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Working Paper

Liquidity and congestion

Staff Report, No. 349

Provided in Cooperation with:

Federal Reserve Bank of New York

Suggested Citation: Afonso, Gara M. (2008) : Liquidity and congestion, Staff Report, No. 349, Federal Reserve Bank of New York, New York, NY

This Version is available at:

<https://hdl.handle.net/10419/60735>

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Federal Reserve Bank of New York
Staff Reports

Liquidity and Congestion

Gara M. Afonso

Staff Report no. 349

October 2008

Revised March 2009

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JEL classification: G12, D40

Abstract

This paper studies the relationship between the arrival of potential investors and market liquidity in a search-based model of asset trading. The entry of investors into a specific market causes two contradictory effects. First, it reduces trading costs, which then attracts new investors (the thick market externality effect). But second, as investors concentrate on one side of the market, the market becomes “congested,” decreasing the returns to participating in this market and discouraging new investors from entering (what we call the *congestion effect*). The equilibrium level of market liquidity depends on which of the two effects dominates. When congestion is the leading effect, some interesting results arise. In particular, we find that diminishing trading costs in our market can impair liquidity and reduce welfare.

Key words: liquidity, search, congestion, asset pricing

Afonso: Federal Reserve Bank of New York (e-mail: gara.afonso@ny.frb.org). The author is indebted to Hyun Song Shin for continuing guidance and encouragement. For very valuable advice, she sincerely thanks Dimitri Vayanos and Markus Brunnermeier. The author is grateful to Jean-Charles Rochet for a constructive discussion of the paper, to Pierre-Olivier Weill for helpful suggestions, and to Max Bruche, Julio Cacho-Díaz, Nicola Cetorelli, Camille Cornand, Amil Dasgupta, Nobuhiro Kiyotaki, Péter Kondor, Albert “Pete” Kyle, Paolo Pasquariello, Xavier Ragot, Ashley Taylor, Laura Veldkamp, S. “Vish” Viswanathan, and Wei Xiong for helpful comments. She also thanks participants at the Cycles, Contagion, and Crises conference, the Princeton Finance Research Camp, and the Cycles and Crises Workshop, as well as seminar participants at Princeton University, the University of Lausanne, the McIntire School of Commerce at the University of Virginia, the Fuqua School of Business at Duke University, the Stephen M. Ross School of Business at the University of Michigan, the Federal Reserve Bank of New York, the Robert H. Smith School of Business at the University of Maryland, the Board of Governors of the Federal Reserve System, and the Workshop in Economic Theory at UCLA. The author is also grateful to the Bendheim Center for Finance at Princeton University for their hospitality, to the Economic and Social Research Council for its support under research grant RES-156-25-0026, and to Fundación Ramón Areces for financial support. The views expressed in this paper are those of the author and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

1 Introduction

Liquidity is sometimes defined as a coordination phenomenon. In financial markets, as investors move into a specific market they facilitate trade for all investors by reducing the cost of participating in this market. At the same time, easier trade and lower trading costs attract potential investors. There is a market externality where new investors provide market liquidity and market liquidity attracts new investors. However, if investors prefer to join one side of a market, i.e. as they become buyers or sellers, this side of the market becomes “congested”, hindering trade. Congestion then discourages investors from entering this market.

One-sided markets arise during financial booms and, more drastically, during market crashes. When a market is in distress, liquidity typically vanishes playing a key role in the build-up of one-sided markets. The study of liquidity in one-sided markets is thus vital to understand the response of financial systems to the threat of market disruptions. Recent episodes of market distress include the LTCM crisis¹ in 1998, the September 11, 2001, events² and the ongoing turbulence in financial markets³.

In this paper we present an alternative view of market liquidity. The main difference with the previous literature is that we consider not only a market externality but also a *congestion effect*. In our model, the arrival of new investors causes two opposite effects. First, it diminishes transaction costs and eases trade, which attracts potential investors. But secondly, if investors concentrate on one side of the market, trade becomes more difficult,

¹For an analysis of the events surrounding the market turbulence in autumn 1998, see BIS (1999) and IMF (1998).

²Cohen and Remolona (2001) presents a summary of the September 11, 2001 episode in global financial markets. Also, McAndrews and Potter (2002) gives a detailed account of the consequences of the September 11, 2001, events on the US payment system and of the actions of the Federal Reserve System to provide liquidity to the financial system.

³Greenlaw et al. (2008) and Brunnermeier (2008) provide a detailed analysis of this episode.

reducing the returns to participating in this market and discouraging potential investors from entering. Market liquidity thus results from the tradeoff between market externalities and a congestion effect.

We assume an infinite-horizon steady-state market where agents can invest in one asset which can be traded only bilaterally. In this market, investors cannot trade instantaneously but it takes some time to find a trading partner resulting in opportunity and other costs. Once an investor buys the asset, he holds it until his preference for the ownership change and he prefers to liquidate the investment and exit the market. To model the search process we adopt the framework introduced in Vayanos and Wang (2007). In our setting though, investors are heterogeneous in their investment opportunities in the sense that some investors have access to better investment options than others.

We compute explicitly the unique equilibrium allocations and the price at which investors trade with each other and show how they depend on the flow of new investors entering the market. Prices negotiated between investors are higher in the flow of potential investors. However, investors' entry decision is endogenous and thus depends on market, asset and investors characteristics. A change in investors' search abilities, for instance, affects both the rate of meetings between trading partners and the flow of investors entering the market, which then determines the distribution of potential partners with whom they can meet.

Moreover, the equilibrium flow of investors arises from a tradeoff between market complementarities and a congestion effect. When congestion is the dominating effect some interesting results come to light. First, *diminishing market frictions can deteriorate market liquidity and reduce welfare*. The reason for this counterintuitive result is the following. In a one-sided market with more sellers than buyers, for example during a fire sale, introducing a measure that improves the efficiency of the search process makes it easier for one of the

few buyers present in the market to acquire the asset. But when the buyer purchases the asset (and a seller exits), the proportion of buyers to sellers falls further and the market becomes more one-sided. As investors cluster on the sell-side of this market, buyers gain a more favorable position in the bargaining process and try to lower the price they pay to acquire the asset. Reducing market frictions in a distressed market thus magnifies the effect of congestion and results in a lower asset price (a higher price discount) and ultimately in a less liquid market. Investors who hold this asset and those trying to sell it are clearly worse-off as the market becomes more one-sided, leading to a decrease in overall welfare.

From this point of view, this paper provides an example of the Theory of the Second Best. Improving the efficiency of the search process, when there are other imperfections in the market such as the ones arising from the congestion effect, is not necessarily welfare enhancing. Stated differently, the paper offers a rationale for measures such as trading halts and circuit breakers which can halt trading when there is a significant imbalance in the pending buy and sell orders in a security or when markets decline beyond trigger levels respectively. For instance, circuit breakers were adopted by the New York Stock Exchange (NYSE) following the 1987 stock market crash to reduce volatility and promote investor confidence.

Second, market illiquidity measured by the price discount can increase while trading volume rises. Reducing search frictions during downswings amplifies the effect of congestion, resulting in a higher price discount and in a less liquid market. But a more efficient search process also increases the frequency of meetings between the investors already present in the market. More frequent meetings then translates into a higher trading volume. Thereby, a measure intended to shorten the waiting times needed to locate a trading partner in a market experiencing distressed selling can cause both higher price discount and higher

trading volume. This second result joins the discussion on the measurement of the effect of liquidity on asset prices and shows how alternative measures capture different dimensions of market liquidity.

The outline of the paper is as follows. In the next section, we discuss the related literature. We introduce a theoretical framework to examine the relationship between market liquidity and the arrival of potential investors to this market in Section 3. Section 4 first determines the population of investors, their expected utilities and the price of the asset, taking as given investors' decision to enter the market. Then, Subsection 4.3 endogenizes the entering rule. Section 5 characterizes the study of the unique market equilibrium and solves the equilibrium in closed-form when search frictions are small. We complement the analysis with a numerical example in Section 6 to illustrate the effect of changes in frictions on liquidity and welfare. Finally, Section 7 concludes. Some proofs and additional results are presented in the appendices.

2 Related Literature

The notion of thick market complementarity is clearly captured in Diamond (1982a). He considers an economy where islanders face production opportunities and decide whether to remain unemployed or to climb a palm tree and retrieve coconuts. Trees differ in their heights (the cost of production). Islanders only climb trees shorter than a certain height and they cannot consume the coconuts they pick. They need to search for a trade to swap the coconuts. The likelihood of meeting a trading partner in this economy increases in the number of potential traders available. This key feature constitutes the basis of the strategic complementarity in Diamond's model. This is highlighted in Cooper and John

(1988), where they discuss the economic relevance of strategic complementarities in agents' payoffs and explain how they can lead to coordination failures. A related argument is presented in Milgrom and Roberts (1990). They show the Diamond-type search model is a supermodular⁴ game, where more production or participation activity by some islanders raises the returns to increased levels of activity by others.

Building on strategic complementarities Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2002) analyze the link between capital and market liquidity. Also, Pagano (1989) focuses on the feedback loop between trading volume and liquidity to study concentration and fragmentation of trade across markets. In Dow (2004), multiple equilibria with different degrees of market liquidity result from informational asymmetries. Plantin (2004) assumes investors can learn privately about an issuer's credit quality by holding an asset. This "learning by trading" also creates a market externality. From a broad perspective, this literature studies liquidity as a self-fulfilling phenomenon where both liquid and illiquid market equilibria may arise. Illiquid markets are thus a consequence of a coordination failure.

Our paper is also related to the search literature. The economics of search have their roots in Phelps (1972). Search-theoretic models such as the frameworks introduced in labor markets⁵ by Diamond (1982a), Diamond (1982b), Mortensen (1982) and Pissarides (1985) have been broadly used in different areas of economics. In asset pricing⁶, Duffie, Gârleanu and Pedersen introduce search and bargaining in models of asset market equilibrium to study the impact of these sources of illiquidity on asset prices. Our paper is related to Duffie

⁴In the unidimensional case, a supermodular game is a game exhibiting strategic complementarities in which each agent's strategy set is partially ordered. See Topkis (1979) and Cooper (1999) for a formal definition.

⁵See Pissarides (2001) for a review of the literature on search in labor markets.

⁶For an excellent review on liquidity and asset prices, see Amihud et al. (2005).

et al. (2005), which presents a theory of asset pricing and marketmaking in over-the-counter markets with search-based inefficiencies. They conclude that risk neutral investors receive narrower bid-ask spreads if they have easier access to other investors and marketmakers. Similarly to Duffie et al. (2005) we consider risk-neutral agents who can only invest in one asset. In our model though, investors can only trade with other investors and our focus, rather than on liquidity and marketmaking, lies on the endogenous relationship between market liquidity and the arrival of potential investors to this market.

Duffie et al. (2007) extends their setting to incorporate risk aversion and risk limits and finds that, under certain conditions, search frictions as well as risk aversion, volatility and hedging demand increase the illiquidity discount. Lagos and Rocheteau (2009) also generalizes Duffie et al. (2005) to allow for general preferences and unrestricted long positions. Unrestricted (positive) asset holdings and free entry of dealers are introduced in Lagos and Rocheteau (2007). Our paper shares with theirs the existence of strategic complementarities and an endogenous entry decision. Lagos and Rocheteau (2007) assumes free entry of dealers and specifies that the contact rate between investors and dealers increases sublinearly in the number of dealers. In our framework, entry is the result of a decision problem where investors compare the benefits derived from investing in this market to their best alternative investment opportunities.

Weill (2008) and Vayanos and Wang (2007) extend the framework of Duffie, Gârleanu and Pedersen to allow investors to trade multiple assets⁷. They show that search frictions lead to cross-sectional variation in asset returns due to illiquidity differences. In Vayanos and Wang (2007) investors are heterogeneous in their trading horizons while in Weill (2008) investors

⁷See also Vayanos and Weill (2008) for an application to the on-the-run phenomenon, by which recently issued bonds have higher prices than older ones with the same cash flows. They develop a multi-asset model where both the spot market and the repo market operate through search.

are homogeneous, but there are differences in the assets' number of tradable shares. From a methodological point of view, our paper is closely related to Vayanos and Wang (2007). The main difference with their work is that we consider only one asset and focus on the analysis of the liquidity in the market for this asset rather than on the liquidity across two assets.

Our paper is close in spirit to Huang and Wang (2009). They also find that decreasing market frictions can diminish the level of market liquidity. However, their framework and the general mechanism that yields this result clearly differ from ours. Rather than a search-based model, they consider a centralized market where exogenous transaction costs take the form of participation costs. Agents can pay an ex-ante cost to trade constantly (and become market makers) or pay a spot cost to trade after observing their trading needs. Huang and Wang (2009) argues that, when there is insufficient supply of liquidity, lowering the cost to enter on the spot can decrease welfare because it reduces investors' incentives to become market makers. In our model, market liquidity results from a tradeoff between market externalities and congestion effects. We show that, when the congestion effect dominates, the market becomes one-sided and improving the efficiency of the search process can diminish market liquidity because it discourages agents from investing in this market.

This paper also relates to the literature on asset pricing with exogenous trading costs studied in Amihud and Mendelson (1986), Vayanos (1998) and Acharya and Pedersen (2005), among others. We complement this literature by endogenizing transaction costs.

3 The Model

Time is continuous and goes from zero to infinity. There is only one asset traded in the market with a total supply S . This asset pays a dividend flow d .

Investors are risk-neutral, infinitely lived and have time preferences determined by a constant discount rate equal to $r > 0$. Investors can hold at most one unit of the asset and cannot shortsell⁸.

In this economy, there are outside investors and three types of inside investors: buyers-to-be, non-searchers and sellers-to-be. At some random time, outside investors can choose to participate in the market for this asset aiming to buy one unit of the asset. They then become buyers-to-be. Once they purchase the asset, buyers-to-be become non-searcher investors. Non-searchers hold the asset and enjoy the full value d of its dividend flow until they receive a liquidity shock which makes them want to liquidate their portfolio and leave the market. At that time, non-searcher investors become sellers-to-be and seek to sell. Upon selling, sellers-to-be exit the market and join the initial group of outside investors. Also, a buyer-to-be, who receives a liquidity shock before purchasing the asset, simply exits the market⁹.

Liquidity shocks arrive with a Poisson rate γ and reduce investors' valuation to a lower level $d - x$ of flow utility, where $x > 0$ captures the notion of a liquidity shock to the investors, for example, a sudden need for cash or the arrival of a good investment opportunity in another market. x could also be understood as the holding cost borne by the investor who receives a liquidity shock and is aiming to exit the market.

The flow of investors entering the economy is defined by a function f . Investors are heterogeneous in their investment opportunities κ , i.e. they differ on their outside options as some investors enjoy better investment possibilities than others. We assume f is a continuous and strictly positive function of the investor's investment opportunity class κ , such that the

⁸Investors are risk neutral and thus have linear utility over the dividend flow d . Consequently, they optimally prefer to hold a maximum long position in the asset (which we can normalize to 1) or zero units of the asset (once they seek to exit the market).

⁹In Duffie et al. (2005)'s framework, a buyer-to-be is an *hn*-investor, i.e. a *high*-type investor (high liquidity state) who does *not* own the asset. A non-searcher corresponds to an *ho*-investor (*high* liquidity state, *owns* asset), a seller-to-be to a *lo*-investor and an outside investor to an *ln*-investor.

total flow of investors entering the economy is given by $\int_{\underline{\kappa}}^{\bar{\kappa}} f(\kappa) d\kappa$, where $[\underline{\kappa}, \bar{\kappa}]$ is the support of $f(\kappa)$. Only a fraction $\nu(\kappa)$ of the flow of investors entering the economy chooses to invest in the market for this asset. At any point in time there is a non-negative flow of every class of investor from the outside investors' group into the market, and hence the total flow of investors entering the market is defined by $g = \int_{\underline{\kappa}}^{\bar{\kappa}} \nu(\kappa) f(\kappa) d\kappa$.

We assume markets operate through search, with buyers and sellers matched randomly over time in pairs. Search is characteristic of over-the-counter markets where investors need to locate trading partners and then bargain over prices. There is a cost associated to this search process. In a market where it is more likely to find a counterpart in a short time, the search cost is smaller and liquidity, measured by search costs, is higher. But we could think of a broader interpretation of the search friction. In a centralized market, it represents the cost of being forced to trade with an outside investor who does not understand the full value of the asset and requires an additional compensation for trading. These investors only buy the asset at a discount and sell it at a premium. This transaction cost decreases in the abundance of investors. In the market of a frequently traded asset, it is less likely that it is necessary to trade with an outside investor who “mis-values” the asset and hence the transaction cost linked to this asset is smaller and its liquidity higher. In this paper, we use the first intuition because of its more transparent interpretation.

We adopt the search framework presented in Vayanos and Wang (2007). To define the search process, we first need to describe the rate at which investors willing to buy meet those willing to sell and once they meet we need to specify how the asset price is determined. The ease in finding a trading partner depends on the availability of potential partners. Let us consider that an investor seeking to buy or sell meets other investors according to a Poisson process with a fixed intensity. Thus, for each investor the arrival of a trading partner occurs at

a Poisson rate proportional to the measure of the partner's group. Denote by η_b the measure of buyers-to-be and by η_s the measure of investors seeking to sell (sellers-to-be). Then, a buyer-to-be meets sellers-to-be with a Poisson intensity $\lambda\eta_s$ and a seller-to-be meets buyers-to-be at a rate $\lambda\eta_b$, where λ measures the efficiency of the search and a high λ represents an efficient search process. The overall flow of meetings¹⁰ between trading partners is then given by $\lambda\eta_b\eta_s$.

Once investors meet they bargain over the price p of the asset. These meetings always result in trade as Proposition 3 shows. For simplicity we assume that either the investor willing to buy or the one willing to sell is chosen randomly to make a take-it-or-leave-it offer to his trading partner. Denoting by $\frac{z}{1+z}$ the probability of the buyer-to-be being selected to make the offer and thus by $\frac{1}{1+z}$ the probability that the seller-to-be makes the offer, $z \in (0, \infty)$ captures the buyer's-to-be bargaining power.

Figure 1 describes this market, specifying the different types of investors and the flows between types. η_0 denotes the measure of non-searcher investors.

4 Steady-State Analysis

We organize the analysis in three steps. We first solve for the steady-state measure of every type of inside investor in Step 1 (Subsection 4.1). Next, in Step 2, we determine inside investors' flow utilities (Subsection 4.2). Both Steps 1 and 2 take as given investors' decision to participate in the market for this asset. Then, in Step 3 (Subsection 4.3), we endogenize the entering rule.

¹⁰See Duffie and Sun (2007) for a formal proof of this result. This application of the exact law of large numbers for random search and matching has previously been used in Duffie et al. (2005), Duffie et al. (2007) and Vayanos and Wang (2007) among others.

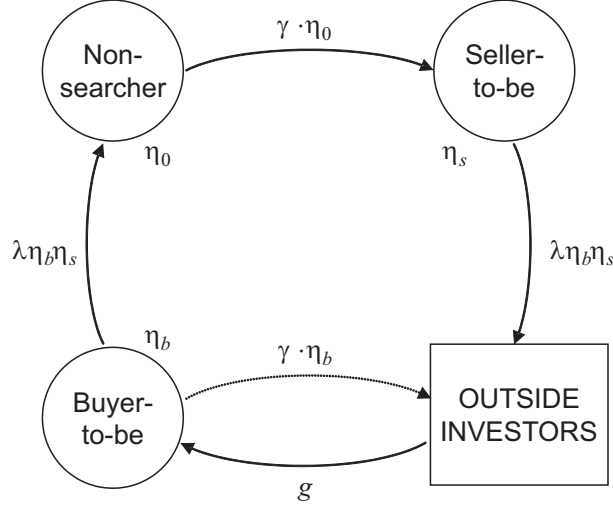


Figure 1: An outside investor enters the market and becomes a buyer-to-be aiming to meet a seller-to-be. If he suffers a liquidity shock before meeting a trading partner, he exits the market. On the contrary, if he meets a seller-to-be, he bargains over the price, buys the asset (pays p) and becomes a non-searcher. He holds the asset until he receives a liquidity shock. At that time, he becomes a seller-to-be seeking a buyer-to-be. When he meets a buyer-to-be, he bargains over the price, sells the asset (receives p) and exits the market returning to the group of outside investors.

4.1 Step 1: Measure of Investors

In this subsection we determine the measure of buyers-to-be (η_b), non-searcher investors (η_0) and sellers-to-be (η_s). Although investors are heterogeneous in their investment opportunities κ , once they enter the market their class does not alter their behavior in this market. Investors develop sudden needs for cash at the same Poisson rate γ , independently of their outside investment opportunities κ . In consequence, we do not need to consider the distribution of investment opportunities within each population but the aggregate measure of buyers-to-be, non-searcher investors and sellers-to-be¹¹.

¹¹This assumption could be generalized by considering γ a function of the outside option κ . The analysis would be similar but the notation more complicated, as we would need to take into account the distribution

In equilibrium, the market needs to clear and thus the supply of the asset equals the measure of investors holding the asset, each of whom holds one unit of the asset. Specifically, the sum of the measures of non-searchers and sellers-to-be is equal to the total supply of the asset:

$$\eta_0 + \eta_s = S \quad \Rightarrow \quad \eta_s = S - \eta_0 \quad (1)$$

In a steady state, the inflow of investors joining a group matches the outflow such that the rate of change of the group's population is zero. The inflow and outflow of the different types of investors are summarized in Figure 1. Let us first consider the non-searcher investors. In this case, inflows are given by the buyers-to-be who meet a trading partner and buy the asset ($\lambda\eta_b\eta_s$), while non-searchers receiving a liquidity shock constitute the outflow ($\gamma\eta_0$). Setting inflow equal to outflow and using equation (1) yields:

$$\eta_b = \frac{\gamma}{\lambda} \frac{\eta_0}{S - \eta_0} \quad (2)$$

We now analyze the population of buyers-to-be. The flows of investors coming from the outside group are defined by g . The outflow is comprised of the buyers-to-be who receive a liquidity shock before meeting a trading partner ($\gamma\eta_b$) and of those who meet sellers-to-be and buy the asset ($\lambda\eta_b\eta_s$). Then,

$$g = \gamma\eta_b + \lambda\eta_b\eta_s$$

Using equations (1) and (2) we can rewrite the previous equation as:

$$g = \gamma \left(1 + \frac{\gamma}{\lambda} \frac{1}{S - \eta_0} \right) \eta_0 \quad (3)$$

of investment opportunities κ within each group of investors rather than the aggregate measures. See Section 3 in Vayanos and Wang (2007) for a particular case.

Equation (3) determines η_0 as a function of g . Then, substituting η_0 in equations (1) and (2) specifies η_s and η_b respectively. Let us first assume the flow of investors entering the market g is constant. We generalize our results in Subsection 4.3.

Proposition 1. *Given g constant, there is a unique solution to the system (1) - (3) given by:*

$$\eta_0 = \frac{1}{2\gamma}A \quad (4)$$

$$\eta_s = S - \frac{1}{2\gamma}A \quad (5)$$

$$\eta_b = \frac{\gamma}{\lambda} \frac{A}{2\gamma S - A} \quad (6)$$

where $A = (g + \gamma S + \frac{\gamma^2}{\lambda}) - \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}$.

The proof is presented in Appendix A. It is interesting to note how the different measures of investors respond to changes in the parameters of our model. For instance, as the flow of investors g entering the market rises, the measures of investors willing to buy (buyers-to-be) and of those passively holding the asset (non-searchers) increase ($\frac{\partial \eta_b}{\partial g}, \frac{\partial \eta_0}{\partial g} > 0$). However, given that there are more investors seeking to buy the asset, it is now easier for a seller-to-be to find a trading partner and hence the measure of investors seeking to sell falls ($\frac{\partial \eta_s}{\partial g} < 0$). This is proven in Appendix B.1.

Given the measures of investors η_b seeking to buy and those η_s seeking to sell, the efficiency of the search process λ defines the overall flow of meetings: $\lambda \eta_b \eta_s$ (and the flow of transactions, according to Proposition 3). However, the measures of the different types of investors (η_b, η_0, η_s) also depend on the efficiency of the search process λ . In particular, for the same level of investors entering the market, if the search process is more efficient, there

will be a lower measure of investors “waiting” to meet a potential seller ($\frac{\partial \eta_b}{\partial \lambda} < 0$). Thus, outside investors, who enter the market, meet a trading partner and become non-searcher investors at a faster rate if the search process is more efficient ($\frac{\partial \eta_0}{\partial \lambda} > 0$). A proportion of non-searcher investors then joins the pool of sellers-to-be and hence there is a higher flow of investors coming from the non-searchers to the group of sellers-to-be. And, although there are more inflows of investors and less investors seeking to buy, if the search process is more efficient, the measure of sellers-to-be “waiting” to sell is reduced ($\frac{\partial \eta_s}{\partial \lambda} < 0$). The proof is in Appendix B.2.

4.2 Step 2: Expected Utilities and Price

We now determine the expected utility of the buyers-to-be (v_b), the non-searcher investors (v_0) and the sellers-to-be (v_s), as well as the price p . Investors exit this market because of a need for cash. We assume that the expected utility of outside investors is zero. Once they are out of the market, investors have different investment opportunities and decide where to invest next. They could even choose to re-enter this market again.

To derive the expected utility of every type of investor we analyze the possible transitions between types. For example, a buyer-to-be can leave the market if he receives a liquidity shock, remain a potential buyer or meet a seller-to-be and become a non-searcher. This is summarized in Figure 2:

The utility flow rv_b of buyers-to-be is thus equal to the expected flow of exiting the market and becoming an outside investor ($(0 - v_b)\gamma$) plus the expected flow derived from meeting a trading partner seeking to sell (which occurs at rate $\lambda\eta_s$), buying the asset (paying

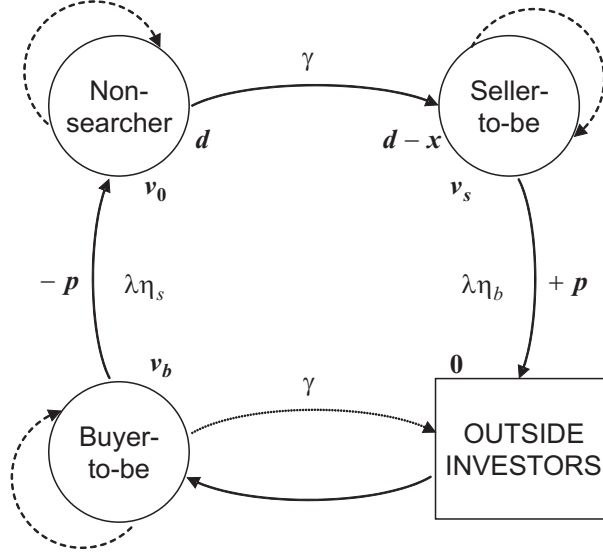


Figure 2: Types of investors and transitions between types.

p) and becoming a non-searcher investor ($\lambda\eta_s(v_0 - v_b - p)$). Then,

$$rv_b = -\gamma v_b + \lambda\eta_s(v_0 - v_b - p) \quad (7)$$

Non-searcher investors can either remain non-searchers enjoying the full value d of the asset's dividend flow or receive a liquidity shock with probability γ and become a seller-to-be. In this case, the flow of utility of being a non-searcher is

$$rv_0 = d + \gamma(v_s - v_0) \quad (8)$$

Sellers-to-be exit the market as soon as they meet a trading partner, i.e., with intensity $\lambda\eta_b$ they sell the asset (receiving p) and become outside investors with zero expected utility.

Meanwhile, they enjoy a low level $d - x$ of utility. Thus,

$$rv_s = (d - x) + \lambda\eta_b(p + 0 - v_s) \quad (9)$$

The asset price is determined by bilateral bargaining between a buyer-to-be and a seller-to-be. We have assumed that with probability $\frac{z}{1+z}$ the buyer-to-be makes a take-it-or-leave-it offer to his trading partner and offers him his reservation value v_s . With probability $\frac{1}{1+z}$, the seller-to-be is chosen to offer the buyer-to-be his reservation value $v_0 - v_b$. As a result,

$$p = \frac{z}{1+z}v_s + \frac{1}{1+z}(v_0 - v_b) \quad (10)$$

where z measures the buyer's-to-be bargaining power which we treat as exogenous. Proposition 2 summarizes this subsection's main result. The proof is in Appendix A.

Proposition 2. *Given g constant, the system of equations (7)-(10) has a unique solution given by:*

$$v_b = k \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \frac{\lambda\eta_s z}{r + \gamma} \quad (11)$$

$$v_0 = \frac{d}{r} - k \left(\frac{x}{r} + \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \right) \frac{\gamma}{r + \gamma} \quad (12)$$

$$v_s = \frac{d}{r} - k \left(\frac{x}{r} + \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \right) \quad (13)$$

$$p = \frac{d}{r} - k \frac{x}{r} \quad (14)$$

where $k = \frac{(r + \gamma + \lambda\eta_s)z + \gamma}{(r + \gamma + \lambda\eta_s)z + (r + \gamma + \lambda\eta_b)}$.

The price of the asset as given by equation (14) is thus equal to the present value of all future dividend flows d , discounted at the rate r , minus a price discount due to illiquidity.

The second term is the product of present value of the holding cost x borne by investors seeking to exit the market and a function k . $k \in (0, 1)$ measures the severity or intensity of the illiquidity discount.

It is interesting to highlight that the asset price will be higher when fundamentals are stronger (i.e. if the asset pays a higher dividend flow d) and whenever the demand for the asset increases ($\frac{\partial p}{\partial d}, \frac{\partial p}{\partial \eta_b} > 0$). On the contrary, the price decreases with investors trying to sell the asset and in the buyer's-to-be bargaining power ($\frac{\partial p}{\partial z}, \frac{\partial p}{\partial \eta_s} < 0$). If during the bargaining process the buyer-to-be holds a more favorable position, he would try to lower the price paid to acquire the asset. The proof of this set of comparative statics is presented in Appendix B.3.

4.2.1 Trade among Investors

In this subsection we prove a result we have assumed so far in our analysis:

Proposition 3. *All meetings between buyers-to-be and sellers-to-be result in trade.*

Proof. Trade between buyers-to-be and sellers-to-be occurs if the gain from trade is strictly positive, i.e., if the buyers'-to-be reservation value $v_0 - v_b$ exceeds the sellers'-to-be reservation value v_s . Let us see if $(v_0 - v_b) - v_s > 0$. Subtracting equations (13) and (11) from (12), we get:

$$(v_0 - v_b) - v_s = \frac{x(1 + z)}{(r + \gamma)(1 + z) + \lambda\eta_s z + \lambda\eta_b}$$

which is always strictly greater than zero since $x, r, \gamma, \lambda, \eta_s, \eta_b, z > 0$.

Therefore, once investors meet, trade among partners always occurs. □

4.3 Step 3: Entering Rule

We now endogenize the entering rule. In our framework, outside investors can choose between entering the market for this asset and investing in an alternative market. Investors are heterogeneous in their outside investment opportunities κ , i.e. each class of investor has access to different investment opportunities. However, once they enter a market, their type no longer influences their decisions in the sense that every buyer-to-be, for instance, enjoys the same expected utility independently of his original outside opportunity. Interestingly, a buyer's-to-be expected utility does depend on the flow of investors who entered this market before him.

Let us refer to the investor who is deciding between moving or not into a market as the marginal investor. And, let us denote by κ' and by $v_{alt}(\kappa')$, respectively, the best outside investment opportunity of the marginal investor and his expected utility from investing in that alternative market. For simplicity, we assume $v_{alt}(\kappa') = \kappa'$, such that an investor with a better outside option (higher κ) enjoys a higher level of expected utility (higher v_{alt}).

When an investor faces the decision to choose a market, he prefers to enter and invest in a specific market if the expected utility v_b of being a buyer-to-be in that market is higher than the expected utility v_{alt} derived from his best outside option. Then, if a market represents the best opportunity for the marginal investor, it is also preferred by any other investor with a worse investment opportunity, i.e. any investor with type $\kappa < \kappa'$ moves into the market too. As a result, when a market is chosen by a marginal investor with a high type, a high flow of investors enters that market. A high flow of investors implies an increase in the measure of buyers-to-be, which then affects the expected utility of being a buyer-to-be. Thus, even though each investor's type does not alter his expected utility, the type of the last investor who enters does. The type of this last investor defines the total flow who invests in this

market and hence determines how concentrated the population of buyers-to-be is.

Let us define the fraction $\nu(\kappa)$ of investors with outside investment opportunity κ who enters the market as follows:

$$\nu(\kappa) = \begin{cases} 0 & \text{if } \kappa > \kappa' \\ [0, 1] & \text{if } \kappa = \kappa' \\ 1 & \text{if } \kappa < \kappa' \end{cases}$$

where $1 - \nu(\kappa)$ represents the fraction of investors with outside option κ who invests in alternative markets. The total flow of investors moving into this market is thus given by: $g(\kappa') = \int_{\underline{\kappa}}^{\bar{\kappa}} \nu(\kappa) f(\kappa) d\kappa$, where f defines the total flow of investors entering the economy and $[\underline{\kappa}, \bar{\kappa}]$ is the support of $f(\kappa)$. In equilibrium, as we discuss in more detail in the next section, the total flow g^* depends on the equilibrium fraction of investors ν^* entering the market. But the equilibrium fraction of investors is determined by the marginal investor who is indifferent between this market and his best outside option. We refer to this investor as the indifferent investor. For the indifferent investor, the expected utility of being a buyer-to-be equals the expected utility of his best outside option:

$$v_b \left(g^* = \int_{\underline{\kappa}}^{\bar{\kappa}} \nu^*(\kappa) f(\kappa) d\kappa \right) = v_{alt}(\kappa^*) \quad (15)$$

Before we proceed, let us introduce the formal definition of market equilibrium.

5 Equilibrium

5.1 Equilibrium Definition

Definition 1. A *market equilibrium* consists of a fraction $\nu(\kappa)$ of investors entering the market, measures (η_s, η_b, η_0) of investors and expected utilities and prices (v_b, v_0, v_s, p) such that:

- $(\eta_s^*, \eta_b^*, \eta_0^*)$ solve the market-clearing condition and inflow-outflow equations given by the system (1) - (3),
- $(v_b^*, v_0^*, v_s^*, p^*)$ solve the flow-value equations for the expected utilities and the pricing condition given by the system (7) - (10),
- $\nu^*(\kappa)$ solves the entering condition given by equation (15).

5.2 Equilibrium Characterization

To analyze the set of equilibria in this market, we need to solve for the fixed points of the system of equations (1) - (3), (7) - (10) and (15). To gain some intuition, let us introduce Figure 3. Figure 3 represents the expected utility of being a buyer-to-be, v_b , and the expected utility v_{alt} derived from investing in an alternative market as a function of the outside investment opportunity κ' of the marginal investor who is deciding between entering or not this market.

Consider, for example, the marginal investor with outside investment opportunity κ'_1 . He compares the utility of his outside option, $v_{alt}(\kappa'_1) = \kappa'_1$, to the utility of being a buyer-to-be, $v_b(g(\kappa'_1))$, given that investors with outside opportunities $\kappa < \kappa'_1$ have already entered.

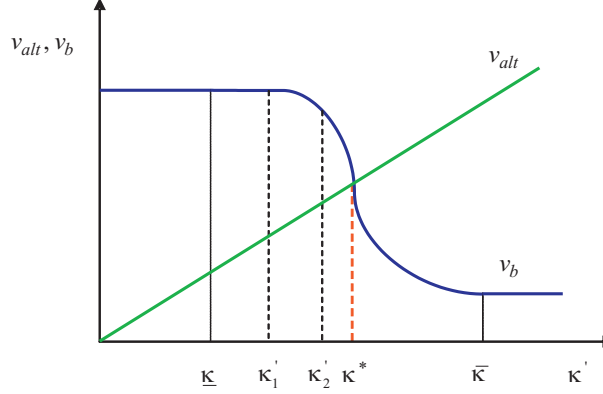


Figure 3: Investors compare expected utilities v_b and v_{alt} and decide to participate in this market if $v_b > v_{alt}$. κ^* defines the outside investment opportunity which makes investors indifferent between entering or not this market. $\underline{\kappa}$ and $\bar{\kappa}$ determine the support of the flow of investors who enter the economy.

He decides to enter since $v_b(g(\kappa'_1)) > v_{alt}(\kappa'_1)$ as shown in Figure 3. Similarly, the marginal investor with investment opportunity κ'_2 prefers to enter too¹². Suppose marginal investor κ^* is now facing the entry decision. For him, $v_b(g(\kappa^*)) = v_{alt}(\kappa^*)$ and he is indifferent between markets. Any investor with a better outside opportunity prefers not to enter.

In the general case there are two types of possible scenarios depending on the behavior of the expected utility v_{alt} of investing in an alternative market and the expected utility v_b of being a buyer-to-be. There is an equilibrium where all investors clearly prefer one market (either all enter (when $\kappa^* > \bar{\kappa}$) or no one enters (if $\kappa^* < \underline{\kappa}$)) or an equilibrium where a fraction of investors is better-off by investing in this market while others prefer not to enter (the case depicted in Figure 3). In either scenario, the market equilibrium is unique.

Theorem 1 summarizes a key result:

¹²Although the expected utility of being a buyer-to-be has decreased because now all investors with $\kappa < \kappa'_2$ are in the market, he is still better-off by moving into this market ($v_b(g(\kappa'_2)) > v_{alt}(\kappa'_2)$).

Theorem 1. *There is a unique market equilibrium.*

The complete proof is in Appendix C but let us provide some guidelines for why the equilibrium is unique. Given non-negative expected utilities, if $v_b(\kappa' = 0) > v_{alt}(\kappa' = 0)$ and v_b decreases in κ' while v_{alt} is strictly increasing, then by continuity there exists a unique threshold κ^* such that expected utilities are equal and investors indifferent between markets. A unique threshold κ^* then defines a unique flow of investors $g^* \equiv g(\kappa' = \kappa^*)$ into this market. And given a unique flow of investors g^* , steady-state measures, expected utilities and the asset price can be determined uniquely as stated in Propositions 1 and 2. Consequently, market equilibrium is unique. It is interesting to note that the expected utility of buyers-to-be decreases as more investors enter this market¹³.

5.3 Market Equilibrium

In general the market equilibrium can only be computed numerically. To gain some intuition we solve the equilibrium in closed-form when search frictions are small¹⁴. Small search frictions corresponds to a large value of the parameter λ that measures the efficiency of the search process. When search frictions are small, the market converges to the Walrasian Equilibrium where the price of the asset is determined by supply and demand for the asset.

¹³An increase in the flow of investors g affects differently the steady-state measures of investors. Specifically, the measures of buyers-to-be and non-searchers increase in the flow of investors while the measure of those seeking to sell is reduced. Buyers-to-be are worse-off as more investors decide to move into a market because their arrival makes it more difficult for them to meet a seller-to-be and purchase the asset. Buyers-to-be thus suffer from a ‘congestion effect’ in the sense that as new investors move in, their side of the market becomes more crowded and there is increasing competition among buyers-to-be to meet one of the fewer sellers-to-be. Similarly, if investors preferred to invest in alternative markets and there were a reduction in the flow of investors into this market, sellers-to-be would be worse-off. It would be more difficult for a seller-to-be to meet a buyer-to-be and sell and exit the market. Sellers-to-be would experience a ‘congestion effect’ as the market becomes more one-sided and their side of the market gets crowded with sellers-to-be looking for a trading partner.

¹⁴We complement the analysis with a numerical example in Section 6.

Let us consider three cases:

- Case 1: $g < \gamma S$ & $\lambda \rightarrow \infty$ - If the flow of investors g is lower than γS , buyers-to-be are the short side of the market and there is “excess supply” of this asset¹⁵. Sellers-to-be are thus marginal and the price is equal to their valuation: $p = \frac{d-x}{r}$.
- Case 2: $g = \gamma S$ - If the flow of investors g equals γS , there is no excess demand or excess supply¹⁶. As $\lambda \rightarrow \infty$ the price of the asset is given by $p = \frac{d}{r} - \frac{z}{z+1} \frac{x}{r}$.
- Case 3: $g > \gamma S$ & $\lambda \rightarrow \infty$ - If the flow of investors g exceeds γS , sellers-to-be are now the short side of the market and there is “excess demand” for this asset¹⁷. Buyers-to-be are thus marginal and the price is equal to their valuation: $p = \frac{d}{r}$.

For simplicity we focus on the case where the flow of investors g equals γS (Case 2).

Proposition 4. *Suppose $g = \gamma S$. When λ goes to infinity the following asymptotics hold:*

$$\eta_s = \eta_b = \frac{\sqrt{g}}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right) \quad (16)$$

$$p = \frac{d}{r} - \frac{z}{z+1} \frac{x}{r} - \frac{\gamma}{z+1} \frac{x}{r} \frac{1}{\sqrt{g}} \frac{1}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right) \quad (17)$$

where $o\left(\frac{1}{\sqrt{\lambda}}\right)$ denotes terms of order smaller than $\frac{1}{\sqrt{\lambda}}$.

When $g = \gamma S$, the measure of buyers-to-be and sellers-to-be are of the same order and the ratio of buyers-to-be to sellers-to-be equals 1 for any level of search frictions. The ratio $\frac{\eta_b}{\eta_s}$ constitutes an important element in our analysis as it captures how “congested” a market is. $\frac{\eta_b}{\eta_s} = 1$ represents a “balanced” market where for every investor willing to buy there is a potential seller.

¹⁵The measure of sellers-to-be is of order 1 while the measure of buyers-to-be is of order $\frac{1}{\lambda}$.

¹⁶The measure of sellers-to-be equals the measure of buyers-to-be ($\frac{\eta_b}{\eta_s} = 1 \forall \lambda$). See Proposition 4.

¹⁷The measure of sellers-to-be is of order $\frac{1}{\lambda}$ while the measure of buyers-to-be is of order 1.

Corollary 1. *Suppose $g = \gamma S$ and λ is large. Liquidity decreases when the buyers'-to-be bargaining power increases.*

When search frictions are small and $\frac{\eta_b}{\eta_s} = 1$, i.e. when $\lambda \rightarrow \infty$ and $g = \gamma S$, the illiquidity discount depends on the buyers'-to-be bargaining power. As buyers-to-be gain a stronger position, they try to lower the price they pay to acquire the asset¹⁸. In a more general case, liquidity depends not only on the buyers'-to-be bargaining power but also on the proportion of potential buyers to sellers in this market. For instance, for small frictions and the same level of bargaining power, the price of the asset would be lower in a market where it is very difficult for an investor seeking to sell to find a trading partner. Put differently, even in the limiting case where search frictions tend to zero, if there are many sellers per buyer the market for this asset might be illiquid¹⁹. Result 1 summarizes this interesting result.

Result 1. *Suppose $g < \gamma S$ and λ is large. Diminishing market frictions does not lead to a more liquid market.*

5.3.1 Welfare

We measure welfare by the weighted sum of investors' expected utilities. Weights are determined by the measure of every type of investor in our economy, including the outside investors. Then, our measure of welfare is:

$$W = \eta_b v_b + \eta_0 v_0 + \eta_s v_s + \int_{\kappa^*}^{\bar{\kappa}} v_{alt} f(\kappa) d\kappa \quad (18)$$

¹⁸We define illiquidity in Subsection 4.2 as measured by the illiquidity discount $k \frac{x}{r}$, where $k \in [0, 1]$ captures the severity of the illiquidity effect. When $\lambda \rightarrow \infty$ and $g = \gamma S$, $k = \frac{z}{z+1} < 1$ for z finite and hence the illiquidity discount is given by $\frac{z}{z+1} \frac{x}{r}$, which is increasing in z .

¹⁹When $\lambda \rightarrow \infty$ and $g < \gamma S$ ($\frac{\eta_b}{\eta_s} = 0$), then $k = 1$. The illiquidity discount equals $\frac{x}{r}$ (its largest value).

where the first three terms represent the welfare of the investors who prefer to enter the market ($W_{\text{inside investors}}$) and the last term reflects the welfare of outside investors ($W_{\text{outside investors}}$). Outside investors (those with investment opportunities above the threshold value κ^*) enjoy the expected utility derived from investing in an alternative market v_{alt} , which for simplicity we assume equal to their outside investment opportunity κ . Substituting equations (11)-(13) and $v_{alt} = \kappa$ we get:

$$W = W_{\text{inside investors}} + W_{\text{outside investors}}$$

where

$$W_{\text{inside investors}} = \frac{d}{r}S - \frac{x}{r + \gamma} \frac{\frac{\gamma}{r}S[(r + \gamma) + \gamma z] + \frac{\gamma}{r}Sz\lambda\eta_s + [(r + \gamma)(z + 1) + \gamma z + z\lambda\eta_s]\eta_s}{(r + \gamma)(z + 1) + z\lambda\eta_s + \lambda\eta_b} \quad (19)$$

$$W_{\text{outside investors}} = \int_{\kappa^*}^{\bar{\kappa}} \kappa f(\kappa) d\kappa \quad (20)$$

The analysis of welfare for any level of market frictions requires (numerically) solving for the equilibrium. To provide some motivation before we introduce a numerical example in Section 6, we now discuss the scenario where search frictions are small.

We can decompose welfare into the welfare of inside investors and the welfare of investors who decide not to invest in this market. The welfare of outside investors, as shown in equation (20), is decreasing in the equilibrium threshold κ^* that defines the indifferent investor and thus in the equilibrium flow of investors g^* entering the market²⁰. However, there is a non-monotonic relationship between the equilibrium flow of investors g^* and the search frictions measured by λ . In particular, in a one-sided market with many potential buyers per

²⁰ g^* increases in κ^* ($g^* = \int_{\underline{\kappa}}^{\kappa^*} f(\kappa) d\kappa$).

seller ($g > \gamma S$), reducing frictions would make the market less attractive to new investors²¹ ($\frac{\partial g^*}{\partial \lambda} \leq 0$). As a result, the welfare of outside investors when there are “too many” buyers-to-be per seller-to-be increases as search frictions are smaller. On the contrary, if there are fewer buyers than potential sellers ($g < \gamma S$), the equilibrium flow of investors into the market increases as search frictions get smaller²² ($\frac{\partial g^*}{\partial \lambda} \geq 0$). Consequently, the welfare of outside investors (when there are “too many” sellers-to-be per buyer-to-be) decreases as search frictions are attenuated. In either case, how relevant the contribution of the welfare of outside investors to total welfare is, depends on specific assumptions on the distribution of f and on the market, investor and asset characteristics. If f were a probability density function with finite support, the welfare of outside investors would be positive and bounded above by the mean of investors’ outside investment opportunities²³. Quantifying the effect would require further assumptions. We postpone a more detailed analysis of the welfare of outside investors to Section 6.

Let us now focus on the welfare of inside investors when search frictions are small. We can consider three different cases:

- Case 1: $g < \gamma S$ & $\lambda \rightarrow \infty$ - Buyers-to-be are the short side of the market. The welfare

²¹Reducing frictions in this one-sided market would amplify the effect of congestion. It would allow sellers-to-be to exit faster leading to a more unbalanced distribution of investors. This would discourage new investors from entering this market.

²²This one-sided market is attractive to some investors as they can easily meet a trading partner and purchase the asset. Diminishing frictions would make it easier. This market would then be preferred by more investors, attracting new investors to this market.

²³ $W_{\text{outside investors}} = 0$ when $\kappa^* = \bar{\kappa}$. $W_{\text{outside investors}} = E[\kappa]$ when $\kappa^* = \underline{\kappa}$.

of inside investors is²⁴:

$$W_{\text{inside investors}} = \frac{d}{r}S - \frac{x}{r} \left[S - \frac{r}{r + \gamma} \frac{g}{\gamma} \right]$$

- Case 2: $g = \gamma S$ - There is no excess demand or excess supply. As $\lambda \rightarrow \infty$, welfare of inside investors is given by²⁵:

$$W_{\text{inside investors}} = \frac{d}{r}S - \frac{x}{r} \left[S - \frac{r}{r + \gamma} \frac{g}{\gamma} \right] \frac{z}{z + 1}$$

- Case 3: $g > \gamma S$ & $\lambda \rightarrow \infty$ - Sellers-to-be are now the short side of the market. Then²⁶,

$$W_{\text{inside investors}} = \frac{d}{r}S$$

where as λ goes to infinity

$$W_{\text{inside investors}}^{g < \gamma S} \leq W_{\text{inside investors}}^{g = \gamma S} \leq W_{\text{inside investors}}^{g > \gamma S}$$

In a market where there is an investor seeking to buy per potential seller, i.e. when $g = \gamma S$, and search frictions are small ($\lambda \rightarrow \infty$) as buyers-to-be gain a stronger bargaining

²⁴When $g < \gamma S$ and $\lambda \rightarrow \infty$, then $\eta_s \rightarrow S - \frac{g}{\gamma} \equiv \text{const.} > 0$, $\lambda \eta_s \rightarrow \infty$ and $\lambda \eta_b \rightarrow \frac{\gamma g}{\gamma S - g} \equiv \text{const.} > 0$.

²⁵When $g = \gamma S$, then $\eta_s = \eta_b = \frac{\sqrt{g}}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right)$. As $\lambda \rightarrow \infty$, $\lambda \eta_s \rightarrow \infty$, $\lambda \eta_b \rightarrow \infty$ and $\lambda \eta_s \eta_b \rightarrow \text{const.} > 0$.

²⁶When $g > \gamma S$ and $\lambda \rightarrow \infty$, then $\eta_s \rightarrow 0$, $\lambda \eta_s \rightarrow \frac{\gamma^2 S}{g - \gamma S} \equiv \text{const.} > 0$ and $\lambda \eta_b \rightarrow \infty$.

position (higher z), the welfare of inside investors decreases. This scenario resembles a market with many sellers-to-be per buyer-to-be. Welfare of inside investors would then be increasingly determined by the contribution of sellers-to-be who enjoy a lower level of utility. Also, in a one-sided market with many investors trying to exit per buyer present in the market ($g < \gamma S$), reducing search frictions may still lead to a low level of welfare.

Result 2. *Suppose $g < \gamma S$ and λ is large. Diminishing market frictions is not necessary welfare enhancing.*

To address how changes in frictions affect liquidity and welfare for any level of search frictions we need to study the general case of the equilibrium.

6 An Example

In this section we present a numerical example to show how diminishing frictions can have an adverse effect on liquidity and welfare. We assume the flow of investors f entering the economy follows a beta distribution²⁷ with support $[\underline{\kappa} \equiv 0, \bar{\kappa} \equiv 5]$ and parameters $a = 8$ and $b = 2$, which is a left-skewed hump-shaped density function. Investors have time preferences with discount rate equal to 1% ($r = 0.01$). The asset pays a dividend flow $d = 2$ and is in total supply $S = 2$. The holding cost is defined as a 40% of the dividend flow to indicate that once an investor receives a liquidity shock his valuation of the asset drops to a 60% of the initial value. Liquidity shocks arrive at a Poisson rate $\gamma = 0.2$ and hence the expected time between shocks is 5. The value of z is chosen such that buyers-to-be and sellers-to-be have the same bargaining power, i.e. $z = 1$. Table I summarizes the set of exogenous parameters:

²⁷The beta distribution is a flexible class of distributions defined on the unit interval $[0, 1]$, whose density function may take on different shapes depending on the choice of the two parameters. These include the uniform density function and hump-shaped densities (See Evans et al. (1993)).

$f \sim f_{beta(8,2)}$	$d = 2$	$S = 2$	$z = 1$
$\underline{\kappa} = 0, \bar{\kappa} = 5$	$x = 0.8$	$\gamma = 0.2$	$r = 0.01$

Table I: Parameter Values.

Market equilibrium is the solution to the system of equations (1)-(3), (7)-(10) and (15). However, for a generic level of search frictions computing the equilibrium can only be done numerically²⁸. The set of parameter values in Table I corresponds to a case where $g < \gamma S$ and there are many investors willing to sell per buyer-to-be present in the market: $\frac{\eta_b}{\eta_s} \ll 1$ as shown in Figure 4(a) and (b) respectively. This scenario represents a one-sided market where there is congestion on the sell-side of the market. An example of such market could be a market experiencing a fire sale.

Note that when search frictions are small (λ large), buyers-to-be are the short side of the market, $\frac{\eta_b}{\eta_s} \rightarrow 0$, as in Case 1 in Subsection 5.3 (Figure 4(b)). Also, the price of the asset, as depicted in Figure 4(c), decreases as search frictions are attenuated (for larger values of λ) and it converges to $\frac{d-x}{r}$ as λ tends to infinity. As a result, illiquidity (measured by the illiquidity discount introduced in Subsection 4.2) increases as search frictions are smaller (Figure 4(d)). Put differently,

Result 3. *Diminishing frictions can deteriorate liquidity.*

In this distressed market, where there are many investors seeking to sell per potential buyer, reducing frictions would amplify the effect of congestion. Lesser frictions allow potential buyers to acquire the asset faster leading to a more unbalanced distribution of investors. As there are even fewer investors left willing to buy, the price of the asset falls and illiquidity rises.

The top panel of Figure 5(c) depicts how the welfare of outside investors decreases when

²⁸See Appendix E.

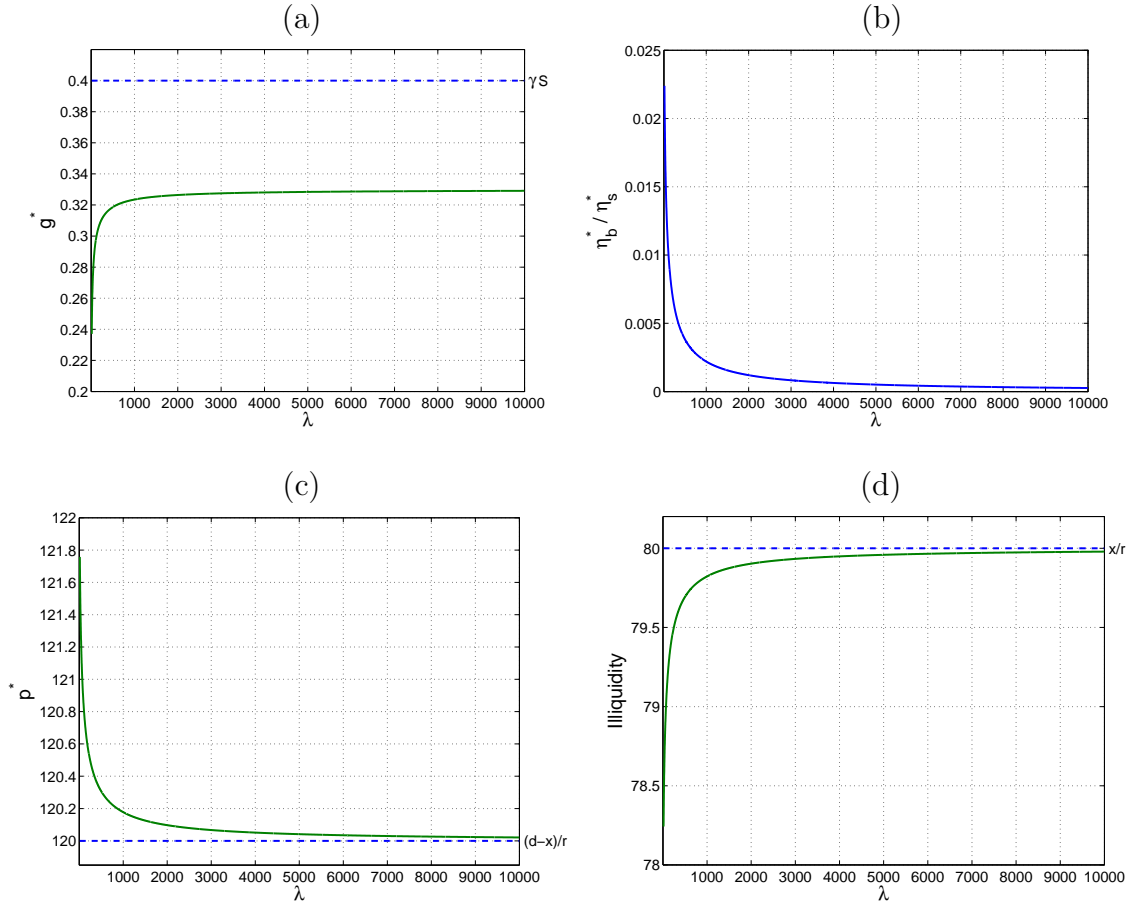


Figure 4: Equilibrium flow of investors g^* entering the market (a), ratio of buyers-to-be to sellers-to-be (b), asset price (c) and illiquidity (measured by price discount) as a function of the efficiency of the search process λ .

search frictions are attenuated as the result of a rise in the equilibrium flow of investors g^* who gives up outside investment opportunities to participate in this market. Also, as shown in the top panel of Figure 5(a), investors holding the asset (non-searchers) and those trying to sell it (sellers-to-be) are worse-off when the distribution becomes more unbalanced as search frictions get smaller. And, although the expected utility of buyers-to-be increases in search efficiency (bottom panel of Figure 5(a)), in equilibrium there are very few potential buyers present in this market (bottom panel of Figure 5(b)). Consequently, the welfare of

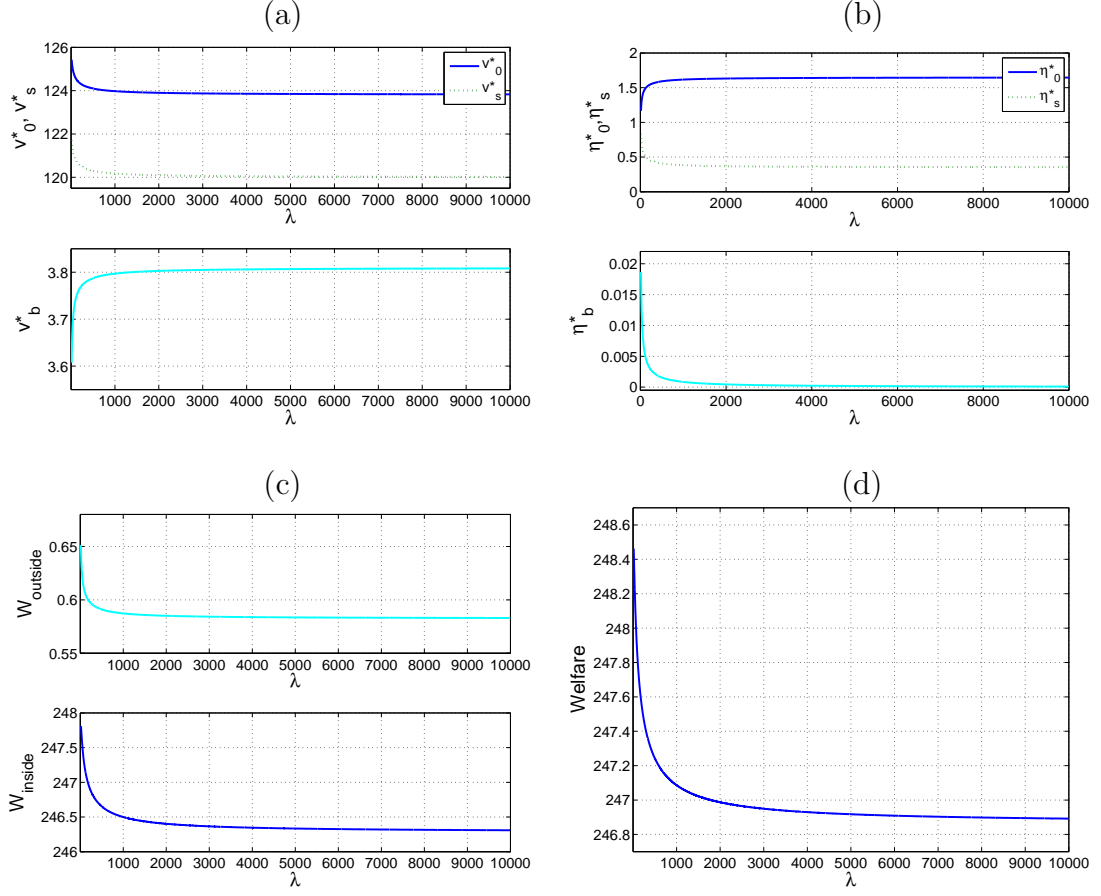


Figure 5: Equilibrium expected utilities (a), measures of investors (b), welfare of outside (c)(top) and inside investors (c)(bottom) and total welfare (d) as a function of the efficiency of the search process λ .

inside investors diminishes in λ as illustrated in the bottom panel of Figure 5(c). Overall, total welfare decreases as search frictions are attenuated (Figure 5(d)). This leads to a key result:

Result 4. *Diminishing frictions can reduce welfare.*

Results 3 and 4 provide a rationale for measures such as circuit breakers that halt (rather than facilitate) trading in distressed markets when a market declines beyond trigger levels.

In this paper we measured liquidity by price discount. Liquidity could also be measured by trading volume: $V = \lambda \eta_b \eta_s$ or by search times, among other measures. Search times capture the expected time it takes to find a trading partner and can hence differ for buyers and sellers. Let us denote by $\tau_b \equiv \frac{1}{\lambda \eta_s}$ the expected time it takes for buyer to meet a seller and by $\tau_s \equiv \frac{1}{\lambda \eta_b}$ the expected time necessary to find a buyer.

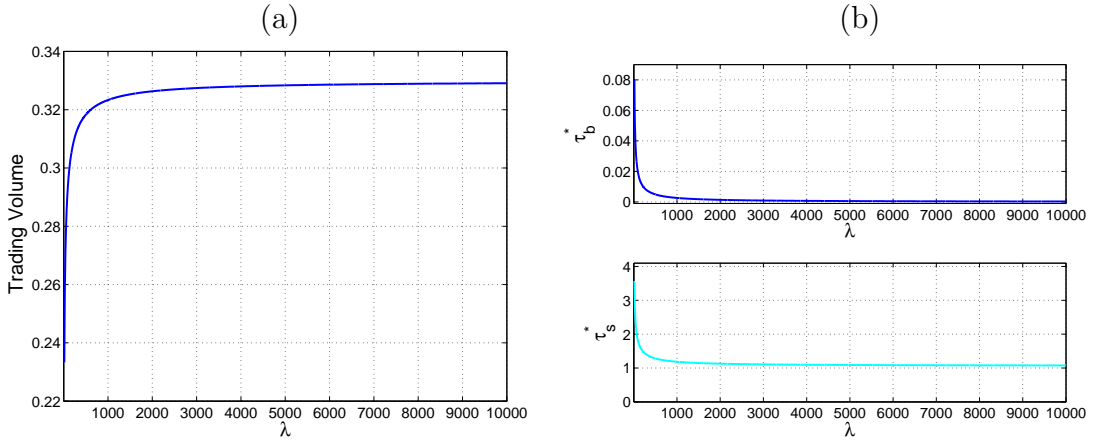


Figure 6: Trading volume (a) and expected trading time for a buyer (b)(top) and expected trading time for a seller (b)(bottom) as a function of the efficiency of the search process λ .

An interesting result is presented in Figure 6. As search frictions are attenuated and liquidity dries up, trading volume rises and the expected time needed to buy and sell in this market diminish. The reason for this counterintuitive result is the following. Facilitating search by reducing frictions yields two consequences. First, it magnifies the effect of congestion leading to a lower price and thus to a less liquid market (as measured by price discount). This is depicted in Figure 4(d). Second, it raises the frequency of meetings between trading partners. Investors meet at a faster rate which translates into an increasing volume and a decreasing trading delay. Consequently, even though the market is less liquid, investors meet

faster and hence expected trading times fall and volume rises as shown in Figure 6. This is our last result:

Result 5. *Illiquidity (measured by price discount) can increase while trading volume rises.*

This result highlights that alternative measures capture different dimensions of liquidity. An example of a dramatic drop in share price and heavy volume followed Lehman Brothers' bankruptcy filing on Monday, September 15, 2008. On Monday, its share price fell over 94% from \$3.65 to \$0.21. Trading volume, however, reached historical levels exceeding thirty times the average daily volume for the last five years.

7 Conclusions

This paper proposes a search-based model to study the relationship between market liquidity and the endogenous arrival of potential investors to a specific market. As investors enter a market, they make trade easier, attracting new investors. This gives rise to a market externality. Interestingly, as investors get attracted to this market, the market becomes crowded and congestion reduces the returns to investing. This paper aims to complement the literature on self-fulfilling liquidity by incorporating a second effect: the congestion effect.

In this market traders can choose to invest in one asset which can be traded only when a pair of investors meet and bargain over the terms of trade. Finding a trading partner takes time and introduces opportunity and other costs. Investors' decision to participate in this market and their ability to trade thus affect the illiquidity discount and ultimately, the equilibrium price. We first solve the equilibrium in closed-form when search frictions are small and then introduce a numerical example to complement the analysis of the implications of changes in frictions on liquidity and welfare.

We find that diminishing frictions in a market with many sellers and too few buyers, as it is the case in markets experiencing a fire sale, induces an adverse effect on both liquidity and welfare. Improving search efficiency leads to a more unbalanced distribution of investors (even less potential buyers per seller trying to exit) magnifying the effect of congestion to the detriment of the overall level of market liquidity and social welfare. From this perspective, this paper presents an example of the Theory of the Second Best, where eliminating one but not all market imperfections does not necessary increase efficiency as it may amplify the effect of the remaining distortions, and provides motivation for measures such as trading halts and circuit breakers.

Appendix

A Proofs of Propositions 1 - 2

Proof of Proposition 1

Proof. Rearranging equation (3), we get

$$h(\eta_0) \equiv \gamma\eta_0^2 - \left(g + \gamma S + \frac{\gamma^2}{\lambda}\right)\eta_0 + Sg = 0$$

where $\eta_0 \in \mathbb{R}_+$. This quadratic function takes positive values as $\eta_0 \rightarrow \infty$, is non-negative at $\eta_0 = 0$ and negative at $\eta_0 = S$. Then, by continuity, the polynomial equation has a root in the interval $[0, S)$ and another one in the interval (S, ∞) . The two solutions $\eta_0^{(1)}$ and $\eta_0^{(2)}$ are given by:

$$\begin{aligned}\eta_0^{(1)} &= \frac{1}{2\gamma} \left[\left(g + \gamma S + \frac{\gamma^2}{\lambda}\right) - \sqrt{\left(g + \gamma S + \frac{\gamma^2}{\lambda}\right)^2 - 4\gamma g S} \right] \\ \eta_0^{(2)} &= \frac{1}{2\gamma} \left[\left(g + \gamma S + \frac{\gamma^2}{\lambda}\right) + \sqrt{\left(g + \gamma S + \frac{\gamma^2}{\lambda}\right)^2 - 4\gamma g S} \right]\end{aligned}$$

where $0 \leq \eta_0^{(1)} < S < \eta_0^{(2)} < \infty$. $\eta_0^{(2)}$ is thus not a valid solution since the total supply of the asset is held either by the non-searchers or by the sellers-to-be and as a result the measure of non-searchers cannot exceed the supply of the asset. Then, there is unique solution to equation (3) given by:

$$\eta_0 = \frac{1}{2\gamma}A \quad (\text{A.1})$$

where $A = (g + \gamma S + \frac{\gamma^2}{\lambda}) - \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}$. Plugging equation (A.1) into equations (1) and (2), we find

$$\begin{aligned} \eta_s &= S - \frac{1}{2\gamma}A \\ \eta_b &= \frac{\gamma}{\lambda} \frac{A}{2\gamma S - A} \end{aligned}$$

which proves Proposition 1. □

Proof of Proposition 2

Proof. Using equation (10), we can rewrite equations (7) and (9) as:

$$rv_b = -\gamma v_b + \lambda \eta_s \frac{z}{1+z} (v_0 - v_b - v_s) \quad (\text{A.2})$$

$$rv_s = d - x + \lambda \eta_b \frac{1}{1+z} (v_0 - v_b - v_s) \quad (\text{A.3})$$

Subtracting equation (A.2) from equation (8) yields:

$$\begin{aligned} r(v_0 - v_b) &= d + \gamma(v_s - v_0) - \left[-\gamma v_b + \lambda \eta_s \frac{z}{1+z} (v_0 - v_b - v_s) \right] = \\ &= d + \gamma(v_s - v_0 + v_b) - \lambda \eta_s \frac{z}{1+z} (v_0 - v_b - v_s) \Rightarrow \\ \Rightarrow v_0 - v_b &= \frac{d + (\gamma + \lambda \eta_s \frac{z}{1+z})v_s}{r + \gamma + \lambda \eta_s \frac{z}{1+z}} \end{aligned} \quad (\text{A.4})$$

We can solve for v_s by plugging equation (A.4) into equation (A.3):

$$\begin{aligned}
rv_s &= d - x + \lambda\eta_b \frac{1}{1+z} \left[\frac{d + (\gamma + \lambda\eta_s \frac{z}{1+z})v_s}{r + \gamma + \lambda\eta_s \frac{z}{1+z}} - v_s \right] = \\
&= d - x + \lambda\eta_b \frac{d - rv_s}{(r + \gamma + \lambda\eta_s)z + r + \gamma} \Rightarrow \\
&\Rightarrow \left[1 + \frac{\lambda\eta_b}{(r + \gamma + \lambda\eta_s)z + r + \gamma} \right] rv_s = \left[1 + \frac{\lambda\eta_b}{(r + \gamma + \lambda\eta_s)z + r + \gamma} \right] d - x \Rightarrow \\
\Rightarrow v_s &= \frac{d}{r} - k \frac{x}{r} - k \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \tag{A.5}
\end{aligned}$$

where

$$k \equiv \frac{(r + \gamma + \lambda\eta_s)z + \gamma}{(r + \gamma + \lambda\eta_s)z + (r + \gamma + \lambda\eta_b)}$$

Given v_s , we can determined v_0 , v_b and p uniquely from equations (8), (A.2) and (10) respectively. Let us compute them. We can solve for v_0 by plugging equation (A.5) into equation (8):

$$\begin{aligned}
rv_0 &= d + \gamma \left[\frac{d}{r} - k \frac{x}{r} - k \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \right] - \gamma v_0 \Rightarrow \\
\Rightarrow v_0 &= \frac{d}{r} - k \frac{x}{r} \frac{\gamma}{r + \gamma} - k \frac{\gamma}{r + \gamma} \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \tag{A.6}
\end{aligned}$$

We now compute v_b by substituting equations (A.5) and (A.6) into equation (A.2):

$$\begin{aligned}
rv_b &= -\gamma v_b + \lambda\eta_s \frac{z}{1+z} \left[\frac{d}{r} - k \frac{x}{r} \frac{\gamma}{r + \gamma} - k \frac{\gamma}{r + \gamma} \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} - \right. \\
&\quad \left. - v_b - \frac{d}{r} + k \frac{x}{r} + k \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \right] \Rightarrow \\
&\Rightarrow \left[r + \gamma + \lambda\eta_s \frac{z}{1+z} \right] v_b = \lambda\eta_s \frac{z}{1+z} \left[k \frac{x}{r + \gamma} + k \frac{x}{(r + \gamma + \lambda\eta_s)z + \gamma} \frac{r}{r + \gamma} \right] \Rightarrow \\
\Rightarrow v_b &= k \frac{x}{r + \gamma} \frac{\lambda\eta_s z}{(r + \gamma + \lambda\eta_s)z + \gamma} \tag{A.7}
\end{aligned}$$

We now solve for the price. Plugging equations (A.5) - (A.7) into equation (10) we get:

$$\begin{aligned}
p &= \frac{1}{1+z} \left[\left(\frac{d}{r} - k \frac{x}{r} - k \frac{x}{(r+\gamma+\lambda\eta_s)z+\gamma} \right) z + \right. \\
&\quad \left. + \frac{d}{r} - k \frac{x}{r} \frac{\gamma}{r+\gamma} - k \frac{\gamma}{r+\gamma} \frac{x}{(r+\gamma+\lambda\eta_s)z+\gamma} - k \frac{x}{r+\gamma} \frac{\lambda\eta_s z}{(r+\gamma+\lambda\eta_s)z+\gamma} \right] = \\
&= \frac{d}{r} - \frac{1}{1+z} \left[k \frac{x}{r} \left(\frac{\gamma}{r+\gamma} + z \right) + k \frac{x}{r+\gamma} \right] \Rightarrow \\
\Rightarrow p &= \frac{d}{r} - k \frac{x}{r}
\end{aligned} \tag{A.8}$$

This concludes the proof of Proposition 2. \square

B Additional Proofs

B.1 Proof of $\frac{\partial \eta_b}{\partial g}, \frac{\partial \eta_0}{\partial g} > 0$ and $\frac{\partial \eta_s}{\partial g} < 0$

Proof. Let us compute the partial derivatives of the measures given by the system of equations (4) - (6) with respect to g :

$$\frac{\partial \eta_0}{\partial g} = \frac{\partial \eta_0}{\partial A} \frac{\partial A}{\partial g} = \frac{1}{2\gamma} \frac{\partial A}{\partial g} \tag{A.9}$$

$$\frac{\partial \eta_s}{\partial g} = \frac{\partial \eta_s}{\partial A} \frac{\partial A}{\partial g} = -\frac{1}{2\gamma} \frac{\partial A}{\partial g} \tag{A.10}$$

$$\frac{\partial \eta_b}{\partial g} = \frac{\partial \eta_b}{\partial A} \frac{\partial A}{\partial g} = \frac{2\gamma^2}{\lambda} \frac{S}{(2\gamma S - A)^2} \frac{\partial A}{\partial g} \tag{A.11}$$

where

$$\frac{\partial A}{\partial g} = 1 - \frac{(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} \tag{A.12}$$

To determine the sign of $\frac{\partial A}{\partial g}$, we check if the second term on the right-hand-side of equation

(A.12) is greater than 1:

$$\frac{(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} > 1 \quad ;$$

$$(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S > \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} \quad ; \quad (\text{A.13})$$

where the right-hand-side of equation (A.13) is strictly positive since

$$\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} = \sqrt{(g - \gamma S)^2 + 2(g + \gamma S)\frac{\gamma^2}{\lambda} + \frac{\gamma^4}{\lambda^2}} > 0 \quad (\text{A.14})$$

We analyze two cases. If $(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S \leq 0$, then equation (A.13) is not satisfied. On the contrary, if $(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S > 0$,

$$\begin{aligned} \left[(g + \gamma S + \frac{\gamma^2}{\lambda}) - 2\gamma S \right]^2 &> \left[\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} \right]^2; \\ (g - \gamma S + \frac{\gamma^2}{\lambda})^2 &> (g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S; \end{aligned}$$

Simplifying we arrive to:

$$4\frac{\gamma^3}{\lambda} S < 0$$

a contradiction, since γ, λ and $S > 0$. Therefore, the second term in equation (A.12) is strictly lower than 1 and as a result:

$$\frac{\partial A}{\partial g} > 0 \quad (\text{A.15})$$

Thus, substituting the previous equation into equations (A.9) - (A.11) yields:

$$\begin{aligned} \frac{\partial \eta_0}{\partial g} &> 0 \\ \frac{\partial \eta_s}{\partial g} &< 0 \\ \frac{\partial \eta_b}{\partial g} &> 0 \end{aligned}$$

since γ, λ and $S > 0$. □

B.2 Proof of $\frac{\partial \eta_b}{\partial \lambda}, \frac{\partial \eta_s}{\partial \lambda} < 0$ and $\frac{\partial \eta_0}{\partial \lambda} > 0$

Proof. Using equations (4) - (6) we can compute the partial derivatives of the measures of every type of investor with respect to the efficiency of the search process λ :

$$\frac{\partial \eta_0}{\partial \lambda} = \frac{\partial \eta_0}{\partial A} \frac{\partial A}{\partial \lambda} = \frac{1}{2\gamma} \frac{\partial A}{\partial \lambda} \quad (\text{A.16})$$

$$\frac{\partial \eta_s}{\partial \lambda} = \frac{\partial \eta_s}{\partial A} \frac{\partial A}{\partial \lambda} = -\frac{1}{2\gamma} \frac{\partial A}{\partial \lambda} \quad (\text{A.17})$$

$$\frac{\partial \eta_b}{\partial \lambda} = \frac{\gamma}{\lambda} \frac{1}{2\gamma S - A} \left[\frac{2\gamma S}{2\gamma S - A} \frac{\partial A}{\partial \lambda} - \frac{1}{\lambda} A \right] \quad (\text{A.18})$$

where

$$\frac{\partial A}{\partial \lambda} = -\frac{\gamma^2}{\lambda^2} \left(1 - \frac{g + \gamma S + \frac{\gamma^2}{\lambda}}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} \right) \quad (\text{A.19})$$

We verify whether the second term in the expression in parenthesis is greater than 1 to determine the sign of $\frac{\partial A}{\partial \lambda}$,

$$\begin{aligned} \frac{(g + \gamma S + \frac{\gamma^2}{\lambda})}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} &> 1; \\ \left(g + \gamma S + \frac{\gamma^2}{\lambda}\right)^2 &> \left[\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}\right]^2; \\ \left(g + \gamma S + \frac{\gamma^2}{\lambda}\right)^2 &> (g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S; \end{aligned} \quad (\text{A.20})$$

where we can square both sides of the expression because, using equation (A.14) and g, γ, S and $\lambda > 0$, the numerator and denominator are strictly positive. Rearranging equation (A.20) we get:

$$4\gamma g S > 0$$

which is true since γ, g and $S > 0$. As a result, the second term in the expression in parenthesis in equation (A.19) is strictly greater than 1 and

$$\frac{\partial A}{\partial \lambda} > 0 \quad (\text{A.21})$$

Thus, plugging the previous equation into equations (A.16) - (A.17) we find:

$$\begin{aligned}\frac{\partial \eta_0}{\partial \lambda} &> 0 \\ \frac{\partial \eta_s}{\partial \lambda} &< 0\end{aligned}$$

The proof that $\frac{\partial \eta_b}{\partial \lambda} < 0$ is not so straightforward. Let us first rearrange equation (A.19) as follows

$$\frac{\partial A}{\partial \lambda} = \frac{\gamma^2}{\lambda^2} \frac{A}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} \quad (\text{A.22})$$

Now, substituting equation (A.22) in equation (A.18) we get:

$$\frac{\partial \eta_b}{\partial \lambda} = \frac{\gamma}{\lambda^2} \frac{A}{2\gamma S - A} \left[\frac{2\gamma S}{2\gamma S - A} \frac{\gamma^2}{\lambda} \frac{1}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} - 1 \right] \quad (\text{A.23})$$

where we need to derive the sign of the expression in brackets to determine the sign of $\frac{\partial \eta_b}{\partial \lambda}$. Let us then verify if the first term of the expression in brackets in equation (A.23) is strictly lower than 1:

$$\begin{aligned}\frac{2\gamma S}{2\gamma S - A} \frac{\gamma^2}{\lambda} \frac{1}{\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}} &< 1; \\ (2\gamma S - A) \lambda \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} - 2\gamma^3 S &> 0; \\ \lambda \left[(2\gamma S - A) \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} - \frac{2\gamma^3 S}{\lambda} \right] &> 0;\end{aligned}$$

Given that $\lambda > 0$ and $A = (g + \gamma S + \frac{\gamma^2}{\lambda}) - \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}$, then

$$\begin{aligned}\left[2\gamma S - (g + \gamma S + \frac{\gamma^2}{\lambda}) \right] \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} + \left[\sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S} \right]^2 - \frac{2\gamma^3 S}{\lambda} &> 0; \\ \left(g - \gamma S + \frac{\gamma^2}{\lambda} \right)^2 - \left(g - \gamma S + \frac{\gamma^2}{\lambda} \right) \sqrt{(g - \gamma S + \frac{\gamma^2}{\lambda})^2 + \frac{4\gamma^3 S}{\lambda}} + \frac{2\gamma^3 S}{\lambda} &> 0;\end{aligned}$$

To simplify the exposition of the proof, let us define $D \equiv g - \gamma S + \frac{\gamma^2}{\lambda}$. Therefore,

$$D^2 - D \sqrt{D^2 + \frac{4\gamma^3 S}{\lambda}} + \frac{2\gamma^3 S}{\lambda} > 0 \quad (\text{A.24})$$

We consider two possible scenarios. If $D \leq 0$, then equation (A.24) is satisfied since λ, γ and $S > 0$. On the contrary, if $D > 0$, then we need to prove that

$$D^2 + \frac{2\gamma^3 S}{\lambda} > D \sqrt{D^2 + \frac{4\gamma^3 S}{\lambda}}$$

Squaring both sides and rearranging, we find

$$D^4 + \frac{4\gamma^3 S}{\lambda} D^2 + \frac{4\gamma^6 S^2}{\lambda^2} > D^2 \left(D^2 + \frac{4\gamma^3 S}{\lambda} \right)$$

Simplifying,

$$\frac{4\gamma^6 S^2}{\lambda^2} > 0$$

and this is always satisfied. Then, we have shown that the first term in the expression in brackets in equation (A.23) is strictly lower than 1 and as a result

$$\frac{\partial \eta_b}{\partial \lambda} < 0$$

which completes the proof. □

B.3 Proof of $\frac{\partial p}{\partial d}, \frac{\partial p}{\partial \eta_b} > 0$ and $\frac{\partial p}{\partial z}, \frac{\partial p}{\partial \eta_s} < 0$

Proof. Using equation (14), the partial derivative of the price with respect to the dividend flow d is

$$\frac{\partial p}{\partial d} = \frac{1}{r} > 0 \quad \Rightarrow \quad \frac{\partial p}{\partial d} > 0 \quad \forall d$$

Let us now compute the partial derivative of the price with respect to the measure of buyers-to-be η_b :

$$\frac{\partial p}{\partial \eta_b} = \frac{\partial p}{\partial k} \frac{\partial k}{\partial \eta_b} = -\frac{x}{r} \frac{\partial k}{\partial \eta_b}$$

where:

$$\frac{\partial k}{\partial \eta_b} = -\lambda \frac{(r + \gamma + \lambda \eta_s)z + \gamma}{[(r + \gamma + \lambda \eta_s)z + (r + \gamma + \lambda \eta_b)]^2}$$

which is strictly lower than zero since $r, \gamma, \lambda, \eta_s, z > 0$. Therefore,

$$\frac{\partial p}{\partial \eta_b} > 0 \quad \forall \eta_b$$

Next, we obtain the partial derivative of the price with respect to the buyer's-to-be bargaining

power z :

$$\frac{\partial p}{\partial z} = \frac{\partial p}{\partial k} \frac{\partial k}{\partial z} = -\frac{x}{r} \frac{\partial k}{\partial z}$$

where:

$$\frac{\partial k}{\partial z} = \frac{r\lambda\eta_s\eta_b}{[(r + \gamma + \lambda\eta_s)z + (r + \gamma + \lambda\eta_b)]^2}$$

which is strictly greater than zero since $r, \lambda, \eta_s, \eta_b > 0$. Then,

$$\frac{\partial p}{\partial z} < 0 \quad \forall z$$

To complete the proof, we calculate the partial derivative of the asset price with respect to the measure of sellers-to-be:

$$\frac{\partial p}{\partial \eta_s} = \frac{\partial p}{\partial k} \frac{\partial k}{\partial \eta_s} = -\frac{x}{r} \frac{\partial k}{\partial \eta_s}$$

where:

$$\frac{\partial k}{\partial \eta_s} = \frac{\lambda z(r + \lambda\eta_b)}{[(r + \gamma + \lambda\eta_s)z + (r + \gamma + \lambda\eta_b)]^2}$$

which is strictly greater than zero since $r, \lambda, \eta_b, z > 0$. Thus,

$$\frac{\partial p}{\partial \eta_s} < 0 \quad \forall \eta_s$$

□

B.4 Proof of $\frac{\partial p}{\partial g} > 0$

Proof. Using equation (14), the partial derivative of the asset price with respect to the flow of investors g entering the market is

$$\frac{\partial p}{\partial g} = -\frac{x}{r} \frac{\partial k}{\partial g}$$

where $k = \frac{(r+\gamma+\lambda\eta_s)z+\gamma}{(r+\gamma+\lambda\eta_s)z+(r+\gamma+\lambda\eta_b)}$. Let us derive the partial derivative of k with respect to the flow of investors g :

$$\frac{\partial k}{\partial g} = \frac{1}{[(r + \gamma + \lambda\eta_s)z + (r + \gamma + \lambda\eta_b)]^2} \left\{ (r + \lambda\eta_b)\lambda z \frac{\partial \eta_s}{\partial g} - [(r + \gamma + \lambda\eta_s)z + \gamma] \lambda \frac{\partial \eta_b}{\partial g} \right\}$$

which is strictly lower than zero since $r, \gamma, \lambda, \eta_s, \eta_b, z > 0$ and $\frac{\partial \eta_s}{\partial g} < 0$ and $\frac{\partial \eta_b}{\partial g} > 0$ as shown Appendix B.1. Then,

$$\frac{\partial p}{\partial g} = -\frac{x}{r} \frac{\partial k}{\partial g} > 0$$

which proves the price increases in the flow of investors entering the market. \square

C Proof of Theorem 1

Proof. In our framework, the marginal investor decides whether to enter or not after comparing the expected utility v_{alt} of investing in an alternative market to the expected utility v_b of a buyer-to-be in the search market. The expected utility of the marginal investor $v_{alt} = \kappa'$ is a non-negative and strictly increasing function of his outside investment opportunity κ' . Also, $v_b(\kappa' = 0) > v_{alt}(\kappa' = 0) = 0$. Hence, if v_b were decreasing in the outside investment opportunity of the marginal investor, κ' , then there would be a unique threshold κ^* satisfying the indifference condition $v_b(g(\kappa^*)) = v_{alt}(\kappa^*)$. Let us show this is the case.

The expected utility v_b of a buyer-to-be is a function of the flow of investors g entering the market. Let us compute the partial derivative of v_b , defined in equation (11), with respect to g :

$$\frac{\partial v_b}{\partial g} = \frac{\lambda z x}{r + \gamma [(1+z)(r + \gamma) + \lambda(z\eta_s + \eta_b)]^2} \left\{ [(1+z)(r + \gamma) + \lambda\eta_b] \frac{\partial \eta_s}{\partial g} - \lambda\eta_s \frac{\partial \eta_b}{\partial g} \right\}$$

which is strictly negative since $r, \gamma, x, z, \lambda, \eta_b, \eta_s > 0$ and $\frac{\partial \eta_s}{\partial g} < 0$ and $\frac{\partial \eta_b}{\partial g} > 0$ as shown Appendix B.1. Hence, the expected utility v_b of a buyer-to-be strictly decreases in the flow of investors g entering the market. However g , as given by $g(\kappa') = \int_{\underline{\kappa}}^{\bar{\kappa}} \nu(\kappa) f(\kappa) d\kappa = \int_{\underline{\kappa}}^{\kappa'} f(\kappa) d\kappa$, is increasing in κ' . As a result,

$$\frac{\partial v_b}{\partial \kappa'} = \frac{\partial v_b}{\partial g} \frac{\partial g}{\partial \kappa'} \leq 0$$

where $\frac{\partial v_b}{\partial g} < 0$ and $\frac{\partial g}{\partial \kappa'} \geq 0$.

Then, by continuity, there exists a unique value of κ' satisfying the indifference condition: $v_b(g(\kappa^*)) = v_{alt}(\kappa^*)$. A unique threshold κ^* thus defines a unique flow of investors $g^* = g(\kappa^*)$ entering the market. But given a flow of investors entering the market, there exists unique equilibrium measures $(\eta_b^*, \eta_0^*, \eta_s^*)$ of each type of investor, expected utilities (v_b^*, v_0^*, v_s^*) and price of the asset, p^* , as proved in Propositions 1 and 2. Consequently, market equilibrium, as presented in Definition 1, is unique. This proves Theorem 1. \square

D Proof of Proposition 4 and Corollary 1

Proof of Proposition 4

Proof. The measures of sellers-to-be and buyers-to-be are given by equations (5) and (6) respectively, where $A = (g + \gamma S + \frac{\gamma^2}{\lambda}) - \sqrt{(g + \gamma S + \frac{\gamma^2}{\lambda})^2 - 4\gamma g S}$. Substituting $g = \gamma S$, we can write A as:

$$A = 2g + 2\gamma\sqrt{g}\frac{1}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right) \quad (\text{A.25})$$

where $\sqrt{1 + \frac{1}{4g}\frac{\gamma^2}{\lambda}} \approx 1 + \frac{1}{4g}\frac{\gamma^2}{\lambda}$ and $o\left(\frac{1}{\sqrt{\lambda}}\right)$ denotes terms of order smaller than $\frac{1}{\sqrt{\lambda}}$.

Plugging equation (A.25) into equations (5) and (6) we find:

$$\begin{aligned} \eta_s &= \frac{\sqrt{g}}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right) \\ \eta_b &= \frac{\sqrt{g}}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right) \end{aligned}$$

Therefore,

$$\eta_s = \eta_b = \frac{\sqrt{g}}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right) \quad (\text{A.26})$$

Let us now derive the expression for the asset price when $g = \gamma S$. The price is given by $p = \frac{d}{r} - k\frac{x}{r}$, where

$$k = \frac{(r + \gamma)z + \gamma + z\lambda\eta_s}{(r + \gamma)(z + 1) + z\lambda\eta_s + \lambda\eta_b} \quad (\text{A.27})$$

Multiplying both sides of equation (A.26) by λ and substituting into equation (A.27) we get:

$$k = \frac{z}{z + 1} + \frac{\gamma}{z + 1} \frac{1}{\sqrt{g}} \frac{1}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right) \quad (\text{A.28})$$

Then,

$$p = \frac{d}{r} - \frac{z}{z + 1} \frac{x}{r} - \frac{\gamma}{z + 1} \frac{x}{r} \frac{1}{\sqrt{g}} \frac{1}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right)$$

which completes the proof of Proposition 4. \square

Proof of Corollary 1

Proof. Illiquidity, measured by price discount, is defined as $k\frac{x}{r}$. In the case of $g = \gamma S$, k is given by equation (A.28). Then,

$$\text{Illiquidity} \equiv k\frac{x}{r} = \left[\frac{z}{z+1} + \frac{\gamma}{z+1} \frac{1}{\sqrt{g}} \frac{1}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right) \right] \frac{x}{r} \quad (\text{A.29})$$

Taking limits in (A.29) as λ goes to infinity

$$\lim_{\lambda \rightarrow \infty} \text{Illiquidity} \equiv \lim_{\lambda \rightarrow \infty} k\frac{x}{r} = \frac{z}{z+1} \frac{x}{r}$$

which is increasing in z . As a result, liquidity decreases in z . \square

E Market Equilibrium: General Case

Market equilibrium is the solution to the system of equations (1)-(3), (7)-(10) and (15). We thus need to solve for the fixed point of this system, which is reduced to solving the indifference condition that defines investors' entry rule. Investors, in our framework, compare the expected utility v_{alt} of investing in an alternative market to the expected utility v_b derived from being a buyer-to-be and they decide to move in whenever $v_b > \kappa' \equiv v_{alt}$. For simplicity, we derive the equilibrium measure of sellers-to-be (η_s^*) as a function of the efficiency of the search process λ and the other nine parameters of the model ($\gamma, r, S, x, z, a, b, \underline{\kappa}$ and $\bar{\kappa}$). To present the indifference condition ($v_b = \kappa'$) as a function of the measure of sellers-to-be (η_s), let us first redefine the measure of buyers-to-be η_b as a function of η_s . Using equations (5) and (6) we find:

$$\eta_b = \frac{\gamma}{\lambda} \frac{A}{2\gamma S - A} = \frac{\gamma}{\lambda} \frac{2\gamma(S - \eta_s)}{2\gamma S - 2\gamma(S - \eta_s)} \quad \Rightarrow \quad \eta_b = \frac{\gamma}{\lambda} \frac{S - \eta_s}{\eta_s} \quad (\text{A.30})$$

We can now express the expected utility v_b of buyers-to-be as a function of η_s by substituting equation (A.30) into equation (11):

$$v_b = \frac{x}{r + \gamma} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \quad (\text{A.31})$$

Next we write κ' as a function of η_s . In this model, the flow of investors g who move into the market is determined by the proportion of the total flow of investors f whose expected utility v_b of being a buyer-to-be exceeds their best outside option κ' . We assume the flow of investors f follows

a beta distribution with support $[\underline{\kappa}, \bar{\kappa}]$ and shape parameters²⁹ a and b . For notational convenience we omit reference to the shape parameters. Then,

$$g(\kappa') = \int_{\underline{\kappa}}^{\kappa'} f_{beta}(\kappa) d\kappa = F_{beta}(\kappa') \quad \Rightarrow \quad \kappa' = F_{beta}^{-1}(g) \quad (\text{A.32})$$

where f_{beta} and F_{beta} denote respectively the probability density function (*pdf*) and the cumulative distribution function (*cdf*) of a beta distribution. F_{beta}^{-1} is the inverse cumulative distribution function. Using equation (5) and the definition of A in Page 13 we can express the flow of investors g as a function of the measure of sellers-to-be η_s :

$$g = \gamma \left(1 + \frac{\gamma}{\lambda \eta_s} \right) (S - \eta_s) \quad (\text{A.33})$$

Substituting equation (A.33) in equation (A.32) yields:

$$\kappa' = F_{beta}^{-1} \left(\gamma \left(1 + \frac{\gamma}{\lambda \eta_s} \right) (S - \eta_s) \right) \quad (\text{A.34})$$

The indifference condition results from equating the expected utility v_b of buyers-to-be (equation (A.31)) to the marginal investor outside option κ' (equation (A.34)):

$$\frac{x}{r + \gamma} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} = F_{beta}^{-1} \left(\gamma \left(1 + \frac{\gamma}{\lambda \eta_s} \right) (S - \eta_s) \right)$$

Rearranging, we get

$$\gamma \left(1 + \frac{\gamma}{\lambda \eta_s} \right) (S - \eta_s) = F_{beta} \left(\frac{x}{r + \gamma} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \right)$$

²⁹The probability density function of the beta distribution defined over the interval $[0, 1]$ with shape parameters a and b is:

$$f_{beta}(y; a, b) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} y^{a-1} (1 - y)^{b-1}$$

where $a, b > 0$ and $\Gamma(\cdot)$ is the gamma function. For integer values of a and b , the cumulative distribution function of the beta distribution is given by:

$$F_{beta}(y; a, b) = \sum_{j=a}^{a+b-1} \binom{a+b-1}{j} y^j (1 - y)^{a+b-1-j}$$

where $\binom{a+b-1}{j} = \frac{(a+b-1)!}{j!(a+b-1-j)!}$.

Then,

$$\gamma \left(1 + \frac{\gamma}{\lambda \eta_s}\right) (S - \eta_s) = \sum_{j=a}^{a+b-1} \binom{a+b-1}{j} \left(\frac{x}{r + \gamma} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \right)^j \left(1 - \frac{x}{r + \gamma} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} \right)^{a+b-1-j} \quad (\text{A.35})$$

Equation (A.35) is a polynomial of degree $2(a + b)$ in the measure of sellers-to-be³⁰. To solve for η_s^* we use the bisection method³¹. Market equilibrium consists of $((\eta_s^*, \eta_b^*, \eta_0^*), (v_b^*, v_0^*, v_s^*, p^*)$ and ν^*). Once we compute η_s^* , we can determine η_0^* and η_b^* by substituting η_s^* in equations (1) and (A.30). Then, we can derive v_b^*, v_0^*, v_s^* and p^* by plugging equation (A.30) and η_s^* into equations (11) - (14) respectively. Finally, there is a one-to-one relationship between g^* and η_s^* . Given η_s^* , we can determine the equilibrium flow of investors g^* entering market (equation (A.33)). Once we derive g^* , equation (15) specifies a unique κ^* and thus a unique equilibrium fraction ν^* of investors who prefer to invest in this market.

³⁰In the simple case of shape parameters of the beta distribution both equal to 1 ($a = 1 = b$), which corresponds to a uniform distribution with support $[\underline{\kappa}, \bar{\kappa}]$, the indifference condition ($v_b = \kappa'$) is:

$$\frac{x}{r + \gamma} \frac{\lambda z \eta_s^2}{\lambda z \eta_s^2 + [(r + \gamma)(1 + z) - \gamma] \eta_s + \gamma S} = \underline{\kappa} + (\bar{\kappa} - \underline{\kappa}) \gamma \left(1 + \frac{\gamma}{\lambda \eta_s}\right) (S - \eta_s)$$

Reorganizing terms yields the following polynomial of degree four in the measure of sellers-to-be η_s :

$$\begin{aligned} \lambda^2 z (\bar{\kappa} - \underline{\kappa}) \gamma \eta_s^4 &+ \left[\lambda (\bar{\kappa} - \underline{\kappa}) \gamma C - \lambda z D + \frac{x}{r + \gamma} \lambda^2 z \right] \eta_s^3 + \left[\lambda (\bar{\kappa} - \underline{\kappa}) \gamma^2 S (1 - z) - CD \right] \eta_s^2 - \\ &- \left[\gamma S D + (\bar{\kappa} - \underline{\kappa}) \gamma^2 S C \right] \eta_s - (\bar{\kappa} - \underline{\kappa}) \gamma^3 S^2 = 0 \end{aligned}$$

where $C = (r + \gamma)(1 + z) - \gamma$ and $D = \lambda \underline{\kappa} + \lambda (\bar{\kappa} - \underline{\kappa}) \gamma S - (\bar{\kappa} - \underline{\kappa}) \gamma^2$. There exists closed-form solution to this equation. In particular, there are at most four solutions but only one, η_s^* , (as proved in Subsection 5.2) lies in the interval $[0, S]$, the set of possible values of the measure of sellers-to-be. Unfortunately, the solution is intractable. We use the bisection method over the interval $[0, S]$ to determine the zero of this equation.

³¹The bisection algorithm is a numerical method for finding the root of a function. It recursively divides an interval in half and selects the subinterval containing the root, until the interval is sufficiently small. Burden and Faires (1993) presents a clear description of this algorithm as well as other numerical methods for solving root-finding problems.

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