Güth, Werner; Ivanova-Stenzel, Radosveta; Königstein, Manfred; Strobel, Martin

Working Paper

Auctions and fair division games under different price rules: Individual bid functions, prices and efficiency rates

SFB 373 Discussion Paper, No. 1999,101

Provided in Cooperation with:
Collaborative Research Center 373: Quantification and Simulation of Economic Processes, Humboldt University Berlin

Suggested Citation: Güth, Werner; Ivanova-Stenzel, Radosveta; Königstein, Manfred; Strobel, Martin (1999) : Auctions and fair division games under different price rules: Individual bid functions, prices and efficiency rates, SFB 373 Discussion Paper, No. 1999,101, Humboldt University of Berlin, Interdisciplinary Research Project 373: Quantification and Simulation of Economic Processes, Berlin, https://nbn-resolving.de/urn:nbn:de:kobv:11-10046888

This Version is available at:
https://hdl.handle.net/10419/61722

Auctions and Fair Division Games Under Different Price Rules: Individual Bid Functions, Prices and Efficiency Rates*

Werner Güth, Radosveta Ivanova-Stenzel,
Manfred Königstein and Martin Strobel

Humboldt-University of Berlin†

22.October, 1999

Abstract

In auctions an outside seller offers a commodity for sale and collects the revenue which is achieved. In fair division games the object is owned by the group of bidders. Consequently the auction's revenue is equally distributed among all bidders. In our experiment participants face four auction types (first versus scond price - auction versus fair division game) repeteadly. Due to the strategy method (one bids before learning one's private value) we can investigate the slope and curvature of individual bid functions, the evidence for risk aversion, the comparative statics with respect to the game type, the price expectations, and the efficiency rates.

JEL classification: D44, C91

Keywords: Auctions, Fair Division Games, Bid Function

*This paper is part of the EU-TMR Research Network ENDEAR. Support from the Deutsche Forschungsgemeinschaft (SFB 373, Quantifikation und Simulation ökonomischer Prozesse) is also gratefully acknowledged. We thank Veronika Grimm for helpful comments.

†Institute for Economic Theory III, Humboldt-University of Berlin, Spandauer Str.1, D-10178 Berlin, Germany, e-mail: {gueth, ivanova, mkoenig,strobel}@wiwi.hu-berlin.de. Part of this research was done while Königstein was visiting the Institute for Empirical Economics at the University of Zürich.

1 Introduction

Auctions and fair division games are two kinds of bidding mechanisms by which an object may be allocated among a group of bidders. In an auction, the object is offered for sale by an outside agent (the seller) who collects the revenue himself. Auctions are widely used to solve allocation problems. Some well–known examples are the "Aalsmeer (Dutch) Flower Auction", art auctions at Christie's or Sotheby's, or online auctions on the internet (see e.g. http://www.ebay.com).

In a fair division game, the object is collectively owned by the bidders. Accordingly, the revenue that is raised gets equally distributed among the bidders. Fair division games may be less familiar, but there are examples like conflict settlements in case of inheritance, divorce, or the termination of a joint venture. For instance, in the latter case usually only the former business partners are interested in buying the firm (e.g. due to private information about the future value of the enterprise). Consequently, these are often the only bidders, who, at the same time, (collectively) own the firm and will split the selling price.[1]

Within this paper we investigate auctions and fair division games within a laboratory experiment in which subjects submit sealed bids. The reselling values are independently and identically distributed. Furthermore, we study these bidding mechanisms under two different price rules: the first price rule, i.e., the selling price of the object is equal to the highest bid, and the second price rule, i.e., the selling price is equal to the second-highest bid.

We think it is important to empirically investigate bidding behavior within these games, since actual behavior might differ substantially from what is usually assumed in auction theory. Theoretical models should consider the restrictions that are imposed by actual bidding behavior. This might also have tremendous implications for social welfare and the revenue that is raised in the different types of games. For instance, given risk neutral equilibrium bidding as proposed in many theoretical models, the first price auction and the second price auction are payoff equivalent (see e.g. Wolfstetter, 1996). So, a risk neutral seller has no reason to prefer one or the other type of auction. However, empirically, there could be a substantial difference in the expected selling price that is induced via choosing an auction type. So, a seller might very well have a strong preference for one or the other mechanism. If bidding mechanisms differ in the social welfare they generate, this can be important e.g. for the government or other public authorities (see e.g. Cox, Roberson, and Smith, 1982). For a discussion of these questions and some evidence from previous laboratory experiments see the survey by Kagel (1995).

While experiments usually follow the "single bid approach" — i.e., each subject submits a single bid for a single, previously drawn reselling value — we employ the "bid function approach"; each subject submits a complete vector of bids (bid function) for each possible reselling value, which is subsequently drawn.[2] Specifically, we ask subjects in the experiment to develop bid functions

[1] Fair division games are related to the so called Hahn-Noll zero-revenue auctions, see Franciosi, Isaac, Pingry, and Reynolds (1993) for an experiemental study.

[2] For other experiments in which subjects had to submit bid functions see Selten and Buchta

Price Rule	Auction	Fair Division Game
price = highest bid	A1	F1
price = 2nd highest bid	A2	F2

Table 1: The four game types.

for the four different types of games, the first (respectively second) price auction and the first (resprectively second) price fair division game. A bid function specifies a bid for each of 11 possible private reselling values. After the bid functions have been submitted a reselling value is drawn randomly and independently for each subject and the game outcome is determined.

Compared to collecting single bids the "bid function approach" offers more information on individual bidding; we observe the bids for each reselling value. Also, submitting a bid function might induce more consistent bidding. For instance, if a subject plans to submit a monotonic bid function, but finds one of its bids (for some reselling value) being unreasonable, it may reconsider and readjust not only that very bid but the entire bid function. This might reduce inconsistencies and thus the noise in the data.

The paper here focusses on the comparative statics and the shape of bid functions as well as on prices and efficiency of the auction types.[3] It is structured as follows: Section 2 describes the experimental games and their theoretical benchmark solutions. Furthermore we inform about the experimental procedures and payments to subjects. In section 3 we investigate the structure of individual bid functions. Sections 4 and 5 compare differences in price and efficiency between the four game types. Section 6 concludes.

2 Auctions and Fair Division Games

2.1 Games and Theoretical Solutions

Experimental economics as well as auction theory distinguishes open and sealed-bid auctions. We will focus here on *sealed bid*–experiments in which a single object is to be allocated and for which each potential buyer has an *independent private value*. We investigate four different allocation rules which we refer to as game types (see table 1): *First Price Auction* (A1), *Second Price Auction* (A2), *First Price Fair Division Game* (F1) and *Second Price Fair Division Game* (F2).

Comparing bidding behavior under the first–price rule with bidding under the second–price rule is a familiar topic in experimental studies.[4] Fair division

(1998) as well as Güth (1998).

[3] In a companion paper (Güth, Ivanova, Königstein and Strobel, 1999) we investigate the adaptation processes and learning aspects of bidding behavior.

[4] See the survey by Kagel (1995).

Price	Auction	Fair Division Game
highest bid	$b_i^*(v_i) = \frac{n-1}{n} v_i$	$b_i^*(v_i) = \frac{n}{n+1} v_i$
	$E(p^*) = \frac{n-1}{n+1}$	$E(p^*) = \left(\frac{n}{n+1}\right)^2$
	$E(\pi_i^*(v_i)) = \frac{v_i^n}{n}$	$E(\pi_i^*(v_i)) = \frac{v_i^n}{n} + \frac{n-1}{n(n+1)}$
2nd highest bid	$b_i^*(v_i) = v_i$	$b_i^*(v_i) = \frac{n}{n+1} v_i + \frac{1}{n+1}$
	$E(p^*) = \frac{n-1}{n+1}$	$E(p^*) = \frac{n^2+1}{(n+1)^2}$
	$E(\pi_i^*(v_i)) = \frac{v_i^n}{n}$	$E(\pi_i^*(v_i)) = \frac{v_i^n}{n} + \frac{n-1}{n(n+1)}$

Table 2: Bid function $b_i^*(v_i)$, expected price $E(p^*)$, and expected payoffs of the equilibrium for risk neutral bidders and the four game types.

games differ from auctions in that the price at which the object is sold is distributed among all bidders. In auctions the price is earned by an outside agent, the seller. While the use of auctions to solve allocation problems is common, fair division games may be less familiar. For instance, allocating inheritance is a real life situation which resembles a fair division game. The object is collectively owned by the heirs who, in many cases, are the only bidders. Similar problems result when a joint venture is terminated.

Let v_i be a bidder's private value for the object to be sold, and suppose v_i is drawn for each player $i = 1, ..., n$ independently from a uniform distribution on the unit interval. If all bidders are risk neutral, the equilibrium bid function $b_i^*(v_i)$, expected equilibrium price $E(p^*)$ and expected equilibrium payoff $E(\pi_i^*)$ are as shown in table 2. For a derivation of these results see Güth and van Damme (1986).

2.2 Experimental Games and Procedures

In our experiment the private values $\tilde{v}_i$ did not vary continuously, but were drawn from the set

$$\tilde{V} = \{50, 60, ..., 150\}$$

with all values $\tilde{v}_i \in \tilde{V}$ being equally likely. These values are denoted in a fictitious currency ECU (experimental currency unit) at which subjects could resell the object to the experimenter. Subjects could choose bids $\tilde{b}_i$ as follows:

$$\tilde{b}_i \in \{0, 1, ..., 200\}.$$

For ease of comparison of the empirical bids $\tilde{b}_i$ and values $\tilde{v}_i$ with the theoretical solution given above all our analysis will be done for normalized bids b_i and

values v_i:

$$\begin{aligned} v_i &= \frac{\tilde{v}_i - 50}{100} \\ b_i &= \frac{\tilde{b}_i - 50}{100} \, . \end{aligned}$$

Accordingly the space of possible values is $V = \{0, 0.1, ..., 1\}$. When we refer to the theoretical benchmark case as described in table 2, we essentially neglect the discreteness of V.

Within a session each subject participated in 36 consecutive games of the four different types. Nine subjects formed a session group. In each of the 36 periods they were randomly partitioned into three groups of three bidders. The number of bidders involved in each game ($n = 3$) was commonly known, but not their identity. All subjects in all sessions played the same sequence of games. Within periods $t = 1$ to 3 they played A1–games, within $t = 4$ to 6 they played A2, in $t = 7$ to 9 the game type was F2 and in $t = 9$ to 12 it was F1. This comprised the first block of 12 games. Then they played block 2 (periods 13 to 24) and 3 (periods 25 to 36) in the same sequence as block 1.

Most participants were students of economics or business administration of Humboldt–University. They had been invited by leaflets to participate in an experiment anounced to last about three hours, and sessions actually took about that long. After entering the laboratory they were placed at isolated computer terminals. Communication among participants was not allowed during the session. While reading the instructions (see the appendix) they could privately ask for clarification or require help in handling the PC.

In each game they had to submit a complete bidding strategy (bid vector) $b_i(v_i)$. Thus, they had to enter a bid for each of the 11 values $v_i \in V$. The actual value v_i' was drawn thereafter. Payments were determined according to the game rules and using the submitted bidding strategies.[5] Subjects were informed on the screen about v_i', whether or not they were buyer, about the price p at which the object was sold and about their own payoff π_i in that game. Then the next game followed.

So, each game type applied nine times. In the first of these nine games the bid screen was blank and each subject had to enter a vector of 11 bids (one for each $v_i \in V$). In later periods the last bid vector for the same game type was displayed as default. It could be revised or submitted as it is. Of course, this may favor the status quo and may work against adjusting behavior over time. We did it for practical reasons. If subjects do not want to always adjust all bids, this saves time and helps to prevent getting bored by the task. Altogether we ran 6 sessions and collected 1944 bidding strategies (54 subjects times 36 games).

[5] The strategy method obviously provides more information than collecting only one bid for a single value. But since ex–post only one component of the bid vector is payoff–relevant, it lowers the incentives of bidding at each single value. By restricting the set V we have tried to achieve a reasonable compromise.

2.3 Payments

Subjects total earnings out of the 36 games ranged between 31 DM and 96 DM with a mean of 56 DM (about 33 US$ at the time of the experiment including a show up fee of 10 DM). In the first three sessions we used the same conversion rate for ECU (experimental currency unit) into cash for all four game types: 1 ECU = 0.05 DM. Theoretically and practically this generates rather asymmetric monetary incentives for auctions compared to fair division games. Güth (1998) tried to guarantee equal monetary incentives by adjusting the conversion rate such that equilibrium profits were equal for $v_i = 0.5$. Instead we used actually observed profits of sessions 1 to 3 to adjust the conversion rate in sessions 4 to 6 in order to induce equal expected payoffs; i.e., we applied the conversion rate that would have lead to equal payoffs in sessions 1 to 3. This meant for sessions 4 to 6 that one ECU was worth DM 0.2857 in auctions and DM 0.02857 in fair division games.

Essentially this means that we had a payoff–treatment: 3 sessions with equal conversion rate and 3 sessions with unequal conversion rate. Theoretically these payoff differences are irrelevant. And, since in all data analyses we ran, we did not find them being relevant, we will not discuss them any further.

3 Individual Bid Functions

3.1 Risk Neutral Equilibrium Bidding

Assuming risk neutral bidders is a theoretical benchmark case. We will first check whether the data are in line with risk neutral equilibrium (RNE) as it is described in table 2. Figures 2 to 5 (see the appendix) show the distributions of bids for $v_i = 0.5$ for the different game types. In this case ($v_i = 0.5$) RNE predicts the following bids:

$$b_i^*(\text{A1}) = \frac{1}{3}, b_i^*(\text{A2}) = \frac{1}{2}, b_i^*(\text{F1}) = \frac{3}{8}, \text{ and } b_i^*(\text{F2}) = \frac{5}{8} \quad .$$

Looking at the data we find that for A1 the mode of the distribution is close to $b_i^*(\text{A1})$. For A2 the mode is in fact at the theoretical prediction $b_i^*(\text{A2}) = \frac{1}{2}$. However, the modes for F1 and F2 are (roughly) at $\frac{1}{2}$ as well, even though this is not in line with RNE. Furthermore, all distributions exhibit considerable dispersion. We conclude:

Result 1: *By and large the data (exept for A2) reject the assumption of risk neutral equilibrium bidding.*

The distributions of bids for other reselling values $v_i \neq 0.5$, which we do not present here, lead to the same conclusion. In principle, the data might be explained by a theory that combines RNE and a sufficiently high error probability ("RNE plus noise"). However, the displayed distributions are quite asymmetric around the equilibrium prediction, especially for A1, F1 and F2, which rather suggests a systematic bias. We will show below that risk aversion may cause such biased bids and may partly explain these observations.

3.2 Slope and Curvature of Bid Functions

Suppose bidders are risk averse and exhibit different constant relative risk aversion (CRRA) utility functions $u_i(x) = x^{1-r_i}$, where the parameter r_i is individual i's coefficient of relative risk aversion. If r_i is randomly drawn from the unit interval with positive density for all risk attitudes between risk neutrality ($r_i = 0$) and extreme risk aversion ($r_i \to 1$), the equilibrium bid function in the first price auction is $b_i^* = \frac{2}{3-r_i} v_i$.[6] The equilibrium describes rational bidding within a heterogenous population of bidders; i.e., when r_i may vary between individuals. So, assuming risk aversion in this sense induces bid functions which are (1) strictly increasing and (2) linear in v_i.

In the following we will investigate the slopes and curvatures of the observed bid functions. Besides for A1 we will do this for A2, F1 and F2 as well. Remember that risk aversion does not influence optimal behavior in A2; equilibrium requires "true value"–bidding regardless which risk attitudes one assumes. So, in A2 risk aversion can not improve the fit of the theory compared to risk neutrality. Nevertheless we want to know whether the observed bid functions are indeed linear.

As far as we know, F1 and F2 have not been solved yet for the case of heterogeneous, risk averse bidders. For explorative reasons we determine slope and curvature of the empirical bid functions also in these cases (F1 and F2). It remains an open question whether such functions can be rationalized by some model of risk preferences (CRRA utility, for instance, *does not* induce linear bid functions in F1). So, while we do not derive bid functions for these cases, in the next section we will derive prediction areas.

To evaluate whether the bid functions were increasing and/or linear we estimated the following regression model:

$$b_i = \alpha_0 + \alpha_1 v_i^{low} + \alpha_2 v_i^{high} \qquad \text{(model 1)}$$

with

$$v_i^{low} = \begin{cases} v_i & \text{if } v_i < 0.5 \\ 0.5 & \text{otherwise} \end{cases}$$

and

$$v_i^{high} = \begin{cases} v_i - 0.5 & \text{if } v_i \geq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Thus, we fitted a piecewise-linear regression line to the data allowing for a kink at $v_i = 0.5$.[7] The rationale for this model is that it produces piecewise-linear approximations of convex or concave individual bid functions. Model 1 was estimated for each individual $i = 1$ to 54 and each period $t = 1$ to 36 separately, giving us a total of 1944 estimated individual bid functions [8].

[6] See e.g. Davis and Holt (1993), p. 287.

[7] A slightly simpler formulation of a piecewise–linear regression model may be: $b_i = \alpha_0 + \gamma_1 v_i + \gamma_2 v_i^{high}$ with all variables as defined above. However, this model is equivalent to model 1, and the latter is advantageous since we want to compare a_1 and α_2 below. For instance, if a bid function is perfectly linear, this results in $\alpha_1 = \alpha_2$.

[8] A separate analysis for each type will follow in section 3 below.

Slope		Curvature	
strictly increasing:	97%	concave:	32%
weakly decreasing:	3%	linear:	48%
		convex:	20%
(100% = 1944 estimated individual bid functions)			

Table 3: Slope and curvature of the estimated individual bid functions

In 1892 cases ($\approx 97\%$) the bid functions were strictly increasing, i.e. $\alpha_1 > 0$ and $\alpha_2 > 0$ (see table 3). The fit of these piecewise-linear approximations was remarkable: quite often the estimated individual bid function explained more than 90% of the variance ($R^2 > 0.9$). In fact, this was the case for 1837 (94%) of the observed bid functions.

To determine the curvature of the bid functions we computed the difference between the two slope coefficients $\Delta \equiv \alpha_2 - \alpha_1$. We consider a bid function as linear if $|\Delta| \leq \pm 0.025$. If $\Delta < -0.025$ ($\Delta > +0.025$), the respective bid function is classified as concave (convex). Due to these criteria about half of the estimated bid functions (48%) are linear, 32% are concave and 20% are convex.[9] Based on these findings we conclude:

Result 2: *The empirical bid functions are strictly increasing in v_i in almost all cases. Many are linear, which in case of A1 and A2 is in line with equilibrium behavior if one assumes heterogenous, risk averse bidders.*[10]

3.3 Bidding Areas

Since subjects may differ in their risk attitudes, behavior should not be expected as uniform. Thus we are interested in a prediction area for rational bid functions, if one allows for various kinds of (weak) risk aversion. More specifically we assume that the population of bidders may be heterogenous and that each bidder exhibits some kind of risk aversion (allowing for risk neutrality as a boundary case). Of course, there is a universe of models for risk averse preferences and it is impossible to explicitly solve them all. However, we want to argue that in all cases the prediction area is bounded above and below by two bid functions: namely, the RNE (of the respective game type) and the "true value"–bid function.

[9] Of course, we could have run formal tests for the theoretical restrictions imposed on α_1 and α_2. We preferred to simply report some descriptive measures since the usual assumptions for statistical testing are hardly satisfied here. Note that each regression is based on 11 data points drawn from a single subject. Furthermore we estimated 36 regressions for the same subject.

[10] Admittedly, the criterion for linearity which we chose is ad hoc. It could have been set more (or less) restrictive. About 11% of the bid functions were exactly linear. On the other hand, if the linearity criterion is set less restrictive, e.g. $|\Delta| \leq \pm \mathbf{0.05}$ (this means that the prediction area has doubled compared to the criterion we report here), the proportion of linear functions increases, but only to 56%.

In case of A2 this is trivial to show, since "true value"–bidding (=RNE) is a dominant strategy. To see why these bounds hold in A1 as well note the following:

1. Overbidding v_i is weakly dominated, and can never be rational. By true value bidding ($b_i = v_i$) a bidder gets zero for sure. Bidding below v_i induces a positive payoff in case i wins the auction and a zero payoff in case i does not win.

2. Any further reduction in b_i increases the payoff in case i wins, but at the same time reduces the probability of winning. In general, a risk neutral bidder will therefore choose a lower bid than a risk averse bidder given that both face the same (possibly mixed) population of bidders.

3. Remember that RNE–bidding maximizes bidder i's expected payoff if all bidders are risk neutral (and therefore submit RNE-equilibrium bids). If any bidder j bids above RNE, a risk neutral bidder i should bid above RNE as well.

Together, statements 1 to 3 establish our conclusion that the area for rational bidding in A1 is bounded below by RNE and bounded above by "true value"–bidding.[11] Similar statements can be given for fair division games. We summarize that the space of bid functions that may be rationalized by risk aversion is:

- bounded below by RNE and above by "true value"–bidding for A1, A2 and F1
- bounded below by "true value"–bidding and above by RNE for F2.

Figure 6 (see the appendix) displays these prediction areas for the differnt game types. Furthermore it reports the percentage of cases in which the estimated, piecewise–linear bid functions (see above) passed through these areas. These numbers will be referred to as hit rates (a hit is a case in which the data can be explained by the respective area theory). The hit rates are: 84% in A1, 37% in A2, 65% in F1 and 39% in F2.[12] We conclude:

Result 3: *Many bid functions pass through prediction areas derived from general models of risk aversion.*

[11] One can check this result even more easily for heterogenous CRRA bidders (see above). Obviously, the bid function $b_i^* = \frac{2}{3-r_i} v_i$ is equal to the RNE for $r_i = \mathbf{0}$ (risk neutrality) and converges to 'true value'–bidding for $r_i \to 1$ (extremely high risk aversion).

[12] Some technical remarks are in place here. First, while the theoretical boundaries of the prediction areas are indicated by the solid lines, hit rates were calculate for slightly larger areas, which is indicated by the dotted lines. This was done to allow for some error. Specifically the boundaries are given by $b_i^* - \varepsilon$ and $v_i + \varepsilon$ for A1, A2 and F1 as well as $b_i^* + \varepsilon$ and $v_i - \varepsilon$ for F2, and with $\varepsilon = \mathbf{0.025}$. This seems reasonable especially in case of A2 were the theoretical prediction area is just a line. Secondly, for similar reasons (allowing for error) we did not require the entire estimated bid function to pass through the prediction area. Rather if this was the case for more than half of the bid function, it was counted a hit.

Game Type	Hit Rate (h)	Relative Area (a)	Predictive Success (m)
A1	84%	11%	73%
A2	37%	3%	34%
F1	65%	9%	56%
F2	39%	9%	30%

Table 4: Predictive success of bidding areas

Obviously, the prediction areas differ in size. So, if one wants to compare the hit rates between different game types, they need to be adjusted for the size of the prediction area. We will do that by applying a measure of predictive success for area theories (m) as proposed by Selten and Krischker (1983). It is defined as $m \equiv h - a$, with h being the hit rate and a representing the size of the prediction area relative to the total outcome space. In our experiment the total outcome space is the space of all feasible bid functions that may be choosen. Remember that the feasible (normalized) bids are between -0.5 and +1.5. The space of feasible bid functions is displayed in figure 6 as well.

Table 4 reports h, a and m for the four game types.[13] Note that m is by definition a number between -100% and $+100\%$. If decisions were equally distributed (i.e., random play) m would be 0, whereas $m = 100\%$ means a perfect fit of a theory (100% of observations on a relative area of size 0). Remember that the areas were constructed assuming that bidders are heterogenous, (weakly) risk averse and rational. We find the predictive success of this concept substantial for all game types. However, it explains behavior much better in A1 and F1 than in A2 and F2. Given that risk preferences and beliefs about other bidders risk preferences are stable, we see no theoretical reason for these strong differences. One may suggest to consider noise, as we did above in the case of investigating risk neutral equilibrium bidding. While we did allow for some noise in measuring h and a by the parameter ε, one could increase ε to allow for larger errors. Obviously, if ε gets sufficiently large the predictive success for the four game types will converge. However, looking again at the distributions of bids for $v_i = 0.5$ (figures 2 to 5) indicates that (roughly) the conclusion we draw from table 4 holds even if ε doubled or tripled. A more detailed analysis of this noise issue shall not be done here.

3.4 Comparative Statics

For risk neutral agents the equilibrium bid functions for the four game types satisfy the following relations (see also figure 1a in the appendix):

$$b_i^*(\mathrm{A1}) \leq b_i^*(\mathrm{F1}) \leq b_i^*(\mathrm{A2}) \leq b_i^*(\mathrm{F2}). \tag{1}$$

[13] In calculating a, again, we considered the prediction areas that allow for some error (boundaries are adjusted by ε).

We refer to this as the "comparative statics" (of game types). Following our analysis above one may object to the assumption that all bidders are risk neutral. However, since we are lacking theoretical bid functions for F1 and F2 for general risk preferences (see above), we think nevertheless that risk neutrality is a theoretical benchmark case that is worthwhile to look at. Furthermore, note that the bidding areas we presented in the last section are partially nested and ordered, and their structure suggests that the relation in (1) will hold also for risk averse bidders. To investigate whether the comparative statics hold for our data we estimated the aggregate bid function for each game according to model 1.[14] The functions are shown in figure 1b (see the appendix) and clearly support the theory.

The estimated bid functions are ordered according to the comparative statics. Again, we do not report the usual statistical information for these regressions (e.g. t-statistics, etc.) since statistical testing is problematic in this environment. However, we will provide a statistical test of the comparative statics based on session information which avoids this problem.

Table 5 reports the percentage of subjects who behave in line with the comparative statics for each session and for three levels of experience (block 1 to block 3). The head row shows the respective two game types under consideration. A subject is considered in line with the comparative statics if its average bids (over all three games in the respective block) satisfy the theoretical order. For example, in block 1 (low experience) 100% of subjects in session 1 chose strictly lower mean bids in A1 than in A2 (first column). 78% of the subjects in session 1 chose strictly lower mean bids in F1 than F2 (second column). The other columns can be read accordingly. Looking down the first column one finds that the percentages vary between sessions and for different levels of experience, but, they are 56% (5 out of 9 subjects) or higher in almost all cases. I.e., in most cases more than half of the subjects chose bids that are in line with the comparative statics. We obtain similar results for the other comparisons in table 5.

One may derive a statistical test of the comparative statics hypothesis as follows: If at least 5 out of 9 subjects (56%) obey the comparative static prediction we consider the respective session in line with the theory. Comparing the two auctions, A1-A2 (first column), we find that this is the case for all six sessions. So a binomial test rejects the null–hypothesis (no systematic shift) in favor of the comparative statics prediction ($p = 0.016$, one–tailed, $N = 6$). The same holds for a comparison of the two fair division games, F1-F2, for experienced bidders (block 3). Similar tests can be derived for A1-F1, F1-A2, A2-F2, and A1-F2. For experienced bidders the tests support the theory for all comparisons but A2-F2 ($p = 0.109$). In summary the individual data analysis supports the finding from the regression analyses shown in figure 1b.

Result 4: *The observed bid functions satisfy the comparative statics derived by assuming risk neutrality.*

[14] I.e. we estimated one piecewise–linear regression for each game instead of one for each subject and each period.

Comparison		A1-A2	F1-F2	A1-F1	F1-A2	A2-F2	A1-F2
Block 1	Session						
Round 1-12	1	100	78	78	67	67	78
	2	67	56	78	56	56	67
	3	67	78	78	44	89	78
	4	89	67	67	78	78	89
	5	78	44	33	78	11	33
	6	89	89	100	56	78	100
Block 2	Session						
Round 13-24	1	89	67	67	67	44	78
	2	78	67	89	78	56	67
	3	89	100	56	78	56	100
	4	78	78	78	78	56	89
	5	100	67	78	78	33	78
	6	89	78	89	67	44	89
Block 3	Session						
Round 25-36	1	100	78	67	78	44	78
	2	78	67	78	78	56	56
	3	89	89	56	78	56	78
	4	89	78	78	78	56	89
	5	100	89	89	89	67	100
	6	89	67	78	67	67	100

Table 5: The percentage of subjects who behave in line with the comparative statics for each session and for three levels of experience (block 1 to block 3).

4 Influence of Game Type on Expected Price

An interesting question in auctions is how the price at which an object is sold is influenced by the price rule. Obviously, this is quite important for the seller. Usually it will be up to him to choose the auction type, and he might want to choose that type (among the two, A1 and A2) which yields the higher price. Since the price depends on the realizations of the randomly drawn values v_i of all bidders i, we can not simply compare the experimentally observed average prices for the two game types. Namely, any difference in the realizations of the reselling values between the game types would induce a bias. Instead, we will determine the respective expected prices.

To do that it is a great advantage that we collected bid functions of each individual rather than single bids. This allows us to determine the expected price for each group of 3 bidders by considering all possible combinations of reselling values that could have ocurred in the experiment. So, we consider each vector $v = (v_1, v_2, v_3)$ where v_1 to v_3 represent the values of the three bidders that form a bidder group. Since each v_i can take one of 11 values this may result in 1331 $(= 11^3)$ different vectors v. We determined the price $p(v)$ for each v. The mean of these prices represents the expected price p_j^e for the respective

Session	Mean Expected Price (Variance)			
	A1	A2	F1	F2
1	.700 (.07)	.574 (.08)	.738 (.10)	.525 (.20)
2	.715 (.03)	.538 (.12)	.770 (.06)	.540 (.10)
3	.698 (.04)	.545 (.08)	.771 (.10)	.715 (.16)
4	.717 (.05)	.568 (.07)	.720 (.08)	.593 (.13)
5	.704 (.03)	.540 (.06)	.743 (.07)	.500 (.17)
6	.633 (.07)	.526 (.07)	.707 (.05)	.552 (.06)
all	.695 (.06)	.549 (.08)	.741 (.08)	.571 (.16)

Table 6: Mean expected price (variance)

bidder group j.

Remember that in each session each type of auction was played 9 times by three groups of three randomly matched bidders. So in each session we collected 27 observations of p_j^e. And, the 6 sessions gave us 162 (= 6 times 27) observations altogether. Figure 7 (see the appendix) shows the cumulative distribution function (CDF) of the prices p_j^e for both auction types.

The CDF for A2 is always above that one for A1 which means that CDF(A1) first–order stochastically dominates CDF(A2).[15] Thus, the expected price is higher for A1 as compared to A2. A seller should clearly prefer the first price auction. This conclusion is in line with the result in Cox et all (1982). It is confirmed by table 6 which reports means of the prices p_j^e for each type of auction and each session (standard deviations in parentheses). The expected price is higher in A1 than in A2 for each of the 6 sessions. A binomial test based on the session means therefore indicates statistical signifance ($p = 0.032$, $N = 6$, two–tailed) of our finding (see figure 7).

One way to justify this result is to refer to the supporting evidence for risk aversion (see above) which pushes the A1-bid functions upwards and leaves the A2-bid functions unchanged (remember that price equivalence between A1 and A2, which is a common result in auction theory, requires risk neutrality[16]).

Regarding fair division games, table 6 and figure 8 report that the expected price is higher in F1 than in F2. Again, this holds for all 6 sessions. One may argue that in fair division games differences in expected prices between these price rules are of minor importance. The price serves only to re–distribute money within the group of bidders. Since, usually, it will be the bidders (e.g. heirs) themselves who choose the price rule, maximizing (or minimizing) the price is, in general, not a common goal of all bidders. And therefore price differences between F1 and F2 do not seem an adequate criterion to solve the bidders' choice problem.

However, there may be third parties who are interested in differences in

[15] See e.g. Laffont (1989), p. 32.

[16] See e.g. Wolfstetter (1996), Güth and van Damme (1986).

expected prices. For instance, lawyers whose fees for conducting the fair division game may depend on the price or public authorities who might want to collect taxes based on price (note that taxes can hardly depend on the net–payoff to the buyer ($v_i - p$), since v_i is unobservable). Such third parties may also influence or even determine the choice of the pricing rule. So, the comparision of prices between fair division games is indeed important. We summarize:

Result 5: *Expected prices are higher under the first–price rule for both auctions as well as fair division games. A risk neutral seller should prefer A1 to A2.*

5 Influence of Game Type on Expected Efficiency

We now look for efficiency differences between game types. Theoretically, an allocation is efficient if the bidder with the highest reselling value gets the object. In the empirical analysis we apply two measures of efficiency: the relative frequency of games that result in an efficient allocation and the "efficiency rate". For a given realization of reselling values the efficiency rate shall be defined as

$$\text{efficiency rate} = \frac{v_i(\text{buyer})}{\max\{v_1, v_2, v_3\}}$$

where v_i(buyer) is the private value of the buyer. Note that the efficiency rate is bounded between 0 and 1. It is 1 if the bidder with the highest value gets the object. Similar to our analysis of price effects (see above) we determined the expected efficiency for each game type.

Figure 9 (see the appendix) shows the CDF of the observed (expected) efficiency rates for A1 versus A2. Figure 10 (see the appendix) is the corresponding presentation for F1 versus F2. Both figures strongly suggest that the first price rule outperforms the second price rule. Expected efficiency is higher in A1 compared to A2 and in F1 compared to F2. This conclusion is confirmed by table 7 which shows the mean efficiency rate for each game type and each session, as well as by table 8 which reports the relative frequency of efficient games for each game type and each session. Accordingly, expected efficiency of A1 is higher than that of A2 in all 6 sessions. So, the shift is statistically significant according to a binomial test ($p = 0.032$, $N = 6$, two–tailed). The data for fair division games show that expected efficiency of F1 is higher than that of F2 in 5 out of 6 sessions. This does not allow to reject the null–hypothesis ($p = 0.219$, $N = 6$, two–tailed) based on session aggregates, despite the distributions of individual decisions displayed in figure 10. Note, however, that we have chosen a rather conservative test procedure. We summarize:

Result 6: *Expected efficiency is higher under the first–price rule compared to the second–price rule for both, auctions and fair division games. Thus, a social planner should prefer the former rule.*

Session	Mean Efficiency Rates (%)			
	A1	A2	F1	F2
1	97.4	93.3	92.9	88.9
2	98.4	94.3	96.9	90.3
3	98.2	95.9	96.5	91.5
4	98.3	90.5	96.9	91.4
5	99.4	97.1	96.6	92.5
6	96.5	95.2	95.1	96.3
all	98.0	94.4	95.8	91.8

Table 7: Mean efficiency rates

Session	Games Resulting in Efficient Allocations (%)			
	A1	A2	F1	F2
1	89.0	83.0	79.2	69.6
2	92.3	79.6	87.2	71.4
3	91.0	84.6	85.7	73.4
4	91.4	77.5	87.0	75.7
5	95.7	90.4	89.0	75.7
6	86.2	84.9	82.0	85.0
all	91.0	83.3	85.0	75.1

Table 8: Percentage of efficient allocations

From a normative viewpoint, the efficiency loss of A2 compared to A1 is puzzling, since it can not be caused by risk aversion. Specifically, in A2 risk aversion does not matter. Truthful bidding leads to an efficiency rate of 100%. Contrary, risk aversion reduces efficiency in A1. So, normatively one should observe quite the opposite of what we found here. A possible, however speculative, explanation we offer is that the second price rule might be more difficult to deal with by boundedly rational bidders than the first price rule. For instance, they might be more familiar with paying their own bid in case of winning an auction rather than paying the second highest bid. This might induce less noisy decisions within A2 compared to A1, despite the fact that A2 is simpler than A1 from a normative perspective. Namely, the former can be solved solely by a dominance criterion whereas the latter requires an equilibrium notion. We don't want to go any further in conjecturing about the causes of our result. The data here are suggestive and if the observed effect shows up persistently within similar studies, this is a clear message regarding institutional design.

6 Concluding Remarks

We reported evidence from an experiment on different types of allocation games: auctions versus fair division games under the first price rule, respectively the second price rule. The "bid function approach" was useful since it gave us more information at the individual level than the usual "single bid approach". We observed that:

- Individual bid functions are monotonically increasing. Monotonicity can be taken as a weak requirement of (boundedly) rational bidding.
- By and large, risk neutral equilibrium bidding is rejected by the data **(result 1)**. However, most bid functions are linear, which is in line with rational behavior by bidders that are endowed with heterogenous CRRA utility functions **(result 2)**. Furthermore, many bid functions pass through prediction areas implied by more general models of risk aversion **(result 3)**.
- Comparing bidding between game types we find that most bid functions are ordered according to comparative statics predictions based on rationality **(result 4)**. In order to derive these predictions we assumed risk neutrality, but we indicated that the ordering of bid functions might result as well for general models of risk aversion. After all, assuming risk neutrality as an approximation did provide comparative statics predictions that are supported by the data.
- The expected price is higher in A1 compared to A2 so that every risk neutral seller should prefer A1 to A2. Furthermore, the expected price in F1 is higher than in F2 **(result 5)**.
- Expected efficiency is higher under the first–price rule compared to the second–price rule for both, auctions and fair division games. So, a social planner should prefer the former rule **(result 6)**.

Results 5 and 6 summarize the implications of our data analysis for institutional design. Behaviorally, the first price-rule yields higher expected prices and allocation efficiency than the second price rule. Normatively, the result on prices can be rationalized by risk aversion. We can not reject such an explanation, at least not if we allow for some error. Also, several features of the individual bid functions are qualitatively in line with this view. The result on the efficiency influences of the different institutions, however, can not be explained by risk aversion. We think that our findings are reliable. Specifically, the results on the aggregate level come together with clearly structured individual bid functions. Remember, while there is a lot of variation between individuals, the bid functions of each individual taken at a time show very regular patterns. This strengthens our confidence in the robustness of our findings, but, of course, as in any other empirical study, these findings should be replicated.

References

[1] Cox, J.C., B. Roberson, and V.L.Smith (1982), Theory and behavior of single object auctions, in: *Research in experimental economis*, Vernon L. Smith, ed., Greenwich, Conn.: JAI Press.

[2] Davis, D.D. and C.A. Holt (1993), *Experimental Economics*, Princeton, New Jersey: Princeton University Press.

[3] Franciosi, R., R. M. Isaac, D. E. Pingry, S. S. Reynolds (1993), An Experimental Investigation of the Hahn-Noll Revenue Neutral Auction for Emmissions Licences, *Journal of Environmental Economics and Management*, 24, 1-24.

[4] Güth, W. (1998), On the effects of the pricing rule in auctions and fair division games - an experimental study, in: D. Budescu, I. Erev, and R. Zwick (eds.), *Games and Human Behavior*, Mahwah (N.J.): Lawrence Erlbaum Ass.

[5] Güth, W. and E. van Damme (1986), A comparison of pricing rules for auction and fair division games, *Social Choice and Welfare, 3*, 177-198.

[6] Güth, W., R. Ivanova, M. Königstein, and M. Strobel, (1999), Learning to bid - an experimental study of bid function adjustments in auctions and fair division games, *Discussion Paper*, Sonderforschungsbereich 373, Humboldt University at Berlin, 70/1999.

[7] Kagel, J. (1995), Auctions: A survey of Experimental Research, in: J. Kagel and A. R. Roth (eds), *The Handbook of Experimentals Economics*, Princeton (N.J.): Princeton University Press.

[8] Laffont, J.-J. (1989), *The Economics of Uncertainty and Information*, Cambridge MA: MIT Press.

[9] Selten, R. and J. Buchta (1998), Experimental sealed bid first price auction with directly observed bid functions, in: D. Budescu, I. Erev, and R. Zwick (eds.), *Games and Human Behavior*, Mahwah (N.J.): Lawrence Erlbaum Ass.

[10] Selten, R. and W. Krischker (1983), Comparison of two theories for characteristic function experiments, in: R. Tietz (ed.), *Aspiration levels in bargaining and economic decision making*, Berlin: Springer, 259-264.

[11] Wolfstetter, E. (1996), Auctions: An Introduction, *Journal of Economic Surveys*, Vol. 10, No. 4, 367-420.

7 APPENDIX

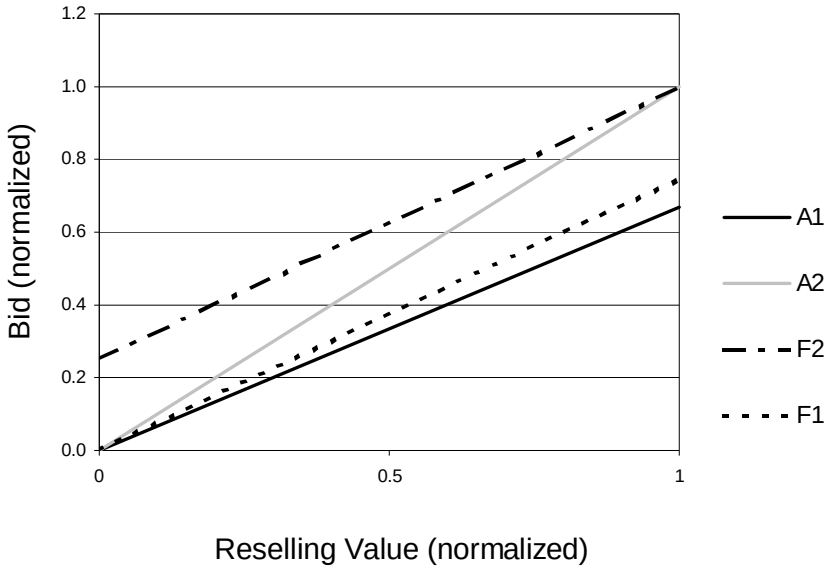

Figure 1a: RNE bid functions for all game types

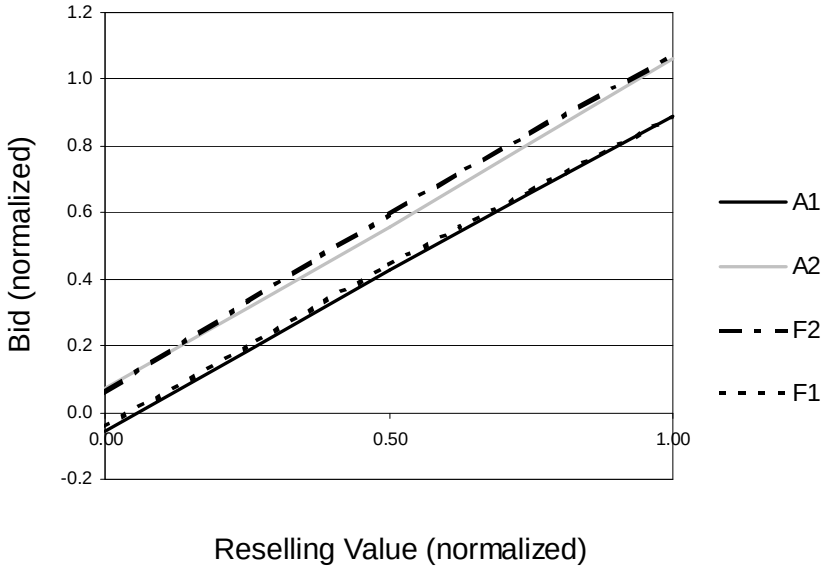

Figure 1b: Estimate aggregate bid functions

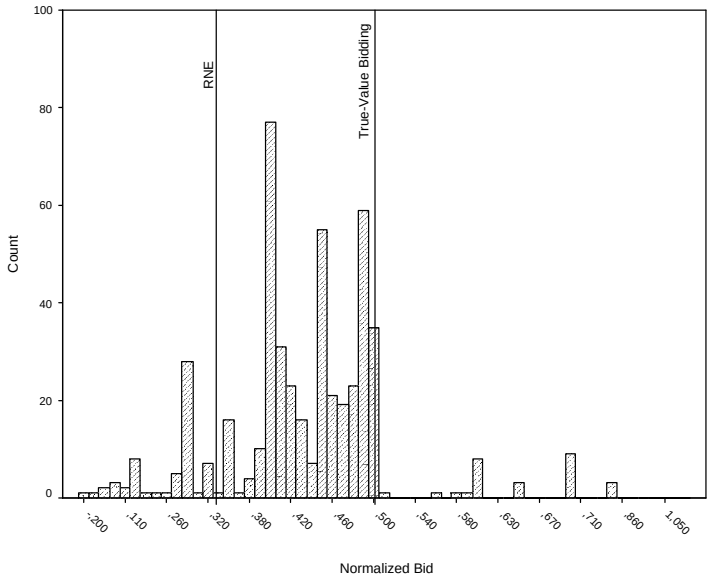

Figure 2: Distribution of bids for $v = 0.5$ (First Price Auction)

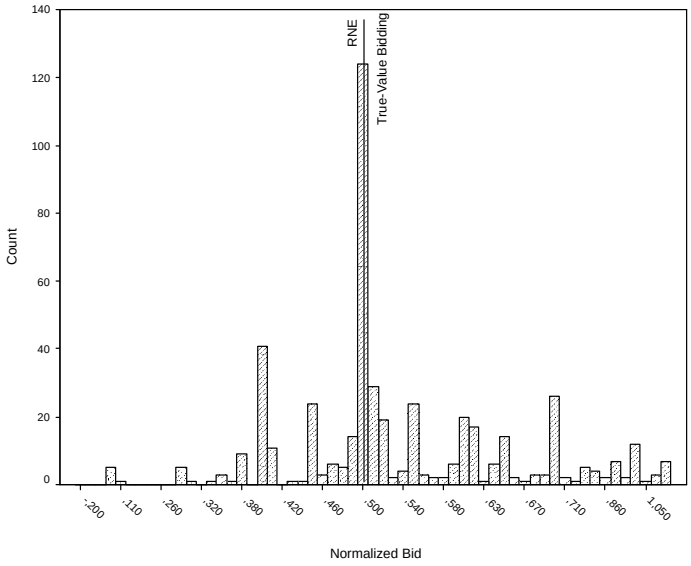

Figure 3: Distribution of bids for $v = 0.5$ (Second Price Auction)

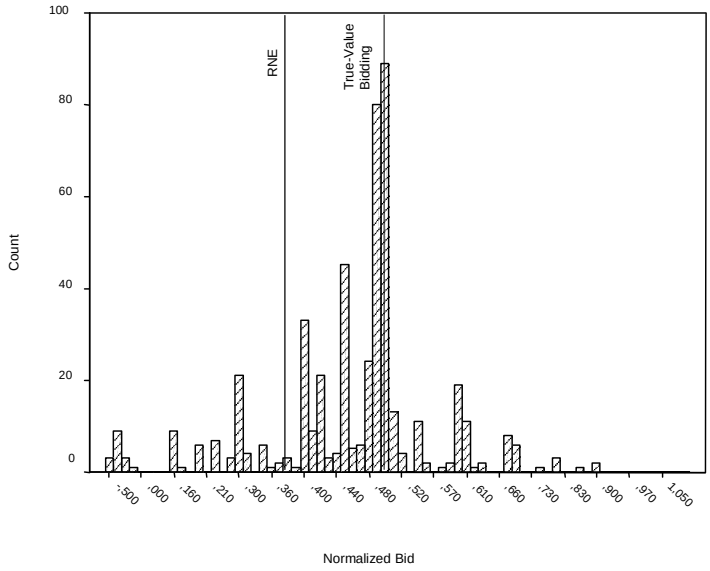

Figure 4: Distribution of bids for $v = 0.5$ (First Price Fair Div. Game)

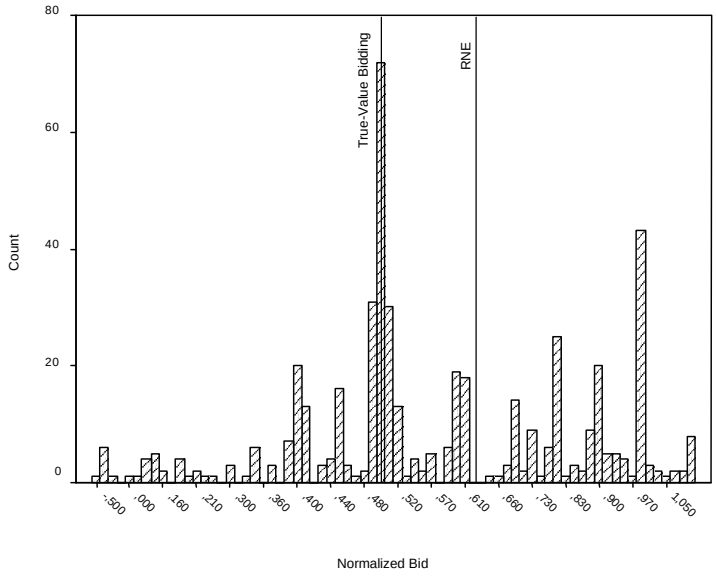

Figure 5: Distribution of bids for $v = 0.5$ (Second Price Fair Div. Game)

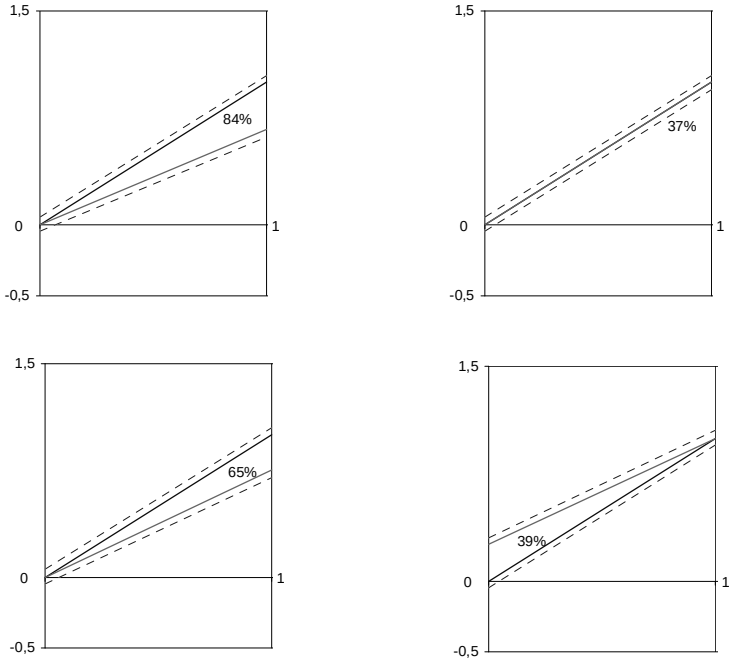

Figure 6: Prediction areas within feasible action space for all four cases

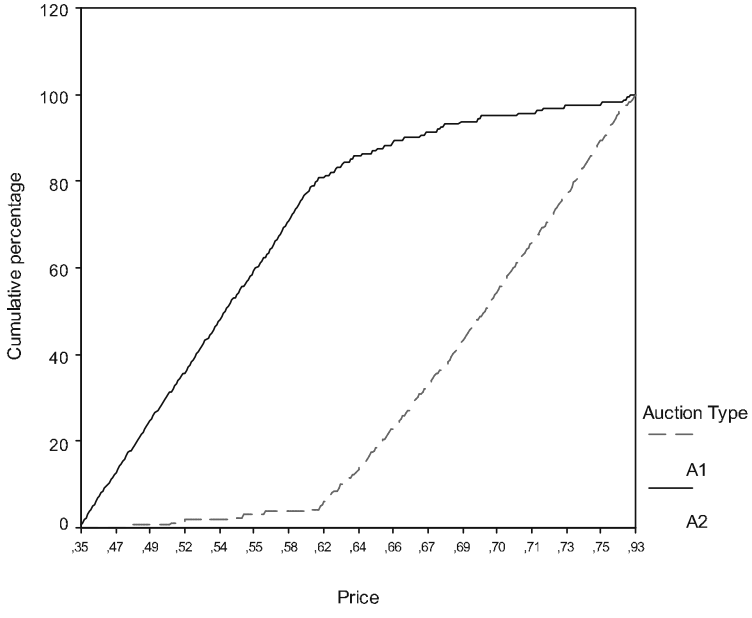

Figure 7: Cumulative Distributions of the prices for both auction types

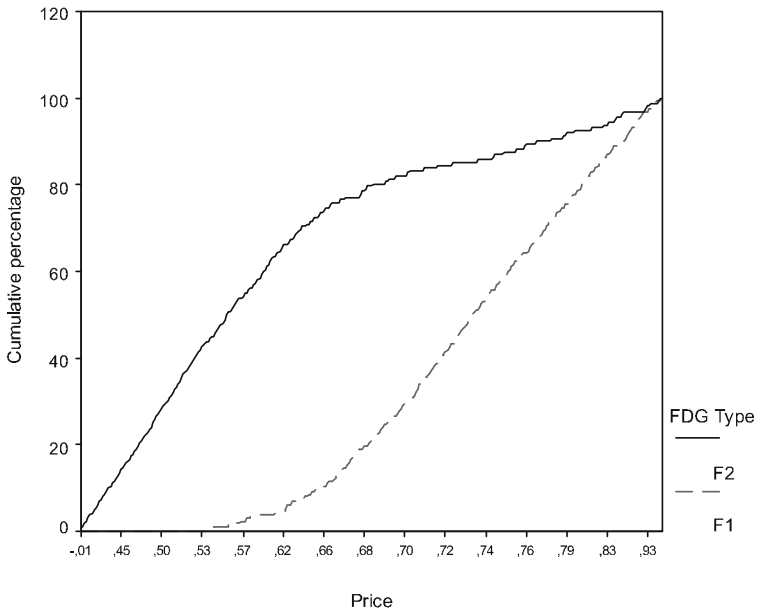

Figure 8: Cumulative Distributions of the prices for both fair division game types (Note: Negative prices are admissible since our analysis is based on normalized bids b_i and values v_i.).

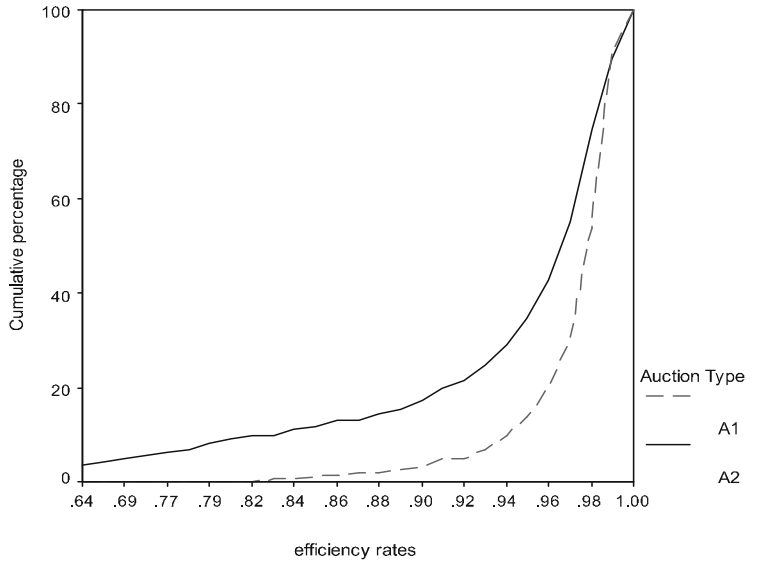

Figure 9: Cumulative distribution function of the observed efficiency rate for both auction types

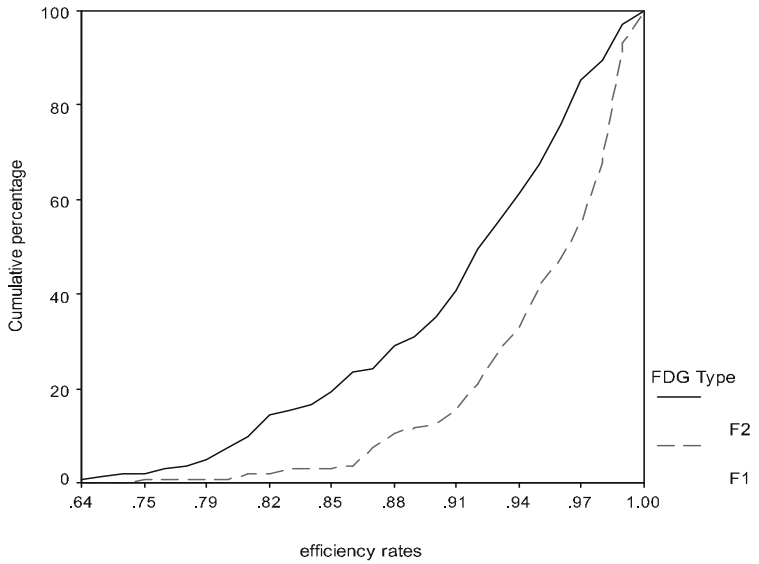

Figure 10: Cumulative distribution function of the observed efficiency rate for both fair divison games