_i x_{i,k} + \eta_{m,k},$$

which is very much related to Piazzesi and Cochrane (2005).

Averaging across maturities gives

$$\frac{1}{M} \sum_{m=1}^{M} (y_{m,k} - x_{m,k}) = \left( \sum_{i=1}^{M} \lambda_i x_{i,k} \right) \left( \frac{1}{M} \sum_{m=1}^{M} \gamma_m \right)$$

or in shorthand notation

$$\frac{1}{T} (\bar{y}_k - \bar{x}_k) = \sum_{i=1}^{m} \lambda_i x_{i,k} = g_k.$$

The left hand side of the latter expression conforms to the way in which Pesaran’s estimator constructs the factor to correct for the omitted bias. The expression in the middle for $g_k$ corresponds to the linear factor structure suggested by the Cochrane and Piazzesi paper. This offers the link between Pesaran’s CCE estimator and Cochrane and Piazzesi’s result that the excess return of a financial asset (in there case a bond) can be explained by a linear combination of forward premia.
Figure 1: $\beta_m$, DM(Euro)/US$ futures contracts - OLS
Figure 2: $\beta_m$, US$/Yen futures contracts - OLS

Figure 3: $\beta_m$, US$/Pound futures contracts - OLS
Figure 4: Trading volume of futures contracts

Figure 5: Annulized volatility of daily futures returns
Figure 6: Average daily return of futures prices

Figure 7: $\beta_m$, DM(Euro)/US$ futures contracts - CCE estimations
Figure 8: $\beta_m$, US$/Yen futures contracts - CCE estimations

Figure 9: $\beta_m$, US$/Pound futures contracts - CCE estimations
Figure 10: Scaled factor loadings across maturity - DM futures contracts
Figure 11: Scaled factor loadings across maturity - Yen futures contracts

Figure 12: Scaled factor loadings across maturity - Pound futures contracts
Figure 13: Scaled factor across time - DM futures contracts
Figure 14: Scaled factor across time - Yen futures contracts

Figure 15: Scaled factor across time - Pound futures contracts
## E Tables

Table 5: GLS estimation results of the Fama regression for 1-week forward contracts.

<table>
<thead>
<tr>
<th></th>
<th>α</th>
<th>p-value</th>
<th>β</th>
<th>p-value</th>
<th>N</th>
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<td>(0.46)</td>
<td>1000</td>
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<td>-1.68</td>
<td>(0.05)</td>
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<td>580</td>
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<td>US$/Yen</td>
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<td></td>
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<tr>
<td></td>
<td>US$/DM</td>
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<td>--------</td>
<td>---------</td>
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<td></td>
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</tr>
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<td>0.015</td>
<td>0.000</td>
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</tr>
<tr>
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<td>(0.66)</td>
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<td>(0.00)</td>
<td>(0.00)</td>
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<tr>
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<td>(1.00)</td>
<td>(1.00)</td>
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<tr>
<td>$\beta'$</td>
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<td>0.309</td>
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Notes: p-values shown in paranthesis.

$^a$) Test results represented in p-values.

<table>
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<th></th>
<th>US$/DM</th>
<th>US$/Yen</th>
<th>US$/Pound</th>
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Notes: p-values shown in paranthesis.

$^a$) Test results represented in p-values.