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## Design of contingent capital with a stock price trigger for mandatory conversion

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Design of Contingent Capital with a Stock Price Trigger  
for Mandatory Conversion

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Zhenyu Wang

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## **Design of Contingent Capital with a Stock Price Trigger for Mandatory Conversion**

Suresh Sundaresan and Zhenyu Wang

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### **Abstract**

The proposal for banks to issue contingent capital that must convert into common equity when the banks' stock price falls below a specified threshold, or "trigger," does *not* in general lead to a unique equilibrium in equity and contingent capital prices. Multiple or no equilibrium arises because both equity and contingent capital are claims on the assets of the issuing bank. For a security to be robust to price manipulation, it must have a unique equilibrium. For a unique equilibrium to exist, mandatory conversion cannot result in any value transfers between equity holders and contingent capital investors. The necessary condition for unique equilibrium is usually not satisfied by contingent capital with a fixed coupon rate; however, contingent capital with a floating coupon rate is shown to have a unique equilibrium if the coupon rate is set equal to the risk-free rate. This structure of contingent capital anchors its value to par throughout the time before conversion, making it implementable in practice. Although contingent capital with a unique equilibrium is robust to price manipulation, the no-value-transfer condition may preclude it from generating the desired incentives for bank managers and demand from investors.

Key words: contingent capital, banking

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# 1 Introduction

One of the lessons learned from the credit crisis of 2007-09 is that the architecture governing the financial insolvency of banks and other financial institutions needs a major overhaul. The bankruptcy of Lehman Brothers and the financial distress experienced by Citigroup, Bank of America, and AIG have demonstrated the need to revisit the financial insolvency procedures that should govern banks and other financial institutions. In particular, the extensive amount of implicit guarantees, outright infusion of taxpayer money, and other direct and indirect benefits extended to large financial institutions has come in for much scrutiny, and a new framework for capital market regulation has been proposed.<sup>1</sup>

An integral part of this debate is the design of the capital structure of banks and related financial institutions. There has been considerable interest in designing debt securities for banks that are forced into equity via mandatory conversion in periods of distress so that 1) the banks are relieved of servicing their debt obligations in bad states of the world, when costly financial distress may result if creditors continue to demand their interest and principal payments, and 2) the equity buffer could increase, further bolstering the bank's capital as a result of the forced conversion of junior debt.<sup>2</sup>

Recently, there have been a few issues of junior debt with such conversion provisions. Lloyds Bank recently issued the so-called contingent convertible (CC, or "Coco bonds"). These bonds will convert into ordinary shares if the consolidated core tier-one ratio of Lloyds falls below 5 percent. The bonds themselves are subordinated bonds, which prior to conversion count as the lower tier-two capital, but count as core tier-one in the context of the Financial Services Authority (FSA) stress tests. They will count as core tier-one for all purposes upon conversion. Swiss regulators are encouraging Swiss banks to issue contingent capital.<sup>3</sup> In Germany, preferred stocks have been issued with similar features.

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<sup>1</sup>See "The Global Financial Crisis, A Plan for Regulatory Reform," May 2009, by the Committee on Capital Markets Regulation.

<sup>2</sup>See Squam Lake Working Group on Financial Regulation (2009).

<sup>3</sup>"New Capital Instruments on Swiss Banks' Radar," by Patrick Jenkins in London and Haig Simonian,

For over 10 years, a number of papers have argued for the issuance of reverse convertible notes (RCN), which are very much in the spirit of CC. Doherty and Harrington (1997) had suggested an RCN whereby the payments to junior debt can be made either in cash or equity at the *discretion* of the issuer at *predetermined* stock prices. Flannery (2002) proposed an RCN structure in which the conversion is *mandatory* and occurs *at the current market price* of equity. In more recent papers, several authors have argued for varying types of contingent capital debt<sup>4</sup>.

Many other papers, however, have identified several potential problems with such contingent debt issues. Acharya, Cooley, Richardson, and Walter (2009) have suggested that, while contingent capital does restore some market discipline, it fails to fully address the fact that banks have deposits, secured debt (repos), noncontingent debt of other types, and liabilities to derivatives transactions that carry either explicit or implicit guarantees. Hence both contingent capital and equity capital may have incentives to take excessive risks at the expense of guaranteed debt (taxpayer money).<sup>5</sup> Hart and Zingalis (2010) have suggested that CC may introduce inefficiency as conversion eliminates default, which forces inefficient businesses to restructure and incompetent managers to be replaced. By eliminating the threat of potential defaults, such bonds, after their conversion, increase inefficiency in the banking sector. If banks have repo and derivatives positions as well, such bonds may not prevent defaults on systemic obligations, thus increasing the risk of systemic crises.<sup>6</sup> Bond, Goldstein, and Prescott (2009) have argued that if agents were to use market prices when taking corrective actions, prices will adjust to reflect such a use and may become potentially less revealing.

Several alternative proposals have also been made with regard to bank capital. For example, Kashyap, Rajan, and Stein (2008) have proposed that banks buy “systemic risk

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*Financial Times*, April 14, 2010.

<sup>4</sup>See papers by Flannery (2009) and McDonald (2009)

<sup>5</sup>Acharya, Cooley, Richardson, and Walter, “Real Time Solutions for Financial Reform,” NYU Stern Working Group on Financial Reform, December 2009.

<sup>6</sup>Hart and Zingalis (2010) propose that CDS spreads be used in trigger mechanisms.

insurance” and secure the payouts on insurance.<sup>7</sup> Admati and Pfleiderer (2010) have argued for increasing the liability of owners (equity holders) and suggest that such a structure will mitigate the conflicts of interests between equity and debt holders and may help reduce the need for bailouts.

Valuation of CC can be performed using the analytical approach developed in structural models of default pioneered by Merton (1974) and significantly extended by Black and Cox (1976) who value default-risky senior and subordinated debt securities. These models work with the (unobserved) asset value of the issuing firm as the state variable and derive *simultaneously the equity and debt values*. The paper by Black and Cox, is particularly relevant as they explicitly model a safety covenant as a trigger for bondholders to take over the firm. The contingent claims approach has been standard for pricing corporate debt and hybrid securities, as presented in detail by Garbade (2001).

Broadly, the valuation of contingent capital falls into the following categories: 1) papers that specify triggers on the asset value to trigger conversion; 2) papers that directly specify triggers on stock prices to force conversion; and 3) papers that specify triggers based on risk-weighted capital, which combines assets with some accounting information. The first category of papers is useful in developing economic intuition about the valuation of CC, but it is difficult to implement in practice. The second category of papers, by specifying the stock price process exogenously, avoids the problem of *simultaneously* determining stock and CC prices.

In the first category of papers, the structural framework has been used by Albul, Jaffee and Tchistyi (2010) and Pennacchi (2010) to analyze CC debt design. Their papers are helpful in explaining the pricing of convertible debt securities, but they do not directly address triggers that are based on equity prices. They place triggers on the underlying (unobserved) asset values. Albul, Jaffee, and Tchistyi consider perpetual debt to derive closed-form so-

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<sup>7</sup>In some catastrophe bond structures, the bond investor (insurance buyer) can put the bond at par if and when a specified catastrophic event occurs during the life of the bond. Banks can also buy such assets from insurance companies so that they can get cash when they need it. In return, they must accept a lower coupon rate.

lutions for optimal capital structure. Pennacchi considers a jump-diffusion formulation and shows that managers may have incentives to shift risk, although this incentive is less with a CC structure.

In the second category of papers, McDonald (2009) addresses the issue of pricing CC when there is a direct equity (and a stock index) trigger. McDonald directly specifies a well-defined process for the stock price and does not relate the stock price *and* the value of CC to the underlying asset values. In reality, both the stock price and the CC price are determined simultaneously, as both are claims to the assets of the underlying bank. In McDonald's formulation, which ignores default, the coupon of the CC is increased by the "conversion premium" so that the CC sells at par upon issue.

The main thrust of our paper is the following: When triggers for mandatory conversion are placed directly on equity prices, there is a need to ensure that conversion does not transfer value between equityholders and CC holders. The economic intuition behind the CC design problem is as follows. In the contingent capital proposed in the literature, junior debt converts to equity shares when the stock price reaches a certain low threshold. This sounds like a normal and innocuous feature. However, the unusual part of the CC design is that conversion into equity is *mandatory* as soon as the stock price hits a trigger level from above. Since common stock is the residual claim of the bank's value, it must be priced *together with* the CC. Keeping firm value fixed, a dollar more for the CC value must be associated with a dollar less for the equity value.<sup>8</sup> Therefore, a value transfer between equity and CC disturbs equilibrium by moving the stock price up or down, depending on the conversion ratio specified. The design of the conversion ratio must ensure that there is no such value transfer. The design proposals in the literature usually ensure that there is no value transfer at maturity, but do not ensure it *before* maturity.

If the value transfer never pushes the stock price across the trigger, there is no problem because, given each asset value, investors always know whether or not there will be a con-

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<sup>8</sup>We assume for exposition that the value of senior bank debt remains unaffected.

version or not. However, it is always possible that the value transfer pushes the stock price across the trigger from above to below. In this case, there are two possible equilibria. In the first one, all investors believe conversion will not happen, leading the stock price to stay above the trigger. In the second one, all investors believe conversion will happen, leading the stock price to hit the trigger. Since two prices are possible whenever the firm's value drops to a certain level, by combining these dual equilibria around the trigger at different times in the future, numerous expected equity values are possible even well before conversion. These numerous values can form a range, and the whole range can be above the trigger. In this range, there is no obvious market force to push the stock price to a particular one. As we will show later, there are also economic conditions in which CC with a stock price trigger may not even have an equilibrium price. The theoretical implication is that arbitrage opportunities may exist no matter what price one assigns to the stock and CC.

The only way to prevent this is to ensure that there is no value transfer at conversion. This requires that, at all possible conversion times, the value of converted shares must be exactly equal to the market value of the nonconverted CC. This requirement implies that the conversion ratio usually cannot be chosen ex-ante once the trigger level has been chosen: This is because the trigger level multiplied by the conversion ratio must equal the market value of the nonconverted CC. However, there is one scenario in which we can select the conversion ratio ex-ante: This corresponds to the design of CC such that the coupon payments are indexed in such a way that the CC always sells at par. In this case, we can set the conversion ratio as simply the par value divided by the trigger level of the stock price at which mandatory conversion will occur. We explore this design possibility further in the paper.

The requirement of no value transfer at conversion also means that the terms of the CC cannot be used to set incentives to either control managers or market manipulation. Albul, Jaffee, and Tchisty (2010) suggest using the conversion ratios to control manipulation. Some practitioners argue that a "dilutive" conversion ratio may incentivize the bank to lower risk

ex-ante and preemptively issue equity ex-post. Albul, Jaffee, and Tchisty (2010), Pennachi (2010), and Flannery (2009) suggest that a particular choice of conversion ratio may have implications for either risk-shifting or for incentives to manipulate. Our paper shows that the design of CC does not allow for freedom to choose the conversion ratio. We show that such provision of incentives may not be feasible with mandatory conversion, as it will transfer value at conversion and hence will not admit a unique equilibrium.<sup>9</sup>

The road map for the paper is as follows. In section 2, we provide an illustration of the problems that arise when conversion ratios are specified ex-ante. The illustration demonstrates that there may be multiple equilibria or no equilibrium, depending on the conversion ratio and coupon rate. In section 3, we derive the condition for an equilibrium to exist and be unique. In section 4, we focus on how to design the CC so that the structure is less prone to multiple equilibria. We present a floating coupon structure, whereby the coupon of the CC is indexed to the risk-free rate so that the market value of the floater is anchored to par. With this structure, CC with a stock price trigger becomes operational. In this context, we discuss the market imperfections that are often cited as reasons for the potential failure of CC. Section 5 concludes.

## 2 Illustration of the Pricing Problem

Before discussing the pricing of contingent capital, we need to describe a bank that has a capital structure with CC. Consider a bank that has senior bondholders and common-equity holders who have claims on an asset (or a business). The asset requires an investment of  $A_0$  dollars today (time 0). The asset is typically risky; its value at time  $t$  is a random number  $A_t$ . At time 0, the bank has also issued a security called “contingent capital.” The security is in the form of a debt (or preferred equity) with face value  $\bar{C}$ , which is junior to the bond

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<sup>9</sup>The possibility of “multiple equilibria” has been identified by Diamond and Dybvig (1983) in the context of bank runs. Chari and Jagannathan (1988) find a unique equilibrium with bank runs and stress the role of suspension of convertibility.

but converts to common equity when certain pre-specified conditions are met.<sup>10</sup>

The *contingent capital with stock price trigger* sets the conversion condition on the bank's stock price. Suppose  $S_t$  is the stock price of this bank. At any time  $t$ , the bank converts the junior debt under the contingent capital to  $m_t$  shares of common equity as soon as the share price  $S_t$  falls to level  $K_t$ . The quantity  $m_t$  is referred to as the *conversion ratio* and  $K_t$  as the *trigger price*. They are either constant or pre-specified functions of observable variables. A particular contract specifies the trigger price  $K(\cdot)$  and conversion ratio  $m(\cdot)$  as functions of market or accounting variables over time.

The following are two examples of contingent capital contracts. The *simple form* of contingent capital can have a constant trigger  $K$  and a constant conversion ratio  $m$ . The contingent capital contracts proposed in the literature typically have a time-varying trigger and conversion ratio. The one suggested by Flannery (2002) specifies that  $K_t = z \cdot \text{RWA}_t/n$ , where  $\text{RWA}_t$  is the risk-weighted asset reported most recently,  $z$  is a constant related to regulatory capital ratios, and  $n$  is the shares outstanding before conversion. The conversion ratio is  $m_t = \bar{C}/K_t$ . Since the risk-weighted asset changes only at the end of each quarter, this contingent capital takes the simple form after its last change of risk-weighted asset, if it is not converted by then.

An important issue with contingent capital is whether it is cheaper and more effective in protecting the bank from bankruptcy than subordinated debt or equity. In fact, the preferred equity can be viewed as contingent capital with a special trigger:  $K_t = 0$  for all  $t \leq T$ . The preferred equity under this contingent capital will never convert to common equity before the firm's default on the senior bond. It will never cause the bank defaults either. Common equity can also be viewed as contingent capital with a special trigger—we set the trigger to infinity  $K_t = +\infty$  and the conversion ratio to  $m$ ; the junior debt always converts to  $m$  shares of common stock. Then, holding this contingent capital is equivalent to holding  $m$  shares

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<sup>10</sup>To keep the analysis simple, we assume in this section that the contingent capital does not pay a coupon or dividend. We make a similar assumption for the bond. Also, we assume that the asset does not generate cash flow. We will relax these assumptions in the next section.

of common equity. The focus of this paper is, however, on the contingent capitals that are not equivalent to preferred or common equity. Thus, we assume that conversion trigger  $K_t$  is always finite and positive.

Although methods for pricing of subordinated debt and equity are established, the pricing of contingent capital that has a stock price trigger as proposed by Flannery (2002) poses special challenges.. In this section, we illustrate these challenges with examples and trinomial trees in discrete time, leaving the formal analysis in dynamic continuous-time models to the next section.

To have a unique equilibrium price, the trigger and conversion ratio cannot be chosen arbitrarily. Let us consider an extremely simple example. Suppose a firm has an asset that has certain value today. On the next day, the value of the asset is a random number. The bank has a senior bond; its par value is \$80 and the maturity date is tomorrow. The bank has contingent capital, and its par value is \$10. The trigger of the contingent capital is assumed to be \$5. The bank has one share of equity. As of today, suppose the contingent capital has not been converted because the asset value has been so high that the per-share price of equity is above the trigger.

Given the trigger, a conversion that is too high can have multiple equilibrium prices. Suppose the conversion ratio is  $m = 3$  shares. If the asset value turns out to be \$100 on the next day, the stock price without conversion is

$$\frac{100 - 80 - 10}{1} = \$10,$$

which is above the trigger. Therefore, if all investors believe that there will be no conversion, they will be willing to pay \$10 per share. As a result, \$10 is an equilibrium price for the stock. However, the stock price with conversion is

$$\frac{100 - 80}{1 + 3} = \$5,$$

which hits the trigger. If all investors believe that conversion will happen, they will want to pay only \$5 per share. This then results in a stock price of \$5 as another equilibrium

price. Therefore, there are two possible equilibrium prices when the asset value turns out to be \$100. This case happens because conversion at a high ratio transfers value from stock holders to contingent capital investors, pushing down the stock price to the trigger.

Similarly, a conversion with a ratio that is too low will pull the stock price above the trigger, leaving no equilibrium price for the market to settle. In the above example, assume that the conversion ratio is  $m = 1$ . If the asset value turns out to be \$95 tomorrow, the stock price without conversion is

$$\frac{95 - 80 - 10}{1} = \$5,$$

which hits the trigger. However, the stock price with conversion is

$$\frac{95 - 80}{1 + 1} = \$7.5,$$

which is above the trigger. In this case, it is difficult for the market to settle down to a price because conversion is expected if all investors believe there is no conversion, and vice versa. This case happens because conversion at a low ratio transfers value from contingent capital investors to equity holders.

To avoid multiple/no equilibriums, we need to set the conversion ratio exactly right so that there is never any value transfer between equity holders and contingent capital investors. In the above example, we need to set the conversion ratio to  $m = 2$ , which equals

$$m = \frac{\text{Value of nonconverted CC}}{\text{Trigger}} = \frac{10}{5} = 2. \tag{1}$$

With this ratio, conversion will not move the stock price such that it causes ambiguity for conversion. For example, if the asset value is \$95, the stock price is  $(95 - 80 - 10)/1 = 5$  without conversion and  $(95 - 80)/3 = 5$  with conversion. Contingent capital must then convert because the stock price hits the trigger of \$5 anyway. If the asset value is \$100, the stock price is  $(100 - 80 - 10)/1 = 10$  without conversion and  $(100 - 80)/3 = 6.66$  with conversion. In that case, contingent capital should not convert because the stock price stays above the trigger regardless of conversion or not.

The contingent capital proposed by Flannery (2002) in fact satisfies this condition at the maturity date. If the contingent capital does not convert at maturity, the value of contingent capital is simply its face value  $\bar{C}$ . The conversion ratio  $m = \bar{C}/K$  is the ratio that does not transfer value on the maturity date because if the stock price hits the trigger, the contingent capital investors receive  $mS_T = mK$  dollars, which is the same  $\bar{C}$  dollars they will receive if it is not converted.

*The conversion ratio that guarantees no value transfer at maturity may still transfer value at some time before maturity, causing multiple equilibria.* In the following example, we set  $m = 2$  and examine pricing of the securities in a one-step trinomial model presented in Figure 1. In the model, we assume that the asset value on the following day can be \$110, \$95, or \$70 with probability 0.3, 0.4, and 0.3, respectively, as in panel (A). To keep calculations simple, we assume that all investors are risk-neutral and that the interest rate is zero. With these assumptions, the asset value today is \$92. The values of the senior bond on the tree are displayed in panel (B). On the top two nodes at maturity, the bond is valued at par (\$80) because the firm does not default. On the bottom node at maturity, the firm defaults and the bond value is the asset value (\$70). These possibilities of bond value at maturity imply that the expected value in the earlier node is \$77.

The values of the contingent capital and per share equity in the simple trinomial model are also displayed in Figure 1. In panel (C), it is assumed that the CC is not converted today. As we have discussed, when the conversion ratio is  $m = 2$ , the CC does not convert when the asset value is above \$100 at maturity and converts when the asset value is \$95. When the asset value is \$70, the CC has zero value because the bank defaults. Therefore, the three possible payoffs of CC at maturity are \$10, \$10, and \$0, corresponding to the cases of no conversion, conversion, and default, respectively. The corresponding stock prices at maturity are \$20, \$5, and \$0. Notice that the stock price hits the trigger on the middle node, on which the contingent capital converts to two shares at \$5 each. Given that the CC does not convert on the node before maturity, its value should be the expected payoff at maturity,

which is  $C = 7$ , using the given probabilities (0.3, 0.4, and 0.3). Therefore, the stock price is  $S = (92 - 77)/1 = 8$ , which is also the expected value of the stock. Notice that the stock price is above the trigger. Therefore, not converting today is consistent with the trigger rule, and  $C = 7$  and  $S = 8$  is an equilibrium if all contingent capital investors and equity holders expect this outcome.

Panel (D) of Figure 1 shows that conversion today is another equilibrium. If the contingent capital converts on the node before maturity, the total number of shares becomes three, and the stock price is  $S = (92 - 77)/3 = 5$ , which hits the trigger. This price is also the expected value of the stock. Since the contingent capital converts to two shares today, its value on this node is  $C = 2 \times 5 = 10$  dollars. Therefore, conversion today is consistent with the trigger rule, and  $C = 10$  and  $S = 5$  is a rational expectations equilibrium.

The two equilibria presented in Figure 1 leave the values of contingent capital and stock price undetermined on the node before maturity. Multiple equilibria occurred on a node before maturity because the conversion ratio,  $m = 2$ , is too high on this node and transfers value from equity holders to contingent capital investors. To prevent such a value transfer, we need to set the conversion ratio so that, on this node, the value of the converted shares at trigger price equals the value of the nonconverted contingent capital. As shown in Figure 1, the nonconverted contingent capital is valued as \$7 on this node. Accordingly, the conversion ratio should be set to  $m = 7/K = 7/5 = 1.4$ . However, to use this conversion ratio, we need to know the value of the nonconverted contingent capital on this node. However, to know the value of nonconverted CC at every time and in every estate is impractical, especially when CC is newly introduced to the market.

The example above was meant to illustrate the potential problems with the design of CC. To formally establish the outcomes of the example, we develop in the next section a continuous-time framework where we show the condition under which we can obtain a unique equilibrium.

### 3 Analysis in a Continuous-Time Model

We develop the ideas in the context of a structural model of default, along the lines of Merton (1974) and Black and Cox (1976). Consistent with these models, we operate in a risk-neutral economy. We denote the assets of the bank by  $A_t$ , which generates cash flow at the rate of  $a_t$  at time  $t$ . One example is a bank whose asset follows a geometric Brownian motion and has a constant  $a_t$ . Another example is a bank whose cash flow  $\alpha_t$  follows a geometric Brownian motion, and  $A_t$  is the value of these future cash flows. If interest rate is constant, it is well known that  $A_t$  follows a geometric Brownian motion, and  $a_t = \alpha_t/A_t$ . These specific assumptions on the stochastic process of  $A_t$  and  $a_t$  are not necessary for the analysis here. Our analysis allows the bank asset value  $A_t$  to have time-varying drift  $\mu_t$  and volatility  $\sigma_t$ . We assume

$$dA_t = \mu_t A_t dt + \sigma_t A_t dZ_t. \quad (2)$$

In risk-neutral probability, we should have  $\mu_t = r_t - a_t$ , where  $r_t$  is the instantaneous risk-less interest rate at time  $t$  and  $Z_t$  is a Wiener process.

We assume that the bank has issued a senior bond with a par value  $\bar{B}$  and maturity  $T$ . The coupon rate of the senior bond is  $b_t$ , which can be constant or time-varying. This allows us to explore both fixed- and floating-rate bank debt. Let  $\delta$  be the time when the senior bond defaults. There are several ways to specify default condition, which then determines the default time. As an example, let the default barrier at time  $t$  be  $\Gamma e^{-\gamma(T-t)}$ , where  $\Gamma$  and  $\gamma$  are positive constants. The time to default is

$$\delta = \inf\{t \geq 0 : A_t \leq \Gamma e^{-\gamma(T-t)}\}. \quad (3)$$

The framework proposed in this section can accommodate various specifications of defaults. The only thing we need for our results is that a default event wipes out the value of equity. This requirement implies a boundary condition for the senior debt:  $B_\delta = A_\delta$ .

The value function of the senior debt can be expressed in terms of the risk-free discount factor and event indicator. Given that the instantaneous risk-free interest rate is  $r_t$ , the

risk-free discount factor from time  $t$  to  $s$  is  $P(t, s) = \exp(-\int_t^s r_u du)$ . The event indicator  $1_{\text{event}}$  equals either 1 or 0, depending on whether or not the event happens. The value of senior debt before default ( $t < \delta$ ) is, in rational expectation,

$$B_t = E_t \left[ \bar{B}P(t, T)1_{\delta > T} + A_\delta P(t, \delta)1_{\delta \leq T} + I_t^B \right], \quad (4)$$

where  $E_t[\cdot]$  denotes the expectation conditional on the information up to time  $t$ , and  $I_t^B$  is the discounted value of interest income:

$$I_t^B = \int_t^{\min\{\delta, T\}} b_t \bar{B}P(t, s) ds. \quad (5)$$

There are  $n$  shares of equity and contingent capital in the capital structure of the bank. Without loss of generality, we assume  $n = 1$  throughout the paper. The par value of contingent capital is  $\bar{C}$  and it pays coupon at a rate  $c_t$  until the equity price hits a trigger level  $K_t$ , which is referred to as the stock price trigger. At any time  $t$ , when the stock price hits the trigger for the first time, the contingent capital converts to  $m_t$  shares. If conversion happens at time  $\tau$ , the outstanding shares of common equity are  $1 + m_\tau$  for all the time after conversion ( $t > \tau$ ). Both  $K_t$  and  $m_t$  are given functions of observable variables at time  $t$ , and they are assumed to be finite and positive.

To specify the terms in a contingent capital contract, we need a stock price  $S_t$  at any time  $t$  for every realization of firm asset value  $A_t$ . Assuming the stock price  $S_t$  exists, the first time the stock price hits the trigger is

$$\tau = \inf\{t \geq 0 : S_t = K_t\}. \quad (6)$$

Since equity value drops to zero when the senior debt defaults, we have  $\tau < \delta$  as long as the trigger is positive, i.e., if  $K_t > 0$  for all  $t \geq 0$ .

After the contractual coupon on the senior bond and contingent capital is paid, the cash flow generated from the assets of the bank will be paid to equity holders as dividends. Therefore, before conversion, the total dividends paid to equity holders during a short period  $dt$  is  $(a_t A_t - b_t \bar{B} - c_t \bar{C})dt$ . After conversion and before default of the senior bond, the total

dividends paid to equity holders (including those new equity holders after conversion) during an infinitesimal period  $dt$  are  $(a_t A_t - b_t \bar{B})dt$ .

At any time  $t$  before contingent capital converts ( $t < \tau$ ), the per-share value of common equity is, in rational expectation,

$$\begin{aligned} S_t &= E_t \left[ (A_T - \bar{B} - \bar{C})P(t, T)1_{\tau > T} + I_t \right] \\ &+ E_t \left[ \frac{1}{1 + m_\tau} \left\{ (A_T - \bar{B})P(t, T)1_{\delta > T} + J_\tau P(t, \tau) \right\} 1_{\tau \leq T} \right], \end{aligned} \quad (7)$$

where  $I_t$  is the present value of total dividends before trigger point, and  $J_\tau$  is the time- $\tau$  value of the total dividends after conversion:

$$I_t = \int_t^{\min\{\tau, T\}} (a_s A_s - b_s \bar{B} - c_s \bar{C})P(t, s)ds \quad (8)$$

$$J_\tau = \int_\tau^{\min\{\delta, T\}} (a_s A_s - b_s \bar{B})P(\tau, s)ds. \quad (9)$$

The value of contingent capital before conversion is

$$\begin{aligned} C_t &= E_t \left[ \bar{C}P(t, T)1_{\tau > T} + H_t \right] \\ &+ E_t \left[ \frac{m_\tau}{1 + m_\tau} \left\{ (A_T - \bar{B})P(t, T)1_{\delta > T} + J_\tau P(t, \tau) \right\} 1_{\tau \leq T} \right], \end{aligned} \quad (10)$$

where  $H_t$  is the present value of coupon interests that the CC holders receive before conversion:

$$H_t = \int_t^{\min\{\tau, T\}} c_s \bar{C}P(t, s)ds. \quad (11)$$

After contingent capital converts to  $m_\tau$  shares and before the senior bond matures or defaults ( $\tau \leq t < \min\{\delta, T\}$ ), the per-share value of stock becomes

$$S_t = \frac{1}{1 + m_\tau} E_t \left[ (A_T - \bar{B})P(t, T) \cdot 1_{\delta > T} + J_t \right]. \quad (12)$$

Since the value function  $B_t$  defined in equation (4) exists and is continuous in  $t$  and  $A_t$ , we focus on the value function of equity share  $S_t$  and the value of nonconverted contingent capital  $C_t$ . Given a set of trigger  $K_t$  and conversion ratio  $m_t$ , a pair of value functions,

$(S_t, C_t)$ , that satisfy equations (6), (7), and (10) is called a *dynamic rational expectations equilibrium* or, simply, an *equilibrium*. The equilibrium is *unique* if each of  $S_t$  and  $C_t$  has a unique value for every realization of  $A_t$  at any time  $t$ . In fact, such an equilibrium does not always exist for arbitrary specification of  $m_t$ . The next theorem presents the restriction on the conversion ratio to ensure that an equilibrium exists.

**Theorem 1** *For any given trigger  $K_t$  and conversion ratio  $m_t$ , a necessary condition for the existence of a unique equilibrium  $(S_t, C_t)$  is  $m_t = C_t/K_t$ .*

This necessary condition is also sufficient in the following sense:

**Theorem 2** *For any given trigger  $K_t$ , there exists a unique equilibrium  $(S_t, C_t)$  that satisfies the conversion ratio  $m_t = C_t/K_t$ .*

These theorems say that a condition for a unique equilibrium is that there should be no transfer of value from CC holders to equity holders, or vice versa. To see this, we can rewrite the condition as  $m_t K_t = C_t$ . At conversion time  $\tau$ , we should have  $S_\tau = K_\tau$ . Then, the value of  $m_\tau$  shares of stock at conversion is  $m_\tau S_\tau$ , which equals  $C_\tau$ .

It is useful to provide some perspective on why the multiple equilibria do *not* arise with convertible bonds or options. With a convertible bond, the investor has the “option” to convert and get a prespecified number of shares of common stock. On each node, the investor can compute the value of the bond when it is not converted and compare it with the value of the bond when converted and select the maximum of the two values. Likewise, the holder of the option can also make the optimal decision on each node. These optimal decisions can be modeled by the “smooth pasting” or the “high contact” condition pioneered by Merton (1973) and further elucidated by Dixit and Pindyck (1994). In such models, the exercise boundary itself is endogenous and not mandated. With mandatory conversion, no agent is allowed to optimally act at the trigger. This absence of a “smooth pasting” condition then leads to the problems we have articulated above. The smoothness breaks down if the

mandatory conversion transfers value between equity holder and CC holders.<sup>11</sup> The state-contingent conversion ratio presented in the theorems prevents the value transfer and, in effect, keeps the prices “smooth” at conversion.<sup>12</sup> However, mandatory conversions that may occur only at maturity do not pose any essential difficulty as the bond trades at par at maturity and hence there will be no value transfer with conversion. The Treasury capital assistance program has such features at maturity.<sup>13</sup>

To examine just how pervasive a problem the lack of a unique equilibrium might be, we provide some numerical results. We first examine the effect of various conversion ratios on the severity of multiple equilibria: We compute the range of prices that can come about as a consequence of setting the conversion ratio arbitrarily. The case where the conversion ratio is “too low” leads to no equilibrium and therefore cannot be demonstrated numerically. To obtain a range of multiple equilibria, we implement the continuous-time model using a large binomial tree. The upper bound of the equity price is obtained by choosing the equilibrium with a higher stock price on every node that has multiple equilibria. Likewise, the lower bound of the equity price is obtained by choosing the equilibrium with a lower stock price. The bounds of the CC value are constructed in a similar way. One intuitive interpretation of these bounds is the following. The equity upper bound reflects circumstances where equity holders somehow have the market power to dictate conversion to maximize their values. The presence of such bounds, and the range associated with the bounds, is an indication of potential market manipulation possibilities with poorly designed bank contingent capital.

The range of prices generated by multiple equilibria seems substantial. As a numerical example, which is presented in Table 1, we assume that the bank’s asset volatility is 6

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<sup>11</sup>The effect of value transfer at mandatory conversion is similar to an exogenous value transfer caused by tax distortion. Albul et al (2010) have shown that differential tax treatment of CC’s coupon interest and equity’s dividend can cause multiple equilibria for mandatory convertible debt with the trigger on asset value.

<sup>12</sup>In the valuation of barrier options, the exercise boundary is exogenous and their structure shares some of the features of CC. But the exercise of such options does not influence the underlying stock price itself, as there are no dilutionary effects to consider. These options are also in zero net supply.

<sup>13</sup>See Glasserman and Wang (2009), who value the capital assistance program.

percent, and its senior bond is valued at 90.43 percent of the current asset value.<sup>14</sup> The par value of the contingent capital is 6 percent of the bank's current asset value, and the conversion trigger is 2 percent of the current asset value.<sup>15</sup> The maturities of the senior bond and contingent capital are both five years. The riskless interest rate is assumed to be 2 percent and constant for five years. With the above parameters, multiple equilibria produce equity values ranging from 3.83 percent to 4.45 percent of current asset value. They are associated with CC values ranging from 5.12 percent to 5.74 percent of current asset value. For a good approximation of the continuous-time model, our trinomial tree contains 2,500 steps, which roughly correspond to two steps per trading day. Figure 2 shows that the range of multiple equilibrium prices converges to a fixed width as the number of steps increases beyond 2,500.

The range of multiple prices depends on the characteristics of the bank and the contracts of the contingent capital. In Figure 3, we let one parameter vary to see how the range of multiple prices changes. Panels (A) and (B) show how the range is related to the bank's leverage with senior debt and its asset volatility. The range is wider for a bank that has higher leverage with senior debt. The range is an increasing function of the bank's asset volatility. Increasing senior debt or asset volatility raises the risk for the part of the firm shared by equity holders and CC holders. An increase in the risk enlarges the range of possible multiple equilibria. Panels (C) and (D) of Figure 3 show how the range is related to the amount of contingent capital and its conversion trigger. The range starts from zero and widens as the par value of contingent capital increases. Therefore, the more contingent capital a bank takes, the wider its range of equity prices. The range is wider for a low trigger than for a high trigger, because the fact that a low trigger is expected to take a longer time to reach and thus has more multiple equilibria to incorporate, pushing up the upper bound

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<sup>14</sup>This value of senior debt can be obtained by setting the par value  $\bar{B}$ , coupon rate  $b_t$ , and default barrier ( $\Gamma$  and  $\gamma$ ) to particular values. The choice of these parameter values that produce a particular value of  $B_0$  is usually not unique. In this example, we choose  $b_t = 0$ ,  $\gamma = 2$  percent, and  $\Gamma = \bar{B}$ , where  $\bar{B}$  is chosen so that  $B_0 = 90.43$  percent.

<sup>15</sup>To keep the calculation simple, we set the coupon of the contingent capital to zero. If the coupon is nonzero, we may run into situations in which there are no equilibrium prices.

and pushing down the lower bound.

These findings demonstrate that the bank-specific information such as leverage and asset volatility play an important role in the nature and severity of multiple equilibria associated with the amount of CC and its trigger. A security with a wide range of multiple equilibria will not have a market force to make the price of the security converge to a particular point in the range. In the absence of such market force, the security will likely be subject to price manipulation by large market players. We can think of a model in which there is a distribution of beliefs by a large number of investors. With this probability distribution of higher order, the market may be able to settle to a particular equilibrium. However, the assumption of a stable distribution of a large number of investors is difficult to apply to the case of contingent capital bonds. Therefore, it is desirable to design contingent capital that gives a unique equilibrium. In the next section, we suggest a design for CC that overcomes the problem of multiple equilibria or no equilibrium.

## 4 Design of Implementable Contingent Capital

The previous section showed that in order to have a unique equilibrium, the conversion ratio must satisfy at conversion times the following condition:  $m_t = C_t/K_t$ . This presents a major roadblock to the implementation of the CC design: The conversion ratio depends on the market value of the contingent capital. Since we cannot tell what the future market value will be, the value  $C_t$  of the nonconverted CC can be different from a pre-specified  $m_t K_t$  at any time before  $t$ . If in the CC contract we set the conversion ratio to  $m_t = C_t/K_t$ , we need to know the value of the nonconverted CC. With CC newly introduced into the market, observing the value of CC is unrealistic. Practically, the only observable “value” of CC is probably only the par value.<sup>16</sup>

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<sup>16</sup>It is well known that secondary markets for corporate debt are highly illiquid and opaque. Therefore, investors often have access only to quotes and indications. Only recently has TRACE enforced post-trade transparency to a subset of corporate bond markets. Hence it is reasonable to design a security that does not presuppose the availability of an active secondary market.

To use the par value for the conversion ratio, we need to focus on a structure that makes the market value of the CC immune to changes in interest rates and default risk. For example, if the CC had no default risk, then by selecting the coupon rate at each instance to be the instantaneously risk-free rate we can ensure that the CC will trade at par. See Cox, Ingersoll, and Ross (1980) for a proof of this assertion.<sup>17</sup> In this case, CC will work well, because we can determine the conversion ratio ex-ante as follows:

$$m_t = C_t/K_t = \bar{C}/K_t. \quad (13)$$

Since  $\bar{C}$  and  $K_t$  are known ahead of time, we can specify the conversion ratio ahead as well.

With default risk, however, no design of floating coupons will actually guarantee that the CC will sell at par. However, by choosing the coupon to reflect the market rates on short-term default-risky bank obligations, it is possible to keep the price close to the par value. For example, if the coupon is tied to the London Interbank Offered Rate (LIBOR), then the price of CC, which is a bank floater, should remain close to par. Putting trigger  $K_t$  on equity reduces the impact of default risk on CC pricing before conversion. Under the assumption that the bank declares default when the equity value is driven to zero, CC holders are default-free prior to conversion and are exposed to default only if they had already converted. Clearly, CC holders still bear large default risk ex ante because the conversion time is uncertain and after conversion they hold equity, which bears default risk.

In the context of these issues, we show that CC can be designed to be default-free during its life, even though the bank may have a positive probability of default on its debt claims subsequent to the expiration of CC. This idea is formalized in Theorem 3 below.

**Theorem 3** *CC designed with a floating coupon rate  $c_t = r_t$  and a conversion ratio  $m_t = \bar{C}/K_t$  will always sell at par. This CC design leads to a unique equilibrium.*

This theorem has two parts. The first part generalizes the immunization results of Cox, Ingersoll, and Ross (1980) to a setting where there is mandatory conversion and a positive

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<sup>17</sup>In the context of a CC that is exposed to default risk, the appropriate indexed coupon may also require a compensation for the mandatory conversion in addition to the risk-free rate.

probability of default after the expiration date of CC. Since the coupons float with the risk-free rate and the principal is guaranteed at conversion, the CC is fully immunized and therefore sells at par. The economic rationale behind the second part of the result is also intuitive. Since the CC sells at par, we can design the CC with an ex-ante conversion ratio that guarantees that, upon conversion, the CC holders will get par.<sup>18</sup> Theorem 3 has an important practical implication: With the design proposed in Theorem 3, the contingent capital certificates suggested by Flannery (2009) become operational and free of multiple equilibria. When mandatory conversion triggers are placed on stock prices, the structure of CC in Theorem 3 offers a practical way to design contingent capital for banks because the specification of the conversion ratio in the contract does not rely on variables that are difficult to observe.

What are the benefits brought about by CC? To address this question within the context of our framework, we can examine the probability of default with and without CC in the capital structure of the bank. We can consider two cases as benchmarks. The first case would be the Black and Cox (1976) benchmark, with the modification that the junior debt is designed as a floater. In the absence of conversion as proposed in our paper, junior debt is subject to default risk and hence will carry a higher coupon rate in order to sell at par. This design ensures that, at the time the junior debt (or CC) is issued, the bank gets the same level of capital, equal to the par value of debt issued. Both the junior debt and the CC will provide tax-shield benefits, but the CC will provide fewer tax benefits as its coupon will be lower than an otherwise identical junior debt, which is subject to credit risk. The probability of default associated with the junior debt is higher than the probability of default associated with the CC. This follows from the fact that default will occur in the Black and Cox (1976) setting whenever the asset value drops below the safety covenant level, which is dependent on the sum of the par values of both senior and junior debt. With CC, the default event depends only on the par value of senior debt. This is one source of the welfare improvement.

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<sup>18</sup>In reality, bank-issued CC will be less liquid than risk-free assets such as Treasury securities, and hence the coupon will have to include a component for the liquidity premium. In addition, if the underlying process of asset price can have discontinuous jumps, then the result will not hold.

Since the probability of default is lower with CC, it stands to reason that the creditors to the bank will benefit at the expense of the equity holders. Since we do not have frictions such as taxes, or costs associated with financial distress, it is clear that any welfare improvement for one subset of claimants must represent a welfare loss for the remaining claimants.

A second benchmark to assess the value added by CC, and perhaps more relevant for policy purposes, is the alternative that the bank issues equity as opposed to CC. With equity issuance, the bank will not be able to obtain the tax shield associated with CC. This can be a source of welfare improvement for the bank with CC. As noted by Pennachi (2010), the issuance of CC may also serve to reduce the risk-shifting incentives, which can be another important source of welfare improvement.

Can we claim that the introduction of CC serves to reduce the overall default risk of the issuing bank? In our model, a bank's default risk is determined by its asset volatility and its choice of leverage. Once we admit frictions such as taxes and bankruptcy costs, it is clear that our model will enable us to compute the optimal leverage, consistent with CC in the capital structure with an equity price trigger. If the optimal (overall) capital structure with CC is less than the optimal capital structure without CC, then one source of default risk would have been lowered as the bank issues more equity in equilibrium.

In addition, if the presence of CC leads to lower risk-shifting incentives, that result can also reduce the bank's overall default risk. Note that although the CC earns the risk-free rate until conversion, once converted it becomes equity and fully bears the default risk of the issuing bank. In this sense, CC serves to mitigate default risk. By accepting a residual claim in the bank's capital structure, the CC holders help the bank overcome a potential crisis.

Finally, as noted by Hart and Zingalis (2010), the CC (once converted) may also free the bank of the discipline imposed by creditors. Once CC is converted, the bank may have to issue new CC to be able to provide that additional layer of protection. Whether the bank will be able to do that depends on its asset value at the time CC is converted. It is conceivable

that the CC may simply “back load” the default risk of the bank rather than reduce the overall default risk by appropriately constraining the bank’s risk-taking propensities. These are potentially interesting areas for further research. Having developed a framework for valuing CC with an equity trigger, we can use it to examine welfare questions. Also, we have not considered in our framework, any aggregate liquidity event, which is an important motivator for designing contingent debt capital. This issue is best modeled in an equilibrium setting with many banks and is beyond the scope of this paper.

## 5 Conclusion

We have not treated the effect of frictions in our analysis. Clearly, one argument as to why debt issuance may add value to the bank is that the interest payments made on debt are subject to tax relief. It stands to reason that CC should be treated as a debt security until its mandatory conversion, in order for such issuance to generate additional value as well as provide the necessary incentives to issuing banks (See Duffie, 2009). A number of papers have also noted that the CC design might set in motion some perverse incentives. For example, Duffie (2009) notes that CC design based on equity design might set in motion a potential attack by short sellers to trigger conversion and benefit from the resulting dilution. Hillion and Vermaelen (2001) have argued that such securities come under selling pressure from the holders of convertible debt. Rajan (2009) has suggested that the conversion should depend on both bank-specific and market-wide systemic risk information, a point explicitly considered by McDonald (2009).

Consistent with many other observers (e.g., Acharya, Thakor, and Mehran, 2010), we note that the mandatory conversion of junior debt should automatically result in suspension of dividends to all holders of common stock. Holding other factors the same, this should serve to alleviate the selling pressure: Any attempt to short the stock by the holders of CC will also result forgone future dividends on their long positions.

In addition, we can use a “look back” period as suggested in the literature as the basis

for conversion instead of using a specific trigger price. Such mechanisms are already used in practice in the context of barrier options. In addition, it may be worthwhile to include a “optional sinking fund” feature in the design of CC so that there is a periodic and orderly retirement of principal (balloon) payment, which should further mitigate the incentive for a speculative short-sale attack.<sup>19</sup> The sinking fund structure reduces the volume of conversion that must occur once the equity trigger is breached by the stock price, leaving the firm more vulnerable to a speculative attack.

We argue that equity and CC are claims on the same assets and that their prices must be determined simultaneously. The conversion ratios must ensure that, at the equity trigger, there are no value transfers between equity holders and CC investors. Our paper provides the condition that the conversion ratio must satisfy in order for a unique equilibrium to exist. Guided by this condition, we present a design that mitigates the problem of multiple equilibria. Since this design does not transfer value between equity holders and CC investors, the security design suggested here does not provide incentives for manipulating stock prices around conversion. We do not suggest that our design is “manipulation proof,” but rather that under the assumptions of the model and in well-functioning markets, this design is less prone to manipulation. In contrast, a security that introduces multiple equilibria adds the problem of price manipulation to the originally well-functioning market.

Finally, we cast some doubts on the efficacy of designing the conversion ratio to mitigate risk-shifting or the propensity for manipulation or coercive equity issues. Our paper suggests that the conversion ratio that admits a unique equilibrium produces no value transfer. Hence, it is not possible to design “dilutive” ratios in order to promote coercive equity issuance, as such a ratio will mean the lack of a unique equilibrium.

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<sup>19</sup>McDonald (2009) suggests a gradual and random retirement policy for CC in order to limit gains from manipulation.

## A Appendix: Proofs of the Theorems

Before proving the theorems, it is useful to make the following observation. If there were no contingent capital, at any time  $t$  before maturity ( $t \leq T$ ) and default ( $t \leq \delta$ ), the equity value would have been

$$U_t = E_t \left[ (A_T - \bar{B})P(t, T) \cdot 1_{\delta > T} + J_t \right], \quad (14)$$

where  $J_t$  is defined in equation (9) by replacing  $\tau$  with  $t$ . Since it is well known that  $U_t$  is a continuous function of  $t$  and  $A_t$ , we can define another hitting time based on  $U_t$  and the given  $K_t$  and  $m_t$ :

$$v = \inf \left\{ t \geq 0 : \frac{1}{1 + m_t} U_t = K_t \right\}. \quad (15)$$

### A.1 Proof of Theorem 1

To prove Theorem 1, we start with a pair  $(S_t, C_t)$  that satisfies equations (6), (7) and (10). Equations (7) and (10) imply

$$S_t = U_t - C_t. \quad (16)$$

For  $\tau \leq T$ , equation (7) implies  $S_\tau = U_\tau / (1 + m_\tau)$ . Since  $S_\tau \leq K_\tau$  by equation (6), we have  $U_\tau / (1 + m_\tau) \leq K_\tau$ , which implies

$$v \leq \tau \quad (17)$$

in view of equation (15). On the other hand, equation (15) implies  $U_v / (1 + m_v) = K_v$ , and thus converting is an equilibrium price at time  $v$ . If  $S_v > K_v$ , then not converting is also an equilibrium price at time  $v$ , contradicting to the assumption of unique equilibrium. Thus, the uniqueness of equilibrium implies  $S_v \leq K_v$ . It follows that

$$\tau \leq v, \quad (18)$$

in view of equation (6). Combining equations (17) and (18), we have

$$\tau = v. \quad (19)$$

It then follows from equations (6), (15) and (16) that

$$\inf\{t \geq 0 : U_t \leq K_t + C_t\} = \inf\{t \geq 0 : U_t \leq (1 + m_t)K_t\}. \quad (20)$$

The above equation holds if and only if  $K_t + C_t = (1 + m_t)K_t$ , which implies  $m_t = C_t/K_t$ .

## A.2 Proof of Theorem 2

To prove Theorem 2, we use the hitting time  $v$  defined in (15) as the conversion time, assuming that the conversion ratio at time  $t$  is given as  $m_t$ . With this conversion rule, the stock price and contingent capital value before conversion ( $t < v$ ) are

$$\begin{aligned} S_t^* &= E_t \left[ (A_T - \bar{B} - \bar{C})P(t, T)1_{v>T} + I_t^* \right] \\ &\quad + E_t \left[ \frac{1}{1 + m_v} \left\{ (A_T - \bar{B})P(t, T)1_{\delta>T} + J_v^*P(t, v) \right\} 1_{v \leq T} \right], \end{aligned} \quad (21)$$

$$\begin{aligned} C_t^* &= E_t \left[ \bar{C}P(t, T)1_{v>T} + H_t^* \right] \\ &\quad + E_t \left[ \frac{m_v}{1 + m_v} \left\{ (A_T - \bar{B})P(t, T)1_{\delta>T} + J_v^*P(t, v) \right\} 1_{v \leq T} \right], \end{aligned} \quad (22)$$

$$H_t^* = \int_t^{\min\{v, T\}} c_s \bar{C}P(t, s) ds \quad (23)$$

$$I_t^* = \int_t^{\min\{v, T\}} (a_s A_s - b_s \bar{B} - c_s \bar{C})P(t, s) ds \quad (24)$$

$$J_v^* = \int_v^{\min\{\delta, T\}} (a_s A_s - b_s \bar{B})P(v, s) ds. \quad (25)$$

The stock price after conversion ( $v \leq t < \min\{\delta, T\}$ ) is

$$S_t^* = \frac{1}{1 + m_v} E_t \left[ (A_T - \bar{B})P(t, T)1_{\delta>T} + J_t^* \right]. \quad (26)$$

Equations (21), (22), and (26) imply

$$S_t^* = U_t - C_t^*. \quad (27)$$

Now, we use  $S_t^*$  to define another hitting time:

$$\tau^* = \inf\{t \geq 0 : S_t^* \leq K_t\}. \quad (28)$$

In view of equation (27), we have

$$\tau^* = \inf\{t \geq 0 : U_t \leq K_t + C_t^*\}. \quad (29)$$

If  $m_t = C_t^*/K_t$ , then

$$\begin{aligned} \tau^* &= \inf\{t \geq 0 : U_t \leq K_t + C_t^*\} \\ &= \inf\{t \geq 0 : U_t \leq (1 + m_t)K_t\} = v. \end{aligned} \quad (30)$$

Therefore,  $(S_t^*, C_t^*)$  satisfies equations (6), (7) and (10) and thus is an equilibrium.

If  $(S_t, C_t)$  is another equilibrium with the conversion ratio  $m_t = C_t/K_t$ , following similar reasoning in the derivation of  $\tau^* = v$ , we can show that the conversion time  $\tau = \inf\{t \geq 0 : S_t \leq K_t\}$  equals  $v$ , which gives  $\tau = \tau^*$ . Therefore, the values of the common stock and contingent capital calculated in equations (7), (10), (21) and (22) imply  $S_t = S_t^*$  and  $C_t = C_t^*$ . This proves the uniqueness of the equilibrium.

### A.3 Proof of Theorem 3

In the floater, we set the interest rate of contingent capital to the riskless rate. That is,  $c_t = r_t$  for all  $t$ . In addition, we set the conversion ratio to  $m_t = C_t/K$ . It follows from Theorem 2 that there is a unique equilibrium. We will show that in this unique equilibrium the value of nonconverted contingent capital equals the par value, i.e.,  $C_t = \bar{C}$  for  $t \leq \tau$ . Then, the conversion ratio  $m_t = \bar{C}/K$  is equivalent to  $m_t = C_t/K$ .

The value of noncontingent capital is

$$\begin{aligned} C_t &= E_t \left[ \bar{C}P(t, T)1_{\tau > T} + H_t \right] \\ &\quad + E_t \left[ \frac{m_\tau}{1 + m_\tau} \left\{ (A_T - \bar{B})P(t, T)1_{\delta > T} + J_\tau P(t, \tau) \right\} 1_{\tau \leq T} \right], \end{aligned} \quad (31)$$

where  $J_\tau$  is defined in equation (9), and  $H_t$  defined in (11) with  $c_t = r_t$ . The riskless floating rate for the coupon of CC implies

$$H_t = \int_t^{\min\{\tau, \min\{\delta, T\}\}} r_s \bar{C}P(t, s) ds = \bar{C} [1 - P(t, T)1_{\tau > T} - P(t, \tau)1_{\tau \leq T}]. \quad (32)$$

Substituting the above expression for  $H_t$  back into the valuation function of  $C_t$  and using the properties of iterated expectations and  $P(t, T) = P(t, \tau)P(\tau, T)$ , we obtain

$$C_t = \bar{C} - E_t [\bar{C}P(t, \tau)1_{\tau \leq T}] \quad (33)$$

$$+ E_t \left[ \frac{m_\tau}{1 + m_\tau} E_\tau [(A_T - \bar{B})P(\tau, T)1_{\delta > T} + J_\tau] P(t, \tau)1_{\tau \leq T} \right]. \quad (34)$$

Noticing

$$\frac{1}{1 + m_\tau} E_\tau [(A_T - \bar{B})P(\tau, T)1_{\delta > T} + J_\tau] = S_\tau = K, \quad (35)$$

the value of contingent capital with the floating coupon rate  $r_t$  equals

$$C_t = \bar{C} - E_t [\bar{C}P(t, \tau)1_{\tau \leq T}] + E_t [m_\tau K P(t, \tau)1_{\tau \leq T}]. \quad (36)$$

Therefore,  $m_t = \bar{C}/K$  for all  $t$  implies

$$C_t = \bar{C} - E_t [\bar{C}P(t, \tau)1_{\tau \leq T}] + E_t \left[ \frac{\bar{C}}{K} K P(t, \tau)1_{\tau \leq T} \right] = \bar{C}. \quad (37)$$

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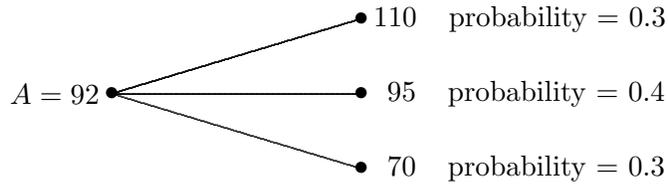
# Table and Figures

Table 1: A Numerical Example for the Range of Multiple Equilibria

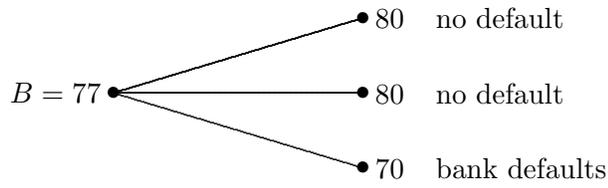
Name	Notation	Value
<i>Parameters</i>		
Asset value	$A_0$	100
Volatility	$\sigma$	6%
Senior bond value	$B_0$	90.43
Riskless rate	$r$	2%
Par value of CC	$\bar{C}$	6
Shares of common equity	$n$	1
<i>equilibria</i>		
Stock price	$S_0$	[3.83, 4.45]
CC value	$C_0$	[5.12, 5.74]

Figure 1: An One-Step Trinomial Tree Model

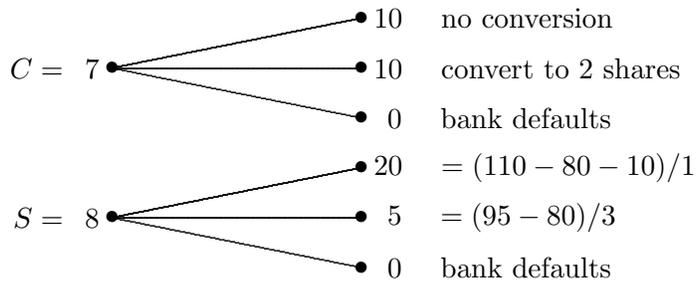
(A) *Bank asset value*



(B) *Senior bond value*



(C) *No conversion is an equilibrium*



(D) *Conversion is another equilibrium*

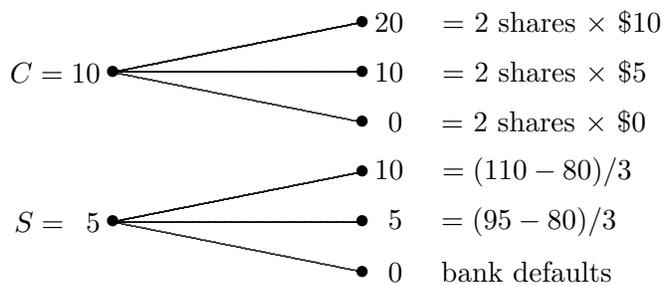


Figure 2: The range of multiple equity and CC prices converges to a fixed width as the number of steps in the binomial tree increases. The solid lines represent the upper and lower bounds of the multiple equity prices, and the dot lines represent the bounds of CC values. The parameters used for the figure are the same as those in Table 1, except the number of steps, which is the horizontal axis. The number of steps used in Table 1 is indicated by the vertical dash line.

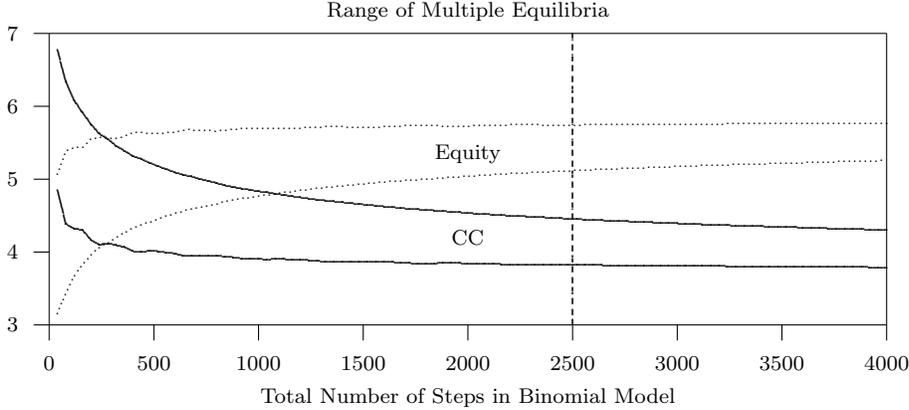


Figure 3: The range of multiple equity and CC prices depend on the bank's characteristics and the CC contract. The solid lines represent the upper and lower bounds of the multiple equity prices, and the dot lines represent the bounds of CC values. The parameters used for the figure are the same as those in Table 1, except the one that varies in a range indicated by the horizontal axis. For the varying parameter, the value used in Table 1 is indicated by the vertical dash line.

