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To commit or not to commit: Endogenous timing in experimental duopoly markets


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To commit or not to commit: Endogenous timing in experimental duopoly markets*

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May 11, 1999

Abstract

In this paper, we experimentally investigate the extended game with action commitment of Hamilton and Slutsky (1990). In their duopoly game, firms can choose their quantities in one of two periods before the market clears. If a firm commits to a quantity in period 1, it does not know whether the other firm also commits early. By waiting until period 2, a firm can observe the other firm’s period 1 action. Hamilton and Slutsky predict the emergence of endogenous Stackelberg leadership. Our data, however, does not confirm the theory. While Stackelberg equilibria are extremely rare we often observe endogenous Cournot outcomes and sometimes collusive play. This is partly driven by the fact that endogenous Stackelberg followers learn to behave in a reciprocal fashion over time, i.e., they learn to reward cooperation and to punish exploitation.

1 Introduction

Starting with papers by Saloner (1987), Hamilton and Slutsky (1990), and Robson (1990), there has been a growing literature studying models of endogenous timing in oligopoly. These papers analyze extended market games which allow to establish conditions specifying whether firms decide on their actions simultaneously or sequentially. The order of output or price decisions is not exogenously specified. Rather, it is derived from firms’ decisions about timing. Results from this literature may indicate whether models of simultaneous output or price decisions (Cournot, Bertrand) or sequential decisions (Stackelberg, price leadership) are preferable.

The games used to determine endogenous timing have, in principle, a simple structure. In Hamilton and Slutsky’s (1990) (henceforth HS) extended game with action commitment, two firms may choose their action in one out of two periods. A firm may move first by committing to an action, or it may wait until the second period and observe the other firm’s first period action. This extended timing game allows, a priori, for simultaneous-move outcomes as well as for sequential-move outcomes.

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What are the equilibrium predictions of the extended game with action commitment? HS show that—if only equilibria in undominated strategies are considered—only sequential-move structures emerge endogenously. With price competition, this result is not surprising as the outcome of the sequential-move price leader game Pareto dominates the outcome of the simultaneous-move Bertrand game. However, the same result also holds with quantity competition where the Stackelberg leader is better off than a firm in Cournot equilibrium while the Stackelberg follower is worse off compared to Cournot. There are two endogenous Stackelberg equilibria with either firm as the Stackelberg leader.\(^1\) While there exists a simultaneous-move Cournot equilibrium in pure strategies, this equilibrium is in weakly dominated strategies.

In closely related timing games with Cournot competition, Ellingsen (1995) (extending Saloner’s (1987) model) and Robson (1990) come to the same conclusion in the sense that only Stackelberg equilibria emerge endogenously.\(^2\) Ellingsen (1995) argues that “only Stackelberg points survive” (p. 87). Similarly, Robson (1990) concludes that an “argument in favor of Stackelberg at the expense of Cournot can be made forcefully” (p. 70). While firms are symmetric in these models, Stackelberg equilibria also emerge endogenously when firms are asymmetrically informed: Again, only Stackelberg equilibria with either the informed or the uninformed firm moving first emerge (see Mailath, 1993, and Normann, 1997). Note that, in all the papers mentioned, the general theoretical support for Stackelberg equilibria crucially depends on equilibrium selection arguments. Simultaneous-move Cournot equilibria in pure strategies typically exist\(^3\)—however, they do not survive the application of equilibrium refinements.

In this paper, we report on an experiment designed to test the HS model with action commitment. We analyze a market with two symmetric firms and with quantity competition. In particular, we check whether there is experimental evidence for endogenous Stackelberg equilibria—or whether some other (if any) equilibrium is selected by subjects.

There are two reasons to assume that the general theoretical evidence for Stackelberg equilibria is not likely to find definite support in experimental markets. First and most importantly, the theory so far has ignored the coordination problem firms face in a duopoly with endogenous timing.\(^4\) There are always two Stackelberg equilibria with either firm as the Stackelberg leader.

\(^1\)Matsumura (1998) shows that this general conclusion does not hold in Cournot oligopolies with more than two firms and with more than two production periods. In an n-firm oligopoly playing HS’s game with action commitment, at least n - 1 firms choose the first production period endogenously. The generalized Stackelberg equilibrium in which each firm chooses a different production period never occurs except in duopoly.

\(^2\)In Saloner’s (1987) model firms may produce their quantity in both periods. Robson’s analysis is restricted to linear demand and cost. Moreover, he assumes an interest rate on production in the first period.

\(^3\)A simultaneous move Cournot equilibrium also exists in Robson (1990) if the interest rate on first period production is equal to zero.

\(^4\)A notable exception is van Damme and Hurkens (in press) who analyze the HS extended game with action commitment in the presence of cost differences. Also their model has two pure strategy Stackelberg equilibria. However, applying the tracing procedure (Harsanyi and Selten, 1988), a unique Stackelberg equilibrium with the efficient firm as the Stackelberg leader is selected.
A priori, there is no reason why one equilibrium is preferable to the other. In an experimental market, severe coordination problems may arise.

The second reason makes the first one more forceful. Since firms are symmetric it is, from a behavioral perspective, difficult to see how players should always coordinate on an asymmetric equilibrium with large payoff differences. It is well known from the ultimatum bargaining literature (Göth, Schmittberger, and Schwarze, 1982) that many subjects exhibit an aversion against disadvantageous inequality in experiments. On top of the coordination problem, this inequality aversion might render the Stackelberg equilibria unappealing candidates for convergence in an experiment.

In a companion paper (Huck, Müller, and Normann, 1998; henceforth HMN), we studied Stackelberg duopoly with exogenous Stackelberg leader and follower roles. We found that followers often punish Stackelberg leaders who try to exploit their first-mover advantage. Given the empirical response function of the followers (which substantially differs from the theoretical prediction), Stackelberg leaders would be much better off producing less than prescribed by the subgame-perfect equilibrium. The parameters of the model and the experimental design underlying the experiment to be reported in this paper are the same as in HMN. The experiments in HMN also include some sessions with simultaneous-move Cournot duopolies. We shall therefore sometimes compare the results of HMN with Stackelberg and Cournot competition to the present study of endogenous timing.

The remainder of the paper is organized as follows. Section 2 discusses the theoretical background and introduces the market used in the experiment. Section 3 illustrates the experimental procedures. Sections 4 and 5 present the experimental results, and Section 6 concludes.

2 Theoretical background

Let us repeat the main characteristics of HS’s extended game with action commitment. This game modifies the standard duopoly model by allowing for two production periods before the market clears. Firms can choose their quantities in one of the two periods, \( t = 1, 2 \). A firm can move in period 1 by committing itself to a quantity—without knowing what its competitor is doing. By waiting until period 2, a firm can observe the other firm’s period-1 quantity (or the decision to wait). It is assumed that the market for the homogeneous good exists only at period 2 and that production costs do not depend on the production period.

Concerning the basic market game, HS rely on a number of rather general assumptions. They assume that there is under simultaneous play as well as under sequential play a unique equilibrium in pure strategies and that these two equilibria differ from each other. Further, they assume that the strategy sets are compact, convex intervals of \( \mathbb{R}^+ \).
A strategy of firm $i$ in the game can be described by the 3-tuple $(q_i^1, f_i(q_j^1), q_i^2)$ where $q_i^1$ either specifies an output for period 1 or indicates that the firm waits, i.e. $q_i^1 \in Q \cup \{W\}$ with $Q$ being the set of possible outputs and $W$ indicating the decision to wait. The function $f_i(q_j^1)$ is a mapping $Q \rightarrow Q$ specifying the firm’s reaction in case it has decided to wait while the other firm has chosen $q_j^1 \neq W$. Finally, $q_i^2$ specifies firm $i$’s quantity decision for the case that both firms have decided to wait.

The analysis of the extended game focuses on subgame-perfect equilibria. Subgame-perfection requires that $f_i(q_j^1)$ is the standard best-reply function of a firm $i$ facing firm $j$’s quantity $q_j^1$ on the basic market. Furthermore, subgame-perfection requires that $q_i^2$ is the Cournot-equilibrium quantity of the basic market game. In the following, we will often simplify notation and will characterize equilibria only by the taken actions.

HS identify three (subgame-perfect) equilibria in pure strategies: The two Stackelberg equilibria in which one firm commits in period 1 to its Stackelberg leader quantity and the other firm waits and reacts with the Stackelberg follower quantity. The third equilibrium has both firms producing the simultaneous play Cournot equilibrium quantities in period 1.

In our experiment we used the following linear inverse demand function

$$p(Q) = \max\{30 - Q, 0\}, \quad Q = q_1 + q_2. \quad (1)$$

Linear costs of production in both periods were given by

$$C_i(q_i) = 6q_i, \quad i = 1, 2. \quad (2)$$

For this specification, the HS predictions are as follows. In the two Stackelberg equilibria the Stackelberg leader chooses $q_i^L = 12$ in period 1 whereas the Stackelberg follower chooses $q_j^F = 6$ in period 2. This implies payoffs of $\Pi_i^L = 72$ and $\Pi_j^F = 36 \ (i, j = 1, 2; \ i \neq j)$ respectively. The simultaneous-move Cournot equilibrium actions are $q_i = 8$ resulting in payoffs of $\Pi_i = 64 \ (i = 1, 2)$ whereas the symmetric joint profit maximizing outputs are $q_i = 6$, implying payoffs of $\Pi_i = 72 \ (i = 1, 2)$.

In our experiment, subjects had to choose their quantities from a truncated and discretized strategy space, yielding a standard payoff bi-matrix. We had two versions—one with a large payoff matrix where subjects had to choose integer quantities between 3 and 15 and one with a smaller strategy space. In the second version subjects could only choose among the quantities 6, 8 and 12. We refer to the first version as the one with a “large payoff matrix” and to the second as the one with a “small payoff matrix”. For the rest of this section, we shall only discuss the large matrix. We will come back to the theoretical predictions for the sessions with the small matrix in Section 5.
The truncated and discretized strategy space is an important difference to HS’s modelling assumptions. First, discretized Cournot matrix games derived from linear demand and cost may exhibit multiple Nash equilibria (see Holt, 1985). To avoid such multiplicity of equilibria, the entries in the payoff table differed slightly from those implied by equations (1) and (2) (see Appendix B). As a consequence, best-replies are unique in the basic game and there is one simultaneous-move Cournot equilibrium and two sequential-move Stackelberg equilibria in the extended game, namely the equilibria mentioned above.

The discretized strategy space has a second consequence: There exists a variety of mixed strategy equilibria for the setup we have chosen. As HS require equilibria to be in undominated strategies, we focus on mixed equilibria fulfilling this property. More specifically, we analyze the truncation of the extended game, in which the function $f_t$ are standard best-response functions and in which $q^*_t$ is the Cournot equilibrium quantity $8$ (see above.) In this truncated game (in which the strategy sets are simply given by \{3, 4, ..., 14, 15, W\}) the strategies 3, 4, 5, 13, 14, and 15 are strictly dominated. Among the remaining strategies, the quantities 6, 7, 8, and 9 are weakly dominated by the wait strategy $W$. This leaves us with the set \{10, 11, 12, W\}. Thus, we can focus on the 4x4 game depicted in Table 1. It is easy to verify that this 4x4 game has only one symmetric mixed equilibrium in which both players choose to wait with probability $3/5$ and produce quantity 10 with the complimentary probability $2/5$. We refer to this equilibrium as the mixed Stackelberg equilibrium.

Table 1: Truncation of the extended game (large matrix).

<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>70</td>
</tr>
<tr>
<td>11</td>
<td>33</td>
<td>22</td>
<td>11</td>
<td>66</td>
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<tr>
<td>12</td>
<td>24</td>
<td>12</td>
<td>0</td>
<td>72</td>
</tr>
<tr>
<td>W</td>
<td>49</td>
<td>42</td>
<td>36</td>
<td>64</td>
</tr>
</tbody>
</table>
3 Experimental procedures

The computerized experiment\(^7\) was conducted at Humboldt-University in November 1998. In the three sessions with the large matrix, each consisting of 30 rounds, 10 subjects were participating.\(^8\) Additionally, we ran four ten-round sessions using a small payoff matrix. Again, 10 subjects were participating in each session. Thus, altogether 70 subjects participated in the experiment. They were students from various fields, mainly students of economics, business administration and law.\(^9\) The sessions with the large matrix lasted about 90 minutes, the sessions with the small matrix about 50 minutes.

In the instructions (see Appendix A) subjects were told that they would act as a firm which, together with another firm, serves one market, and that in each round both were to choose the period of production and the quantity. In all sessions subjects were informed that in each round pairs of participants would be randomly matched.\(^10\) After having read the instructions, participants could privately ask questions to the experimenters.

Subjects were informed that at the end of the experiment three of the thirty rounds (large matrix) would be randomly selected to determine the actual monetary profit in German marks. The numbers given in the payoff tables were measured in a fictitious currency unit called “Taler”. The monetary payment was computed by using an exchange rate of 10:1 and adding a flat payment of DM 5.\(^11\) (In the sessions with the small payoff matrix (see below) two out of ten rounds were randomly selected to determine real payment.) Subjects’ average earnings were DM 20.60 (\$ 11.44) in the thirty-round sessions and DM 17.22 (\$ 9.57) in the ten-round sessions (including the flat payment).

In the sessions with the large payoff matrix, before the first round was started, subjects were asked to answer two control questions (which were checked) in order to make sure that everybody had full understanding of the payoff table. After each round (with both small and large matrix) subjects got individual feedback about what happened in their market, i.e., the computer screen showed the production period, the quantity, and the profit of both duopolists.

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\(^7\)We thank Urs Fischbacher for letting us use his software toolbox “z-Tree”.

\(^8\)In one of these sessions only the results up to round 29 were saved. After the play of round 30 of this session the network broke down such that the results of the last round were not saved.

\(^9\)Subjects were either randomly recruited from a pool of potential participants or invited by leaflets distributed around the university campus.

\(^8\)We think that randomly matched duopoly pairs, rather than fixed pairs, are appropriate when testing the predictions of the HS model. In HMN (with exogenous timing), the sessions with fixed duopoly pairs were considerably collusive, particularly in the simultaneous-move treatment. Even when firms moved sequentially à la Stackelberg there was some collusion. It is doubtful that, with fixed pairs and with endogenous timing, less collusion would be observed.

\(^11\)This payment was made since subjects could have made losses in the game.
<table>
<thead>
<tr>
<th></th>
<th>in period 1</th>
<th>explicit followers</th>
<th>simult. dec. in period 2</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aver. quantity</td>
<td>9.15</td>
<td>8.39</td>
<td>8.40</td>
<td>17.70</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>1.91</td>
<td>1.75</td>
<td>1.67</td>
<td>1.93</td>
</tr>
<tr>
<td># of observations</td>
<td>543</td>
<td>207</td>
<td>140</td>
<td>890</td>
</tr>
<tr>
<td>HMN aver. quant.</td>
<td>10.19</td>
<td>8.32</td>
<td>8.07&lt;sup&gt;b&lt;/sup&gt;</td>
<td>18.51&lt;sup&gt;a&lt;/sup&gt; / 16.14&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>2.45</td>
<td>2.07</td>
<td>1.61</td>
<td>2.86 / 3.21</td>
</tr>
<tr>
<td># of observations</td>
<td>220</td>
<td>220</td>
<td>240</td>
<td>220 / 240</td>
</tr>
</tbody>
</table>

Table 2: Aggregate results (<sup>a</sup> Stackelberg market, <sup>b</sup> Cournot market)

4 Experimental results (large matrix)

The results of sessions with the large matrix are reported in three subsections. Section 4.1 presents aggregated results. Group effects are examined in Section 4.2 and individual behavior is explored in Section 4.3. We will concentrate on preemptive commitments in the first period of a round, on the reaction of endogenous Stackelberg followers, on the behavior of two waiting firms deciding simultaneously in the second period, and on overall market outcomes. As mentioned in the introduction, in HMN we investigated Stackelberg and Cournot duopoly markets in which roles were exogenously fixed. In these experiments, 10 successive rounds were played using the same payoff matrix.<sup>12</sup> Whenever useful we will compare the results of the current experiment with the results of HMN.

4.1 Aggregated results

Table 2 presents a summary of experimental results on an aggregate level. Table 2 also shows the results of the Stackelberg and Cournot markets with random matching as observed in HMN. Inspection of Table 2 reveals that in the endogenous timing sessions in 543 out of 890 cases (61%) subjects committed themselves in period 1. In 347 out of 890 cases (39%) subjects decided to wait.

When committing themselves in \( t = 1 \), subjects chose on average about one unit less than in the Stackelberg experiment with exogenous timing. Since subjects who endogenously got into the position of a Stackelberg follower chose about the same quantity as exogenous Stackelberg followers the differences between total quantities in both versions (17.70 vs. 18.51) seems to be entirely due to the fact that exogenous Stackelberg leaders committed to higher quantities. Note furthermore that the average quantity chosen in markets in which decisions were made simultaneously in the second period are slightly higher than in the Cournot duopolies in HMN (8.40 vs. 8.07).

<sup>12</sup>The experiments were run with pen and paper.
Behavior in the first period: To illustrate first-period behavior, Figure 1 shows absolute frequencies (across all sessions) of quantities chosen in the first period of a round. In the left panel of Figure 1 these frequencies are shown separately for the first (rounds 1-15) and the second half (rounds 16-30) of the experiment. The right panel of Figure 1 shows absolute frequencies for all rounds of the experiment. First of all, recall that choosing quantities of 3, 4, 5, 13, 14 and 15 are strictly dominated actions in the 2-stage quantity commitment game. According to Figure 1 these quantities are rarely chosen in the first period. Altogether, choices in the first period are quite dispersed over the range of quantities from 6 to 12. The Stackelberg leader action, 12, was chosen in only 53 out of 543 cases (9.8 %). Instead, we observe that the quantities 8 and 10 were chosen most often. This is true with regard to earlier and later play and, as a consequence, it is also true over the whole experiment (#8 = 142 (26.2 %), #10 = 139 (25.6 %)). Moreover, whereas the absolute frequencies with which quantities 8, 10 and 12 were chosen remain rather constant over the two halves this is not true for quantities 6, 7, 9 and 11. Here we observe that quantities of 9 and 11 were chosen less often in the second half whereas quantities of 6 and 7 are chosen more often in the second half of the experiment. In fact, the frequency of choosing quantities 6 and 7 increases from 9.9 % in the first to 24.4 % in the second half. Thus behavior becomes more cooperative over time. Regarding the high frequency of $q = 10$, recall that playing this quantity in the first period is part of the symmetric mixed-strategy equilibrium in undominated strategies.

Figure 1: Absolute frequencies of quantities chosen in the first period in the first (rounds 1-15) and second half (rounds 16 - 30) (left) and for all rounds (right).

Behavior of endogenous Stackelberg followers: Figure 2 shows best responses as well as average observed responses of endogenous Stackelberg followers. Additionally, it shows average responses of exogenous Stackelberg followers as observed in HN. The empirical response function of exogenous Stackelberg followers virtually coincides with the theoretical best response
function as long as the leader’s quantity is smaller or equal to 7. However, exogenous Stackelberg followers produce on average more than one unit more than prescribed by the best response function if Stackelberg leaders produce more than 7 units. With regard to the empirical response function of endogenous Stackelberg followers we observe that it first lies below the response function of exogenous followers (for \(q^L < 7\)), then almost coincides with it (for \(7 \leq q^L \leq 9\)) and, finally, lies above it (for \(q^L > 9\)). Thus one clearly sees that (i) endogenous Stackelberg followers reward cooperation more often and (ii) endogenous Stackelberg followers punish harder than exogenous Stackelberg followers if leaders try to exploit the first-mover advantage.

The best reply function is given by \(q^F = 12 - 0.5q^L\) (for continuous actions)\(^{13}\). Estimating the followers’ actual response function by a simple linear regression model for the exogenous timing experiment one gets \(q^F = 6.98 + 0.154q^L\) (for a more complex regression, see the next subsection). Surprisingly, the response function is upward sloping. Even more interesting is to look at the response function for the first and the second half of the experiment separately. For the first half (rounds 1–15) we get \(q^F = 9.596 - 0.149q^L\) whereas for the second half (rounds 16–30) of the experiment we get \(q^F = 4.59 + 0.442q^L\). The striking result is that, over time, the empirical response function clearly moves away from the best response function. In the second half the reward-for-cooperation-and-punishment-for-exploitation scheme second movers apply

\(^{13}\)A linear regression estimation of the best reply function for the discretized game yields \(q^F = 12.1 - .49q^L\).
becomes more pronounced which probably explains the higher frequency of collusive choices taken in the first period (see above).

**Behavior in case of simultaneous decisions in the second period:** When deciding simultaneously in the second period, subjects play a standard Cournot market. The average quantities chosen is 8.40 with a standard deviation of 1.67. This does not vary significantly across the first and the second half (8.30 (1.88) and 8.55 (1.91)). Interestingly, the average quantity is larger than the observed average in the simultaneous-move Cournot duopolies of HMN. That is, though subjects are strategically in exactly the same situation, they apparently perceive the situation differently which leads to different results.

**Market outcomes:** We shall distinguish between rational outcomes and boundedly rational outcomes. Table 3 shows absolute and relative frequencies of outcomes classified along these lines. We define rational outcomes as outcomes which stem from strategies which are either part of one of the pure equilibria or part of the mixed equilibrium in undominated strategies. These strategies are all those in which firms choose in \( t = 1 \) the quantities 8, 10, or 12 or opt to wait, and in which they play best replies in \( t = 2 \). To the choice of 8 we refer as the Cournot action, to the commitment of higher quantities in \( t = 1 \) we refer as Stackelberg actions. Playing rational strategies might lead to an equilibrium, but coordination failures can also occur, e.g., both firms could play Stackelberg leader (that is, Stackelberg warfare), or one firm could play Cournot in \( t = 1 \) while the other plays Stackelberg leader.

We refer to collusive strategies (i.e., to produce 6 or 7 in either period) and to punishment strategies of followers (i.e., to produce strictly more than the best reply in \( t = 2 \)) as boundedly rational strategies. Collusion may be successful, it may be exploited in \( t = 2 \), or it may fail when one firm plays 6 or 7 in \( t = 1 \) while the second firm plays Cournot or Stackelberg leader in \( t = 1 \).

Among the remaining strategies (3, 4, 5, 9, 11, 13, 14 and 15), only 9 and 11 are chosen frequently. It is not very surprising that subjects choose 9 and 11 more often than, say, 3 or 14 since it seems reasonable to assume that subjects are more likely to choose non-equilibrium actions that are close to equilibrium actions (Simon and Stinchcombe, 1995). For this reason, we also report the results (in parenthesis) when 9 and 11 are viewed as quasi equilibrium strategies. Somewhat arbitrarily, we count 9 as a Cournot action and 11 as a Stackelberg leader action.

Out of 445 outcomes, 257 (337) can be classified in our scheme. The remaining 188 (108) outcomes involve the choice of a dominated strategy, or a non-best reply in \( t = 2 \) not punishing the leader. The most striking fact is that Stackelberg equilibria occur only rarely (24 (33) outcomes or 5.4% (7.4%)). A subject committing itself in \( t = 1 \) faces the risk of a coordination failure (48 (71) cases), or being punished (43 (55) cases). Even successful collusion occurs more often than a Stackelberg equilibrium. However, the collusive strategies are likely to be exploited or to fail coordination.
<table>
<thead>
<tr>
<th>Market outcome</th>
<th>Type</th>
<th># of cases</th>
<th># of cases incl. quant. 9 and 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cournot</td>
<td>equilibrium</td>
<td>64 (14.4 %)</td>
<td>93 (20.9 %)</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>equilibrium</td>
<td>24 (5.4 %)</td>
<td>33 (7.4 %)</td>
</tr>
<tr>
<td>Stackelberg/Cournot</td>
<td>coord. failure</td>
<td>27 (6.1 %)</td>
<td>41 (9.2 %)</td>
</tr>
<tr>
<td>Stackelberg warfare</td>
<td>coord. failure</td>
<td>21 (4.7 %)</td>
<td>30 (6.7 %)</td>
</tr>
<tr>
<td>Stackelberg punished</td>
<td>rational/bound.</td>
<td>43 (9.7 %)</td>
<td>55 (12.4 %)</td>
</tr>
<tr>
<td>Collusion (successful)</td>
<td>boundedly rational</td>
<td>25 (5.6 %)</td>
<td>25 (5.6 %)</td>
</tr>
<tr>
<td>Collusion (exploited)</td>
<td>bound. /rational</td>
<td>19 (4.3 %)</td>
<td>19 (4.3 %)</td>
</tr>
<tr>
<td>Collusion (failed)</td>
<td>bound. /rational</td>
<td>34 (7.6 %)</td>
<td>41 (9.2 %)</td>
</tr>
<tr>
<td>others</td>
<td></td>
<td>188 (42.2 %)</td>
<td>108 (24.3 %)</td>
</tr>
<tr>
<td><strong>∑</strong></td>
<td></td>
<td>445 (100 %)</td>
<td>445 (100 %)</td>
</tr>
</tbody>
</table>

Table 3: Number of outcomes (large matrix)

<table>
<thead>
<tr>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>54.97</td>
<td>58.49</td>
<td>59.42</td>
<td>56.63</td>
<td>52.24</td>
<td>44.00</td>
<td>32.53</td>
<td>53.54</td>
</tr>
</tbody>
</table>

Table 4: Average earnings of actions in period $t = 1$ (large matrix)

The Cournot equilibrium is the most frequent outcome (64 (93 cases). Playing Cournot is also (ex post) the most successful strategy across all sessions; in contrast to playing Stackelberg leader, it is not punished by followers in $t = 2$, and, when clashing with a collusive firm in $t = 1$, it yields a profit at least as high as an (equilibrium) Stackelberg leader. Table 4 contains the average earnings certain actions chosen in the first period yield. Playing a Stackelberg leader action (10, (11,) 12) yields a profit strictly worse than the collusive strategies. The fact that the wait strategy does worse than any quantity in $t = 1$ smaller than 10 is explained by the high costs followers inflicted on themselves by punishing greedy leaders.

### 4.2 Group effects

In this subsection we will briefly examine group effects by looking at the results for each of the sessions separately. Figure 3 shows absolute frequencies of quantities chosen in earlier and later rounds of each session. We also estimated, separately for the three sessions and for the pooled data, the simple regression model given in Table 5. The dependent variable in the equation given in Table 5 is the observed quantity, $q^f$, of followers. The two explanatory variables included are the quantity of Stackelberg leaders, $q^L$, and a dummy, $Half$, representing the first respectively second half of the session. The dummy was introduced in order to control for experience effects. It turns out that behavior is quite different across sessions.

In session 1, the quantities 7 and 10 were chosen most often whereas the Cournot quantity of 8 is rarely chosen. Comparing behavior in the two halves of this session, the most striking result is that quantity 7 was chosen more than twice as often in the second half than in the first half. Thus there is a clear shift towards more cooperative behavior. Inspecting Table 5,
we find that followers chose a rather flat response function in the first half. More or less, they played the Cournot quantity regardless of what leaders did. So, it seems that followers tried to educate leaders to play Cournot. (After all the best reply to such a response strategy is playing Cournot!) In the second half the response function is upward sloping and has a smaller intercept. As we have seen on the aggregate level, endogenous followers learn to behave in a reciprocal fashion. In turn, this is learnt by endogenous leaders who choose more collusive actions in the second half.

Next, consider session 2. In contrast to session 1 we observe that the quantity 7 was chosen only twice whereas the Cournot quantity 8 is the one that was chosen most often. We also observe that the frequency with which quantity 6 was chosen clearly increases from the first to the second half. Note, furthermore, that quantities smaller than 6 or larger than 12 were virtually never chosen during the course of this session. Regarding followers’ behavior, session 2
Estimating equation: 
\[ q^F = \beta_0 + \beta_1 q^F + \beta_2 \text{Half} + \beta_3 \text{Half} \times q^F + \varepsilon \]

<table>
<thead>
<tr>
<th></th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>8.46**</td>
<td>-0.05</td>
<td>-5.52**</td>
<td>0.70**</td>
</tr>
<tr>
<td></td>
<td>(6.89)</td>
<td>(-0.43)</td>
<td>(-3.31)</td>
<td>(3.93)</td>
</tr>
<tr>
<td>Session 2</td>
<td>8.41**</td>
<td>0.00</td>
<td>-5.57*</td>
<td>0.70**</td>
</tr>
<tr>
<td></td>
<td>(4.11)</td>
<td>(-0.01)</td>
<td>(-2.21)</td>
<td>(2.62)</td>
</tr>
<tr>
<td>Session 3</td>
<td>11.73**</td>
<td>-0.38**</td>
<td>-3.38*</td>
<td>0.36*</td>
</tr>
<tr>
<td></td>
<td>(10.27)</td>
<td>(-3.06)</td>
<td>(-2.04)</td>
<td>(2.00)</td>
</tr>
<tr>
<td>Pooled data</td>
<td>9.60**</td>
<td>-0.15</td>
<td>-5.01**</td>
<td>0.59**</td>
</tr>
<tr>
<td></td>
<td>(11.63)</td>
<td>(1.72)</td>
<td>(4.48)</td>
<td>(4.95)</td>
</tr>
</tbody>
</table>

Table 5: Regression results. 
Note: ** (*) significant at the 1 % (5 %) level. Absolute value of asymptotic t-statistics in parentheses.

is virtually identical to session 1. As in session 1, followers start by playing Cournot (regardless of the leader’s choice) and then shift to an upward sloping response function.

Finally, consider session 3. Whereas the collusive quantity 6 was chosen only once we observe that quantities 8 and 10 were chosen most often. Interestingly, the number of choices of quantity 8 decreases whereas the number of choices of quantity 10 increases from the first to the second half of the experiment. With respect to followers we find that they start with a response function very “close” to the rational one. However, they change their behavior in the second half where their response function is similar to those of followers in the first halves of sessions 1 and 2: It is more or less flat and prescribes the Cournot quantity. It would have been interesting to see whether this process would have continued if there had been more rounds. In any case, the more aggressive behavior of leaders in session 3 can be explained by the more rational response function of followers in the first half and the less reciprocal one in the second.

### 4.3 Individual behavior

An interesting question is whether behavior converged on the individual level. Do some subjects always commit themselves? If so, which quantity do they play? Are there pure followers, possibly playing best reply? More specifically, we searched for subjects who had chosen the same production period in at least 25 of the 30 periods. As a result, we found only few subjects who chose the same production period over most rounds of the experiment: Five subjects almost always committed themselves and three subjects almost always waited until \( t = 2 \). As the total number of subjects was 30, this number is relatively small. However, the behavior of these subjects is quite telling.
The five subjects almost always committing can not generally be classified as pure Stackelberg leaders. One subject produced a quantity of 6 in 28 out of 30 rounds. This subject is thus a pure collusive player. A second subject produced a quantity of 8 in 22 out of 28 rounds in which he or she committed in $t = 1$ (average quantity produced in $t = 1$ was 8.23) — a Cournot player. A third subject must also be classified as a Cournot player (average quantity 8.38), though this person also experimented with the quantities 7 and 10. Another subject chose 12 in 15 out of 30 rounds and 8 in the remaining rounds. In accordance with our aggregate and group data, this person played 12 in the beginning and, being discouraged, played exclusively 8 over the last 10 rounds. Only one subject may be classified as a Stackelberg leader (average quantity: 10.00), but even this subject occasionally produced 8 in $t = 1$. He or she started with producing 12, but then reduced the output to 10 or 8 over the last third of the experiment.

The behavior of the subjects who almost always waited is strikingly homogenous. Looking at the periods in which they actually became Stackelberg followers, they played tit-for-tat to a very large extent. One subject strictly played tit-for-tat. That is, this person produced exactly the same quantity as the Stackelberg leader in every period except one. A second subject very often did so, though occasionally punishing Stackelberg leaders even more severe than plain tit-for-tat would have prescribed. The third subject played tit-for-tat in each of the last 14 rounds of the experiment (and occasionally earlier on).

To summarize, looking at these individuals consistently committing or waiting, there is no support for subgame perfect play except for the Cournot players.

4.4 Discussion

In this subsection we summarize the main results of the experiments with the large payoff matrix and discuss their implications. In view of the theory we embarked on testing, the most important result is the following:

**Result 1** HS’s predication fails. Endogenous Stackelberg equilibria are extremely rare and their frequency does not increase with experience.

The next two results implicitly offer explanations for this.

**Result 2** Subjects have problems to coordinate their actions. In roughly 25% of all encounters we find evidence for coordination failure.

**Result 3** Endogenous Stackelberg followers exhibit an aversion against disadvantageous inequality. Over time they learn to employ reciprocal (upward sloping) response functions, rewarding cooperation and punishing exploitation.

As a consequence of this we find
**Result 4** Cooperation and collusion are increasing over time.

Finally, we find

**Result 5** Cournot equilibria are the most frequent outcomes.

In spite of these five results, it is difficult to offer a complete description of behavior. Although we can indicate some trends, we do not find convergence. Rather, behavior is quite dispersed, also when subjects have gathered experience. Furthermore, it is not perfectly clear how to interpret some of the frequently chosen actions. For example, the choice of $q_1^1 = 10$ might be interpreted as a compromise between full exploitation of the theoretical first-mover advantage but it can also be seen as the outcome of mixed-equilibrium play. With strategies not in the support of the equilibria we focused on it is even harder to assess their precise meaning. As a consequence of this, we conducted four further sessions with a smaller payoff matrix which are discussed in the following section.

**5 Experimental results (small matrix)**

In the sessions with the small payoff matrix, subjects had to choose their quantities from the set $\{6, 8, 12\}$. The following reduced matrix was the basis for these sessions.\(^{14}\)

$$
\begin{array}{c|ccc}
\text{Firm 1} & 6 & 8 & 12 \\
\hline
6 & 72 & 80 & 36 \\
8 & 60 & 64 & 48 \\
12 & 72 & 48 & 0 \\
\end{array}
$$

The time horizon was reduced to 10 periods. Everything else in the design remained unchanged.

The equilibrium predictions with the small matrix, concerning the pure strategy equilibria, are similar to those of the large matrix. However, now there exists a symmetric mixed equilibrium in which firms randomize over committing to 12 and waiting.\(^{15}\) The equilibrium probability for a commitment is $p = 2/11$. As with the large matrix, the Cournot-like equilibria are in weakly dominated strategies.

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\(^{14}\)In the instructions, we actually labelled the strategies 6, 8 and 12 by 1, 2 and 3. The labels 6, 8, and 12 are meaningless for the subjects (recall that they did not know the demand and cost parameters of the model). Moreover, the difference between 8 and 12 is larger than the difference between 6 and 8. So the action 12 might appear as a rather extreme choice to subjects and, hence, they might be biased against this action. In order to avoid confusion, here in the paper, we refer to quantities 6, 8 and 12.

\(^{15}\)There exists also a continuum of mixed equilibria (in weakly dominated strategies) in which firms randomize over committing to quantity 8 in period 1 and the wait strategy.
<table>
<thead>
<tr>
<th></th>
<th>in period 1</th>
<th>explicit followers</th>
<th>simult. dec. in period 2</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aver. quantity</td>
<td>8.65</td>
<td>7.89</td>
<td>7.60</td>
<td>16.05</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>2.24</td>
<td>1.22</td>
<td>1.21</td>
<td>1.64</td>
</tr>
<tr>
<td># of observations</td>
<td>136</td>
<td>94</td>
<td>170</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 6: Aggregate results

5.1 Aggregated results

Table 6 presents a summary of experimental results on an aggregate level. We observe that in 136 out of 400 cases (34%) subjects committed themselves in period 1. In 264 out of 400 cases (66%) subjects decided to wait. The proportion of committing firms is much smaller than that observed with the large matrix.

Table 7 summarizes behavior for the first five (top), the second five (middle), and all ten rounds (bottom), respectively. The table consists of 3x3 matrices. In each line the left matrix shows all quantity decisions for the case of both firms producing in period 1, the middle matrix shows output decisions in the case of endogenous Stackelberg leaders and followers, and the right matrix shows output combinations for the case of both firms producing in the second period.

Table 8 shows average payoffs of the four choices possible in the first period. Finally, Table 9 classifies the market outcomes according to the scheme we developed above. Note that, with the small payoff matrix, the classification of market outcomes is unique since there are no actions which are close to the collusive, Cournot or Stackelberg leader action.

With respect to our main question, the result is clear-cut. Endogenous Stackelberg equilibria occur even less frequently (5%) than with the large matrix. If players are to commit themselves in the first period they rather choose the Cournot or the collusive action instead of the Stackelberg leader action. Thus, HS’s theoretical predictions clearly fail although the game is now considerably less complex than before.

The increased simplicity of the game has further effects: Unclassifiable (not even boundedly rational) outcomes virtually disappear and coordination failure becomes less of an issue (4.5% vs. 15.8%). At the same time Cournot outcomes become much more frequent (45% vs. 20.7%).

The frequencies of successful and unsuccessful collusion are roughly similar to those in the large-matrix version. Furthermore, we find that endogenous Stackelberg followers punish harder in the second half of the experiment than in the first. However, positive reciprocity does not increase, i.e. followers play almost always best replies when confronted with leaders who made collusive choices.

Ex post, the best first-period choice has been to wait as Table 8 reveals. This explains why commitment in the first period is much rarer in the sessions with the small matrix than in the sessions discussed above where commitment paid more than waiting (34.5% vs. 61%).
First half (rounds 1-5):

\[
\begin{array}{ccc}
6 & 8 & 12 \\
t = 1 & 6 & 1 \\
& 2 & \\
t = 1 & 8 & 4 \\
& 12 & - 0 \\

t = 1 & 6 & 8 & 12 \\
t = 2 & 6 & 1 & 11 \\
& 8 & 0 \\
& 12 & 6 & 6 \\	n = 2 & 8 & 0 & 12 \\
\end{array}
\]

Second half (rounds 6-10):

\[
\begin{array}{ccc}
6 & 8 & 12 \\
t = 1 & 6 & 0 \\
& 5 & \\
t = 1 & 8 & 1 \\
& 15 & \\
& 12 & - 0 \\

t = 1 & 6 & 8 & 12 \\
t = 2 & 6 & 1 & 8 \\
& 24 & 1 \\
& 12 & 4 & 8 \\
t = 2 & 8 & 1 \\
& 12 & - 0 \\
\end{array}
\]

All rounds:

\[
\begin{array}{ccc}
6 & 8 & 12 \\
t = 1 & 6 & 1 \\
& 6 & 2 \\
& 8 & -3 \\
& 12 & -0 \\

t = 1 & 6 & 8 & 12 \\
t = 2 & 6 & 19 & 1 \\
& 8 & 3 \\
& 12 & 10 \\
t = 2 & 6 & 28 & 0 \\
& 12 & -0 \\
\end{array}
\]

Table 7: Summary of experimental results in the sessions with the small payoff matrix: Numbers of outcomes in case of simul. decisions in period 1 (left), in case of seq. decisions (middle) and in case of simul. decisions in period 2 (right).

<table>
<thead>
<tr>
<th>6</th>
<th>8</th>
<th>12</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>59.25</td>
<td>61.33</td>
<td>51.79</td>
<td>62.62</td>
</tr>
</tbody>
</table>

Table 8: Average earnings (small matrix)

<table>
<thead>
<tr>
<th>Market outcome</th>
<th>Type</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cournot</td>
<td>equilibrium</td>
<td>90 (45 %)</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>equilibrium</td>
<td>10 (5 %)</td>
</tr>
<tr>
<td>Stackelberg/Cournot</td>
<td>coord. failure</td>
<td>9 (4.5 %)</td>
</tr>
<tr>
<td>Stackelberg warfare</td>
<td>coord. failure</td>
<td>0 (0 %)</td>
</tr>
<tr>
<td>Stackelberg punished</td>
<td>rational/bound.</td>
<td>17 (8.5 %)</td>
</tr>
<tr>
<td>Collusion (successful)</td>
<td>boundedly rational</td>
<td>10 (5 %)</td>
</tr>
<tr>
<td>Collusion (exploited)</td>
<td>bound./rational</td>
<td>19 (9.5 %)</td>
</tr>
<tr>
<td>Collusion (failed)</td>
<td>bound./rational</td>
<td>36 (18 %)</td>
</tr>
<tr>
<td>other</td>
<td></td>
<td>9 (4.5 %)</td>
</tr>
<tr>
<td>[ \sum ]</td>
<td></td>
<td>200 (100 %)</td>
</tr>
</tbody>
</table>

Table 9: Number of outcomes (small matrix)
5.2 Group effects

Consider choices made in the first period. Although starting from different levels, average quantities chosen rise from the first to the second half in three of the four sessions (from 7.5 to 8.1 in session 1, from 8.0 to 8.9 in session 2 and from 8.9 to 9.1 in session 4). Only in session 3 in which subjects commit to high quantities in the first half, the average quantity decreases in the course of the experiment (from 10.1 to 9.2).

Furthermore, we observe that endogenous Stackelberg followers react essentially as prescribed by the best response function as long as endogenous Stackelberg leaders commit to quantities of 6 or 8. The best reply to both actions is to choose a quantity of 8 (see payoff matrix on page 15). Remarkably, average responses are rather homogeneous across both halves of a single session as well as across different sessions. Average responses to quantities of 6 or 8 in each half of the four sessions deviate from 8, if at all, by at most .5 units.

The only differences worth mentioning are due to reactions to the Stackelberg leader quantity of 12, the best response to which is choosing an output of 6. In sessions 1 and 2 endogenous Stackelberg leaders committing to quantity 12 in the first period are punished in both halves of these sessions (average response is 8.0). However, endogenous Stackelberg followers in sessions 3 and 4 react rather gently in the first half (6.7 vs. 7.0) whereas they punish much harder in the second half of the experiment (8.5 vs. 8.7).

5.3 Individual behavior

We selected subjects who either committed or waited in at least 9 of the 10 rounds. As the total number of commitments in \( t = 1 \) is smaller compared to the large matrix, it is not surprising that we found fewer subjects almost always committing (3 out of 40) and more subjects almost always waiting (9).

As with the large matrix, the subjects who committed themselves in \( t = 1 \) are by no means Stackelberg leaders. Instead, they must be classified as Cournot players. One subject chose the Cournot quantity in \( t = 1 \) in 10 out of 10 rounds. A second subject chose a quantity of 8 in 8 of 10 rounds while attempting to collude in two rounds. The third subject produced the Stackelberg leader quantity in \( t = 1 \) twice, but, in 6 out of 9 commitments, he or she played Cournot; particularly over the second half of the experiment (average quantity 8.60).

The behavior of the subjects who waited does not yield much insight because of the large proportion of Cournot outcomes on the aggregated level (45%). Occasionally, a Stackelberg leader was punished or an attempt to collude was exploited by these subjects. But most of the time, Cournot was answered by Cournot.
5.4 Discussion

What can we conclude from these results? In our view, the most important aspect of the small-matrix data is that the failure of HS’s theoretical predictions which we observed in the large game is not due to its complexity. Given the small amount of unclassifiable outcomes in the small-matrix game we can be sure that subjects understood the game well. Nevertheless, they did not play Stackelberg games. Furthermore, the failure of the theory cannot be exclusively attributed to the coordination problem. With the small matrix coordination failures are rare. Rather, it seems that subjects prefer symmetric Cournot outcomes over asymmetric outcomes.

6 Conclusion

Recent theoretical contributions have made forceful arguments supporting endogenous Stackelberg equilibria. The data of our experimental test show, however, that endogenous Stackelberg leadership does not occur to the degree theory predicts. The theoretical criterion to prefer pure strategy equilibria in undominated strategies over other equilibria turns out to be of little behavioral importance. Rather, we see the emergence of Cournot outcomes and, sometimes, collusive outcomes.

An important driving force for this result is the behavior of endogenous Stackelberg followers who learn to behave in a reciprocal fashion over time. In games with an exogenous first-mover advantage it is sometimes claimed that non-rational response functions of second movers are likely to disappear (or, at least, to become “more rational” when subjects have the opportunity to learn). Our data show that when timing decisions are endogenous the opposite may happen. In so far, the framework we studied here offers some hints about why the behavioral rule of reciprocity may have evolved. In our case, it helps subjects to resurrect initial symmetry.

Although our data refute HS’s predictions, this does not imply that endogenous Stackelberg leadership is generally unlikely to arise. In all our sessions we focused on symmetric firms and introducing cost asymmetries could change the picture. However, an examination of this hypothesis requires a fully fledged study of its own. We are currently preparing a new series of experiments to investigate this matter. There are more options for future research. For example, we have pointed out in the introduction that endogenous price leadership might be more likely to be observed in a laboratory than endogenous Stackelberg leadership as sequential decisions may increase the payoffs of both firms when their actions are prices. This is an interesting hypothesis, to be tested in experimental research.
References


**APPENDIX**

**A  Translated instructions of the 30-rounds sessions**

Welcome to our experiment! Please read these instructions carefully! Do not talk to your neighbors and keep quiet during the entire experiment. If you have any questions, give notice. We will answer them privately.

In our experiment you can earn different amounts of money, depending on your behavior and that of other participants matched with you. All participants read identical instructions.

You have the role of a firm which produces the same product as a second firm in the market. First you have to decide, at which time you want to produce. There are two possibilities: the first and the second production period. Afterwards, you decide on the quantity you want to produce.

If you choose the first production period, you decide about your production quantity immediately afterwards. At this point of time you will not know how the other firm has decided about its production period. If the other firm has chosen the second production period, it will be informed about the amount you have chosen before it decides about its own quantity.

If you choose the second production period, you get the following information before you decide on your quantity: If the other firm has made a decision about its quantity on the first period of production, you will be informed about this quantity. If the other firm has also chosen the second production period you will be informed about this.

Note that the profit of the rounds depend only on the chosen quantities, not on the choice of production periods.

In the attached payoff table, you can see the resulting profits of both firms for all possible choices of quantity.

The table reads as follows: At the head of a row the quantity of your firm is indicated, at the head of a column the quantity of the other firm is stated. In the cell at which row and column intersect, your profit is noted in the upper left and the other firm’s profit is stated in the lower right. All profits are expressed in a fictional currency, which we call "Taler".

The experiment consists of 30 rounds. After each round, you will be informed about the period of production, the quantity, and the profit of the other firm. You do not know with which
participant you serve the same market. You will be randomly matched with a participant each round. The decisions are made at the computer.

Anonymity is kept among participants and instructors, as your decisions will only be identified with your code number. You will discreetly receive your payment by showing your code number at the end of the experiment.

Concerning the payment note the following: At the end of the experiment three out of the thirty rounds will be randomly drawn to determine your payment. The sum of your profits in "Taler" of (exclusively) these three rounds determines your payment in DM. For 10 "Taler" you will receive 1 DM. In addition to this money, you will receive 5 DM independently of your profit during the thirty rounds.
### B Payoff table

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<th>Quant.</th>
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</tr>
</tbody>
</table>

The head of the row represents one firm’s quantity and the head of the column represents the quantity of the other firm. Inside the box at which row and column intersect, one firm’s profit matching this combination of quantities stands up to the left and the other firm’s profit stands down to the right.