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Is Increased Price Flexibility Stabilizing? Redux

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Abstract

We study the implications of increased price flexibility on aggregate output volatility in a dynamic stochastic general equilibrium (DSGE) model. First, using a simplified version of the model, we show analytically that the results depend on the shocks driving the economy and the systematic response of monetary policy to inflation: More flexible prices amplify the effect of demand shocks on output if interest rates do not respond strongly to inflation, while higher flexibility amplifies the effect of supply shocks on output if interest rates are very responsive to inflation. Next, we estimate a medium-scale DSGE model using post-WWII U.S. data and Bayesian methods and, conditional on the estimates of structural parameters and shocks, ask: Would the U.S. economy have been more or less stable had prices been more flexible than historically? Our main finding is that increased price flexibility would have been destabilizing for output and employment.

Key words: increased price flexibility, aggregate volatility, systematic monetary policy, DSGE model, Bayesian estimation
1 Introduction

How sticky are nominal prices? In recent years we have seen an explosion in empirical research addressing this question using micro-data (see e.g. Bils and Klenow (2004), Gopinath and Rigobon (2008), Klenow and Kryvtsov (2008), and Nakamura and Steinsson (2008)). Answers have ranged from prices adjusting once every five months on average to more than one year.\(^1\) From a theoretical perspective, the primary motivation behind this work, either implicitly or explicitly, appears to be to settle the following question: Can monetary policy actions lead to large and persistent movements in output? Researchers seem to have in mind that modeling sticky prices is important if prices adjust infrequently in the data, while not if they adjust all the time. The first case corresponds to a “Keynesian economy” in which prices adjust only infrequently, and thus monetary policy and nominal frictions matter for real outcomes. The second case corresponds to a perfectly “classical economy” in which prices adjust at all times, and thus monetary policy and pricing frictions plays little or no role in stabilizing or destabilizing the business cycle. The degree to which one view is correct, then, is presumably determined by the degree to which a “flexible” or “fixed” pricing strategy of firms is closer to reality.

For a casual reader of this recent empirical literature, then, the question posed by this paper may strike as odd: Can increasing price flexibility be destabilizing for output? It might seem obvious that more flexible prices make monetary frictions as given by rigid prices – and more specifically monetary policy – play little or no role in stabilizing or destabilizing the business cycle. In this paper, however, we will argue that not only is that conclusion not obvious, but in fact, one can easily make the opposite case, both in theory and in an empirically estimated model.

Moreover, the question – whose answer is sometimes taken as being self evident – is in fact an old and classic question in macroeconomics and one that we argue remains unsettled. To make this clear we have stolen the title from De Long and Summers (1986), a paper published 25 years ago. They use a dynamic IS-LM model with rational expectations and Taylor-type wage contracts to show that an increase in flexibility can increase output volatility for reasons we make clear shortly. But this argument goes even farther back as these authors point out. A similar observation is made, for example, by Tobin (1975) using a more old-style Keynesian model. Similarly, Keynes (1936), declared that “it would be much better that wages should be rigidly fixed and deemed incapable of material changes, than the

\(^1\)For a survey of this literature, see Klenow and Malin (2010).
depression should be accompanied by a gradual downward tendency of money-wages” using more informal arguments. In fact, the question about the relationship between price flexibility and output volatility even pre-dates Keynes. As early as 1923, Irving Fisher (1923) saw the business cycle as “largely a dance of the dollar.” According to Fisher, expected deflation leads to high anticipated real interest rates and thus suppresses investment and output.

In this paper we address this classic question in macroeconomics in a modern micro-founded DSGE model with infrequent price adjustment. Our analysis proceeds in two steps. We first analyze a simplified model, basically the prototypical three-equation New Keynesian model, where we can show several key results in closed form and are able to characterize explicitly the conditions under which higher price flexibility, given by an increase in the frequency of price adjustment by firms, can be destabilizing and vice versa. We then move on to a quantitative medium scale DSGE model with a rich set of nominal and real rigidities popularized by Smets and Wouters (2007), building on Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005). We estimate this model using Bayesian methods and then ask the following question: Taking the various structural shocks and other structural and policy parameters from the estimated model as given, would a counterfactual history in which prices were more flexible in the post-WWII U.S. have led to more or less output volatility?

To summarize our main analytical results, we find that there are several circumstances under which a higher degree of price flexibility is destabilizing. Generally the result will depend upon two factors: (i) the source of a given disturbance and (ii) the policy reaction function of the central bank. We find it useful to classify disturbances into two broad classes: “demand” and “supply” shocks. Examples of demand shocks include monetary policy shocks (measured as unexpected movements in the nominal interest rate) and shocks such as preference shocks that mainly affect the natural rate of interest without having much effect on the natural level of output. Supply shocks are disturbances that mainly affect the natural level of output without having much effect on the natural rate of interest. Examples of supply shocks include variations in distortionary taxes, variations in the monopoly power of firms and/or unions, and technology shocks.

First, consider demand shocks. For an increase in price flexibility to be destabilizing we find that the key condition is that the central bank does not raise/cut the nominal interest aggressively enough in response to movements in inflation. Intuitively, what is going

2 The natural level of output is the output that would prevail if prices were flexible while the natural rate of interest is the real interest rate that would prevail if prices were flexible.
on is that a higher price flexibility can trigger unstable inflation expectations if monetary policy does not act aggressively to counteract this by raising/cutting interest rates. If the interest rate response is weak, the result is precisely of the form analyzed by De Long and Summers (1986) and Tobin (1975) and anticipated by Keynes (1936) and Fisher (1925): Higher price flexibility will destabilize the real interest rate – the difference between the short term nominal interest rate (the policy rate) and expected inflation. Since aggregate demand depends upon the real interest rate, this destabilizes output as well.

In contrast to this earlier literature, however, we find that the condition under which this destabilization occurs is quite special. In particular, one needs to a assume an interest rate reaction function for the central bank that does not correspond well to the one estimated on U.S. post-WWII data since it would require an interest rate response to inflation that is less than one-to-one. However, we find that this particular condition – i.e. a small reaction of the policy rate in response to inflation – is satisfied if the demand shocks are large enough for the zero bound on the short-term nominal interest rate to be binding. This situation, of course, is faced by large parts of the world today. Interestingly, it was also the state of affairs at the time Keynes conjectured that increased price flexibility would be destabilizing. These results thus relate to a similar finding in the literature on the zero bound on the short-term nominal interest rate, see for example Eggertsson (2010), Christiano, Eichenbaum, and Rebelo (2011) and Werning (2011). In a distinct contribution to the literature, we show that our results can hold even at positive interest rates and away from the zero bound if the central bank cares excessively about output stabilization at the expense of inflation. While this may not apply to U.S. in the post war data, one can plausibly argue that examples of this may include several developing countries where a heavily politicized central bank targets output stabilization at the expense of inflation volatility.

Second, consider supply shocks. Here we find that an increase in price flexibility is destabilizing under relatively conventional estimates of the monetary policy reaction function, in contrast to the result for demand shocks. Intuitively, consider a shock that increases the natural level of output. Under a standard monetary policy specification, this would imply that the price of goods needs to decrease as the economy expands towards its natural level. The more rigid prices are, however, the more difficult this adjustment becomes. Thus, actual output does not immediately increase with the natural rate of output, since not all firms will lower their prices right away when their ability to produce is enhanced. The same applies in the other direction, i.e. if the natural rate of output drops for some reason, output is slower to fall the more prices are rigid. An implication of this is that output moves more,
the more flexible are prices, in the presence of supply shocks. Importantly, this result applies whether or not a reduction/increase in the natural level of output is efficient (as in the case of productivity shocks) or inefficient (as in the case of an increase in monopoly power and/or tax distortions).

We find that our key analytical results continue to apply in the estimated model. For the estimated model, we find that output would have been less stable if prices had counterfactually been more flexible than the historical estimates. Given our estimated policy rule – and since the zero bound was never binding in our sample (which runs from 1966 to 2004) – this reflects that according to the estimated model “supply shocks” were a quite important driving force of the business cycle during this period. In particular, inefficient supply shocks driven by markup variations are non-negligible according to the estimation. Moreover, under the estimated parameter values, for supply shocks, the variance in output is quite sensitive to changes in the level of price stickiness.

The result from the estimated model is the same as in the numerical experiment reported in De Long and Summers (1986). The reason for our result, however, is different. Under their specification, price flexibility is destabilizing when demand shocks are perturbing the economy – but stabilizing when the driving force is supply shocks. Our estimated model, however, implies the opposite. Price flexibility becomes destabilizing when supply shocks are mainly perturbing the economy, which account for a non-trivial fraction of total output volatility.

Should one care about our results? For one thing, they should caution one against claiming that higher degree of price flexibility necessarily implies more output stability. For example, our analytical results show that it is not even obvious that a monetary policy shock (as measured by unexpected reduction/increase in FFR rates) has a smaller effect as prices become more flexible – in general this depends upon the assumed policy reaction function of the central bank, a fact that is not captured by much of the recent literature due to the fact that it assumes a reduced form aggregate demand specification.3 Furthermore, some caution is needed when evaluating proposals to increase the “flexibility” of markets. For supply shocks, this may not lead to more output stability, and for demand shocks it can also be destabilizing, provided the demand shocks are large enough for the zero bound to be binding or if the central bank is in general unresponsive to inflation. Finally, what our paper illustrates, we believe, is that care needs to be taken when interpreting micro-data on

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3See for example Caballero and Engel (2007), Dotsey, King, and Wolman (1999), Gertler and Leahy (2008), Golosov and Lucas Jr. (2007), Klenow and Kryvtsov (2008), and Midrigan (2011)).
the frequency of price changes. Higher flexibility in price adjustment, in general, does not provide conclusive evidence one way or the other about the importance of nominal frictions in explaining output volatility over the business cycle.

2 A Simple New Keynesian Model

We start by showing our key results in the standard three-equation New Keynesian model with price stickiness. Since this model has become standard by now, we do not write up the microfoundations, which can be found in textbooks such as Woodford (2003).

Thus, consider the standard New Keynesian model with time-dependent pricing as in Calvo (1983). From the optimization problem of the firm, which chooses its price anticipating that it only gets to revisit this choice with an exogenous probability $1 - \alpha$ in every period, we can derive the optimal pricing equation. We can do a log-linear approximation of the model and the firms’ pricing decisions that implies the New Keynesian Phillips curve, or the “AS” equation

$$\pi_t = \kappa \hat{Y}_t - \kappa \hat{Y}_n^t + \beta E_t \pi_{t+1}$$

where $\pi_t$ is inflation, $\hat{Y}_t$ is output in log-deviation from steady state, and $\hat{Y}_n^t$ is a disturbance term that has the following interpretation: It corresponds to the output that would have been produced in case prices were flexible. Thus, it is a composite that includes multiple shocks, such as productivity shocks, shocks to markup power of firms, and so on. The parameter $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\phi+\sigma^{-1}}{1+\phi \theta} > 0$ measures the slope of the Phillips curve, where $\beta$ is the discount rate, $\phi^{-1}$ is the Frisch elasticity of labor supply, $\sigma$ is the intertemporal elasticity of substitution, and $\theta$ is the elasticity of substitution among different varieties of goods. We are primarily interested in what happens as we increase “price flexibility.” We interpret this as increasing the (exogenous) probability of adjusting prices, that is $1 - \alpha$, which results in a higher $\kappa$. Thus, the higher the parameter value of $\kappa$, the larger the impact on inflation of the gap between output and the “natural level” of output.

Let us start with the exercise which has become relatively standard in the empirical and theoretical literature on price rigidities. Consider an exogenous path for nominal spending

$$D_t \equiv Y_t P_t$$

given by

$$\Delta \hat{D}_t = \rho \Delta \hat{D}_{t-1} + \epsilon_{t}^d$$
where $\Delta \hat{D}_t$ is growth rate in log deviation from steady state, $0 < \rho < 1$, and $\epsilon^d_t$ is an exogenous iid disturbance. The definition of nominal spending given by eqn.(2) implies that

$$\Delta \hat{D}_t = \pi_t + \hat{Y}_t - \hat{Y}_{t-1}. \tag{4}$$

What does this equation say? It says that an increase in nominal spending must, by definition, result in one of two things: Either an increase in the growth rate of prices (inflation), or an increase in the growth rate of output. Given that we take the path of demand as an exogenous process, it is then not altogether surprising that the higher the degree of price flexibility, that is, higher the $\kappa$, then the “easier” it is to meet an increase in nominal spending by inflation without changing output much. Indeed, by using the method of undermined coefficients and solving eqns.(1), (3), and (4), output can be expressed as

$$\hat{Y}_t = \frac{(1 - \rho \beta)}{(1 + \kappa - \rho \beta) + \beta \frac{(\kappa + \beta)}{(1 + \kappa + \beta)}} \Delta \hat{D}_t + \frac{1}{1 + \kappa + \beta} \hat{Y}_{t-1}$$

which implies that

$$VAR(\hat{Y}_t) = \Gamma \ast VAR(\Delta \hat{D}_t)$$

where $\Gamma = \frac{(1 - \rho \beta)}{(1 + \kappa - \rho \beta) + \beta \frac{(\kappa + \beta)}{(1 + \kappa + \beta)}}^2$ and $VAR(X_t)$ represents variance of variable $X_t$. The proof is in Appendix A.

It is now easy to see that $\Gamma$ is decreasing in $\kappa$, that is, the higher is the degree of price flexibility, the lower is the the variance in output. The intuition is straight-forward. As price flexibility increases, the instability in nominal demand is reflected in inflation rather than in output. Assuming a cash-in-advance constraint, it is common to interpret $D_t$ as the nominal stock of money. On the basis of this, then, it would be tempting to conclude that the degree of “instability” in the economy due to “monetary instability” is a simple function of the duration of price rigidities. Indeed, this is often implicitly or explicitly what the literature does. As we show now, however, this conclusion is not warranted once we model monetary policy more explicitly and subject the economy to other shocks.

Instead of treating nominal demand as exogenous, let us now model it explicitly from the households maximization problem. In the standard New Keynesian framework this leads to an “IS” relationship given by

$$\hat{Y}_t - \hat{Y}^n_t = E_t \hat{Y}_{t+1} - E_t \hat{Y}^n_{t+1} - \sigma(i_t - E_t \pi_{t+1} - \hat{i}^n_t) \tag{5}$$
where $\hat{i}_t$ is the nominal interest rate and $\hat{r}_n^t$, the “natural rate” of interest, is an exogenous disturbance term we give the following interpretation: It corresponds to the real interest rate that would take place if prices were flexible. Thus, it is a composite that includes multiple shocks and we will give some explicit examples below.

Meanwhile, we assume that monetary policy is given by a policy reaction function, that is the standard “Taylor rule” augmented by the zero bound

$$\hat{i}_t = \max(\beta - 1, \phi_\pi \pi_t + \phi_y \hat{Y}_t + \eta_t)$$

(6)

where $\phi_\pi, \phi_y > 0$ and $\eta_t$ is a monetary policy shock. Since we define each variable in terms of deviation from steady state, the zero bound is now $\hat{i}_t \geq \beta - 1$. This closes the model. In principle we can add to this system of equations a “money demand equation,” for example by adding money in the utility function, but we will abstract from this.

To ensure a determinate equilibrium, see Woodford (2003), we assume that the following condition is satisfied

$$\phi_\pi + \frac{1 - \beta}{\kappa} \phi_y > 1.$$  

(7)

Finally, we assume that each of the exogenous processes $\hat{r}_n^t, \hat{Y}_n^t$, and $\eta_t$ follow a first order AR process with persistence $\rho_i$ and iid component $\epsilon_i^t$ where $i$ indexes $r^n, Y^n$, or $\eta$. Also for the moment we assume that the exogenous shocks are independent from each other, but once we start being more specific about the driving forces, we drop this simplifying assumption.

What is important here, is that this system of equations implies that nominal demand, $\Delta \hat{D}_t$, is endogenous. It is determined by a host of factors, and in particular the monetary policy reaction function and the aggregate demand specification. Recall that before, the question was posed as: How does an exogenous shock to nominal demand influence output and prices? Since nominal demand is by definition

$$\Delta \hat{D}_t = \pi_t + \hat{Y}_t - \hat{Y}_{t-1}$$

it has to show up in either prices and output. In the model above, where nominal demand is endogenous, we see that there is no particular reason for any “shock”, be it monetary disturbance or any other kind, to show up in either inflation or output. In particular, it could just as well change the path for $\Delta \hat{D}_t$ which was exogenous before. Hence, we cannot conclusively state how the variance of output will change with an increase in $\kappa$. In fact,

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4Determinacy is required for the comparative static we study to be well-defined.
increasing price flexibility can now even increase volatility in nominal demand endogenously depending on how we specify monetary policy.

We will proceed by first analyzing each of the composite shocks of the model in turn. Consider first $\hat{r}_n^t$. Proposition 1 shows how the variance of output depends on $\kappa$ when the shock perturbing the economy is $\hat{r}_n^t$. Let the variance of output that can be attributed to a shock $x_t$ be given by $VAR(\hat{Y}_t/x_t)$.

**Proposition 1** Suppose $\hat{r}_n^t$ follows an AR(1) process with persistence $\rho_r$. Then the variance of output that can be attributed to $\hat{r}_n^t$ is given by

$$VAR(\hat{Y}_t/\hat{r}_n^t) = \left(\frac{\sigma}{(1 - \rho_r + \sigma \phi_y)(1 - \beta \rho_r) + \sigma \kappa [\phi_\pi - \rho_\pi]}\right)^2 VAR(\hat{r}_n^t)$$

**Proof.** In Appendix A. 

The proof of this proposition is a straight-forward application of the method of undetermined coefficients. Taking the partial derivative of this expression with respect to $\kappa$ then allows us immediately to state the following result in Proposition 2.

**Proposition 2** The effect of higher price flexibility on output variance is given by the following:

If $\phi_\pi - \rho_\pi > 0$, then \(\frac{\partial VAR(\hat{Y}_t/\hat{r}_n^t)}{\partial \kappa} < 0\)

If $\phi_\pi - \rho_\pi < 0$, then \(\frac{\partial VAR(\hat{Y}_t/\hat{r}_n^t)}{\partial \kappa} > 0\).

**Proof.** In Appendix A. 

Thus, when the underlying shock is $\hat{r}_n^t$, Proposition 2 shows that if monetary policy is responsive enough, as given by $\phi_\pi - \rho_\pi > 0$, output volatility decreases with price flexibility. If monetary policy is not responsive enough, as given by $\phi_\pi - \rho_\pi < 0$, then output volatility instead increases with price flexibility.

To provide intuition, it is useful to write out explicitly the solution and graph it up, assuming that $r_n^t$ is the only source of economic fluctuations. Under this assumption, since the model is linear and $r_n^t$ is the only state variable, eqn.(7) guarantees a unique bounded solution which takes the form

$$\hat{Y}_t = Y_t r_n^t, \quad \pi_t = \pi_\pi r_n^t, \quad \text{and} \quad \hat{r}_n^t = \rho_\pi \hat{r}_n^{t-1} + \epsilon_t$$
where \( Y_r \) and \( \pi_r \) are coefficients. This implies that

\[
E_t \hat{Y}_{t+j} = \rho_r^j Y_r \hat{r}_t^n \\
E_t \hat{\pi}_{t+j} = \rho_r^j \pi_r \hat{r}_t^n .
\]

Consider now the solution in period \( t \), which we subscript with \( S \) (for short run): \( \hat{Y}_S = \hat{Y}_t = Y_r \hat{r}_t^n \) once the economy has been perturbed by a shock \( \hat{r}_t^n = \hat{r}_S^n \neq 0 \). The IS equation can be combined with the policy rule to yield an aggregate demand equation of the following form

\[
(1 - \rho_r + \phi_y \sigma) \hat{Y}_S = -\sigma(\phi_\pi - \rho_r) \pi_S + \sigma \hat{r}_S^n \quad (8)
\]

where we have substituted \( E_t \hat{Y}_{t+1} = \rho_r \hat{Y}_S \) and \( E_t \pi_{t+1} = \rho \pi_S \) and the AS equation is similarly

\[
(1 - \rho_r \beta) \pi_S = \kappa \hat{Y}_S . \quad (9)
\]

For later purposes, note here that the slope of the aggregate demand equation given by eqn.(8) depends on whether \( (\phi_\pi - \rho_r) \gtrless 0 \) while the slope of the AS equation given by eqn.(9) is always positive since \( \kappa > 0 \).

The two relationships are plotted up in Figure 1 for the case in which \( \phi_\pi > \rho_r \). The figure shows the effect of a negative demand shock, from \( AD_1 \) to \( AD_2 \) under two assumptions, i.e. when prices are rigid or more flexible (shown via a steeper AS curve). We see that under rigid prices, a given drop in demand results in a steeper contraction (point A) than if prices are more flexible (point B). The reason for this is relatively simple: Consider first the AD equation which pins down the number of goods purchased by the consumers. In this economy, production is demand determined, i.e. the firms produce as many goods as are demanded by the customers that show up in front of their doors. This demand, however, depends not on any measure of price rigidity, but instead (as we see in IS) only on expectations about future output and the difference between the real interest rate and the natural rate of interest, \( \hat{r}_t^n \).

To clarify things further, let us for a moment assume that \( \rho = 0 \). Then the expectation terms drop out since the economy is in steady state the next period. The central bank responds to a negative demand shock in the short run by cutting the nominal interest rate (since \( \phi_\pi > 0 \)). This cut, however, will be bigger the greater is the drop in inflation associated with the demand shock. As prices become more flexible, then, the central bank cuts the nominal interest rate by more, and thus has a bigger effect on demand. That is essentially the logic that underlies Figure 1.
Consider now the case when $\rho_r > 0$, in which case the shock becomes more persistent. The logic described above still applies: the central bank will try to offset the demand shock by interest rate cuts and thus stimulate demand. But now some additional effects come into play due to the persistence of the shock. A persistent shock influences aggregate demand in two ways, as can be seen in IS eqn.(5): a more persistent shock changes both expected inflation and expected output. In particular, we see from eqn.(8) that once we have substituted out for the policy rule, then a persistent negative shock can potentially reduce future inflation expectation to such an extent that it actually destabilizes demand. To see this, consider an increase in $\rho$ for a given $\phi_\pi$. As we see from eqn.(8), this means that the AD curve becomes steeper, suggesting that a given nominal interest rate cut (in response to a reduction in $\pi_S$) now leads to a smaller increase in demand because once the shock is persistent, it not only triggers a reduction in current nominal interest rate today, it also triggers expectations of lower inflation in the future. The lower expected inflation in the future, in turn, increases the real interest rate, thus offsetting some of the expansionary effect of the decline in the nominal interest rate today. If the shock is persistent enough, and the interest response (given by $\phi_\pi$) weak enough, the effect given by lower expected inflation can be so strong that it dominates and the aggregate demand become upward sloping in output and inflation. We turn to this case next.

Figure 2 shows the effect of a demand shock in the short run when $\phi_\pi - \rho_r < 0$. We see that in this case an increase in price flexibility leads to a bigger output contraction,
Figure 2: Effect of Price Flexibility Under Non-Responsive Monetary Policy, Demand Shock

from point A to point B. The reason for this has already been hinted at above: the more flexible are prices, the more inflation expectations drop, thus leading to an increase in the real interest rate. Because $\phi_\pi - \rho_r < 0$, this drop in inflation expectations is not met by an aggressive enough reduction in the central bank nominal interest rate at time $t$. This is an example, then, of price flexibility being destabilizing in face of aggregate demand shocks.

What is the interpretation of $\phi_\pi - \rho_r < 0$? Because $\rho_r$ has to be between zero and one for the model to be stationary, this condition implies that $\phi_\pi < 1$. Recall our condition for determinacy, eqn.(7), which then implies that for a determinate equilibrium we require that

$$\phi_y > \frac{\kappa}{1 - \beta}(1 - \phi_\pi) > 0.$$  

Intuitively, the condition implies that if the central bank puts little weight on inflation $\phi_\pi$, and thus correspondingly larger weight on output $\phi_y$ (which is required for determinacy), then this leads to instability in aggregate demand in response to more price flexibility. In particular, a higher price flexibility will trigger instability in inflation expectations, which in turn leads to more unstable demand. Is this a realistic description of a central bank behavior? As we will see once we estimate the model, this condition is not satisfied in the U.S. postwar data. It may, however, be satisfied in some countries where the central bank does not react sufficiently to inflation, but excessively to output volatility. But an even more important point, perhaps, (at least from a U.S. standpoint) is that the basic logic of
the proposition does carry over to an empirical specification of U.S. policy if we take one additional property of the policy reaction function into account.

The key mechanism behind proposition 2 is that the central bank does not respond sufficiently strongly to deviation of inflation from target. In the proposition this occurs due to a low $\phi_\pi$. As we can see in the policy rule given by eqn.(6) however, monetary policy can be unresponsive to inflation for two reasons: either because $\phi_\pi$ is small, or alternatively, if there are shocks so that the zero bound is binding and the central bank cannot respond to reduction in inflation. We analyze this case next, which also helps us connect to the recent literature on the zero bound.

We consider the case when $\hat{r}_n^n$ becomes negative enough so that the zero bound on the nominal interest rate binds. To be more specific, let us consider the case in which $\hat{r}_n^n < 0$ and let us put a slightly different structure on the shock for tractability. In particular, consider a shock as in Eggertsson and Woodford (2003) in which $\hat{r}_n^n = \hat{r}_S^n < 0$ in period 0 and which reverts back to steady state $r_S = \bar{r} > 0$ with a fixed probability $1 - \mu$ every period thereafter. Call the period in which it reverts back to steady state $\tau$. Then it is easy to confirm (see e.g. Eggertsson (2010)) that the solution for inflation and output is

$$\pi_S = \frac{\kappa \sigma}{(1 - \mu)(1 - \beta \mu) - \mu \sigma \kappa} \hat{r}_S^n < 0$$

$$\hat{Y}_S = \frac{\sigma}{(1 - \mu)(1 - \beta \mu) - \mu \sigma \kappa} \hat{r}_S^n < 0.$$

The next proposition follows.

**Proposition 3** Conditional on $r_n^n < 0$, and the shock process outlined above, output drops the more, the higher $\kappa$.

**Proof.** In Appendix A. □

Hence, if the shock to $\hat{r}_t^n$ is large enough so that the zero bound is binding, an increase in price flexibility is no longer stabilizing, it instead is destabilizing regardless of the value of $\phi_\pi$ and $\phi_y$. The logic of this proposition is in fact the same as we showed in Figure 2. The intuition for this relies heavily on the fact that the nominal interest rate does not respond strongly to the drop in inflation and output since it is stuck at zero. Consider now what an increase in price flexibility does. Not only does it lead to a drop in the price level today but because the shock is persistent, it also leads to expectation of future deflation. Because the real interest rate is the difference between the nominal interest rate and expected deflation,
higher price flexibility thus leads to expectation of more deflation in the future, thus even increasing the real interest rate by more, creating a vicious deflationary spiral. As discussed in Eggertsson (2010), this may not even converge.

We now move on to other shocks that we have introduced above. Proposition 4 summarizes how the variance of output depends on $\kappa$ under different assumptions about shocks perturbing the economy. It does so for the exogenous shocks $\hat{Y}_t^n$ and $\eta_t$ assuming they are independent of one another and without considering the zero lower bound.

Proposition 4 Suppose each of the following shocks are independent of one another $(\hat{Y}_t^n, \eta_t)$, and follow an AR(1) with persistence $\rho_i$ $i = n, y$. Then the variance of output that can be attributed to each shock can be written as follows

$$VAR(\hat{Y}_t^n) = \left(\frac{(1 - \rho_y)(1 - \beta \rho_y) + \kappa \sigma [\phi_n - \rho_y]}{[(1 - \rho_y + \sigma \phi_y)(1 - \beta \rho_y) + \kappa \sigma [\phi_n - \rho_y]]}\right)^2 VAR(\hat{Y}_t^n)$$

$$VAR(\hat{Y}_t/\eta_t) = \left(\frac{\sigma (1 - \beta \rho_y)}{(1 - \rho_y + \sigma \phi_y)(1 - \beta \rho_y) + \sigma \kappa [\phi_n - \rho_y]}\right)^2 VAR(\eta_t).$$

Proof. In Appendix A. ■

The proof of this proposition is a straightforward application of the method of undetermined coefficients. We can see from this proposition right away that the partial derivative of the variance of output with respect to $\kappa$ depends fundamentally on what is the source of the variation in output and the responsiveness of monetary policy. We summarize these signs in the next proposition.

Proposition 5 The effect of higher price flexibility on output variance is given by the following:

If $(\phi_n - \rho_y) > 0$, then $\frac{\partial VAR(\hat{Y}_t^n)}{\partial \kappa} > 0$

If $(\phi_n - \rho_y) < 0$, then $\frac{\partial VAR(\hat{Y}_t^n)}{\partial \kappa} < 0$

and

If $(\phi_n - \rho_y) > 0$, then $\frac{\partial VAR(\hat{Y}_t/\eta_t)}{\partial \kappa} < 0$

If $(\phi_n - \rho_y) < 0$, then $\frac{\partial VAR(\hat{Y}_t/\eta_t)}{\partial \kappa} > 0$.

Proof. In Appendix A. ■
The proof of this proposition is obtained by taking a partial derivative of the expressions in Proposition 4 with respect to $\kappa$.

Let us now comment upon the intuition for the result. For the case of $\eta_t$, the intuition is exactly the same as for $\hat{r}_t^n$. To see this, just note that if we substituted for the monetary policy reaction function given by eqn.(6) (assuming positive interest rates) into the IS equation given by eqn.(5), then the shock $\eta_t$ appears exactly in the same way as $\hat{r}_t^n$ but with a different sign. Thus a monetary disturbance – defined in this way – works in the same way as a “demand shock.” Let us now turn to the supply shock.

For the supply shock, consider first the case $\phi_\pi - \rho_y > 0$. Proposition 5 says that then, if supply shocks are perturbing the economy, an increase in price flexibility increases output volatility. Again, to provide intuition, it is useful to explicitly write out the solution and graph it up assuming $\hat{Y}_t^n$ is the only source of economic fluctuations. Under this assumption

$$\hat{Y}_t = Y_t \hat{Y}_t^n, \quad \pi_t = \pi_t \hat{Y}_t^n, \quad \text{and} \quad \hat{Y}_t^n = \rho_y \hat{Y}_{t-1} + \epsilon_t^y$$

where $Y_t$ and $\pi_t$ are coefficients. This implies that

$$E_t \hat{Y}_{t+j} = \rho_y^j Y_t \hat{Y}_t^n$$
$$E_t \pi_{t+j} = \rho_y^j \pi_t \hat{Y}_t^n$$

Consider now the solution in period $t$, which we denote $S$ (for short run), $\hat{Y}_S = \hat{Y}_t = Y_t \hat{Y}_t^n$ once the economy has been perturbed by a shock $\hat{Y}_t^n = \hat{Y}_S^n \neq 0$. The IS equation can be combined with the policy rule to yield an aggregate demand equation of the following form

$$(1 - \rho_Y + \phi_y \sigma) \hat{Y}_S = -\sigma(\phi_\pi - \rho_y) \pi_S + (1 - \rho_y) \hat{Y}_S^n$$

(10)

where we have substituted $E_t \hat{Y}_{t+1} = \rho_y Y_S$ and $E_t \pi_{t+1} = \rho_y \pi_S$ and the AS equation is similarly

$$(1 - \rho_y \beta) \pi_S = \kappa \hat{Y}_S - \kappa \hat{Y}_S^n.$$ 

(11)

Again for later purposes, note here that the slope of the aggregate demand equation given by eqn.(10) depends on whether $(\phi_\pi - \rho_y) \gtrless 0$ while the slope of the AS equation given by eqn.(11) is always positive since $\kappa > 0$.

The two relationships are plotted up in Figure 3 for the case in which $\phi_\pi > \rho_y$. Without the shock, the equilibrium is point A (we omit the AD1 curve there so as not to clutter the figure). A negative supply shock will shift the AS equation, as well as AD equation to the
new location towards the middle of the figure, with the equilibrium taking place in either point B or C depending on how flexible prices are. As we see from the figure, the output drop is bigger the steeper the AS curve is (point C), i.e. the more flexible price are. The logic is as follows, and here again, it is useful to consider the case when $\rho = 0$ so that we can ignore expectations for the moment (since the economy is expected to go to steady state in the next period). If natural output drops then the flexible price allocation would imply that firms should increase their prices today. But this cannot be done if some firms fix their prices. So even if there is some inflation in the economy, as seen in eqn.(10), it is not big enough to get the economy to contract to the flexible price equilibrium due to the rigidities. The bigger the rigidities, the less the adjustment. The only way the reduction in output can take place, since prices cannot increase fast enough, is via monetary contraction, that is, by an increase in the nominal interest rate. But this will never be done to a full extent under the policy reaction function assumed (only if $\phi_\pi \to \infty$ this would be achieved but this policy would be equivalent to inflation being zero at all times).

The conclusion, then, is that if the source of instability is variation in the natural level of output, rigidities in prices make output fluctuate less, not more. Observe that there are many shocks that can affect the natural level of output and we will be more specific below. But perhaps more importantly, there is no reason to expect that all shocks that change the flexible price level of output are efficient. Variations in the markup power of firms or unions, for example, are in general inefficient, as are what have been termed more generally as “labor
Meanwhile shocks to productivity are often efficient, although this, too, depends very much on the details.

While the intuition is relatively simple for why output drops more under flexible prices with supply shocks, this result goes the other way if we assume persistent enough shocks, i.e. $\phi_\pi < \rho_y$. Graphically, what happens is that in this case the AD curve will once again take the backward-bending shape we saw with demand shocks as seen in Figure 4, where the drop in output (point C) is now smaller when prices are more flexible. In this case, we need to pay attention to a force we glossed over earlier by assuming that $\rho_y = 0$. When $\rho_y - \phi_\pi > 0$ we need to analyze what happens to inflation expectations. A negative supply shock has two effects, it reduces the natural rate of output, but it also increases inflation expectations if the shock is persistent enough. This increase in inflation expectations reduces the real rate of interest and thus increases demand at any given nominal rate. This latter force only becomes strong enough (to offset the contraction due to increase in the nominal rate) when $\phi_\pi < \rho_y$ because then the current interest rate does not increase fast enough to offset the increase in inflation expectations.

It is worth commenting upon how one should interpret the shocks $\hat{Y}_t^n$ and $\hat{r}_t^n$. These correspond to composite disturbances that can be due to a host of factors. To be more specific, consider the following four structural shocks: A multiplicative shock to the production function (productivity shock), $\hat{A}_t$, a shock to the market power of firms (or workers), $\hat{\mu}_t$, a change in a proportional labor tax, $\hat{\tau}_t^w$, and a multiplicative shock to time preferences, $\hat{\psi}_t$. It
is easy to show that the natural level of output can then be expressed as a function of these shocks as

\[ \hat{Y}_n^t = \frac{1 + \phi}{\sigma^{-1} + \phi} \hat{A}_t - \frac{1}{\sigma^{-1} + \phi} \hat{\mu}_t + \frac{1}{\sigma^{-1} + \phi} \hat{\tau}_w^t \]

and the natural rate of interest as

\[ \hat{r}_n^t = -\sigma^{-1}(\hat{Y}_n^t - E_t \hat{Y}_{n+1}) + \sigma^{-1} \left( \hat{\psi}_t - E_t \hat{\psi}_{t+1} \right). \]

We can see here that any of the shocks considered can make the natural rate of interest negative, although \( \hat{\psi}_t \) is perhaps the most natural candidate. Similarly all the shocks except for \( \hat{\psi}_t \) change the natural level of output. Notice that variations in both \( \hat{\mu}_t \) and \( \hat{\tau}_w^t \) are examples of inefficient movements in output.\(^5\)

In contrast to our experiments above, the shocks \( \hat{A}_t, \hat{\mu}_t, \) and \( \hat{\tau}_w^t \) will affect both the natural level of output and the natural rate of interest (unless the shock \( \hat{\psi}_t \) offsets it exactly). It is easy to confirm, however, that the same results (and basic intuition) as given in Proposition 5 applies: if \( (\hat{A}_t, \hat{\mu}_t, \hat{\tau}_w^t) \) are the sources of output variation, then increasing the frequency of price changes increases output volatility rather than reducing it when monetary policy is responsive enough. To take one example, assume variation in \( \hat{A}_t \) is independent of other disturbances and follows an \( AR(1) \) with persistence \( \rho_A \). Then, we show in the Appendix that

\[ VAR(\hat{Y}_t / \hat{A}_t) = \left( \frac{\sigma[\phi_\pi - \rho_A]}{(1 - \rho_A + \sigma \phi_\pi)(1 - \beta \rho_A) + \sigma[\phi_\pi - \rho_A] \kappa} \right)^2 VAR(\hat{A}_t) \]

where \( \gamma_A = \frac{1 + \phi}{\sigma^{-1} + \phi} \). It is easy to verify that the derivative of \( VAR(\hat{Y}_t / \hat{A}_t) \) with respect to \( \kappa \) is positive if \( (\phi_\pi - \rho_A) > 0 \) and vice-versa. Similar statistics can be computed for \( \hat{\mu}_t \) and \( \hat{\tau}_w^t \), replacing \( \gamma_A \) with \( \gamma \equiv \frac{1}{\sigma^{-1} + \phi} \) and \( \rho_A \) with \( \rho_\mu \) or \( \rho_{\tau^w} \).

In Appendix B, we consider an extension of the simple model presented above where we allow for wage stickiness in addition to price stickiness. In such a set-up, similar to eqn. (1) which determines price inflation as a function of the difference between actual output and the natural level of output, there will be an equation that determines wage inflation as a function of the difference between real wage and the natural level of real wage \( w_n^t \), that is, the real wage that would prevail were wages flexible. This extended model cannot be solved analytically but using numerical methods we have verified that the results of this section

\(^5\)Although how inefficient it is depends on if the steady state around which the equilibrium conditions are linearized is efficient or not.
carry over for shocks to $\hat{Y}_t^n$, $\hat{r}_t^n$, and $\eta_t$. Moreover, for shocks to $w^n_t$, volatility in output increases with increased price flexibility if monetary policy is responsive and vice-versa. In this model, one can also analyze the aggregate implications of increased wage flexibility and we present some results in Appendix B.

We now turn to our quantitative experiment where we assess the aggregate effects of increased price flexibility using an estimated medium-scale DSGE model.

3 Quantitative Experiment

In this section, we conduct a quantitative evaluation of the effects of greater price flexibility on aggregate volatility. As we emphasized in the last section, the effect on output volatility of increased price flexibility depends crucially on the underlying shocks driving the economy and the endogenous response of monetary policy. To explore if the results of the previous section are empirically relevant, we fit the well-known Smets and Wouters (2007) model, a new Keynesian model with rich sources of nominal and real rigidities that features an endogenous monetary policy rule, to the data and estimate the structural parameters and the underlying shocks. Conditional on the estimated values for all other parameters of the model, we then conduct the following counterfactual comparative statics exercise: does increasing the frequency of price adjustment lead to higher or lower output volatility in the model? In addition, complementary to the previous exercise, after backing out the entire time-series of the estimated shocks, we also conduct counterfactual exercises to see, what would have been the historical path for output had prices been more flexible than the estimated value.

3.1 A Medium-Scale DSGE Model

We refer the reader to the Smets and Wouters (2007) paper for a detailed description of the model and its log-linear approximation. Here we only lay out the basic model features and introduce relevant notation. Households in the model face an infinite horizon problem and maximize expected discounted utility (discount factor given by $\beta$) over consumption and leisure. The utility function is non-seperable over consumption and labor effort. There is a time-varying external habit formation in consumption. The intertemporal elasticity of substitution is given by $\sigma_c$, the elasticity of labor supply by $\sigma_l$, and the habit parameter by $h$.

Households supply labor to a labor union, which differentiates the homogenous labor input. The elasticity of substitution over the differentiated labor services is time-varying
and modeled as in Kimball (1995), where $\varepsilon_w$ represents the curvature of the aggregator function. The union enjoys some monopoly power over setting wages, which are sticky in nominal terms. Wage stickiness is modeled following Calvo (1983). The constant probability of not adjusting wages is given by $\xi_w$, with wages that do not adjust partially indexed to past inflation, with the extent of indexation given by $\iota_w$. The steady-state mark-up in the labor market is given by $\lambda_w$.

Households also rent capital services to firms and in deciding how much capital to accumulate, take into account capital adjustment costs which enter as a function of change in investment. The steady-state elasticity of the capital adjustment cost function is given by $\varphi$. Moreover, the model features variable capital utilization rate, with the dependence of the degree of capital utilization on the rental rate of capital a function of the parameter $\psi$. Capital depreciates at the rate $\delta$.

Firms produce differentiated goods using labor and capital as inputs, with $\alpha$ denoting the share of capital in production. The share of fixed cost in production is given by $1 - \Phi$. Like for labor, the elasticity of substitution over the differentiated goods is time-varying and modeled as in Kimball (1995), where $\varepsilon_p$ represents the curvature of the aggregator function. Firms have some degree of monopoly power over setting prices, which are sticky in nominal terms. Price stickiness is modeled following Calvo (1983). The constant probability of not adjusting prices is given by $\xi_p$, with prices that do not adjust partially indexed to past inflation, with the extent of indexation given by $\iota_p$.

Government behavior is specified in terms of fiscal and monetary policies. The government levies lump-sum taxes and government spending follows an exogenous path, with some response to the productivity process. In particular, government spending responds by $\rho_{ga}$ to an innovation to total factor productivity. Monetary policy is modeled using an empirical endogenous interest rate rule which features interest rate smoothing, given by $\rho$, feedback from inflation, given by $r_\pi$, and feedback from output gap, given by $r_y$. Moreover, there is also some short-run feedback from the change in the output gap, given by $r_{\Delta y}$. It is important to note here that the output gap is the difference between actual output and potential output, where potential output is defined as the output that would prevail under flexible prices and in the absence of the price and wage markup shocks.

The economy is driven by seven fundamental aggregate shocks. The total factor productivity, investment-specific technology, risk premium, exogenous government spending, and monetary policy shocks follow AR(1) processes. The persistence parameters of the shocks are given by $\rho_a$, $\rho_I$, $\rho_b$, $\rho_g$, and $\rho_r$ and the standard deviations of the innovations by $\sigma_a$, $\sigma_I$,
\( \sigma_b, \sigma_g, \) and \( \sigma_r \) respectively. The price and wage markup shocks are assumed to follow ARMA \((1,1)\) processes, with \( \rho_p \) and \( \rho_w \) representing the corresponding AR parameters and \( \mu_p \) and \( \mu_w \) the corresponding MA parameters. The standard deviations of the innovations are given by \( \sigma_p \) and \( \sigma_w \).

Finally, the model features deterministic growth driven by labor-augmenting productivity with the quarterly trend growth rate in real GDP given by \( \gamma \). Similarly, the quarterly steady-state inflation rate is given by \( \pi \), the steady-state hours worked by \( \bar{I} \), and the steady-state government spending to GDP ratio by \( g_y \).

### 3.2 Estimation Results

#### 3.2.1 Data

We directly follow Smets and Wouters (2007) in our estimation exercise. We use quarterly U.S. data from 1966:I - 2004:IV on log difference of real GDP, real consumption, real investment, real wage, and the GDP deflator, log hours worked, and the federal funds rate. We therefore use seven observables along with seven fundamental shocks. In particular, our estimation exercise does not feature measurement errors.

#### 3.2.2 Methodology

We follow a standard Bayesian estimation and model comparison procedure for linearized models. All the details are in Appendix C. The likelihood function is evaluated using the Kalman filter. We compute the mode of the posterior and then use a random walk Metropolis algorithm to sample from the posterior distribution. A scaled version of the inverse of the Hessian computed at the mode is used as a proposal density in the random walk Metropolis algorithm. The results we report are based on 2.5 million draws, with one third of the draws burned-in. We assess convergence using multiple chains and trace-plots. For model comparison purposes, we compute the marginal likelihoods of the various model specifications using the methodology in Geweke (1998).

#### 3.2.3 Priors

As in Smets and Wouters (2007) we calibrate a few parameters. \( \delta \) is set at 0.025, \( g_y \) at 0.18, \( \lambda_w \) at 1.5, and \( \varepsilon_p \) and \( \varepsilon_w \) at 10. For the rest of the parameters that are estimated, the prior distributions are described in tables (1) and (2). We directly follow Smets and Wouters (2007) in the priors we pick, except for the price-markup shock. Smets and Wouters (2007)
estimate a scaled version of the markup shock, which is a combination of the true markup shock and various structural parameters, in particular, the probability of price adjustment. Since our main goal is to conduct a comparative statics exercise on the probability of price adjustment, we estimate the “true” price markup shock. For this reason, the prior mean of the standard deviation of the price-markup shock is set at a quite high value.

3.2.4 Model Comparison

We estimate various versions of the model outlined above with respect to wage and price indexation. This is a pertinent exercise especially since Smets and Wouters (2007) conjecture that some forms of indexation might not be essential to explain the data. In table (3) we report the marginal likelihoods of the four model specifications that we estimate and find that a model that features wage but not price indexation provides the best fit to the data.\(^6\) We report the posterior estimates of this specification below.

3.2.5 Posterior Estimates

The posterior estimates of the various parameters of the model that features wage but not price indexation are given in tables (4) and (5). Since our entire exercise is extremely close to that of Smets and Wouters (2007), the estimates are in line with their results. The only exceptions are the estimates pertaining to the price markup shock for reasons described above.

3.3 Price Flexibility Counterfactual Experiment

With the posterior estimates of the structural parameters and the shocks at hand, we then conduct the following counterfactual exercise. Fixing all the other parameters at the posterior mean values, we compute the model implied variance of the growth rate of output when we change the probability of price adjustment from the posterior mean (estimated at $\xi_p = 0.6270$). In particular, a lower value of $\xi_p$ implies a higher frequency of price changes and hence greater price flexibility. Table (6) reports the results. We find that increased price flexibility would indeed have led to higher variance and been destabilizing. This is the main result of our quantitative exercise.

Next, we also verify the results of the analytical model regarding individual shocks. For this exercise, we fix all other parameters and shut down all shocks except the shock in

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\(^6\)Note that using marginal likelihoods to compare models penalizes over parameterization of the model.
question. Then we ask the same question as above: what would have happened to the model implied variance of the growth rate of output had the probability of price adjustment been different from the estimated value. We report the results in tables (7) and (8). Since our estimates imply a responsive monetary policy, consistent with our analytical results, we find that had prices been more flexible than historically, aggregate volatility would have increased in the case of total factor productivity, price mark-up, and wage mark-up shocks while it would have decreased in the case of risk premium, investment specific technology, exogenous government spending, and monetary policy shocks.

To get more insights, we also conduct complementary counterfactual exercises. We recover the historical realizations of all the (smoothed) shocks from the estimated model and feed these into the model where prices are more flexible than the historical estimate. We then simulate the implied historical path for output growth and compare it with the actual path. Figures 5 and 6 report the results of this exercise when we lower price durations by 1 and 1/2 quarters. As expected we see that for the bulk of history, output growth would have been more volatile, particularly in the 1970s, while it would have been less volatile in the Volcker disinflation period of early 1980s. Figure 7 shows the results for inflation. Consistent with our analytical results, inflation is always more volatile when price are more flexible. Finally, to help understand the different results across various time-periods we repeat the exercise above for individual shocks one at a time. Figure 8 shows the results for monetary shock. As is intuitive, the results show that the slowdown in output growth during the Volcker disinflation period, caused by a contractionary monetary shock, would have been less severe. Figures 9 and 10 report the results for price and wage markup shocks. It is clear that particularly during the 1970s, in response to these shocks, output growth would have been more volatile had prices been more flexible.

4 Conclusion

In this paper, we study the aggregate implications in a DSGE model of increased price flexibility. Our analytical results highlight the importance of the source of shocks, the modeling of monetary policy, and the general equilibrium environment in assessing the aggregate implications of increased price flexibility. In a quantitative exercise using an estimated DSGE model and U.S. data, we find that conditional on the estimated values of the structural parameters and shocks, increased price flexibility would indeed have been destabilizing.

While the following two points are a bit speculative, we believe the mechanism we have
uncovered is likely to explain at least two empirical phenomena. First, the current paper may also shed some light on why the Great Recession triggered a far smaller drop in output than the U.S. economy experienced during the Great Depression: the Great Recession was associated with a relatively modest decline in inflation, while the Great Depression was characterized by excessive deflation, and the model suggests the former should trigger a smaller drop in output than the latter. Second, our model may shed light on cross-country variation in output volatility. One important factor there may be that in certain countries monetary policy is relatively unstable, which may trigger prices to be more flexible. The model suggests that if certain shocks are driving the business cycle, then this may be one factor in explaining cross-country variations in output volatility.
References


5 Tables

Table 1: Prior Distribution of Structural Parameters

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Table 2: Prior Distribution of Shock Processes

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<td>$\sigma_r$</td>
<td>$\mathbb{R}^+$</td>
<td>InvG</td>
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</tr>
<tr>
<td>$\sigma_p$</td>
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<td>InvG</td>
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<tr>
<td>$\sigma_w$</td>
<td>$\mathbb{R}^+$</td>
<td>InvG</td>
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Table 3: Model Comparison

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<tr>
<td>Both price and wage indexation</td>
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<tr>
<td>Only price indexation ($t_w = 0$)</td>
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</tr>
<tr>
<td>Only wage indexation ($t_p = 0$)</td>
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<tr>
<td>Neither price nor wage indexation ($t_w, t_p = 0$)</td>
<td>-925</td>
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<tr>
<td>Parameters</td>
<td>Prior Mean</td>
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<tr>
<td>-----------</td>
<td>------------</td>
</tr>
<tr>
<td>( \phi )</td>
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<td>( \sigma_c )</td>
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<td>( h )</td>
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<td>( \iota_w )</td>
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<td>( \psi )</td>
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<td>( \Phi )</td>
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<td>( r_{\Delta y} )</td>
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<td>( 100(\beta^{-1} - 1) )</td>
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<td>( \bar{l} )</td>
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<td>( \bar{\gamma} )</td>
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<td>( \alpha )</td>
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Table 5: Posterior Estimates of Shock Processes

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<td>[0.0702 0.3546]</td>
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<td>$\rho_g$</td>
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<td>0.9755</td>
<td>[0.9620 0.9896]</td>
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<td>[0.6118 0.8075]</td>
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<td>$\rho_w$</td>
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<td>[0.9540 0.9910]</td>
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<td>$\rho_{ga}$</td>
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<td>$\sigma_b$</td>
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<td>[0.2035 0.2809]</td>
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<td>[0.4775 0.5794]</td>
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<td>$\sigma_I$</td>
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Table 6: Price Flexibility Experiment (all shocks)

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<tr>
<th>Prob (no price adj.)</th>
<th>Variance of Output Growth</th>
<th>% Change to Base Case</th>
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<tr>
<td>0.6270</td>
<td>1.0197%</td>
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</tr>
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<td>0.5415</td>
<td>1.0791%</td>
<td>5.83%</td>
</tr>
<tr>
<td>0.4051</td>
<td>1.2161%</td>
<td>19.26%</td>
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</table>
Table 7: Price Flexibility Experiment

<table>
<thead>
<tr>
<th>Prob (no price adj.)</th>
<th>Variance of Output Growth</th>
<th>% Change to Base Case</th>
<th>Shocks</th>
</tr>
</thead>
<tbody>
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<td>0.6270</td>
<td>0.1417%</td>
<td>-</td>
<td>TFP</td>
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<td>TFP</td>
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<td>0.1827%</td>
<td>28.96%</td>
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<td>0.0618%</td>
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</table>
Table 8: Price Flexibility Experiment

<table>
<thead>
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<th>Prob (no price adj.)</th>
<th>Variance of Output Growth</th>
<th>% Change to Base Case</th>
<th>Shocks</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-</td>
<td>Government Spending</td>
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<td>Government Spending</td>
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<td>Risk-Premium</td>
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<td>Risk-Premium</td>
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<td>0.1924%</td>
<td>-3.15%</td>
<td>Risk-Premium</td>
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Figure 5: Actual and Counterfactual Output Growth under 1 Quarter Lower Price Durations
Figure 6: Actual and Counterfactual Output Growth under 1/2 Quarter Lower Price Durations
Figure 7: Actual and Counterfactual Inflation Rate under 1/2 and 1 Quarter Lower Price Durations
Output Growth due to Monetary Shocks under Increased Price Flexibility

Figure 8: Output Growth due to Monetary Shocks under 1 Quarter Lower Price Durations and for Base Case
Figure 9: Output Growth due to Price Markup Shocks under 1 Quarter Lower Price Durations and for Base Case
Figure 10: Output Growth due to Wage Markup Shocks under 1 Quarter Lower Price Durations and for Base Case
7 APPENDIX A: Proofs

7.1 Expositional Model

Here, we detail some of the calculations of the expositional model in the text which is based on exogenous nominal spending. We have the following key equations

\[ \Delta \hat{D}_t = \rho \Delta \hat{D}_{t-1} + \epsilon_t^D \]
\[ \pi_t = \kappa \hat{Y}_t + \beta E_t \pi_{t+1} \]
\[ D_t = P_t Y_t. \]

The latter implies
\[ \Delta \hat{D}_t = \pi_t + \hat{Y}_t - \hat{Y}_{t-1}. \]

The system has a solution of the following form

\[ \hat{Y}_t = Y_d \Delta \hat{D}_t + Y_y \hat{Y}_{t-1} \]
\[ \pi_t = \pi_d \Delta \hat{D}_t + \pi_y \hat{Y}_{t-1}. \]

Some algebra using the method of undetermined coefficients implies that

\[ \hat{Y}_t = \frac{(1 - \rho \beta)}{(1 + \kappa - \rho \beta) + \beta \frac{(\kappa + \beta)}{(1 + \kappa + \beta)}} \Delta \hat{D}_t + \frac{1}{1 + \kappa + \beta} \hat{Y}_{t-1} \]

and

\[ \text{VAR}(\hat{Y}_t) = \frac{(1 - \rho \beta)^2}{(1 + \kappa - \rho \beta) + \beta \frac{(\kappa + \beta)}{(1 + \kappa + \beta)} \Delta \hat{D}_t} \text{VAR}(\Delta \hat{D}_t). \]

7.2 Proofs of Propositions 1-5

Under endogenous nominal demand and Taylor rule, the following equations hold

\[ \hat{Y}_t = E_t \hat{Y}_{t+1} + (\hat{Y}_t^n - E_t \hat{Y}_{t+1}^n) - \sigma (\hat{i}_t - E_t \pi_{t+1} - \hat{r}_t^n) \]
\[ \pi_t = \kappa \hat{Y}_t - \kappa \hat{Y}_t^n + \beta E_t \pi_{t+1} \]
\[ \hat{i}_t = \phi_x \pi_t + \phi_y \hat{Y}_t + \eta_t \]
\[ \hat{t}_t \geq \beta - 1 \]
\[ \hat{Y}_t^n = \rho_y \hat{Y}_{t-1}^n + \epsilon_t^y \]
\[ \hat{r}_t^n = \rho_r \hat{r}_{t-1}^n + \epsilon_t^r \]
\[ \eta_t = \rho_\eta \eta_{t-1} + \epsilon_t^\eta. \]

This system has a solution of the following form

\[ \hat{Y}_t = Y_Y \hat{Y}_t^n + Y_r \hat{r}_t^n + Y_\eta \eta_t \]
\[ \hat{\pi}_t = \pi_Y \hat{Y}_t^n + \pi_r \hat{r}_t^n + \pi_\eta \eta_t \]
\[ \hat{i}_t = i_Y \hat{Y}_t^n + i_r \hat{r}_t^n + i_\eta \eta_t \]

\[ E_t \hat{Y}_{t+1} = E_t \left[ Y_Y \hat{Y}_{t+1}^n + Y_r \hat{r}_{t+1}^n + Y_\eta \eta_{t+1} \right] \]
\[ = Y_Y \rho_y \hat{Y}_t^n + Y_r \rho_r \hat{r}_t^n + Y_\eta \rho_\eta \eta_t \]

\[ E_t \hat{\pi}_{t+1} = E_t \left[ \pi_Y \hat{Y}_{t+1}^n + \pi_r \hat{r}_{t+1}^n + \pi_\eta \eta_{t+1} \right] \]
\[ = \pi_Y \rho_y \hat{Y}_t^n + \pi_r \rho_r \hat{r}_t^n + \pi_\eta \rho_\eta \eta_t. \]

First, consider \( Y_t^n \) as the only shock. Some algebra directly implies that

\[ Y_Y = \frac{(1 - \rho_y)(1 - \beta \rho_y) + \kappa \sigma [\phi_\pi - \rho_y]}{[(1 - \rho_y + \sigma \phi_y)(1 - \beta \rho_y) + \kappa \sigma [\phi_\pi - \rho_y]]} \]

and

\[ VAR(\hat{Y}_t) = \left( \frac{(1 - \rho_y)(1 - \beta \rho_y) + \kappa \sigma [\phi_\pi - \rho_y]}{[(1 - \rho_y + \sigma \phi_y)(1 - \beta \rho_y) + \kappa \sigma [\phi_\pi - \rho_y]]} \right)^2 \]

\[ VAR(\hat{Y}_t^n). \]

Second, consider \( r_t^n \) as the only shock. Some algebra directly implies that

\[ Y_r = \frac{\sigma}{(1 - \rho_r + \sigma \phi_y)(1 - \beta \rho_r) + \sigma \kappa [\phi_\pi - \rho_r]} \]

and

\[ VAR(\hat{Y}_t) = \left( \frac{\sigma}{(1 - \rho_r + \sigma \phi_y)(1 - \beta \rho_r) + \sigma \kappa [\phi_\pi - \rho_r]} \right)^2 \]

\[ VAR(\hat{r}_t^n). \]
Third, consider $\eta_t$ as the only shock. Some algebra directly implies that

$$Y_\eta = -\sigma \left(1 - \beta \rho_\eta \right) \frac{1 - \beta \rho_\eta}{(1 - \rho_\eta + \sigma \phi_y)(1 - \beta \rho_\eta) + \sigma \kappa [\phi_\pi - \rho_\eta]}$$

and

$$VAR(\hat{Y}_t) = \left(\sigma \left(1 - \beta \rho_\eta \right) \frac{1 - \beta \rho_\eta}{(1 - \rho_\eta + \sigma \phi_y)(1 - \beta \rho_\eta) + \sigma \kappa [\phi_\pi - \rho_\eta]}\right)^2 VAR(\eta_t).$$

Next, take the derivatives with respect to $\kappa$ in the above expressions:

$$\frac{\partial VAR(\hat{Y}_t/\hat{R}_n^t)}{\partial \kappa} = -2\sigma^2 \frac{\sigma (\phi_\pi - \rho_\eta)}{((1 - \rho_y + \sigma \phi_y)(1 - \beta \rho_y) + \sigma \kappa [\phi_\pi - \rho_y])^3}$$

If $(\phi_\pi - \rho_\eta) > 0$, then this derivative is always negative. The sign of the derivative flips iff $(\phi_\pi - \rho_\eta) < 0$. This follows from the bounds implied by the determinacy condition.

$$\frac{\partial VAR(\hat{Y}_t/\hat{Y}_t^\pi)}{\partial \kappa} = 2((1 - \rho_y)(1 - \beta \rho_y) + \kappa \sigma [\phi_\pi - \rho_y]) \sigma^2 \phi_y (1 - \beta \rho_y)(\phi_\pi - \rho_y) \frac{1}{((1 - \rho_y + \sigma \phi_y)(1 - \beta \rho_y) + \sigma \kappa [\phi_\pi - \rho_y] (1 - \rho_y + \sigma \phi_y)(1 - \beta \rho_y) + \sigma \kappa [\phi_\pi - \rho_y])^2}$$

If $(\phi_\pi - \rho_y) > 0$, then this derivative is always positive. The sign of the derivative flips iff $0 > (\phi_\pi - \rho_y)$. This follows from the bounds implied by the determinacy condition.

$$\frac{\partial VAR(\hat{Y}_t/\eta_t)}{\partial \kappa} = -2\sigma^2 \frac{\sigma (\phi_\pi - \rho_\eta)(1 - \beta \rho_\eta)^2}{((1 - \rho_\eta + \sigma \phi_y)(1 - \beta \rho_\eta) + \sigma \kappa [\phi_\pi - \rho_\eta])^3}$$

If $(\phi_\pi - \rho_\eta) > 0$, then this derivative is always negative. The sign of the derivative flips iff $0 > (\phi_\pi - \rho_\eta)$. This follows from the bounds implied by the determinacy condition.

For the liquidity trap case, the expression for $\hat{Y}_t$ is given by

$$\hat{Y}_S = \frac{\sigma}{(1 - \beta \mu)(1 - \mu) - \sigma \mu \kappa \hat{R}_S}$$

using standard solution methods. See for example, Eggertsson (2010). This then implies

$$VAR(\hat{Y}_t) = \left(\frac{\sigma}{(1 - \beta \mu)(1 - \mu) - \sigma \mu \kappa}\right)^2 VAR(\hat{r}_S)$$

The proof then follows directly by taking the derivative with respect to $\kappa$. 

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7.3 Proof of Productivity Result

The following equations hold

\[ \hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma (\hat{t}_t - E_t \pi_{t+1}) - \hat{\psi}_t + E_t \hat{\psi}_{t+1} \]

\[ \pi_t = \kappa \hat{Y}_t - \kappa \hat{Y}_t^n + \beta E_t \pi_{t+1} \]

\[ \hat{t}_t = \phi_{\pi} \pi_t + \phi_Y \hat{Y}_t \]

\[ \hat{Y}_t^n = \frac{1 + \phi}{\sigma^{-1} + \phi} \hat{A}_t - \frac{1}{\sigma^{-1} + \phi} \hat{\mu}_t + \frac{1}{\sigma^{-1} + \phi} \hat{\tau}_t w. \]

The solution to the system takes the following form, allowing for all shocks

\[ \hat{Y}_t = Y_a \hat{A}_t + Y_\mu \hat{\mu}_t + Y_\tau \hat{\tau}_t + Y_\psi \hat{\psi}_t \]

\[ \pi_t = \pi_a \hat{A}_t + \pi_\mu \hat{\mu}_t + \pi_\tau \hat{\tau}_t + \pi_\psi \hat{\psi}_t \]

\[ E_t \pi_{t+1} = \rho_a \pi_a \hat{A}_t + \rho_\mu \pi_\mu \hat{\mu}_t + \rho_\tau \pi_\tau \hat{\tau}_t + \rho_\psi \pi_\psi \hat{\psi}_t \]

\[ E_t \hat{Y}_{t+1} = \rho_a Y_a \hat{A}_t + \rho_\mu Y_\mu \hat{\mu}_t + \rho_\tau Y_\tau \hat{\tau}_t + \rho_\psi Y_\psi \hat{\psi}_t. \]

Consider \( \hat{A}_t \) as the only shock. Some algebra directly implies that

\[ Y_a = \frac{\sigma [\phi_\pi - \rho] \kappa}{(1 - \rho_a + \sigma \phi_y)(1 - \beta \rho_a) + \sigma [\phi_\pi - \rho] \kappa} a \]

and

\[ VAR(\hat{Y}_t) = \left( \frac{\sigma [\phi_\pi - \rho] \kappa}{(1 - \rho_a + \sigma \phi_y)(1 - \beta \rho_a) + \sigma [\phi_\pi - \rho] \kappa} \right)^2 VAR(\hat{A}_t). \]

An analogous exercise as above shows directly that \( \frac{\partial VAR(\hat{Y}_t/\hat{A}_t)}{\partial k} > 0 \) if \( \phi_\pi - \rho_A > 0 \) and vice versa.
8 APPENDIX B: Wage Flexibility

While the focus of this paper is on the aggregate implications of increased price flexibility, we briefly consider below an extension of the basic model, where both prices and wages are sticky, to analyze the aggregate implications of increased wage flexibility. The model is the same as in Woodford (2003). In this case, the dynamics of the economy can be captured by the following five equations

\[ \pi_t^w = \kappa_w \left( \hat{Y}_t - \hat{Y}_t^n \right) - \xi_w (w_t - \log w_t^n) + \beta E_t \pi_{t+1}^w \]

\[ \pi_t = \kappa_p \left( \hat{Y}_t - \hat{Y}_t^n \right) + \xi_p (w_t - \log w_t^n) + \beta E_t \pi_{t+1} \]

\[ w_t - w_{t-1} = \pi_t^w - \pi_t \]

\[ \hat{Y}_t - Y_t^n = E_t \left( \hat{Y}_{t+1} - \hat{Y}_{t+1}^n \right) - \sigma \left( i_t - E_t \pi_{t+1} - r_t^n \right) \]

\[ i_t = \phi\pi_t + \phi_y Y_t + \eta_t \]

where \( \kappa_w = \xi_w (\omega + \sigma^{-1}) \), \( \kappa_p = \xi_p \omega_p \), \( \xi_w = \frac{(1-\alpha_w)(1-\alpha_w \beta)}{\alpha_w(1+\omega \theta_w)} \), and \( \xi_p = \frac{(1-\alpha_p)(1-\alpha_p \beta)}{\alpha_p(1+\omega_p \theta_p)} \). Here \( \pi_t^w \) now denotes wage inflation, \( w_t \) the real wage, and \( w_t^n \) the natural level of real wage. Moreover, now \( 1 - \alpha_w \) represents the probability of wage adjustment, \( 1 - \alpha_p \) represents the probability of price adjustment, \( \theta_w \) the elasticity of substitution among the different types of labor, and \( \theta_p \) the elasticity of substitution among the different types of goods.

We solve this model numerically first under the simplifying assumption that the driving forces follow independent processes. In that case, for a wide range of parameter values, we can establish that when monetary policy is responsive, if wages are more flexible, output becomes more volatile when the underlying disturbance is \( Y_t^n \) while it becomes less volatile when the underlying disturbance is either \( r_t^n \) or \( \eta_t \). Thus, these results remain the same as our results for price stickiness in the simple model. On the other hand, for the new shock \( w_t^n \), if wages are more flexible, output becomes less volatile.

Given the different results for \( Y_t^n \) and \( w_t^n \) it is then instructive to consider structural shocks that introduce a correlation between \( Y_t^n \) and \( w_t^n \). For example, as shown in Woodford (2003), the following relationship holds for a technology shock \( \hat{A}_t \)
\[
\hat{Y}_t = \frac{(1 + (\omega_w + \omega_p))}{((\omega_w + \omega_p) + \sigma^{-1})} \hat{A}_t \\
\log w_t^n - \log \bar{w} = \left(1 + \frac{\omega_p (\sigma^{-1} - 1)}{(\omega_w + \omega_p) + \sigma^{-1}}\right) \hat{A}_t.
\]

Then, given the comparative statics on \(\hat{Y}_t^n\) and \(\log w_t^n\), for a technology shock, output volatility can either increase or decrease when wages become more flexible. For instance, Figure 11, where we plot \(VAR(\hat{Y}_t)\) against \(\alpha_w\), shows that increasing wage flexibility can lead to either higher or lower output volatility.

![Graph](image)

Figure 11: Variance of Output following Technology Shock for Given Price Stickiness
We use a Bayesian framework for estimation. The first-order approximation to the equilibrium conditions of the model can be written as

\[ \Gamma_0(\theta) s_t = \Gamma_1(\theta) s_{t-1} + \Gamma_\varepsilon(\theta) \varepsilon_t + \Gamma_\eta(\theta) \pi_t \]

where \( s_t \) is a vector of model variables and \( \varepsilon_t \) is a vector of shocks to the exogenous processes. \( \pi_t \) is a vector of rational expectations forecast errors, which implies \( E_{t-1} \pi_t = 0 \) for all \( t \), and \( \theta \) contains the structural model parameters. The solution to this system is given by

\[ s_t = \Omega_1(\theta) s_{t-1} + \Omega_\varepsilon(\theta) \varepsilon_t. \]

which can be obtained using standard methods in the literature. Finally, the model variables are related to the observables by the measurement equation

\[ y_t = Bs_t \]

where \( y_t \) is the vector of observables.

Let \( Y = \{y\}_{t=1}^T \) be the data. In a Bayesian framework, the likelihood function \( L(Y \mid \theta) \) is combined with a prior density \( p(\theta) \) to yield the posterior density

\[ p(\theta \mid Y) \propto p(\theta)L(Y \mid \theta). \]

Assuming Gaussian shocks, it is straightforward to evaluate the likelihood function using the Kalman filter. A numerical optimization routine is used to maximize \( p(\theta \mid Y) \) and find the posterior mode. Then, we can generate draws from \( p(\theta \mid Y) \) using the Metropolis-Hastings algorithm where we use a Gaussian proposal density in the algorithm, using a inverse of a scaled Hessian computed at the posterior mode as the covariance matrix.

The Metropolis-Hastings algorithm works as follows. Let the posterior mode computed from the numerical optimization routine be \( \tilde{\theta} \). Let the inverse of the Hessian computed at \( \tilde{\theta} \) be \( \tilde{\Sigma} \).

(a) Choose a starting value \( \theta^0 \). Then use a loop over the following steps (b)-(d).

(b) For \( d = 1, \ldots, D \), draw a \( \theta^* \) from the proposal distribution \( N(\theta^{d-1}, \ c\tilde{\Sigma}) \).

(c) Accept \( \theta^* \), that is \( \theta^d = \theta^* \), with probability \( \min\{1, r(\theta^{d-1}, \theta^* )\} \). Reject \( \theta^* \), that is \( \theta^d = \theta^{d-1} \), otherwise.
(d) \( r(\theta^{d-1}, \theta^*) \) is given by:

\[
r(\theta^{d-1}, \theta^*) = \frac{p(\theta^*)L(Y \mid \theta^*)}{p(\theta^{d-1})L(Y \mid \theta^{d-1})}
\]

The scale parameter \( c \) is chosen to lead to acceptance rates of around 30%.

To settle on a model specification, we do Bayesian model comparison using the marginal data densities of the models. In comparing models \( A \) and \( B \) we are interested in the relative posterior probabilities of the models given the data. That is, \( \frac{p(A|Y)}{p(B|Y)} = \frac{p(A) p(Y|A)}{p(B) p(Y|B)} \) where \( p(A) \) and \( p(B) \) are the prior probabilities of the models \( A \) and \( B \). Since we do not specifying different prior probabilities over the models, we just compare the marginal data densities given by \( p(Y \mid A) \) and \( p(Y \mid B) \). The marginal data density of a model is given by

\[
p(Y) = \int p(\theta)L(Y \mid \theta) \, d\theta.
\]

Note that this measure penalizes overparameterized models.

The marginal data density is approximated by the Geweke (1999) modified harmonic-mean estimator. First note that we can write

\[
\frac{1}{p(Y)} = \int \frac{f(\theta)d\theta}{p(\theta)L(Y \mid \theta)} \, d\theta
\]

where \( f \) is a probability density function such that \( \int f(\theta)d\theta = 1 \). Then, we can use the following estimator

\[
\hat{p}(Y) = \left[ \frac{1}{D} \sum_{d=1}^{D} \frac{f(\theta^d)}{p(\theta^d)L(Y \mid \theta^d)} \right]^{-1}
\]

where \( d \) denotes the posterior draws obtained using the Metropolis-Hastings algorithm. For \( f \), Geweke (1999) proposed a truncated multivariate normal distribution.

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