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Bank Capital Regulation and Structured Finance

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Abstract

We construct a model in which bank capital regulation and financial innovation interact. Innovation takes the form of pooling and tranching of assets and the creation of separate structures with different seniority, different risk, and different capital charges, a process that captures some stylized features of structured finance. Regulation is motivated by the divergence of private and social interests in future profits. Capital regulation lowers bank profits and may induce banks to innovate in order to evade the regulation itself. We show that structured finance can improve welfare in some cases. However, innovation may also be adopted to avoid regulation, even in cases where it decreases welfare.

Key words: bank regulation, financial innovation, structured finance
1 Introduction

The purpose of this paper is to model the interaction between bank capital regulation and financial innovation. A large literature argues that taxes and regulations have provided major impetuses to successful financial innovations (Silber 1983, Miller 1986). Kane (1986) coined the expression “regulatory dialectic” to describe the process of financial innovations arising as responses to the cost of regulatory constraints—for example, bank capital requirements.

More recently, Goodhart (2008) and Brunnermeier et al. (2009) have argued that any effective financial regulation lowers the regulated entities’ profitability and return on capital by preventing them from achieving their preferred, unrestricted position, the so called “boundary problem of financial regulation”. If returns on the regulated sector fall relative to those available on substitutes outside, then there may be a switch toward nonregulated businesses. For example, banks may set up conduits and other financial vehicles outside the regulated sector in order to expand their balance sheets. In good times, funds flow from the regulated to the nonregulated sectors, while during crises the flow will reverse in boom-bust cycles. Brunnermeier et al. (2009) argue that it is surprising, at least in retrospect, that the regulators appeared to be largely unaware of banks’ reliance on a variety of legally separate, but reputationally connected, off-balance-sheet entities set up to increase leverage and circumvent capital regulation. Hanson et al. (2011) argue that a by-product of higher capital requirements to address new concerns of macroprudential regulation will be the pressure it creates for activities to migrate outside the regulated banking sector.

The financial innovation we focus on takes the form of pooling and tranching of assets and the creation of separate structures, special-purpose vehicles, with different seniority, different risk, and different capital charges, a process that captures some stylized features of structured finance. A main reason for financial innovation is indeed to repackage risks in order to manufacture safer securities (Allen and Gale 1994). Similarly, Gennaioli, Shleifer, and Vishny (2010) argue that intermediaries have engineered CDOs to meet investors’ demand for safer cash flows.

Innovation may be welfare improving because repackaging risk to create a safer senior tranche limits banks’ opportunistic behavior, which may increase a bank’s borrowing capacity and profits. In our model, pooling and tranching per se cannot improve welfare. Only when pooling and tranching are combined with the allocation of the different tranches to structures that adopt different risk profiles can welfare increase.

Innovation may be desirable even if it entails costs in terms of future profits. Future profits could be lost if the originating bank sells the project to entities that can only extract a lower value from its continuation, as may happen in the structured finance sector when assets are placed off the bank’s balance sheet. This, in turn, could affect incentives. However, innovation can be adopted even if it does not increase welfare, because it can serve the purpose of avoiding regulation.
In our model, lack of enforceability of contracts allows bankers to abscond and enjoy part of the profits instead of repaying depositors in full (Calomiris and Kahn 1991) and fund new projects. Anticipating this possibility, depositors demand a higher risk premium and constrain deposit size given bank capital.

To consider the interaction between innovation and regulation, there needs to be a role for regulation. In a multi-period setting, capital regulation is motivated by the fact that the bank may fail to internalize future profits, for example, because it may go out of business. We assume that, with some exogenous probability, a bank goes out of business after it has funded new projects. In such a case, the bank cannot reap the benefits of its investment, but society does. This is the only source of divergence of private and social interests in our model. Capital regulation limiting bank size reduces the incentive to abscond, although potentially at the cost of lower profits. Lower profits in the regulated sector, in turn, may induce financial innovation to avoid regulation.

Of course, other sources of externalities, related to financial stability, systemic risk, and protection of uninformed depositors may motivate banking regulation, but are not considered in this paper. We focus on microprudential regulation and, since we assume that a bank’s capital is an endowment, regulation takes the form of a constraint on a bank’s size. However, even if from a microprudential standpoint this is isomorphic to a constraint on the bank’s capital, taking the bank’s size as given, from a macroprudential perspective it matters if the bank complies with the capital ratio by raising capital or by shrinking assets potentially exerting downward pressure on asset prices in a crisis (Hanson et al. 2011).

While our model of the interaction between capital regulation and financial innovation is stylized, we believe that its building blocks are general enough to shed light on the interactions between bank capital regulation and financial innovation.

The rest of the paper is organized as follows. In Section 2 we set up the model. Section 3 describes financial innovation. Section 4 analyzes the welfare properties of different bank sizes and financial innovation. Section 5 introduces regulation to address the divergence of private and social interests, and examines the incentive to innovate in order to evade regulation. Section 6 concludes and discusses some extensions. The proofs are in the appendix.

2 The model

2.1 Preferences

The economy has three dates, 0, 1, and 2, and is populated by \(N\) banks, a large number of depositors, and a regulator. All agents are risk-neutral and do not discount the future. There is a unique good at each date. Each bank is endowed with \(K\) units of the good at date 0, which we associate with the bank’s capital.
Banks can invest in projects that we describe below. In addition to investing their own endowment, banks can invest on behalf of depositors. Depositors care only about consumption at date 1. They can invest their endowment in storage or in deposits at banks. Storage returns 1 unit at date 1 for each unit invested at date 0. We assume that the supply of deposits to banks is larger than the demand for deposits from banks, so that banks offer depositors an expected return of 1.

2.2 Technology

We focus on a representative bank. At date 0, the bank collects deposits \( D \geq 0 \) to be repaid at date 1, which it invests, together with its capital \( K \), to finance a project of size \( S = K + D \). The bank can manage only one project, perhaps because of limited managerial skills, but can use its capital to originate more than one project, bundle the projects, and resell them. At date 1, the project may be either productive, with probability \( \lambda \), or unproductive. For each unit invested at date 0, a productive project returns \( y_H \), in case of success which happens with probability \( p \), and \( y_L > 0 \) with \( \Delta y = y_H - y_L > 0 \), in case of failure which happens with probability \( 1 - p \) at date 1. We assume \( py_H + (1 - p)y_L - 1 > 0 \). An unproductive project returns 0 at date 1 and at date 2.

Depositors have no possibility to enforce loan repayments because they cannot observe cash flows. Hence, at date 1 the bank repays depositors only if it is profitable to do so. Formally, the bank can abscond with the proceeds from the project at date 1. If the bank absconds, it is able to consume \( S\omega y_i \), with \( 0 < \alpha < 1 \), where \( S\omega y_i \) is the realized cash flow of the project, \( i \in \{ L, H \} \) (Calomiris and Kahn 1991). Depositors obtain nothing and the bank cannot make further investment.

As in Calomiris and Kahn (1991), absconding can be interpreted more generally as a costly ex post fraudulent action that the bank undertakes if it is more profitable than making the promised repayments to depositors. For example, such an action could consist of paying an abnormally high fraction of profits as bonuses to top executives, possibly even threatening the bank’s survival. If the bank does not abscond, it repays its depositors the agreed upon gross interest \( R \). Observe that the temptation to abscond is greater with the lower realization of the cash flows \( S\omega y_i \), which is consistent with the prevalence of fraud in times of crisis (Calomiris and Kahn 1991).

If the project is productive and the bank does not abscond, it can choose to continue the project at date 1 by making an additional investment of size \( SI \), where \( I \) is the continuation investment per unit of investment at date 0. We assume that \( SI < S\omega y_L - R \) so that the continuation investment is funded with internal funds. The continuing project yields a nonstochastic net return at date 2 that we define as \( SV > 0 \). Observe that \( S[(1 - \alpha)y_i + V] \) is the social cost of absconding.

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1 Throughout the paper, we use the terms assets and projects interchangeably to describe the output from a bank’s investment.
Only the bank that made the investment at date 0 can make the additional investment at date 1. The idea is that the initial financier acquires the skills to put the assets to their best use and everyone else can generate only a fraction of this value, as in Diamond and Rajan (2000). This assumption implies that if the bank sells the investment, as in the case of structured finance, the possibility to make the date 1 investment is lost. In other words, structured finance involves arm’s length transactions, as observed by Rajan (2006).

Between dates 1 and 2, the bank goes out of business with an exogenous probability $\mu$. The role of this assumption is to create a wedge between the bank’s and society’s objective function. Society reaps the benefit of the productive project at date 2, $SV$, whether or not a bank goes out of business. In contrast, the bank obtains that benefit only if it is still around at date 2. If $m = 1$, then the bank and society’s objective functions are the same. If $m < 1$, then the bank values future profits less than society does, so the incentives of the bank and society are not aligned. The bank does not abscond at date 2, because no outside resources are invested at date 1. The sequence of events is represented in Figure 1.

### 2.3 Optimal bank size

A bank chooses its size to maximize profits. The constraint on bank size is that depositors understand that a larger bank has a higher incentive to abscond. In equilibrium, the probability that the bank repays depositors is common knowledge. We denote this probability $\bar{p}$. The bank’s total gross interest cost is $R = D/\bar{p}$, where $D = S - K$ is the amount borrowed at the gross interest rate of $1/\bar{p}$ per unit of deposits so the expected return per unit of deposits is 1.

When the investment is unproductive, the bank’s cash flows are zero, the bank fails, and depositors receive nothing. If the investment is productive, the bank must decide whether to abscond. The possibility of absconding when the project cash flow is $Sy_H$ imposes a ceiling on the size of the bank. Indeed, if a bank would abscond even when the project’s cash flow is $Sy_H$, it could not raise any funding. If the bank is sufficiently small, it does not abscond even if the project cash flow is $Sy_L$. Hence, a bank can either be high risk, if it absconds when the cash flow is $Sy_L$ and fails when the project is unproductive, or low risk, if it never absconds but fails when the project is unproductive.

The low-risk bank faces a gross interest rate of $1/\lambda$ per unit. The incentive constraint guaranteeing that the low-risk bank will not abscond can be written as

$$S[y_L + mV] - R = S[y_L + mV] - \frac{S - K}{\lambda} \geq S\gamma y_L.$$  

(1)

Rearranging, we can derive $S_{tr}$, the maximum size that a low-risk bank can achieve, which is given by

$$S_{tr} = \frac{K}{1 - \lambda[(1 - \alpha)y_L + mV]}.$$  

(2)
The profit function of the low-risk bank is

\[ \pi_{lr} = \lambda [S_{lr}(py_H + (1-p)y_L + mV) - R] = S_{lr} [\lambda(py_H + (1-p)y_L + mV) - 1] + K. \]  

Hence, the bank will want to achieve the maximum size if

\[ \lambda(py_H + (1-p)y_L + mV) - 1 > 0. \]

Observe that the constraint (1) is conceptually similar to a margin requirement on the bank holdings (Krishnamurthy 2009) or to a limit on the bank’s investment based on the pledged collateral (Kyotaki and Moore 1997).

The high-risk bank repays depositors only when the investment returns \( S_{yH} \), which happens with probability \( p\lambda \) and pays a gross interest rate of \( \frac{1}{p\lambda} \) per unit. The no-absconding incentive constraint it faces is

\[ S [y_H + mV] - R = S [y_H + mV] - \frac{S}{p\lambda} \geq \alpha S_{yH}. \]

The maximum size consistent with this constraint, which we denote \( S_{hr} \), is

\[ S_{hr} = \frac{K}{1 - p\lambda [(1-\alpha)y_H + mV]}. \]

Thus the profit function for a high-risk bank is

\[ \pi_{hr} = p\lambda [S_{hr}(y_H + mV) - R] + \lambda (1-p)\alpha S_{hr}y_L = S_{hr} [\lambda p(y_H + mV) + \lambda (1-p)\alpha y_L - 1] + K. \]

Hence, the bank will want to achieve the maximum size if

\[ \lambda p(y_H + mV) + \lambda (1-p)\alpha y_L - 1 > 0. \]

It is easy to verify that the profit per unit of size is higher for a low-risk bank than for a high-risk bank. Indeed, high-risk banks pay a higher interest rate on their liabilities because they are less likely to repay them. Hence, for the problem to be interesting, it must be the case that \( S_{lr} < S_{hr} \). Notice that

\[ S_{lr} < S_{hr} \Leftrightarrow (1-p) mV < (1-\alpha) (py_H - y_L). \]

For these parameter values, a bank faces a trade-off between a large size, but a small profit per unit of size, or a low size, but a high profit per unit of size. A further implication of this model is that larger, and more leveraged, banks are riskier and must compensate their liability holders by paying a higher return.

### 2.4 Bank profits

We calculate the profits for a bank at each threshold level. We do this under the assumption that the bank chooses the maximum size possible for each risk
profile. Observe that the conditions for which banks choose the largest possible size are nested: namely \( \text{LHS of (4)} \geq \text{LHS of (8)} \). If one of the conditions is not satisfied, then the bank will not try to reach the largest possible size for a given risk profile. In this paper, we focus on parameters consistent with banks wanting to reach the maximum possible size. Hence, we assume that (8) holds.

For each set of thresholds, we then plug in the maximum size in the profit functions (3) and (7) and obtain the maximum level of profits for each risk profile. With (2) and \( R = \frac{S_{tr} - K}{\lambda} \) the profits for a low-risk bank are

\[
\pi_{tr} = \lambda[S_{tr}(p y_H + (1 - p) y_L + m V) - R] = \lambda K \frac{p \Delta y + \alpha y_L}{1 - \lambda[y_L(1 - \alpha) + m V]}.
\]

With (6) and \( R = \frac{S_{hor} - K}{\lambda} \) the profits of a high-risk bank are

\[
\pi_{hr} = p \lambda[S_{hor}(y_H + m V) - R] + \lambda(1 - p)\alpha S_{hor} y_L = \lambda \alpha K \frac{p \Delta y + y_L}{1 - p \lambda[y_H(1 - \alpha) + m V]}.
\]

3 Financial innovation: Structured finance

We now apply the above setup to investigate a particular type of financial innovation: the pooling and tranching of assets with different seniority and risk, along with the creation of separate structures to house them. Indeed, one of the main motives for financial innovation is to repackage risk in order to manufacture safer securities (Allen and Gale 1994, Shleifer and Vishny 2010). We model this idea following Coval et al. (2009). In the sequel, we also consider the pooling of assets without tranching.

Recall that we assumed that structured finance entails a loss of future profits; hence, there is no divergence between social and private interests in the structured finance sector and no need for regulation. The following matrix summarizes the relative importance of future profits for society and for banks, depending on the financial structure:

<table>
<thead>
<tr>
<th>Value of future profits</th>
<th>Traditional banking</th>
<th>Structured finance</th>
</tr>
</thead>
<tbody>
<tr>
<td>to society</td>
<td>( m_0 V )</td>
<td>( m_2 V )</td>
</tr>
<tr>
<td>to banks</td>
<td>( m_1 V )</td>
<td>( m_2 V )</td>
</tr>
</tbody>
</table>

In this paper, we set \( m_2 = 0 \), since with structured finance the possibility to make date 1 investment is lost. We set \( 0 \leq m_1 < 1 \) because a bank may go out of business between dates 1 and 2 and would not reap the benefits of the date 1 investment in this case. Finally, we set \( m_0 = 1 \) because society enjoys the benefits of date 1 investment even if the bank goes out of business.

3.1 Pooling and tranching

The process of pooling and tranching has two steps. First, the originating bank invests in two assets, pools them in a portfolio, and creates two tranches with
different seniority. Second, the bank sells the two tranches to two “special-purpose vehicles” (SPVs) off its balance sheet.

Consider a bank that pools two projects. We assume that the productivity of the assets is akin to an aggregate shock, so either both assets are productive or neither is. Conditional on the assets being productive, we assume that the returns of the assets are uncorrelated. As before, each asset’s returns per unit invested are \( y_H, y_L \), with probability \( p \) and \( 1 - p \), respectively, and they can be pooled in a portfolio. The portfolio’s cash flows are represented as follows:

<table>
<thead>
<tr>
<th>Asset 1</th>
<th>Asset 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success</td>
<td>Success</td>
</tr>
<tr>
<td>Failure</td>
<td>Failure</td>
</tr>
<tr>
<td>( y_H )</td>
<td>( y_L )</td>
</tr>
<tr>
<td>( y_H ), ( y_L ) case 1</td>
<td>( y_H ), ( y_L ) case 2</td>
</tr>
<tr>
<td>( y_H ), ( y_L ) case 3</td>
<td>( y_L ), ( y_L ) case 4</td>
</tr>
</tbody>
</table>

In what follows, we assume that there are no agency problems between the bank and the SPVs. This is, of course, an extreme assumption because each stage of the structured finance process may offer scope for opportunistic behavior. This assumption, which provides the most favorable environment for structured finance to have benefits, allows the bank to operate with zero capital when it originates projects and places them off its balance sheet. This assumption could be justified if the SPVs belong to the same bank and, more broadly, in a context in which the bank does not perform any ex ante screening or ex post monitoring that may be hampered by risk transfer.

To develop the latter point, we could assume that there are two groups of providers of outside funds to banks: financiers, who are in charge of SPVs, and households. Households provide deposits to either banks or financiers. Financiers have the skills to enforce contracts and prevent absconding by banks, but households don’t because they cannot observe cash flows. Therefore, the funds provided by households are repaid only if the banks or SPVs find it convenient. This corresponds to the reduced form of a model in which both financiers and households face enforcement costs, but the cost that financiers incur to make bankers pay is smaller than that incurred by households. We normalize this difference in such a way that the enforcement costs of financiers are zero and the enforcement costs of the households are prohibitively large such that households cannot enforce contracts.²

Consider two tranches against the portfolio’s cash flows: The senior tranche pays \( y_H \) per unit invested in cases 1,2,3, and pays \( y_L \) in case 4 only. The junior tranche pays \( y_L \) per unit invested in cases 2,3,4 and pays \( y_H \) in case 1 only. Under the maintained assumption that the returns of the underlying assets are uncorrelated, the probability that the senior tranche pays the smallest amount, \( y_L \), is \( (1 - p)(1 - p) < (1 - p) \), and the probability that the junior tranche pays \( y_L \) is \( 1 - pp > 1 - p \). Thus the probabilities of the cash flows of the synthetic securities have been modified even if, of course, the overall probability of paying \( y_L \) remains \( 1 - p + 1 - p \).

²Alternatively, we could assume that there are economies of scale in monitoring costs. These economies of scale are exploited by financiers who spread the fixed cost, but cannot be exploited by each zero-measure depositor who therefore faces high enforcement costs.
This setup allows us to investigate the potential benefit of repackaging risk by creating safer and riskier assets and, also, of placing them in different entities with different capital charges. This process captures some stylized features of structured finance. In particular, we assume that, after manufacturing the two tranches, the originating bank creates two separate SPVs and sells to the SPVs the senior and junior tranches. We can also think of the originating bank as having retained one of the tranches. For exposition purposes, however, it is convenient to think of each tranche being assigned to a separate SPV. One SPV houses the senior tranche and will receive $y_H$ per unit invested with probability $p^G = 1 - (1 - p)^2$; the other houses the junior tranche and will receive $y_H$ per unit invested with probability $p^B = p^2$. Note that the sizes of each tranche cannot be chosen independently since for each junior tranche there must be one senior tranche of equal size. Therefore the senior tranche raises deposits $D^G = S/2 - K^G$, and the junior raises deposits $D^B = S/2 - K^B$, where $K^G$ and $K^B$ are the capital of the senior and junior tranches, respectively with $K^G + K^B = K$ the bank endowment. The two SPVs make the absconding decision independently, each facing its own incentive constraint.

Although the results that we will present hold for generic values of $0 \leq m \leq 1$, as mentioned above it is convenient to think of the structured finance sector as one in which the value of $m = 0$. The balance sheets of the originating bank and of the SPVs are represented in Figure 2. The following lemma shows that pooling and tranching per se does not increase profits if both SPVs choose the same risk profile.

**Lemma 1** The profits of a whole bank are at least as great as the sum of profits of the junior and senior SPVs if they choose the same risk profile as the whole bank.

**Proof.** See the appendix.

Since in expectation the sum of the cash flows of the two tranches has to be equal to the cash flows of the original assets, the expected payoff of the whole bank is identical to the expected payoff of the sum of the tranches, when they all choose the same risk profile. Thus, pooling and tranching alone cannot increase welfare because it does not affect the asset side of the SPVs. Since the value of $m$ for the bank and for society in the structured finance sector is no greater than in the traditional banking sector, the sum of the profits of the two tranches could be smaller than the profit of the whole bank if the two tranches choose the same risk profile.

### 3.2 SPVs with different risk

Now we turn to the case where the junior and senior SPVs choose different risk profiles so that they may abscond in different states of nature. We assume that the junior SPV chooses to be low risk and the senior SPV chooses to be high risk, in the sense defined above.
Let $S_{lr}$ ($K^B, p = p^B$) and $S_{hr}$ ($K^G, p = p^G$) denote the sizes of the low- and high-risk tranches, where the probability of the return being $y_H$ is $p^B, p^G$, respectively. Since the sizes of both SPVs must be equal, then

$$S_{lr} (K^B, p = p^B) = \frac{K^B}{1 - \lambda [(1 - \alpha) y_L + mV]} = (12)$$

$$S_{hr} (K^G, p = p^G) = \frac{K^G}{1 - \lambda [(1 - \alpha) y_H + mV]} = (13)$$

Observe that

$$p^G \lambda [(1 - \alpha) y_H + mV] > \lambda [(1 - \alpha) y_L + mV] \Leftrightarrow K^B > K^G, \quad (14)$$

Hence, there are parameters for which $S_{lr}$ ($K^B, p = p^B$) = $S_{hr}$ ($K^G, p = p^G$) and $K^B > K^G$. This implies that, under these parameters, the junior SPV is less leveraged, in the sense of size divided by capital, than the senior SPV. Notice that the probability of depositors being repaid is higher in the junior SPV than in the senior SPV, since $\lambda > p^G \lambda$. The appendix presents the profits of the junior and senior SPVs, as well as their sum.

### 4 The bright side of structured finance

In this section, we show that structured finance can improve social welfare in some cases. We abstract from the potential divergence of private and public interests and focus on the type of financial structure a social planner would choose. Specifically, we ask whether the planner would choose structured finance or traditional banks.

**Proposition 1**

1) There exists a function $\alpha(p)$, with $\frac{d\alpha(p)}{dp} < 0$, that defines the locus of points in the space $0 < \alpha < 1; 0 < p < \min \left\{ \frac{1}{\lambda [(1 - \alpha) y_L + mV]}, 1 \right\}$ such that for any $0 \leq m \leq 1$ welfare from the low-risk whole bank and the high-risk whole bank is the same. For higher (lower) $\alpha$, the high- (low-) risk whole bank yields higher welfare.

2) Along the frontier $\alpha(p)$ if

$$p^B [(1 - \alpha) y_H + mV] = (1 - \alpha) y_L + mV \quad (15)$$

then structured finance yields higher welfare than either high- or low-risk whole banks. By continuity, structured finance yields higher welfare than the whole bank even if the “m” in the structured finance sector is smaller than that in the whole bank; therefore welfare may be higher under pooling and tranching despite the loss of future profits that structured finance may entail.

Proof. See the appendix.

For an illustration of proposition 1, see Figure 3. Several comments are in order. First, high- or low-risk banks may be preferred by society, depending on
parameters. For low $\alpha$—that is, when the social cost of absconding is high—the low-risk bank is preferred. For high $p$—that is, when the probability that the productive investment returns $y_H$ is high—the high-risk bank is preferred.

Second, structured finance, in the form of pooling and tranching of assets combined with the allocation of the tranches to structures with different risk profiles, has economic value because it increases the probability that $y_H$ happens in the senior tranche and decreases the probability that $y_H$ happens in the junior tranche. The increase of the probability of $y_H$ in the senior tranche lowers the incentive to abscond and, hence, allows the senior tranche to achieve a higher size per unit of capital—that is, to borrow more per unit of capital. However, decreasing the probability of $y_H$ has no effect on incentives for the junior tranche and, hence, on borrowing per unit of capital for that tranche. Indeed, because the junior tranche is low risk, it repays when the project is productive, which does not depend on the probability of $y_H$. Thus pooling and tranching adds value by allowing the SPVs to redeploy capital where there is more need to satisfy incentives—that is, to move capital from the senior to the junior tranche. This enhances the flexibility of the liability side of the SPVs.

Third, the need to combine structured finance with the allocation of the tranches to structures with different risk profiles can be seen from lemma 1, since this lemma showed that pooling and tranching alone could not increase welfare.

Fourth, the value added by pooling and tranching can be traded off against the loss of future profits.

Fifth, absent a screening or monitoring role for the bank, the model is neutral to the possibility that the bank should retain either the junior or the senior tranche and place the other off its balance sheet, or place both tranches off its balance sheet.

Sixth, applying an additional layer of pooling and tranching to the cash flows of the senior tranche allows for the creation of an asset with an even higher probability of $y_H$, as in the process of manufacturing CDOs (Coval et al. 2009).

Finally, proposition 1 is established for the case where the risks of the underlying assets are uncorrelated. Of course, if the risks of the underlying assets are positively correlated, the benefit of structured finance is reduced.

### 4.1 Pooling

To conclude this section, we show that pooling assets alone cannot achieve higher welfare. The return of the pooled assets is given in the following:

<table>
<thead>
<tr>
<th>Return</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>$y_H$</td>
</tr>
<tr>
<td>Medium</td>
<td>$\frac{y_H + y_L}{2}$</td>
</tr>
<tr>
<td>Low</td>
<td>$y_L$</td>
</tr>
<tr>
<td>Zero</td>
<td>0</td>
</tr>
</tbody>
</table>
In the case of pooling, the bank can choose three different sizes, depending on whether it never absconds (low risk), absconds only when the return of the pooled asset is $y_L$ (medium risk), or absconds when the return of the assets is either $y_L$ or $\frac{y_L + y_H}{2}$ (high risk). The appendix provides the expression for the bank’s size and profits in each case.

**Lemma 2** Pooling cannot achieve higher welfare than either traditional banking or pooling and tranching.

**Proof.** See the appendix.

As in the case of lemma 1, the lemma above shows that changing the payoffs of assets is not sufficient to improve welfare in our model. The key economic reason structured finance improves welfare in our model is that it allows changes to the liability side of the bank’s balance sheet.

## 5 The dark side of structured finance

Notice that the risk-neutral depositors adjust the interest rate they require from the bank to its size and, thus, its risk so that their risk-adjusted return is always the same. Therefore, the social cost of the opportunistic behavior of the bank is borne by the bank itself. From this standpoint, there is no divergence from private and society’s interests. In this model, as noted, divergence between the two arises because the regulator recognizes that society benefits from the date 2 profits of the bank, even if the bank is around only with probability $m < 1$. Apart from their weight on date 2 profits, the regulator and the bank share the same objective function. This allows us to compare the welfare of low- and high-risk traditional banks, as well as SPVs with pooled or pooled-and-tranched assets. See also Figure 4.

**Proposition 2** If the regulator gives more weight to the future profits than the bank does, and if $p$ is sufficiently small, then a bank prefers the larger size (high risk) but the planner prefers the smaller size (low risk).

**Proof.** See the appendix.

The above proposition establishes a rationale for limiting bank size, since private and public interests may diverge. In our model, because we assume that a bank’s capital is an endowment, regulation takes the form of a constraint on a bank’s size. However, this is isomorphic to a constraint on the bank’s capital, if we take the bank’s size as given. Our assumption to take the bank’s capital, rather than its size, as given is due to ease of exposition. Hence, we will interpret regulation in our model as a constraint on the bank’s capital. In this model, regulation is needed to give the bank the incentive to take a long-term view, which model-wise means to give the bank incentives not to abscond.
Maximum-size regulation, however, lowers bank’s profits, which may induce it to venture outside the regulated sector and into structured finance. We establish the following proposition. See also Figure 4.

**Proposition 3** There exist parameters such that 1) the planner prefers the bank to be low risk, but the bank prefers structured finance, and 2) structured finance lowers welfare with respect to the case where size regulation is absent.

Proof. See the appendix.

Several comments are in order. First, proposition 3 shows that structured finance can be used to evade capital regulation and that an unintended consequence of capital regulation may be welfare-reducing structured finance. Second, however, the widespread belief that structured finance was motivated by regulatory arbitrage, whereby banks used off-balance-sheet SPVs to increase leverage and circumvent capital regulation, must be tempered with the result of our model showing that part of the motivation behind structured finance is an efficient use of capital.

Proposition 3 shows that a well-meaning but "naive" regulator could worsen welfare if it does not realize that regulation meant to align public and private incentives may result in a welfare-reducing evasion of the regulation. This suggests that a more sophisticated regulator could choose one of two approaches. It may decide not to regulate the bank, despite the divergence between private and public interests in that sector. Alternatively, it may decide to regulate the structured finance sector despite the absence of divergence between private and public interest in that sector.

We conclude this section by showing that if \( p \) is not too large, pooling and tranching is preferred to pooling only.

**Lemma 3** If \( p < 2/3 \), a bank that must reduce its size because of regulation finds pooling and tranching more attractive than only pooling assets.

Proof. See the appendix.

For values of \( p \) that are not too large, which are the main focus of this paper, pooling alone is not an attractive option, even when trying to evade regulation. This reinforces the message of this paper, which is that changes to the liability side of financial intermediary are key.

### 6 Extensions and Conclusion

In this paper, we have constructed a model in which bank capital regulation and financial innovation can interact. Regulation is motivated by the divergence of private and social interests in future profits. Innovation takes the form of pooling and tranching of assets and the creation of separate structures with different seniority, different risk, and different capital charges, a process that
captures some stylized features of structured finance. Capital regulation lowers bank profits and may induce banks to innovate in order to evade the regulation itself.

On the one hand, we have established that financial innovation can improve welfare. We have also shown that changes to the liability side of the financial intermediary are important for this result. Changes that affect only the asset side cannot achieve higher welfare.

On the other hand, we have shown that financial innovation may be adopted for the purpose of evading regulation. This can happen even if financial innovation decreases welfare. In such cases, innovation increases the bank’s profits and is individually rational. However, it induces a loss of future profits that reduces social welfare.

In our analysis, we assumed that the regulator adopts a “naive” view, in the sense that it does not anticipate the bank’s reaction. The regulator may fail to anticipate the bank’s reaction because it leads to financial innovation that could not have been foreseen at the time size regulation was set in place.

However, we could also assume that the regulator anticipates the bank’s reaction. Here, we briefly discuss how regulation could change. If financial innovation in the form of pooling and tranching can be foreseen, then a “sophisticated” regulator may find it optimal to prohibit welfare-reducing structured finance. That is, the perimeter or regulation may have to be extended beyond the point at which private and social interests conflict. The reason for prohibiting structured finance, an activity where, in our model, there is no direct conflict between private and social interests, is because it may be an attractive outside option if returns in the regulated sector fall relative to those available with structured finance. Alternatively, the regulator may choose not to impose regulation, despite the divergence between private and public interests. The question of which form would lead to optimal “sophisticated” regulation is left for future research.

We have abstracted from stability considerations that may come into play because of the increased leverage that SPVs allow. In a crisis, the increased leveraged may threaten the stability of the originating banks if, for reputation reasons, they absorb the losses of the SPVs even in the absence of contractual obligations.

7 References


8 Appendix

8.1 Proof of lemma 1

We show that when the two tranches abscond in the same state of nature pooling and tranching does not increase profits. First, consider the case where the junior and senior SPVs abscond in the same state of the high-risk bank with sizes:

\[ S_{hr}(K^B, p = p^B) = \frac{K^B}{\lambda(1 - \alpha)^2} \]  
\[ S_{hr}(K^G, p = p^G) = \frac{K^G}{\lambda(1 - \alpha)^2} \]  

The junior SPV and senior SPV profits are

\[ \pi_{hr} \text{(junior)} = K^B \lambda \alpha \frac{p^B \Delta y + y_L}{\lambda(1 - \alpha)^2} \]  
\[ \pi_{hr} \text{(senior)} = K^G \lambda \alpha \frac{p^G \Delta y + y_L}{\lambda(1 - \alpha)^2} \]

Observing that

\[ S_{hr}(K^B, p = p^B) = S_{hr}(K^G, p = p^G) \]

the sum of the profits of junior and senior is

\[ \lambda K^B \frac{p^2 \alpha \Delta y + \alpha y_L}{\lambda(1 - \alpha)^2 y_L + \lambda(1 - \alpha)^2 m} + \lambda K^G \frac{p(2 - p) \alpha \Delta y + \alpha y_L}{\lambda(1 - \alpha)^2 y_L + \lambda(1 - \alpha)^2 m} = \]

\[ K \lambda \frac{p \alpha \Delta y + \alpha y_L}{\lambda(1 - \alpha)^2 y_L + \lambda(1 - \alpha)^2 m} \]

which is (smaller or) equal to profit for the high-risk whole bank if \( m \) of the structured institution is (smaller or) equal to that of the high-risk whole bank, given by (11).

Next, consider the case where the junior and senior SPVs abscond in the same state of the low-risk bank with sizes:

\[ S_{lr}(K^B, p = p^B) = \frac{K^B}{\lambda(1 - \alpha)^2} \]  
\[ S_{lr}(K^G, p = p^G) = \frac{K^G}{\lambda(1 - \alpha)^2} \]  

Since neither \( p^G \) nor \( p^B \) appear in the denominator, \( K^G = K^B = K/2 \). Next step is to compare

\[ \pi_{lr} = \lambda K \frac{p \alpha \Delta y + \alpha y_L}{\lambda(1 - \alpha)^2 y_L + \lambda(1 - \alpha)^2 m} = \]

\[ \lambda (K^G + K^B) \frac{p \alpha \Delta y + \alpha y_L}{\lambda(1 - \alpha)^2 y_L + \lambda(1 - \alpha)^2 m} \]
and the sum of junior SPV and senior SPV profits:

$$\pi_{tr} \text{ (junior)} = K^B \lambda \frac{p^B \Delta y + \alpha y_L}{1 - \lambda [y_L (1 - \alpha) + mV]}.$$ 

(24)

and

$$\pi_{tr} \text{ (senior)} = K^G \lambda \frac{p^G \Delta y + \alpha y_L}{1 - \lambda [y_L (1 - \alpha) + mV]}.$$ 

(25)

Summing the profits of junior and senior we have

$$\lambda \frac{K^B (p^2 \Delta y + \alpha y_L) + K^G ((2p - p^2) \Delta y + \alpha y_L)}{1 - \lambda [y_L (1 - \alpha) + mV]} = \lambda K (p \Delta y + \alpha y_L) \frac{1}{1 - \lambda [y_L (1 - \alpha) + mV]}$$

(26)

which is (smaller or) equal to the profit for the low-risk whole bank if $m$ of the structured institution is (smaller or) equal to that of the low-risk whole bank which is given by (10). Therefore, when the two tranches abscond in the same states of nature, profits with pooling and tranching can never be strictly better than without pooling and tranching. End of proof.

8.2 Profits of junior and senior banks/SPVs

Junior and senior SPVs abscond in different states: junior is low risk and senior is high risk. Notice that from (12) and (13) we have

$$K^B \{1 - p^G \lambda [y_H (1 - \alpha) + mV]\} =$$

$$= (K - K^B) \{1 - \lambda [y_L (1 - \alpha) + mV]\},$$

from which

$$K^B \{2 - \lambda (1 - \alpha) [y_H (2p - p^2) + y_L] - \lambda mV (2p - p^2 + 1)\}$$

$$= K \{1 - \lambda [y_L (1 - \alpha) + mV]\},$$

(28)

$$K^B = \frac{1 - \lambda [y_L (1 - \alpha) + mV]}{2 - \lambda (1 - \alpha) [y_H (2p - p^2) + y_L] - \lambda mV (2p - p^2 + 1)}.$$ 

(29)

The profits of the junior and senior SPVs are given by (24) and (19). Using (29) the sum of the profits of junior and senior SPVs yield

$$\pi_{P&T} = \pi_{tr} \text{ (junior)} + \pi_{tr} \text{ (senior)} =$$

$$K^B \frac{\lambda p^2 \Delta y + \lambda \alpha y_L}{1 - \lambda [y_L (1 - \alpha) + mV]} + K^G \frac{\lambda p(2 - p) \alpha \Delta y + \lambda \alpha y_L}{1 - \lambda [y_H (1 - \alpha) + mV]} =$$

$$K^B \frac{\lambda p^2 \Delta y + 2 \alpha y_L + 2 \alpha p \Delta y - p^2 \alpha \Delta y}{1 - \lambda [y_L (1 - \alpha) + mV]} =$$

$$K^B \frac{(1 - \alpha)p^2 \Delta y + 2 \alpha y_L + 2 \alpha p \Delta y}{1 - \lambda [y_L (1 - \alpha) + mV]} =$$

$$K \lambda \frac{(1 - \alpha)p^2 \Delta y + 2 \alpha (y_L + p \Delta y)}{2 - \lambda (1 - \alpha) [y_H (2p - p^2) + y_L] - \lambda mV (2p - p^2 + 1)} =$$
8.3 Proof of Proposition 1

To prove part 1. Observe that

\[
\pi(0 \leq m \leq 1; S = S_{tr}) > \pi(0 \leq m \leq 1; S = S_{hr}) \iff \lambda \frac{p \Delta y + \alpha y_L}{1 - \lambda [y_L (1 - \alpha) + mV]} > \lambda \alpha \frac{p \Delta y + y_L}{1 - \lambda [y_H (1 - \alpha) + mV]}
\]

which is satisfied if \( \alpha \to 0 \) as the LHS \( \to 0 \) and the RHS \( \to 0 \) \( \forall m \). Observe also that

\[
\pi(0 \leq m \leq 1; S = S_{tr}) < \pi(0 \leq m \leq 1; S = S_{hr}) \iff \lambda \frac{p \Delta y + \alpha y_L}{1 - \lambda [y_L (1 - \alpha) + mV]} < \lambda \alpha \frac{p \Delta y + y_L}{1 - \lambda [y_H (1 - \alpha) + mV]}
\]

which is satisfied if

\[
1 > p\lambda [y_H (1 - \alpha) + mV] > \lambda [y_L (1 - \alpha) + mV]
\]

and \( p\lambda [y_H (1 - \alpha) + mV] \to 1 \), as RHS \( \to \infty \) while LHS is finite and positive. Because of the two conditions (32) and (34) for any \( 0 \leq m \leq 1 \) there exists a function \( \alpha (p, m) \) with \( \frac{\partial \alpha(p, m)}{\partial p} < 0 \) that defines the locus of points in the space

\[
0 < \alpha < 1; 0 < p < \min \left\{ \frac{1}{\lambda [y_H (1 - \alpha) + mV]}, 1 \right\}
\]

such that the profits from the low risk whole bank and the high-risk whole bank are the same, that is

\[
\lambda \frac{p \Delta y + \alpha y_L}{1 - \lambda [y_L (1 - \alpha) + mV]} = \alpha \lambda \frac{p \Delta y + y_L}{1 - \lambda [y_H (1 - \alpha) + mV]}
\]

(36)

Observe that this result is valid for any \( 0 \leq m \leq 1 \), thus it is also valid for the weight of the regulator, \( m = 1 \).

To prove part 2. Define the bank size per unit of capital as

\[
Z_{tr} \ (\text{junior}) \equiv \frac{1}{1 - \lambda [(1 - \alpha) y_L + mV]},
\]

(37)

\[
Z_{hr} \ (\text{senior}) \equiv \frac{1}{1 - \lambda p^{\beta} [(1 - \alpha) y_H + mV]},
\]

(38)

\[
Z_{hr} \ (\text{junior}) \equiv \frac{1}{1 - \lambda p^{\beta} [(1 - \alpha) y_H + mV]}
\]

(39)
Profits per unit of capital are

\[ \pi_{tr} (\text{junior}; K = 1) \equiv \lambda \frac{p^B \Delta y + \alpha y_L}{1 - \lambda [(1 - \alpha) y_L + mV]}, \]  
(40)

\[ \pi_{hr} (\text{junior}; K = 1) \equiv \lambda \alpha \frac{p^B \Delta y + y_L}{1 - p^B \lambda [(1 - \alpha) y_H + mV]}, \]  
(41)

\[ \pi_{hr} (\text{senior}; K = 1) \equiv \lambda \alpha \frac{p^G \Delta y + y_L}{1 - p^G \lambda [(1 - \alpha) y_H + mV]}. \]  
(42)

Observe that since \( \alpha < 1 \), then (41) < (40) if

\[ p^B [(1 - \alpha) y_H + mV] \leq (1 - \alpha) y_L + mV \]  
(43)

\[ \Leftrightarrow p^B = p^2 \leq \frac{(1 - \alpha) y_L + mV}{(1 - \alpha) y_H + mV}, \]

for or any \( 0 \leq m \leq 1 \). Notice that conditions (43) and (14) are compatible since \( p^B < p^G \).

Recall that capital must be allocated to the junior and senior tranches in such a way that the sizes of the SPVs are equal. Observe that if condition (15) is satisfied then \( Z_{hr} (\text{junior}) = Z_{tr} (\text{junior}) \), so that splitting a high-risk whole bank into junior and senior tranches that abscond in the same state of nature of the whole high-risk bank we have

\[ S_{hr} (\text{junior}) = Z_{hr} (\text{junior}) K^{B*} = Z_{hr} (\text{senior}) K^{G*} = S_{hr} (\text{senior}) \]  
(44)

and with structured finance we have

\[ S_{tr} (\text{junior}) = Z_{tr} (\text{junior}) K^{B*} = Z_{tr} (\text{senior}) K^{G*} = S_{tr} (\text{senior}) \]  
(45)

where \( K^{B*} + K^{G*} = K \). Using the capital quantities \( K^{B*}, K^{G*} \) and recalling that at condition (15) we have \( \pi_{tr} (\text{junior}; K = 1) > \pi_{hr} (\text{junior}; K = 1) \), it follows that

\[ \pi_{tr} (\text{junior}; K = 1) K^{B*} + \pi_{hr} (\text{senior}; K = 1) K^{G*} > \]  
(46)

\[ \pi_{hr} (\text{junior}; K = 1) K^{B*} + \pi_{hr} (\text{senior}; K = 1) K^{G*} \]

where the LHS is the profit of the sum of the junior and senior pooled and tranched SPVs, and the RHS is the profits of the high risk whole bank split into a junior and senior pooled and tranched banks that abscond in the same state of nature. Notice that (46) holds if (43) is satisfied. Since this result is valid for a generic \( 0 \leq m \leq 1 \) it is also valid if we interpret the structured finance sector as one where there are no date 2 profits, e.g. \( m = 0 \).

End of proof.
8.4 Bank size and profits in the case of pooling

Recall that pooling assets entails \( m = 0 \). The bank that pool assets can choose three sizes: 1) low risk, 2) medium risk, and 3) high risk. We consider each one in turn.

**Low risk.** The probability of repayment in this case is \( \lambda \). The incentive constraint is the same as in the case without pooling:

\[
Sy_L - R = Sy_L - \frac{S - K}{\lambda} \geq S \alpha y_L. \tag{47}
\]

Thus the maximum size consistent with a low-risk bank is given by (2) and its profits are given by (3), with \( m = 0 \).

**Medium risk.** The probability of repayment in this case is \( \frac{p}{2} \). The incentive constraint is

\[
S\frac{y_H + y_L}{2} - R = S\frac{y_H + y_L}{2} - \frac{S - K}{\lambda p(2 - p)} \geq S \alpha \frac{y_H + y_L}{2}. \tag{48}
\]

The maximum size consistent with a medium-risk bank is

\[
S_{mr} = \frac{K}{1 - \lambda p(2 - p) \frac{y_H + y_L}{2} (1 - \alpha)} \tag{49}
\]

and its profits are

\[
\pi_{mr_{\text{pooling}}} = \lambda S \left[ p^2 y_H + p(1 - p)(y_H + y_L) + (1 - p)^2 \alpha y_L \right] - \lambda p(2 - p) R
\]

\[
= S \left[ \lambda p y_H + (1 - p)(y_L + \alpha(1 - p)) \right] - 1 + K
\]

\[
= \lambda K \left[ py_H \left( \frac{\alpha}{2} + \frac{\alpha}{2}(1 - \alpha) \right) + y_L \left( \alpha(1 - p) - \frac{p}{2}(1 - \alpha) \right) \right] \tag{50}
\]

**High risk.** The probability of repayment in this case is \( \lambda p^2 \). The incentive constraint is

\[
Sy_H - R = Sy_H - \frac{S - K}{\lambda p^2} \geq S \alpha y_H. \tag{51}
\]

The maximum size consistent with a high-risk bank is

\[
S_{hr} = \frac{K}{1 - \lambda p^2 y_H (1 - \alpha)} \tag{52}
\]

and its profits are

\[
\pi_{hr_{\text{pooling}}} = \lambda S \left[ p^2 y_H + p(1 - p)\alpha(y_H + y_L) + (1 - p)^2 \alpha y_L \right] - \lambda p^2 R
\]

\[
= S \left[ \lambda p y_H (p + (1 - p)\alpha) + \lambda (1 - p)\alpha y_L - 1 \right] + K
\]

\[
= \lambda \alpha K \frac{p \Delta y + y_L}{1 - p^2 \lambda y_H (1 - \alpha)} \tag{53}
\]
8.5 Proof of lemma 2

First, since the profits of the high risk SPV are given by (53), then putting pooled assets into a high-risk SPV achieves lower welfare than a high-risk traditional bank. Indeed pooling reduces the probability of obtaining the output $y_H$ and, hence, forces a high-risk SPV with pooled assets to have a smaller size than a high-risk traditional bank.

Second, a low-risk SPV with pooled assets is identical to a low-risk traditional bank with $m = 0$.

Finally, we show that a medium size SPV with pooled assets achieves lower welfare than pooling and tranching. From equation (30) we know that the profits from pooling and tranching can be written as

$$
\pi_{P&T} = K\lambda \frac{\alpha y_L + \frac{\alpha}{2} \Delta y (1 - \alpha) + p \Delta y \alpha}{1 - \lambda (1 - \alpha) \left[ \frac{y_H p (2 - p) + y_L}{2} \right]}. \tag{54}
$$

Notice that equation (50), the profits of a medium size SPV with pooled assets, simplifies to

$$
\pi_{mrPOOLING} = \lambda K \frac{\Delta y \frac{\alpha}{2} (1 - \alpha) + p \alpha \Delta y + y_L \alpha}{1 - \lambda (1 - \alpha) \left[ \frac{y_H \alpha}{2} \right]}. \tag{55}
$$

Thus the numerators of (55) and of (54) are the same. As for the denominators we have:

$$
1 - \lambda (1 - \alpha) \left[ \frac{y_H \alpha}{2} \right] > 1 - \lambda (1 - \alpha) \left[ \frac{y_H p (2 - p) + y_L}{2} \right] \iff
$$

$$
\lambda (1 - \alpha) \left[ \frac{y_H \alpha}{2} \right] > \lambda p (2 - p) y_H + y_L \left( 1 - \alpha \right) \iff
$$

$$
\frac{y_H \alpha}{2} > p (2 - p) \left( \frac{y_H + y_L}{2} \right) \iff
$$

$$
y_L > p (2 - p) y_L
$$

which is always true. Hence $\pi_{mrPOOLING} < \pi_{P&T}$.

End of proof.

8.6 Proof of Proposition 2

We prove that if $p$ is sufficiently small there exists parameters such that

$$
\pi(m_0 = 1; S = S_r) > \pi(m_0 = 1; S = S_h) \tag{56}
$$

$$
\pi(0 < m < 1; S = S_r) < \pi(0 < m < 1; S = S_h) \tag{57}
$$
Recall that from proposition 1, for any \( 0 \leq \mu \leq 1 \) there exists a function \( \alpha(p,m) \) with \( \frac{d\alpha(p,m)}{dp} < 0 \) that defines the locus of points in the space \[ 0 < \alpha < 1; 0 < p < \min \left\{ \frac{1}{\lambda[y_H(1-\alpha)+mV]} , 1 \right\} \] such that the profits from the low-risk whole bank and the high-risk whole bank are the same. Therefore there exists a value of \( m \) denoted \( m^* \) such that the profits from the low-risk whole bank and the high-risk whole bank are the same, which is (36) occurs.

The derivatives w.r.t. \( m \) of the LHS of and of RHS of (36) are, respectively,
\[ \lambda^2 \frac{(p\Delta y + \alpha y_L)V}{(1 - \lambda[y_L(1-\alpha) + mV])^2}, \]
and
\[ \alpha \lambda^2 \frac{(p\Delta y + y_L)\rho V}{(1 - p\lambda[y_H(1-\alpha)+mV])^2}. \]
Notice that from (36) we can write (59) as
\[ \lambda \frac{AV}{1 - \lambda[y_L(1-\alpha) + mV]}, \]
and (60) as
\[ \lambda \frac{BpV}{1 - p\lambda[y_H(1-\alpha) + mV]} \]
where \( A \) and \( B \) are defined by (36) so that using that \( A = B \) we have (59) > (60) \( \Leftrightarrow \)
\[ \frac{1}{1 - \lambda[y_L(1-\alpha) + mV]} > \frac{p}{1 - p\lambda[y_H(1-\alpha) + mV]} \Leftrightarrow \]
\[ \frac{1}{1 - p\lambda[y_H(1-\alpha) + mV]} > p \Leftrightarrow \]
\[ 1 - p > p\lambda(1-\alpha)\Delta y \Leftrightarrow \]
\[ 1 > p[1 + \lambda(1-\alpha)\Delta y] \Leftrightarrow \frac{1}{1 + \lambda\Delta y(1-\alpha)} > p. \] (63)

End of proof.

### 8.7 Proof of Proposition 3

To prove part 1. Recall that there exists a value of \( m \) denoted \( m^* \), with \( m^* < 1 \) such that the profits from the low risk whole bank and the high risk whole bank are the same. From proposition 2 if
\[ p < \frac{1}{1 + \lambda\Delta y(1-\alpha)} \] (64)
for \( m > (\leq) m^* \) a low-risk bank has more (less) profit than a high-risk bank, which implies that the regulator prefers the low-risk bank while if the bank has \( m < m^* \) it prefers to be high risk. If the bank has \( m < m^* \) from (43) if
\[
p \leq \sqrt{\frac{(1-\alpha)y_L + mV}{(1-\alpha)y_H + mV}}
\] (65)
there exist parameters such the bank prefers pooling and tranching to being whole and high risk, which in turn is preferred to be whole and low risk, that is
\[
\pi(m_2; \text{structured finance}) > \pi(m_1; hr) > \pi(m_1; lr).
\] (66)

Note that we must make sure our parameters satisfy both equation (36) and equation (15). Let \( \Gamma \equiv (1-\alpha)y_H + mV \). Using equation (15) to eliminate \((1-\alpha)y_L + mV\) from the denominator of the LHS of equation (36), we can write
\[
\frac{p\Delta y + \alpha y_L}{1 - \lambda p_2 \Gamma} = \frac{p\Delta y + y_L}{1 - \lambda p}.
\] (67)
Rearranging this expression, we obtain
\[
(1-\alpha)\Delta y = \lambda \Gamma [p\Delta y(1 - \alpha p) + \alpha y_L(1 - p)].
\] (68)
Therefore restricting parameters in such a way that (68) is satisfied, if
\[
p < \min \{ (65), (64) \}
\] (69)
then we have that regulation limiting bank size induces the bank to innovate.

To prove part 2. We established in proposition 1 that structured finance has economic value, that can be traded off against loss of future profits. Therefore we can show that there exists parameters such that
\[
\pi(m_0; hr) = \pi(m_2; \text{structured finance}).
\] (70)
Recall that welfare and profits coincide in the structured finance sector. Because of proposition 1 there exists parameters such
\[
\pi(m_2; \text{structured finance}) > \pi(m_1; hr)
\] (71)
and if \(0 \leq m_1 < m^*\)
\[
\pi(m_1; hr) > \pi(m_1; lr)
\] (72)
and if \(m^* < 1\)
\[
\pi(m_0; lr) > \pi(m_0; hr).
\] (73)
Thus we can lower \( m_2 \) so that
\[
\pi(m_1; hr) > \pi(m_2; \text{structured finance}) > \pi(m_1; lr).
\] (74)
Thus we have
\[
\pi(m_0; lr) > \pi(m_0; hr) > \pi(m_1; hr) > \pi(m_2; \text{structured finance}) > \pi(m_1; lr).
\] (75)
That is an optimally chosen size regulation may induce structured finance that ends up lowering welfare with respect to absence of size regulation.

End of proof.
8.8 Proof of lemma 3

In lemma 2, we have already shown that pooling and tranching is preferred to pooling assets in an medium-risk SPV.

We still have to check if pooling assets in a high-risk SPV gives higher profit than pooling and tranching. The profits of a high-risk SPV with pooled assets are given by (53), while the profits from pooling and tranching are given by (54). Observe that the numerator of (54) > the numerator of (53). Comparing denominators:

\[
1 - p^2 \lambda y_H (1 - \alpha) > 1 - \lambda (1 - \alpha) \left( \frac{y_H p (2 - p) + y_L}{2} \right) \Leftrightarrow \\
\frac{y_H p (2 - p) + y_L}{2} > p^2 y_H \Leftrightarrow y_H p (2 - p) + y_L > 2 p^2 y_H \Leftrightarrow \\
y_H p^2 - p^2 y_H + y_L > 2 p^2 y_H \Leftrightarrow y_H p (2 - 3p) + y_L > 0.
\]

Thus a sufficient condition for pooling and tranching to be preferred to a high-risk SPV with pooled assets is \( p < 2/3 \).

Under the restriction \( p < 2/3 \) a bank that is forced to shrink because of capital regulation finds pooling and tranching more attractive than simply pooling assets.

End of proof.
Figure 1: Sequence of events

<table>
<thead>
<tr>
<th>Step 1. Pooling and tranching</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
<td><strong>Liabilities</strong></td>
</tr>
<tr>
<td>$y_H S/2 + y_H S/2$ with prob. $p^2$</td>
<td>$y_H S/2$ with prob. $1-(1-p)^2 = p^G$</td>
</tr>
<tr>
<td>$y_H S/2 + y_L S/2$ with prob. $2p(1-p)$</td>
<td>$y_L S/2$ with prob. $(1-p)^2 = 1-p^G$</td>
</tr>
<tr>
<td>$y_L S/2$ with prob. $(1-p)^2$</td>
<td>$y_H S/2$ with prob. $p^2 = p^B$</td>
</tr>
<tr>
<td>$y_L S/2$ with prob. $1-(1-p)^2 = 1-p^B$</td>
<td>$y_L S/2$ with prob. $p = p^A$</td>
</tr>
</tbody>
</table>

Step 2. Housing tranches in separate structures with different capital, $K^B + K^G = K$

<table>
<thead>
<tr>
<th>Step 2. Structured finance</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets (Junior Tranche)</strong></td>
<td><strong>Liabilities</strong></td>
</tr>
<tr>
<td>$y_H S/2$ with prob. $p^2 = p^B$</td>
<td>$D^B$</td>
</tr>
<tr>
<td>$y_L S/2$ with prob. $1-p^2 = 1-p^B$</td>
<td>$K^B = S/2 - D^B$</td>
</tr>
<tr>
<td><strong>Senior SPV</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Assets (Senior Tranche)</strong></td>
<td><strong>Liabilities</strong></td>
</tr>
<tr>
<td>$y_H S/2$ with prob. $1-(1-p)^2 = 1-p^G$</td>
<td>$D^G$</td>
</tr>
<tr>
<td>$y_L S/2$ with prob. $(1-p)^2 = 1-p^G$</td>
<td>$K^G = S/2 - D^G$</td>
</tr>
</tbody>
</table>

Figure 2: Structured finance
Objective function: welfare and profits

Figure 3: Proposition 2

Figure 4: Propositions 3 and 4