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How competition in investing, hiring, and selling affects (un)employment – An analysis of equilibrium scenarios*–

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Abstract

Most models of labor markets and (un)employment neglect how competition among firms or sectors of the economy affects their hiring of workers and working times. Our approach pays special attention to such effects by proposing a complex stage game where firms invest in capital equipment before hiring workers. Working times are adjusted to demand which is implied by price competition. A special advantage of such a framework is that one can distinguish two kinds of employment effects, namely the numbers of workers as well as their working times, and that one can locate which firm or sector suffers from unemployment. Instead of solving the model in full generality we offer several equilibrium scenarios showing that certain economic phenomena are consistent with subgame perfect equilibrium behavior.

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1 Introduction

There exist many models of labor markets (e.g. Layard et al., 1991, Phelps, 1994). Whereas the main trend in the recent past is dominated by the macro-economic structuralistic approach, inspired by the literature just mentioned, our model is more in tradition of industrial economics, i.e. relies on a sound micro-economic foundation. Another motivation was to develop a model which lends itself to an experimental validation (for experimental studies of labor markets see Fehr, Kirchsteiger, and Riedl, 1996, Burda et al., 1998, and Berninghaus et al., 1997, for a more abstract aspect of collective wage bargaining). To analyse unemployment seriously there should be more than just one (representative) potential employee so that unemployment can be captured by shorter working times, but also by somebody being laid off.

We also wanted to capture the competition of employers who pay the same wages, but compete otherwise in several ways, e.g. in investing, in hiring employees, and in price setting. Minimal requirements for such a model are at least two firms which can hire more or less workers. Unfortunately such models become usually too complex to solve them analytically. Here we propagate and apply the method of equilibrium scenarios: Instead of solving a general model one asks whether for a usually rather complex model a certain type of economic results is consistent with equilibrium behavior.

An innovative aspect of our model is that it distinguishes clearly the two components of (un)employment, namely the numbers of employees in the various firms as well as their working times. It will be shown that this implies (at first sight) surprising and even counterintuitive results, e.g. that there exists no trade off between an increase of wages and (narrowly defined) employment (empirically such a trade off has been claimed by Phillips, 1958). In our view, the political debates and many (macro-economic) models of labor markets hide crucial aspects of employment or labor market policy since they do not (allow to) distinguish between the two forms of unemployment, namely shorter working times and fewer employees.

The details of our model which relies on four successive decision stages, namely wage determination, investment competition, hiring competition, and finally price competition, are presented in section 2. How backward induction allows to solve these games (with continuous action sets only on the last stage) is demonstrated then in section 3. The equilibrium scenarios, presented in section 4, are “union’s paradise” (a higher wage increases employment), “union of no effect” (unemployment cannot be reduced by wage cuts), “union endangered” (a high wage would endanger a centralized trade union due to its detrimental effects for one sector), and in most detail “union’s trade off” (higher wages cause more unemployment). We conclude by comparing our model to the literature and by outlining the perspectives for future research.

2 The basic model

For the sake of simplicity, especially in order to limit the number of structural parameters, we assume symmetry of firms. Let $n (\geq 1)$ denote the number of firms (when solving the multi stage-game we will rely on $n = 2$ although our analysis in principle can be performed for larger numbers of competing firms),

$p_i (\geq 0)$ firm i 's sales price, and

$x_i (\geq 0)$ firm i 's sales amount.

We rely on a heterogeneous market with the linear **demand function**

$$x_i = x_i(p) = \alpha - \beta p_i + \gamma \sum_{j \neq i} p_j.$$

Here $p = (p_1, \dots, p_n)$ is the price vector. The positive parameters α , β , and γ must satisfy

$$\gamma < \frac{\beta}{n-1}$$

guaranteeing lower individual sales amounts when all individual sales prices are increased by the same positive constant.

Production of x_i requires investments $I_i (> 0)$ and labor input where the latter is determined by the number of working hours $L_i (> 0)$ per worker and by the number of workers $N_i (\in \mathbb{N})$ in firm i . For all firms $i = 1, \dots, n$ we assume **production functions** of the form

$$x_i = x_i(I_i, N_i, L_i) = I_i N_i \sqrt{L_i}.$$

The main assumption is that for given I_i production is linear in the number of employees whereas an additional working hour is less productive than the previous one. Our justification for this assumption is that one can always find another production site with similar technological conditions. If at all, decreasing marginal productivity makes more sense when applied to an individual worker rather than to hiring more workers. Of course, the assumption can be weakened. What matters for some of our results is only that hiring more workers may allow to lower the cost of serving additional demand. For given demand = production levels x_i and a given number N_i of workers in firm i this implies the **work time demand**

$$L_i = (x_i / I_i N_i)^2.$$

Labor supply is not explicitly modelled, i.e. we assume that every labor demand is satisfied. Denote by $w_i (\geq 0)$ the wage level of firm i and by $K(N_i)$ the fixed labor costs for a given number N_i of employees in firm i . With the help of this notation we can write firm i 's **profit function** G_i as

$$G_i = p_i x_i(p) - w_i \frac{x_i(p)^2}{I_i^2 N_i} - I_i - K(N_i).$$

Inserting the demand function the profit G_i can be expressed as

$$G_i = \left[p_i - \frac{w_i}{I_i^2 N_i} \left(\alpha - \beta p_i + \gamma \sum_{j \neq i} p_j \right) \right] \left(\alpha - \beta p_i + \gamma \sum_{j \neq i} p_j \right) - I_i - K(N_i).$$

By the **decision process** we want to allow that firms anticipate labor's reactions when investing in technical equipment, i.e. when choosing their investments I_i and thus the productivity of labor input. On the other hand labor must anticipate the firms' reactions regarding labor demand via the hiring decisions N_i as well as by the work time demands L_i resulting from sales prices. More specifically, we rely on several successive decision stages where it is generally assumed that all previous decisions are commonly known and that all choices on the same stage are made independently. The **stages** are as follows:

1. The union player U determines the wage level w which is the valid wage for all firms $i = 1, \dots, n$ where $w \in \{\underline{w}, \bar{w}\}$ with $0 < \underline{w} < \bar{w}$.
2. For all $i = 1, \dots, n$ employers E_i select I_i with $I_i \in \{\underline{I}, \bar{I}\}$ where $0 < \underline{I} < \bar{I}$.
3. For all $i = 1, \dots, n$ employers E_i choose their number N_i of employees with $N_i \in \{1, 2\}$ for $i = 1, \dots, n$.

4. For all $i = 1, \dots, n$ employers E_i determine their sales price $p_i (\geq 0)$ what ends the game.

The essential restriction is that firms know the wages when determining their investments. By this we wanted to induce trade unions to anticipate the effects of wage policy on investments and thereby on future employment prospects. Regarding payoffs we assume that G_i is employer E_i 's utility ($i = 1, \dots, n$). For U the payoff function is

$$U(w, N)$$

where N is the sum of N_i over all firms $i = 1, \dots, n$ and where U is supposed to increase with both its arguments. Ideally a trade union should also care for the working time L which together with w and N determines wage income $w \cdot L \cdot N$. In our view, trade unions are, however, usually judged by which wage increases they achieve and how many members they have. What one finds here are the typically divergent interests of the principals (the workers) and their agents (their trade unions). Our approach does, of course, not hinge on this assumption, i.e. we could as well have relied on a more general function $U(\cdot)$.

Whereas we allow for continuous actions on the last stage (4.), players are restricted to discrete choice sets on earlier stages. By this we want to avoid all the computational difficulties which one encounters by deriving repeatedly interior equilibrium solutions which anticipate the interior equilibrium solutions of all later stages. Of course, discrete action sets lead often to mixed strategy solutions which are generally hard to justify. Such problems, however, never show up in the analysis below. The parameters of our model are

$$\alpha, \beta, \gamma, \underline{I}, \bar{I}, \underline{w}, \bar{w} \text{ as well as the cost levels } K(N_i)$$

for all $N_i \in \mathbb{N}$.

3 How to solve the model

Due to our information requirements all earlier choice trajectories of stage t define a subgame whose first decision stage is stage t . To determine the **subgame perfect equilibrium behavior** (Selten, 1965 and 1975) we proceed by backward induction.

a) Price competition

On the last stage $t = 4$ employer E_i tries to maximize G_i by his choice of p_i .

From

$$\frac{\partial}{\partial p_i} G_i = -\beta \left[p_i - \frac{w_i}{I_i^2 N_i} \left(\alpha - \beta p_i + \gamma \sum_{j \neq i} p_j \right) \right] + \left(1 + \frac{\beta w_i}{I_i^2 N_i} \right) \left(\alpha - \beta p_i + \gamma \sum_{j \neq i} p_j \right) = 0$$

and

$$\frac{\partial^2}{\partial p_i^2} G_i = -2\beta \left(1 + \frac{\beta w_i}{I_i^2 N_i} \right) < 0$$

for $i = 1, \dots, n$ follows that the **equilibrium price vector** $p^* = (p_1^*, \dots, p_n^*)$ is the solution of the linear equation system

$$\alpha \left(1 + 2 \frac{\beta w_i}{I_i^2 N_i} \right) = 2\beta \left(1 + \frac{\beta w_i}{I_i^2 N_i} \right) p_i^* - \gamma \left(1 + 2 \frac{\beta w_i}{I_i^2 N_i} \right) \sum_{j \neq i} p_j^*$$

for $i = 1, \dots, n$. We concentrate on the parameter region where all components p_i^* of the price vector p^* satisfy $p_i^* > 0$ for all earlier choice trajectories of stage $t = 5$.

Notice that p^* determines work time demands via

$$\begin{aligned} L_i^* &= (x_i(p^*) / I_i N_i)^2 \\ &= \left(\frac{\alpha - \beta p_i^* + \gamma \sum_{j \neq i} p_j^*}{I_i N_i} \right)^2 \end{aligned}$$

for all firms $i = 1, \dots, n$.

On the earlier stages the model allows only for two decision alternatives since this offers a practible way of solving such a complex multi stage-oligopoly market. We further seriously restrict the generality of analysis by assuming $n = 2$ and by relying on numerically specified decision alternatives $I_i = \underline{I}$ and $I_i = \bar{I}$ with $0 < \underline{I} < \bar{I}$ for $i = 1, 2$ as well as $w = \underline{w}$ and $w = \bar{w}$ with $0 < \underline{w} < \bar{w}$. Whereas N_i for $i = 1, 2$ is restricted to $N_i \in \{1, 2\}$, we will allow the specific investment levels \underline{I} and \bar{I} to depend on the previously chosen wage w . The high wage level \bar{w} may, for instance, allow only smaller investment levels \underline{I} and \bar{I} than the low wage \underline{w} since banks are not willing to grant large credit lines in case of too high wages. The restriction to $N_i \in \{1, 2\}$ for $i = 1, 2$ is chosen since in an experiment one may actually want to explicitly include worker participants who, in case of $N_i = 1$, would be partly employed and partly unemployed.

b) Competition in hiring

For stage 3 when both firms $i = 1, 2$ determine independently the number $N_i \in \{1, 2\}$ of workers there exist altogether four different I_1, I_2 -constellations for each wage level. Thus if the two wage levels $w = \underline{w}$ and $w = \bar{w}$ imply different \underline{I} and \bar{I} -alternatives one has to solve eight different 2 x 2-bimatix games on stage 3 whose payoff functions can be described as bimatrices of the form:

	N_2	1	2
N_1			
1			
2			

For the numerical constellation

$$\alpha = 4.83, \beta = 1.2, \gamma = 1, K(N_i) = N_i^2/2, \underline{w} = 25, \underline{I}(\underline{w}) = 10, \bar{I}(\underline{w}) = 12$$

the four games are, for instance, given by

5.25, 5.25	3.56, 4.81	2.87, 2.87	1.61, 2.14	4.25, 3.89	3.00, 3.16
4.81, 3.56	3.10, 3.10	2.14, 1.61	.87, .87	3.80, 2.18	2.53, 1.45
$I_1 = \underline{I}, I_2 = \underline{I}$		$I_1 = \bar{I}, I_2 = \bar{I}$		$I_1 = \underline{I}, I_2 = \bar{I}$	

Due to the basic symmetry of both firms $i = 1, 2$ the case $I_1 = \bar{I}$ and $I_2 = \underline{I}$ can be obtained by exchanging player indices. The entries of these matrices are simply obtained by computing the working times L_i^* , derived above, and by substituting them into the profit functions $G_i(\cdot)$ for $i = 1, 2$.

If one assumes the same parameters α, β , and γ as well as $K(N_i) = N_i^2/2$ for

$$\bar{w} = 50, \underline{I}(\bar{w}) = .5, \bar{I}(\bar{w}) = 1.5$$

the corresponding three games are

- .29, - .29	- .30, -1.10	3.32, 3.32	2.89, 6.09	- .35, 3.77	- .40, 6.87
-1.10, - .30	-1.11, -1.11	6.09, 2.89	5.35, 5.35	-1.21, 3.71	-1.32, 6.77
$I_1 = \underline{I}, I_2 = \underline{I}$		$I_1 = \bar{I}, I_2 = \bar{I}$		$I_1 = \underline{I}, I_2 = \bar{I}$	

For each of these bimatrix games the equilibrium is indicated by fat entries. Whereas for \underline{w} and all possible $I_1(\underline{w}), I_2(\underline{w})$ -constellations the unique equilibrium is always $(N_1^*, N_2^*) = (1, 1)$, for $w = \bar{w}$ the equilibrium is also always unique, but depends on the $I_1(\bar{w}), I_2(\bar{w})$ -constellation: It is $(N_1^*, N_2^*) = (1, 1)$ only for $I_1(\bar{w}) = I_2(\bar{w}) = \underline{I}(\bar{w})$, the strategy vector $(N_1^*, N_2^*) = (2, 2)$ for $I_1(\bar{w}) = I_2(\bar{w}) = \bar{I}(\bar{w})$, and $(N_1^*, N_2^*) = (1, 2)$ for $I_1(\bar{w}) = \underline{I}(\bar{w})$ and

$I_2(\bar{w}) = \bar{I}(\bar{w})$, i.e. the firm, which invests more, will also employ more workers.

In the same way one can derive the solution for stage 3 for other numerical specifications of the parameters $\alpha, \beta, \gamma, K(N_i), \underline{w}, \bar{w}, \underline{I}(\underline{w}), \bar{I}(\bar{w}), \underline{I}(\bar{w})$, and $\bar{I}(\underline{w})$. In all our numerical calculations we never encountered more than one strict equilibrium or no strict equilibrium at all, i.e. we always had to look only for the unique strict equilibrium which, by definition, is in pure strategies.

c) Competition in investing

As the solution of stage 4 allowed us to determine the optimal hiring decisions, the solutions of stage 3 (and stage 4) allow to derive the optimal investment level for both firms. How this can be done is illustrated by the numerical example considered in the previous section. For $w = \underline{w}$ and $w = \bar{w}$ the implications of all possible I_1, I_2 -constellations are represented by the two 2 x 2-bimatrices.

	I_2	$\underline{I}(\underline{w})$	$\bar{I}(\underline{w})$		I_2	$\underline{I}(\bar{w})$	$\bar{I}(\bar{w})$
I_1				I_1			
$\underline{I}(\underline{w})$		5.25, 5.25	4.25, 3.89	$\underline{I}(\bar{w})$		-.29, -.29	-.40, 6.87
$\bar{I}(\underline{w})$		3.89, 4.25	2.87, 2.87	$\bar{I}(\bar{w})$		6.87, -.40	5.35, 5.35
		$w = \underline{w} = 25$				$w = \bar{w} = 50$	

Again we have indicated the unique strict equilibrium by fat entries. The result, namely high investments at the high wage \bar{w} and low investments at the low wage \underline{w} , may look counterintuitive at first sight. But this neglects that the two investment alternatives $\underline{I}(w)$ and $\bar{I}(w)$ depend on w . Since $\underline{I}(\underline{w}) = 10 > \bar{I}(\bar{w}) = 1.5$, the solution prescribes much larger investment levels $\underline{I}(\underline{w}) = 10$ by both firms in case of the low wage $\underline{w} = 25$ than the optimal investment levels $\bar{I}(\bar{w}) = 1.5$ of both firms for the high wage $\bar{w} = 50$.

What does this imply for the employment levels of both firms? For $w = \underline{w}$ the investment levels $I_1 = (\underline{w}) = I_2(\underline{w}) = \underline{I}(\underline{w})$ imply $N_1^* = N_2^* = 1$ whereas

for $w = \bar{w}$ and $I_1(\bar{w}) = I_2(\bar{w}) = \bar{I}(\bar{w})$ the result is $N_1^* = N_2^* = 2$. Thus our numerical example seems to describe **trade union U 's paradise**: The higher wage $\bar{w} = 50$ implies also higher employment levels in both firms rendering $w = \bar{w} = 50$ as the unambiguously better wage level in view of trade union U .

One may not believe in (trade union's) paradise and may therefore ask what causes its possibility. Since $\bar{w} = 50$ implies the investment levels $\bar{I}(\bar{w}) = 1.5$ and $\underline{w} = 25$ investments $\underline{I}(\underline{w}) = 10$, the high wage essentially crowds out capital and crowds in labor. How this happens in detail is not captured by our model, but can be imposed by appropriately specifying how $\underline{I}(w)$ and $\bar{I}(w)$ depend on the wage level w . Since our results rely on strict inequalities, the effects are locally robust in the sense that they remain valid in case of small changes in the parameters. Of course, globally a union's paradise will not exist since labor costs are bounded from above (one can increase both wages and employment, only ad infinitum when L goes to 0, a rather unlikely assumption).

Of course, even the union's paradise has a flaw: Since the high wage \bar{w} implies lower investments, it could endanger future employment. The result that a wage increase from \underline{w} to \bar{w} reduces investment may at first sight appear counterintuitive since a higher wage should lead to substituting labor by capital. In an open economy lower investments due to high wages are, however, a rather likely scenario (as shown by the recent political debate in Germany about what endangers the "Standort Deutschland", i.e. the future employment prospects in Germany). But even in a rather closed economy high wages may decrease future profits to such an extent that present investments are endangered.

The analysis so far illustrates the flexibility of our basic model which allows us to capture quite differently structured situations. Especially by letting the two investment alternatives $\underline{I}(w)$ and $\bar{I}(w)$ with $0 < \underline{I}(w) < \bar{I}(w)$ depend on $w \in \{\underline{w}, \bar{w}\}$ with $0 < \underline{w} < \bar{w}$ one can capture situations like a trade

union's paradise and compare it with situations where the trade union U encounters a trade off between high wage and high employment or one where employment is low regardless of the chosen wage level w .

Of course, one may want to impose further natural restrictions as $\underline{I}(w) > \bar{I}(\bar{w})$ like in our numerical example when exploring such other situations. Such restrictions can reflect reasonable assumptions regarding credit rationing as they might result in a general equilibrium framework. To illustrate that also these other situations can be more or less easily induced by appropriately specifying the parameters the following section illustrates some equilibrium scenarios, offering interesting treatments for an experimental study.

4 Equilibrium scenarios

In economic modelling one often must either rely on simply structured models, which can be solved analytically without having to restrict parameters numerically, or study institutionally richer models which are too complex to allow for a general, analytic solution by reasonable efforts. In the latter case the method of equilibrium scenarios offers an alternative: Rather than solving a general model structure analytically one specifies a usually rather complex model and explores whether or not certain behavioral constellations, often empirically valid ones, are consistent with rational, e.g. subgame perfect equilibrium, behavior in such a model world.

In essence one explores whether there exist parameter constellations or functional specifications which render a certain behavior as the solution outcome. In order to avoid results which are highly special, e.g. in the sense of border cases, one often will require that small changes in the parameters do not endanger such a result. If, for instance, all structural relationships are continuous, this often can be guaranteed by relying on strict equilibria only. A strict equilibrium reflecting a certain economic phenomenon in this sense is called an equilibrium scenario.

a) Union's paradise

Union's paradise exists when the higher wage \bar{w} implies maximal employment in both firms. That this is a possible result in our model world has been illustrated above. Furthermore, as can be checked numerically or deduced since all our results relied on strict equilibria for the various subgames to be solved in the course of backward induction, the result is robust in the sense of being insensitive to small parameter changes. Thus union's paradise is an equilibrium scenario. Its existence presupposes, of course, a drastic reduction of the possible investment levels $\underline{I}(w)$ and $\bar{I}(w)$ when w increases.

b) Union's trade off

If the high wage \bar{w} implies lower employment levels than the low wage \underline{w} , the union faces a trade off between high wages or high employment levels N_i in both firms $i = 1, 2$. Such a trade off is often seen behind the so-called Phillips curve (Phillips, 1958).

In view of the strong empirical evidence for a trade off between increasing wages and employment it may be surprising that for our model this trade off can be hardly derived. If one, for instance, assumes that the investment levels $\underline{I}(w)$ and $\bar{I}(w)$ do not depend on w , all our many numerical results have implied higher employment levels for higher wages.

What can explain such a surprising effect? One obvious idea would be that higher wages imply higher investments like in section 2.c) above and that these higher investments overcompensate the marginal cost increase due to the higher wages. Whether this is true can be easily analysed by letting the positive differences $\bar{I}(w) - \underline{I}(w)$ for $w \in \{\underline{w}, \bar{w}\}$ approach 0. In our numerical attempts this has been tried out systematically without success. Thus the difficulties in generating the union's trade off cannot be attributed to an investment increase.

This leaves essentially only one explanation for the non-existing trade off between wage and employment increases. Since higher wages imply higher marginal costs with respect to working time L_i in both firms $i = 1, 2$, such an increase of marginal labor cost can be (over) compensated by increasing firms i 's number N_i of employees. In essence this means that higher wages imply higher employment in the sense of N_i , but shorter working times L_i in both firms $i = 1, 2$ (see Hart, 1987, for a discussion of working time and employment).

In our view, it is a major advantage of our model, respectively a disadvantage of (most macro and at least many micro) models of labor markets, that it distinguishes between the two components of employment, namely the numbers of workers and their working times. The usual assumption that higher wages reduce employment holds for working times L_i , but not for employment numbers N_i . If marginal productivity decreases with L_i , one can fight an increase of marginal cost with respect to working time by hiring more employees.

This raises the question whether one can generate a union's trade off in the sense of

$$(N_1 \cdot L_1 + N_2 \cdot L_2) (\underline{w}) > (N_1 \cdot L_1 + N_2 \cdot L_2) (\bar{w})$$

when both investment levels do not depend on w .

For the parameter constellation

$$\alpha = 4.83, \beta = 1.2, \gamma = 1, \text{ and } I_i \in \{.5, 1.5\}, i = 1, 2,$$

and all possible pairs $\underline{w}, \bar{w} \in \{25, 50, 100\}$ such that $\bar{w} > \underline{w}$ the solution play relies on

$$I_1 = I_2 = \bar{I} = 1.5 \text{ and } (N_1, N_2) = (2, 2).$$

By deriving the equilibrium price vector for these previous moves and the implied working times L_1 and L_2 it follows that

$$\begin{aligned} (N_1 L_1 + N_2 L_2)(w = 25) &= .9 \\ &> (N_1 L_1 + N_2 L_2)(w = 50) = .33 \\ &> (N_1 L_1 + N_2 L_2)(w = 100) = .1 \end{aligned}$$

If one substitutes $I_i \in \{.5, 1.5\}$ by $I_i \in \{10, 12\}$ for $i = 1, 2$ the analogous results are

$$I_1 = I_2 = \underline{I} = 10 \text{ and } (N_1, N_2) = (1, 1)$$

as well as

$$\begin{aligned} (N_1 L_1 + N_2 L_2)(w = 25) &= .29 \\ &> (N_1 L_1 + N_2 L_2)(w = 50) = .25 \\ &> (N_1 L_1 + N_2 L_2)(w = 100) = .13 \end{aligned}$$

Both numerical constellations illustrate that an increasing wage can leave the numbers N_1 and N_2 of employees unchanged, but reduces the working times L_1 and L_2 . Thus, in total the result is a typical union's trade off.

One may argue that the trade union is less interested in the total working time $N_1 \cdot L_1 + N_2 \cdot L_2$, but only in the level $N_1 + N_2$ of (un)employment.

Is a trade union's trade off possible when the conflict is between increasing $N_1 + N_2$ and w ? Even by adjusting $\underline{I}(w)$ and $\bar{I}(w)$ in many ways we were not able to find an equilibrium scenario in this sense. Even very extreme parameters did not generate such a result.

Thus one has to add even more flexibility in order to generate a union's trade off in this more narrow sense. One possibility is to let the parameter q of the cost function

$$K_i(N_i) = \frac{N_i^2}{q} \text{ with } q > 0 \text{ for } i = 1, 2$$

decrease when the wage level w increases. The quite intuitive interpretation for such a dependency is that the fixed costs of employees increase with their wages. The following tables allow to analyse as in section 2b) the numerical constellation

$$\alpha = 180.83, \beta = 1.2, \gamma = 1, \underline{I} = 10, \bar{I} = 100$$

for $\underline{w} = 50$ and $q(\underline{w}) = .5$, respectively for $\bar{w} = 100$ and $q(\bar{w}) = .01$: For the first case we get

23331, 23331	19527, 25979	19969, 19996	19909, 19998	14840, 28915	14786, 28941
25979, 19527	22061, 22061	19998, 19909	19938, 19938	17107, 25045	17049, 25073
$I_1 = \underline{I}, I_2 = \underline{I}$		$I_1 = \bar{I}, I_2 = \bar{I}$		$I_1 = \underline{I}, I_2 = \bar{I}$	

whereas the corresponding results for the latter case are

24315, 24315	19126, 28483	19922, 19922	19802, 19691	11681, 34078	11590, 33834
28483, 19126	22933, 22933	19691, 19802	19571, 19571	14548, 28752	14442, 28517
$I_1 = \underline{I}, I_2 = \underline{I}$		$I_1 = \bar{I}, I_2 = \bar{I}$		$I_1 = \underline{I}, I_2 = \bar{I}$	

Thus competition in investing can be analysed as in section 2c) by analysing the following two $I_1 \times I_2 - b$ -bimatrices with $I_i \in \{\underline{I}, \bar{I}\}$, $i = 1, 2$:

22061, 22061	17049, 25073	22933, 22933	14548, 28752
25073, 17049	19938, 19938	28752, 14548	19922, 19922
$w = \underline{w} = 50, q(\underline{w}) = .5$		$w = \bar{w} = 100, q(\bar{w}) = .01$	

For both wage levels the unique strict equilibrium is $(I_1, I_2) = (\bar{I}, \bar{I})$. But whereas this implies $(N_1, N_2) = (2, 2)$ for $w = \underline{w}$, the result is $(N_1, N_2) = (1, 1)$ for $w = \bar{w}$ as required by a trade union's trade off in the narrow sense.

c) Union of no effect

If employment is low in both firms regardless whether the wage w is high or low, the union does not encounter a decision conflict as for trade union's trade off, but faces a dilemma regarding employment: Low employment and thus unemployment cannot be prevented by the trade union ...

To demonstrate that this qualifies as an equilibrium scenario consider the situation with

$$\alpha = 180.83, \beta = 1.2, \gamma = 1, q = .01, \underline{I} = 10, \bar{I} = 100, \underline{w} = 50, \bar{w} = 100.$$

The bimatrices for deriving I_1 and I_2 for $w = \underline{w}$ are

23233, 23233	19429, 25587	19871, 19871	19811, 19606	14742, 28817	14688, 28549
25587, 19429	21669, 21669	19606, 19811	19546, 19546	16715, 24947	16657, 24681
$I_1 = \underline{I}, I_2 = \underline{I}$		$I_1 = \bar{I}, I_2 = \bar{I}$		$I_1 = \underline{I}, I_2 = \bar{I}$	

and for $w = \bar{w}$

24315, 24315	19126, 28483	19922, 19922	19802, 19691	11681, 34078	11590, 33834
28483, 19126	22933, 22933	19691, 19802	19571, 19571	14548, 28752	14442, 28517
$I_1 = \underline{I}, I_2 = \underline{I}$		$I_1 = \bar{I}, I_2 = \bar{I}$		$I_1 = \underline{I}, I_2 = \bar{I}$	

Consequently, competition in investing can be analysed with the help of the two bimatrices

21669, 21669	16715, 24947	22933, 22933	14548, 28752
24947, 16715	19871, 19871	28752, 14548	19922, 19922
$w = \underline{w} = 50$		$w = \bar{w} = 100$	

Thus one has $(I_1, I_2) = (\bar{I}, \bar{I})$ for both wage levels \underline{w} and \bar{w} . Since this implies $(N_1, N_2) = (1, 1)$ regardless whether $w = \underline{w}$ or $w = \bar{w}$, the wage does not influence the level $N_1 + N_2$ of employment at all as required by a union of no effect.

This, of course, does not exclude employment effects in the sense that total working time $N_1 \cdot L_1 + N_2 \cdot L_2$ will react to changes of w . Actually for the example one gets $(N_1 \cdot L_1 + N_2 \cdot L_2)(\underline{w}) = 4.788$ and $(N_1 \cdot L_1 + N_2 \cdot L_2)(\bar{w}) = 4.772$ (see also the second constellation of a union's trade off in the sense that $N_1 L_1 + N_2 L_2$ decreases with an increase of w although $(N_1, N_2) = (1, 1)$ results for all wage levels).

d) Union endangered

Union is endangered if the low wage \underline{w} implies full employment (here $N_i = 2$ for $i = 1, 2$), whereas at the high wage \bar{w} employment is maximal only in one firm. Here a central union in the sense of our model, which dictates the wage level w in both firms, is endangered since there is the threat that the work force of each firm could prefer to bargain for its own firm specific wage level w_i . More specifically, both such firm specific trade unions U_1 and U_2 would then induce wage levels $w_1 = w_2 = \underline{w}$ even when for the central union U

the wage level \bar{w} is preferable, e.g. when $[N_1^*(\bar{w}) + N_2^*(\bar{w})]\bar{w} = 3\bar{w}$ exceeds $[N_1^*(\underline{w}) + N_2^*(\underline{w})]\underline{w} = 4\underline{w}$.

What a “union endangered”-scenario requires is a constellation where $w = \underline{w}$ implies $(N_1, N_2) = (2, 2)$ and $w = \bar{w}$ the constellation $(N_1, N_2) = (1, 2)$ or $(N_1, N_2) = (2, 1)$. Several numerical attempts with symmetric firms did not generate a “union endangered”-scenario. Thus such a scenario seems to require a fundamental asymmetry. One simple way to create such an asymmetry is by postulating different costs of hiring, e.g.

$$K_1(N_1) = \frac{N_1^2}{q_1}, K_2(N_2) = \frac{N_2^{16}}{q_2}.$$

For the parameter constellation

$$\alpha = 480.83, \beta = 1.2, \gamma = 1, I_i \in \{10, 12\} \text{ for } i = 1, 2$$

one obtains

165032, 165032	138138, 151028	160120, 160120	138830, 141281
183794, 138138	156094, 123328	174047, 138830	152347, 119581
$I_1 = \underline{I}, I_2 = \underline{I}$		$I_1 = \bar{I}, I_2 = \bar{I}$	

149328, 176119	128608, 157393	176119, 149328	148648, 134972
167693, 148648	146130, 129627	190159, 128608	162393, 113364
$I_1 = \underline{I}, I_2 = \bar{I}$		$I_1 = \bar{I}, I_2 = \underline{I}$	

for $\bar{w} = 50$ and $q_1(\bar{w}) = q_2(\bar{w}) = 2$. This implies $I_1(\bar{w}) = I_2(\bar{w}) = \bar{I}$ and $(N_1, N_2)(\bar{w}) = (2, 1)$ since

I_2	\underline{I}	\bar{I}
I_1		
\underline{I}	183794, 138138	167693, 148648
\bar{I}	190159, 128608	174047, 138830

If, however, $\underline{w} = 10$ and $q_1(\underline{w}) = 2$, $q_2(\underline{w}) = 30$ the implications are

148188, 148189	140460, 150599	146283, 146284	140749, 147355
152781, 140460	145015, 142833	149537, 140749	143984, 141802
$I_1 = \underline{I}, I_2 = \underline{I}$		$I_1 = \bar{I}, I_2 = \bar{I}$	

143520, 150967	138006, 152048	150966, 143520	143213, 145909
148092, 143213	142546, 144275	154231, 138006	146457, 140364
$I_1 = \underline{I}, I_2 = \bar{I}$		$I_1 = \bar{I}, I_2 = \underline{I}$	

an thus $I_1(\underline{w}) = I_2(\underline{w}) = \bar{I}$ and $(N_1, N_2)(\underline{w}) = (2, 2)$ due to

I_2	\underline{I}	\bar{I}
I_1		
\underline{I}	145015, 142833	142546, 144275
\bar{I}	146457, 140364	143984, 141802

The low wage $\underline{w} = 10$ induces full employment in the sense of $(N_1, N_2) = (2, 2)$ whereas the high wage $\bar{w} = 50$ implies unemployment in firm or sector 2 of the economy. If the central union would choose \bar{w} in spite of the unemployment effect in sector 2 and if there is a lot of solidarity among the employed and unemployed worker of sector 2, this could result in an attempt to found a separate trade union for firm or sector 2. By lowering the wage from \bar{w} to \underline{w} for firm 2 this firm or sector specific trade union can ensure full employment in firm/sector 2 of the economy.

One may object against this scenario that it relies on a dramatic asymmetry of the cost function $K_i(N_i)$. In our view, this is, however, more a strength than a weakness of the scenario. It is one of the crucial problems of centralized trade unions that they determine the same wage tariffs for a large variety of firms which differ greatly in their economic success. And this causes a

serious threat for centralized wage bargaining as illustrated by our “union endangered”-scenario.

A different story is how the coefficients q_1 and q_2 depend on w , i.e. how hiring costs are influenced by the hourly wage w . It is not so much this dependency as such, but that this is different for both firms (q_1 , respectively q_2 does not, respectively does depend on w). Potential workers of firm 2 require lower hiring costs at the lower wage, what can be justified by benefits depending on wages, whereas in firm 1 such benefits remain unchanged. One can imagine that firm 1 and firm 2 have a different legal structure and are subject to different legal obligations (e.g. firm 1 could have private owners whereas firm 2 is owned by its employees).

5 Final remarks

Whereas most models of labor markets (e.g., Phelps 1994) are macro-models, the model, introduced and analysed above, is more in the tradition of oligopoly theory, i.e. the (game) theoretic approach to industrial economics. Our model focuses on firm competition instead of excluding it by distinguishing various layers of competition: Firms strategically interact not only on their sales markets via marketing strategies (here via price competition), but also in investing and in hiring employees.

Another, in our view, innovative feature is that employment, both at the firm level as well as for the whole economy, is split up in its two crucial components, the number of employees and their working times. The political debates often seem to suggest that the essential conflict is the “union’s trade off” in the narrow sense, i.e. that one needs lower wages for having more people employed. But such a conflict, if it exists at all, would be an extremely rare phenomenon in our model world. Actually we could not generate it as an equilibrium scenario without adding more flexibility.

None of the structural relationships of our model is producing the counter-intuitive result. Although one may question their functional specifications, which are mostly aiming at simplicity, qualitatively all the ingredients of our model appear as quite intuitive. Like others we were initially and unconsciously misled to specify the “union’s trade off” in the (too) narrow sense. If one considers both components of employment, the numbers of employees and their working times, also our model can easily account for the “union’s trade off” which according to Phillips (1958), seems to be a stable empirical phenomenon.

Of course, one could have distinguished between the two notions of employment, namely the number of people employed and the product of this number and working time, also when discussing the other equilibrium scenarios. In our view, they are, however, less likely as empirical phenomena and therefore also less relevant to justify the repeated exercise.

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