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Working Paper

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Staff Report, Federal Reserve Bank of New York, No. 541

Provided in Cooperation with:
Federal Reserve Bank of New York

Suggested Citation: Ferrero, Andrea (2012) : House price booms, current account deficits, and low interest rates, Staff Report, Federal Reserve Bank of New York, No. 541, Federal Reserve Bank of New York, New York, NY

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House Price Booms, Current Account Deficits, and Low Interest Rates

Andrea Ferrero

Staff Report no. 541
January 2012
Revised February 2012

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Abstract

One of the most striking features of the period before the Great Recession is the strong positive correlation between house price appreciation and current account deficits, not only in the United States but also in other countries that have subsequently experienced the highest degree of financial turmoil. A progressive relaxation of credit standards can rationalize this empirical observation. Lower collateral requirements facilitate access to external funding and drive up house prices. The current account turns negative because households borrow from the rest of the world. At the same time, however, the world real interest rate counterfactually increases. The two key ingredients that reconcile a demand-based explanation of house price booms and current account deficits with the evidence on real interest rates are nominal interest rates lower than the predictions of a standard monetary policy rule in leveraged economies and foreign exchange rate pegs in saving countries.

Key words: borrowing constraints, monetary policy shocks, exchange rate pegs
1 Introduction

The boom-bust in U.S. house prices has been a fundamental determinant of the recent financial crisis. The securitization process that eventually lead the financial sector to the brink of collapse crucially relied on expectations of ever-increasing house prices. Understanding the causes of these house price dynamics is crucial for preventing a repeat of a similar situation in the future.

Large and widening current account deficits accompanied soaring house prices, especially during the five years before the eruption of the crisis (figure 1). These two variables were perhaps the most discussed indicators of U.S. imbalances (Greenspan, 2005). Interestingly, the negative correlation between house price dynamics and current account balances is not specific to the U.S. but rather a robust global phenomenon, affecting advanced and emerging market economies alike (figure 2).1 Countries that witnessed house prices booms and substantial external deficits (such as Greece, Iceland, Ireland, Spain and the U.S.) also experienced among the highest degrees of financial turmoil during the crisis.2

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1Bernanke (2010) plots the cumulative change between 2001Q4 and 2006Q4 in current account balances and house prices for advanced economies. The August 2007 ECB Monthly Bulletin features a similar figure for the period 1997-2005. Figure 2 extends the sample to include emerging market economies such as China, which play a key role in financing the U.S. current account deficit.

2Similar dynamics for capital inflows and real estate prices occurred before the Asian crisis in the late 1990s (see Obstfeld and Rogoff (2010) and the references therein).
A popular explanation for the negative correlation between current account balance and house prices is the so-called “global saving glut” hypothesis (Bernanke, 2005). In particular, building on their earlier work, Caballero, Fahri and Gourinchas (2008b) argue that a global demand for liquidity can generate capital flows from the rest of the world toward the U.S., where asset prices – and especially house prices, due to the securitization process – take off.

This paper takes a different perspective and argues that a progressive relaxation of borrowing constraints can generate a strong negative correlation between house prices and the current account.

The analysis relies on a two-country model with tradable consumption goods and housing. The expected value of housing represents the collateral for private debt. This endogenous borrowing constraint is buffeted by a time-varying parameter which controls the loan-to-value (LTV) requirement and constitutes the key shock in the model. An increase in this threshold, for given value of the collateral, leads households to lever up and demand more housing, hence driving up house prices. To the extent that the relaxation of credit constraints affects the whole economy, the increase in domestic borrowing must be financed from abroad, thus generating a current account deficit.

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3 Caballero, Fahri and Gourinchas (2008a) and Mendoza, Quadrini and Rios-Rull (2009) formalize the idea of a global saving glut, with particular focus on the implications for the U.S. current account deficit.

4 This model essentially interprets the non-tradable sector in Ferrero, Gertler and Svensson (2010) as housing and introduces an endogenous borrowing constraint, as in Kiyotaki and Moore (1997), in the household problem. Alternatively, this economy translates Iacoviello (2005) into an open economy framework by adding two types of goods.
This story is certainly consistent with several pieces of both anecdotal and hard evidence but raises an important issue. In many respects, an exogenous relaxation of borrowing constraints is akin to a demand shock. The real interest rate typically increases in response to this type of shock. To the contrary, in the data, real interest rates have progressively declined for at least the last 20 years.

Not surprisingly, then, other papers that have studied the role of a relaxation of borrowing constraints for house prices and external deficits also feature some role for the global saving glut hypothesis. In Boz and Mendoza (2011), agents in a small open economy experience a switch to a new regime (financial liberalization) that allows for higher LTV ratios. Imperfect knowledge (learning), combined with early realizations of this high-leverage regime, lead agents to overestimate the true probability of persistence of easy credit and creates a boom in asset prices. One interpretation of the small open economy assumption is that saving glut shocks exactly compensate the upward pressure on the real interest rate due to the relaxation of borrowing constraints. This interpretation is explicit in Adam, Kuang and Marcet (2011), in the sense that the learning mechanism of this paper interacts with exogenous shocks that reduce the world real interest rate. Finally, in Favilukis, Ludvigson and Van Nieuwerburgh (2011), the relaxation of borrowing constraints, together with the reduction in transaction costs for housing and business cycle factors, quantitatively accounts for almost the entire increase in the observed price-to-rent ratio. In their paper, exogenous capital flows from the rest of the world are close to irrelevant for the dynamics of house prices but necessary to keep the equilibrium real interest rate low as in the data.

This paper abstracts from saving glut shocks and suggests a different explanation for low interest rates, related to monetary policy in the U.S. and in the rest of the world. During the early 2000s, nominal interest rates were low for a “considerable period” (mentioned for the first time in the August 2003 FOMC statement). If inflation expectations are stable, low nominal rates translate into low real rates.

According to Taylor (2008), loose U.S. monetary policy was the key determinant of house price appreciation. Developments in mortgage markets and the securitization process only contributed to worsen the problem. Departures of the measured Federal Funds Rate (FFR) from the interest rate implied by a standard monetary policy (Taylor) rule in the U.S. during the period 2000-2005 explain low real interest rates. From a qualitative perspective, these shocks do contribute to amplify the boom in house prices as well as to widen the current account deficit. However, their quantitative contribution is extremely small.

A second implication of monetary policy for the correlation between house prices and the current account concerns the choice of the exchange rate regime by foreign economies. After the Asian crises of the late 1990s, several emerging markets in that region and elsewhere (most notably China) started pegging their nominal exchange rate to the U.S. dollar. Therefore, in a small open economy, Iacoviello and Minetti (2003) relate the strength of the impact of monetary policy shocks on house prices to the degree of financial liberalization.

Dooley, Folkerts-Landau and Garber (2008) labeled the resulting international monetary regime “Bretton Woods II”. Their work emphasizes the interplay between managed exchange rate regimes in Asian countries and U.S. current account deficits. The basic idea is that emerging economies stimulate their exports (their main source of growth) by keeping the domestic currencies artificially undervalued relative to fundamentals.
these countries import U.S. monetary policy. As a consequence, low U.S. interest rates lead to low global interest rates. The quantitative analysis shows that foreign pegs, coupled with over-expansionary U.S. monetary policy, exert additional downward pressure on the real interest rate and impair a real depreciation of the dollar that would help rebalance the U.S. current account deficit.

Taken together, the relaxation of borrowing constraints and low interest rates in the U.S. (coupled with foreign pegs) account for about two-thirds of the increase in real house prices and almost one-half of the deterioration of the current account during the first half of the 2000s. These quantitative findings complement the role of other factors in accounting for the correlation between the house price boom and the deterioration of the current account in the U.S. during the early 2000s, such as the global saving glut hypothesis discussed above or preference shocks for housing (Gете, 2010).

The rest of the paper proceeds as follows. Section 2 provides some evidence on the relaxation of credit standards induced by the process of financial innovation. Section 3 presents the model and develops some intuition using the steady state of a tractable special case. Section 4 discusses the calibration and the basic quantitative experiment. Section 5 addresses the quantitative importance of overly-accommodative U.S. monetary policy and foreign exchange rate pegs. Section 6 evaluates the robustness of the results to several changes in the parameters. Finally, the seventh section concludes.

2 Evidence on the Relaxation of Collateral Constraints

Few existing empirical studies discuss the relation between house prices and external imbalances. Ahearne et al. (2005) document the co-movement between house price dynamics and current account balance since 1970. Aizenman and Jinjarak (2009) provide a precise estimate of the relation between the two variables: a one standard deviation increase in lagged current account deficits is associated with a 10% appreciation of real estate prices. Fratzscher, Juvenal and Sarno (2010) adopt the opposite perspective: according to their estimates, together with equity market shocks, house price shocks account for up to 32% of the movements in the U.S. trade balance over a 20−25 quarter horizon. Arguably, both house prices and the current account are endogenous variables. From a more structural perspective, the key question becomes which underlying fundamentals drive the correlation between these two variables.

The key shock that generates a house price boom and a contemporaneous current account deficit in the model below is a reduction in the parameter that measures LTV requirements.

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7 Punzi (2006) considers a two-country, two-type (borrower and saver in each country) model with residential investment. The impulse responses from a VAR provide broad support for LTV shocks to play an important role. Midrigan and Philippin (2010) use credit constraint shocks in an island economy to match the distribution of house prices across U.S. counties. Eggertsson and Krugman (2010) and Guerrieri and Lorenzoni (2010) argue that a tightening of borrowing constraints (relative to pre-crisis levels) can lead to a substantial drop in aggregate demand and potentially create depression-like scenarios.

8 Iacoviello and Neri (2010) estimate a DSGE model with housing and find that slow technological progress in the housing sector explains the long-run upward trend in U.S. house prices. Housing preference and technology shocks account for about 50% of the variance of housing investment and prices at business cycle frequencies.

9 Kole and Martin (2009) find slightly smaller results.
At a broader level, however, lower collateral requirements also capture easier access to housing finance for households previously excluded from credit markets. The next two sections present some evidence on developments in mortgage markets and the evolution of LTV ratios in the U.S. and in the rest of the world.

2.1 United States

Rajan (2010) argues that easy credit in the U.S. is the consequence of the political response to increasing income inequality. According to this view, the growing role of government-sponsored enterprises (primarily Fannie Mae and Freddie Mac) and the expansion of subprime lending follow from the need to guarantee affordable housing to low income households who are falling behind.

Favara and Imbs (2010) trace the increase in supply of mortgage credit back to the deregulation of cross-state ownership of banks that started with the Interstate Banking and Branching Efficiency Act of 1994. Loosely speaking, interstate branching makes banks more efficient. More narrowly, the mortgage deregulation process creates the possibility to offer credit products across regions with less correlated housing cycles.

While the rationale and the origins of the credit boom are certainly interesting per-se and well-worth investigating, the analysis below starts from the presumption that a relaxation of borrowing constraints occurred and studies the consequences on asset prices and macroeconomic quantities.

In the early 2000s, subprime lending was clearly the most significant development in mortgage finance. In practice, no legally binding definition of subprime lending exists. The Office of the Comptroller of the Currency, the Board of Governors of the Federal Reserve System, the Federal Deposit Insurance Corporation and the Office of Thrift Supervision (2001) jointly issued a document defining as ‘subprime’ those borrowers who “...display a range of credit risk characteristics that may include one or more of the following:

- Two or more 30-day delinquencies in the last 12 months, or one or more 60-day delinquencies in the last 24 months
- Judgment, foreclosure, repossession, or charge-off in the prior 24 months
- Bankruptcy in the last 5 years
- Relatively high default probability as evidenced by, for example, a credit bureau risk score (FICO) of 660 or below (depending on the product/collateral), or other bureau or proprietary scores with an equivalent default probability likelihood
- Debt service-to-income ratio of 50% or greater, or otherwise limited ability to cover family living expenses after deducting total monthly debt-service requirements from monthly income.”

Pinto (2008a,b), a former Fannie Mae’s chief credit officer, points out that official classifications severely underestimate the extent of subprime lending, which could easily include most

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10. See Kumhof and Ranciere (2010) for a formalization of this hypothesis.
11. A similar assumption underlay the rating agency valuation models.
Table 1:

<table>
<thead>
<tr>
<th>Year</th>
<th>FHA/VA</th>
<th>Conv/Conf</th>
<th>Jumbo</th>
<th>Subprime</th>
<th>Alt A</th>
<th>HEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>8</td>
<td>57</td>
<td>20</td>
<td>7</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2002</td>
<td>7</td>
<td>63</td>
<td>21</td>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2003</td>
<td>6</td>
<td>62</td>
<td>16</td>
<td>8</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2004</td>
<td>4</td>
<td>41</td>
<td>17</td>
<td>18</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>2005</td>
<td>3</td>
<td>35</td>
<td>18</td>
<td>20</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>2006</td>
<td>3</td>
<td>33</td>
<td>16</td>
<td>20</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>2007</td>
<td>4</td>
<td>48</td>
<td>14</td>
<td>8</td>
<td>11</td>
<td>15</td>
</tr>
</tbody>
</table>


“Alternative-to-Agency” (Alt-A) loans and “Home Equity Loans” (HEL). A large portion of Federal Housing Administration (FHA), Veteran Administration (VA) and rural housing loans also conform to subprime standards. Between 2002 and 2007, approximately 83% of FHA loans consisted of LTV ratios higher than 90% and approximately 70% had a FICO lower than 660.12

The growth in subprime lending can thus be seen not only as a direct relaxation of credit constraints for households previously able to borrow but also, and perhaps more importantly, as an opportunity to participate for households previously excluded from mortgage markets.

The share of subprime lending in the U.S. mortgage market grew from 0.74% to almost 9% during the 1990s (Nichols, Pennington-Cross and Yezer, 2005). After slowing down in 2001 (7%) and 2002 (1%), subprime origination soared from 8% in 2003 to 20% in 2005 and 2006. In the same period, Alt-A mortgages and HELs also gained popularity. Table 1 shows that by 2006 the higher risk segment accounted for 48% of securitized origination (equivalent to 34% of the dollar volume).13

Table 2 presents direct evidence on the evolution of LTV ratios, broken down by type of mortgage (Prime, Alt A and Subprime) and rate (Fixed and Adjustable). For the prime segment, the average LTV ratio increased by 10 percentage points between 2002 and 2006, for both the fixed- and adjustable-rate types. In both categories, the fraction of mortgages with LTV ratios higher than 80% increased from less than 5% in 2002 to about 25% in 2006. Alt A fixed-rate mortgages featured a similar increase, with LTV ratios higher than 80% roughly doubling between 2002 and 2006. Over the same period, the increase in LTV ratios for Alt A adjustable-rate mortgages was smaller (of the order of 5 percentage points) but LTV ratios higher than 80% more than doubled, reaching 55% in 2006. For the subprime segment, the increase in LTV ratio was also of the order of 5 percentage points and subprime LTV ratios higher than 80% soared too, especially for the adjustable-rate type.

Several other papers document similar, if not more extreme, patterns for LTVs. Using data from DataQuick that covers 89 metro areas in the United States, Glaeser, Gottlieb, and

12While similar data are not available for the smaller volume VA and rural housing loan programs, original LTV distributions were believed to be similar.
13Mian and Sufi (2010) show that HELs are responsible for a significant fraction of the increase in U.S. household leverage between 2002 and 2006.
<table>
<thead>
<tr>
<th>Year</th>
<th>Fixed-Rate CLTV</th>
<th>Fixed-Rate CLTV &gt; 80%</th>
<th>Adjustable-Rate CLTV</th>
<th>Adjustable-Rate CLTV &gt; 80%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>65.4</td>
<td>3.0</td>
<td>66.5</td>
<td>4.1</td>
</tr>
<tr>
<td>2003</td>
<td>63.8</td>
<td>4.4</td>
<td>68.2</td>
<td>10.1</td>
</tr>
<tr>
<td>Prime</td>
<td>2004</td>
<td>67.4</td>
<td>73.5</td>
<td>20.7</td>
</tr>
<tr>
<td>2005</td>
<td>70.9</td>
<td>13.4</td>
<td>74.1</td>
<td>21.7</td>
</tr>
<tr>
<td>2006</td>
<td>74.5</td>
<td>23.1</td>
<td>75.3</td>
<td>26.2</td>
</tr>
<tr>
<td>2002</td>
<td>74.7</td>
<td>22.0</td>
<td>74.3</td>
<td>20.8</td>
</tr>
<tr>
<td>2003</td>
<td>71.5</td>
<td>21.4</td>
<td>78.0</td>
<td>33.3</td>
</tr>
<tr>
<td>Alt A</td>
<td>2004</td>
<td>75.3</td>
<td>82.6</td>
<td>46.9</td>
</tr>
<tr>
<td>2005</td>
<td>76.2</td>
<td>31.3</td>
<td>83.5</td>
<td>49.6</td>
</tr>
<tr>
<td>2006</td>
<td>79.4</td>
<td>39.6</td>
<td>85.0</td>
<td>55.4</td>
</tr>
<tr>
<td>2002</td>
<td>77.3</td>
<td>38.0</td>
<td>81.2</td>
<td>46.8</td>
</tr>
<tr>
<td>2003</td>
<td>78.0</td>
<td>41.7</td>
<td>83.5</td>
<td>55.6</td>
</tr>
<tr>
<td>Subprime</td>
<td>2004</td>
<td>77.7</td>
<td>85.3</td>
<td>61.1</td>
</tr>
<tr>
<td>2005</td>
<td>78.7</td>
<td>44.5</td>
<td>86.6</td>
<td>64.4</td>
</tr>
<tr>
<td>2006</td>
<td>78.7</td>
<td>44.6</td>
<td>86.7</td>
<td>64.0</td>
</tr>
</tbody>
</table>

Table 2: Evolution of LTV ratios (in %). Source: Abraham, Pavlov and Wachter (2008). CLTV stands for combined (i.e. first and second mortgage) LTV ratio. CLTV > 80% refers to the fraction of combined LTV ratios larger than 80%.

Gyourko (2010) find that the median combined LTV ratio on all housing purchases increased from 80% in 2004 to 90% in 2006. Moreover, extreme leverage, in the form of 100% LTV, was available and used by at least 10% of borrowers. This fraction became at least 25% in 2006. Perhaps not surprisingly, extreme leverage was concentrated outside the prime segment. Using First American CoreLogic Loan Performance data, Haughwout, Lee, Tracy and Van der Klaauw (2011) find that the median LTV on securitized non-prime mortgages increased from 95% in 2004 to 99% in 2006 and was equal to 100% for the 75th percentile throughout this period.

Interestingly, the literature attributes different importance to this phenomenon. Glaeser, Gottlieb and Gyourko (2010) argue that the magnitude of the LTV changes was not large enough to account for a significant fraction of the increase in house prices. Geanakoplos (2009, 2010) challenges this view, emphasizing the possibility that the increase in leverage increases house prices not only directly but also through a shift in the composition of buyers. Haughwout, Lee, Tracy and Van der Klaauw (2011) support this interpretation by documenting the key role of investors, especially “buy and flip” ones, at the peak of the crisis.

In addition, as mortgage products became riskier due to the increasing participation of subprime borrowers and lower LTV ratios, interest rates did not increase. In fact, the spreads between subprime and prime mortgages of similar maturities uniformly decreased between 2000 and 2005.

The bottom line is that financial innovation, supported by the securitization process, provided greater access to mortgage finance at affordable prices for a broader pool of households, both at the extensive (higher share of subprime mortgages) and intensive (lower LTV require-
<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>21.7</td>
<td>22.6</td>
<td>24.2</td>
<td>26.0</td>
<td>29.3</td>
<td>32.2</td>
</tr>
<tr>
<td>Germany</td>
<td>53.1</td>
<td>53.2</td>
<td>53.5</td>
<td>52.4</td>
<td>51.9</td>
<td>51.3</td>
</tr>
<tr>
<td>Greece</td>
<td>11.8</td>
<td>14.8</td>
<td>17.2</td>
<td>20.2</td>
<td>25.1</td>
<td>29.3</td>
</tr>
<tr>
<td>Iceland</td>
<td>59.4</td>
<td>61.1</td>
<td>66.2</td>
<td>71.0</td>
<td>80.8</td>
<td>75.5</td>
</tr>
<tr>
<td>Ireland</td>
<td>32.8</td>
<td>36.3</td>
<td>42.7</td>
<td>52.2</td>
<td>61.4</td>
<td>70.1</td>
</tr>
<tr>
<td>Spain</td>
<td>32.5</td>
<td>35.9</td>
<td>40.0</td>
<td>45.7</td>
<td>52.3</td>
<td>58.1</td>
</tr>
<tr>
<td>U.K.</td>
<td>58.9</td>
<td>63.9</td>
<td>69.3</td>
<td>74.1</td>
<td>78.4</td>
<td>83.1</td>
</tr>
<tr>
<td>U.S.</td>
<td>60.5</td>
<td>66.1</td>
<td>71.1</td>
<td>76.1</td>
<td>81.1</td>
<td>84.8</td>
</tr>
</tbody>
</table>

Table 3: Residential mortgage debt in % of GDP. Source: European Mortgage Federation Hypostat, 2008.

ments) margin.\(^{14}\)

2.2 Rest of the World

Direct evidence on the relaxation of households’ borrowing constraints for countries other than the U.S. is much more scattered.

The European Mortgage Federation provides some information on housing finance in Europe, although data on LTV ratios are generally not available. One notable exception is Iceland, where LTV ratios increased from 65% to 90% in 2003 before going back to 80% in 2006. Iceland experienced a 60% increase in real house prices between 2001 and 2006, together with one of the largest deteriorations of the current account (more than 20% as a percentage of GDP) among Western economies.

The U.K experienced an early wave of mortgage market liberalization at the beginning of the 1980s, when down-payment requirements dropped from 25% to 15% (Ortalo-Magné and Rady, 2004). During that decade, real house prices increased by about 70%.

Outside Europe, Williams (2009) finds evidence that financial liberalizations in the 1980s and 1990s account for about half of the trend increase in real house prices in Australia over the period 1972-2006.

More indirect evidence also points in the direction of a large boom in housing finance in several European countries. Table 3 reports residential mortgage debt as fraction of GDP for a selected group of countries over the period 2001-2006. Iceland, the U.S. and the U.K. featured a similar pattern with mortgage debt growing from about 60 to about 80% of GDP or more. Countries like Spain and Ireland started from lower levels (approximately 30%) but roughly doubled their shares. Mortgage finance in Greece accounted for a small fraction of GDP (12%) in 2001 but reached about 30%, close to the level of France, where mortgage finance increased a more moderate 10% over the sample period.

\(^{14}\)Although not explicitly modeled here, the reduction in housing transaction costs provides further evidence in support of the process of liberalization in real estate financing. See Favilukis, Ludvigson and Van Nieuwerburgh (2011) for details.
All these examples of significant growth in mortgage debt contrast with the case of Germany, where the share of GDP remained roughly constant just above 50%. The increase in mortgage finance relative to GDP was also small in Japan, from 25% in 1990 to 36% in 2006 (IMF World Economic Outlook, 2008). Finally, while on the uprise from essentially zero in 1998, mortgage debt was still a small 10% GDP in China as of 2004 (Jain-Chandra and Chamon, 2010).

Obviously, the boom in mortgage debt can capture several factors, not only financial deregulation. In the case of Spain, for example, the common explanation for the housing boom relies on factors related to housing demand, such as strong income growth, foreign demand for vacation homes and immigration flows (Cortina, 2009). Spanish authorities explicitly limited LTV ratios for securitized mortgages. However, inflated appraisals may have contributed to circumvent these limits so that higher leverage may have amplified demand shocks (Duca, Muehlbauer and Murphy, 2010).

To summarize, credit market liberalizations have greatly stimulated housing finance. The evidence is quite clear for the U.S. and is at least suggestive for several other countries that have experienced contemporaneous house prices booms and current account deficits. Conversely, countries where the process of financial innovation has been less abrupt have experienced a much lower degree of house price appreciation and often current account surpluses. The next section develops a model in which the relaxation of borrowing constraints plays a key role to account for these facts.

3 An Open Economy Model with Borrowing Constraints

Time is discrete and indexed by \( t \). The world consists of two countries, Home and Foreign, of equal size. In each country, a continuum of measure one of firms produce a final tradable good using a labor aggregate as the only factor of production. The representative household in each country comprises a continuum of measure one of workers who supply differentiated labor inputs and consume a composite of the tradable goods produced in each country as well as housing services, which are assumed to be proportional to the fixed housing stock. An endogenous collateral constraint limits the maximum amount of private credit to a fraction of the expected value of housing. Goods and labor markets are imperfectly competitive. Prices and wages are set on a staggered basis. The law of one price holds but home bias in consumption implies that purchasing power parity is violated. International financial markets are incomplete. The only asset traded across countries is a one-period nominal risk-free bond denominated in the Home currency.

This section presents the household and firms’ problems from the perspective of the Home country. An asterisk denotes foreign variables when relevant.

**Household’s Preferences and Constraints**

The representative household maximizes

\[
U_t = E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left\{ \frac{X^{1-s}}{1-s} - \frac{1}{1+\nu} \int_0^1 L_{t+s}(i)^{1+\nu} dt \right\} \right\}.
\]  

(1)
Per-period utility depends positively on the consumption index $X_t$ and negatively on hours worked by each member of the representative household $L_t(i)$. The parameter $\sigma > 0$ is the coefficient of relative risk aversion while $\nu > 0$ is the inverse Frisch elasticity of supply of a specific labor input.

The index $X_t$ combines consumption of goods $C_t$ and housing services $H_t$ with constant elasticity of substitution $\varepsilon > 0$

$$X_t \equiv \left[ \eta C_t^{\frac{1}{\varepsilon}} + (1 - \eta) H_t^{\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}}, \ (2)$$

where $\eta \in (0, 1)$ represents the share of tradable goods in total consumption.

The tradable bundle $C_t$ combines consumption of goods produced in the Home ($C_{ht}$) and Foreign ($C_{ft}$) country with constant elasticity of substitution $\gamma > 0$

$$C_t \equiv \left[ \alpha \gamma C_{ht}^{\frac{1}{\gamma}} + (1 - \alpha) \gamma C_{ft}^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{\gamma - 1}}, \ (3)$$

where $\alpha \in [0.5, 1)$ is the share of domestic tradable goods.\(^{15}\)

The budget constraint for the representative household in nominal term is

$$P_{ht}C_{ht} + P_{ft}C_{ft} + Q_t H_t - B_t \leq \int_0^1 W_t(i)L_t(i)di + P_t + Q_t H_{t-1} + T_t - (1 + i_{t-1})B_{t-1}, \ (4)$$

where $P_{jt}$ is the Home price of good $j = \{h, f\}$, $Q_t$ is the price of housing, $W_t(i)$ is the wage for the specific labor input supplied by the $i^{th}$ household member, $P_t$ are profits from ownership of intermediate goods producers, $T_t$ are lump-sum transfers and $i_t$ is the net nominal interest rate on an internationally-traded one-period risk-free debt instrument $B_t$, denominated in the Home currency.

Household’s members perfectly pool their consumption risk within each country. The representative household can smooth consumption intertemporally by borrowing and lending in international financial markets, subject to a collateral constraint that depends on the expected value of housing

$$(1 + i_t)B_t \leq \Theta_t E_t(Q_{t+1}H_t), \ (5)$$

where the borrowing constraint parameter $\Theta_t$ is an exogenous shock with mean $\Theta$ and support over the unit interval. The idea behind the borrowing constraint is that the Foreign household can only recover a fraction $\Theta_t$ of the collateral in case of default, possibly due to various costs associated with the bankruptcy process.\(^{16}\)

**Labor Agencies and Wage Setting**

Perfectly competitive labor agencies hire differentiated labor inputs from household members

\(^{15}\)If $\alpha > 0.5$, preferences for tradable goods exhibit home bias. The Foreign tradable bundle places a weight $\alpha$ on consumption of Foreign tradable goods.

\(^{16}\)See, for instance, Kiyotaki and Moore (1997) or Kocherlakota (2000).
and supply intermediate goods producers with a composite

\[ L_t = \left[ \int_0^1 L_t(i)^{\frac{\phi_w-1}{\phi_w}} di \right]^{\frac{1}{\phi_w}}, \]

(6)

where \( \phi_w > 1 \) is the elasticity of substitution among differentiated labor inputs. Profit maximization gives the demand for the \( i \)th labor input

\[ L_t(i) = \left[ \frac{W_t(i)}{W_t} \right]^{-\phi_w} L_t, \]

(7)

where \( W_t \) is the aggregate wage index implied by the zero profit condition for labor agencies

\[ W_t = \left[ \int_0^1 W_t(i)^{1-\phi_w} di \right]^{\frac{1}{1-\phi_w}}. \]

Household members are monopolistic supplier of their labor inputs and set wages on a staggered basis. In each period, independently of previous adjustments, the probability of not being able to reset the wage is \( \zeta_w \). A household member who is able to reset the wage at time \( t \) solves

\[
\max_{W_t(i)} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_w)^s \left[ \lambda_{t+s} \tilde{W}_t(i)L_{t+s}(i) - \frac{1}{1 + \nu} L_{t+s}(i)^{1+\nu} \right] \right\},
\]

subject to (7) conditional on no further wage changes, where \( \lambda_t \) is the marginal utility of consumption at time \( t \). Appendix A.1 reports the details on the first order condition and derives the associated wage Phillips curve.

**Firms and Production**

Competitive retailers pack intermediate goods according to a constant returns technology with elasticity of substitution \( \phi_p > 1 \)

\[ Y_{ht} = \left[ \int_0^1 Y_t(h)^{\frac{\phi_p-1}{\phi_p}} dh \right]^{\frac{1}{\phi_p}}. \]

(8)

Profit maximization gives the demand for the \( h \)th good

\[ Y_t(h) = \left[ \frac{P_t(h)}{P_{ht}} \right]^{-\phi_p} Y_{ht}, \]

(9)

where \( P_{ht} \) is the aggregate price index for goods produced in the Home country implied by the zero profit condition for final goods producers

\[ P_{ht} = \left[ \int_0^1 P_t(h)^{1-\phi_p} dh \right]^{\frac{1}{1-\phi_p}}. \]

All intermediate goods producing firms have access to the same constant return technology
which uses the labor aggregate $L_t$ as the only factor of production

$$Y_t(h) = AL_t,$$  \hspace{1cm} (10)

where $A$ is a constant productivity factor. Intermediate goods producers set prices on a staggered basis, where $\zeta_p$ is the probability of not being able to adjust the price in the future, independently of previous adjustments. A firm that can reset its price at time $t$ solves\footnote{The representative household in each country owns the domestic firms. Therefore, the marginal utility of consumption, i.e. the lagrange multiplier on the budget constraint, is the appropriate measure to convert the value of future profits in units of current consumption.}

$$\max_{P_t(h)} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_p)^s \lambda_{t+s} \left[ \bar{P}_t(h)Y_{t,t+s}(h) - W_{t+s}L_{t+s} \right] \right\}$$

subject to the technology constraint (10) and to the demand for their product (9) conditional on no further price changes in the future, which the firm takes as given. Appendix A.1 reports the details on the first order condition and derives the associated price Phillips curve.

Finally, the stock of housing (land) is assumed to be fixed

$$H_t = H.$$ \hspace{1cm} (11)

This assumption gives the model a better chance to match the increase of house prices in response to the financial deregulation experiment described below and, more generally, in response to any shock that would lead to higher housing demand. In practice, of course, the housing boom was also accompanied by a large increase in residential investment. However, interpreting housing as land fits well the evidence in Davis and Heathcote (2007), who find that land prices, rather than the price of structures, explain the bulk of both trend growth and cyclical house price fluctuations between 1975 and 2006.

**Monetary Policy**

The central bank sets the short-term nominal interest rate in response to deviations of inflation and output from their targets

$$(1 + i_t) = (1 + i_{t-1})^{\rho_i} \left[ (1 + i) \left( \Pi_t / \Pi \right) \bar{\psi}_n \left( \bar{Y}_{ht} / \bar{Y}_n \right) \bar{\psi}_y \right]^{1-\rho_i} e^{\varepsilon_{it}},$$ \hspace{1cm} (12)

where $\rho_i$ is the degree of interest rate smoothing, $\Pi_t \equiv P_t / P_{t-1}$ is the inflation rate of goods prices $P_t$, $\Pi$ and $\bar{Y}_{ht}$ and $\bar{Y}_n$ are the targets for inflation and output respectively and $\varepsilon_{it}$ is an i.i.d. normal innovation to the interest rate rule with mean zero and standard deviation $\sigma_i$.

**3.1 Equilibrium and Steady State**

An imperfectly competitive equilibrium for the world economy is a sequence of prices and quantities such that:
1. The representative household in each country maximizes utility subject to the budget constraint and the collateral constraint, taking prices as given. Household’s members set wages on a staggered basis, taking labor demand for their specific labor input as given.

2. Intermediate goods producing firms set prices on a staggered basis to maximize the present discounted value of profits, taking the demand for their variety as given. Final goods producing firms minimize costs, taking prices as given.

3. The housing market clears in each country. Goods and financial markets clear internationally.

Appendix A.1 describes the details of the optimization problem of households and firms. The list of equilibrium conditions is in appendix A.2.

The model admits two types of steady state. If the borrowing constraint is not binding in either country, a symmetric steady state exists in which all relative prices (those of tradable goods, the terms of trade and the real exchange rate) are equal to one and foreign debt is zero. In this steady state, each country is in autarchy and the level of productivity pins down output (and hence consumption). House prices are equal to the present discounted value of the marginal utility of housing services while the real return is equal to the inverse of the discount factor.

The unattractive feature of a perfectly symmetric steady state is that, up to a linear approximation, borrowing constraints are irrelevant for house prices dynamics. Real house prices \( Q_t \equiv \frac{Q_t}{P_t} \) obey the following forward looking relation

\[
Q_t = \left( \frac{1 - \eta}{\eta} \right) \left( \frac{H}{C_t} \right)^{-\frac{1}{\sigma}} + \beta \mathbb{E}_t \left[ \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{\sigma} - \sigma} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\sigma}} Q_{t+1} \right] + \Xi_t \Theta_t \mathbb{E}_t \left( Q_{t+1} \right) .
\]

The first two terms of the right-hand side of (13) are standard. Real house prices are equal to the current marginal utility of housing services in units of marginal utility of consumption plus the discounted expected value of future house prices. The third term measures the contribution of the shadow value of the borrowing constraint to current house prices. If the borrowing constraint is not binding in steady state, the multiplier is equal to zero (\( \Xi = 0 \)). Therefore, up to a first order approximation, changes in the LTV ratio \( \Theta \) would have no effects on real house prices.

An asymmetric steady state (characterized in appendix A.3) resuscitates a role for borrowing constraints in affecting house prices dynamics. Even with identical preferences and technologies, simply imposing that one country’s borrowing constraint is binding is enough to generate an asymmetric steady state. However, assuming a different degree of patience across countries provides a more fundamental reason why one country’s steady state borrowing constraint may be binding. In what follows, the Foreign country representative household is assumed to be relatively more patient (\( \beta^* > \beta \)). This assumption implies that the Foreign country is a net saver in international financial markets.

Interestingly, the presence of a binding borrowing constraint also solves the problem of indeterminacy of the net foreign asset position typical of open economy models with incomplete international financial markets (Schmitt-Grohé and Uribe, 2003). The borrowing constraint at equality pins down the steady state level of net foreign assets as a function of house prices and
the real interest rate.

The steady state of a simplified version of the model helps develop the intuition for the main result. Suppose for a moment the Home country is a small open economy which takes the world interest rate as given. Further, abstract from nominal rigidities and assume the Home country receives a fixed endowment of a single consumption good. Finally, preferences are log-separable in consumption and housing. In steady state, the real value of the housing stock in this economy is

$$Q = \frac{(1 - \eta)C}{\eta(1 - \beta - \Xi\Theta)}.$$ \hspace{1cm} (14)

The shadow value of the borrowing constraint introduces a wedge in the consumption Euler equation

$$\Xi = \frac{1 - \beta R}{R}.$$ \hspace{1cm} (15)

The resource constraint for the simplified economy is

$$C = Y - (R - 1)B.$$ \hspace{1cm} (16)

Finally, the borrowing constraint at equality requires that debt is equal to a fraction of the discounted real value of the housing stock

$$B = \frac{\Theta Q}{R}.$$ \hspace{1cm} (17)

Holding consumption constant, a permanent increase in the LTV ratio $\Theta$ increases the real value of the housing stock (equation 14). At the same time, a higher LTV ratio increases foreign debt (equation 17). The increase of real house prices amplifies this mechanism. These two effects (higher house prices and higher debt) are mitigated by the drop in consumption due to the fact that higher foreign debt must be eventually paid back by running trade surpluses. Along the transition, or if the positive shock to the LTV ratio is persistent but not permanent, consumption initially increases too because higher debt allows agents to spend more resources both on housing and goods consumption. The mitigating effect of intertemporal solvency on foreign liabilities kicks in only at a later stage.

\section*{4 Quantitative Results}

This section discusses the calibration of the parameters and presents the central quantitative experiment of the paper – a relaxation of the borrowing constraint parameter $\Theta$.

\footnote{See Boz and Mendoza (2011) for a quantitative analysis of the dynamics of this economy when agents must learn the true persistence of the LTV ratio parameter, which follows a two-state Markov process.}
4.1 Calibration

The Foreign discount factor pins down the steady state real return on the internationally traded asset. A target of 4% for the annualized real return implies $\beta^* = 0.99$. The Home country is a net borrower in international financial markets because of a lower discount factor ($\beta = 0.98$).

The coefficient of risk aversion $\sigma$ and the inverse Frisch elasticity of labor supply are both set equal to 2, within the range of common practice in macroeconomics (see, for instance, Hall, 2010). Also standard are the values for the elasticity of substitution among goods and labor varieties ($\phi_p = \phi_w = 11$), which are calibrated to match steady state a 10% markup in both the goods and labor market. The price and wage stickiness parameters are chosen to match an average duration of price and wage contracts of four quarters ($\zeta_p = \zeta_w = 0.75$).

The parameters of the goods consumption basket are fairly standard in the international macroeconomics literature (see, for instance, Obstfeld and Rogoff, 2007). The domestic share of tradable consumption $\alpha$ is set to 0.7 (home bias). The elasticity of substitution between Home and Foreign tradable goods $\gamma$ equals 2.

The intratemporal elasticity of substitution between goods consumption and housing services $\varepsilon$ is set equal to one. A Cobb-Douglas specification of the aggregator $X_t$ is consistent with the micro evidence from the Decennial Census of Housing in Davis and Ortalo-Magné (2010), indicating that expenditure shares on housing are constant over time and across U.S. metropolitan areas. The calibration of this parameter is not uncontroversial. Section 6.2 discusses the robustness of the results to alternative values for this elasticity sometimes used in the literature.

Conditional on the elasticity of substitution, the parameter $\eta$ is chosen to match a consumption share of total expenditure of about 80%, which is in line with the average for the U.S. from 1929 to 2001 (Piazzesi, Schneider and Tuzel, 2007). The steady state consumption share of total expenditure in the Home country is

$$
\left(1 + \frac{QH}{C}\right)^{-1} = \left[1 + \left(\frac{1 - \eta}{\eta}\right) \frac{(H/C)^{1-\frac{1}{\gamma}}}{1 - \beta - \Xi \Theta}\right]^{-1}
$$

In the Cobb-Douglas case, the mapping between $\eta$ and the consumption share of total expenditure is independent of the stock of housing and the steady state level of consumption (except for the small indirect effect via the steady state Lagrange multiplier $\Theta$). The relative stock of housing is adjusted so that in steady state the level of house prices in the two countries is the same.

Based on the evidence in section 2, a steady state borrowing constraint parameter $\Theta$ of 70% seems to characterize quite appropriately the period before credit market deregulation. The stochastic process for the borrowing constraint is assumed to follow an AR(1) process with persistence close to one (0.9999) and i.i.d. innovations $\sim \mathcal{N}(0,1)$. The idea behind a near unit root process for the LTV ratio is to capture, in a reduced form, the “regime switch” effect.

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19 These values coincide with the assumed discount factors of savers and borrowers in the closed economy model of Monacelli (2009).

20 The Cobb-Douglas specification is the baseline case also in Fernandez-Villaverde and Kruger (2001), who study life-cycle consumption and portfolio decisions in a quantitative general equilibrium model with borrowing constraints.

21 A similar expression holds for the Foreign country, with the difference that $\Xi^* = 0$. 

15
emphasized in Boz and Mendoza (2011). Agents in the model perceive the financial deregulation process essentially as permanent (i.e. a new regime). This assumption plays a crucial role for the quantitative results. If agents anticipate that shocks to the borrowing constraint are mean-reverting, albeit fairly persistent, the boom in house prices becomes about 40% smaller. Conversely, the consumption boom remains about 90% of the baseline case. As the persistence of the financial liberalization process declines, agents switch their consumption from durables (housing) to non-durables (goods).

For simplicity, the steady state value of the terms of trade (and hence of the real exchange rate and the relative prices of Home and Foreign tradable goods) are normalized to one by appropriately picking the steady state productivity ratio $A/A^*$. Finally, the targets and parameters of the monetary policy rule take fairly conventional values (e.g. Galí and Gertler, 2007). The inflation target is normalized to zero and the target for output is its steady state value. The interest rate smoothing parameter $\rho_i$ is set equal to 0.7. As in Taylor (1993), the response to inflation $\psi_\pi$ equals 1.5 while the response to output $\psi_y$ equals 0.5.

### 4.2 The Effects of Relaxing the Borrowing Constraint

The model is approximated up to the first order about the asymmetric steady state described in section 3.1. Appendix A.4 lists the system of log-linear equations that characterize the equilibrium.

The financial deregulation experiment corresponds to a relaxation of the collateral constraint parameter $\Theta$ so that households can borrow a higher fraction of the expected value of their house. The increase in the LTV ratio occurs gradually over time – a reduced-form approach to capture the possibility that in reality households slowly “learned” the transition to a new regime of more relaxed credit standards. The borrowing constraint parameter starts at 70% in the initial steady state and progressively moves up to 99% over a five-year horizon (2001-2005). At its peak, the borrowing constraint parameter takes more extreme values than the evidence on the median LTV ratios suggest. However, these extreme values capture the fact that by the end of 2005 the marginal borrowers were mostly in the subprime segment and were often able to obtain a mortgage with zero down-payments. Additionally, while the model has a stationary population of households who continuously refinance their loans, in practice high LTVs allowed many new borrowers who previously could not afford a loan to become homeowners.

Figure 3 shows the results of this experiment for a number of selected variables. A higher value of $\Theta$ corresponds to a higher LTV ratio in the Home country. Because of the financial deregulation process, households in the Home country can now borrow more. By construction, borrowing occurs in international financial markets only. Therefore, foreign debt increases (top-left) and the current account turns negative (middle-left). At the same time, higher leverage translates into higher demand for consumption of both goods (middle-right) and housing.

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22To obtain a profile of house prices that resembles the data, the borrowing constraint parameter remains at its peak value for one year (2006) and returns toward its initial steady state over the next five years (2007-2011). The results are not particularly sensitive to the assumption about the speed of reversion to the initial steady state. Households perceive changes in the value of the borrowing constraint as permanent due to its near unit root process.
Because the stock of housing is fixed, house prices absorb the adjustment in full (top-right), providing the endogenous component to the relaxation of borrowing constraints. As resources flow into the Home country and the current account turns negative, the real exchange rate appreciates (bottom-left), while external imbalances are partly mitigated by the increase in the real interest rate (bottom-right).

The simulation accounts for about 2/3 of the increase in the real FHFA house price index reported in figure 1 and for almost 1/2 of the deterioration of the U.S. current account between 2001q1 and 2005q4. The model generates an almost perfect (-0.98) negative correlation between house prices and the current account balance. Interestingly, the simulation is also broadly consistent with two other features of the data before the recent crisis: (i) a level of consumption well above trend for the entire duration of the house price boom and deterioration of the current account and (ii) the appreciation of the real dollar against a basket of currency of U.S. trading partner, at least for the early 2000s.

The main counterfactual feature of the simulation in figure 3 is the behavior of the real interest rate. In the model, the real interest rate increases because the relaxation of the collateral constraint stimulates aggregate demand and induces households to anticipate their consumption. However, all available empirical measures point to a decline in short and long-term real interest rates during the early 2000s (see, for example, figure 4).

In the recent literature on global imbalances, the persistent drop in the real interest rate is a consequence of the “saving glut” that originated in Asian economies after the financial crisis of the late 1990s (Bernanke, 2005; Caballero, Fahri and Gourinchas, 2008a). A few papers that investigate the role of lower collateral requirements for the negative correlation between house prices and current account have also built up on this idea. Favilukis, Ludvigson and Van Nieuwerburgh (2011) explicitly combine exogenous capital flows from the rest of the world with a relaxation of borrowing constraints and lower transaction costs. Boz and Mendoza (2011) and Adam, Kuang and Marcet (2011) justify the use of a small open economy model based on the idea that saving glut shocks in the rest of the world balance the upward pressure on the real interest rate deriving from financial deregulation. The next section explores a different (and potentially complementary) rationale for low real interests at the world level.

5 The Role of Monetary Policy in the U.S. and Abroad

The financial deregulation process that started in the U.S. starting in the late 1990s, and that gained full traction during the first half of the 2000s, can account for the strong negative correlation between house prices and the current account. However, this explanation implies a counterfactual path for the real interest rate. The global saving glut hypothesis, either in isolation or in conjunction with other stories, provides one rationale for the low real interest rates observed in the data. This section investigates the role of monetary policy as an alternative

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23 To the extent that house prices in the model reflect the value of land, the increase generated in the simulation is consistent with the finding in Davis and Heathcote (2007) that the value of land, and not the value of structures, accounts for most of the run up in house prices observed in the data.

24 One caveat is that part of this decline may be attributable to a longer trend (see Ferrero, 2010).
Figure 3: Simulated path of selected variables in response to a temporary change in the borrowing constraint parameter $\Theta$ from 70% to 99% over five years.
Figure 4: The short-term real rate is the nominal yield on a 1-year T-bill minus expected inflation from the Survey of Professional Forecasters. TIPS are U.S. Treasury Inflation Protected Securities that pay a given interest rate (the implied real interest rate) plus the realized CPI.
mechanism that keeps world real rates low.

The basic idea is that, if inflation expectations are well-anchored, a central bank that sets the nominal interest rate essentially controls the real rate. Some observers (most notably Taylor, 2008) have argued that the Federal Reserve, as well as central banks in other countries, kept nominal interest rates artificially low for too long after the 2001 recession. According to this interpretation, monetary policy shocks may have contributed to stimulate demand beyond what would be normally considered appropriate according to a standard interest rate rule (Taylor, 1993). Therefore, monetary policy may be responsible not only for low interest rates but also for generating the boom in house prices and contributing to the deterioration of the current account.

While the role of the dollar as a reserve currency may justify the prominent role of U.S. monetary policy in influencing the world real interest rate, overly accommodative U.S. monetary policy alone may not be enough to keep the world real interest rate low for a prolonged period. One notable feature of the late 1990s and early 2000s period is that several emerging market economies (which also financed the U.S. current account deficit) were pegging their nominal exchange rate to the dollar, thus effectively importing U.S. monetary policy. In this environment, low U.S. interest rates spread globally as pegging countries lose their control on domestic interest rates.

The question then becomes whether foreign exchange rate pegs have exacerbated the magnitude of the adjustment due to domestic (U.S. regulatory and monetary policy) factors.

The next two sections formalize these ideas in the context of the model.

5.1 Easy U.S. Monetary Policy

Figure 5 compares the effective Federal Funds Rate (FFR) in blue with the nominal interest rate predicted by a standard interest rate rule (Taylor, 1993), similar to the linearized version of equation (12) in the model

\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) (\psi_\pi \pi_t + \psi_y y_{ht}) + \epsilon_t, \]

where \( i_t \) is the effective FFR, \( \pi_t \) is the year-over-year CPI inflation rate and \( y_t \) is the deviation of real GDP from potential output as measured by the CBO. The difference between the red line and the green line is the value of the smoothing parameter \( \rho_i \), which is set equal to zero in first case and equal to 0.7 as in the baseline calibration in the second case.

Figure 5 captures the essence of the criticism in Taylor (2008). Between 2001 and 2005, U.S. monetary policy was excessively accommodating compared to the prescriptions of an interest rate rule that characterized well monetary policy in the previous two decades. According to this view, easy monetary policy is a primary suspect for the house price boom.

The U.S. is not the only country with significant deviations from a standard monetary policy rule. Taylor (2008) presents evidence on the correlation between housing investment and deviations from a Taylor rule among European countries.\(^{26}\)

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\(^{25}\)Countries can retain some control on domestic monetary policy while pegging their exchange rate by imposing restrictions on foreign capital flows, as in the case of China.

\(^{26}\)Deviations from the Taylor rule differ among Euro countries because inflation and output gaps are country-
largest deviations have also the highest changes in housing investment as percentage of GDP. These countries are also the very same with a high correlation between house price and current account changes during the period 2001-2006.

One limitation of this argument is that the correlation between the departures from a standard Taylor rule and house price appreciation in the cross section is much weaker than for residential investment (Bernanke, 2010). While this evidence may question the importance of easy monetary policy in causing the housing boom, low interest rates may still play an important role as an amplification mechanism (Adam, Kuang and Marcet, 2011). Furthermore, combining a relaxation of borrowing constraints with an easy monetary policy stance allows for a quantitative evaluation of the relative importance of these two potential explanations for the boom in house prices and the deficit on the current account.

Figure 6 compares the baseline financial deregulation experiment (dashed red line) with a simulation (continuous blue line) that combines the relaxation of borrowing constraints with the monetary policy shocks calculated as departures of the effective FFR from the prescriptions of (18).\(^{27}\)

The figure highlights how little monetary policy shocks contribute to house price appreciation and the deterioration of the current account. The process of financial deregulation remains the driving force. Monetary policy shocks, however, play an important role in the dynamics of consumption, the real interest rate and the real exchange rate. Because inflation expectations

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\(^{27}\)For consistency between model and data, the series of monetary policy shocks is calculated using (18) with \(\rho_i = 0.7\).
Figure 6: Simulated path of selected variables in response to the baseline change in the borrowing constraint parameter combined with monetary policy shocks derived from the Taylor rule (continuous blue line). The dashed red line corresponds to the change in the borrowing constraint parameter only.
are anchored (the systematic part of the monetary policy rule is unchanged), low nominal rates translate into low real rates (bottom-right panel) which stimulate consumption on top of the boost provided by the financial deregulation process. The depreciation of the real exchange rate largely reflects the depreciation of the nominal exchange rate induced by the expansionary monetary policy shocks in the Home country.

The limited role of monetary policy shocks in accounting for the correlation between house prices and the current account is even more strikingly visible from figure 7, that compares the simulation with both LTV and monetary policy shocks (continuous blue line) against a simulation with monetary shocks only (dashed red line). At a qualitative level, easy monetary policy does generate a negative correlation between house prices and the current account, although significantly smaller than in the data (equal to -0.29). The magnitudes of the changes in each of the variables, however, are negligible. House prices increase slightly more than 2% at the peak while the maximum current account deficit is 0.2%. Conversely, monetary policy shocks lead to a much larger boom in consumption than in the baseline experiment.

Another limitation of considering only monetary policy shocks as the main driver of the adjustment process is the unequivocal depreciation of the real exchange rate. This feature characterizes, except for few periods, also the simulation that combines monetary policy and LTV shocks. The intuition is simple. As monetary policy becomes excessively accommodative in the U.S., the real value of the dollar tends to depreciate due to the uncovered exchange rate parity condition. To the extent that in the data such a relation is systematically violated, this model confronts the same issue as the vast majority of open economy macroeconomic frameworks. This problem, however, will not be present in the next section where the Foreign country is assumed to peg its exchange rate to the Home currency.

5.2 Foreign Exchange Rate Pegs

One important feature of the international monetary system since the early 2000s is the fact that many emerging economies, mostly in East Asia (and most notably China) have pegged their exchange rate to the U.S. dollar. Dooley, Folkerts-Landau and Garber (2008) have called this extensive peg arrangement “Bretton Woods II”. In their view, pegged exchange rates among fast-growing, export-oriented economies are responsible for the large external imbalances between the U.S. and the rest of the world.

The intuition is that pegged exchange rates keep foreign currencies significantly below their true market value, hence stimulating exports and growth abroad. From the perspective of the emerging economies, the peg may be a reasonable policy. The consequence for the U.S., however, has been a series of widening current account deficits. For the purpose of this paper, the key question is how much foreign exchange rate pegs have contributed to exacerbate the boom in house prices generated by the financial liberalization process.

Figure 8 compares the simulation with LTV and monetary policy shocks under a flexible exchange rate as in the previous section (continuous blue line) with the case (dashed red line) in which the monetary authority of country F follows an exchange rate peg ($E_t = \bar{E}$).

The fixed exchange rate arrangement in the rest of the world does not change the dynamics
Figure 7: Simulated path of selected variables in response to the baseline change in the borrowing constraint parameter combined with monetary policy shocks derived from the Taylor rule (continuous blue line). The dashed red line corresponds to the monetary policy shocks only.
Figure 8: Simulated path of selected variables in response to the baseline change in the borrowing constraint parameter combined with monetary policy shocks derived from the Taylor rule under flexible (continuous blue line) and fixed (dashed red line) exchange rates.
of house prices and the current account. The real interest rate, however, substantially drops by about 4% at the trough. This magnitude is fairly consistent with the data (compare with the top panel of figure 4). Similarly, the real exchange rate significantly appreciates, roughly by 3%. The appreciation of the real exchange rate limits the consumption boom, especially during the two years of the simulation.

A pure peg in the rest of the world may be an extreme characterization of the true international system of exchange rates. A perhaps more realistic approach would be to assume that the Foreign country Taylor rule places a non-zero weight on the depreciation of the nominal exchange rate

\[ i_t^* = \rho_i i_{t-1}^* + (1 - \rho_i)(\psi_{\pi_t^*} + \psi_{y_y} y_t) - \psi_e \Delta e_t + \varepsilon_{it}^*, \]

with \( \psi_e > 0 \). In this case, not surprisingly, an intermediate adjustment would obtain. For example, if \( \psi_e = 3 \), both the appreciation of the real exchange rate and the drop in the real interest rate are 1% smaller than in the case of a pure peg, while the increase in consumption is about 1% bigger.

All in all, assuming the Foreign country pegs its nominal exchange rate to the Home currency (whether fully or partially) helps the model align better with the data in terms of the behavior of the real interest rate and of the real exchange rate, without substantially affecting the dynamics of house prices and the current account.

5.3 Implications for Inflation and Output

Most of the analysis so far has focused on the evolution of international variables, such as the current account and the real exchange rate, and asset prices, such as house prices and the real interest rate. Because monetary factors (shocks and exchange rate regime) play an important role for the results, this section discusses some implications for domestic variables, such as inflation and output, which are at the core of the Federal Reserve’s mandate.

5.3.1 Inflation

An important aspect of the period 2000-2005 is the behavior of inflation. Part of the reason why the Federal Reserve decided to keep nominal interest rates low for “a considerable period” was the deflationary scare of 2001, when year-over-year headline CPI inflation declined from 3.4% to 1.2% within twelve months. If monetary policy was indeed overly accommodative, the excessive stimulus could have induced inflation to take off. In the data, inflation remained well contained, moving from the lows of late 2001 back up to 3% over the course of the next four years.

Figure 9 shows that the simulation produces a path of inflation fairly consistent with the data, except for the disinflationary pressures that are not part of the model. Over the five-year horizon of the simulation, CPI inflation rises between 1 and 1.5% on an year-over-year basis.\(^{28}\)

\(^{28}\)Before the financial deregulation experiment inflation is assumed to be at its steady state value. Therefore, year-over-year inflation equals zero for the first three quarters of the simulation.
Figure 9: Year-over-year percentage change of headline Consumer Price Index Inflation in the data (top) and in the model (bottom) under flexible exchange rates (continuous blue line) and under foreign peg (dashed red line).
Under flexible exchange rates, the increase progressively continues after the initial spike as a consequence of the expansionary monetary policy shocks and the depreciation of the exchange rate. Conversely, if the Foreign country pegs its nominal exchange rate to the Home currency, domestic CPI stabilizes around 1% after the initial increase. The peg prevents the nominal exchange rate from depreciating, thus moderating the increase in inflation.

5.3.2 Output

A second implication for domestic variables is related to the potential response of monetary policy to asset prices. Whether central banks should set interest rates taking into account exchange rate or stock market movements has been the subject of a long-standing debate. The consensus before the recent financial crisis, both among academics and policymakers, had been that monetary policy acts more effectively by “mopping after the fact” (Bernanke and Gertler, 2001; Greenspan, 2002).\(^{29}\) The crisis has led several observers to reconsider the consensus (see, for example, Rudebusch (2005) for an early dissenting argument). Perhaps, the Federal Reserve could have prevented the excessive house price appreciation of the first half of the 2000s by increasing the FFR early on.

A simple approach to evaluate this hypothesis is to augment the baseline interest rate rule

\(^{29}\)Albeit dominant, this view was far from universal even before the recent crisis. See, for instance, Roubini (2006) and the references therein.
with a response to house price appreciation

\[ i_t = \rho_i i_{t-1} + (1 - \rho_i)(\psi_{\pi} \pi_t + \psi_{y} y_{ht}) + \psi_q \Delta q_t + \varepsilon_{it}, \tag{19} \]

with \( \psi_q > 0 \). As house prices rise, the central bank hikes the nominal interest rate. Besides the standard channel, the monetary contraction makes debt more costly for households, hence dampening the increase in debt.

Consider the same financial deregulation experiment as in the original simulation, hence abstracting from any expansionary monetary policy shocks. Suppose further the central bank in the Home country follows the modified interest rate rule (19) while the Foreign country pegs its nominal exchange rate to the domestic currency. The rest of the model remains unchanged. The feedback parameter \( \psi_q \) is chosen so that the maximum tolerated increase in house prices is 10\%, half of the increase absent any response.

Figure 10 shows the behavior for output in this case. Clearly, monetary policy is contractionary enough to eventually induce a recession. The consequences of the central bank response to house prices are even more dramatic for inflation, which becomes substantially negative. 30

6 Robustness

This section shows that the results are robust to variations in a number of parameters of particular interest.

6.1 The Role of Nominal Rigidities

The negative correlation between house prices and the current account does not rely on the presence of nominal rigidities. These two variables largely reflect real forces that are independent of whether prices and wages are sticky or not. Nominal rigidities, however, do play a role in the adjustment of consumption, output and inflation. This result is intuitive. By limiting the adjustment of prices and wages to the financial deregulation experiment, nominal stickiness leads to a boom in aggregate demand which translates into higher consumption and domestic production.

Figure 11 compares the evolution of consumption, output and inflation in the Home country under the baseline financial deregulation experiment with (continuous blue line) and without (dashed red line) nominal rigidities. The consumption boom discussed earlier essentially disappears when prices and wages are fully flexible. Absent nominal rigidities, the higher volatility in prices and wages reduces the need for changes in real quantities. In this case, output in country H actually falls because Home goods become more expensive much faster. As a result, the increase in inflation becomes non-negligible.

Interestingly, this simulation also highlights how the deviations of inflation from trend discussed in the previous section and illustrated in the bottom panel of figure 9 almost entirely depend on monetary shocks. The model, therefore, attributes a very clear-cut role to financial

30One caveat is that the price index considered here to calculate headline inflation puts no weight on house prices.
deregulation in explaining the boom in house prices and the current account deficits, and to monetary policy in explaining the increase in inflation.

6.2 Goods and Housing: Complements or Substitutes?

As mentioned earlier, the value of the elasticity of substitution between consumption of goods and housing services is the subject of an open debate. While several papers have adopted a Cobb-Douglas specification (i.e. an elasticity equal to one) like in the baseline calibration, values both higher and lower than one have been used.

Piazzesi, Schneider and Tuzel (2007) argue that a Cobb-Douglas formulation for $X_t$ may be too restrictive. Using annual U.S. data since 1929, these authors show that the non-housing share of total consumption is not constant, although its volatility is fairly low. Their calibration focuses on values of $\varepsilon$ slightly bigger than one, consistent with the estimates in Ogaki and Reinhart (1998) that lie in the 95% confidence interval $[1.04, 1.43]$.

At the opposite end of the spectrum, Lustig and Van Nieuwerburgh (2004) need a low value of the intertemporal elasticity to match the volatility of U.S. rental prices in an asset pricing model with housing collateral. These authors choose a benchmark is $\varepsilon = 0.05$ and explore values up to 0.75.

Figure 12 repeats the baseline experiment for values of the elasticity equal to 0.05 (dashed green line), 1 (continuous blue line – the benchmark calibration) and 1.5 (dashed-dotted red line).
Figure 12: Robustness of baseline simulation to the elasticity of substitution between consumption of goods and housing services $\varepsilon$. 
An elasticity higher than one, at the upper bound of the confidence interval in the estimates of Ogaki and Reinhart (1998), does not produce significant differences compared to the Cobb-Douglas benchmark calibration. Some differences are evident if the elasticity of substitution is very small, in line with the value in Lustig and Van Nieuwerburgh (2004). A low elasticity of substitution generates a smaller initial increase in house prices. The house price boom, however, continues well after the financial deregulation process is over (about ten quarters). In this case, house prices rise by 40% at the peak. The deterioration of the current account is also larger, in spite of a smaller increase in consumption. The reason is that domestic output falls, due to the larger appreciation of the real exchange rate. Finally, a further consequence of the more moderate behavior of consumption is the modest rise in the real interest rate.

The bottom line is that the results are quite robust to alternative values of the elasticity of substitution between consumption of goods and housing services, a fairly controversial parameter in the literature. If anything, low values of this elasticity generate larger quantitative effects of financial deregulation on house prices and the current account than in the baseline calibration and limit the counterfactual increase in the real interest rate.

6.3 Other Parameters

The results are also robust to changes in several other parameters. For example, changes in the elasticity of substitution between Home and Foreign goods $\gamma$ are almost irrelevant for the dynamics.

Lower degrees of relative risk aversion $\sigma$ imply more responsiveness of Foreign consumption to domestic financial liberalizations (and vice versa). Interestingly, however, Home consumption is much less sensitive to changes in this parameter. For the Home country, the wedge in the Euler equation introduced by the borrowing constraint absorbs most of the adjustment. The other variables are almost unaffected by different calibrations of this parameter.

Changes in the inverse Frisch elasticity of labor supply $\nu$ matter, mostly for inflation, if this parameter takes a relatively low value (e.g. close to one). In this case, domestic inflation in both countries becomes more sensitive to the financial deregulation experiment because the wealth effect associated with the shock reduces labor supply, thus increasing firms marginal costs. Remember, however, that in the absence of monetary policy shocks, the inflationary consequences of relaxing LTV ratio are modest. The dynamics of house prices and the current account coincide with the baseline case.

Values of the share of domestic consumption goods $\alpha$ in the range $[0.6, 0.8]$ also do not substantially alter the main picture. Again, the evolution of house prices and the current account in response to the baseline financial deregulation experiment is unchanged. Domestic consumption responds more the higher home bias is. Consequently, the real interest rate increases more and the appreciation of the Home currency is larger. When home bias is large, the Home country experiences some deflationary pressures, partly driven by the appreciation of the domestic currency. Additionally, the high degree of home bias implies that the consumption boom induced by the relaxation of borrowing constraints mostly pertains to domestic goods. The monetary authority leans against the higher demand increasing the nominal interest rate. As a consequence,
domestic inflation falls in response to the more aggressive response of monetary policy.

7 Conclusions

A relaxation of borrowing constraints, in the form of lower collateral requirements, can explain a significant fraction of the increase in U.S. house prices and, at the same, give rise to substantial external imbalances. This explanation rationalizes the negative correlation between house prices and current account balances in the U.S. and in several other developed and developing economies. The counterfactual implication of this story, however, is that financial deregulation shocks lead the real interest rate to increase, contrary to the data. The two empirical observations, negative correlation between house prices and current account and low real rates, can be reconciled by considering accommodative monetary policy shocks as departures of the nominal interest rate from a conventional monetary policy rule. An exchange rate regime based on foreign pegs to the dollar exports U.S. monetary policy to the rest of the world, amplifying the effect of domestic shocks.

Except for the role of foreign exchange rate pegs, this explanation of house prices booms and current account deficits has its origins in U.S. policies. This approach contrasts with recent explanations based on the idea of a foreign saving glut. The two theories are not mutually exclusive. If interpreted as a preference shock (more patient foreign households), the foreign saving glut has the effect of further depressing the real interest rate, thus strengthening the mechanism at play in this paper. A more structural interpretation of the foreign saving glut phenomenon would require explicit modeling of the securitization process that generates safe assets in the U.S. but not elsewhere. Nevertheless, even in this case, the effects of financial flows from the rest of world would likely amplify the consequences of looser borrowing constraints and monetary policies in the home country.
References


A  Optimality, Equilibrium, Steady State and Approximation

This appendix presents details on the derivation of the optimality conditions for the Home country representative households and firms, lists the equilibrium conditions, briefly discusses the asymmetric steady state and finally provides the first order approximation of the system of equations that characterizes the equilibrium.

Given the assumption of a representative household in each country, borrowing and lending occurs in equilibrium only at the international level. In what follows, the borrowing constraint is always assumed to bind for the Home economy and never for the Foreign economy.

A.1 Optimality Conditions for Households and Firms

Cost Minimization

Expenditure minimization determines the allocation of total consumption between Home and Foreign tradable goods as a function of their relative prices and total demand. Formally, the problem is

\[
P_t C_t = \min_{C_{ht}, C_{ft}} P_{ht} C_{ht} + P_{ft} C_{ft},
\]

subject to (3). The first order conditions for this problem are

\[
C_{ht} = \alpha \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} C_t \quad \text{and} \quad C_{ft} = (1 - \alpha) \left( \frac{P_{ft}}{P_t} \right)^{-\gamma} C_t,
\]

(20)

where the resulting price of the aggregate consumption bundle \( P_t \) is

\[
P_t = \left[ \alpha P_{ht}^{1-\gamma} + (1 - \alpha) P_{ft}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}.
\]

(21)

Final goods producers are perfectly competitive. Their cost minimization problem generates the demand for intermediate goods. The problem for these firms is

\[
P_{ht} Y_{ht} = \min_{Y_t(h)} \int_0^1 P_t(h) Y_t(h) dh,
\]

subject to (8). The first order condition for this problem is

\[
Y_t(h) = \left[ \frac{P_t(h)}{P_{ht}} \right]^{-\phi_p} Y_{ht},
\]

(22)

where the implied price index of the tradable bundle \( P_{ht} \) is

\[
P_{ht} = \left[ \int_0^1 P_t(h)^{1-\phi_p} dh \right]^{\frac{1}{1-\phi_p}}.
\]

(23)
Labor agencies are also perfectly competitive. Their cost minimization problem generates the demand for differentiated labor inputs. The problem for these firms is

$$W_t L_t = \min_{L_t(i)} \int_0^1 W_t(i) L_t(i) di,$$

subject to (6). The first order condition for this problem is

$$L_t(i) = \left[ \frac{W_t(i)}{W_t} \right]^{-\frac{1}{\phi_w}} L_t,$$  \hfill (24)

where $W_t$ is the implied aggregate wage index

$$W_t = \left[ \int_0^1 W_t(i)^{1-\phi_w} di \right]^{\frac{1}{1-\phi_w}}.$$  \hfill (25)

**Utility Maximization**

The representative household maximizes utility (1) subject to the budget constraint (4) and the borrowing constraint (5). Let $\beta_t \lambda_t$ and $\beta_t \lambda_t \Xi_t$ be the lagrange multipliers on the two constraints. Workers operate in monopolistic competition taking the demand for their generic labor input as given. Therefore, equation (24) becomes an additional constraint for the household problem.

The first order condition for consumption is

$$\eta X_t^{\frac{1}{\gamma} - \sigma} C_t^{-\frac{1}{\gamma}} - \lambda_t P_t = 0.$$  \hfill (26)

The first order condition for housing services is

$$(1 - \eta) X_t^{\frac{1}{\gamma} - \sigma} H_t^{-\frac{1}{\gamma}} - \lambda_t Q_t + \beta E_t(\lambda_{t+1} Q_{t+1}) + \lambda_t \Xi_t E_t(Q_{t+1}) = 0.$$  \hfill (27)

The first order condition for debt is

$$\lambda_t - \beta(1 + i_t) E_t(\lambda_{t+1}) - \lambda_t \Xi_t(1 + i_t) = 0.$$  \hfill (28)

Wages are set on a staggered basis (Calvo, 1983). The probability of not being able to adjust the wage is $\zeta_w$. The optimality condition for a worker who is able to adjust the wage at time $t$ is

$$E_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_w)^s L_{t+s}(i) \left[ \lambda_{t+s} \tilde{W}_t(i) - \frac{\phi_w}{\phi_w - 1} L_{t+s}(i)^{\nu} \right] \right\} = 0,$$  \hfill (29)

where $\tilde{W}_t(i)$ is the optimal reset wage at time $t$ conditional on no future adjustments. Using the labor demand equation (69) and the expression for the marginal utility of consumption (26)
into the previous expression yields
\[ \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta w)^s \left[ \bar{W}_t(i) \left( \frac{\bar{W}_t(i) L_{t+s}}{P_{t+s}} - \frac{\phi_w}{\phi_w - 1} \left( \frac{\bar{W}_t(i) L_{t+s}}{P_{t+s}} \right)^{(1+\nu)} \right) \right]^s \right\} = 0. \] (30)

Equation (30) can be rearranged as to express the relative wage of type \( i \) as a function of the ratio between the present discounted value of the marginal disutility of labor and the present discounted value of the real wage in units of marginal utility of consumption
\[ \left( \frac{\bar{W}_t(i)}{W_t} \right)^{1+\phi_w \nu} = \frac{K_{wt}}{F_{wt}}. \] (31)

The terms on the right-hand side of the last expression can be written recursively as
\[ K_{wt} = \frac{\phi_w}{\phi_w - 1} L_t^{1+\nu} + \beta \zeta w \mathbb{E}_t \left[ (\Pi_{wt+1})^{\phi_w (1+\nu)} K_{wt+1} \right] \] (32)
and
\[ F_{wt} = \eta X_t^{\frac{1}{\epsilon}} C_t^{\frac{-\sigma}{\epsilon}} L_t \frac{\bar{W}_t}{P_t} + \beta \zeta w \mathbb{E}_t \left[ (\Pi_{wt+1})^{\phi_w - 1} F_{wt+1} \right], \] (33)
where \( \Pi_{wt} \equiv W_t/W_{t-1} \) represents wage inflation. Expressions (31)-(33) show that the optimal choice of household members who optimally reset their wage in any given period is a function of aggregate variables only. Therefore, in a symmetric equilibrium, all household members who are able to reset their wage at time \( t \) make the same choice, i.e. \( \bar{W}_t(i) = \bar{W}_t \). The aggregate wage index (25) can then be rewritten as to link the optimal reset relative wage to wage inflation
\[ \zeta w (\Pi_{wt})^{\phi_w - 1} + (1 - \zeta w) \left( \frac{\bar{W}_t}{W_t} \right)^{1-\phi_w} = 1. \] (34)

Using the first order condition for consumption (26), the first order conditions for housing services (27) becomes
\[ Q_t = \frac{1 - \eta}{\eta} \left( \frac{H_t}{C_t} \right)^{-\frac{1}{\epsilon}} + \beta \mathbb{E}_t \left[ X_t^{\frac{1}{\epsilon}} C_t^{\frac{-\sigma}{\epsilon}} \left( \frac{X_{t+1}}{C_{t+1}} \right)^{-\frac{1}{\epsilon}} \right] Q_t + \Xi_t \Theta_t \mathbb{E}_t (\Pi_{t+1} Q_{t+1}), \] (35)
where \( Q_t \equiv Q_t/P_t \) defines real house prices. Equation (35) consists of a standard part, according to which real house prices are equal to the marginal utility of housing services in units of marginal utility of consumption plus expected discounted future house prices, and a second part which measures the contribution of the borrowing constraint via the shadow price \( \Xi_t \).

Similarly, using again the first order condition for consumption (26), the first order condition
for debt (28) becomes

\[(1 + i_t)\Xi_t = 1 - \beta(1 + i_t)IE^t \left[ \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{\sigma}} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\sigma}} \frac{1}{\Pi_{t+1}} \right] \tag{36} \]

Equation (36) shows that the shadow price \(\Xi_t\) represents a wedge in the standard consumption Euler equation due to the borrowing constraint.

**No Arbitrage**

The representative household in the Foreign country solves the same maximization problem with one substantial difference. While the Foreign representative household can purchase Home debt, Foreign debt only circulates domestically. No arbitrage then implies the consumption-based uncovered interest parity condition

\[IE^t \left\{ \left( \frac{X^*_t}{X_t} \right)^{\frac{1}{\sigma}} \left( \frac{C^*_t}{C_t} \right)^{-\frac{1}{\sigma}} \frac{1}{\Pi_{t+1}} \left[ (1 + i^*_t) - (1 + i_t) \frac{E_t}{E_{t+1}} \right] \right\} = 0, \tag{37} \]

where \(E_t\) is the nominal exchange rate, defined as the price in Home currency of one unit of Foreign currency. Because of the representative household assumption, Foreign debt is in zero net supply in equilibrium. Additionally, the Foreign country is assumed to be a net saver in international financial markets so that the Foreign borrowing constraint to never bind \((\Xi^*_t = 0, \forall t)\).

**Profit Maximization**

The optimality condition for a firm able to adjust its price at time \(t\) is

\[IE^t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_p)^s \lambda_{t+s} Y_{t+s}(h) \left[ \hat{P}_t(h) - \left( \frac{\hat{W}_t}{\hat{W}_{t+s}} \right) \frac{1}{\frac{A}{A^s}} \right] \right\} = 0. \tag{38} \]

Using the demand for intermediate goods (22) and the expression for the marginal utility of consumption (26) into the previous expression yields

\[IE^t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_p)^s X^{\frac{1}{\sigma}}_{t+s} C^{-\frac{1}{\sigma}}_{t+s} \left[ \frac{\hat{P}_t(h)}{\hat{P}_{ht+s}} \right]^{-\phi_p} \frac{Y_{ht+s}}{A^s} \left[ \hat{P}_t(h) - \left( \frac{\hat{W}_t}{\hat{W}_{t+s}} \right) \frac{1}{\frac{A}{A^s}} \right] \right\} = 0. \tag{39} \]

As for wages, equation (39) can be rearranged as to express the optimal reset relative price of variety \(h\) as a function of the ratio between the present discounted value of the real marginal cost and the present discounted value of the real marginal revenues

\[\left[ \frac{\hat{P}_t(h)}{P_{ht}} \right] = \frac{K_{pt}}{F_{pt}}. \tag{40} \]

The terms on the right-hand side of the last expression can be written recursively as

\[K_{pt} = \frac{\phi_p}{\phi_p - 1} X^{\frac{1}{\sigma}} t C^{-\frac{1}{\sigma}}_t W_{ht} \frac{Y_{ht}}{AP_t} + \beta \zeta_p IE^t \left[ (\Pi_{ht+1})^{\phi_p} K_{pt+1} \right] \tag{41} \]
and
\[
F_{pt} = X_t^{1-\sigma} C_t^{-\frac{1}{2}} \frac{P_{ht}Y_{ht}}{P_t} + \beta \zeta_p \mathbb{E}_t \left[ (\Pi_{ht+1})^{\phi_p-1} F_{pt+1} \right].
\] (42)

Expressions (40)-(42) show that the optimal choice of firms who reset their price in any given period is a function of aggregate variables only. Therefore, in a symmetric equilibrium, all firms that reset their price at time \( t \) make the same optimal choice, i.e. \( \bar{P}_t(h) = \bar{P}_t \). The aggregate price index (23) can be rewritten as to link the relative price of variety \( h \) to price inflation

\[
\zeta_p (\Pi_{ht})^{\phi_p-1} + (1 - \zeta_p) \left( \frac{\bar{P}_t}{P_{ht}} \right)^{1-\phi_p} = 1,
\] (43)

where \( \Pi_{ht} \equiv P_{ht}/P_{ht-1} \) represents domestic inflation.

**Market Clearing**

The law of one price holds for tradable goods

\[
P_{ht} = \mathcal{E}_t P_{ht}^*.
\] (44)

Home bias, however, implies that purchasing power parity does not hold (i.e. \( P_t \neq \mathcal{E}_t P_t^* \)).

Final goods producing firms sell their products in the Home and Foreign market. Goods market clearing requires

\[
Y_{ht} = C_{ht} + C_{ht}^* = \alpha \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} C_t + (1 - \alpha) \left( \frac{P_{ht}^*}{P_t^*} \right)^{-\gamma} C_t^*,
\] (45)

where the second part of (45) uses (20) and its Foreign country counterpart.

As mentioned, the housing stock is fixed in both countries

\[
H_t = H \quad \text{and} \quad H_t^* = H^*.
\] (46)

Market clearing for financial assets requires

\[
B_t + B_t^* = 0,
\] (47)

where \( B_t^* \) represents Foreign country holdings of international debt.

**A.2 Equilibrium**

The goods market equilibrium pins down Home and Foreign consumption as a function of relative prices and the real exchange rate \( (S_t \equiv \mathcal{E}_t P_t^* / P_t) \)

\[
Y_{ht} = \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} [\alpha C_t + (1 - \alpha) S_t^\gamma C_t^*].
\] (48)
The Foreign country counterpart of the last equation is
\[ Y_{ft} = \left( \frac{P_{ft}}{P_{ft}^*} \right)^{-\gamma} \left[ (1 - \alpha)S_t^{-\gamma}C_t + \alpha C_t^* \right]. \] (49)

Real house prices are
\[ Q_t = \frac{1 - \eta}{\eta} \left( \frac{H}{C_t} \right)^{-\frac{1}{\eta}} + \beta E_t \left[ \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{\lambda}} \left( \frac{C_{t+1}}{C_t} \right) - \frac{1}{\Pi_{t+1}} \right] + \Xi_t \Theta_t E_t (\Pi_{t+1} Q_{t+1})\] (50)
The Foreign counterpart of equation (50) is
\[ Q^*_t = \frac{1 - \eta}{\eta} \left( \frac{H^*}{C^*_t} \right)^{-\frac{1}{\eta}} + \beta^* E_t \left[ \left( \frac{X^*_{t+1}}{X^*_t} \right)^{\frac{1}{\lambda}} \left( \frac{C^*_{t+1}}{C^*_t} \right) - \frac{1}{\Pi^*_t} \right] Q^*_{t+1}. \] (51)

Differently from the Home economy, the borrowing constraint never binds in the Foreign country, therefore \( \Xi^*_t = 0 \) at all times.

The borrowing constraint (5) pins down the stock of internationally-traded real debt \( B_t \equiv B_t/P_t \)
\[ (1 + i_t)B_t = \Theta_t E_t (Q_{t+1} H \Pi_{t+1}). \] (52)
The shadow price of the borrowing constraint is
\[ (1 + i_t) \Xi_t = 1 - \beta (1 + i_t) E_t \left[ \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{\lambda}} \left( \frac{C_{t+1}}{C_t} \right) - \frac{1}{\Pi_{t+1}} \right]. \] (53)

No arbitrage pins down the return in international financial markets
\[ E_t \left[ \left( \frac{X_{t+1}}{X_t} \right)^{\frac{1}{\lambda}} \left( \frac{C_{t+1}}{C_t} \right) - \frac{1}{\Pi_{t+1}} - \frac{1 + i_t}{\Pi_{t+1}} \frac{S_t}{S_{t+1}} \right] = 0, \] (54)
while the Euler equation for the Foreign country pins down the return in the Foreign country
\[ 1 = \beta^* (1 + i_{t}^*) E_t \left[ \left( \frac{X^*_{t+1}}{X^*_t} \right)^{\frac{1}{\lambda}} \left( \frac{C^*_{t+1}}{C^*_t} \right) - \frac{1}{\Pi^*_{t+1}} \right]. \] (55)
The wage determination process yields a non-linear wage Phillips curve, which combines the optimal choice of household members who reset their wage in any given period and their mass with the aggregate wage index
\[ \left( \frac{1 - \zeta_w \Pi_{wt}^{-1}}{1 - \zeta_w} \right)^{\frac{1 + \phi_{ww}}{1 + \phi_{ww}}} = K_{wt}/F_{wt}. \] (56)
According to expression (56), wage inflation \( \Pi_{wt} = W_t/W_{t-1} \) is a non-linear function of the
present discounted value of the marginal disutility of labor $K_{wt}$

$$K_{wt} = \frac{\phi_w}{\phi_w - 1} L_t^{1+\nu} + \beta \zeta_w E_t \left[ \left( \Pi_{wt+1} \right)^{\phi_w (1+\nu)} K_{wt+1} \right]$$ (57)

and of the present discounted value of the real wage in units of marginal utility of consumption $F_{wt}$

$$F_{wt} = \eta X_t^{\frac{1}{\epsilon} - \sigma} C_t^{-\frac{1}{\epsilon}} \frac{W_t L_t}{P_t} + \beta \zeta_w E_t \left[ \left( \Pi_{wt+1} \right)^{\phi_w - 1} F_{wt+1} \right].$$ (58)

Price setting decisions yield a non-linear price Phillips curve, which combines the optimal choice of firms who reset their price in any given period and their mass with the price index for domestic tradable goods

$$\left( \frac{1 - \zeta_p \Pi_{ht}^{\phi_p - 1}}{1 - \zeta_p} \right)^{\frac{1}{1-\rho_i}} = \frac{K_{pt}}{F_{pt}}.$$ (59)

According to expression (59), inflation in the domestic tradable good sector $\Pi_{ht} \equiv P_{ht}/P_{ht-1}$ is a non-linear function of the present discounted value of real marginal costs $K_{pt}$

$$K_{pt} = \frac{\phi_p}{\phi_p - 1} X_t^{\frac{1}{\epsilon} - \sigma} C_t^{-\frac{1}{\epsilon}} \frac{W_t Y_{ht}}{AP_t} + \beta \zeta_p E_t \left[ \left( \Pi_{ht+1} \right)^{\phi_p} K_{pt+1} \right]$$ (60)

and of the present discounted value of real marginal revenues

$$F_{pt} = X_t^{\frac{1}{\epsilon} - \sigma} C_t^{-\frac{1}{\epsilon}} \frac{P_{ht} Y_{ht}}{P_t} + \beta \zeta_p E_t \left[ \left( \Pi_{ht+1} \right)^{\phi_p - 1} F_{pt+1} \right].$$ (61)

In each country, the central bank determines the inflation rate via the interest rate rule

$$(1 + i_t) = (1 + i_{t-1})^{\rho_i} \left[ (1 + i) \left( \frac{\Pi_t}{\Pi_{t-1}} \right)^{\phi_x} \left( \frac{Y_{ht}}{Y_{ht-1}} \right) \right]^{1-\rho_i} e^{\varepsilon_{it}},$$ (62)

and

$$(1 + i_t^*) = (1 + i_{t-1}^*)^{\rho_i} \left[ (1 + i) \left( \frac{\Pi_t^*}{\Pi_{t-1}^*} \right)^{\phi_x} \left( \frac{Y_{ft}}{Y_{ft-1}} \right) \right]^{1-\rho_i} e^{\varepsilon_{it}},$$ (63)

The law of motion of foreign debt (from the resource constraint) pins down the relative price

$$- B_t = - \frac{(1 + i_t) B_{t-1}}{\Pi_t} + \left( \frac{P_{ht}}{P_t} \right) Y_{ht} - C_t.$$ (64)

The world resource constraint pins down the real exchange rate

$$\left( \frac{P_{ht}}{P_t} \right) Y_{ht} + \left( \frac{P_{ft}^*}{P_t^*} \right) S_l Y_{ft} = C_t + S_l C_t^*.$$ (65)
Equations (71) to (65) characterize the equilibrium in terms of domestic relative prices and the real exchange rate. The terms of trade \( T_t \equiv P_{ft}/P_{ht} = P_{ft}^*/P_{ht}^* \) link domestic relative prices in the two countries

\[
\left( \frac{P_{ht}}{P_t} \right)^{-(1-\gamma)} = \alpha + (1 - \alpha)T_t^{1-\gamma} \quad \text{and} \quad \left( \frac{P_{ft}^*}{P_t^*} \right)^{-(1-\gamma)} = \alpha + (1 - \alpha)T_t^{-(1-\gamma)} \tag{66}
\]

Finally, first-differencing the definition of the real exchange rate allows to pin down the nominal exchange rate

\[
\frac{S_t}{S_{t-1}} = \frac{\varepsilon_t}{\Pi_t^*}. \tag{67}
\]

### A.2.1 Flexible Prices and Wages

If wages are flexible \( (\zeta_w \to 0) \), the optimal labor supply decision implies that the real wage equals the marginal disutility of labor in units of marginal utility of consumption

\[
\frac{W_t}{P_t} = \left( \frac{\phi_w}{\phi_w - 1} \right) \frac{L_t^\nu}{\eta X_t^{\frac{1-\sigma}{\nu}} C_t^{\frac{1}{\nu}}}. \tag{68}
\]

On the firm side, if prices are flexible \( (\zeta_p \to 0) \), optimality implies

\[
P_{ht} = \left( \frac{\phi_p}{\phi_p - 1} \right) \frac{W_t}{A}. \tag{69}
\]

Therefore, equilibrium in the labor market requires

\[
\left( \frac{P_{ht}}{P_t} \right) = \frac{\Phi L_t^\nu}{\eta AX_t^{\frac{1-\sigma}{\nu}} C_t^{\frac{1}{\nu}}}, \tag{70}
\]

or

\[
\left( \frac{P_{ht}}{P_t} \right) = \frac{\Phi Y_t^\nu}{\eta A^{1+\nu} X_t^{\frac{1-\sigma}{\nu}} C_t^{\frac{1}{\nu}}}, \tag{71}
\]

where the last equation makes use of the production function to eliminate labor and

\[
\Phi \equiv \left( \frac{\phi_w}{\phi_w - 1} \right) \left( \frac{\phi_p}{\phi_p - 1} \right).
\]

Similarly, the labor market equilibrium in the Foreign country gives

\[
\left( \frac{P_{ft}^*}{P_t^*} \right) = \frac{\Phi Y_t^\nu}{\eta A^{1+\nu} X_t^{\frac{1-\sigma}{\nu}} C_t^{\frac{1}{\nu}}}. \tag{72}
\]
A.3 Asymmetric Steady State

To build an asymmetric steady state in which country H is a net borrower but relative prices, terms of trade and real exchange rate are still equal to one, start with the assumption that the Home country representative household is relative more impatient ($\beta < \beta^*$). Assume that the borrowing constraint is binding for country H but not for country F ($\Xi > 0$ and $\Xi^* = 0$).

Nominal rigidities are absent in steady state. The Home country labor market equilibrium is

$$1 = \frac{\Phi Y_h^\nu}{A^{1+\nu}X^{\frac{1}{2}-\sigma}C^{1-\frac{1}{2}}}. \quad (73)$$

Equilibrium in the market for goods produced in the Home country is

$$Y_h = \alpha C + (1 - \alpha)C^*. \quad (74)$$

These two equations, together with their Foreign country counterpart, pin down $C$, $C^*$, $Y_h$ and $Y_f$ as a function of productivity and the housing stock (through $X$ and $X^*$). The appropriate choice of $A$ and $A^*$, conditional on the housing stock, ensures that in steady state relative prices are equal to one. Obviously, in this asymmetric steady state, trade is not balanced ($Y_h \neq C$ and $Y_f \neq C^*$). From the perspective of country H, the steady state trade balance must be in surplus to repay the positive stock of foreign debt.

No arbitrage implies

$$R = R^* = \frac{1}{\beta^*}. \quad (75)$$

Since the borrowing constraint is binding for country H, debt is equal to

$$B = \Theta \beta^* QH. \quad (76)$$

The house price equation yields

$$Q = \left(\frac{1 - \eta}{\eta}\right) \frac{(H/C)^{-\frac{1}{2}}}{1 - \beta - \Xi \Theta}. \quad (77)$$

Holding consumption constant, higher LTV ratios increase house prices and debt, both directly and indirectly. In the Foreign country, the borrowing constraint is not binding, thus house prices are

$$Q^* = \left(\frac{1 - \eta}{\eta}\right) \frac{C^*/H^*}{1 - \beta^*}. \quad (78)$$

The ratio between the housing stocks in the two countries can be chosen so that the steady state house prices are the same.
A.4 Log-Linear Approximation of the Model

Unless otherwise noted, for any given variable $Z_t$ define $z_t \equiv \log(Z_t/Z) \simeq (Z_t - Z)/Z$, where $Z$ is the steady state of $Z_t$.

The log-linear approximation of the index (2) for the Home and Foreign country gives

$$x_t = \eta \left( \frac{C}{X} \right) \frac{z_t}{c_t} \quad \text{and} \quad x^*_t = \eta \left( \frac{C^*}{X^*} \right) \frac{z^*_t}{c^*_t} \quad (79)$$

Equilibrium in goods markets can be approximated as

$$y_{ht} = -\gamma p_{ht} + s_H [\alpha c_t + (1 - \alpha) c^{-1}_R (\gamma s_t + c^*_t)] \quad (80)$$

and

$$y_{ft} = -\gamma p^*_ft + s_F [(1 - \alpha) c^*_t - \gamma s_t + \alpha c^*_t]. \quad (81)$$

where $s_i \equiv C_i/Y_i$ is the steady state consumption share of output in country $i = \{H, F\}$ and $c_R \equiv C/C^*$ is relative consumption across countries.

Next, the approximation of the house price equations (50) yields

$$q_t = (1 - \beta - \Xi \Theta) c_t + \beta \left[ \left( \frac{1}{\varepsilon} - \sigma \right) (E_t x_{t+1} - x_t) - \frac{1}{\varepsilon} (E_t c_{t+1} - c_t) \right]$$

$$+ \Xi \Theta (\xi_t + \theta_t + E_t \pi_{t+1}) + (\beta + \Xi \Theta) E_t q_{t+1}. \quad (82)$$

The lagrange multiplier on the borrowing constraint introduces a wedge in the Home country Euler equation. A first order approximation of equation (53) gives

$$i_t + \beta R \left[ \left( \frac{1}{\varepsilon} - \sigma \right) (E_t x_{t+1} - x_t) - \frac{1}{\varepsilon} (E_t c_{t+1} - c_t) - E_t \pi_{t+1} \right] + (1 - \beta R) \xi_t = 0. \quad (83)$$

In the Foreign country, the slack borrowing constraint implies that equation (51) becomes

$$q^*_t = (1 - \beta^*) c^*_t + \beta^* \left[ \left( \frac{1}{\varepsilon} - \sigma \right) (E_t x^*_{t+1} - x^*_t) - \frac{1}{\varepsilon} (E_t c^*_{t+1} - c^*_t) \right] + \beta^* E_t q^*_{t+1}. \quad (84)$$

The approximation of the borrowing constraint (52) is

$$i_t + b_t = \theta_t + E_t q_{t+1} + E_t \pi_{t+1}. \quad (85)$$

A first order approximation to country F Euler equation (55) gives

$$i^*_t + \left( \frac{1}{\varepsilon} - \sigma \right) (E_t x^*_t - x^*_t) - \frac{1}{\varepsilon} (E_t c^*_t - c^*_t) - E_t \pi^*_{t+1} = 0. \quad (86)$$

Up to the first order, the no-arbitrage relation (54) can be written as

$$i_t - E_t \pi^*_{t+1} = i^*_t - E_t \pi_{t+1} + E_t s_{t+1} - s_t. \quad (87)$$
The Fisher parity defines the real interest rate in each country
\[ r_t \equiv i_t - E_t \pi_{t+1} \quad \text{and} \quad r^*_t \equiv i_t^* - E_t \pi^*_{t+1}. \] (88)

A first order approximation of the non-linear wage Phillips curve (56) gives
\[ \frac{\zeta_w(1 + \phi_w \nu)}{1 - \zeta_w} \pi_{wt} = k_{wt} - f_{wt}. \] (89)

Up to a first order approximation, the present discounted value of the marginal disutility of labor (32) and the real wage in units of marginal utility of consumption (33) are
\[ k_{wt} = (1 - \beta \zeta_w)(1 + \nu)\ell_t + \beta \zeta_w E_t[\phi_w (1 + \nu)\pi_{wt+1} + k_{wt+1}] \] (90)
and
\[ f_{wt} = (1 - \beta \zeta_w) \left[ w_t + \left( \frac{1}{\varepsilon} - \sigma \right) x_t - \frac{1}{\varepsilon} c_t + \ell_t \right] + \beta \zeta_w E_t[(\phi_w - 1)\pi_{wt+1} + f_{wt+1}], \] (91)
where \( w_t \equiv \log[(W_t/P_t)/(W/P)] \) stands for the log-deviation of the real wage from its steady state value. Combining the last three expressions gives a standard forward looking wage Phillips curve
\[ \pi_{wt} = \kappa_w \left[ \nu \ell_t - w_t - \left( \frac{1}{\varepsilon} - \sigma \right) x_t + \frac{1}{\varepsilon} c_t \right] + \beta E_t(\pi_{wt+1}), \] (92)
where \( \kappa_w \equiv (1 - \beta \zeta_w)(1 - \zeta_w)/[\zeta_w(1 + \phi_w \nu)]. \)

For prices, a first order approximation the non-linear Phillips curve (59) gives
\[ \frac{\zeta_p}{1 - \zeta_p} \pi_{ht} = k_{pt} - f_{pt}. \] (93)

Up to a first order approximation, the present discounted value of marginal costs (41) and marginal revenues (42) are
\[ k_{pt} = (1 - \beta \zeta_p) \left[ \left( \frac{1}{\varepsilon} - \sigma \right) x_t - \frac{1}{\varepsilon} c_t + w_t + y_{ht} \right] + \beta \zeta_p E_t(\phi_p \pi_{ht+1} + k_{pt+1}) \] (94)
and
\[ f_{pt} = (1 - \beta \zeta_p) \left[ \left( \frac{1}{\varepsilon} - \sigma \right) x_t - \frac{1}{\varepsilon} c_t + p_{ht} + y_{ht} \right] + \beta \zeta_p E_t[(\phi_p - 1)\pi_{ht+1} + f_{pt+1}], \] (95)
Combining the last three expressions gives a standard forward looking price Phillips curve
\[ \pi_{ht} = \kappa_p(w_t - p_{ht}) + \beta E_t(\pi_{ht+1}), \] (96)
where \( \kappa_p \equiv (1 - \beta \zeta_p)(1 - \zeta_p)/\zeta_p. \)
In each country, the central bank determines inflation via a standard interest rate rule

\[ i_t = \rho i_{t-1} + (1 - \rho)(\psi \pi_t + \psi y_{ht}) + \varepsilon_{it} \]  

(97)

and

\[ i^*_t = \rho i^*_{t-1} + (1 - \rho)(\psi \pi^*_t + \psi y_{ft}) + \varepsilon^*_{it}. \]  

(98)

The dynamics of debt (64) can be approximated as

\[-b_t = -R(i_{t-1} - \pi_t + b_{t-1}) + b_{y}^{-1}(p_{ht} + y_{ht} - s_H c_t), \]  

(99)

where \( b_y \equiv B/Y_h \) is the steady state ratio between net foreign debt and GDP for the Home country.

Up to a first order approximation, the world resource constraint (65) gives

\[ \frac{1}{s_H}(p_{ht} + y_{ht}) + \frac{1}{c_R s_F}(p^*_{ft} + s_t + y_{ft}) = c_t + \frac{1}{c_R}(s_t + c_t^*). \]  

(100)

The approximation of equations (66) that link the terms of trade to domestic relative prices is

\[ p_{ht} = -(1 - \alpha)\tau_t \]  

and \[ p^*_{ft} = (1 - \alpha)\tau_t. \]  

(101)

Finally, the approximation of equation (67) that links real and nominal exchange rates is

\[ s_t = s_{t-1} + e_t - e_{t-1} + \pi^*_t - \pi_t. \]  

(102)

**A.4.1 Flexible Prices and Wages**

Under flexible prices and wages, a first order approximation to labor market equilibrium conditions (71) and (72) is

\[ p_{ht} = \nu y_{ht} + \frac{1}{\varepsilon}c_t - \left(\frac{1}{\varepsilon} - \sigma\right)x_t \]  

and \[ p^*_{ft} = \nu y_{ft} + \frac{1}{\varepsilon}c_t^* - \left(\frac{1}{\varepsilon} - \sigma\right)x_t^*. \]  

(103)