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Global Bond Risk Premiums

Rebecca Hellerstein

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## **Global Bond Risk Premiums**

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### **Abstract**

This paper examines time-varying measures of term premiums across ten developed economies. It shows that a single factor accounts for most of the variation in expected excess returns over time, across the maturity spectrum, and across countries. I construct a global return forecasting factor that is a GDP-weighted average of each country's local return forecasting factor and show that it has information not spanned by the traditional level, slope, curvature factors of the term structure, or by the local return forecasting factors. Including the global forecasting factor in the model produces estimates of spillover effects that are consistent with our conceptual understanding of these flows, both in direction and magnitude. These effects are illustrated for three episodes: the period following the Russian default in 1998, the bond conundrum period from mid-2004 to mid-2006, and the period since the onset of the global financial crisis in 2008.

Key words: term premium, bond risk premiums, international spillover effects.

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## 1 Introduction

This article studies time-varying risk premiums in ten developed countries' government bonds. I examine a model that produces term premium estimates that are comparable across countries and that also accounts for various spillover effects of the pricing of risk across national borders. To account for the global pricing of risk in such a multi-country framework, a problem arises in using a traditional three- or five-factor affine term-structure model. In this type of model, the first three principal components account for the behavior of both the cross-section and the time-series of yields, so it is difficult to combine them (e.g. in a simple linear combination) across countries without positing a single global factor driving the domestic factors, which in turn drive domestic yields (as in Diebold, Li, and Yue 2008), which increases the computational complexity of the model substantially. In contrast, recent work by Cochrane and Piazzesi (2005, 2008, hereafter, CP) identifies a *single* return-forecasting factor with negligible information about the cross section of yields, but with most of the economically important information about their movements over time (across maturities). Using data for the U.S., CP (2005) show that although different maturity bond returns may vary by different amounts, they all vary together with movements in this common return forecasting factor which, in turn, is not fully characterized by the three factors (level, slope, curvature) traditionally used in term structure models.

This paper shows that a similar return forecasting factor (RFF) plays an analogous role in the time-series variation of the excess returns of the government bonds of nine additional developed economies for the period from 1990 to 2011, as well as the empirical relevance of a global return forecasting factor (hereafter, GFF) that is a GDP-weighted average of each country's local return forecasting factor (hereafter, LFF).<sup>1</sup> I define the 10-year term premium as the sum of expected future one-year term premiums of declining maturity.

I construct two panel datasets of nominal zero-coupon yields with maturities from one month to ten years. The monthly data set includes ten countries (the U.S., U.K., Germany, Japan, Canada, Australia, Switzerland, Sweden, Finland, and Norway) and runs from 1990 to the present – with the exception of the Scandinavian countries, whose monthly data begin in the mid to late 1990s. The daily data set spans six countries (the U.S., U.K., Germany, Japan, Canada, and Australia) and begins in January 1998 to maximize country coverage.

Almost all of the current research on term premiums uses data from one country, usually the United States. This paper's estimates, across a range of developed economies, enable one to exploit cross-country variation in term-premiums' behavior to identify their relationships to various macroeconomic and financial variables. Over the sample period, the economies included in my data set exhibit marked differences in, for example, their fiscal outlook, their production or use of commodities, their openness, and the history of their monetary institutions.

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<sup>1</sup>Dahlquist and Hasseltoft (2011) explore a similar return-forecasting factor across four major developed economies.

To account for variation in the global pricing of risk that may have spillover effects across countries, I introduce two new elements into the Cochrane-Piazzesi framework. The first one is to modify the model to allow the pricing of risk over time to be affected by a global return forecasting factor, which is orthogonalized to each country’s local return forecasting factor. (For each country, I verify that the local return forecasting factor is a valid predictor of excess bond returns locally before incorporating it into the global return forecasting factor.) This global factor is meant to capture those aspects of “global risk appetite” that may not be evident in the behavior of each country’s forwards alone. I am agnostic about exactly what process characterizes this global pricing of risk: Its effects may include short-term capital flows associated with flight-to-quality motives or with global portfolio rebalancing, as well as some of the more persistent cross-border effects associated with global liquidity conditions, such as the global savings glut which has been identified as a driver of low risk premiums in the mid 2000s by Ben Bernanke.

The second innovation of the paper is on the data side. In order to identify the potential role of international spillover effects I propose using higher frequency (daily) data on yields than is common in the term structure literature. The advantage of using high-frequency data is that I observe many episodes during which variation in yields appears quiescent, followed by a news announcement in one country which appears to lead to a rise (or fall) in risk premiums across countries. Daily estimates of term premiums enable one to identify any discrete jumps following sudden increases or decreases in global risk appetite, as I discuss in more detail in the examples in Section 5. It is this discreteness in the adjustment of term premiums that I exploit in order to identify the role of international spillover effects, such as flight-to-quality flows in periods of financial and economic turmoil. I find that the effect of the global forecasting factor on U.S. and German term premiums estimates appears to correspond, in both sign and magnitude, to narrative evidence about periods in which flight-to-quality, savings-glut, or analogous international capital flows had a significant impact on the pricing of their government bonds.

The basic idea behind my approach is as follows. I extend the model of one-year risk premia in Cochrane and Piazzesi (2005) by modeling the term structure of risk premia, and forecasting the return forecasting factors along the lines described in Cochrane Piazzesi (2008), via the traditional level, slope, and curvature yield curve factors. This of course implies that the movement of yields over time is captured by the return forecasting factor, and that the variation across yields in the cross section is adequately characterized via these traditional three yield curve factors. The estimation procedure, therefore, uses these yield-curve factors to forecast the return forecasting factors, which in turn forecast excess returns, over time and across the maturity spectrum.

The model exploits information from both domestic and international bond markets to predict the future behavior of excess returns. This approach is based on the insight that the difference between an estimate of the term premium that accounts for this global pricing of risk, and one identified exclusively

off of variation in the local (defined here as country-specific) pricing of risk may reflect spillover effects across countries, the effects of short-term international portfolio capital flows, and the like.

Across countries, the model's term premium estimates appear reasonable and are consistent with estimates from other well-known term-structure models for the U.S. Like Wright (2010) and others in this literature, I find that term premiums appear to have declined gradually across developed economies since the early 1990s. The analysis yields several other interesting findings. First, at the descriptive level, and as mentioned previously, I document that a single factor accounts for almost all of the variation in bond excess returns across all the countries in the sample. Second, I show that this factor has information not spanned by the traditional level, slope, and curvature factors used in term-structure models. Third, I find that a global factor, constructed by combining each country's RFF into a single GFF, each weighted by its respective GDP, has information not spanned by these traditional factors, or by the local RFFs. I find that the including the GFF in the model produces estimates of spillover effects that appear consistent with our conceptual understanding of these flows, both in direction and magnitude. For example, following the Russian default and LTCM bailout in the fall of 2008, one finds a sharply negative impact of this GFF on U.S. term premiums, which conforms to the conventional wisdom of flight-to-quality motives driving international capital flows during that period. Similarly, in the bond conundrum period from mid 2004 to mid 2006, the GFF effects suggest that the U.S. term premium, and so its long-term yields, were roughly 50 basis points lower than they otherwise would have been, an estimate that is consistent with the gap left unexplained by the literature.

The approach I just described does not apply no-arbitrage constraints in estimating term premiums across the countries in the sample. While estimating a full affine term-structure model across countries would of course be desirable, the data of many of the countries studied do not allow one to do so (not without imposing a degree of inflexibility that the data do not appear to support, at least at a daily frequency, where liquidity issues can lead to dislocations across forward rates). However, I do estimate an affine term-structure model for four of the countries that have sufficient liquidity to support the no-arbitrage restrictions on their daily zero-coupon yields, and whose market prices of risk appear to be determined by the covariance of the level shock with excess returns: the U.S., U.K., Germany, and Japan. The results, reported in Appendix A, appear quite close to those in the paper.

The remainder of the paper is organized as follows. To set the stage, I start by providing a brief description of the data, then discuss the evidence across countries of a single factor accounting for most of the economically relevant variation in excess returns. Section 3 describes the model and Section 4 the steps of its empirical implementation. Section 5 presents the term-premium estimates across countries, and Section 6 concludes.

## 2 Bond Return Regressions

### 2.1 Data

I obtained or estimated local currency zero-coupon government yield curves at the monthly frequency for all ten countries from the early to mid 1990s to April 2011, and at the daily frequency for six of those countries from January 1998 to April 2011. Table 1 lists the sources, frequency, and sample periods of these ten yield curves. All the yields used are continuously compounded and at maturities of 1 to 10 years. Quarterly GDP data to construct the GFF come from the OECD.

### 2.2 Notation

Suppose  $p_t^{(n)}$  is the log price at time  $t$  of an  $n$ -period zero-coupon bond, and  $y_t^{(n)} = \frac{\log(p_t^{(n)})}{n}$  is its log yield, where maturity  $n$  and  $t$  are defined in years. Let the one-year log forward rate between periods  $t + n - 1$  and  $t + n$  be the differential in log bond prices,  $f_t^n = p_t^{(n-1)} - p_t^{(n)}$  and the excess (over the alternative of holding a one-year bond to maturity) log holding period return (here an annual return) from buying an  $n$ -year bond in period  $t$  and selling it as an  $n-1$ -year bond at time  $t+1$  be:

$$rx_{t+1}^{(n)} = p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)}$$

I define the term premium of an  $n$ -year bond as the excess return from buying the bond in period  $t$  and holding it until maturity relative to the alternative of rolling over 1-year bonds over the same period

$$tp_t^{(n)} = y_t^{(n)} - \frac{1}{n} E_t \left( y_t^{(1)} + y_{t+1}^{(1)} \dots + y_{t+n-1}^{(1)} \right)$$

This should equal the sum of excess holding period returns from an  $n$ -year bond over the next  $n-1$  years, as Equation (6) in CP (2008) states:

$$tp_t^{(n)} = y_t^{(n)} - \frac{1}{n} \sum_{h=0}^{n-1} E_t \left( y_{t+h}^{(1)} \right) = \frac{1}{n} \sum_{h=1}^{n-1} E_t \left( rx_{t+h}^{(n-h+1)} \right) \quad (1)$$

This implies that a reasonable estimate of future expected excess holding period returns will also be a reasonable estimate of the expected term premium. I turn next to estimating this term structure of excess returns.

### 2.3 Estimating Return Forecasting Factors

Cochrane and Piazzesi (2005, 2008) identify a return forecasting factor with considerable forecasting power for future excess bond returns that is not fully spanned by the first three principal components

(level, slope, curvature) traditionally used in TS models.<sup>2</sup> In related work, Duffie (2008) estimates a five-factor TS model for the U.S., identifying a fifth factor with a negligible impact on the cross section of yields, but with important information about expected future short rates and excess bond returns. One advantage of CP over models such as Duffie (2008) is the possibility to use their return forecasting factor to identify a global return forecasting factor, but via a term-structure model whose parameters are tailored to the cross-section of each country. It appears difficult to get robust estimates of the fourth and fifth principal components across models and data sets. For example, Dai, Singleton, and Yang (2004) find that the fourth and fifth principal components are quite sensitive to the smoothing technique used to construct the zero coupon data.<sup>3</sup> A second advantage of the CP model is that it appears to capture some of the forecasting power of these fourth and fifth principal components, while avoiding the volatility and possible lack of robustness from introducing them separately.

CP's (2008) model draws on two stylized facts which I replicate in three steps for the ten countries in the sample:

1. The first principal component from the covariance matrix of excess returns accounts for almost all of the variation in excess returns over time.
2. There is considerable information in forward rates that can be used to forecast bond excess returns and that is not spanned by the traditional level, slope, and curvature factors of term structure models.

First, like CP (2008) I examine the following relationship

$$rx_{t+1}^{(n)} = \alpha^{(n)} + \beta_1^{(n)} y_t^{(1)} + \beta_2^{(n)} f_t^{(2)} + \beta_3^{(n)} f_t^{(3)} + \beta_4^{(n)} f_t^{(4)} + \beta_5^{(n)} f_t^{(5)} + \epsilon_{t+1}^{(n)} \quad (2)$$

finding that a single factor accounts for most of the variation in expected excess returns across maturities across the countries in the sample. In Figure 1, I display the coefficients from running this regression for the sample countries, all of which exhibit the familiar tent-shaped pattern identified by CP for the U.S. This elegant result across countries implies that one can harness the predictive power of all these forward rates via a single linear combination, so that:

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<sup>2</sup>Litterman and Scheinkman (1991) review this literature.

<sup>3</sup>They find that the coefficients on the first two principal components are very similar across the four data sets they consider, unsmoothed Fama-Bliss (UFB), Fisher-Waggoner cubic spline (FW), Nelson-Siegel-Bliss (NSB), and smoothed Fama-Bliss (SFB), but that "as we move out the list of PCs, the magnitudes of the coefficients become increasingly different across data sets. For PC5, the differences are large with the magnitudes being positive for the choppiest data (UFB) and then declining monotonically to large negative numbers as the zero data becomes increasingly smooth. That the variation in yields associated with the fifth PC in data set UFB is 'excess' relative to the variation in the yields from other datasets is seen from Table 5. The volatilities of the first three PCs are quite similar across data sets. However the volatilities of PC4 and PC5 are larger in data set UFB than in the other data sets... These differences, that largely show up on the properties of the fourth and fifth PCs, are entirely attributable, of course, to the choice of spline methodology used to construct the zero coupon yields. What seems striking is how much even small differences in the smoothness of the zero curves affects the properties of the PCs" (DSY, 2004, pp. 8-9).



$$rx_{t+1}^{(n)} = b_{(n)}x_t + \epsilon_{t+1}^{(n)} \quad (3)$$

CP interpret this forecasting power of lagged yields as resulting from measurement error (that is, small i.i.d. measurement errors over time) rather than reflecting an economic phenomenon.

CP (2008) also show that a single factor accounts for over 99 percent of the variation in 1-year excess returns in U.S. Treasuries. They measure this fraction as the ratio of the largest eigenvalue of the covariance matrix of excess returns relative to the sum of all the other eigenvalues. I run this exercise for the countries in the sample, with results reported in the last column of Table 2. I find that a single factor accounts for at least 98 percent of the variance in excess returns for all the countries in the sample except Finland and Australia, where it still accounts for around 90 percent.

Second, I construct local return forecasting factors for each country in the sample. CP (2008) construct their return forecasting factor  $x_t$  by weighting the expected excess returns for each maturity by the eigenvector  $q'_r$  corresponding to the largest eigenvalue of the first principal component of forward rates:

$$x_t = q'_r E_t(rx_{t+1}) = q'_r(\alpha + \beta f_t) \quad (4)$$

As the  $\beta$ 's are tent shaped, and  $q_r$  is made up of positive numbers, CP (2008) show that because the regression coefficients of each maturity return on forward rates are all proportional, then if I start with the regression forecast of each excess return,

$$rx_{t+1} = \alpha + \beta f_t + \epsilon_{t+1} \quad (5)$$

and premultiply by  $q'_r$  I get that the return forecast factor is the linear combination of forward rates that forecasts the portfolio  $q'_r rx_{t+1}$

$$E_t(q'_r rx_{t+1}) = q'_r(\alpha + \beta f_t) = x_t \quad (6)$$

With the single factor restriction, then, I can combine all the excess returns across the maturity spectrum into a single weighted average, with  $q_r$  serving as the weights.<sup>4</sup>

Third, I confirm that the RFF's have information that is not spanned by the traditional level, slope, and curvature factors of conventional term structure models. Table 2 reproduces the  $R^2$  from Table 2 of CP (2008), showing that the local RFF's account for a similar share of the total variation in other countries' excess returns as CP find for the U.S. The first three columns of Table 2 report the  $R^2$  from regressing average excess returns across maturities on the traditional level, slope, and

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<sup>4</sup>When the zero coupon data are constructed using a method that smooths yields across maturities, like NSS or SS, this can lead to multicollinearity across the forward rates, which is important in any study of excess returns, as differences of differences.

curvature factors. As one can see, while these conventional factors do have some power to forecast excess returns, the  $R^2$  reported in the fourth and fifth columns, from regressing average excess returns on the local and global RFF's clearly indicate that some orthogonal movement in expected returns remains. As their forecasting power is not spanned by the traditional three factors, both RFF's should be included in the model.

Figure 2 displays monthly estimates of the global and local RFF's across all ten countries in the sample, which exhibit a striking degree of comovement over time. Table 3 reports the correlations between each of these monthly local RFF's and the monthly global RFF, which in general appear quite intuitive. The U.S.'s RFF has the highest correlation with the GFF, with the U.K. and Germany's correlation coefficients both above 0.75. Not surprisingly, there is a higher correlation between the European RFF's in the sample than between each of them and Japan, whose RFF has the highest correlation with Australia's, at 0.58.

### 3 Model

CP (2008) document that their return forecasting factor shares important dynamics with the level, slope, and curvature factors of the yield curve. Hence, one can run a vector autoregression on the RFF and these three factors to predict the RFF a few periods ahead, and on the basis of this prediction, construct expected excess holding period returns. These additional factors are formed by an eigenvalue decomposition of the covariance matrix of forward rates, after orthogonalizing them with respect to the local RFF. This procedure also ensures that each of these factors retains virtually no information to forecast excess returns.

**Local Return Forecasting Factor Model** Consider a matrix of variables  $X_t$  made up of the local RFF,  $x_t$ , and the three eigenvalue decomposition factors of the forward covariance matrix, each orthogonalized to  $x$ . Let the dynamics of  $X_t$  be characterized by a Gaussian vector autoregression:

$$X_{t+1} = c + \rho X_t + \Sigma \varepsilon_{t+1} \quad (7)$$

One can predict future values of the return forecasting factor  $x_t$  by estimating the parameters of this VAR via ordinary least squares and iterating it forward. In particular:

$$E_t(x_{t+h} - c) = \Omega'_1 \rho^h (X_t - c) \quad (8)$$

or

$$E_t(x_{t+h}) = \Omega'_1 \left[ \left( \sum_{j=0}^{h-1} \rho^j \right) c + \rho^h X_t \right] \quad (9)$$

where  $\Omega'_1 = [1 \ 0 \ 0 \ 0]$ . From Equation (3) it follows that

$$E_t(rx_{t+1}) = b_{(n)}x_t$$

which, in turn, implies that

$$E_t \left( rx_{t+h}^{(n-h+1)} \right) = b_{(n-h+1)} E_t(x_{t+h-1})$$

$$E_t \left( rx_{t+h}^{(n-h+1)} \right) = b_{(n-h+1)} \Omega'_1 \left[ \left( \sum_{j=0}^{h-2} \rho^j \right) c + \rho^{h-1} X_t \right] \quad (10)$$

I use Equation (10) to model expected future excess holding period returns and sum them up to get an estimate of today's term premium using Equation (1).

**Global Return Forecasting Factor Model** The global return-forecasting-factor model differs from the local only in the addition of the GFF to the matrix of variables  $X_t$ . The remaining equations go through, provided one redefines the  $\Omega'_1 = [1 \ 1 \ 0 \ 0 \ 0]$ . Whereas in the local version of the model, setting  $\Omega'_1$  equal to  $[1 \ 0 \ 0 \ 0]$  restricts variation in expected excess returns, (that is, the market price of risk is restricted to be a function of the single local return-forecasting factor), in the global version of the model, it is a function of both this local return forecasting factor and the global return forecasting factor, with the latter orthogonalized to the former. Hence  $\Omega'_1$  is redefined to equal  $[1 \ 1 \ 0 \ 0 \ 0]$  in this case.

## 4 Estimation

The steps to estimate the model are as follows:

1. Estimate the local return forecasting factor, LFF, as described in Section 2, along with the three traditional term-structure factors;
2. Estimate a VAR of the LFF, level, slope, and curvature factors orthogonalized to the LFF to predict future values of the LFF, which in turn predicts future excess returns.
3. Iterate forward the LFF VAR to compute implied forecasts of the LFF: Use the LFF prediction to compute expected excess holding returns. Compute the estimated term premium of a 10-year

bond as the average expected excess return of declining maturity for  $n=2:10$  for the non-GFF model.

4. Combine the LFF's, each weighted by its country's GDP, into a single GFF. Assess how much of variation over time in excess returns can be attributed to the global as opposed to the local return forecasting factor for each country. Orthogonalize the GFF to each country's LFF before estimating a VAR of the LFF, the orthogonalized GFF, and the orthogonalized level, slope, and curvature factors.
5. Iterate forward the GFF VAR to compute implied forecasts of the GFF: Use the GFF prediction to compute expected excess returns. Compute the estimated term premium of 10-year bond as the average expected excess holding period return of declining maturity for  $n=2:10$  for the GFF model.

#### 4.1 Monthly to Daily Model

For the monthly model, the vector autoregressions described in Section 3 are estimated as written, via ordinary least squares. However, I must also fit the model to the yield curves of six countries at a daily frequency. To obtain these real-time term-premium estimates, I follow the empirical strategies of Adrian-Moench (2010) and CP (2008), estimating most of the model's parameters at a lower (monthly) frequency, and then apply these parameters to the higher frequency data of interest – in this case, daily data. Measurement error appears to be i.i.d. in the daily yield data which suggests that I will get a better fit for the daily TP estimates from principal components whose weights are identified using monthly rather than daily data. (CP, 2005, make a similar point about the use of monthly versus quarterly data in term-structure estimation).

In the daily version of the model, I aggregate daily yields to a monthly frequency by taking monthly averages. I then compute the local and global return forecasting factors, and extract principal components from the (de-meaned) error term after regressing forward rates on the return forecasting factor. I apply the weights from these monthly principal components to the dataset of daily yields to obtain daily estimates of the model's factors. As the principal components are extracted from de-meaned errors, I must make an adjustment to the daily factors – I apply the monthly principal component weights to the sample average of the error term from the monthly version of the model, and then subtract this vector from the daily factors obtained above.

### 5 Results

Figure 3 plots the model's 10-year term premium estimates and compares them to zero-coupon yields for 10-year government bonds for each of the countries in the sample. The daily estimates come from

the global forecasting factor model, while the monthly estimates use only the individual country return forecasting factors. The left column of charts in Figure 3 reports estimates from July 1997 to April 2011, and the right column from April 2008 to April 2011, to provide a closer look at their variation in recent years.

The estimates appear reasonable and are consistent with estimates from other well-known term-structure models. The model’s term premium estimates are generally (though not always) positive. Like Wright (2010), I find that term premiums have gradually declined across developed economies since the early 1990s.

The term premiums of countries thought to be relatively insulated from the financial crisis such as Canada or Japan do not jump dramatically after 2008. In those countries that were more exposed, either directly through their financial sector, as in the case of the U.S. and U.K., or indirectly, through the sovereign debt crisis, as in the case of Germany, term premiums have been higher than before the crisis. The U.K., which is facing a particularly unwieldy fiscal outlook, has seen its term premium rise on a sustained basis by even more than those of the U.S. and Germany.

Figure 4 compares the model’s term premiums estimates across countries. It shows that the model’s term premium estimate is somewhat higher for the United States than for Germany over most of the sample period. The chart on the bottom right of Figure 4 entitled “Term Premium Comparisons” compares the model’s estimates to the Kim-Wright (2005) and Adrian-Moench (2010) term-premium measures for the U.S. While the levels of the GFF term-premiums tend to be slightly higher than those of the other models, their variation over time appears quite similar.

## 5.1 Cross-Border Effects

In periods of financial and economic turmoil, such as the period since the onset of the recent financial crisis, or during the Asian crisis in 1998, one finds a sharply negative impact of the global forecasting factor on U.S. term premiums, which conforms to the conventional wisdom of flight-to-quality flows driving international capital flows during such periods.

The charts in Figure 5 illustrate how including the global forecasting factor in the model provides some estimates of international spillover effects. Its top left chart plots the difference in the estimated term premiums with and without the GFF for the U.S., U.K. and Germany in the months following the Russian default, in August of 1998, and the failure of LTCM, in September of 1998. U.S. term premium estimates were about 40 basis points lower than they otherwise would have been, according to the model, while German and U.K. bond risk premiums were largely unaffected.

The next two charts in Figure 5 plot these differences for the U.S., U.K., Canada, and Germany since the onset of the recent financial crisis, in 2008. They illustrate the global factor’s current downward pressure on the U.S. term premium, a trend that has intensified since the onset of the

sovereign debt crisis in early 2010. One can see how, following an initial period of panic, U.S. yields have been lower than they otherwise would have been over the past few years, by roughly 50 basis points. One implication is that if global risk appetite strengthens, it may lead to a rise in long-term U.S. yields, even in the absence of any changes in U.S. monetary policy.

Similarly, in the bond conundrum period from mid 2004 to mid 2006, the global-forecasting-factor effects – reported in the bottom right chart of Figure 5 – suggest that the U.S. term premium was about 50 basis points lower than it otherwise would have been, an estimate that is consistent with the gap left unexplained by the literature, after accounting for the fall in implied volatility of longer-term Treasuries over that period. This chart also shows that this difference is negatively correlated with total (but not official) purchases of U.S. Treasuries, with a correlation coefficient of almost -0.70 from 2004 to 2006.

## **6 Conclusion**

I estimate time-varying measures of government bond term premiums for ten major developed economies. In future work, I plan to expand the model to include estimates for some of the peripheral European countries, to assess the magnitude of spillover effects of their distress on the pricing of risk in the sample countries. I also plan to connect the findings regarding the global forecasting factor to the literature on real and financial integration, for example Kose et al (2003) who find a common global business cycle factor to be an important source of economic volatility in most countries and Ehrmann and Fratscher (2004) who document significant comovement between U.S. and European financial markets.

## References

- [1] Adrian, Tobias and Emanuel Moench. 2010. "Pricing the Term Structure with Linear Regressions," Federal Reserve Bank of New York Staff Report No. 340, July.
- [2] Cochrane, John and Monika Piazzesi. 2008. "Decomposing the Yield Curve," Working Paper
- [3] Cochrane, John and Monika Piazzesi. 2005. "Bond Risk Premia," *The American Economic Review*, vol. 95, pp. 138-60.
- [4] Dahlquist, Magnus and Henrik Hasseltoft. 2011. "International Bond Risk Premia," Working Paper.
- [5] Dai, Qiang, Kenneth J. Singleton, and Wie Yang. 2004. "Predictability of Bond Risk Premia and Affine Term Structure Models," Working Paper.
- [6] Diebold, Francis, Canlin Li, and Vivian Z. Yue. 2008. "Global Yield Curve Dynamics and Interactions: A Dynamic Nelson-Siegel Approach," *Journal of Econometrics*, vol. 146, pp. 351-63.
- [7] Duffee, Gregory R. 2008. "Information in (and not in) the Term Structure," Working Paper.
- [8] Ehrmann, Michael and Marcel Fratscher. 2004. "Equal Size, Equal Role? Interest Rate Interdependence between the euro area and the United States" Working Paper.
- [9] Ehrmann, Michael, Marcel Fratscher, and Roberto Rigobon. 2005. "Stocks, Bonds, Money Markets and Exchange Rates: Measuring International Financial Transmission," Working Paper.
- [10] Kim, Don H. and Wright, Jonathan H. 2005. "An Arbitrage-Free Three-Factor Term Structure Model and the Recent Behavior of Long-Term Yields and Distant-Horizon Forward Rates." Finance and Economics Discussion Series 2005-33, Board of Governors of the Federal Reserve System.
- [11] Kose, Ayhan, Christopher Otrok, and Charles H. Whiteman. 2003. "International Business Cycles: World, Region, and Country-Specific Factors" *The American Economic Review*, vol. 93, pp. 1216-39.
- [12] Ludvigson, Sydney C., and Serena Ng. 2009. "Macro Factors in Bond Risk Premia" *Review of Financial Studies*, vol. 22, pp. 5027-67.
- [13] Wright, Jonathan. 2010. "Term Premia and Inflation Uncertainty: Empirical Evidence from an International Panel Dataset," *The American Economic Review*, forthcoming.

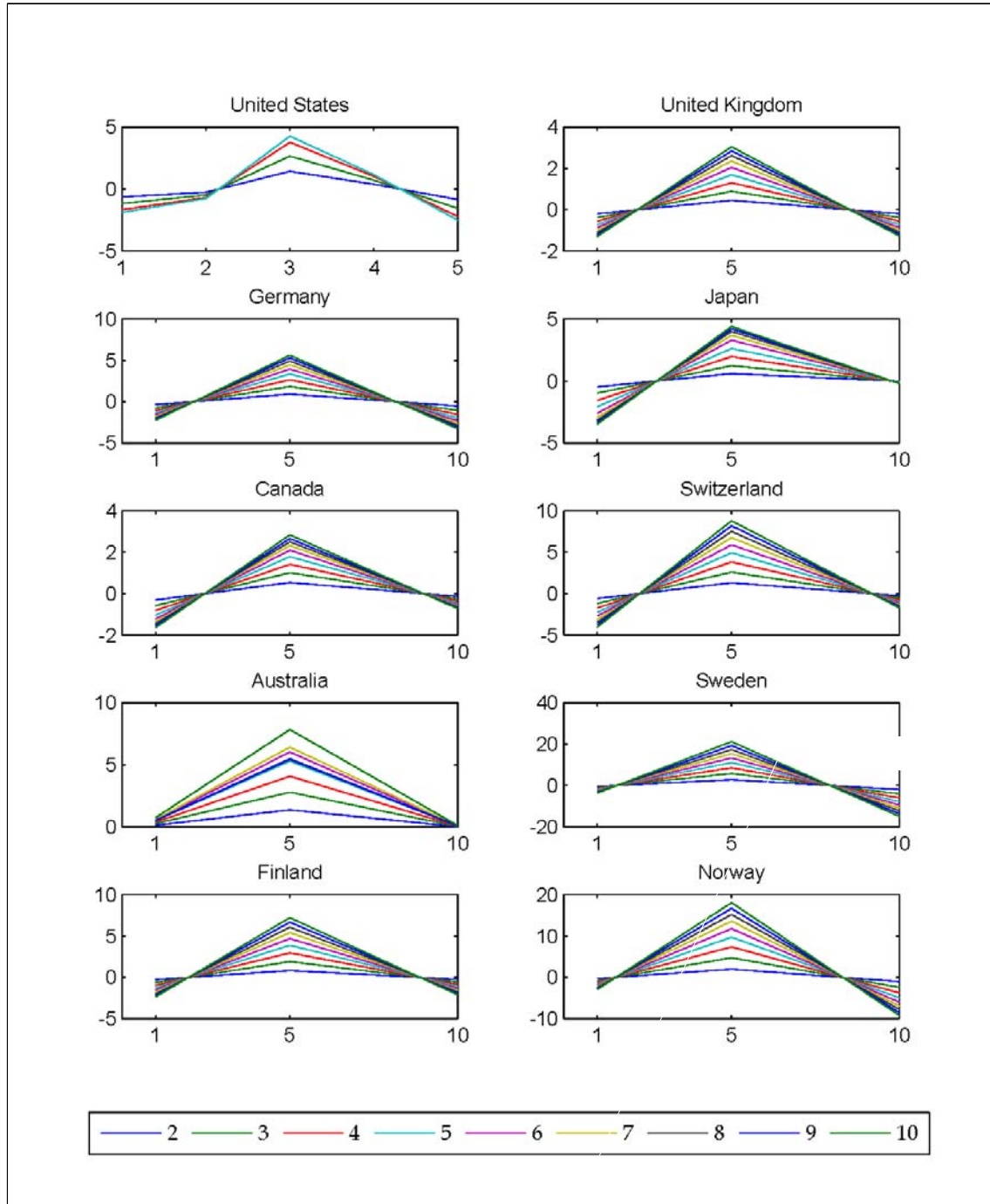
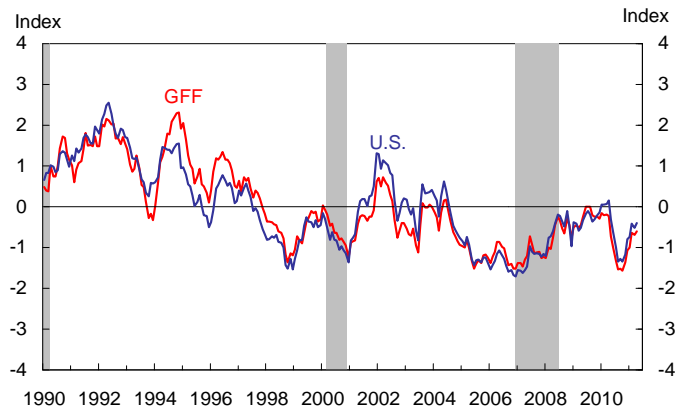


Figure 1: *Regression Coefficients of Excess Returns on Forward Rates*. Parameter estimates from the single-factor model. The legend denotes the maturity of the bond whose excess return is forecast. The x-axis reports the maturity of the forward rate which is the right-hand side variable.



## Figure 2. Global and Local Return Forecasting Factors

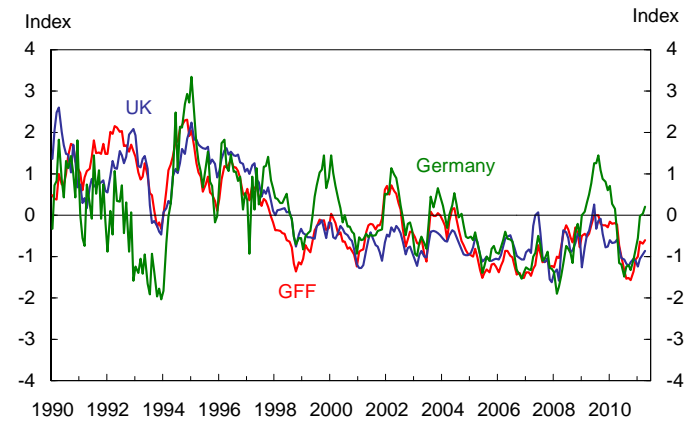
### United States



Source: Author's Calculations

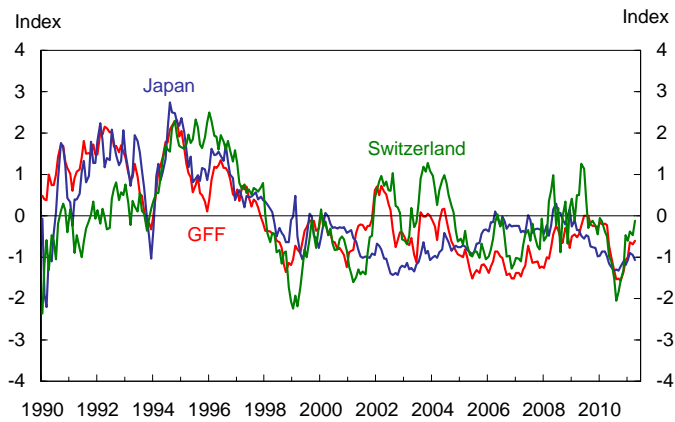
Note: Shading represents NBER recessions.

### United Kingdom, Germany



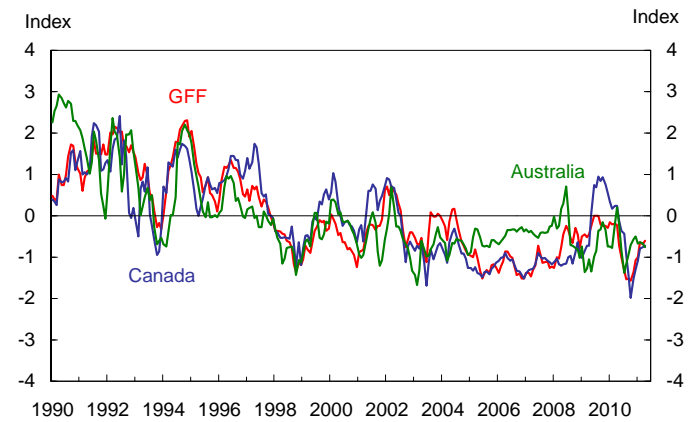
Source: Author's Calculations

### Japan, Switzerland



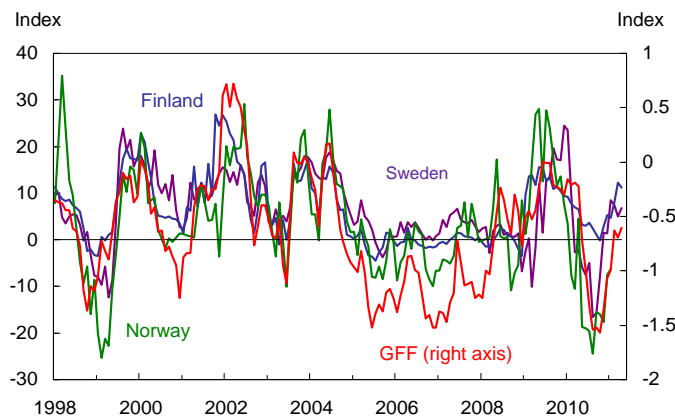
Source: Author's Calculations

### Canada, Australia



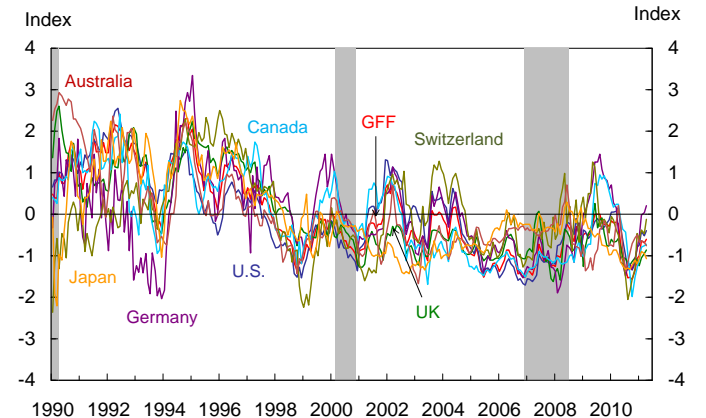
Source: Author's Calculations

### Finland, Norway, Sweden



Source: Author's Calculations

### Comovement of Expected Excess Returns

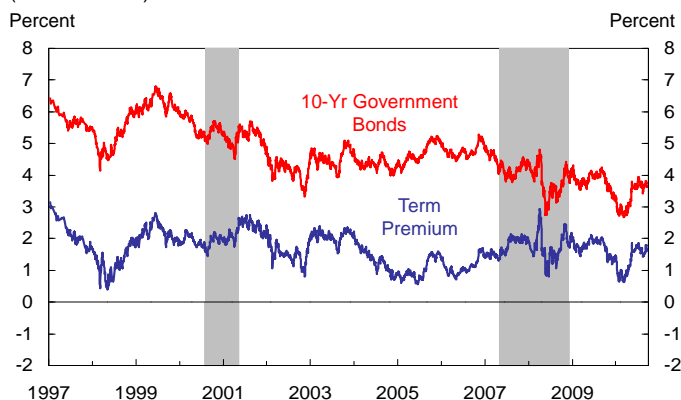


Source: Author's Calculations

Note: Shading represents NBER recessions.

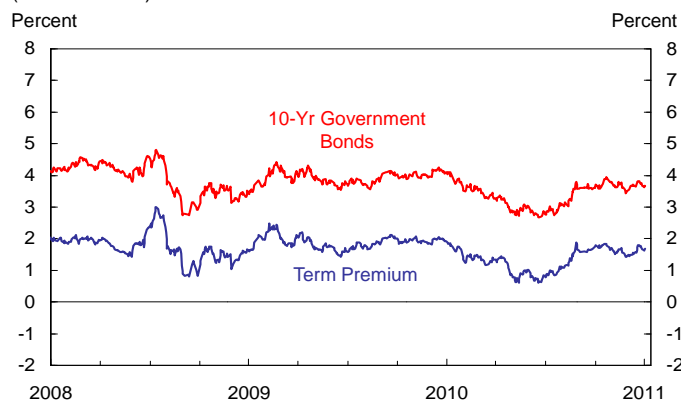
# Figure 3. 10-Year Government Bond Yields and Term Premiums

## 10-Year Government Bond Yields and Term Premiums (United States)



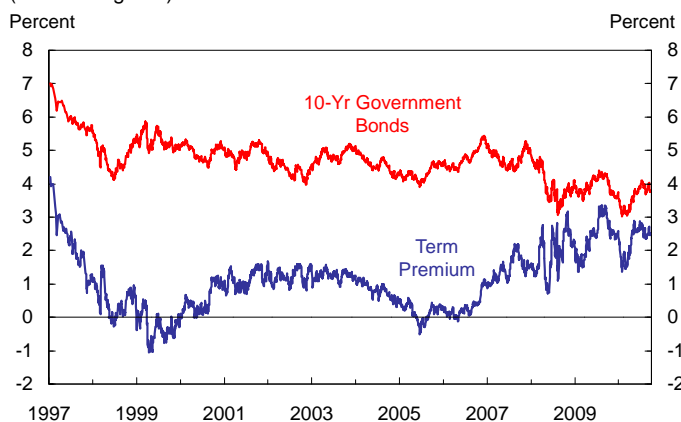
Source: Author's Calculations      Note: Shading represents NBER recessions.

## 10-Year Government Bond Yields and Term Premiums (United States)



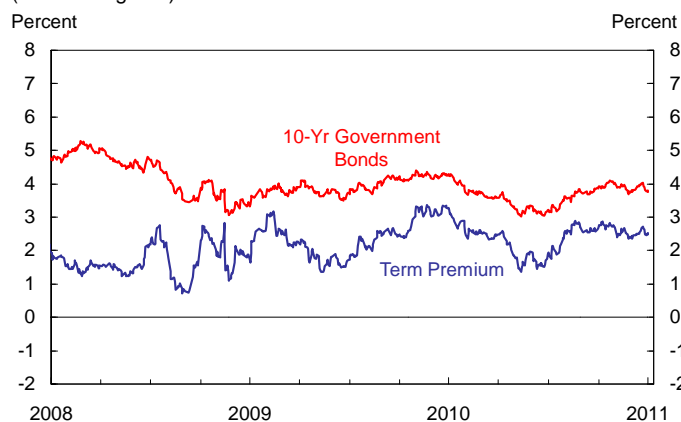
Source: Author's Calculations

## 10-Year Government Bond Yields and Term Premiums (United Kingdom)



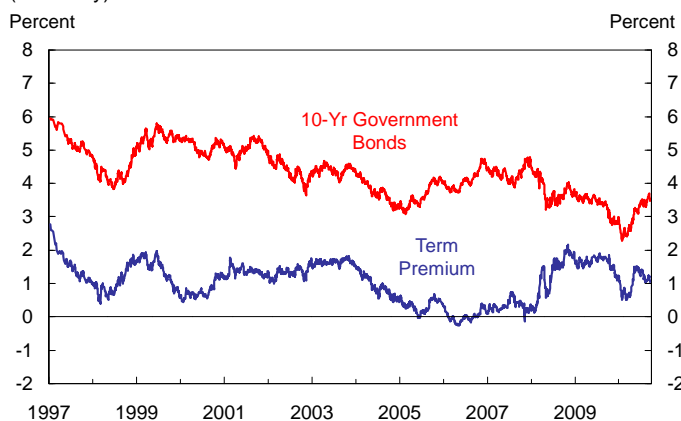
Source: Author's Calculations

## 10-Year Government Bond Yields and Term Premiums (United Kingdom)



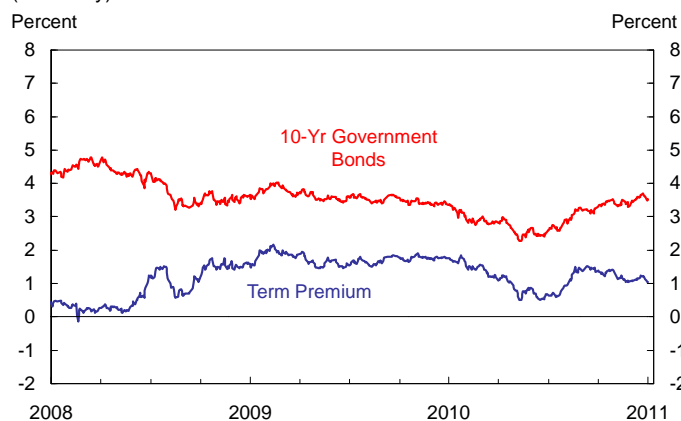
Source: Author's Calculations

## 10-Year Government Bond Yields and Term Premiums (Germany)



Source: Author's Calculations

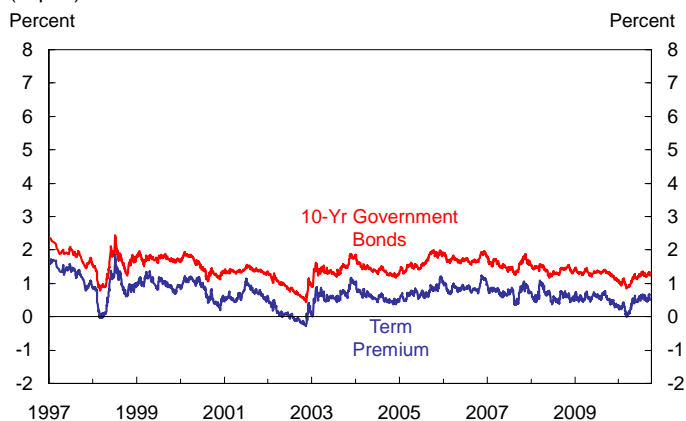
## 10-Year Government Bond Yields and Term Premiums (Germany)



Source: Author's Calculations

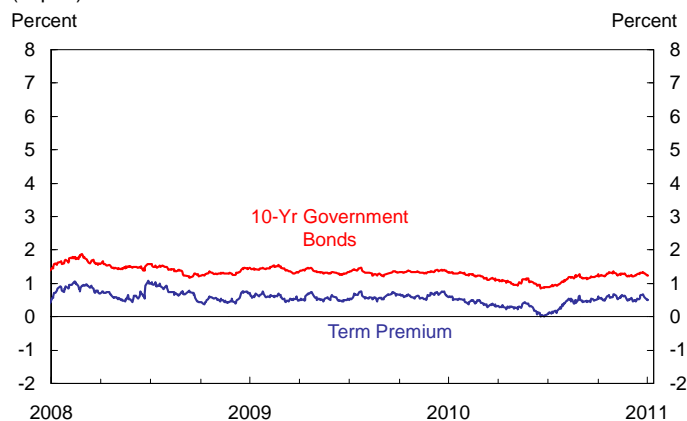
# Figure 3. 10-Year Government Bond Yields and Term Premiums

## 10-Year Government Bond Yields and Term Premiums (Japan)



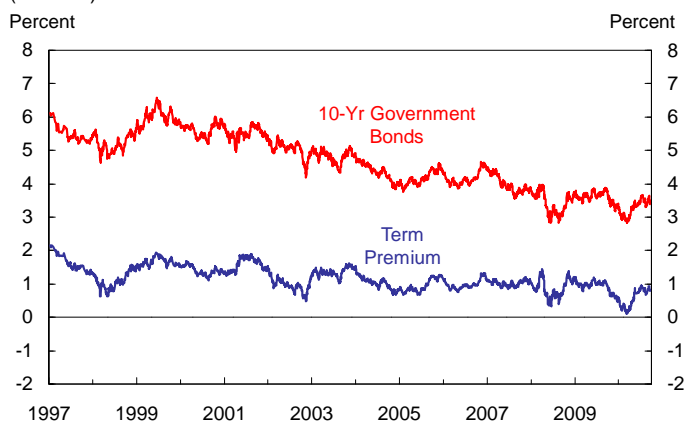
Source: Author's Calculations

## 10-Year Government Bond Yields and Term Premiums (Japan)



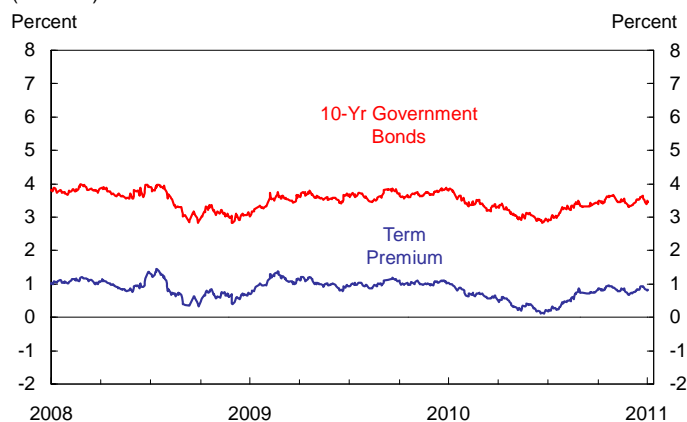
Source: Author's Calculations

## 10-Year Government Bond Yields and Term Premiums (Canada)



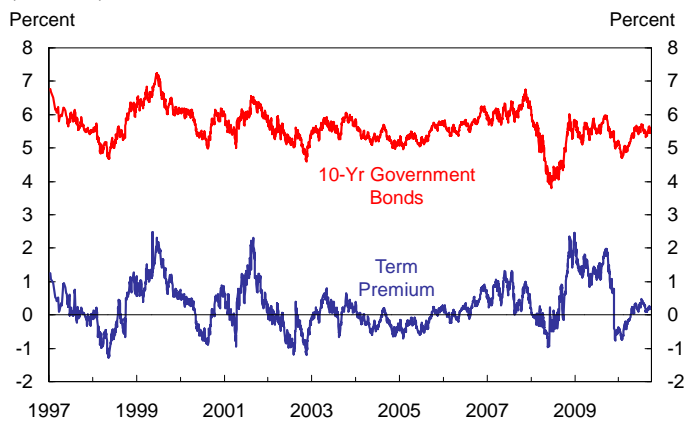
Source: Author's Calculations

## 10-Year Government Bond Yields and Term Premiums (Canada)



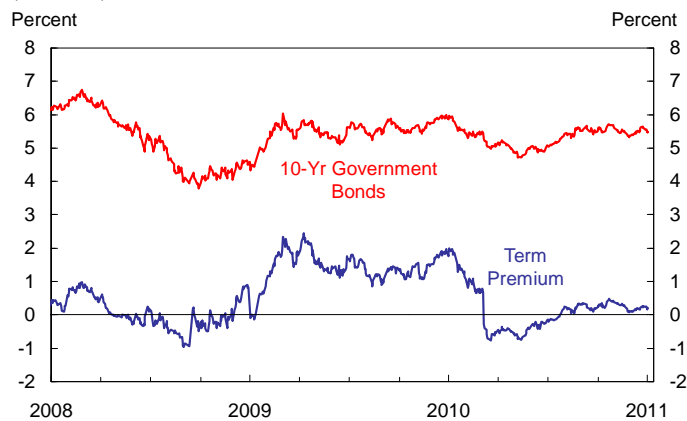
Source: Author's Calculations

## 10-Year Government Bond Yields and Term Premiums (Australia)



Source: Author's Calculations

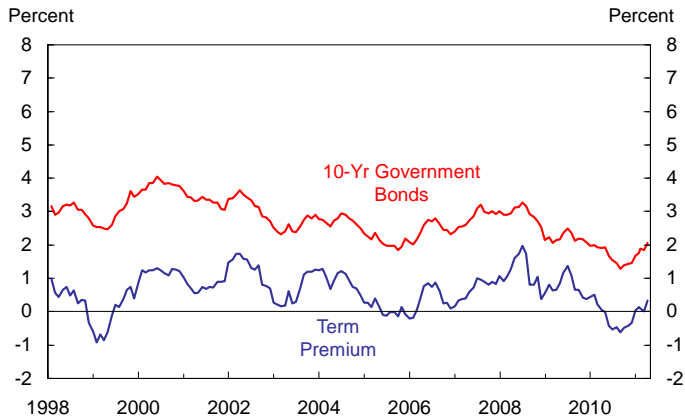
## 10-Year Government Bond Yields and Term Premiums (Australia)



Source: Author's Calculations

## Figure 3. 10-Year Government Bond Yields and Term Premiums

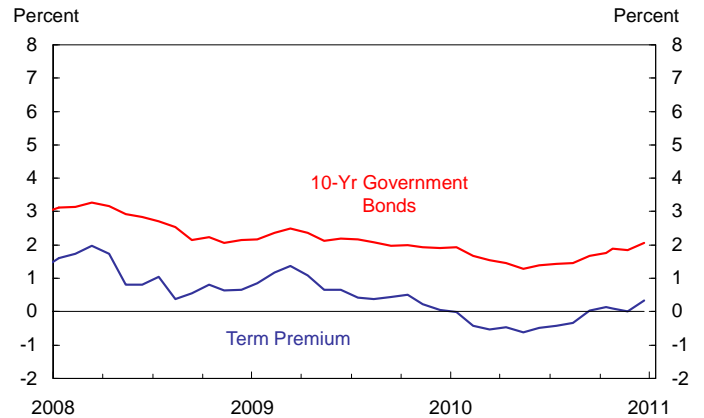
10-Year Government Bond Yields and Term Premiums  
(Switzerland)



Source: Author's Calculations

Note: Monthly estimates

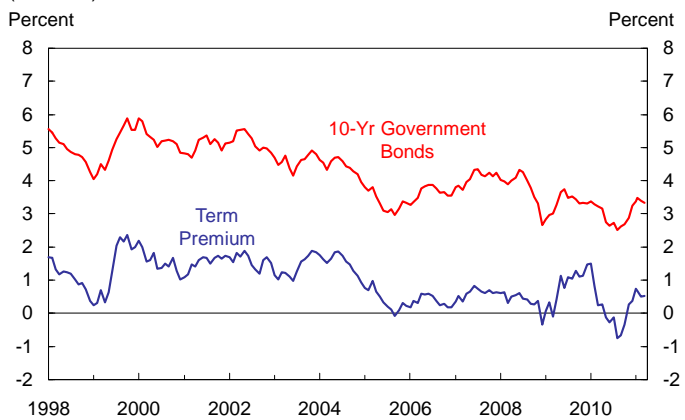
10-Year Government Bond Yields and Term Premiums  
(Switzerland)



Source: Author's Calculations

Note: Monthly estimates

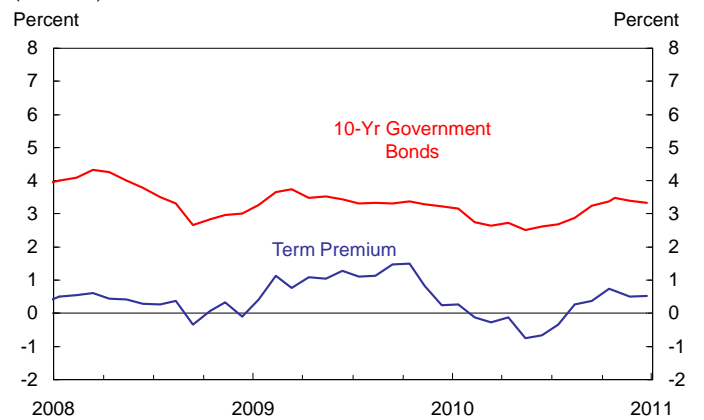
10-Year Government Bond Yields and Term Premiums  
(Sweden)



Source: Author's Calculations

Note: Monthly estimates

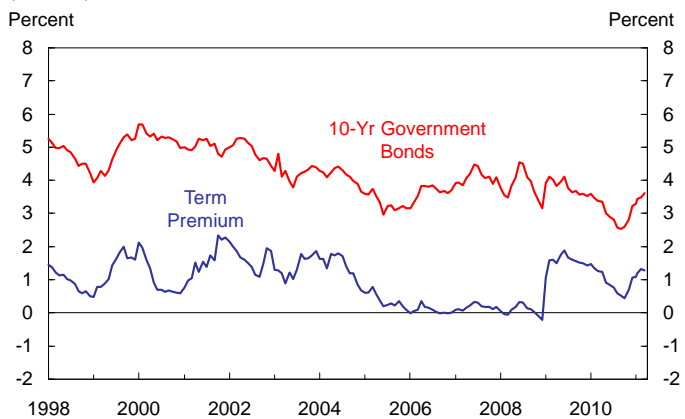
10-Year Government Bond Yields and Term Premiums  
(Sweden)



Source: Author's Calculations

Note: Monthly estimates

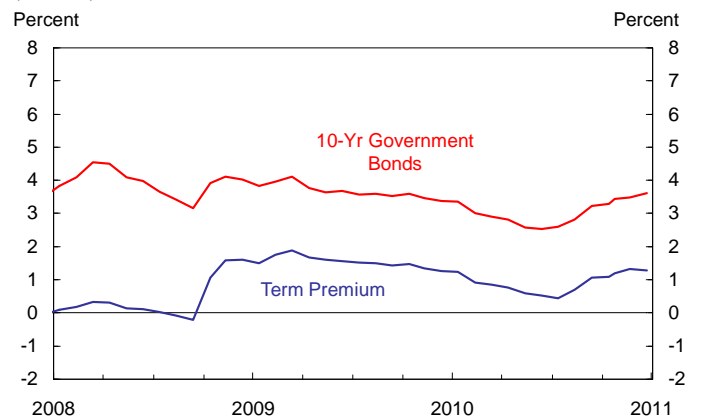
10-Year Government Bond Yields and Term Premiums  
(Finland)



Source: Author's Calculations

Note: Monthly estimates

10-Year Government Bond Yields and Term Premiums  
(Finland)



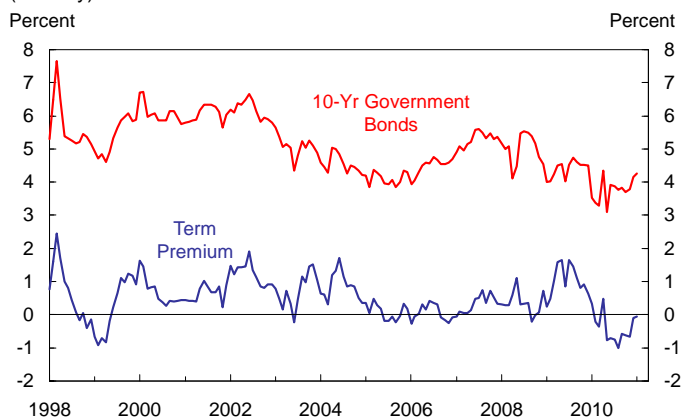
Source: Author's Calculations

Note: Monthly estimates

### Figure 3. 10-Year Government Bond Yields and Term Premiums

#### 10-Year Government Bond Yields and Term Premiums

(Norway)

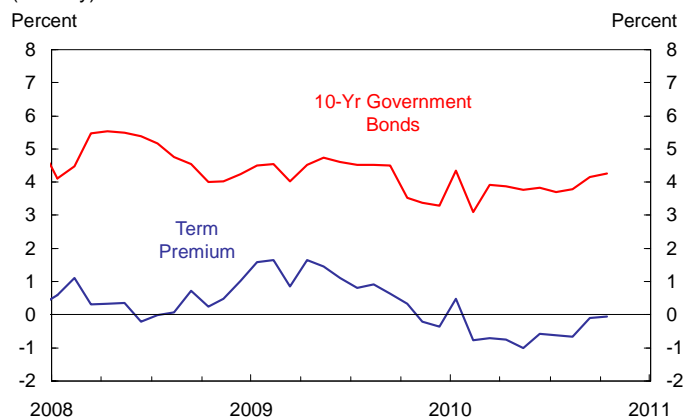


Source: Author's Calculations

Note: Monthly estimates

#### 10-Year Government Bond Yields and Term Premiums

(Norway)

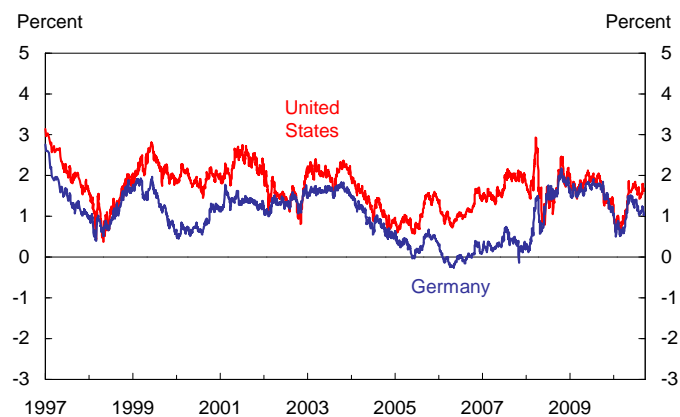


Source: Author's Calculations

Note: Monthly estimates

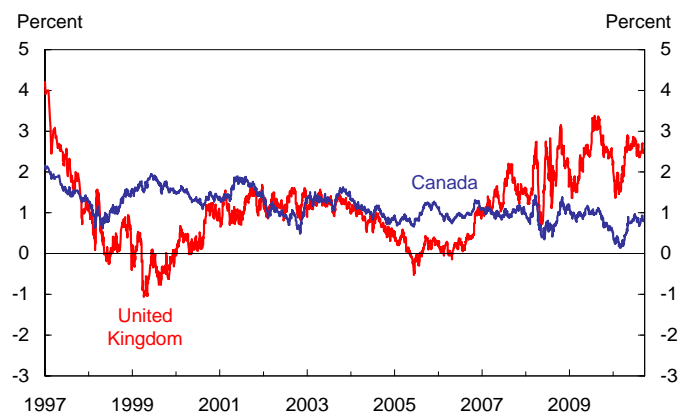
# Figure 4. Comparisons of 10-Year Term Premiums

## United States, Germany



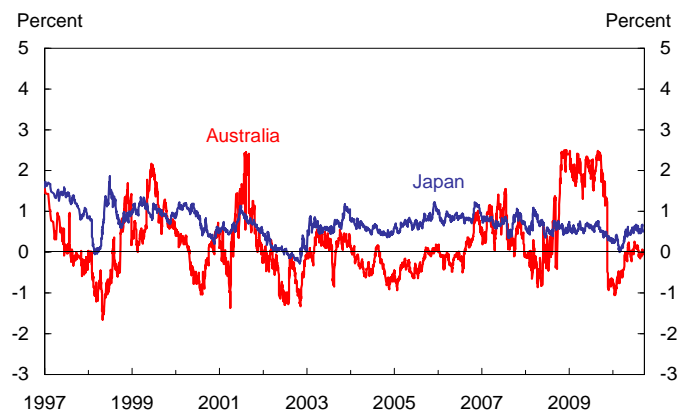
Source: Author's Calculations      Note: Shading represents NBER recessions.

## United Kingdom, Canada



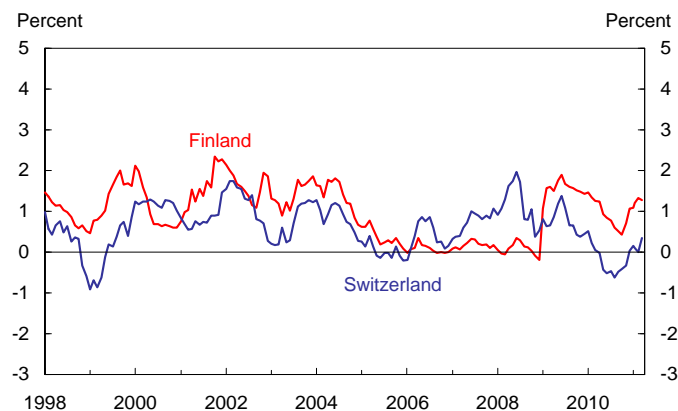
Source: Author's Calculations

## Australia, Japan



Source: Author's Calculations

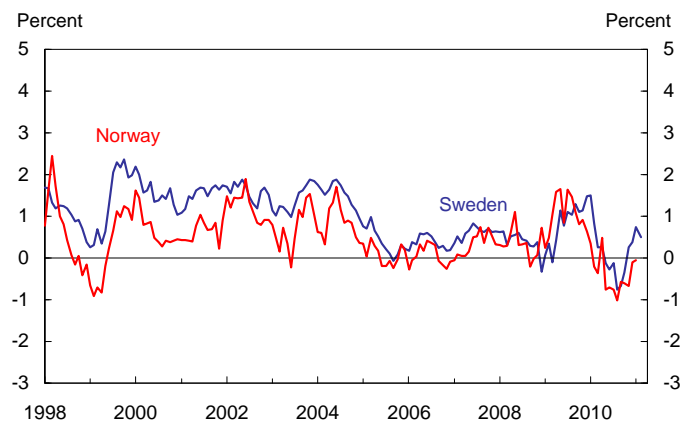
## Finland, Switzerland



Source: Author's Calculations

Note: Monthly Estimates

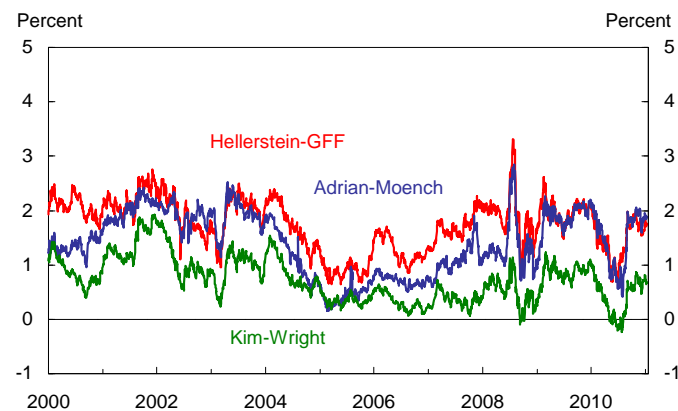
## Norway, Sweden



Source: Author's Calculations

Note: Monthly Estimates

## Term Premium Comparisons

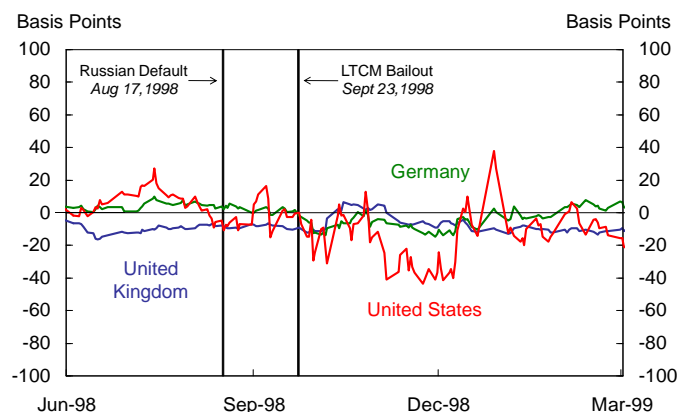


Source: Author's Calculations

Note: Shading represents NBER recessions.

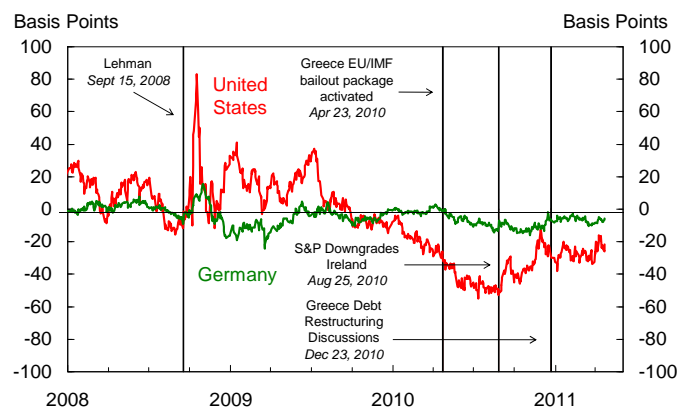
# Figure 5. Comparing GFF and Non-GFF Term Premium Estimates

## Differences in Term Premiums with GFF



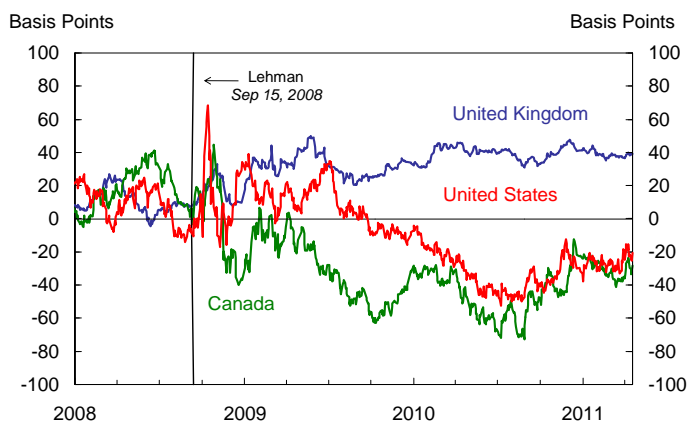
Source: Author's Calculations

## Differences in Term Premiums with GFF



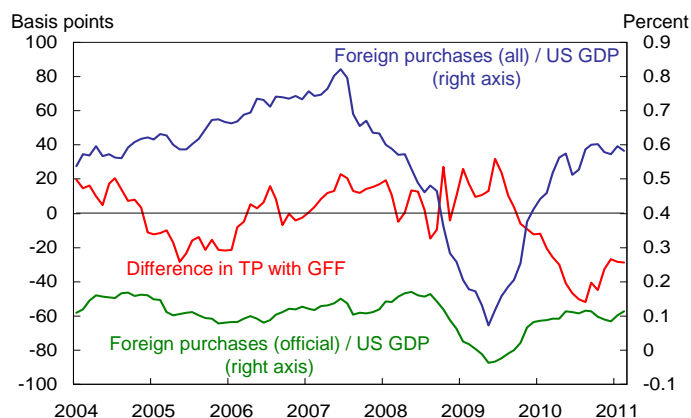
Source: Author's Calculations

## Differences in Term Premium with and without GFF



Source: Author's Calculations

## Foreign Purchases of U.S. Long-term Securities and Difference in U.S. Term Premium with GFF



Source: Author's Calculations and Treasury International Capital System

Table 1. Zero-Coupon Yield Data Sources

Country	Source	Start Date	Frequency	Method
U.S.	Gurkaynak, Sack, and Wright (2007)	November 1971	Daily	Svensson
U.K.	Anderson and Sleath (1999) and BoE database	January 1975	Daily	VRP/Spline
Germany	Bundesbank and BIS database	January 1973	Daily	Svensson
Japan	Bank of Japan and author's calculations	January 1987	Daily	Bootstrap
Canada	Bank of Canada and BIS database	January 1986	Daily	Spline
Australia	Bloomberg and author's calculations	January 1990	Daily	Bootstrap
Switzerland	Swiss National Bank and BIS database	January 1988	Weekly	Svensson
Sweden	Riksbank and BIS database	December 1992	Weekly	Svensson
Finland	Finlands Bank and BIS database	January 1998	Weekly	Svensson
Norway	Norges Bank and BOS database	January 1998	Monthly	Svensson

Notes: Zero-coupon yields are available out to ten-year maturities for each country. For Australia and Japan, we downloaded the prices of sovereign non-callable fixed-rate government bonds from Bloomberg and the Bank of Japan, respectively, and used bootstrap techniques to compute zero-coupon yields.



Table 2.  $R^2$  for Forecasting Average (Across Maturity) Excess Returns

Country	Level	Slope	Curvature	$X_t$	GFF	1st PC $R^2$
U.S.	0.11	0.15	0.30	0.44	0.45	0.98
U.K.	0.18	0.18	0.18	0.23	0.40	0.99
Germany	0.08	0.10	0.12	0.31	0.33	0.98
Japan	0.30	0.29	0.34	0.61	0.67	0.98
Canada	0.15	0.16	0.17	0.24	0.28	0.98
Switzerland	0.04	0.11	0.25	0.36	0.36	0.98
Australia	0.43	0.43	0.43	0.51	0.52	0.88
Sweden	0.11	0.13	0.27	0.31	0.33	0.98
Finland	0.19	0.21	0.25	0.34	0.46	0.96
Norway	0.08	0.12	0.39	0.45	0.46	0.99

Each entry gives the share of variation in excess returns explained by each of the factors, cumulatively for the first three columns. The fourth column shows the share of variation explained by the local return forecasting factor alone, the fifth column by the local and global return forecasting factor, and the final column the share of the variation in excess returns accounted for by their first principal component. The sample goes from January 1990 to April 2011 except for the Norwegian data, which end in January 2011.

Table 3. Correlation between the Global and Local Return Forecasting Factors

Country	GFF	U.S.	U.K.	Germany	Japan	Canada	Switzerland
U.S.	0.95						
U.K.	0.77	0.71					
Germany	0.76	0.69	0.68				
Japan	0.38	0.14	0.05	0.03			
Canada	0.46	0.39	0.48	0.51	-0.04		
Switzerland	0.60	0.58	0.45	0.54	0.11	0.26	
Australia	0.60	0.45	0.32	0.36	0.58	0.29	0.43

Sample runs from January 1998 to April 2011.

## A Affine Model

In this appendix, I decompose forward rates into average future expected one-month interest rates and the term premium by fitting a homoskedastic, discrete-time affine term structure model of the type considered by Ang and Piazzesi (2003) and Cochrane and Piazzesi (2008) to U.S., U.K., German, and Japanese yields.

### A.1 Basic Framework

Consider an  $(M \times 1)$  vector of variables  $X_t$  whose dynamics are characterized by a Gaussian vector autoregression:

$$X_{t+1} = c + \rho X_t + \Sigma v_{t+1} \quad (\text{A.1})$$

with  $v_{t+1} \sim \text{i.i.d. } N(0, I_M)$  with a conditional distribution that is  $\sim \text{i.i.d. } N(\mu_t, \Sigma \Sigma')$  for

$$\mu_t = c + \rho X_t. \quad (\text{A.2})$$

Let  $r_t$  denote the risk-free one-period interest rate. If  $X_t$  contains all the variables of importance to investors, then the price of a pure discount asset (e.g. a zero coupon bond) at time  $t$  should be a function  $P_t(X_t)$  of the current state vector. If investors are risk neutral, then the price they would be willing to pay should satisfy

$$P_t(X_t) = \exp(-r_t) E_t[P_{t+1}(X_{t+1})] \quad (\text{A.3})$$

For risk-averse investors, Equation (A.3) becomes

$$P_t(X_t) = E_t[P_{t+1}(X_{t+1}) M_{t,t+1}] \quad (\text{A.4})$$

with  $M_{t,t+1}$  defined as its nominal pricing kernel. Affine term structure models are derived from a particular pricing kernel which is conditionally lognormal:

$$M_{t,t+1} = \exp\left(-r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' v_{t+1}\right) \quad (\text{A.5})$$

where  $r_t = \delta_0 + \delta_1' X_t$  is the risk-free one-period interest rate,  $v_{t+1}$  is i.i.d. normally distributed  $N(0, I)$ , and  $\lambda_t'$  is an  $(M \times 1)$  vector that characterizes investors' attitudes towards risk, with  $\lambda_t' = 0$  for risk-neutral investors. Let  $X_t$  be an  $(M \times 1)$  vector of state variables:

$$M_{t,t+1} \phi(X_{t+1}; \mu_t, \Sigma') = \exp(-r_t) \phi(X_{t+1}; \mu_t^Q, \Sigma \Sigma')$$

which confirms that for this specification of the pricing kernel, risk-averse investors value any asset as risk-neutral investors would if the latter thought the conditional mean of  $X_{t+1}$  was

$$\mu_t^Q = \mu_t - \Sigma \lambda_t \quad (\text{A.6})$$

rather than  $\mu_t$ . To give an example, a positive value for the first element of  $\lambda_t'$  indicates that an asset that delivers the quantity  $X_{1,t+1}$  dollars in period  $t+1$  would have a lesser value in period  $t$  for a risk-averse than a risk-neutral investor, with the size of this difference determined by the size of the (1,1) element of  $\Sigma$ . The price of an asset delivering  $X_{i,t+1}$  dollars is reduced by  $\Sigma_{i1} \lambda_{1t}'$  relative to a risk-neutral valuation, through the covariance between factors  $i$  and 1. The term  $\lambda_{1t}'$  might therefore be described as the market price of factor 1 risk. As affine TS models also assume that this market price of risk is itself an affine function of  $X_t$ ,

$$\lambda_t' = \lambda_0' + \lambda_1' X_t \quad (\text{A.7})$$

then substitution of Equations (A.7) and (A.2) into Equation (A.6) yields

$$\mu_t^Q = c^Q + \rho^Q X_t \quad (\text{A.8})$$

for

$$c^Q = c - \Sigma \lambda_0' \quad (\text{A.9})$$

and

$$\rho^Q = \rho - \Sigma \lambda_1'. \quad (\text{A.10})$$

If the risk-free one-period interest rate is also an affine function of the factors:  $r_t = \delta_0 + \delta_1' X_t$ , then as Ang and Piazzesi (2003) show, the price on an  $n$ -period pure-discount bond can be calculated as a function of the state variables.

$$p_t^n = A_n + B_n' X_t \quad (\text{A.11})$$

where  $A_0 = 0$ ,  $B_0 = 0$ ,  $A_1 = \delta_0$  and  $B_1' = \delta_1'$  (from the short rate equation) and

$$b_n = \left( I_M + \rho^{Q'} + \dots + (\rho^{Q'})^{n-1} \right) \delta_1 \quad (\text{A.12})$$

and

$$a_{n+1} = \delta_0 + a_n + b_n' c^Q + \frac{1}{2} (b_n' \Sigma \Sigma' b_n) \quad (\text{A.13})$$

The  $n$ -year forward rate is then a function of the difference in these parameters for each period:  $a_n^f = a_{n-1} - a_n$  and  $b_n^f = b_{n-1} - b_n$

$$f_t^n = p_t^{n-1} - p_t^n = a_n^f + b_n^f X_t \quad (\text{A.14})$$

where

$$b_n^f = -(\rho^{Q'})^{n-1} \delta_1 \quad (\text{A.15})$$

and

$$a_n^f = \delta_0 - b_{n-1}' c^Q - \frac{1}{2} (b_{n-1}' \Sigma \Sigma' b_{n-1}) \quad (\text{A.16})$$

So if we know  $X_t$  and the values of  $c$ ,  $\rho$ ,  $\delta_0$ ,  $\delta_1$ , and  $\Sigma$ , we can use (A.11), (A.12), and (A.13), to predict the yield for any maturity  $n$ . There are, therefore, three sets of parameters in our model, where if one knows any of the two sets, one can calculate the third:

1. the parameters  $c$ ,  $\rho$ , and  $\Sigma$  that characterize the dynamics of the factors in Equation (A.1)
2. the parameters  $\lambda_0'$  and  $\lambda_1'$  that characterize the price of risk
3. the Q parameters  $c^Q$  and  $\rho^Q$

## A.2 Estimation

Our four factors are observed. We follow this multi-step algorithm to estimate the models' parameters:

1. Estimate Equation (A.1) by OLS, regressing each demeaned factor on the lagged values of the other factors:

$$X_{t+1} = c + \rho X_t + \Sigma v_{t+1}$$

which gives the physical representation of the transition matrix for the model's state variables.

2. Use one-month yields to estimate  $\delta_0$  and  $\delta_1$  via OLS.
3. Choose the market prices of risk to match the cross-section of bond expected returns. Our model states that all but the first column of  $\lambda_1$  must equal zero, or for those countries (or cases) where we include the global forecasting factor, the first two columns. We denote the first column  $\lambda_{1x}$ . We want to estimate the market prices of risk so the model reproduces the forecasting regressions that describe bond expected returns. We have 9 expected returns, each a function of a constant and  $x_t$ , which we want to match with two numbers (up to 8 in other specifications):  $\lambda_{01}$  and  $\lambda_{1L}$ . To do so, we will have to choose a portfolio to match, so we choose one weighted by  $q_r$ , as it recovers the return-forecasting factor. The assumption of no arbitrage

implies  $1 = E_t(M_{t+1}R_{t+1}^n)$  (Dybvig and Ross, 1987) where  $R_{t+1}^n$  are holding period returns, which, together with the assumption in Equation (A.5) that the pricing kernel is exponentially affine, also noting that  $R_{t+1}^n = \exp(rx_{t+1}^n + r_t)$  implies that

$$E_t(rx_{t+1}) + \frac{1}{2}var(rx_{t+1}) = cov_t(rx_{t+1}, v_{t+1})\lambda_t$$

We have a regression model for  $E_t(rx_{t+1})$ , the time series of excess returns to estimate the variance term, and the time series of factor innovations  $v_{t+1}$  so we can estimate the covariance term. So we have all the ingredients necessary to determine the market prices of risk. We estimate the market price of risk by setting the regression coefficient of excess returns weighted by  $q_r$  on  $x_t$  to 1, so given that

$$E_t(q'rx_{t+1}) = x_t$$

$$E_t(q'rx_{t+1}) + q'\frac{1}{2}var(rx_{t+1}) = q'cov_t(rx_{t+1}, v_{t+1})(\lambda_{0l} + \lambda_{1l}x_t)$$

from imposing the one-factor restriction for expected returns on the right-hand side and the one-factor model for expected returns on the left-hand side, it follows that from isolating the terms that vary with  $x_t$  that

$$1 = q'cov_t(rx_{t+1}, v_{t+1})\lambda_{1l}$$

so

$$\lambda_{1l} = \frac{1}{q'cov_t(rx_{t+1}, v_{t+1})} \quad (\text{A.17})$$

where  $v_{t+1} = v_{t+1}^l$  for those countries in which level shocks dominate, in which case  $\lambda_{1l}$  will be  $1 \times 1$ . We identify the constant portion of the market price of risk as the value that sets the intercept in the forecasting regression of  $E_t(q'rx_{t+1})$  equal to zero,

$$q'\frac{1}{2}var(rx_{t+1}) = q'cov_t(rx_{t+1}, v_{t+1})\lambda_{0l}$$

and substituting in (A.17) we get an expression for  $\lambda_{0l}$

$$\lambda_{0l} = q'\frac{1}{2}var(rx_{t+1})\lambda_{1l}$$

With  $\lambda_{0l}$  and  $\lambda_{1l}$  estimated, we can now recover risk-neutral dynamics:  $\rho^Q = \rho - \Sigma\lambda_{1l}$  and  $c^Q = 0 - \Sigma\lambda_{0l}$ .

4. Given the set of  $M$  observed forwards rates we then compute the following recursions:

$$f_t^2 = \widehat{A}^f + \widehat{B}^f X_t + \Sigma_e u_t^e \quad (\text{A.18})$$

$$[u_t^e] \sim N([0], [\Sigma_e \Sigma_e']) \quad (\text{A.19})$$

The value for the row  $i$  of  $\widehat{B}$  is:

$$\widehat{B}_i^f = -(\rho^{Q'})^{n-1} \delta_1 \text{ for } n = 1, \dots, M.$$

The value for the row  $i$  of  $\widehat{A}$  is:

$$\widehat{A}_i^f = \delta_0 - b'_{n-1} c^Q - \frac{1}{2} (b'_{n-1} \Sigma \Sigma' b_{n-1}) \text{ for } n = 1, \dots, M. \quad (\text{A.20})$$

I define the term premium as the difference between the observed five-to-ten-year forward rate and the model-predicted one-month interest rate from five to ten years hence under the Q measure. Figures A.1 through A.4 plot this term premium measure for the U.S., U.K., Germany, and Japan from 1998 to the present.

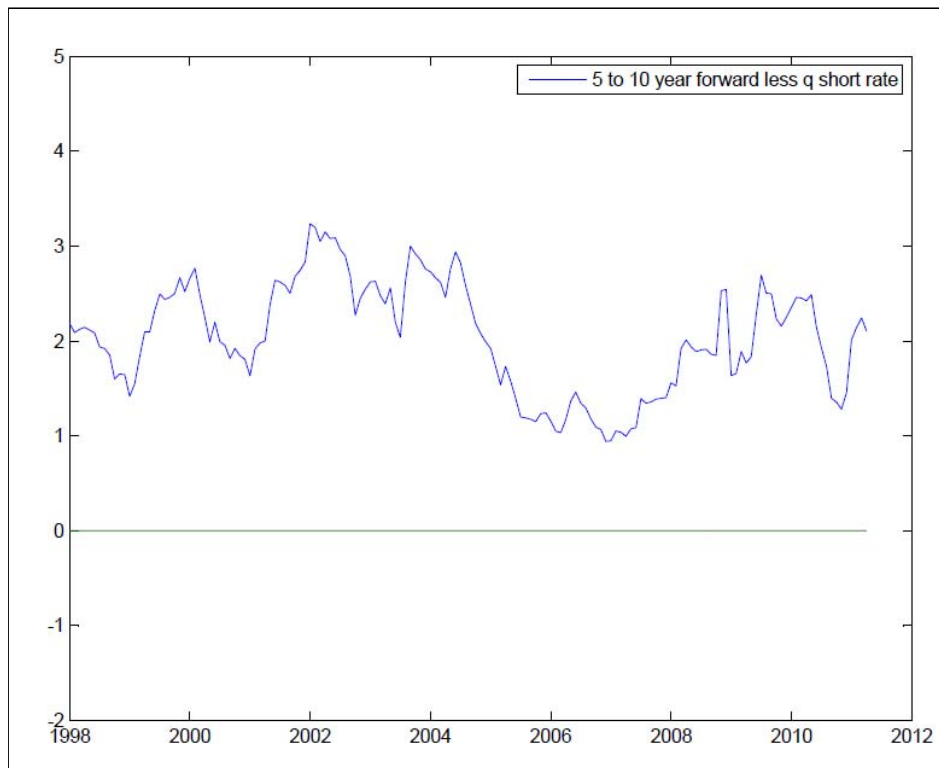


Figure A.1: *Term Premium Estimate for the United States from the Affine Model.* Monthly estimates.  
Source: Author's calculations.



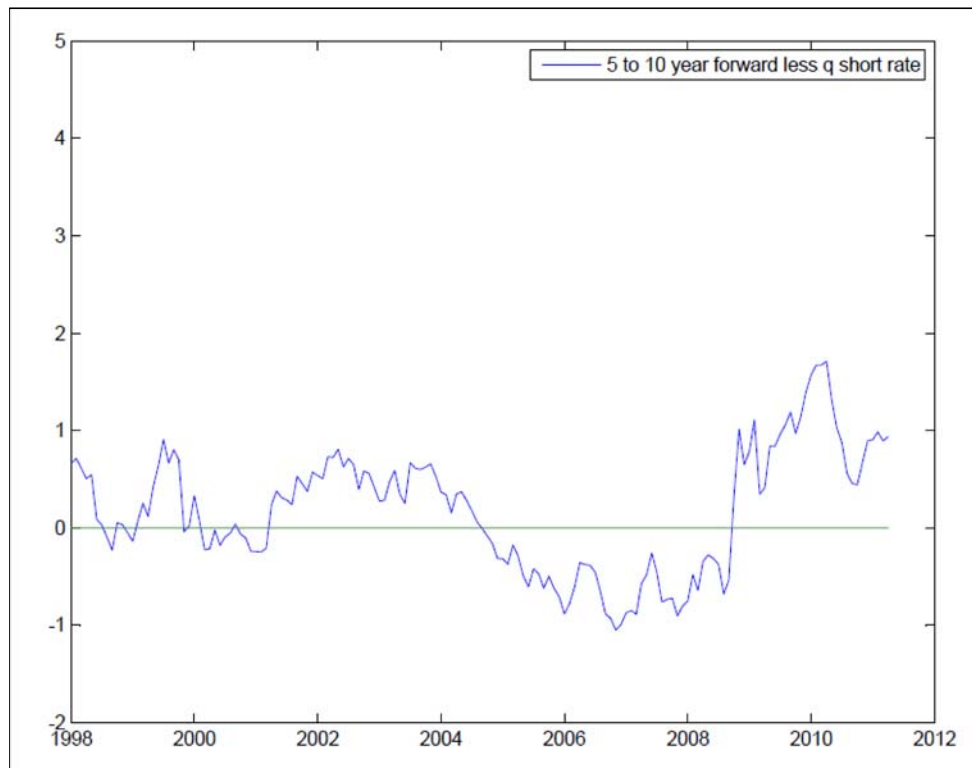


Figure A.2: *Term Premium Estimate for the United Kingdom from the Affine Model.* Monthly estimates. Source: Author's calculations.

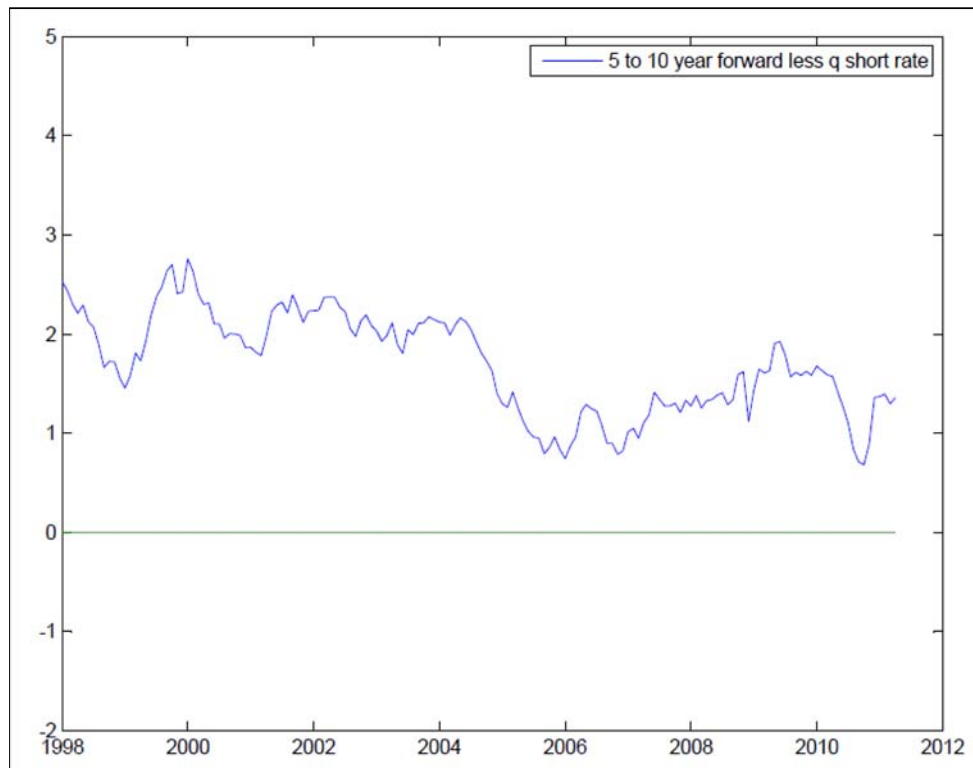


Figure A.3: *Term Premium Estimate for Germany from the Affine Model*. Monthly estimates. Source: Author's calculations.

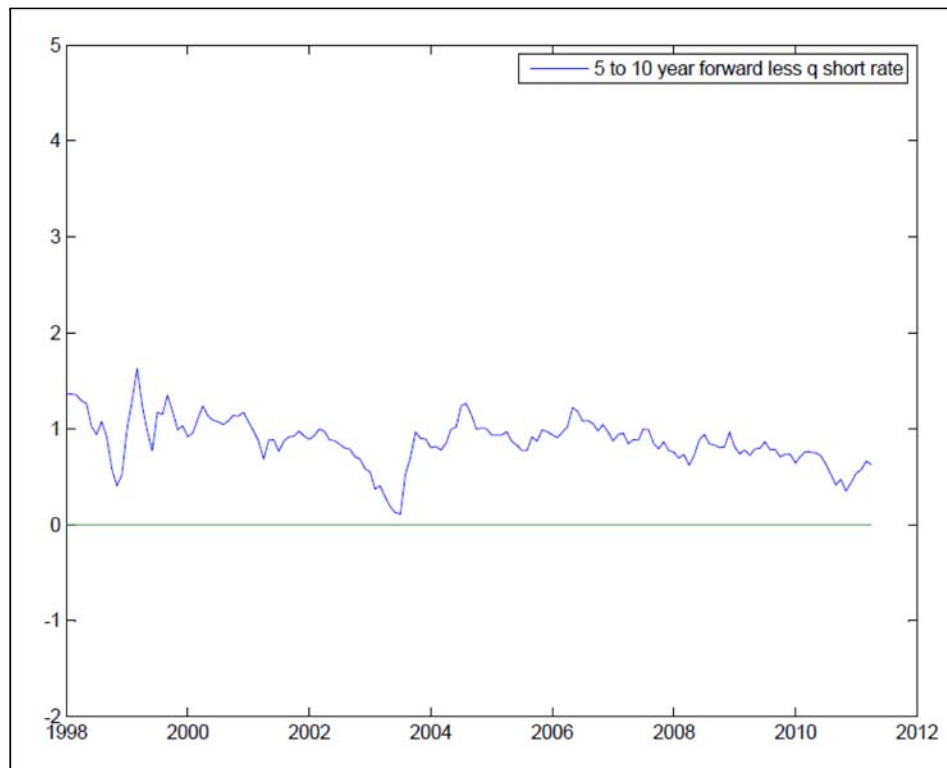


Figure A.4: *Term Premium Estimate for Japan from the Affine Model*. Monthly estimates. Source: Author's calculations.