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# Tax Clientele Effects in the German Bond Market

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## Abstract

This paper presents an analysis of tax clientele effects in the German government bond market from the viewpoint of private investors. The methods developed here allow the identification of bonds that are over-valued from the viewpoint of a certain tax class, the estimation of tax-specific term structures, and the identification of representative investors. Regression and no-arbitrage approaches are unified. The empirical results presented have important implications for the estimation of the term structure from coupon bond prices and the valuation of interest rate derivatives.

**Keywords:** term structure of interest rates, tax clientele, arbitrage bounds, linear programming, duality theory, smoothing splines

**JEL classification:** E43, C14, C61

**Mathematics Subject Classification:** 90A12, 62-07, 90C05

# Tax Clientele Effects in the German Bond Market

## Introduction

Our understanding of how security prices are related was improved tremendously by the Arbitrage Pricing Principle. A modern version of this concept is, that a cash flow  $A$  that is in some sense “as least as good as” some cash flow  $B$  cannot have a lower price as  $B$ . If two such cash flows exist, profit-oriented investors will exploit that by buying  $A$  and selling  $B$ . In a perfect market the Law of One Price, which says that identical cash flows must have identical prices, follows as a consequence of the absence of arbitrage. The Arbitrage Pricing Principle works best in a world without taxes, transactions costs, and short-selling restrictions. Applied to the market for government bonds, it may be used to derive a unique pricing system, if a sufficient number of bonds exists. One way to express this unique pricing system is the term structure of interest rates for zero-coupon bonds.

Implicit in the above argument is, that all securities available in the market offer identical cash flows for all investors. Once taxes are introduced, this assumption is no longer realistic. An important aspect of many tax systems is that the after-tax cash flows investors receive from a specific security may differ. Even though with differential taxation a specific security offers different after-tax cash flows to different investors they all face the same market price. As a consequence, tax-clientes may exist, that is, some bonds are only held by investors of specific tax classes.

Hodges and Schaefer (1977) and Schaefer (1982) identified and described such tax-induced clientele effects in the British gilts market. They argue that with heterogeneous taxation and short-selling constraints the concept of arbitrage may be replaced by the concept of risk-free portfolio improvement and concluded, that under these conditions a unique term structure that is simultaneously valid for all investors will not exist. Schaefer (1981) estimated tax-specific term structures for the British gilts market.

Under transaction costs – and short-selling constraints can be interpreted as transaction costs – the Arbitrage Pricing Principle does not lead to a unique term structure, even without taxes. This was pointed out by Dermody and Prisman (1988) who introduced the concept of *individual no-arbitrage term structure packets*, which are packets of possible shadow prices in certain individual portfolio optimization problems. These term structure packets depend on an investor’s transaction costs, on his current portfolio, and on his trading constraints. If there are taxes, the term structure packets also depend on how coupons and capital gains are taxed, on whether capital losses and interest payments are deductible, and on the prices at which securities in the current portfolio were bought in the past. Larger transaction costs lead to more bloated packets whereas different tax schedules lead to

differently located packets. Dermody and Rockafellar (1991, 1995) provided a rather complete arbitrage theory for the valuation of deterministic cash flows in imperfect markets.

Differences in the tax treatment of individual market participants are rather large in Germany, compared to other industrial countries. Thus the German government bond market is an interesting example for the study of tax clientele effects and an ideal testing ground for theories about markets with taxes. Bühler and Rasch (1994) and Rasch (1996) described tax clientele effects in the DM-Eurobond market.

The existence of a more or less uniquely determined term structure is fundamental to many economic theories, especially those for the valuation of interest rate derivatives like (Cox et al.; 1985; Ho and Lee; 1986; Heath et al.; 1992), and (Björk et al.; 1996). In the presence of tax clienteles, is there any hope that some of the methods developed in the context of perfect markets can still be used?

A tax class is called *representative* if investors of this class do not generally object to holding all bonds. Equivalently, a tax class is representative if there is a term structure such that observed prices can be explained as the present values of the after-tax cash flows of that specific class. If such a class exists, if it can be identified with a really existing influential investor group, and if its term structure packet is not too fat, then the present value principle is rescued. All cash flows belonging to the same tax regime as the considered bond market can be valued by the present value principle from the viewpoint of the representative tax class. This case was succinctly called “clientele effect in volumes but not in prices” by Bühler and Rasch (1994). So, one of the most important questions in markets with different tax classes is whether a representative investor exists.

This paper presents an analysis of tax clientele effects in the German bond market that goes beyond the analysis in (Bühler and Rasch; 1994) and (Rasch; 1996). It is based on the theoretical framework of Dermody and Rockafellar (1995) and the crucial question whether a representative tax class exists is investigated. As far as we know, this is the first such analysis for the German bond market.

In section 1 we recall the arbitrage theory for bond markets with taxes as developed by Dermody and Prisman (1988) and Dermody and Rockafellar (1991, 1995). Section 2 briefly describes the taxation of bonds for several groups of investors. Tax clienteles are described in section 3. We propose one way to compute *generic term structure packets* for different tax classes in section 4. Finally, we investigate representative tax classes in section 5.

## 1 A Theory of Bond Markets with Different Taxes

Consider a family of tax classes indexed by  $k$ .

## 1.1 Arbitrage

Let  $S_i$  denote the ask price for bond  $i$  and  $p_i$  the net receipt of short-selling bond  $i$ . If  $x$  is a vector of transactions,  $x^+ = \max(x, 0)$  is the vector of buy orders and  $x^- = \max(-x, 0)$  is the vector of sell orders. Let  $C_i^k$  denote the after-tax cash flow received by an investor of class  $k$  who buys bond  $i$  (at time 0). Let  $g_i^k$  denote the net cash flow an investor of class  $k$  has to pay if he short-sells bond  $i$  (at time 0 and holds the short-position until maturity).

Define  $\bar{S} = \begin{pmatrix} 1 \\ S \end{pmatrix}$ ,  $\bar{x} = \begin{pmatrix} x_0 \\ x \end{pmatrix}$ ,  $\bar{p} = \begin{pmatrix} 0 \\ p \end{pmatrix}$ ,  $\bar{C} = \begin{pmatrix} 1 & 0 \\ 0 & C \end{pmatrix}$  and  $\bar{g} = \begin{pmatrix} 1 & 0 \\ 0 & g \end{pmatrix}$ .

Now  $\pi(\bar{x}) = S'x^+ - p'x^- + x_0$  is the net cost of the transaction  $\bar{x}$  and  $(\bar{C}^k)' \bar{x}^+ - (\bar{g}^k)' \bar{x}^-$  is the generated cash flow. For any cash flow  $z$ ,

$$\bar{\pi}^k(z) = \min_{\bar{x}} \{ \pi(\bar{x}) \mid (\bar{C}^k)' \bar{x}^+ - (\bar{g}^k)' \bar{x}^- \succeq \bar{z} \}$$

is the *upper arbitrage bound (imputed long price)* of  $z$  for class  $k$ . (A cash flow  $y$  is said to be *at least as good as* a cash flow  $z$  ( $y \succeq z$ ) if for each payment date  $t$   $\sum_{s=1}^t (y_s - z_s) \geq 0$  holds.) The market is *arbitrage-free*<sup>1</sup> for investors of class  $k$  ( $NA^k$ ) if  $\bar{\pi}^k(0) = 0$ .  $NA^k$  holds if and only if the *no-arbitrage term structure packet*

$$D^k := \{u \mid p \leq g^k u, C^k u \leq S, u \in K\}$$

is nonempty. In this case the arbitrage bounds can be computed as

$$\bar{\pi}^k(z) = \max_{u \in D^k} u'z \quad \text{and} \quad \underline{\pi}^k(z) = \min_{u \in D^k} u'z.$$

The name ‘‘arbitrage bound’’ is justified because an investor of class  $k$  can arbitrage if someone offers to buy a cash flow  $z$  for more than  $\bar{\pi}^k(z)$  or sell  $z$  for less than  $\underline{\pi}^k(z)$ . The market is called *arbitrage-free (NA)*, if  $NA^k$  holds for all  $k$ .

Bond  $i$  is called *unattractive long for class  $k$*  (or *dominated* in Schaefer’s terminology) if

$$\bar{\pi}^k(C_i^k) < S_i.$$

Bond  $i$  is called *unattractive short for class  $k$*  if

$$\bar{\pi}^k(g_i^k) > p_i.$$

The *long (resp. short) clientele of bond  $i$*  is the set of all classes for which  $i$  is not unattractive long (short). The price  $S_i$  is called *unsupported* if  $i$  has no long clientele. The price  $p_i$  is called *unsupported* if  $i$  has no short clientele.

A market with rational investors should be arbitrage-free and should have no unsupported prices. Under these conditions

$$S_i = \max_k \bar{\pi}^k(C_i^k) \quad \text{and} \quad p_i = \min_k \underline{\pi}^k(g_i^k)$$

holds.

<sup>1</sup>This is called weak no-arbitrage by Dermody and Rockafellar (1991). We do not need the concept of strong no-arbitrage here and drop the ‘‘weak’’ attribute for simplicity.

## 1.2 Risk-Free Portfolio Improvement

In economic theory, arbitrage is defined in the following way: an investor goes to the market with nothing, initiates a transaction  $\bar{x}$  that gives an immediate profit ( $\pi(\bar{x}) < 0$ ), and walks off with the portfolio  $\bar{x}$  that entails no future obligations ( $(\bar{C}^k)' \bar{x}^+ - (\bar{g}^k)' \bar{x}^- \succeq 0$ ). If there is a difference (in prices or in after-tax cash flows) between selling a bond and short-selling a bond, a slightly different question is whether an investor with initial portfolio  $\bar{X}$  can do a transaction  $\bar{x}$  that gives him an immediate profit and still the new portfolio  $\bar{X} + \bar{x}$  is at least as good as the old one  $\bar{X}$ . This is called *risk-free portfolio improvement* or *quasi-arbitrage*.

Assume  $s_i$  is the bid price and  $P_i$  is the price for closing out a short-position. Let  $c_i^k$  denote the class  $k$  after-tax cash flow of bond  $i$  (as if it was not sold) plus the actual taxes due when bond  $i$  is sold (e.g., taxes on realized capital gains). Let  $G_i^k$  denote the after-tax cash flow gained by closing out a short position in  $i$ . Define the index sets  $I^+ := \{i | X_i > 0\}$  and  $I^- := \{i | X_i < 0\}$ . Again,  $\bar{\pi}^{k,X}(z)$  is defined to be the lowest net cost at which an investor of class  $k$  can improve his initial portfolio  $\bar{X}$  by at least  $z$ . Investor  $(k, X)$  can *improve his portfolio*<sup>2</sup> if  $\bar{\pi}^{k,X}(0) < 0$ . Further, one can define *customized term structure packets*<sup>3</sup>

$$D^{k,X} := D^k \cap \{u | s_i \leq u' c_i^k \forall i \in I^+, u' G_i^k \leq P_i \forall i \in I^-\}$$

$D^{k,X}$  is the set of consistent term structures from the viewpoint of investor  $(k, X)$  in the following sense:

**Lemma 1** *Quasi-arbitrage is possible for an investor of class  $k$  with an initial portfolio  $X$  if and only if  $D^{k,X}$  is empty.*

Unlike the arbitrage bounds, the customized bounds  $\bar{\pi}^{k,X}$  and  $\underline{\pi}^{k,X}$  are not necessarily linear in scale. However, for small portfolio changes  $z^4$ , the equations

$$\bar{\pi}^{k,X}(z) = \max_{u \in D^{k,X}} u' z \quad \text{and} \quad \underline{\pi}^{k,X}(z) = \min_{u \in D^{k,X}} u' z$$

hold.

For investors without initial portfolio there is no difference between arbitrage and quasi-arbitrage ( $D^k = D^{k,0}$ ). In a market with rational investors there should be no quasi-arbitrage.

<sup>2</sup>This improvement is strict in the sense that it is independent of expectations and preferences and provides a sure profit now with no additional obligations later.

<sup>3</sup>Dermody and Rockafellar (1995) are a bit more general. We assume for simplicity here that  $c_i^k$  does not depend on the time when the investor bought the bond  $i$ . Generally, matrices  $c_i^k$  have to be constructed for each date bonds of type  $i$  were bought in the past.

<sup>4</sup>for  $z$  in a neighborhood of 0



Part of the transaction costs is in the prices  $p, P, s$  and  $S$ . If there are additional transaction costs, the term structure packets get fatter. Namely, the no-arbitrage packet becomes

$$D^{k,\alpha,\beta} := \{u \mid p - \beta \mathbf{1} \leq g^k u, C^k u \leq S + \alpha \mathbf{1}, u \in K\}$$

under linear transaction costs ( $\alpha$  for buying and  $\beta$  for short-selling).

So much for theory. In practice there is no market on which one can enter short-positions and hold them until maturity. At least, repo markets do not support it.<sup>5</sup> Without information on (maybe OTC) short-selling costs, no-arbitrage term structure packets cannot be computed and the question whether an investor had arbitrage possibilities cannot be answered. Without information on the portfolios held by different investors, customized term structure packets cannot be computed and the question whether an investor had quasi-arbitrage possibilities cannot be answered. The next sections present questions that can be answered on the basis of the available bond market data.

## 2 Optimization of After-Tax Cash Flows

### 2.1 Private Investors

Long-term capital gains ( $\geq 6$  months holding period) are tax-free for private investors. Before 1994, capital gains on German government bonds had generally been tax-free for private investors. Coupons are taxed as income, so the marginal income tax-rate applies. The marginal tax is zero for investors whose income from coupons and dividends remains below the allowance for capital income. For the portfolio optimization of private investors we make the following simplifying assumptions:

- Taxes on coupons are due the same day the coupons are paid.
- The nonlinear optimization problem – taxes are a nonlinear function of income – is approximated by the linear optimization problem in which coupons are taxed with the investor’s marginal tax rate.

Let  $Z$  denote the matrix of coupon and accrued interest payments and  $N$  the matrix of payments of the principal. Then we set

$$C^{priv(t)} = c^{priv(t)} = (1 - t)Z + N,$$

for  $t \in \{0\} \cup [0.19, 0.58]$ .

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<sup>5</sup>Typical terms of repo contracts are a couple of days (Ray; 1993). Security lending is generally restricted to 6 months or less with the *Deutsche Kassenverein* (Corrigan and Pohlmann; 1991).

When we present examples for certain optimization problems for private investors, we will do this for the marginal tax rates 0% and 50%. To shorten the notation, we will refer to these investors as the *tax-free investor* and the *highly taxed investor*, respectively.

## 2.2 Mutual Funds

Owners of mutual fund shares have to pay the same taxes on coupon payments from the bonds in the fund's portfolio, as if they would hold the bonds themselves. So, mutual funds should optimize their portfolios for certain tax classes and advertise this "target tax class".

## 2.3 Firms

An important notion in the accounting and taxation of German firms is the "principle of the lower of cost or market" (*Niederstwertprinzip*), which is derived from the "prudence principle" (*Vorsichtsprinzip*).<sup>6</sup> Approximately<sup>7</sup>, it says that the book value of a financial asset is the lowest market price since the date of acquisition. This leads to an asymmetry of the taxation of capital gains and losses:

Unrealized capital **gains** do not appear in the bookkeeping. Their taxation is deferred to the year of realization.

Unrealized capital **losses** enter the books and reduce the amount of tax in the year they occur.<sup>8</sup>

## 2.4 International Investors

Under certain conditions, international investors have to pay taxes in Germany. Typically, however, they pay taxes under the rules of their home country and no taxes in Germany (Corrigan and Pohlmann; 1991). In countries where capital gains are taxed and capital losses are deductible (in the U.S., for example) investors have a *tax-timing option* (the option to realize capital losses, see (Constantinides and Ingersoll; 1984)), which leads to similar after-tax cash flows as the "principle of the lower of cost or market" for German firms.

A general problem now is that the after-tax cash flows of German firms as well as the cash flows from the tax timing option of international investors

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<sup>6</sup>It seems to be typical of continental European accounting rules that the prudence principle is partly overriding the principle of the "true and fair view", whereas the American accounting rules are closer to the "true and fair view".

<sup>7</sup>For the precise rules see (Jaschke; 1998a).

<sup>8</sup>Firms can choose to not activate capital losses if the security is in the *Anlagevermögen*. This is called *gemildertes Niederstwertprinzip*. They must write off capital losses if the security is in the *Umlaufvermögen* (*strenges Niederstwertprinzip*).

are not deterministic. So, they cannot be considered in the framework of arbitrage bounds for deterministic cash flows implied by buy-and-hold portfolios. Hence, the following empirical results on tax clienteles apply to private investors only.

### 3 Tax Clienteles

Quasi-arbitrage implies that investors of class  $k$  remove bonds  $i$  from their portfolio if

$$\bar{\pi}_i^k(c_i^k) < s_i.$$

They refrain from buying bonds  $i$  if

$$\bar{\pi}_i^k(C_i^k) < S_i.$$

(Investors of class  $k$  belong to the *weak long clientele* of bond  $i$  if they would not buy new bonds of this type but possibly continue to hold bonds they already own.) We call  $S_i - \bar{\pi}_i^k(c_i^k)$  the *relative dearness* of bond  $i$  from the viewpoint of class  $k$ . In the following, we call a bond (or rather its ask price  $S_i$ ) *dominated by  $\varepsilon$*  if its relative dearness is larger than  $\varepsilon$ .

#### 3.1 Tax Clienteles and Coupons

Consider the market on March 16, 1984, shown in figure (1). The clienteles depend, as we expect, on the coupon. The highly taxed investor prefers low-coupon bonds. The unbalanced demand for low-coupon bonds leads to higher prices (from a tax-free view) of these bonds, so the tax-free investor holds the high-coupon bonds.

By 1997, however, the effect changed a bit. On July 24, no bonds were dominated (by more than 5 ticks) for the tax-free investor while more than half of the bonds were dominated (by more than 10 ticks) for the highly taxed investor. (See figure (2).)

#### 3.2 Clienteles across Time

Another way to look at the same data is to fix a bond and plot its clientele as a time series. Figures (3), (4) and (5) show tax-clienteles for specific bonds. The clientele of the bond (tax classes for which the bond is not dominated by more than 0.1% of face value) is marked with fat stars, while the other tax classes are indicated by small dots.

Bond 103000, for example, has a typical clientele. Investors with a low tax rate would hold the bond while highly taxed investors got the same after-tax cash flow cheaper. Bond 103000 was issued in 1990 as a 10-year bond with a coupon of 8.75%. Earlier 10-year bonds had lower coupons and later 10-year bonds had slightly higher coupons.

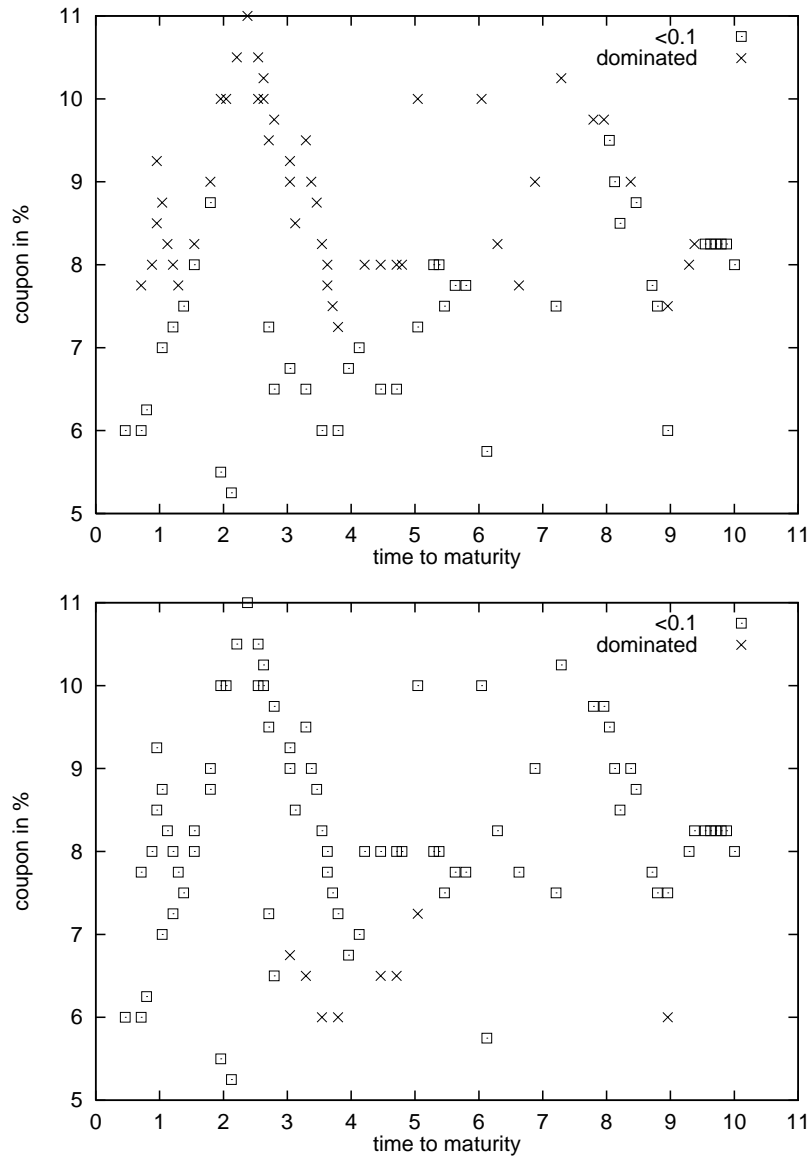


Figure 1: **Dominated Bonds on March 16, 1984, marginal tax rates 0% and 50%**

Every point in this coupon-versus-time-to-maturity-plot is a bond. If it is dominated by more than 0.1% of face value it is crossed out, otherwise it is boxed. The first plot shows the view of the highly taxed investor, the second that of the tax-free investor.

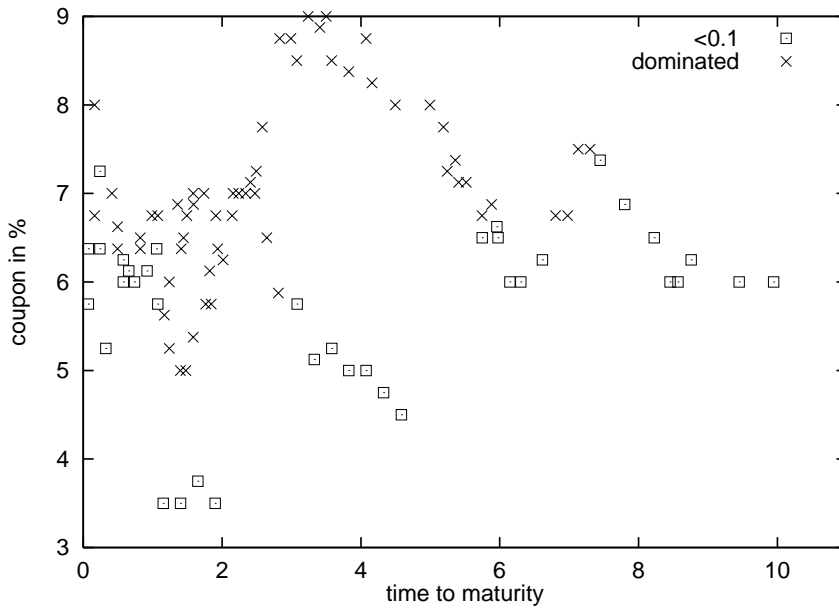


Figure 2: **Dominated Bonds on July 24, 1997, marginal tax rate 50%**

Every point in this coupon-versus-time-to-maturity-plot is a bond. If it is dominated by more than 0.1% of face value it is crossed out, otherwise it is boxed.

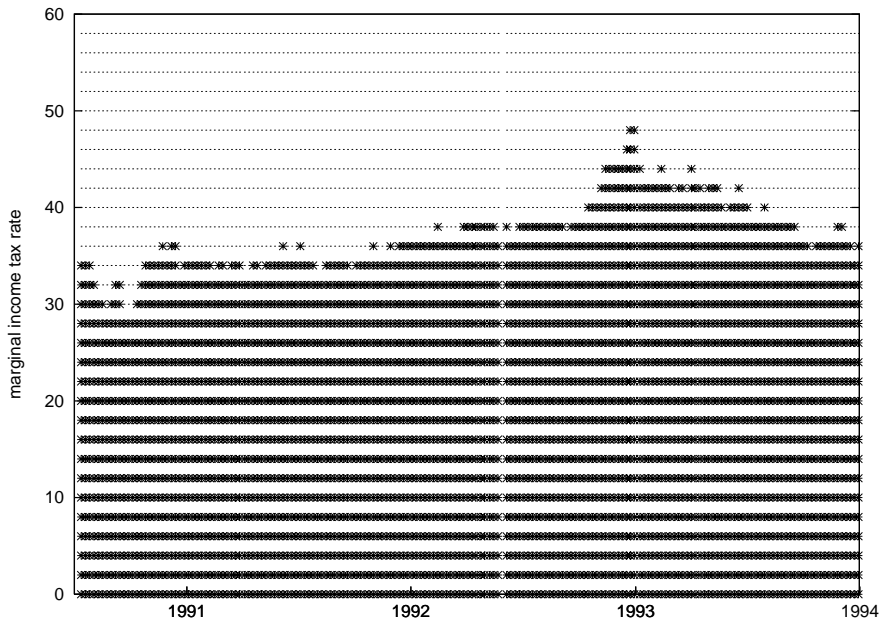


Figure 3: **Clientele of Bond 103000**

This bond was issued in 1990 and matures 07/20/00. The annual coupon is 8.75%. A star denotes that the bond is not dominated by more than 10 ticks for an investor in the given tax bracket at the given point in time.

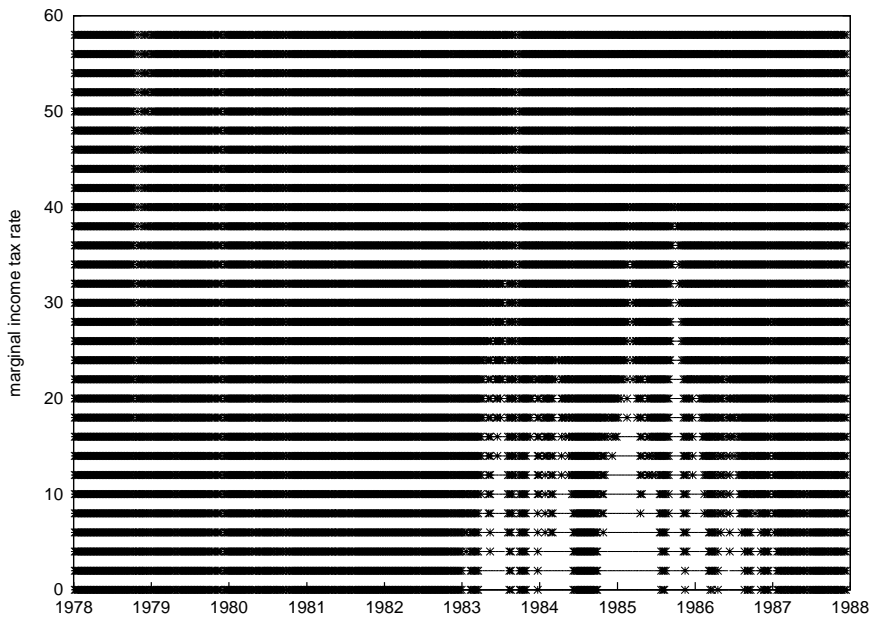


Figure 4: **Clientele of Bond 110078**

This bond was issued in 1978 and matured 01/01/88. The annual coupon is 6%. A star denotes that the bond is not dominated by more than 10 ticks for in an investor in the given tax bracket at the given point in time.

Bond 110078 was issued in 1978 as a 10-year bond. It had a coupon of 6%. In 1983 it competed with freshly issued 5-year bonds, which had higher coupons (7-9%). Clearly, it was so popular among highly taxed investors that it became too expensive for tax-free investors.

Bond 110045 presents an example of a more exotic clientele time series, that features almost any possible form of clienteles, including empty ones.

All in all, these examples show that a bond can have very different clienteles:<sup>9</sup>

**all investors:** 110078 before 1983, there were no competing bonds with a similar maturity;

**low-taxed investors:** 103000 throughout;

**high-taxed investors:** 110078 in 1983-1986;

**investors with an intermediate tax rate:** 110045 at some dates in 1982-1985;

**no clientele:** the price of 110045 is occasionally *unsupported* in 1981-1983.

<sup>9</sup>We conjecture that in a market of coupon bonds the relative dearness as a function of the tax rate is convex. This would imply that the clientele of a bond is always an interval.

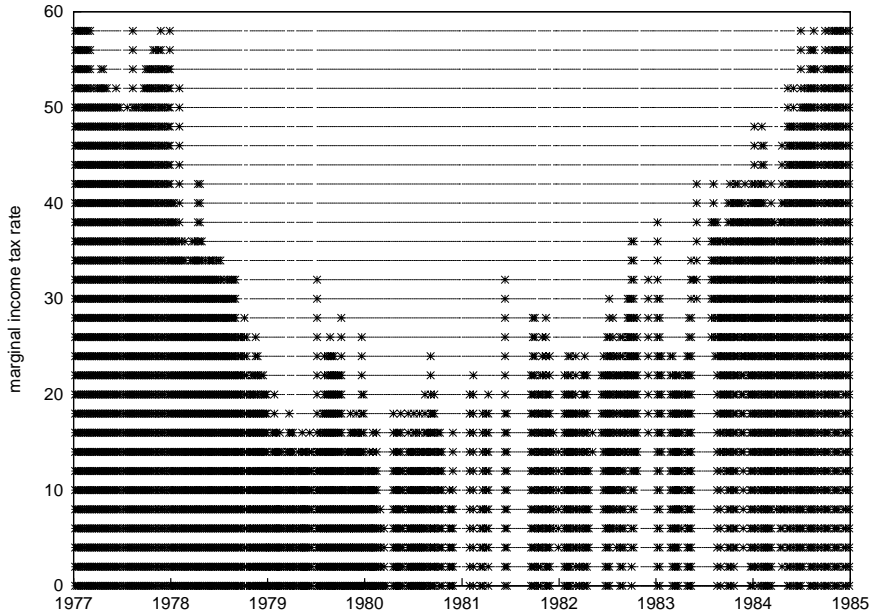


Figure 5: **Clientele of Bond 110045**

This bond was issued in 1973 and matured 03/01/85. The annual coupon is 8.5%. A star denotes that the bond is not dominated by more than 10 ticks for in an investor in the given tax bracket at the given point in time.

It should be noted that the concepts of dominance and clientele (based on dominance) are a bit short-sighted on one eye. The super-replication of high-coupon bonds (in terms of  $\succeq$ ) is close to a perfect replication: they can be nearly replicated by a series of low-coupon bonds with decreasing times to maturity. In order to super-replicate a bond with a minimal coupon, however, one has to match the principal payment with another bond that has a higher coupon, thereby throwing away a lot of money in the  $\succeq$ -comparison. In other words, the dominance concept fails to detect bonds that are over-valued (in a common sense of the word) if they have the minimal coupon of all bonds with the same or slightly smaller time to maturity.<sup>10</sup>

## 4 Customized Term Structure Packets

If we assume that short-selling is prohibitively expensive ( $p = 0, P = \infty$ ), the customized term structure packet for an investor of class  $k$ , with initial portfolio  $X$ , and transaction costs  $\varepsilon$  is

$$D^{k,I,\varepsilon} := \{u \mid s_i - \varepsilon \leq (c_i^k)'u \forall i \in I, C^k u \leq S + \varepsilon \mathbf{1}, u \in K\},$$

where  $I = \{i \mid X_i > 0\}$ .

<sup>10</sup>This observation was first made by Rasch (1996).

In order to compute *generic term structure packets for a tax class  $k$* , we have to determine index sets  $I$  of bonds that could possibly be held by investors of that class.

One possibility is to assume that an investor holds all bonds that are not dominated for his class:  $I_1^k = \{i | i \text{ is not dominated for class } k\}$ . This might, however, lead to portfolios that allow quasi-arbitrage, that is,  $D^{k, I_1^k}$  may be empty.

Another way to arrive at a generic set of bonds is to start with an initial portfolio  $M$ <sup>11</sup> and then do portfolio improvement (quasi-arbitrage) as long as no short-selling is required:

$$\min_{\bar{x}} \{ \pi(\bar{x}) | (\bar{C}^k)' \bar{x} \succeq 0, x^+ \geq 0, x_0 \geq 0, 0 \leq x^- \leq M \}. \quad (1)$$

(Here we assume transaction costs  $\varepsilon$ , i.e. the cost of a transaction  $x$  is  $\pi(\bar{x}) = x_0 + (S + \varepsilon \mathbf{1})' x^+ - (s - \varepsilon \mathbf{1})' x^-$ .) If  $\tilde{x}$  is an optimal solution of (1),  $\{i | \tilde{x}_i^- = M_i\}$  is the set of bonds sold out completely. So,  $I^{k, \varepsilon}(M) = \{i | \tilde{x}_i^- < M_i\}$  defines a set of bonds an investor of class  $k$  (and with transaction costs  $\varepsilon$ ) could possibly hold if he started out from  $M$ . The optimal solution  $\tilde{x}$  need not be unique. In terms of bonds  $i$  and  $j$ , for example, it may be equally optimal to throw out bond  $i$  and hold  $j$ , to throw out bond  $j$  and hold  $i$ , or to hold a convex combination of both bonds. An LP solver using a simplex method gives out one of the first two (extremal) solutions while an interior point solver gives out a solution of the last form. This means that the index set  $I^{k, \varepsilon}(M)$  is properly defined if we restrict ourselves to interior solutions of (1) (and it can be properly computed by an interior point solver).

The quasi-arbitrage problem (1) and the corresponding index sets  $I^{k, \varepsilon}(M)$  provide an alternative clientele concept: Let us say that  $k$  belongs to the *M-clientele of bond  $i$*  if  $i \in I^{k, \varepsilon}(M)$ . This concept does not suffer from the one-sidedness of the clientele concept based on dominance. Overvalued low-coupon bonds can be thrown out together with other bonds of shorter time to maturity and replaced by high-coupon bonds without losing too much in the  $\succeq$ -comparison. On the other hand, this clientele concept depends on preferences, as the quasi-arbitrage problem (1) implicitly assumes that the investor wants to receive a cash flow that is close to the cash flow that is generated by the market portfolio  $((C^k)'M)$ .

To make the two clientele concepts empirically comparable observe that a relative dearness  $\Delta_i > 0$  leads to the obvious quasi-arbitrage to sell bond  $i$  and buy the dominating portfolio if both bid and asked prices for that bond are available and the relative dearness is greater than the bid-ask-spread plus double the transaction cost  $\varepsilon$ , approximately. So, we compare the Schaefer-clientele defined as “not dominated by more than 20 ticks” with

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<sup>11</sup>  $M$  could be the “market portfolio”, ie  $M_i$  is the proportion of the issue of bond  $i$  to the whole market in terms of nominal value outstanding.  $M$  could also be one unit of each bond ( $M = \mathbf{1}$ ).



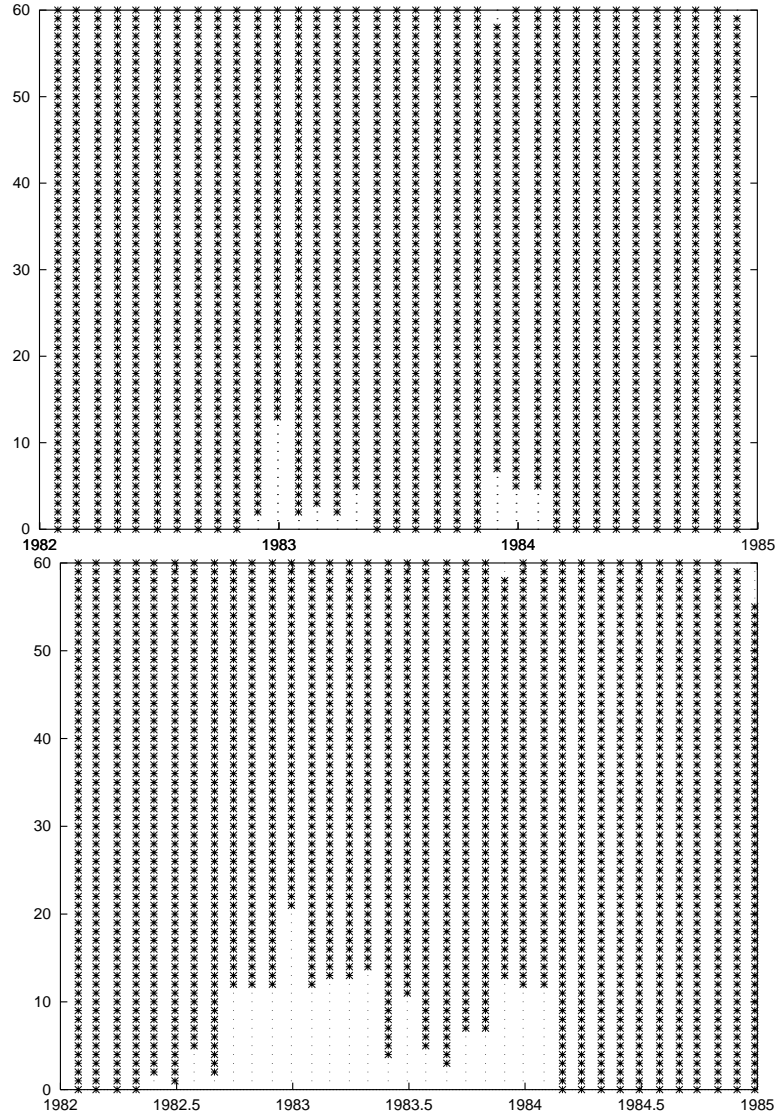


Figure 6: **Two Clientele Concepts for Bond 110070**

The comparison shows the clientele defined as “not dominated by more than 20 ticks for this class” (in the upper picture) with the  $M$ -clientele defined in terms of the quasi-arbitrage problem (1) with transactions costs  $\varepsilon = 10$  ticks (in the lower picture).  $M = 1$ .

the  $M$ -clientele defined in terms of the quasi-arbitrage (1) with transactions costs  $\varepsilon = 10$  ticks.

On our dataset, the two clientele concepts give very similar pictures for most bonds<sup>12</sup>. For only few bonds – those with an unmatched low coupon in a certain range of maturities – the clientele and the  $M$ -clientele differ by several classes. Bond 110070 is such an example (figure 6).

The other important feature of the index set  $I^{k,\varepsilon}(M)$  is to define a non-empty term structure packet

$$D^{k,I^{k,\varepsilon}(M),\varepsilon} := \{u \mid s_i - \varepsilon \leq u'c_i^k \forall i \in I^{k,\varepsilon}(M), C^k u \leq S + \varepsilon \mathbf{1}, u \in K\}. \quad (2)$$

If  $M$  is the market portfolio (or  $M = \mathbf{1}$  for simplicity), the index set  $I^{k,\varepsilon}(M)$  is a rather large set of bonds, so the corresponding term structure packet (2) can be considered a *generic term structure packet* for class  $k$ .

We computed bounds for the zero-bond yield curve with respect to these generic term structure packets. Figure (7) shows the (bounds for the zero-bond yield curve corresponding to) generic term structure packets of tax-free investors and highly taxed investors. It reveals several interesting empirical facts. A first guess at the relation between the term structures of a tax-free investor and an investor who receives only half of the coupons is that they are proportionate (2 to 1). Figure (7) demonstrates that this need not be the case. The term structure for the highly taxed investor is very ragged (and the bounds show that this is not a spurious effect of an estimation procedure). In certain maturity ranges (2-3 years and 5-6 years) there are no bonds that a highly taxed investor would rationally hold (see also figure(2)). In this sense, the market is “quite incomplete” (even for maturities less than 10 years) from the viewpoint of a highly taxed investor. In other ranges (0-1 years) there are only bonds that trade above par, so highly taxed investors have to pay, say, 2.5% tax on a bond with a coupon of 5.5%, but the expected capital loss of 2.2% cannot be deducted and only 0.3% remains after taxes.<sup>13</sup>

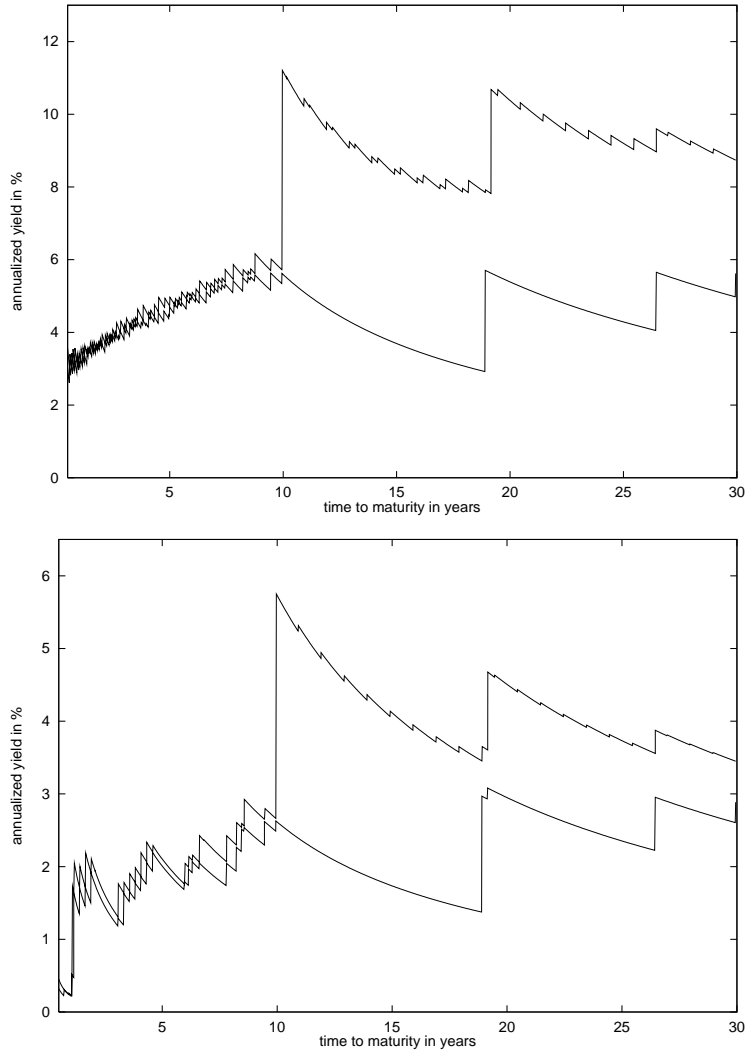
## 5 Is There a Representative Investor?

We call  $k$  a *representative tax class*, if investors of class  $k$  would not object to holding all bonds. More precisely, if the set

$$D^{k,all,\varepsilon} = \{u \mid s - \varepsilon \mathbf{1} \leq c^k u, C^k u \leq S + \varepsilon \mathbf{1}, u \in K\}$$

<sup>12</sup>Actually they differ a bit due to a peculiarity of our dataset. When there is only a bid-price, the  $M$ -clientele is defined, but the Schaefer-clientele (“strong long clientele” in Dermody and Rockafellar (1991)) is not. If there is only an asked price, it is the other way round. Since the first case happens more often, the  $M$ -clientele pictures show less missing values than the Schaefer-clientele pictures.

<sup>13</sup>Highly taxed investors would not hold coupon bonds with such a short time to maturity as they could buy other money market products providing 3% pre-tax and 1.5% after tax. This is just to illustrate how tax-specific term structure packets implied by coupon bond prices can differ.



**Figure 7: Generic Term Structure Packets for Tax Classes 0% and 50%, July 24, 1997**

The starting portfolio  $M$  is set to one unit of each Government bond, excluding Post bonds, Bahn bonds, and the newly introduced STRIPS. Transactions costs  $\varepsilon$  are set to  $10^{-3}$ . Then the quasi-arbitrage optimization (1) is run. The resulting term structure packet is the set of term structures that are consistent with asked prices of all bonds and the bid prices of those bonds that were not thrown out in the quasi-arbitrage. The bounds show the minimal and maximal yield to maturity of zero-bonds with respect to this term structure packet. The first picture is the packet for 0% marginal tax and and the second for 50%.

is nonempty for realistic transaction costs  $\varepsilon$ .

If a representative tax class exists and it can be identified with an important group of investors, *the* term structure alias the present value principle is rescued.

The notion of a representative tax class is closely related to attempts to include taxes into the term structure regression, like in (McCulloch; 1975), (Jordan; 1984), or (Green and Ødegaard; 1996):

$$\min_k \min_u \|\tilde{S} - C^k u\|^2$$

( $\tilde{S} = \frac{1}{2}(S + s)$ ). In terms of arbitrage theory this translates to

$$\min_k \min_{\varepsilon, u} \{\varepsilon \mid s - \varepsilon \mathbf{1} \leq C^k u, C^k u \leq S + \varepsilon \mathbf{1}, u \in K\}. \quad (3)$$

We call the tax classes reaching the minimum in (3) *best-fit tax classes*. If a representative tax class exists, (3) can be used to find it. If, however, no representative tax class exists, the best-fit tax class has no economical meaning<sup>14</sup>.

Figure (8) shows the minimax fitting error for all tax classes in the period 1976-1996. Figure (9) shows more clearly the representative tax classes and additionally the best-fit tax class. If we allow for 0.2% minimax fitting error, investors with low tax rates were representative in 1976-1979, 1988-1990, 1995-1996, and some other weeks. The rest of the time, however, there is no representative tax class among private investors and the best-fit tax class and the corresponding estimate of the term structure have no economical meaning.

In (Jaschke et al.; 1996) we analyze in more detail the reasons why certain bonds appear to be mispriced from a pre-tax viewpoint and whether these price discrepancies could have been exploited. We show there, that the price discrepancies in the period 1982-1985 seem to be mainly caused by tax clientele effects.

Once it is accepted that one cannot get around tax clienteles even if one wants to determine a general-purpose (pre-tax) term structure from German government bonds in the period 1976-1993, one could call a tax class *almost representative* if investors of that class would not object to holding almost all bonds, say 90%.

As before, we compute sets of bonds  $I^{k,\varepsilon}(M)$  that allow no further quasi-arbitrage for class  $k$ . Figure (10) shows the tax classes that were *almost representative*. That the tax-free investor has been *almost representative* since 1987 (with a few exceptions around 1992) justifies the method to determine a generic (pre-tax) term structure that was proposed by Jaschke (1998b):

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<sup>14</sup>There is a general message from the arbitrage theory for bond markets. One should be very careful in applying standard regression techniques based on some form of the present value equation to bond market data. The outcome may be economically meaningless in the presence of market segmentation.

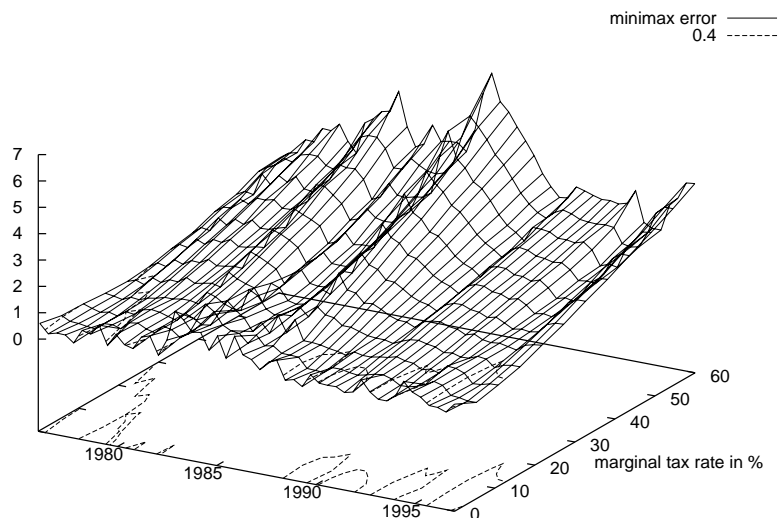


Figure 8: **The Minimax Fitting Error for All Tax Classes** (in % of face value)

The minimax fitting error is computed with respect to all German government bonds except Post and Bahn bonds. The contour plot on the bottom shows the representative tax classes (at a certain level of transaction costs  $\varepsilon$ ).

1. Exclude (the few) bonds that are not rationally hold by tax-free investors.
2. Display the resulting term structure packet by computing arbitrage bounds for the zero-bond yield curve.
3. Determine the “smoothest” term structure in that term structure packet.

## 6 Conclusion and Open Problems

In markets with differently taxed investors the usual assumption of the existence of a term structure that explains *all* prices as present values of pre-tax cash flows may be invalid:

- Clientele effects in the sense that some investors would not rationally hold certain bonds are omnipresent. That private investors with high marginal income tax prefer low-coupon bonds over high-coupon bonds is well known. These effects are so large that the rather crude super-replication ( $\succeq$ ) comparison can be used to accurately describe them. This has the advantage over regression techniques that one can *prove*

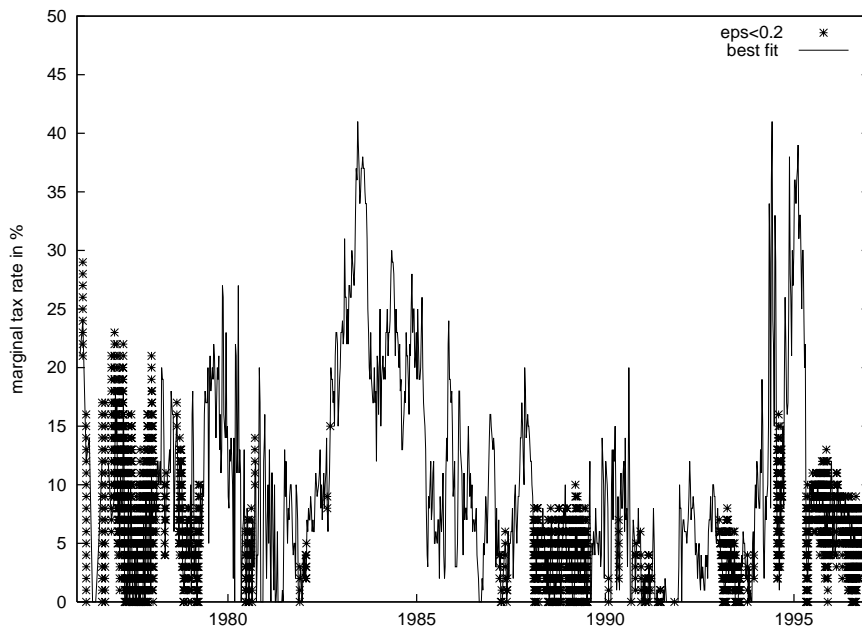


Figure 9: **Representative and Best-Fit Tax Classes**

Representative classes (i.e. classes whose minimax fitting error does not exceed 0.1% of face value) are marked with stars. The best-fit tax rate is drawn with a solid line.

the tax effects independently of choices of a parametric or nonparametric model and statistical assumptions.

- Tax-specific term structures can differ quite a lot and have “strange shapes” (see figure 7).
- The most interesting question from a theoretical-economical point of view is whether a representative investor exists. If no representative investor exists, computing best-fit tax classes is not the way to deal with tax effects.

Most of the ideas to use the dominance/arbitrage-principle to investigate tax clientele effects go back to (Hodges and Schaefer; 1977; Schaefer; 1981, 1982). However, tax effects have often been ignored or dealt with by estimating a marginal (best-fit) tax rate. Often one can get away with that in terms of precision. The German bond market in the mid-eighties is an example that *requires* to explicitly deal with tax-effects. Dermody and Rockafellar’s theory (1991,1995) is a nice framework to use.

A consequence of that theory is that the absence of arbitrage net of proportional transaction costs implies that the residuals in a term structure regression are *bounded* and the bound equals the transaction costs factor. If the minimax-residuals are too large, one can *prove* that the regression equa-

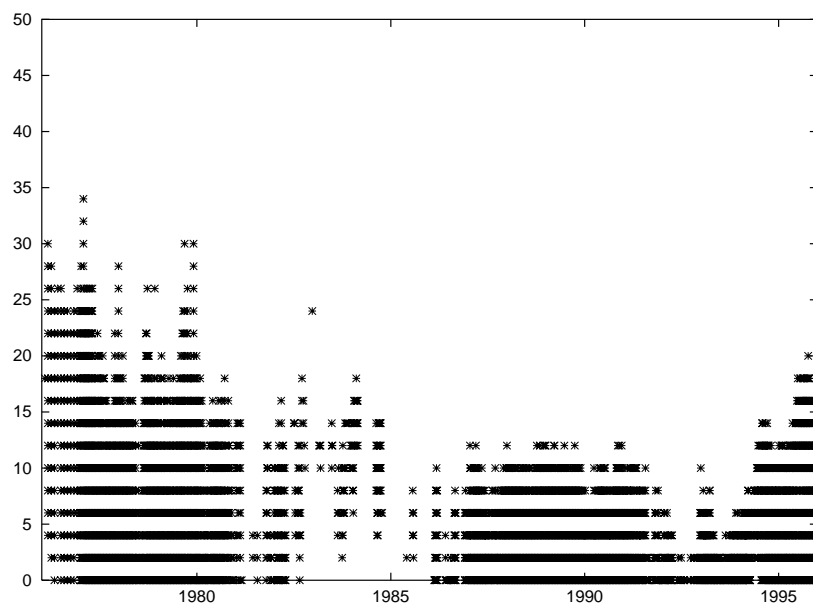


Figure 10: **Almost Representative Tax Classes**

When a term structure can be fitted to 90% of all bonds with a minimax fitting error of no more than 0.1% of face value then the date/tax pair is marked.

tion is invalid. In the other case – when there is a representative investor and present value regression makes sense – the fact that residuals are bounded, questions the wide-spread use of certain least-squares fitting techniques that are based on the assumption that residuals are *Gaussian*.

Since 1987 the tax-free investor has been almost representative. This means that only few bonds are somehow “special” from a pre-tax viewpoint, but “special” enough to invalidate the assumption that prices are given by the present value equation plus noise. Excluding the special bonds allows to fit a term structure with a small minimax fitting error.

From table (1) we see that domestic financial institutions and foreign investors are the most important investors in the German bond market. Those investors, however, have non-deterministic after-tax cash flows, which we cannot deal with in the context of arbitrage bounds implied by buy-and-hold portfolios. It is an open question whether domestic banks are representative in the sense that there is a price functional for their *stochastic* after-tax cash flows such that observed prices are explained as *expected* present values under this price functional.

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<sup>15</sup> *Kreditinstitute*

	1988	1994
	percentages	
domestic banks <sup>15</sup>	41.8	37.3
foreign investors	15.3	26.6
private investors	11.9	11.6
insurance companies	10.0	7.0
mutual funds	6.0	7.4
non-financial organizations	4.9	3.9
non-profit organizations	2.5	1.9
social security	1.2	0.9
other public sectors	0.9	1.2
Deutsche Bundesbank	0.4	0.1
rest	5.1	2.1
sum (in thousand million DM nominal value)	1150.5	2604.5

Table 1: **Important Investors**

Important Investors in Fixed Income Securities issued by domestic debtors and denominated in DM (*auf DM lautende Inhaberschuldverschreibungen inländischer Emittenten*). Source: Deutsche Bundesbank (1995).



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