Bougheas, Spiros

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Pooling, tranching and credit expansion

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Pooling, Tranching and Credit Expansion

Abstract

Traditionally banks have used securitization for expanding credit and thus their profitability. It has been well documented that, at least before the 2008 crisis, many banks were keeping a high proportion of the securities that they created on their own balance-sheets. Those securities retained included both the high-risk ‘equity’ tranche and the low-risk AAA-rated tranche. This paper builds a simple model of securitization that accounts for the above retention strategies. Banks in the model retained the equity tranche as skin in the game in order to mitigate moral hazard concerns while they post the low-risk tranche as collateral in order to take advantage of the yield curve. When variations in loan quality are introduced the predicted retention strategies match well those found in empirical studies.

JEL-Code: G210, G240.

Keywords: securitization, tranching, credit expansion.

Spiros Bougheas
School of Economics
University of Nottingham
UK – Nottingham, NG7 2RD
United Kingdom
spiros.bougheas@nottingham.ac.uk

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1. Introduction

As time goes on banks rely less on deposits for financing their activities and more on securitization and leverage. These options for raising funds have allowed them to substantially expand their balance sheets and thus their profitability albeit, as the crisis of 2008 has made clear, at higher levels of risk exposure (Brunnermeier, 2009; Dell’Ariccia, Ingan and Laeven, 2009; Mian and Sufi, 2009). Securitization itself has also been the subject of financial innovation. Some of the securities are straight pass-throughs as, for example, in the case of some types of loan sales (Pennacchi, 1988; Gorton and Pennacchi, 1995) while other securities are created by pooling and tranching the cash-flows of banking assets.¹ In the latter case, a variety of new securities are formed differentiated by their default risk and then sold to investors according to their risk appetite.

The initial objective of securitization was to boost liquidity by enabling banks to sell their assets and use the funds raised from these sales to offer new loans. However, it has been well documented that, at least before the crisis, many banks were keeping a high proportion of the securities that they created on their own balance-sheets. What is more surprising is that those securities retained included both the high-risk ‘equity’ tranche (Acharya, Philippon, Richardson and Roubini, 2009) and the low-risk AAA-rated tranche (Acharya and Schnabl, 2009).² The same banks, especially those that are large and grow fast, have also increasingly relied on short-term wholesale financial markets for raising funds (Demirgüç-Kant and Huizinga, 2010).

In this paper, I provide a theoretical account for the above observations by introducing in the Shleifer and Vishny (2010) banking model a monitoring role for banks, similar to Holmström and Tirole (1997). In the latter work, the form of contracts related to the sale of securities to investors and the form of contracts agreed between the bank and its lenders are both exogenously given. In particular, investors require the bank to keep in its books as ‘skin in the game’ a fixed fraction of the securities that it creates while lenders impose a fixed ‘haircut’ on the size of the loan in relation to the value of the securities that the bank posts as collateral. Using the extended version of the model, I am able to derive the optimal contractual forms that the bank agrees with its investors and lenders. In particular, I demonstrate that, when the returns of the loans are not perfectly correlated, pooling and

¹ This method of security creation has attracted an extensive literature reviewed by Gorton and Metrick (forthcoming) who also provide a detail description of the institutional (including the legal) environment within which this process takes place.

² Acharya and Schnabl (2009) report that if someone includes those AAA-rated asset backed securities that banks held off their balance-sheets in ABCP conduits and SIVs then the fraction retained rises above 50%. See also Jaffee, Lynch, Richardson and Van Nieuwerburgh (2009) and Krishnamurthy (2008).
tranching of the cash-flows generated by the loans that the bank offers to its clients is optimal. By keeping a fraction of the equity tranche in its books, the bank assures investors that it still has an incentive to monitor its clients. I further show that, when I allow for projects of different quality the skin in the game declines as quality improves. What the bank does with the AAA-rated tranche depends on the relative cost of raising funds between selling these securities to investors and increasing its leverage by posting them as collateral. Before the crisis banks used the overnight market to ‘ride the yield curve’ and thus financed a significant part of their activities by rolling over low-cost short-term debt (Calomiris, 2009). The model predicts that when the bank uses leverage to finance its activities it can reduce haircuts, and thus boost credit expansion, by posting as collateral the high-quality AAA-rated tranche.

The model rationalizes the practices that for a long time banks have been using to expand their activities. However, the global financial crisis has made painfully clear that many institutions around the world that have adopted those practices only survived the crisis because of, very expensive to taxpayers, government bailouts. There is a very fast growing literature devoted not only to identifying the causes of the crisis but also to the design of appropriate policy responses. Along with lax monetary conditions, regulatory failure, underestimation of systemic risk, poor performance by rating agencies and a weak banking governance structure, there are aspects of financial innovation that have also been regarded responsible for the financial crisis. But as the FED chairman Ben Bernanke has suggested it is important to distinguish financial innovation from its implementation. Financial innovation can lead to new products that offer efficient solutions to agency problems in financial markets. In the model below, securitization by pooling and tranching of asset returns in conjunction with certain retention strategies ensure investors that the bank has a strong incentive to monitor its clients, thus, enabling it to expand its balance sheet and hence its profits.

I develop the model in Section 3 and in the next two sections I focus on the case when all loan returns are perfectly correlated. In Section 4, I analyze credit expansion for the case

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4 Gennaioli, Shleifer and Vishny (forthcoming) have shown that when consumer neglect small probability risks financial innovation can indeed lead to higher financial market volatility.

5 The following quote is taken from his speech “Financial Innovation and Consumer Protection” that he delivered at the Federal Reserve System's Sixth Biennial Community Affairs Research Conference, Washington, D.C. in April, 2009: “…as we have seen only too clearly during the past two years, innovation that is inappropriately implemented can be positively harmful. In short, it would be unwise to try to stop financial innovation, but we must be more alert to its risks and the need to manage those risks properly.”
when the bank creates straight pass-through securities while in Section 5, I derive the optimal form of securitization contracts. I show that by using tranching the bank can (a) create high-risk securities which, by keeping them in its books, enhance its incentives to monitor its clients, and (b) some very low-risk securities that can post as collateral. In Section 6, I extend the analysis to the case where loan returns are independently distributed and show that incentives can be further improved by pooling, in addition to tranching, loan payoffs. In the context of the present model, where all parties are risk-neutral, the benefits of pooling are not the result of risk diversification. By combining pooling and tranching the bank conditions security payoffs on the proportion of projects that succeed. The bank by keeping in its books the high-risk tranche that pays out only when a sufficiently high fraction of projects succeed has even stronger incentives to monitor its clients. In section 7, I allow for project quality variations and address issues related to the financial crisis. I conclude in Section 8.

2. Related Literature

One old method for tranching payoffs is their separation into seniority claims. The advantages of this practice for mechanism design have been the subject of a very long literature and the work that is most closely related to the present one is Innes (1990). In his model the lender cannot observe the level of effort exerted by the entrepreneur. Given that expected profits increase with effort, Innes (1990) shows that, if (a) the entrepreneur has limited liability, and (b) the contract is restricted to be non-decreasing in profits, it is optimal for the lender to offer a standard-debt contract. Thus, the entrepreneur holds the risky equity tranche that pays out in states that become more likely as she exerts higher levels of effort. Similarly, in this model the bank’s incentives to monitor its clients are strongest when it holds the risky-equity tranche. Of course, the observation that financing loan sales using debt increases the bank’s incentives to monitor is not new. One of the contributions of this paper is to show that the combination of pooling and tranching when bank asset returns are not perfectly correlated can further boost these incentives.

What drives the results in Innes (1990) and in the present work is the assumption that the return distribution conditional on the level of effort (monitoring) satisfies the monotone likelihood ratio property (MLRP). Chiesa (2008) and Fender and Mitchell (2009) analyze the role of securitization for environments where the property is violated. In Chiesa (2008) banks

6 The purpose of contract design is to provide a solution to delegated monitoring, a problem previously analyzed by Diamond (1984), Ramakrishnan and Thakor (1984), Boyd and Prescott (1986) and Winton (2003). These papers are part of a very extensive literature that analyzes the role of banks as monitors; for a review see Bhattacharya and Thakor (1993).

perform a monitoring role similar to the one in this paper. When monitoring is most valuable in those states where the systemic risk is high (economic downturns) she finds that it is optimal for the bank to sell its entire portfolio to investors and in addition offer them the option to sell it back to the bank at a pre-specified price. In Fender and Mitchell (2009) banks, rather than monitoring their clients after the signing of contracts, they screen them in advance in order to separate those with high-quality projects from the rest. They restrict their analysis to two types of securitization, namely, straight pass-through securities (what they call ‘vertical slice’) and securities created by pooling and tranching. They find that when MLPR is violated pooling and tranching is the best option along with a retention strategy where the bank keeps medium-risk securities (mezzanine tranche) in its books. In this paper, I analyze securitization within a mechanism design framework, albeit for a simpler environment where MLPR is satisfied, and demonstrate the optimality of pooling and tranching. In particular, I show that the optimal mechanism reduces both the ‘skin in the game’ required by investors and the ‘haircut’ required by lenders.

DeMarzo and Duffie (1999) also find that retention of the equity tranche by the bank is optimal. In their model the bank has superior information about the quality of the loans that attempts to sells to investors and it uses the size of the ‘skin in the game’ as signal. In particular, the skin of the game increases with quality indicating that the bank is willing to hold in its books better quality assets. In contrast, when projects of different quality are introduced in the present model the skin of the game declines as quality improves. The evidence, which I review in Section 7, is very thin but it suggests that the relationship might be negative. Finally, DeMarzo (2005) demonstrates the advantages of pooling and tranching for the case when the seller of securities is informed. In his model, the advantages of pooling are due to the benefits of diversification. In this paper, I show that pooling can be optimal even in the absence of diversification concerns.9

3. The Model

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8 The two approaches are quite similar. Under the supposition that the bank needs to screen a fixed number of potential customers in order to identify one with a high-quality project the two models become isomorphic. The screening model is more suitable for the case when the bank securitizes mortgages while the monitoring model fits better the case of collaterized loan obligations (CLOs) structured by pooling a variety of assets that, more recently, have included business loans (Jobst, 2003). See also Keller (2008) for a description of various agency costs related to CLOs.

9 The advantages of tranching are also considered by Riddiough (1997). He considers the securitization of a single loan and therefore there is no pooling. In contrast, Glaeser and Kallal (1997) consider the advantages of pooling but allow for only pass-through securities.
There are four types of risk-neutral agents in the model: entrepreneurs who need funds to finance projects, banks who provide funds to entrepreneurs and then use the loans to create securities, investors who buy the securities and lenders who offer loans to the banks accepting as collateral securities that banks keep in their books. I would like to use the model to understand not only the contractual arrangements between the banks and the other agents but also the process of credit expansion allowed by these arrangements. Then, it will be convenient to analyze an environment where the period of credit expansion is relatively short in comparison to the duration period of projects. With that in mind, I consider a model with three dates: 0, 1 and T. All contracts are agreed during the period between dates 0 and 1 and all projects financed during this initial period mature at T. The risk-free interest rate is equal to zero.

3.1. Projects

All projects are identical and require an investment of one unit of the single good in the economy. Projects can either succeed in which case they yield pledgeable income $R_H$ or fail in which case they yield pledgeable income $R_L$, where $R_H > 1 > R_L$. The probability of success of a project depends on the behaviour of its owner (entrepreneur) who can either choose to exert effort or shirk. In the former case the probability of success is equal to $p_H$ while in the latter case the probability of success is equal to $p_L$, where $p_H > p_L$. The returns of all projects are perfectly correlated.

3.2. Banks

A bank by monitoring a project at a cost $c$ can ensure that its entrepreneur exerts effort. If a bank decides to finance a project it collects an up-front fee $f$ and an expected repayment of $1 + m$ at date T. The following condition ensures that a project will only be financed when its entrepreneur exerts effort:

$$p_H R_H + (1 - p_H) R_L < 1 < p_H R_H + (1 - p_H) R_L - m.$$  \hspace{1cm} \mbox{(C1)}

Banks offer entrepreneurs debt contracts. Let $V$ denote the repayment when the project succeeds. The repayment must satisfy the condition $(1 - p_H)R_L + p_H V = 1 + m$ where the bank collects the whole payoff when the project fails. Solving for the repayment it yields $V = \frac{1 + m - (1 - p_H)R_L}{p_H}$.

3.3. Credit Expansion without Securitization
Let $E_0$ denote a bank’s equity at date 0. The bank will be able to fund $\frac{E_0}{1 + m}$ projects and given that the only profits that the bank earns are due to the up-front fees it collects, the bank’s expected final equity is given by $E_T = E_0 \left(1 + \frac{f}{1 + m}\right)$.

4. Fractional Securitization and Leverage

In this section, I am going to expand the activities of banks by allowing them to securitize their loans. Through securitization banks will be able to expand credit, and thus profits, by selling a fraction of each security to the market. In addition, I will allow banks to post those securities that they keep in their books as collateral so that they can obtain loans from the market. Before I examine securitization and leverage together, I will analyze each one separately.

4.1. Fractional Securitization without Leverage

Securitization involves the sale in the market of cash flow claims associated with loan repayments. The contact between the buyers and the bank must be designed so that the bank still has an incentive to monitor its clients. It is clear, that if a buyer were to buy an entire loan, that is its total payoffs in both states, the bank would not have an incentive to monitor the loan. But then (C1) implies that the price the buyer would be willing to pay for the loan is less than one, which is an offer that the bank would deny given that it had loaned one unit to the entrepreneur. This argument suggests that the bank must keep part of the payoffs of each loan in its books for a sale to take place.

Let the bank keep a fraction $d$, known as ‘skin in the game’, of each loan on its books. Thus, the buyer has a claim on a fraction $1 - d$ of the payoff for each loan that she buys a share.\(^\text{10}\) Competition among buyers ensures that the equilibrium price of each share is equal to $1 + m$, and thus the bank’s sales revenues from each security is equal to $(1 - d)(1 + m)$.

Next, I derive the optimal value of $d$. If the bank monitors the loan then it will earn a net payoff equal to $f$. If the bank does not monitor the loan then it will earn a net payoff equal to $f + d(1 - p_t)R_L + p_tV) + (1 - d)(1 + m) - 1$. The second term is equal to the bank’s payoff from the fraction of the loan that it kept in its books, the third term is equal to its revenues from the sale of the rest of the loan and the last term is equal to its initial investment. Comparing the two net payoffs shows that the bank will monitor the loan if the following incentive compatibility holds:

\[^{10}\text{Without any loss of generality, I assume that the number of securities that the bank creates equals the number of projects it finances.}\]
\[ d \geq \frac{m}{1-\rho+m}, \]  where \( \rho \equiv (1 - p_t)R_t + p_t V. \] (IC1)

The incentive compatibility constraint defines all values for \( d \) that offer incentives for the bank to monitor. Among those values the bank will choose the one that maximizes its expected final equity. A bank with equity \( E_0 \) at date 0 initially provides \( \frac{E_0}{1+m} \) loans. From the first round of securitization receives revenues \((1 - d)E_0\) and provides \((1 - d) \frac{E_0}{1+m} \) additional loans. Repeating the process the bank by date 1 will provide in total \( \frac{E_0}{1+m} \sum_{\tau=0}^{\infty} (1 - d)^\tau = \frac{1}{d} \frac{E_0}{1+m} \) loans. Given that the bank’s profits come from the fees it collects, these profits are proportional to the number of loans it provides and thus in equilibrium the incentive compatibility constraint will bind. The following proposition summarizes the results of this section:

**Proposition 1:** The optimal value of the skin in the game without leverage is given by

\[
 d^F = \frac{m}{1-\rho+m}. \tag{1}
\]

The bank’s expected final equity under fractional securitization without leverage is given by

\[
 E^F_T = E_0 \left( 1 + \frac{1}{d^F \frac{1}{1+m}} \right). \tag{2}
\]

The bank’s incentive to monitor declines as the gap between the expected payoff of a loan that has been monitored and the expected payoff of a loan that has not been monitored gets smaller. This is the case because the bank’s decision to monitor affects its own payoff only through the securities that it keeps in its books.

### 4.2. Leverage without Securitization

Banks can borrow funds from the market, and thus expand credit, by posting securities that they keep in their books as collateral. I assume that potential lenders offer to banks only secured loans. This, for example, will be the case if it is too costly for them to verify the bank’s stochastic payoffs. In this section, I do not allow the bank to sell to the market any of the securities that it creates but keeps them in its books.

Once more, I consider a bank that at date 0 has equity \( E_0 \). The bank will initially finance, and thus securitize, \( \frac{E_0}{1+m} \) loans. Given that (a) the returns of all loans are perfectly correlated, and (b) the bank raises funds in a competitive market, the collateral value of these

\[ 11 \] \( \rho \) is equal to the expected payoff of the loan given that bank does not monitor.
initial loans, and thus the amount the bank can borrow, is equal to \( \frac{E_0}{1+m} R_L \). With the new funds the bank can finance \( \frac{E_0}{(1+m)^2} R_L \) additional loans which can also be used as collateral. The process repeats itself \textit{ad infinitum}. The bank by date 1 will have in total created \( \frac{E_0}{1+m} \sum_{\tau=0}^{\infty} \left( \frac{R_L}{1+m} \right)^\tau = \frac{E_0}{1+m-R_L} \) loans. Notice, that given that the loans are secured, lenders do not care about whether the bank monitors its clients. Having said that, as the bank keeps all the loans it its books and, moreover, only receives a payoff when the loans succeed, it has an even stronger incentive to monitor relative to the case when it does not lever its equity.

The competitive market value of each security is equal to \( 1 + m \). The collateral value of the same security is equal to \( R_L \). The ratio \( h = \frac{1+m-R_L}{1+m} \), that is the ratio of the difference between market value and collateral value to market value, is known as the ‘haircut’. The following proposition summarizes the results of this section:\(^{12}\)

**Proposition 2:** The optimal value of the haircut is given by

\[
h = \frac{1+m-R_L}{1+m}.
\]

The bank’s expected final equity under leverage is given by

\[
E_T^L = E_0 \left( 1 + \frac{1}{h} \frac{1}{1+m} \right).
\]

4.3. Fractional Securitization with Leverage

Now, I will allow the bank to engage in both fund raising activities. It can sell securities in the market and borrow funds by posting as collateral those securities it holds in its books. The bank is not allowed to sell any securities that it has already posted as collateral for a loan unless it does so to repay the loan. However, the bank is allowed to post as collateral securities that keeps in its books as skin in the game.

I begin by showing that the buyers of securities set the skin in the game lower when the bank posts the corresponding securities as collateral. The bank’s net gain by not monitoring a loan is equal to \( m - d(p_h - p_l) V \). Given that the bank is posting the security created from the loan as collateral it will earn a payoff only when the corresponding project succeeds. Then, the bank will monitor the loan if the following incentive compatibility constraint holds:

\[
d \geq \frac{m}{(p_h - p_l) V}.
\]

\(^{12}\) I have assumed throughout that the bank repays all its loans at date T. The results in the paper will not be affected if I allow the bank to borrow for shorter periods and then rollover its debt.
Given that the bank’s profits are increasing with credit expansion the incentive compatibility constraint binds in equilibrium. The next lemma compares the two skins in the game:

**Lemma 1:** The optimal value of the skin in the game with leverage is given by

\[ d^{FL} = \frac{m}{(p_h-p)v} < d^F. \]

Now that the bank earns a payoff only when the project succeeds it has a stronger incentive to monitor and thus the buyers respond by asking for a lower skin in the game.13

Next, I examine the level of credit expansion when the bank uses both securitization and leverage to finance its activities. Remember that the bank cannot sell any securities that it has already posted as collateral. Once more a bank with equity \( E_0 \) at date 0 will initially finance \( \frac{E_0}{1+m} \) projects and create the same number of securities. The bank will keep a fraction \( d^{FL} \) of these securities as skin in the game and also post them as collateral in order to borrow from the market. Thus, the bank will be able to borrow \( d^{FL} \frac{E_0}{1+m} R_L \) from lenders and receive \((1 - d^{FL})E_0\) from the sale of securities. The bank can now use the revenues from the sale of securities and the amount borrowed from the market to finance \((1 - d^{FL}) \frac{E_0}{1+m} + d^{FL} 1+m2RL\) new projects. The bank will repeat the process by keeping a fraction \( d^{FL} \) of the new loans as skin in the game and at the same time posting them as collateral to finance

\[ 2d^{FL}(1 - d^{FL}) \frac{E_0}{(1+m)^2} R_L + (1 - d^{FL})^2 \frac{E_0}{1+m} + (d^{FL})^2 \frac{E_0}{(1+m)^3} (R_L)^2 = \frac{E_0}{1+m} (1 - d^{FL}) + d^{FL} 1+m2RL\]

additional projects. By repeating the process \( ad\ infinitum \) the bank by date 1 will have created in total \( \frac{E_0}{1+m} \sum_{t=0}^{\infty} \left( (1 - d^{FL}) + d^{FL} \frac{1}{1+m} R_L \right)^t = \frac{1}{d^{FL} (1,R_L,1+m)} = \frac{1}{d^{FL}_f} \) new loans. The following proposition summarizes the results of this section:

**Proposition 3:** The bank’s expected final equity under fractional securitization with leverage is given by

\[ E_T^{FL} = E_0 \left( 1 + \frac{f}{d^{FL}_h (1+m)} \right). \]

5. Optimal Securitization Contracts

13 There is an alternative way to derive (IC2). If the bank monitors the loan will earn a net payoff equal to \( f - (1 - p_h)R_L \) given that it has posted the corresponding security as collateral (I disregard the continuation payoffs here as they are not affected by the decision of the bank to monitor). If the bank does not monitor the loan will earn a net payoff equal to \( f + dp_v V - (1 - d)(1 + m) - 1 \). As earlier, the second term is equal to the bank’s payoff from the fraction of the loan that it kept in its books, the third term is equal to its revenues from the sale of the fraction of the loan and the last term is equal to its initial investment. (IC2) follows from subtracting the second expression from the first one.
Up to this point, I have assumed that when a bank sells securities it keeps a share $d$ of each security in its books and, thus, it receives a fraction $d$ of its payoffs in each state of the world. Now, I will demonstrate how a bank can further boost credit expansion, and thus increase its profits, by choosing an alternative way to split the payoffs of the projects.

5.1. Optimal Securitization Contracts without Leverage

For the moment, I will ignore leverage which I will reconsider in the following section. Suppose that the bank creates two securities out of each loan. The first security, which I denote $X_B$, will pay off $z_L$ if the project succeeds and $z_H$ if the project fails and the bank keeps it in its books. The second security, which I denote $X_I$, will pay off $R_L - z_L$ if the project succeeds and $V - z_H$ if the project fails and the bank sells it to investors. If the bank monitors the loan its payoff will be equal to $p_hz_H + (1 - p_h)z_L - m$ and if it does not monitor the loan its payoff will be equal to $p_lz_H + (1 - p_l)z_L$. The bank’s incentives to monitor increase with $z_H$ and decline with $z_L$. Then optimality requires:

$$z_L = 0 \text{ and } z_H = \frac{m}{p_h - p_l}. \quad (6)$$

Next, I consider credit expansion for a bank with equity at date 0 equal to $E_0$ that it has initially used to finance $\alpha \beta_{1 + m}$ projects. The bank will keep in its books $\frac{E_0}{1 + m}$ securities of type $X_B$ and sell to investors $\frac{E_0}{1 + m}$ securities of type $X_I$ at price $P^S \equiv (1 - p_h)R_L + p_h(V - z_H)$, raising revenues $P^S \frac{E_0}{1 + m}$. With these revenues the bank will be able to finance $P^S \frac{E_0}{(1 + m)^2}$ additional loans. Repeating the earlier process the bank will sell $P^S \frac{E_0}{(1 + m)^2}$ raising revenues equal to $(P^S)^2 \frac{E_0}{(1 + m)^2}$. The process will repeat ad infinitum and by date 1 will have created $\frac{E_0}{1 + m} \sum_{\tau=0}^{\infty} \left( \frac{P^S}{1 + m} \right)^\tau = \frac{E_0}{1 + m} \left( \frac{1}{1 - \frac{P^S}{1 + m}} \right)$ loans in total. It is straightforward to prove the following proposition:

**Proposition 4:** The equilibrium price of security $X_I$ is given by

$$P^S = 1 + m - \frac{p_h m}{(p_h - p_l)} \quad (7)$$

The optimal value of the skin in the game under optimal securitization but without leverage is given by

$$d^S = \frac{p_h m}{(p_h - p_l)(1 + m)} \quad (8)$$

The bank’s expected final equity under optimal securitization but without leverage is given by
\[ E_T^S = E_0 \left(1 + \frac{f}{d^S} \frac{1}{1+m}\right). \] (9)

From the solution for \( z_H \) it follows that the bank’s expected payoff from each security of type \( X_B \) is equal to \( \frac{p_h m}{(p_h - p_l)(1+m)-(1-p_h)R_L} \). Given that each loan’s expected payoff is equal to \( 1 + m \), the equilibrium price of a security of type \( X_l \) is equal to the difference between these two payoffs. Under the optimal arrangement the payoffs of the two securities created from each loan are not proportional to the payoffs of the loan (as in the fractional securitization case) but we can still define as skin in the game the ratio of the bank’s expected payoff from each loan to the total expected payoff of the loan. Doing so makes it easy to compare the implications of each type of bank activity for credit expansion.

**Proposition 5:** (a) \( d^{FL} h > d^S \), and (b) \( E_T^S - E_T^{FL} > 0 \).

**Proof:** (a) Substituting for \( V \) in the expression for \( d^{FL} \) given in Lemma 1 and multiplying by \( h \) given by (3) yields \( \left(\frac{p_h m}{(p_h - p_l)(1+m)-(1-p_h)R_L}\right) \left(\frac{1+m-R_L}{1+m}\right) \) which after some straightforward algebraic manipulation can be shown to be greater than \( \frac{p_h m}{(p_h - p_l)(1+m)} = d^S \). Part (b) follows after comparing (5) and (9).

The split of payoffs under optimal securitization by enhancing the bank’s incentives to monitor lowers the skin of the game and thus boosts credit expansion.

5.2. Optimal Securitization with Leverage

The split of payoffs examined in the previous section does not allow the bank to lever its equity. The \( X_B \) security that the bank keeps in its books has a zero payoff when the project fails and, given that the returns of all projects are perfectly correlated, it has zero collateral value. However, there is an alternative way for splitting the payoffs that would allow the bank to borrow from the market.

The bank can do three things with the securities that it creates. It can sell them or keep them in its books and post them as collateral or keep them in its books without posting them as collateral. Furthermore, as the collateral value of any security is equal to its minimum payoff between the two states of the world it will be optimal for the bank to post as collateral securities that offer the same payoff in the two states. Therefore, there are potentially five types of securities that the bank might want to issue.\(^{14}\) However, it is never optimal for the bank to issue securities that pay off only when the project fails and keep them in its books.

\(^{14}\) I preclude the possibility of securities with lotteries as state-contingent payoffs.
This is because these types of securities reduce the bank’s incentive to monitor and would only keep them in its books if it can post them as collateral. Then, there are only four types of securities that we need to consider.

The first security, which I denote, \( Y_C \) will pay off \( \bar{v} \) in both states of the world and the bank will keep it in its books and post it as collateral. The second security, which I denote \( Y_L \), will pay off \( R_L - \bar{v} \) only when the project fails and the bank will sell it to investors. The third security, which I denote \( Y_H \), will pay off \( v_H \) only when the project succeeds and the bank will keep it in its books. The fourth security, which I denote \( Y_L^H \), will pay off \( V - v_H - \bar{v} \) only when the project succeeds and the bank will sell it to investors.

If the bank monitors the loan its payoff will be equal to \( p_h v_H - m \) and if it does not monitor the loan its payoff will be equal to \( p_l v_H \). Keep in mind that when the project succeeds the bank has to repay \( \bar{v} \) its lenders. The bank’s incentive to monitor increases with \( v_H \) but its ability to expand credit decreases with \( v_H \) and increases with \( \bar{v} \). Then, optimality requires:

\[
\bar{v} = R_L \text{ and } v_H = \frac{m}{p_h - p_l},
\]

(10)

The bank’s credit expansion ability is given by

\[
C_S = R_L + p_h \left( V - \frac{m}{p_h - p_l} - R_L \right) = 1 + m - \frac{p_h m}{p_h - p_l},
\]

(11)

The first term is equal to the amount borrowed by posting security \( Y_C \) as collateral. The second term is equal to the price of security \( Y_L^H \) and thus equal to the revenues received by selling it to investors. \(^{15}\)

Therefore, a bank with equity at date 0 equal to \( E_0 \) that it has used to finance \( \frac{E_0}{1+m} \) projects will receive \( \frac{E_0}{1+m} \left( 1 + m - \frac{p_h m}{p_h - p_l} \right) \) from lenders and investors. With these funds the bank will be able to finance \( \frac{E_0}{1+m} \left( 1 - \frac{p_h}{1+m} \frac{m}{p_h - p_l} \right) \) additional projects. Repeating the process \( ad \ infinitum \) the bank will, by date 1, have created in total

\[
\frac{E_0}{1+m} \sum_{t=0}^{\infty} \left( 1 - \frac{p_h}{1+m} \frac{m}{p_h - p_l} \right)^t = \frac{E_0}{1+m} \frac{1}{p_p m (p_h - p_l)(1+m)}
\]

loans. The following proposition summarizes:

**Proposition 6:** The bank’s expected final equity under optimal securitization

---

\(^{15}\) Security \( Y_L^H \) does not show up as the bank keeps it in its books.
$$E_T^{St} = E_T^S = E_0 \left( 1 + \frac{f}{p_{hm}} \frac{1}{1+m} \right) = E_0 \left( 1 + \frac{f}{d^S 1+m} \right).$$  

(12)

is independent of its level of leverage.

Under the optimal securitization arrangement the bank does not derive any additional benefits by borrowing from the market. This result is the securitization version of the Modigliani-Miller theorem. The bank can either sell the safe securities or post them as collateral. Given that, in the model, the required rate of return of investors and lenders is identical the bank is indifferent about how it finances its activities. In practice, banks have been financing a great deal of their activities by borrowing. To keep the analysis simple I have assumed that all loans are repaid at date T and thus the investment horizon of investors and lenders is the same. In reality, banks take advantage of the yield curve by borrowing and rolling over short-term debt that allows them to further boost their profitability.

6. Independently Distributed Project Returns, Pooling and Tranching

Up to this point, I have focused on the case where all project returns are perfectly correlated. In this section, I relax this assumption. The distribution of returns of each project is exactly the same as before, however, these returns are now independently distributed. To keep things simple, I assume that there are two types of projects, namely type I and type II and four states of the world and that half of the projects in the bank’s portfolio are type I while the other half of the projects are type II. The returns of projects of the same type are perfectly correlated. I assume that project types are observable and thus I eliminate any adverse selection considerations from the security design problem. The following table shows the distribution of returns:

<table>
<thead>
<tr>
<th>State of the World</th>
<th>Outcome</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>All projects succeed</td>
<td>$(p_h)^2$</td>
</tr>
<tr>
<td>$I$</td>
<td>Only type I projects succeed</td>
<td>$p_h(1 - p_h)$</td>
</tr>
<tr>
<td>$II$</td>
<td>Only type II projects succeed</td>
<td>$p_h(1 - p_h)$</td>
</tr>
<tr>
<td>$L$</td>
<td>All projects fail</td>
<td>$(1 - p_h)^2$</td>
</tr>
</tbody>
</table>

Given that the returns of projects of the same type are perfectly correlated, without any loss of generality, I assume that if the bank monitors a project it also monitors all other projects of the same type. Then, I consider the securitization of a portfolio that comprises of two projects
I also consider the case where the bank is able to post the securities that it keeps in its books as collateral.

I will divide the analysis of rest of this section into two parts. I will begin by considering the case of optimal securitization without pooling and tranching. Then, I will introduce these possibilities and by comparing the two cases the benefits of pooling and tranching will become clear.

6.1. Securitization without Pooling and Tranching

Following the analysis in the last section, the bank can potentially issue the following types of securities. Security $Y_c(i)$, where $i = I, II$, will pay off $\tilde{v}(i)$ in all states of the world and the bank will keep it in its books and post it as collateral. Security $Y^H_i(i)$ will pay off $R_L - \tilde{v}(i)$ only when the project fails and the bank will sell it to investors. Security $Y^H_H(i)$ will pay off $v_H(i)$ only when the project succeeds and the bank will keep it in its books. Security $Y^H_L(i)$ will pay off $V - v_H(i) - \tilde{v}(i)$ only when the project succeeds and the bank will sell it to investors.

After the bank signs an agreement with the investors it has three choices. It can monitor both projects or it can monitor only one project or it will not monitor any of the two projects. If the bank monitors both loans its payoff will be equal to $(p_h)^2 + p_h(1 - p_h))(v_H(I) + v_H(II)) - 2m$, keeping in mind that the bank receives a payoff only when a project succeeds. If the bank monitors only the type $I$ project its payoff will be equal to $(p_h p_I)(v_H(I) + v_H(II)) + p_h (1 - p_I)v_H(I) + p_I(1 - p_h)v_H(II) - m$ and if the bank monitors only the type $II$ project its payoff will be equal to $(p_h p_I)(v_H(I) + v_H(II)) + p_h (1 - p_I)v_H(II) + p_I(1 - p_h)v_H(I) - m$. Lastly, if the bank does not monitor any project its payoff will be equal to $(p_h)^2 + p_h(1 - p_I))(v_H(I) + v_H(II))$.

Given that $\tilde{v}(I)$ and $\tilde{v}(II)$ do not appear in the above payoffs it is optimal to set them equal to $R_L$, so that credit expansion is maximized. It is also optimal to set $v_H(I) = v_H(II) = \tilde{v}_H$. To see this suppose that $v_H(I) > v_H(II)$. In this case the bank’s payoff is greater when it only monitors the type $I$ project than when it only monitors the type $II$ project. Then it is only the former payoff that matters for incentives. By decreasing $v_H(I)$ by $\delta$ and increasing $v_H(II)$ by $\delta \frac{p_h(1-p_I)}{p_I(1-p_h)}$ that payoff remains the same but the corresponding incentive compatibility constraint would be relaxed. A similar argument eliminates $v_H(I) < v_H(II)$.

The discussion above implies that the optimal arrangement must satisfy the following incentive compatibility constraints:
\[(p_h)^2 + p_h(1 - p_h))2\tilde{\sigma}_H \geq (p_hp_l)2\tilde{\sigma}_H + (p_h(1 - p_l) + p_l(1 - p_h))\tilde{\sigma}_H + m\]

and

\[(p_h)^2 + p_h(1 - p_h))2\tilde{\sigma}_H \geq (p_l)^2 + p_l(1 - p_l))2\tilde{\sigma}_H + 2m.\]

It is straightforward to show that each of the two constraints implies that \(\tilde{\sigma}_H \geq \frac{m}{p_h-p_l}\). The two constraints are identical as both the benefits and the costs are doubled when the bank decides not to monitor at all rather than monitoring only one of the projects. The solution is also the same as the one derived for the case of perfect correlation. This is because the incentives for monitoring are designed at the project level.

6.2. Securitization with Pooling and Tranching

In this section, I show how the bank can further boost credit expansion through the creation of new types of securities designed by pooling and tranching the payoffs of the loans on its balance sheet. Although these additional benefits are only possible when the project returns are not perfectly correlated they are not the result of diversification as all agents are risk neutral. The bank will issue four types of securities. The following table shows their state-contingent payoffs.

<table>
<thead>
<tr>
<th>Security</th>
<th>State-Contingent Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(H)</td>
</tr>
<tr>
<td>(W_{C0})</td>
<td>2(R_L)</td>
</tr>
<tr>
<td>(W_{L}^{L})</td>
<td>(V - R_L)</td>
</tr>
<tr>
<td>(W_{I}^{H})</td>
<td>(V - w_{H} - R_L)</td>
</tr>
<tr>
<td>(W_{B}^{H})</td>
<td>(w_{H})</td>
</tr>
</tbody>
</table>

Security \(W_{C0}\) is riskless having a payoff equal to \(2R_L\) in every state of the world. Therefore, this security is created by pooling together the two \(Y_C(i)\) securities discussed in the previous section. This security is either posted as collateral or is sold to investors. \(W_{L}^{L}\) is a low-risk security that has a payoff equal to zero in state \(L\) and a payoff equal to \(V - R_L\) in all other states. The security is created by pooling the payoffs of the two loans in states \(I\) and \(II\) and half the payoff of the two loans in state \(H\) after subtracting first the payoff of security \(W_{C0}\). This security is sold to investors. The other two securities are high-risk and they only offer a
positive payoff in state $H$. Security $W^H_I$ is sold to investors and has a payoff equal to $V - w^H_I - R_L$ while security $W^H_B$ has a payoff equal to $w^H_I$ and the bank keeps it in its books.

If the bank monitors the two loans its net payoff will be equal to $(p_h)^2 w^H_I - 2m$ and if it does not monitor the two loans its net payoff will be equal to $(p_l)^2 w^H_I$. Given that the bank’s credit expansion ability is decreasing with $w^H_I$, optimality requires that:

$$\hat{w}^H_I = \frac{2m}{(p_h)^2 - (p_l)^2}. \quad (13)$$

It is crucial that the payoff of security $W^L_I$ in state $H$ is created by pooling and halving the payoffs of the two loans in that state. The same payoff can be created by separating the payoffs of the two loans but in that case the bank would not have an incentive to monitor the loan that is not in its books.

The bank’s per loan credit expansion ability under pooling and tranching is given by:

$$C^{PT} = \frac{2R_L + (1 - (1 - p_h)^2)(V - R_L) + (p_h)^2(V - \hat{w}^H_I - R_L)}{2} = (1 + m) - \frac{(p_h)^2 m}{(p_h)^2 - (p_l)^2} \quad (14)$$

The three terms on the numerator after the first inequality are equal to the bank’s revenues from the sale of securities $W^0_C$, $W^L_I$ and $W^H_I$, respectively, created from the pooling of the payoffs of two loans. Following the same steps as for the case of optimal securitization it can be shown that:

**Proposition 7:** The bank’s expected final equity under pooling and tranching is given by:

$$E^{PT}_T = E_0 \left( 1 + \frac{f}{(p_h)^2 m} \frac{1}{1 + m} \right) = E_0 \left( 1 + \frac{f}{d^{PT}} \frac{1}{1 + m} \right). \quad (15)$$

Next, I show that when returns are not perfectly correlated, pooling and tranching dominate the securitization process that is optimal when returns are perfectly correlated.

**Proposition 8:** (a) $d^S > d^{PT}$, and (b) $E^{PT}_T - E^{S}_T > 0$. 

---

16 If the parameters of the model are such that $\frac{2m}{(p_h)^2 - (p_l)^2} > V - R_L$ then $\hat{w}^H_I = V - R_L$ and the bank also receives a payoff in states $I$ and $II$.

17 With pooling the bank does not have an incentive to monitor only one loan. A sufficient condition to prevent this is to have $w^H_I \geq \frac{m}{(p_h)^2 - p_h p_l}$. However, $\hat{w}^H_I = \frac{2m}{(p_h)^2 - (p_l)^2} = \frac{2m}{(p_h - p_l)(p_h + p_l)} > \frac{m}{p_h(p_h - p_l)} = \frac{m}{(p_h)^2 - p_h p_l}$ and, thus, the constraint does not bind.

18 In the Appendix, I derive sufficient conditions for pooling and tranching to be optimal, i.e. it cannot be dominated by any other incentive compatible mechanism.
Proof: (a) \[ \frac{(p_h)^2}{(p_h)^2 - (p_l)^2} = \left( \frac{p_h}{p_h - p_l} \right) \left( \frac{p_h}{p_h + p_l} \right) > \left( \frac{p_h}{p_h - p_l} \right). \] (b) It follows after comparing (9) and (15).

The intuition for the above result is that pooling and tranching concentrate the bank’s payoffs on those states where the incentives for monitoring are the strongest. The securitization arrangement that is optimal when project returns are perfectly correlated is not optimal when the same returns are imperfectly correlated as it offers a payoff to the bank on those states where only one project is successful.

7. Project Quality and the Skin in the Game

Up to this point, I have assumed that the return distribution of all projects is identical. Now, I relax this assumption in order to consider how the quality of the project affects the skin in the game, that is, the portion of the corresponding loan that the bank keeps in its books. To keep the analysis simple, I consider the case of independent returns so that there is no pooling. From (12) the skin in the game is given by:

\[ d^s = \frac{p_{hm}}{(p_h - p_l)(1 + m)}. \]  

(16)

Better project quality is captured by either a higher \( p_h \) or a lower \( p_l \). There are stronger incentives to monitor better quality projects. Then, (16) implies that the skin in the game should increase as project quality deteriorates.

In contrast, DeMarzo and Duffie (1999) find that when the bank has superior information about project quality the skin of the game rises as project quality improves. In their model the bank by showing a willingness to keep a higher fraction of the securities in its books it signals that the underlying assets are of better quality.

There is quite a lot of empirical work on securitization related to moral hazard and adverse selection issues and the evidence is mixed. The empirical works by Berndt and Gupta (2009) and Elul (2009) suggest that securitization has a detrimental effect on loan quality but it cannot discriminate between the causes, namely, between moral hazard and adverse selection. The studies by An, Deng and Gabriel (2009) and Downing, Jaffee and Wallace (2009) provide empirical support for the adverse selection interpretation. In contrast, Keys, Mukherjee, Seru and Vig (2009) present evidence that is consistent with the presence of moral hazard in the subprime loan market (lack of screening and monitoring) but not with strategic adverse selection. The last term refers to the advantages confer to banks by their ability to select the quality of the loans that offer for sale to investors in addition to those due
to asymmetric information.\textsuperscript{19} Most of the work uses data from mortgage-backed securities. Benmelech, Dlugosz and Ivashina (2010) analyzing collateralized loan obligations find that syndication significantly reduces adverse selection concerns.

There are a couple of papers that look directly at the relationship between loan quality and the skin and the game (Chen, Liu and Ryan, 2008; Demiroglu and James, 2012). Both find that the skin in the game decreases as quality improves a finding consistent with the prediction of the present model.

8. Conclusions

The goal of this paper is to provide a simple model that captures the practices that modern banks employ to expand credit and thus profitability. It is well understood now that many banks that adopted such practices have been principal players in the recent financial turmoil. But one point that this paper emphasizes is that financial innovation is not necessarily responsible for the crisis. By creating securities of different risk classes by pooling and tranching the payoffs of various types of loans banks have managed to expand their activities either by selling these securities to investors or convincing lenders to accept them as collateral. Using a simple model, I have shown how the banks can mitigate moral hazard by using pooling and tranching that leads to a decline in the skin in the game and thus credit expansion. I have also shown that the same process creates low-risk securities that lenders are willing to accept as collateral. Banks can further expand their activities by riding the yield curve and using leverage to their advantage.

In addition to creating incentives for banks to either screen or monitor their clients securitization had also to address issues arising because of asymmetries in information about loan quality between banks on one hand and lenders and investors on the other. While there is no doubt that banks have used various credit enhancement mechanisms to reduce the impact of adverse selection they have also relied heavily on certification and, in particular, the reports of rating agencies.\textsuperscript{20} To keep things simple in my model the low-risk tranche is riskless. The model can be easily extended to allow for some risk. As long as the risk level is

\textsuperscript{19} Put differently, the bank’s type (portfolio quality) is endogenous. This problem is addressed by the endogenous screening literature; see, for example, González (2004). Gorton and Souleles (2006) analyze issues related to the legal structure of special purpose vehicles within a framework that allows for both moral hazard and strategic adverse selection.

\textsuperscript{20}Gorton and Metrick (1997) compare the performance of AAA-rated asset-backed securities to AAA-rated corporate bonds and find that the default rates of the former group exceeded the rates of the latter. When the lenders realized that the ratings did not reflect the true default risk they kept increasing their demand for collateral till eventually the overnight fund market froze. See Acharya, Gale and Yorumalzer (2011) for a theoretical model that captures this process.
below the maximum risk that lenders are willing to take then banks can still post the low-risk securities as collateral. Within this extended version it would be straightforward to show that as the quality of the collateral deteriorates lenders would require a higher haircut.²¹

There is no doubt that an increase in the complexity of financial products can exacerbate problems related to asymmetric information.²² But it is important to separate those aspects of financial innovation that aim to mitigate agency problems in financial markets from those that do not.

²¹ See Brennan, Hein and Poon (2009) for an analysis of the relationship between the prices and the ratings of different tranches of asset-backed securities.
²² Here I have in mind some by-products of securitization such as synthetic CDOs and CDO²s.
Appendix A: Optimality of Pooling and Tranching

Without any loss of generality, I shall ignore leverage and, thus, assume that the bank shares the loan payoffs only with investors. Let \( u_i^j \), where \( i = I, II \) and \( j = H, I, II, L \), be non-negative real numbers denoting the bank’s payoff from the loan repayment of project \( i \) in state \( j \). These payoffs must satisfy the following constraints:

\[
\begin{align*}
&u_L^I \leq R_L, \quad u_L^{II} \leq R_L, \quad u_I^I \leq R_L, \quad u_I^{II} \leq R_L, \\
&u_L^I \leq V, \quad u_I^{II} \leq V, \quad u_I^I \leq V, \quad u_I^{II} \leq V.
\end{align*}
\]  

(A1)

The objective is to choose \( u_i^j \) to maximize the bank’s revenues from loan sales subject to the participation constraint of investors, the set of inequalities (A1) and the incentive compatibility constraints that ensure that the bank has an incentive to monitor the projects. Given that investors make zero profits in equilibrium the above objective is equivalent to choosing \( u_i^j \) to minimize the bank’s expected payoff subject to (A1) and the incentive compatibility constraints:

\[
\begin{align*}
\text{Min}\{&p_h^2(u_i^I + u_i^{II}) + p_h(1 - p_h)(u_i^I + u_i^{II} + u_i^{III} + u_i^{IV}) + (1 - p_h)^2(u_L^I + u_L^{II})\} \\
\text{subject to:} &\quad (p_h)^2(u_i^I + u_i^{II}) + p_h(1 - p_h)(u_i^I + u_i^{II} + u_i^{III} + u_i^{IV}) + (1 - p_h)^2(u_L^I + u_L^{II}) - m \geq \sum p_h \left( u_i^I + u_i^{II} + u_i^{III} + u_i^{IV} \right) + (1 - p_h)(1 - p_l)(u_L^I + u_L^{II}) \\
&\quad (p_l)^2(u_i^I + u_i^{II}) + p_l(1 - p_l)(u_i^I + u_i^{II} + u_i^{III} + u_i^{IV}) + (1 - p_l)^2(u_L^I + u_L^{II}) - 2m \\
&\quad \geq (p_l)^2(u_i^I + u_i^{II}) + p_l(1 - p_l)(u_i^I + u_i^{II} + u_i^{III} + u_i^{IV}) + (1 - p_l)^2(u_L^I + u_L^{II})
\end{align*}
\]  

(A2)

and

\[
\begin{align*}
&\text{subject to:} \\
&\quad (p_h)^2(u_i^I + u_i^{II}) + p_h(1 - p_h)(u_i^I + u_i^{II} + u_i^{III} + u_i^{IV}) + (1 - p_h)^2(u_L^I + u_L^{II}) - m \geq \sum p_h \left( u_i^I + u_i^{II} + u_i^{III} + u_i^{IV} \right) + (1 - p_h)(1 - p_l)(u_L^I + u_L^{II}) \\
&\quad (p_l)^2(u_i^I + u_i^{II}) + p_l(1 - p_l)(u_i^I + u_i^{II} + u_i^{III} + u_i^{IV}) + (1 - p_l)^2(u_L^I + u_L^{II}) - 2m \\
&\quad \geq (p_l)^2(u_i^I + u_i^{II}) + p_l(1 - p_l)(u_i^I + u_i^{II} + u_i^{III} + u_i^{IV}) + (1 - p_l)^2(u_L^I + u_L^{II})
\end{align*}
\]  

(A3)

Constraint (A2) ensures that the bank does not have an incentive to monitor only one project. Although, in principle, there are two such constraints, symmetry implies that they are identical. Constraint (A3) ensures that the bank does not have an incentive not to monitor at all. Linearity implies that if (A3) is satisfied so is (A2). Symmetry also implies that \( u_H^I = u_H^{II} = u_H, \quad u_I^I = u_I^{III} = u_I, \quad u_H^I = u_H^{III} = u_H, \quad u_L^I = u_L^{III} = u_L \). Furthermore the inequality \( p_h > p_l \) implies that it is optimal to set \( u_L = 0 \). The above arguments imply that the optimization problem can be simplified to:

\[
\begin{align*}
\text{Min}\{&p_h^2 2u_H + p_h(1 - p_h)(u_I + u_{II})\} \\
\text{subject to:} &\quad (p_h)^2 2u_H + p_h(1 - p_h)(u_I + u_{II}) - 2m \geq (p_l)^2 2u_H + p_l(1 - p_l)(u_I + u_{II})
\end{align*}
\]
Given that \((p_h)^2 - (p_l)^2 > p_h(1 - p_h) - p_l(1 - p_l)\), if at the optimum \(u_H < V\) then \(u_I = u_{II} = 0\). Then, the following proposition describes the optimum sharing arrangement between the bank and its investors:

**Proposition A1:**

1) If \(\frac{m}{(p_h)^2 - (p_l)^2} \leq V\) then \(\hat{u}_H = \frac{m}{(p_h)^2 - (p_l)^2}\) and \(u_I + u_{II} = 0\), and

2) if \(\frac{m}{(p_h)^2 - (p_l)^2} > V\) then \(\hat{u}_H = V\) and \(\hat{u}_I + \hat{u}_{II} = \frac{2((p_h)^2 - (p_l)^2)V - m}{p_l(1 - p_l) - p_h(1 - p_h)}\)

Lastly, the following result identifies a sufficient condition for the optimum to be implemented by pooling and tranching:

**Corollary A1:**

If \(\frac{2m}{(p_h)^2 - (p_l)^2} \leq V - R_L\) then the optimum can be implemented by pooling and tranching where \(w_H = \frac{2m}{(p_h)^2 - (p_l)^2}\).

The reason that the condition is not necessary is because even when the inequality is not satisfied, it might be still possible to partially implement the optimal solution by pooling and tranching. To see this, suppose that \(V > \frac{2m}{(p_h)^2 - (p_l)^2} > V - R_L\). In this case, the bank will issue only one type of high-risk security that pays off \(\frac{2m}{(p_h)^2 - (p_l)^2}\) in state \(H\), which the bank keeps in its books. Moreover, the safe security now has a payoff equal to \(2R_L - \left(V - \frac{2m}{(p_h)^2 - (p_l)^2}\right)\) while the low-risk security has the same payoff as above. Finally, the bank will issue an additional security with that pays off \(V - \frac{2m}{(p_h)^2 - (p_l)^2}\) in states \(I, II, and L\).

Thus, the only way to increase the bank’s payoff in state \(H\) so that to ensure it will monitor its loan portfolio is by reducing the payoff of the safe security. But for the latter to remain riskless its payoffs in all states must be reduced by the same amount. The new security is created by the residual payoffs in all other states.
References


