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Vesting and Control in Venture Capital Contracts

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Abstract

Vesting of equity payments to an entrepreneur, which is a form of time-contingent compensation, is very common in venture capital contracts. Empirical research suggests that vesting is used to help overcome asymmetric information and agency problems. We show in a theoretical model that vesting equity to an entrepreneur over a long period of time acts as a screening device against a bad entrepreneur type. But incomplete contracts due to hold-up by the venture capitalist imply that equity compensation, in the form of either short-term or long-term vesting, cannot provide standard contractible equity incentives for the entrepreneur to take an unobservable action involving effort. We introduce a new model of effort based on a verifiable choice of an effort-intensive project, for which the short-term vesting of equity can provide incentives, but which results in a trade-off between incentives and screening. Contingent control rights are a substitute for short-term vesting and provide the largest incentives for effort by fully protecting the entrepreneur from hold-up. We also show that a new link between equity cash flow claims and control rights is that residual equity control rights over the firm are necessary to protect residual equity claims from hold-up.

Key words: venture capital, vesting, control rights, contingent compensation, hold-up problem
1. Introduction

This paper is motivated by venture capital contracting to examine time-contingent compensation in a principal-agent model. Contracted payments of equity shares to an entrepreneur are often vested over time, or paid out only after the entrepreneur has remained with a firm for a specified time period. If the entrepreneur quits or is fired from the firm prematurely, unvested shares are not paid to the entrepreneur; however, the entrepreneur keeps any shares already vested. Kaplan and Stromberg (2004) show that vesting is extensively used in venture capital contracts in association with the risks of general uncertainty, asymmetric information, project complexity, and potential hold-up between the venture capitalist and the entrepreneur. In addition, Kaplan and Stromberg (2003) show that when contingent control rights are used, vesting is most common.

We study a model in which an entrepreneur has inalienable human capital required for a project to succeed. Project cash flows and the entrepreneur’s effort actions and quality type are fully revealed in the long run. Paying the entrepreneur contingent on the project’s final outcome would provide the greatest effort incentives and screening against asymmetric information of types. Correspondingly, equity compensation vested over the long run would pay the entrepreneur contingent on the project outcome, conditional on the entrepreneur not being fired for being a bad type.

However, we show that a hold-up problem between the entrepreneur and venture capitalist implies that contracts are incomplete and subject to repudiation. Equity paid to the entrepreneur, whether vested over a short or long period of time, has limited incentives to induce effort actions. Short-term vesting, which is equity compensation paid contingent on the entrepreneur not leaving the firm over a short term, has no standard equity incentives for future effort due to ex-post bargaining. Long-term vesting, which is equity compensation paid contingent on the entrepreneur not leaving the firm over a long term, has fixed equity incentives for effort but is not ex-ante contractible due to hold-up.

In order to examine the ability to induce effort when there is hold-up, we introduce an innovative model of effort in which a contractible project choice determines the optimum effort that is consistent with repudiation-proof contracts. Short-term vesting can give the entrepreneur incentives to choose an effort-intensive project under which effort is taken. However, short-term vesting reduces the screening against bad entrepreneur types, giving a trade-off between effort incentives and a separating equilibrium with the screening out of bad types.

Contingent control rights protect the entrepreneur from hold-up and are complementary to long-term vesting of equity. We show that a new explanation for the link between equity cash flow claims and control rights is that residual equity control is necessary to protect residual equity claims from hold-up. Control rights provide the greatest incentives for effort, beyond
that which can be provided by short-term vesting. But control rights operate over “bundled”
actions, for which individual control rights cannot be distinguished. The result is that control
rights give the holder the ability to divert cash flows, which implies that control rights reduce
screening more than short-term vesting. Short-term vesting is a partial substitute for control
rights that gives lower effort incentives and greater screening capability.

We examine the trade-off between incentives and screening within the context of a model of
venture capital contracting, but this result is likely to hold in corporate contracting in general.
For example, consider a CEO who has inalienable human capital due to firm-specific knowledge.
To provide full effort incentives, the CEO’s entire future compensation has to be contracted
ex-ante with full credibility. Though typically some components of CEO compensation, such
as equity or options, may be granted for several years in advance, often the salary and bonus
are determined each year, implying that contracts are not complete. Numerous cases of CEOs
receiving ex-post readjusted option grants and terms suggest the potential for hold-up by CEOs.
Cases of CEOs losing previously contracted compensation ex-post, such as Jack Welch of GE,
Richard Grasso of NYSE, Christos Cotsakos of ETrade, and John Antioco of Blockbuster,
suggest the potential for hold-up by the firm. If either the CEO or the firm can hold up the
other, then ex-post bargaining limits the incentives of ex-ante contracted compensation. Total
compensation in the form of up-front as well as contracted future equity compensation has
limited ex-post sensitivity to the stock price and hence the CEO’s effort if there is hold-up.

In contrast, this paper suggests that large up-front compensation, such as backdated in-
the-money-options, or control rights, such as appointing the CEO as chairman of the board,
which may appear to be excessive if contracts were complete, may actually help alleviate hold-
up problems by giving the CEO incentives to choose an effort-intensive project choice ex-ante.
Regardless, there may be a trade-off in that these methods for providing greater effort lessen
the screening against bad CEO types.

We also derive results about the trade-off of ex-ante and interim asymmetric information.
Greater incentives through short-term vesting or contingent control to the entrepreneur reduce
screening and lead to pooling equilibria. The cost of pooling increases with the ex-ante prob-
ability that an entrepreneur is a bad type. This is due to both the higher chance the bad type
receives financing, and the chance the bad type is not detected by a noisy interim signal and
so receives short-term vesting or contingent control by which he can divert funds. The cost of
pooling also increases with the noise of the signal. This is because bad types are more likely
to look for financing knowing that they are less likely to be caught by the noisy signal later,
and bad types are indeed caught less often and so receive short-term vesting or contingent
control allowing them to divert funds. When the cost of pooling is large enough from either
more bad types or a noisier signal, contingent control is not given to the entrepreneur, but
the entrepreneur is given large short-term vesting. As the cost increases further, short-term vesting is reduced. Withholding control from the entrepreneur and reducing short-term vesting increasingly lowers effort incentives, until finally no effort occurs.

There is an opposite effect between the two causes for increased costs of pooling on whether bad types look for financing in equilibrium. As the probability of bad types increases, the reduction in short-term vesting and ability to divert funds lowers bad types’ desire to look for financing, resulting in a separating equilibrium. Conversely, as the signal quality decreases, bad types find it increasingly attractive to look for financing and bear the decreasing risk of being caught even though short-term vesting or ability to divert funds is decreased. The difference in the direction of the effects is because in the limit, when there are only bad types, effort incentives are worthless and so not worth paying for due to the loss from funding bad types. But even when the signal loses all quality and becomes uninformative, it may still be worthwhile to provide incentives for effort for the good types despite the losses of funding bad types.

Section 2 discusses the related literature. Section 3 presents the model. Section 4 shows repudiation results, limited optimal results, and focuses on suboptimal results that obtain for more typical entrepreneurial projects. This section analyzes short-term vesting and control in the trade-off of the cost of pooling and incentives for effort, and highlights the special features of control. Section 5 concludes with further interpretations of the model. The repudiation assumption and robustness to the timing of events are discussed in Appendix A, and proofs are in Appendix B.

2. Related Literature

In a model of capital structure and security design, Aghion and Bolton (1992) show control may need to be contingent to implement optimal actions in the firm. We reach this result and also show how short-term vesting of equity or long-term vesting that is protected by entrepreneur control also helps implement optimal actions when there is asymmetric information. In our model, no action is dependent on the state as in Grossman and Hart (1994) or Aghion and Bolton (1992), but we have unverifiability of actions that leads to incomplete contracts and hold-up. The venture capitalist does not observe the state, but she does observe actions. Hart (2001) questions why the agent necessarily receives control in a good state and the principal receives control in a bad state, which is also empirically demonstrated in Kaplan and Stromberg (2003), rather than based solely on personal benefits as in Aghion and Bolton (1992). He suggests what may be missing from the model is effort that needs to be rewarded and implicitly requires entrenchment. We find this result, and also show that ex-ante asymmetric information drives the result as well.
Models of debt with unverifiable cash flows and liquidation rights given to investors, including Bolton and Scharfstein (1990) and Hart and Moore (1994, 1998), are somewhat similar to our model in that ours has a potential interim unverifiable cash flow and entrepreneurial human capital with potential hold-up of the investor, which necessitates that the entrepreneur cannot receive full payments up-front. However, this is counter-balanced by the entrepreneur’s needs for up-front payments or control rights for effort incentives due to potential reverse hold-up by the investor. Dewatripont and Tirole (1994) and Berkovitch and Israel (1996) examine the joint aspect of cash flow and control rights in the context of multiple classes of security holders, but they do not explain the timing of cash flows. Diamond (1991) shows that better borrower types prefer short-term debt because their liquidity risk of losing non-pledgeable rents is smaller.

The structure of our model is related to Neher (1999), who examines a hold-up problem involving an entrepreneur’s required human capital in the financing of a project over periods. The venture capitalist stages financing to protect herself from hold-up by the entrepreneur. In our model, the entrepreneur is compensated over time to protect the venture capitalist from hold-up, but the entrepreneur may also need protection from the venture capitalist which requires short-term vesting or control.

Shleifer and Vishny (1989) recognize the potential benefit to purposely entrenching the manager with a contract to achieve efficient investment. They do not examine the potential agency issues of diversion and asymmetric information, nor do they consider the possibilities of short-term vesting as a substitute for entrenchment. Gorton and Grundy (1996) show that when long-term vesting of manager equity pay is needed so the manager does not quit and free ride his equity off the replacement manager’s effort, there is a benefit to entrapping the manager (through control of the firm). This is so he cannot be fired but rather will stay to vest his equity even when he is less efficient as a manager ex-post, in order to provide him with effort incentives ex-ante. We show similar entrepreneur control and entrenchment is beneficial even if his human capital is vital so he cannot quit and free ride, but we also show when it is preferable to give the entrepreneur short-term vesting instead of control due to the larger agency costs of control from diversion and asymmetric information.

In Hellman (1998), the entrepreneur relinquishes control (including firing rights) of the firm in the initial contract to the venture capitalist and accepts a later payment because it protects the venture capitalist from hold-up if she later needs to search for superior management, which increases firm value ex-ante. We show the entrepreneur may relinquish control in trade for short-term vesting, but for long-term vesting the entrepreneur requires control, otherwise long-term vesting is repudiated. Dessein (2001) is similar to our model in examining ex-ante asymmetric information of the entrepreneur’s quality. He predicts that a good entrepreneur
signals by giving up formal control in exchange for more de facto control, but does not examine the timing of compensation. We predict the opposite: a good entrepreneur receives more formal control, because he gives up short-term for long-term vesting. Kirilenko (2001) also models a venture capital firm with asymmetric information, bargaining, and control rights, but also has no implications about the timing of the entrepreneur’s compensation.

Landier (2001) models venture capital versus bank financing as depending on career concerns and project risk to explain U.S. versus European venture capital markets. In the conclusion, we show our model can be applied to explain venture capital versus bank (or analogously U.S. versus European) financing as depending on the ex-ante probability of good entrepreneur types and the quality of interim information.

Kaplan and Stromberg (2003) show empirically our prediction that when the entrepreneur receives cash flows and control based on performance, vesting is most common. Our model predicts that control is needed to protect long-term vesting. Kaplan and Stromberg (2001) show empirically that cash flows, control and liquidation rights shift to the entrepreneur with performance in interrelated ways as complements, not substitutes, as we obtain, and they also show that venture capitalists are concerned about the entrepreneur’s ability as a manager. Kaplan and Stromberg (2004) show empirically that more complexity in contracts, which they interpret as an entrepreneur’s human capital requirements, leads to more time contingent vesting. Greater asymmetric information between the entrepreneur and the venture capitalist (primarily management quality risk) leads to more performance-based vesting and venture capital control. While our model has vesting as a function of time and performance, we do not analyze these aspects separately. Nevertheless, our model results are in alignment generally with their results on vesting and control rights. Dahiya and Yermack (2007) show empirically that the vesting schedules for corporate management equity and options compensation vary tremendously across firms. Kole (1997) shows empirically that corporate management compensation contracts are highly complex and variable across firms.

3. Model

The model is presented here, and detailed discussion of the model assumptions, robustness and timing is in Appendix A. A timeline illustrates the steps of the project in Figure 3.1.

At $t = 0$, an entrepreneur ("E" or “he”) has a project that requires external funding $K$. E chooses whether to pay irreversibly his only wealth $M$ as seed costs to initiate the project, which makes him visible to venture capitalists. $M$ is the monetary cost of developing a business model or building a prototype that E must perform before he can be known and approached by a venture capitalist. (An alternative interpretation is that $M$ is E’s pecuniary or non-pecuniary opportunity cost to pursue an entrepreneurial project). If E pays $M$, a competitive venture
capitalist ("VC" or "she") then offers E take-it-or-leave-it funding of $K$ and contract

$$\{W_1(c,s), W_T(c,s,\tau, V), \gamma(s)\},$$

the elements of which are defined below. Contracts can be written as functions of verifiable variables. $E$ has type $\theta \in \{\theta_G, \theta_B\}$, corresponding to "good" or "bad." With probability $p \in (0,1)$, $\theta = \theta_G$, and with probability $1-p$, $\theta = \theta_B$. $E$ knows his type at $t = 0$, but VC does not. If the contract is accepted by $E$, VC invests $K$. $E$ then makes a verifiable project choice $c \in \{i, r\}$ between mutually exclusive project variants, where $i$ is an effort-intensive project and $r$ is a regular (non-effort intensive) project.

At $t = 1$, a verifiable signal $s \in \{0,1\}$ correlated with $E$’s type is realized, where $\Pr(s = 1 | \theta = \theta_G) = 1$, which implies $\Pr(\theta = \theta_B | s = 0) = 1$. The quality of the signal is measured by $q \equiv \Pr(s = 0 | \theta = \theta_B) \in (0, \bar{q})$. We assume $L > M$ and define $\bar{q} \equiv \frac{L-M}{L} \in (0,1)$. Thus, $q$ gives the probability that the bad type is revealed by the signal. Control rights contingent on $s$ are given to either the VC or $E$: $\gamma(s) \in \{VC, E\}$. We refer to VC control if $\gamma(s) = VC$ for $s \in \{0,1\}$, and E control or contingent control if $\gamma(1) = E$. After the signal, $E$ chooses unobservable effort $e \in \{0,1\}$. The cost of effort is $\kappa(e)$, where $\kappa(0) = 0$.

At $t = 2$, the party with control rights, $\gamma(s)$, chooses whether to operate the assets prematurely to divert value $L \in (0, K)$ from the project: $\delta \in \{y, n\}$. This action is observable to the other party but not verifiable. Next, in an independent action, either $E$ or $\gamma(s)$ chooses whether to terminate the project: $\tau \in \{y, n\}$. Termination is verifiable, and if $\gamma(s) = E$, it is verifiable that $E$ terminated the project. If $\gamma(s) = VC$, it is observable but not verifiable who terminated the project. Termination by $E$ is interpreted as $E$ quitting, while termination by VC is interpreted as VC firing $E$. If the project is terminated, $\gamma(s)$ re-chooses $\delta \in \{y, n\}$ if $\delta = n$ was previously chosen. Regardless of $\gamma(s)$, VC cannot be replaced with a different investor during the project.
At \( t = 3 \), the payoff value of the project as shown in Figure 3.2 is realized as \( V(\theta, c, e, \delta, \tau) \in \{V_1, V_r, V_0, 0\} \), where

\[
\begin{align*}
V_1 &\equiv V(\theta_G, i, 1, n, n) \\
V_r &\equiv V(\theta_G, r, e, n, n) \\
V_0 &\equiv V(\theta_G, i, 0, n, n) \\
V(\theta, c, e, \delta, \tau) &= 0 \text{ if } \theta = \theta_B, \quad \delta = y \text{ or } \tau = y \\
V_1 - \kappa(1) &> V_r > V_0 > K + M > 0.
\end{align*}
\]

The interpretation of a verifiable positive payoff at project completion is an IPO or private sale of the firm. Because the funds of an IPO or sale do not accrue privately to an individual party but are based on a legal contract of sale, the payoff is considered verifiable.

VC and E are risk neutral. E has no liability beyond \( W_1 \) since he has no further verifiable wealth after paying \( M \), while VC has unlimited liability due to unlimited wealth. Without loss of generality, VC receives the payoff \( V \) and E receives payment \( W_T \) from VC at \( t = 3 \), where \( W_T(c, s, \tau, V) \equiv W_1(c, s) + W_3(c, s, \tau, V) \). \( W_1(c, s) \) is called “short-term vesting” and is determined at \( t = 1 \), and \( W_3(c, s, \tau, V) \) is called “long-term vesting” and is determined at \( t = 3 \). The contract specifies \( W_1 \) and \( W_T \), which implicitly gives \( W_3 \). The utility for E is

\[
U_E = \begin{cases} 
W_T - \kappa(e) + 1_{\gamma(s)=E}1_{\delta=y}L - M & \text{if E invests } M \text{ and accepts contract} \\
-M & \text{if E invests } M \text{ and doesn’t accept contract} \\
0 & \text{if E does not invest } M,
\end{cases}
\]

where \( 1_[] \) denotes the indicator function. The utility for VC is

\[
U_{VC} = \begin{cases} 
V - K - W_T + 1_{\gamma(s)=VC}1_{\delta=y}L & \text{if E accepts contract} \\
0 & \text{if E does not accept contract}.
\end{cases}
\]
We define $k \equiv \frac{\kappa(1)}{\nu} \in (\frac{1}{2}, 1)$, where $\nu \equiv V_1 - V_r$ defines the product of effort. Since $k < 1$, effort is always efficient in an effort-intensive project. We also assume $\kappa(1) < \frac{1}{2}(V_1 - V_0)$, for reasons discussed below.

At the beginning of $t = 2$, E, and VC if $\gamma(s) = VC$, can repudiate the contract with the threat of terminating the project, and $\gamma(s)$ can repudiate with the threat of diverting $L$. If there is a repudiation, the parties bargain over the surplus value from continuing rather than terminating the project, resulting in a Nash bargaining solution and efficient ex-post termination and diversion actions. The outcome of the Nash bargaining solution is that each party receives its threat point, equal to its utility from termination and/or diverting $L$, plus half the surplus amount from continuing the project. Since the continuation value is not yet determined but will be verifiable at $t = 3$, E and VC can write a new contract to split the surplus contingent on $V$.

Control rights may be separately delegated from verifiable cash flows, but control itself is a complex bundle of rights over bundled actions. The nature of a non-contractible action is that it either cannot be defined in a contract ex-ante or verified ex-post. If there are more than one of these actions within a firm, we argue that control over the actions may not be assignable to separate parties. In the present model, an interpretation is that control over the assets includes both the action of operating the assets for a secondary value and the action of operating the assets to impose non-pecuniary costs on E. An example of the latter under VC control is VC relocating the assets and E’s office to Antartica, such that E would prefer to quit. This also explains why it is not verifiable which party terminates the project under VC control. Assigning control rights to VC is not symmetric to assigning control rights to E. Under VC control, VC termination is not verifiable as due to VC, but VC diversion of assets is verifiable. Under E control, E diversion of funds is not verifiable (due to the bad type’s project paying zero), but termination is verifiable as due to E.

An innovation of the model is the choice between mutually exclusive project variants: a regular (non-effort intensive project) or an effort-intensive project. A good type E can choose $c = r$ to produce payoff $V_R$ without special effort. Alternatively, he can choose $c = i$ to produce a higher payoff $V_i$ with effort $e = 1$. Without effort, the effort-intensive project produces payoff $V_0$. An interpretation of the project choice is that for $c = r$, E chooses to spend funds $K$ on regular assets that give a payoff of $V_R$. For $c = i$, E verifiably spends $V_R - V_0$ of the funds $K$ on special effort-intensive assets. If E gives unverifiable effort $e = 1$, the project pays off $V_1$. If $e = 0$, the assets are wasted and reduce the project payoff by their cost, so the payoff is $V_0$. A further interpretation of this project choice is that more expensive high capacity assets can be purchased, which will give greater profit if E gives effort to produce at full capacity. If regular capacity assets are purchased, money is saved and can be spent to produce at lower
capacity without the effort of E. But high capacity assets without E’s effort wastes money and results in the lowest production. An alternative interpretation of the project choice is that in the effort-intensive project, \( V_R - V_0 \) is spent to enhance E’s effort productivity, which increases output if \( e = 1 \) but is wasted if \( e = 0 \).

4. Results

Before examining the general optimization problem and formally defining an equilibrium, we first show some basic results. We denote equilibrium values with asterisks. Since \( s = 0 \) implies E is bad, E will never receive control contingent on \( s = 0 \), VC will fire E and divert \( L \), and short-term and long-term vesting are zero: \( W_1^*(s = 0) = 0 \) and \( W_3^*(s = 0) = 0 \). For simplicity, we suppress the argument \( s \), so \( W_1(c) \) and \( W_3(V) \) refer to \( s = 1 \).

If the project completion value is zero, either E is bad or E has diverted funds, so long-term vesting is always zero: \( W_3^*(0) = 0 \). Since the project completion payoff outcome of \( V \in \{ V_r, V_1, V_0 \} \) is determined by verifiable project choice, we write long-term vesting \( W_3(V) \) as \( W_3 \) when long-term vesting as determined by project choice is understood. Similarly, we write \( W_1(c) \) as \( W_1 \), when short-term vesting as determined by project choice is understood. Since project choice is clear given outcome \( V \), total vesting \( W_T(V, c) \) can be expressed as \( W_T(V) \), which can be written without the argument as \( W_T \) when the project outcome is clear.

4.1. Hold-up and Repudiation Results

Under VC control, a termination results in threat point payoffs of \( W_1(c, s) \) to E and \( L - W_1(c, s) \) to VC. E’s repudiation-proof long-term vesting is half the surplus of \( V - L \), where \( V \in \{ 0, V_r, V_1, V_0 \} \). E and VC can split the surplus contingent on the outcome, even though the outcome of the surplus as a function of \( e \) is known by E but not necessarily by VC, because the effort decision has already been made at the time of bargaining, the renegotiated compensation can be contracted upon the realization of effort revealed through \( V(c, e) \), and E can take no action based on his asymmetric information after bargaining other than his threat point of quitting. Bargaining ensures ex-post efficiency. Since a bad E type’s project always has a negative surplus outcome of \(-L\), he prefers to quit. A bad project is always terminated and the bad type receives only short-term vesting \( W_1 \). For the good type, the project is not terminated or diverted. This gives under VC control

\[
\delta^* = \tau^* = y \quad \text{if } \theta = \theta_B
\]
\[
\delta^* = \tau^* = n \quad \text{if } \theta = \theta_G.
\]
The repudiation-proof constraint under VC control is
\[ W_T = W_1 + \frac{1}{2}(V(c, e) - L), \] (RP_{VC})
where \( V(c, e) \in \{V_1, V_r, V_0\} \). This implies repudiation-proof long-term vesting is \( W_3 = W_T - W_1 = \frac{1}{2}(V(c, e) - L) > 0 \).

Under E control, VC cannot fire E so an early termination is verifiably due to E quitting. Long-term vesting can be contracted contingent on the final outcome and VC cannot repudiate it. Short-term vesting is set as \( W_1^* = 0 \), in order to reduce the amount that the bad type can receive and the good type can use as bargaining power, as seen below. This does not restrict the maximum amount for which \( W_T \) can be contracted. E can repudiate with the threat of diverting funds \( L \), which would lead to a final firm payoff of zero. E’s threat point is \( L \) and VC’s threat point is zero. Again, the solution is that E and VC split the surplus contingent on the outcome. The bad type’s project has a negative surplus outcome of \(-L\). Since he has limited liability, long-term vesting can only be zero, and the bad type diverts \( L \).

For the good type, since VC cannot repudiate long-term vesting, E always chooses the effort-intensive project and gives effort. This gives under E control
\[
\begin{align*}
\delta^* &= \tau^* = y \quad \text{if } \theta = \theta_B \\
\delta^* &= \tau^* = n \quad \text{if } \theta = \theta_G.
\end{align*}
\]
As above, the surplus from bargaining is \( \frac{1}{2}(V(c, e) - L) \). The E control repudiation proof constraint is:
\[
W_T \geq L + \frac{1}{2}(V(c, e) - L). \] (RP_{E})

At \( t = 1 \), E chooses
\[
e^* = \text{argmax}_e U_E(e). \] (IC_e)
If \( c^* = r \), \( e^* = 0 \) since effort is costly and provides no additional return. If \( c^* = i \), our assumption of \( \kappa(1) < \frac{1}{2}(V_1 - V_0) \) implies that the good type always supplies effort, as follows. E’s short-term vesting \( W_1 \) is paid before effort and so cannot depend on it. Under VC control, E’s utility subject to (RP_{VC}) is \( U_E = W_1 + \frac{1}{2}(V(e) - L) - \kappa(e) \). Since
\[
U_E(e = 1) - U_E(e = 0) = \frac{1}{2}(V_1 - V_0) - \kappa(1) > 0,
\]
due to our assumption of \( \kappa(1) \leq \frac{1}{2}(V_1 - V_0) \), \( e^* = 1 \). If on the contrary \( \kappa(1) > \frac{1}{2}(V_1 - V_0) \), E would never give effort under VC control. Since effort cannot be incentivized with \( W_1 \), our
assumption about $\kappa(1)$ allows us to focus on the effort-intensive project choice. We examine the trade-off of short-term vesting and $E$ control as incentives for $E$ to choose the effort-intensive project choice.

Under $E$ control, constraint $(\text{RP}_E)$ requires $W_T(e) \geq \frac{1}{2}(V(e) + L)$. There is a pooling equilibrium and the binding budget constraint for VC (discussed further below) is $p(V(e) - W_T(e)) + (1 - p)qL = K$, which implies $W_T(e) = V(e) + \frac{(1-p)qL - K}{p}$. Substituting this into $E$’s utility function implies $U_E(e) = V(i, e) + \frac{(1-p)qL - K}{p} - \kappa(e)$. Since

$$U_E(e = 1) - U_E(e = 0) = (V_1 - V_0) - \kappa(1)$$

$$> 0,$$

e^* = \arg\max_e U_E(e) = 1. Thus, $E$ always gives effort. Since the good type is indifferent to a contract that pays $W_3'(V_0) = 0$ under VC or $E$ control, we assume this for simplicity.

We introduce the effort-related project choice $c \in \{i, r\}$ to consider a new variant of contracting for effort when there are incomplete contracts. $E$’s decision to choose the effort-intensive project at $t = 0$ is different than his choice to give effort at $t = 1$ after the effort-intensive project has been chosen. The repudiation-proof results show that under VC control, unverifiable effort cannot be contracted. $W_1$ is paid before effort, and $W_3 = \frac{1}{2}(V(e) - L)$ gives $E$ a fixed, non-contractible exposure to the product of his effort of $\frac{1}{2}e$. Instead, we show how short-term vesting can give incentives to $E$ to choose an effort-intensive project. Under VC control, $E$ will choose the effort-intensive project if the extra compensation he receives from short-term vesting for choosing the effort-intensive project is greater than the cost of effort he will take. Under $E$ control, long-term vesting can be contracted so $E$ can be paid the full cost of his effort in long-term vesting, and $E$ will choose the effort-intensive project.

Our assumption that $k > \frac{1}{2}$ allows us to focus on interesting cases, seen below, in which either some amount of short-term vesting or $E$ control is always necessary for $E$ to recover his cost of effort so that he is willing to choose the effort-intensive project. If instead $k \leq \frac{1}{2}$, $E$ would always choose an effort-intensive project and effort during the project since he always receives at least half the product of his effort from his split over the bargaining surplus.

4.2. Equity Shares Interpretation

The general contract that pays $W_1$ and $W_3$ contingent on any verifiable information can be interpreted in terms of vesting of an actual equity compensation contract. This shows why under VC control, apparent short-term vested equity is insensitive to effort, and long-term vesting has a non-contractible sensitivity to effort. Consider a contract at $t = 0$ that specifies VC control and nominal equity shares of the project paid to $E$ of $\tilde{\alpha}_1(s = 1, c)$ short-term
vesting and $\tilde{\alpha}_3(s = 1, c)$ long-term vesting. If the project is terminated, $E$ receives $\tilde{\alpha}_1 L$. Thus, the repudiation-proof value of $E$'s equity is $(\tilde{\alpha}_1 + \tilde{\alpha}_3)V(c) = \tilde{\alpha}_1 L + \frac{1}{2}(V(c) - L)$, rather than necessarily $(\tilde{\alpha}_1 + \tilde{\alpha}_3)V(c)$. $E$'s repudiation-proof equity vesting can be interpreted as $\tilde{\alpha}_1 V(c) = \tilde{\alpha}_1 L = W_1$ and $\tilde{\alpha}_3 V(c) = \frac{1}{2}(V(c) - L) = W_3$. The sensitivity for a given $c$ to $e$ of $\tilde{\alpha}_1 V(c)$ is zero and of $\tilde{\alpha}_3 V(c)$ is fixed at $\frac{1}{2} \nu$.

Thus, short-term vesting cannot give $E$ incentives for taking effort, though it does give incentives for choosing the effort-intensive project. The effort incentives from long-term vesting are not contractible, since they are fixed at $\frac{1}{2} \nu$.

### 4.3. Optimization Problem

Since the venture capital market for financing is competitive, and projects accepted by the good type are profitable, while those accepted by the bad type are not, VC offers a competitive contract to maximize the good type's utility, subject to constraints. The optimization problem is given as:

\[
\max_{W_1(c^*, s), W_T(c^*, s, \tau^*, V), \gamma(s)} \quad W_T(c^*, s, \tau^*, V) - \kappa(e^*) - M \tag{4.1}
\]

s.t.

\[
W_T - \kappa(e^*) \geq M \tag{IR}
\]

\[
e^* = \text{argmax}_c W_T^*(c, V(c, e^*)) - \kappa(e^*) \tag{IC_c}
\]

\[\text{(RP}_{\text{VC}}) \text{ if } \gamma(1) = VC \tag{4.2}\]

\[\text{(RP}_{E}) \text{ if } \gamma(1) = E \tag{4.3}\]

\[\text{(VC}_{P}) \text{ if } W_1 + L_1 1_{\gamma(1)=E} > \frac{M}{1-q} \tag{PL}\]

\[\text{(VC}_{S}) \text{ if } W_1 + L_1 1_{\gamma(1)=E} \leq \frac{M}{1-q}. \tag{SP}\]

where VC's individual rationality constraints (or budget constraints) for pooling and separation are, respectively,

\[
p(V(c^*, e^*) - W_T) + (1 - p)[L - (1 - q)(W_1 + L_1 1_{\gamma(1)=E})] \geq K \tag{VC_P}
\]

\[V(c^*, e^*) - W_T \geq K. \tag{VC_S}\]

The good type's individual rationality constraint is given by (IR). His incentive constraint for the effort-intensive project choice is given by (IC_c), and for effort is given by (IC_e). Constraints (4.2) and (4.3) state whether the (RP_{VC}) or (RP_{E}) repudiation-proof constraint applies. Constraints (PL) and (SP) state whether the (VC_P) or (VC_S) individual rationality constraint for
VC applies, according to the bad type’s individual rationality constraint to pool or separate, respectively.

We define a “solution” as

\[ \{W_1(c^*, s), W_T(c^*, s, \tau^*, V), \gamma(s)\}, c^*, e^*, \delta^*, \tau^*, V(\theta, c^*, e^*, \delta^*, \tau^*)\],

which is pooling if \(W_1^*(c^*) + L1_{\gamma(1)=E} > M\) or is otherwise separating, and in which (ICc) and (ICe) hold, and \(\delta^*\) and \(\tau^*\) are chosen as above. This implies \(e^* = 1\) if \(c^* = i\), \(e^* = 0\) if \(c^* = r\), \(\delta^* = \tau^* = n\) if \(\theta = \theta_G\), and \(\delta^* = \tau^* = y\) if \(\theta = \theta_B\). We define a solution as “viable” if (IR), (4.2), (4.3), (PL) and (SP) also hold. Viability can also refer to whether any viable solution exists for a particular value for one or more specified variables, and/or for a separating or pooling outcome. We define an equilibrium as a viable solution that satisfies (4.1) among all viable solutions. We denote equilibrium values with asterisks, including \(W_1^*\), \(W_T^*\) and \(\gamma^*\), which implicitly defines the value of long-term vesting in equilibrium as \(W_3^* = W_T^* - W_1^*\).

4.4. Optimal Results with Relaxed Model Assumptions

Optimal results are a separating equilibrium, in which the good type chooses the effort-intensive project and gives effort. We first show how optimal results obtain if any assumption of the model is relaxed.

**Verifiable Termination Actions** If the cause of termination is verifiable as due to either VC firing or E quitting (or alternatively, if E and VC are able to commit not to terminate E after a high signal \(s = 1\)), optimal results obtain. If the fire and quit actions are verifiable, long-term vesting can be contracted since the party causing termination can be contracted to receive none of the surplus rather than being able to bargain for half. With full commitment, long-term vesting can be contracted since commitment precludes the ability to repudiate the contract. Either way, contracts are complete. Contracting on long-term vesting allows for full incentives for the effort-intensive project and screening of the bad type. VC offers E a contract with VC control and \(W_1^* = 0\), which implies separation, and long-term vesting that pays E the full residual, \(W_T^* = V_1 - K\). This gives E full incentives to choose the effort-intensive project for any cost of effort \(k \in (\frac{1}{2}, 1)\).

**Ex-Ante Symmetric Information** If E’s type is known by VC at \(t = 0\), VC does not offer the bad type a contract and optimal results obtain. If the value of the project is large enough such that \(V_1 \geq 2K + L\), VC offers a contract with E control, \(W_1 = 0, W_T = V_1 - K\), and RP_E is satisfied so that E does not hold up VC. If \(V_1 < 2K + L\), VC offers the good type a contract that gives full up-front vesting to provide E with incentives to choose the effort-intensive project.
VC offers no contract to the bad type and a repudiation-proof contract to the good type with VC control, \( W^*_1 = \frac{1}{2}(V_1 + L) - K \), \( W^*_T = V_1 - K \).

**No Diversion of Funds** If the agent in control of the assets is not able to operate the assets early to divert funds, E control is efficient because the bad type cannot divert and E cannot hold up VC with the threat to divert under E control. VC offers a contract with E control, \( W^*_1 = 0 \) and \( W^*_T = V_1 - K \).

**High Quality Interim Signal** If the signal quality \( q \) is high enough, \( q \geq \bar{q} = \frac{L - M}{L} \), and \( V_1 \geq 2K + L \), VC can offer E control. The bad type does not pool because of the probability that he will be caught, and the good type cannot hold up VC because \( \text{(RP}_E \text{)} \) is satisfied. VC offers an effort-intensive contract with E control, \( W^*_1 = 0 \), and \( W^*_T = V_1 - K \).

4.5. Limited Optimal Results

With the combination of non-verifiable termination actions, ex-ante asymmetric information, diversion of funds, and a noisy interim signal, the model has limited optimal results. The bad type pools for any contract that gives E control or large enough short-term vesting, so optimal results require that the contract has VC control with limited short-term vesting. However, E will only choose the effort-intensive project if the short-term vesting incentive is large enough to overcome the effort cost. We define the level of incentives for E to choose the effort-intensive project as \( W_T(V_1) - W_T(V_1) - kv \), which is the amount of slack in constraint \( \text{(IC}_c \text{)} \). Our discussion of the model often focuses on the level of incentives, which is continuous, rather than just on the choice of the effort-intensive project, which is discrete, because a higher level of incentives will induce the effort-intensive project choice for a higher cost of effort \( k \).

For a contract with E control, E can be contracted with long-term vesting the entire product of his effort \( \nu \). E will always choose the effort-intensive project and give effort. But the expected value of pooling to the bad type is \( (1 - q)L > M \) due to diverting \( L \), so the bad type always pools. There is no separating equilibrium and hence no optimal results.

To illustrate a range of parameter values for which the surplus value and the product of effort are not too large, such that optimal results hold with VC control, separation and the effort-intensive project, consider

\[
\frac{1}{2}(V_1 + L) - K \in [0, \frac{M}{1 - q}), \tag{4.4}
\]

where \( \frac{1}{2}\nu < \frac{M}{1 - q} \), and the contract \( W^*_1(i) = \frac{1}{2}(V_1 + L) - K \), \( W^*_1(r) = \frac{1}{2}(V_R + L) - K \), \( W^*_T(V) = V - K \) and \( \gamma^*(1) = VC \). Since \( W^*_3(V(c)) = \frac{1}{2}(V(c) - L) \) for \( c \in \{i, r\} \), \( \text{(RP}_VC \text{)} \) is satisfied. \( W^*_1(c) < \frac{M}{1 - q} \) for \( c \in \{i, r\} \), so \( \text{(SP)} \) implies a separating equilibrium and requires \( \text{(VC}_s \text{)} \) to
hold. Substituting $W_T^*(V)$ into (VC$_S$) shows that (VC$_S$) holds for $V \in \{V_1, V_R\}$ and binds, implying that the objective function in (4.1) is maximized for $c \in \{i, r\}$. Finally,

$$W_T^*(i) - W_T^*(r) = \nu > \kappa(1),$$

(4.5)

hence (IC$_c$) implies $c^* = i$. Also,

$$W_T^*(i) - \kappa(1) = V_1^* - K - \kappa(1) \geq M,$$

so (IR) holds.

A key reason that optimal results are achieved for this parameter is that $W_1^*(c)$ is sensitive to the value of $c$, as shown in (4.5), even under the constraint for separation that $W_1^*(c) \leq \frac{M}{1-q}$. Optimal results are limited to projects that are not what we define as “highly entrepreneurial.” Highly entrepreneurial projects are those for which E’s human capital value of the project, based on $V_r$, is relatively large, such that the short-term vesting that can be paid for a regular project according to VC’s individual rationality constraint under separation and (RP$_{VC}$) is binding:

$$\frac{1}{2}(V_r - L) - \frac{M}{1-q} \geq K - L.$$  (HE)

Any projects that are not highly entrepreneurial we call “semi-entrepreneurial” projects since E still adds value: $V_r > K > L$. The group of all highly entrepreneurial and semi-entrepreneurial projects are called entrepreneurial projects.

4.6. Results for Highly Entrepreneurial Projects

4.6.1. Tradeoff of Effort-Intensive Project and Separation

Under (HE), separation with VC control is viable for all $p$ and $q < \bar{q}$, but implies that the effort-intensive project is not chosen. The surplus on a regular project is large enough that VC can recover all her investment from the half of the surplus she receives from repudiation, and E receives the maximum possible amount under separation, $\frac{M}{1-q}$. For $W_1(r) = \frac{M}{1-q}$, the bad type does not pool. Since $W_1(r) \geq M$, (IR) is satisfied. The return to VC of $V_r - W_T(r) = -\frac{M}{1-q} + \frac{1}{2}(V_r + L) \geq K$ due to (RP$_{VC}$).

However, separation restricts $W_1(i) = \frac{M}{1-q}$. Short-term vesting cannot increase for the effort-intensive project to compensate E for his cost of effort, despite the increase in project payoff from effort. $W_1(c)$ is insensitive to $c$, since $W_1(i) - W_1(r) = 0$. The increase of $W_3$ due to $c$
through the return to the effort-intensive project choice is $\frac{1}{2} \nu < k \nu$. Thus, $W_T(i) - k \nu - W_T(r) < 0$, and $E$ does not have enough incentives to choose the effort-intensive project. If VC offered a contract with $W_1(r) < W_1(i) = \frac{M}{1-q}$ to induce the effort-intensive project choice, another VC could break that contract with an alternative contract of $W_1(r) = W_1(i) = \frac{M}{1-q}$.

Instead, VC makes a profit of $\frac{1}{2}(V_r + L) - K - \frac{M}{1-q} \geq 0$ under separation. Even though the venture capital market for financing entrepreneurs is competitive at $t = 0$, a VC cannot commit not to hold up $E$ other than by offering $E$ control, which implies pooling. If VC’s individual rationality constraint under pooling (VC$_P$) does not hold, pooling is not viable. In this case, no viable alternative VC contract can break a separating equilibrium by paying $W_1(c) > \frac{M}{1-q}$ or by $\gamma(1) = E$. Thus, no VC can compete away positive profits.

If $W_1(c) > \frac{M}{1-q}$ is viable according to (VC$_P$), then any equilibrium must be pooling, since larger short-term vesting maximizes the good type’s utility in the objective function of (4.1) regardless of $c \in \{i, r\}$. Under pooling, (VC$_P$) always binds. No alternative VC separating contract can break a pooling equilibrium, since a good type $E$ could only separate by taking lower $W_1(c)$. Under pooling, the good type chooses the effort-intensive project if and only if there is $E$ control, or VC control with $W_1(i) - W_1(r) \geq k \nu$. The bad type chooses the same $c$ as the good type since $W_1(c^*) \geq W_1(c')$ for $c' \neq c^*$. Thus, highly entrepreneurial projects have no optimal results. There is a trade-off in which short-term vesting, or $E$ control that protects long-term vesting, give $E$ incentives to choose the effort-intensive project, but they reduce screening of bad types. With greater short-term vesting in a viable pooling solution, there is a trade-off of the benefit of greater effort incentives with the cost of pooling, defined below.¹

**Proposition 1.** Under assumption (HE), no projects in which the effort-intensive project choice is taken have separating equilibria.

### 4.6.2. Equilibrium Results

The possible types of equilibria for highly entrepreneurial projects are illustrated in Figure 4.1, with the probability of a good type $p$ on the x-axis and the signal quality $q$ on the y-axis. The divided regions illustrate the resulting equilibria classified as separating or pooling, effort-intensive projects or regular projects, and VC control or $E$ control. In Region 4, the

¹If a contract with a non-equity based payment $W_0 > 0$ to $E$ was possible conditional on $E$ revealing he was a bad type at $t = 0$, before VC invested $K$, there would be no pooling equilibria in which projects were invested with bad types. Instead, bad types would either pool or separate regarding investing $M$ to then receive $W_0 = (1-q)W_1$. Thus, the general results of separating and pooling solutions implying a trade-off of effort incentives versus screening would remain. There is no $W_0 > 0$ possible in the present model with equity-based compensation. Any equity share $\alpha_0 > 0$ would have zero value $W_0 = 0$, since there would be no residual value. The project would not be started and hence would have zero liquidation value.
separating equilibrium, VC always has control and there is never an effort-intensive project. Within the pooling equilibria, Regions 1-3, E has control in Region 1 and chooses the effort-intensive project. Under VC control, E chooses the effort-intensive project in Region 2 and the non-effort intensive project in Region 3.

Region 4, separation, is viable for all highly entrepreneurial projects, for all $p$ and $q$. However, the good type prefers the solution specified in Regions 1-3 for their corresponding levels of $p$ and $q$, when they give him greater profit, as shown in Figure 4.1, and so these are the resulting equilibria solving (4.1) when they are viable for values of $p$ and $q$. Region $i$ is the
equilibrium rather than Region \( j \) if \( p \) is large enough such that \( p > p_{i,j} \), where

\[
p_{1,2} = \frac{K - qL}{\frac{1}{2}(V_r + \nu - L) - qL} \quad (4.6)
\]

\[
p_{1,3} = \frac{\mu_2 - \sqrt{(\mu_2)^2 - 4\mu_1\mu_3}}{2\mu_1} \quad (4.7)
\]

where \( \mu_1 = \frac{1}{2}(V_r - L) - qL \)
\( \mu_2 = -\mu_1 - \mu_3 - \frac{(1-k)}{1-\nu} \)
\( \mu_3 = K - qL - (1-k)\nu \)

\[
p_{1,4} = \frac{1}{2}(V_r - L) + (1-k)\nu + (1-q)L - \frac{1}{1-q}M
\]

\[
p_{2,3} = \frac{(k-\frac{1}{2})(1-q)}{\frac{1}{2} - (k - \frac{1}{2})q}
\]

\[
p_{2,4} = \frac{K - L + (k-\frac{1}{2})(1-q)\nu + M}{\frac{1}{2}(V_r - L) + \frac{1}{2} - (k - \frac{1}{2})q\nu - \frac{q}{1-q}M}
\]

\[
p_{3,4} = \frac{K - L + M}{\frac{1}{2}(V_r - L) - \frac{q}{1-q}M}
\]

The viability of each region in Figure 4.1 depends on conditions implicit in \( p > p_{i,j} \) from the appropriate \( p_{i,j} \) above, such that the surplus \( V_r - L \) is large enough and the product of effort \( \nu \) is of appropriate size.

**Proposition 2.** The unique equilibrium under assumption \( (HE) \) is:

1. Region 1, pooling with E control and an effort-intensive project, for the highest levels of \( p \):
   - if \( p > p_{1,2} > 0, p > p_{1,3} > 0 \) and \( p > p_{1,4} > 0 \);

2. Region 2, pooling with VC control and an effort-intensive project, for moderate levels of \( p \) and moderate to low levels of \( q \):
   - if (1) does not hold, \( p > p_{2,3} > 0 \) and \( p > p_{2,4} > 0 \);

3. Region 3, pooling with VC control and a non-effort intensive project, for low levels of \( p \) and \( q \):
   - if (1) and (2) do not hold and \( p > p_{3,4} > 0 \);

4. Region 4, separation with VC control and a non-effort intensive project, for the lowest levels of \( p \), or for moderate levels of \( p \) and high levels of \( q \):
   - if (1), (2) and (3) do not hold.

**Proof.** See the Appendix. □
4.7. Effort-Intensive Project and Cost of Pooling

We define the cost of pooling to be the expected amount of financing of a bad project that is not recovered by VC plus the expected amount of vesting and diversion of funds that is received by the bad type in a pooling equilibrium. Both of these occur in a pooling equilibrium in which bad types pool due to the benefits of E control or high short-term vesting, but would not occur in a separating equilibrium. We call this amount a cost because it both decreases VC’s ability to recover $K$ from the project, decreasing the project’s viability regarding VC’s individual rationality constraint, and inefficiently decreases the amount of short-term vesting or E control, decreasing a good type’s incentives to choose the effort-intensive project. The expected cost of pooling due to inefficient investment is $(1 - p)(K - L)$, due to short-term vesting paid to the bad type under VC control is $(1 - q)(1 - p)W_1$, and due to diversion by the bad type under E control is $(1 - q)(1 - p)L$.

VC profits under separation with VC control and no effort-intensive project are also inefficient. They are amounts that under separation cannot be paid to the good type for incentives to choose the effort-intensive project. The equilibrium contract is the one that maximizes the good type’s utility and depends upon the trade-off of the cost of pooling and cost of VC profit under separation.

The expected costs of pooling are relatively low for high $p$ and $q$, when the ex-ante probability of bad types is low and the probability of catching bad types with the signal after contracting is high. When costs of pooling are relatively low, E control is viable. This gives E the greatest incentives to choose the effort-intensive project, which he always chooses under E control, as seen in Region 1 of Figure 4.1. As costs of pooling increase with the decrease of $p$ and $q$, and E control becomes too costly, large short-term vesting may instead be still viable. This may still give E large enough incentives to choose the effort-intensive project, as shown in the VC control pooling equilibrium with effort-intensive project in Region 2. As costs of pooling increase even further, short-term vesting is decreased, decreasing incentives until the effort-intensive project is not chosen. However, there may still be a pooling equilibrium where the good type receives higher short-term vesting despite not choosing the effort-intensive project, as seen in Region 3. Finally, for large enough costs of pooling, E chooses separation (and profits to VC) rather than pooling with losses to bad types, as shown in Region 4.

The levels of $p$ and $q$ may also be loosely interpreted as corresponding to levels of due diligence and monitoring, respectively. Greater due diligence performed by VC before the project would increase the chance of weeding out bad types and so increase $p$. This is independent of the screening role played by the contract offered. If bad types know there is a chance of being denied for funding even if they pay $M$ to look for funding, fewer will try. Greater monitoring by VC during the life of the project may increase $q$, the quality of the interim signal. Hence,
results of the model may give insight into the trade-off of better due diligence versus better monitoring by VC. However, this is only a loose interpretation, since $p$ and $q$ are exogenous. Also, actual due diligence would occur after $M$ is paid but before $K$ is invested, whereas $p$ is the probability of $E$’s type before $E$ decides to pay $M$.

The following two propositions give comparative statics for changes in $p$ and $q$. The results for changes in $p$ hold for all $p$. The results for changes in $q$ hold for all $q$ except for between Regions 1 and 4, 2 and 4, and 1 and 3, due to nonlinearities in $q$ there and the switch from pooling to separating equilibria.

**Proposition 3.** Under assumption (HE), as $p$ decreases:

- $E$ control, short-term vesting plus $E$ diversion of funds, incentives for and choice of the effort-intensive project, and level of effort decrease weakly, and the equilibrium changes weakly from pooling to separating.

**Proof.** See the Appendix.

A decrease in $p$ or $q$ has similar effects on $E$ control plus diversion, short-term vesting, incentives, and effort-intensive project choice, but they have opposite effects on a pooling versus separating equilibrium obtaining, as seen in the next proposition.

**Proposition 4.** For highly entrepreneurial projects, as $q$ decreases within any region and between Regions 1 and 2, Regions 2 and 3, and Regions 3 and 4:

- $E$ control, short-term vesting plus possible $E$ diversion of funds, incentives for and choice of the effort-intensive project, and level of effort decrease weakly, and the equilibrium changes weakly from separating to pooling.

**Proof.** See the Appendix.

We now explain the difference in comparative statics between $p$ and $q$. As $p$ decreases to zero in the limit, high short-term vesting becomes infinitely costly in relation to the benefit to good types, so separation is eventually preferable for the good type. As $q$ decreases to zero, pooling may still allow for large enough short-term vesting that it is preferred over separation, and may even allow for large enough effort incentives that the effort-intensive project is chosen.

However, Figure 4.1 shows that as $q$ decreases the equilibrium may turn from separating to pooling (from Region 4 to 3) even without the effort-intensive project being chosen. As $q$ decreases, both pooling and separation costs increase, reducing the good type’s possible short-term vesting under pooling, or under separation as the $\frac{M}{1-q}$ cap decreases. However, capped short-term vesting available under separation decreases with $q$ faster than possible short-term vesting under pooling decreases with $q$. This is because the cost of pooling due to losses in investment $(1-p)(K-L)$ is independent of $q$. Only the cost of pooling due to the ex-post
short-term vesting paid to the bad type, \((1 - p)(1 - q)W_1\), increases with \(q\). Thus, short-term vesting under pooling, \(W_1 = \frac{\frac{3}{2}p(V_1 - L) - (K - L)}{1 - (1 - p)q}\), does not decrease as fast as capped short-term vesting under separation, \(\frac{M}{1 - q}\), the entire amount of which is sensitive to \(q\). When \(q\) is so low as to be a nearly informationless signal, the good type may lose more in profit to VC under separation than he would lose in the cost of pooling to the bad type under pooling, so pooling is selected. For a high \(q\), separation gives a high cap making it more attractive than pooling. As \(q\) decreases, pooling is eventually chosen when the separation-capped vesting eventually falls below the possible pooling short-term vesting amount. Although the bad types who avoid detection at \(t = 1\) receive lower short-term vesting, their ex-ante expected profit increases because their decreasing chance of being caught dominates.

In the case of \(p\) decreasing, the cost of pooling also increases. This decreases short-term vesting to the good type under pooling. However, the capped short-term vesting available under separation is unchanged with \(p\). Thus, for low enough \(p\), separation is eventually preferred. Furthermore, \(q\) occurs after the investment of \(K\), while \(p\) occurs before \(K\). The decrease in \(q\) implies no increase in the cost to investment within the pooling region, whereas the decrease in \(p\) does, so separation is eventually required for low enough \(p\). Since the signal is after investment, marginal decreases in \(q\) do not increase the initial investment amount in bad projects. Conversely, marginal decreases in \(p\) increase the investment in bad projects since \(p\) is before the investment decision.

### 4.8. Importance of Control Rights

The model shows not only the importance of short-term versus long-term vesting but gives a new explanation for control rights. Control rights and short-term vesting are imperfect substitutes for providing effort-intensive project incentives, whereas control rights and long-term vesting are compliments. E control acts to entrench E in the firm in order to protect long-term vesting from hold-up. Thus, E control over residual actions in the firm allows for contracting E to receive the verifiable residual cash flows at \(t = 2\) without hold-up, giving full effort-intensive project incentives.

Without E control, no amount of short-term vesting gives full effort-intensive project incentives. Short-term vesting incentives may be large enough to induce E to choose the effort-intensive project for a given \(p\) if the fractional cost of effort \(k\) is not too large, but the next proposition shows that for any \(p\), there is some \(\overline{k}\) above which for all \(k > \overline{k}\), short-term vesting does not induce the effort-intensive project. Under VC control, short-term vesting can never viably pay the good type the full value of the product of effort if there is any asymmetric information \((p < 1)\). VC receives half of the product of effort due to hold-up, and pays the good type for this through additional short-term vesting. However, bad types receive some
of this additional short-term vesting, limiting the extent to which the good type can capture proceeds from the product of his effort. E control always induces the effort-intensive project because it makes E the residual claimant so he receives the full product of effort $\nu$.

**Proposition 5.** Under E control, E has full effort-intensive project incentives and the effort-intensive project is chosen for all $p < 1$ and $k < 1$. Under VC control and assumption (HE), for any $p^{'}, 0 < p < 1$, there exists a $k(p^{'}) < 1$ such that, for all $k > k(p^{'})$, E would not choose the effort-intensive project.

**Proof.** See the Appendix. ■

This result that assigning control rights to E is sometimes necessary for optimal actions is similar to that found in the incomplete contracts literature. However, this literature typically relies on non-pecuniary private benefits from control for which monetary payments cannot compensate. We show that even without private benefits, contractible monetary payments to E are not large enough to compensate for the monetary benefits gained from E control. Control is worth more in pecuniary terms to good types than to bad types or to VC. Due to the special problems of asymmetric information combined with hold-up, there is a distinction between contingent control and contingent compensation, because short-term vesting is an imperfect substitute for E control. With greater short-term vesting, bad types would not purchase control but would keep the payment. Short-term vesting of $W_1(i)$ marginally greater than $L$ is more costly for satisfying VC’s individual rationality constraint than E control, and worth less to E than control.

Although for simplicity we assume in our model that E can choose effort levels of only zero or one, if effort choice were a continuous variable between zero and one (with appropriate variable effort costs and effort-intensive project choices and payoffs) we would achieve intermediary results. A graph of equilibria regions with continuous levels of effort would look similar to Figure 4.1. Under continuous effort, constant-effort level curves separating regions of greater effort within the VC control pooling regions would look similar to the curve separating Regions 2 and 3 in Figure 4.1. A continuous choice of effort and effort-intensive project choices would particularly demonstrate the importance of control rights. Short-term vesting could never give incentives to induce as large of effort and effort-intensive project choice as could E control. Only E control could induce optimal effort and the effort-intensive project in a continuous model.

An interpretation of E control is that by having equity-like residual control rights over the firm, E also has full equity-like residual cash flow rights and thus equity-like incentives. When VC cedes control rights, she has a debt-like claim. Under VC control, VC has more equity or ownership-type control rights and cash flow rights and E has more of an employment-type
fixed wage. The contingent nature of the control rights helps make the contingent employee or owner nature of E’s position possible, so that the owner incentives are possible to give to E on a contingent basis.

5. Conclusion

We examine venture capital contracting to show how a hold-up problem constrains contracting from providing incentives for hidden action effort and from screening against asymmetric information. A new model of a verifiable effort-intensive project choice is introduced. Short-term vesting of equity provides incentives for the effort-intensive project choice but reduces screening. The equilibrium depends upon the value of effort versus the cost of pooling, according to ex-ante levels of asymmetric information and an interim signal. Control rights are complementary to long-term vesting and give full effort incentives. Short-term vesting is only a partial substitute for control rights.

An application of the model is that since the entrepreneur and the venture capitalist can extract rents through repudiation, control and up-front payments must be given to the more valuable party to protect their larger claim to payoffs. During times when entrepreneurial projects are expected to have smaller profits, the entrepreneur is not as essential to the firm, or managerial skill or the entrepreneur’s ability is hard to distinguish, the entrepreneur would extract too much profit. He has to give the venture capitalist control and accept smaller short-term vesting. The control rights that the venture capitalist requires to allow her to break even actually leads to the venture capitalist making larger than competitive profits in a separating equilibrium. This may provide a new explanation to the empirical puzzle regarding the apparent excess returns attained by venture capitalists.

Alternatively, during times when entrepreneurial projects have very high profits or technologies for which the entrepreneur is essential, the venture capitalist would extract too many rents. The venture capitalist must give up control and give easy investment terms that ex-post looks inefficient for the cases of bad entrepreneurs who were financed and failed, but ex-ante is constrained-efficient in order to satisfy and protect the highly valuable efforts of the good entrepreneurs. This may partially explain in the dot-com boom, which was heavily venture capital-financed, why many of the failed startups may appear to have been given excessive financing and management control, but several firms had very large success.

The model can also be interpreted as a comparison of banking or debt markets versus venture capital financing, in a setup of only one round of financing and one cash flow. When asymmetric risk is low, a large fraction of entrepreneurs are good, or when a signal is correlated highly enough with the continuation value of the firm (such as if loan default signals insolvency and not just illiquidity), banks or debt markets can finance entrepreneurs (which is equivalent
to entrepreneur control in our model). In this case, the financing party has no control over the firm except as provided by contracting on the signal. Giving up the control to fire or renegotiate with the entrepreneur is acceptable when the signal protects the outside investor enough. However, when asymmetric risk is high, many entrepreneurs are bad, or the quality of the signal is low, firms should be financed by venture capitalists who hold control of the firm (which is equivalent to venture capitalist control in our model), and who can more actively manage and renegotiate when needed.

Finally, the model can be seen as a hybrid of the property rights model of the firm (Grossman and Hart, 1986; Hart and Moore, 1990) and the agency model of the firm. Our model combines the question of which party should have control rights in the face of hold-up and ex-ante investment decisions with the problem of the separation of ownership and management in the face of asymmetric information and hidden action. In our model, the entrepreneur has the up-front choice of taking the effort-intensive project, which is similar to an up-front investment decision, and the entrepreneur is the party with valuable human capital. The property rights perspective implies that the entrepreneur should receive control rights to protect him from hold-up so that he maximizes his “investment” of taking the effort-intensive project. However, the party receiving control is not an owner-manager as in the property rights literature but rather the agent of the other party who is the owner. As an agent his type is unknown and the diversion action is not observable. From the agency perspective, the venture capitalist should hold control rights to mitigate the asymmetric information and diversion problems. We show how the assignment of control rights depends on the extent of the hold-up versus agency problems in a combined model. We also show when assigning short-term vesting can be a substitute tool that provides a better solution than assigning control rights to the party with the greatest human capital and investment needs, in order to overcome hold-up in the face of agency problems.
Appendix A: Model Assumptions and Robustness

Repudiation and Bargaining Assumption  We assume parties may repudiate a contract based on the approach taken in Hart and Moore (1994, 1998), Hart (1995), Bulow and Rogoff (1989), and Neher (1999). In Hart and Moore (1994), repudiation starts a Rubinstein bargaining game of alternating offers under which repudiation is subgame perfect. A Rubinstein bargaining game of alternating offers with a positive probability of exogenous breakdown delivers the Nash bargaining outcome that we assume.

DeMarzo and Fishman (2000) argue that repudiation is not a credible threat and that the other party should be able to enforce the original contract in court. Resorting to a court enforcement is not possible in the present model. Under VC control, the only court-verifiable information is whether there is a termination, not who the terminating party is. Gromb (1994) shows that in an infinitely repeated game of lending without collateral, the principal’s inability to commit not to renegotiate implies she can make only zero profit, because no outcome of the game can rely on a threat that is Pareto dominated by the outcome (the latter shown by Farrell and Maskin (1989)).

Other approaches to modeling long-term contracts as inherently non-contractible due to indescribability or unverifiable actions would result in a similar outcome as our model. Assume that continuation of the firm at \( t = 2 \) depends on multiple rounds of staged financing (as shown in several papers, e.g. Neher (1999)), and that long-term vesting cannot be contracted until the staged financing is completed. The renegotiation that is voluntary and welfare-improving for both parties gives the same results as repudiation in our model. Although long-term VC contracts including vesting are typically used in reality, they are commonly updated due to events such as refinancing. In practice, long-term vesting and equity ownership is often either formally or implicitly renegotiated. Baker and Gompers (1999) show empirically that equity investments by venture capitalists just before an IPO reduce CEO ownership by about half, and this dilution is only partially mitigated by measures undertaken that are designed to do so. This implies that VC contracts are not complete and are subject to unilateral renegotiation as we assume.

Robustness to the Timing of Events  The timing of the unverifiable termination action by E or VC and divert action by the party in control is not important for our results. Since repudiation depends on the threat of taking one of the unverifiable actions, it occurs at the time of the unverifiable actions. However, the unverifiable termination and divert actions could occur at any time or multiple times during the life of the project and not significantly change the results.
The unverifiable actions could occur any time up to the final verifiable sale of the firm at $t = 3$. If the agent in control were to operate the assets and divert the cash flows just before $t = 3$, the diversion value is still only $L$ and the assets expire worthless. If the assets have not been diverted by $t = 3$, the firm is sold for a verifiable price and thus the agent loses control at the time of the sale and no longer can divert the assets. If the unverifiable actions were to occur before the signal at $t = 2$, repudiation would also occur earlier. Since short-term vesting must be paid at or before the possibility of repudiation, short-term vesting would be paid before the signal and could not be conditional on the signal. The signal then could not be contracted upon, and the outcome would be the same as when the quality of the signal is zero. If the unverifiable actions were to occur at the beginning of the project, short-term vesting would be paid at the signing of the contract at $t = 0$. If the unverifiable actions were to occur multiple times or continuously throughout the life of the project, the model results would be unaffected. Once short-term vesting has been paid and repudiation determines long-term vesting given by $(RP_{VC})$ or $(RP_{E})$, any further repudiation only gives the same long-term vesting.

The timing of E’s effort may also occur at any time throughout the life of the project without qualitatively changing the results. If effort were to occur at any time before the unverifiable actions and repudiation, the model results are the same. If effort were to occur after repudiation, the only change to results would be that the cost of effort $\kappa(1)$ would be shared by E and VC as a part of the surplus bargained over for effort-intensive projects rather than born solely by E. $(RP_{VC})$ would be replaced by $W_T = W_1 + \frac{1}{2}[V(i) - L + \kappa(1)]$, and $(RP_{E})$ would be replaced by $W_T \geq L + \frac{1}{2}[V(i) - L + \kappa(1)]$. Therefore, effort-intensive projects would be chosen by E more easily and without as much short-term vesting or contingent control necessary, but results are qualitatively similar.

The signal could occur after effort and be a function of effort as well as of E’s type. This would give some ability to contract short-term vesting on realized effort directly. Modeling the signal as uncorrelated with effort simplifies the model and makes the goal of inducing effort more difficult, to highlight the trade-off of better effort incentives at the cost of pooling using short-term vesting and contingent control.
Appendix B: Proofs

**Proof of Proposition 2.** Proposition 1 implies that there can be no viable solutions with separation and the effort-intensive project. E control implies pooling, so there cannot be a viable solution with separation and E control. We examine the remaining potential classes of viable solutions as follows. We define a region \( R_i \in \{R_1, R_2, R_3, R_4\} \) as a solution with specified values of separation or pooling, \( \gamma(1) \) and \( c^* \), which satisfies (4.1) among all solutions with such specified values. The regions have specified values as follows. \( R_1 \) is pooling, \( \gamma(1) = E \) and \( c^* = i \). \( R_2 \) is pooling, \( \gamma(1) = VC \) and \( c^* = i \). \( R_3 \) is pooling, \( \gamma(1) = VC \) and \( c^* = r \). \( R_4 \) is separation, \( \gamma(1) = VC \) and \( c^* = r \). Let \( \kappa(e^*, R_i) \) equal the cost of effort, and let \( W_1(R_i) \) equal the vested wage paid for \( t \in \{1, 3, T\} \), for \( R_i \in \{R_1, R_2, R_3, R_4\} \).

First we show that \( R_4 \) is always viable. (VC\(_S\)) and (RP\(_VC\)) imply that \( (V_r - L) \geq 2(K - L) \) must hold, which does by (HE). Wages determined by (VC\(_S\)), (RP\(_VC\)) and the separation condition that \( W_1 \leq \frac{M}{1-q} \), is given by \( W_1(R_4) = \min\{\frac{1}{2}(V_r + L) - K, \frac{M}{1-q}\} \) and \( W_T(R_4) = \min\{\frac{1}{2}(V_r - L) + \frac{M}{1-q}, V_r - K\} \). This by (HE) implies \( W_1(R_4) = \frac{M}{1-q} \) and \( W_T(R_4) = \frac{1}{2}(V_r - L) + \frac{M}{1-q} \). E’s individual rationality constraint (IR) holds: \( W_T(R_4) \geq M \). Hence, \( R_4 \) is always viable. Thus, an equilibrium always exists.

Consider any contract \( R_i \in \{R_1, R_2, R_3\} \). If

\[
W_T(R_i) - \kappa(e^*, R_i) \geq W_T(R_4),
\]

(5.1)

then \( R_i \) satisfies (4.1) over \( R_4 \). Additionally, (5.1) implies that \( R_i \) satisfies (IR) since \( R_4 \) does.

Under \( R_3 \), solving (VC\(_P\)) and (RP\(_VC\)) for wages gives

\[
W_T(R_3) = \frac{p + \frac{1}{2}(1 - p)(1 - q)(V_r - L) - (K - L)}{p + (1 - p)(1 - q)}
\]
\[
W_1(R_3) = \frac{\frac{1}{2}p(V_r - L) - (K - L)}{p + (1 - p)(1 - q)}.
\]

Since

\[
W_3(R_3) = W_3(R_4) = \frac{1}{2}(V_r - L),
\]

if \( W_T(R_3) > W_T(R_4) \), then \( W_1(R_3) > W_1(R_4) \).

By (HE), \( W_1(R_4) = \frac{M}{1-q} \). Hence, if \( W_T(R_3) > W_T(R_4) \), then \( W_1(R_3) > \frac{M}{1-q} \). The bad type’s incentive constraint to pool is satisfied. Hence, all constraints for (4.1) are satisfied, so \( R_3 \) is viable. Substituting for \( W_T(R_3) \) and \( W_T(R_4) \) in \( W_T(R_3) > W_T(R_4) \) and solving for \( p \) gives \( p > p_{3,4} \).
Under $R_2$, solving (VC$_P$) and (RP$_V$) for wages gives

$$W_T(R_2) = \frac{[p + \frac{1}{2}(1 - p)(1 - q)][V_1 - L] - (K - L)}{p + (1 - p)(1 - q)}$$

$$W_1 = \frac{1}{2}p(V_1 - L) - (K - L)$$

Since

$$W_3(R_2) - \kappa(1) = \frac{1}{2}(V_r - L) - (k - \frac{1}{2})\nu$$

$$< \frac{1}{2}(V_r - L)$$

$$< W_3(R_4)$$

$$< W_3(R_3),$$

if $W_T(R_2) - \kappa(1) > W_T(R_4)$, then $W_1(R_3) > W_1(R_4) = \frac{M}{1 - q}$. The bad type’s incentive constraint to pool is satisfied. Thus, all constraints for (4.1) are satisfied, so $R_2$ is viable.

Substituting and solving for $p$, $W_T(R_2) - \kappa(1) > W_T(R_4)$ is equivalent to $p > p_{2,4}$, and $W_T(R_2) - \kappa(1) > W_T(R_3)$ is equivalent to $p > p_{2,3}$.

Under $R_1$, solving (VC$_P$) and (RP$_E$) for wages gives $W_T(R_1) = V_1 + \frac{(1 - p)qL - K}{p}$. Under E control, the bad type will always divert assets for $L$, and so the bad type’s incentive constraint to pool is always satisfied. Hence, all constraints for (4.1) are satisfied, so $R_1$ is viable.

Substituting and solving for $p$, $W_T(R_1) - \kappa(1) > W_T(R_4)$ is equivalent to $p > p_{1,4}$, and $W_T(R_1) - \kappa(1) > W_T(R_2)$ is equivalent to $p > p_{1,2}$.

The inequality $R_1 > R_3$ is equivalent to

$$(1 - q)(1 - p)[p[\frac{1}{2}p(V_r - L) + (1 - k)\nu] + p(1 - k)\nu]$$

$$> (1 - q)(1 - p)[K - (1 - p)qL]$$

$$(1 - q)(1 - p)[p[\frac{1}{2}(V_r - L) - qL] - (K - qL)$$

$$+ (1 - k)\nu(\frac{q}{1 - q})] + (1 - k)\nu > 0.$$

Sufficient for (5.2) is:

$$\frac{1}{2}p(V_r - L) + (1 - k)\nu > K - (1 - p)qL$$

$$p > \bar{p}_{1,3} \equiv \frac{K - qL}{\frac{1}{2}(V_r - L) + (1 - k)\nu - qL}$$

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The solution of (5.2) is given by the solution to the quadratic equation

\[ Q \equiv \mu_1 p^2 + \mu_2 p + \mu_3 < 0, \tag{5.3} \]

where

\[
\begin{align*}
\mu_1 &= \frac{1}{2}(V_r - L) - qL \\
\mu_2 &= -\mu_1 - \mu_3 - \left(\frac{1-k}{1-q}\right)\nu \\
\mu_3 &= K - qL - (1-k)\nu.
\end{align*}
\]

When there exists a real solution \([\mu_2]^2 - 4\mu_1\mu_3 > 0\], define the roots of \(Q = 0\) as \(\{p^-, p^+\}\), where \(p^- = \frac{-\mu_2 - \sqrt{(\mu_2)^2 - 4\mu_1\mu_3}}{2\mu_1}\) and \(p^+ = \frac{-\mu_2 + \sqrt{(\mu_2)^2 - 4\mu_1\mu_3}}{2\mu_1}\). The roots are difficult to analyze directly. At \(k = 1\), the roots are \(\{p^-, p^+\} = \{\frac{K-qL}{2(V_r-L)-qL}, 1\}\). Since \(\mu_1 > 0\) by (HE), if \(p^- < 1\), then the quadratic inequality (5.3) is graphed as a parabola with vertex below zero and roots \(p^- < 1\) and \(p^+ = 1\), so the solution to (5.3) is \(p \in (p^-, p^+)\). Since \(\frac{dQ}{dk} < 0\) and \(\frac{dQ}{dp}|_{k \leq 1, p \geq 1} < 0\), this implies that for \(k < 1\), \(p^+ \geq 1\) in order to be a root of \(Q = 0\). Hence, when \(p^- < 1\), the solution to (5.3) is \(p \in (p^-, p^+)\), so the relevant solution is \(p > p_{1,3} \equiv p^- > 0\).

If \(p^- \geq 1\), then since \(\frac{dQ}{dk} < 0\) and \(\frac{dQ}{dp}|_{k \leq 1, p \geq 1} < 0\), \(p^- < 1\) for all \(k < 1\), and \(p^+ \geq 1\), so there is no solution for \(p < 1\).

The viable solution that uniquely solves (4.1), and is hence the unique equilibrium, is:

\[
\begin{align*}
R_1 & \quad \text{if } p > p_{1,2} > 0, \ p > p_{1,3} > 0 \text{ and } p > p_{1,4} > 0 \\
R_2 & \quad \text{if } p \leq |p_{1,2}|, \ p > p_{2,3} > 0 \text{ and } p > p_{2,4} > 0 \\
R_3 & \quad \text{if } p \leq |p_{1,3}|, \ p \leq |p_{2,3}| \text{ and } p > p_{3,4} > 0 \\
R_4 & \quad \text{if } p \leq |p_{1,4}|, \ p \leq |p_{2,4}|, \text{ and } p \leq |p_{3,4}|,
\end{align*}
\]

where the condition for \(R_4\) always holds if the conditions for \(R_i \in \{R_1, R_2, R_3\}\) do not hold. ■

**Proof of Proposition 3.** For any \(q\) and for any Regions \(i\) and \(j\) defined by Proposition 2 such that \(i < j\), where \(i \in \{1, 2, 3\}\) and \(j \in \{2, 3, 4\}\), any point \((p^i, q)\) in Region \(i\) must be such that \(p^i > p_{i,j}(q)\), and any point \((p^j, q)\) in Region \(j\) must be such that \(p^j \leq p_{i,j}(q)\), hence \(p^j < p^i\). Thus, for a fixed \(q\), as \(p\) decreases, the equilibrium region increases in cardinal value. For an increase in cardinal value of region numbers, Regions 2, 3, and 4 have VC control and Region 1 has E control, so E control decreases weakly among regions. Within regions there is no change. Regions 1 and 2 have effort-intensive projects and \(e^* = 1\) while Regions 3 and 4 do not, so the effort-intensive project and the level of effort decrease weakly among regions.
Within regions, there is no change. Region 4 has separation while Regions 1, 2 and 3 have pooling, so there is a change from a pooling equilibrium to a separating equilibrium. Within regions there is no change.

The level of incentives for each region is the good type’s utility value of choosing the effort-intensive project in the given region, as follows. For $R_1$:

$$W_T(R_1, i) - \kappa(1) - W_T(R_1, r) = (1 - k)\nu.$$  

$R_2$:

$$W_T(R_2, i) - \kappa(1) - W_T(R_2, r) = \left[\frac{p + \frac{1}{2}(1 - p)(1 - q)}{p + (1 - p)(1 - q)} - k\right] \nu |_{\{p, q: p \leq p_{1,2}, p > p_{2,3} > 0, p > p_{2,4} > 0\}} \in (0, (1 - k)\nu).$$

$R_3$:

$$W_T(R_3, i) - \kappa(1) - W_T(R_3, r) = \left[\frac{p + \frac{1}{2}(1 - p)(1 - q)}{p + (1 - p)(1 - q)} - k\right] \nu |_{\{p, q: p \leq p_{1,3}, \ p \leq p_{2,3}, \ p > p_{3,4} > 0\}} \leq 0.$$ 

$R_4$:

$$W_T(R_4, i) - \kappa(1) - W_T(R_4, r) = -k\nu \leq 0.$$ 

Thus, the level of incentives decreases weakly with Region number. Within the regions of $R_1$, $R_3$, and $R_4$ there is no change. Within the region of $R_2$,

$$\frac{d}{dp} \left\{ \left[\frac{p + \frac{1}{2}(1 - p)(1 - q)}{p + (1 - p)(1 - q)} - k\right] \nu \right\} = \frac{\frac{1}{2}(1 - q)\nu}{[p + (1 - p)(1 - q)]^2} > 0,$$

so the level of incentives decreases.

Diversion of funds in $R_1$ is $L$ and zero for other regions, $W_1(R_1) = 0$. Comparing diversion of funds plus short-term vesting across regions, again holding $q$ constant,

$$L + W_1(R_1) = L$$

$$> W_1(R_2) = \frac{\frac{1}{2}p(V_1 - L) - (K - L)}{p + (1 - p)(1 - q)}|_{\{p, q: p \leq p_{1,2}, p > p_{2,3} > 0, p > p_{2,4} > 0\}}$$

$$> W_1(R_3) = \frac{\frac{1}{2}p(V_r - L) - (K - L)}{p + (1 - p)(1 - q)}|_{\{p, q: p \leq p_{1,3}, p \leq p_{2,3}, p > p_{3,4} > 0\}}$$

$$> W_1(R_4) = \frac{M}{1 - q},$$
so the amount of short-term vesting plus diversion of funds decreases with Region number. Short-term vesting plus diversion of funds is constant within Regions 1 and 4. Within Region 2,
\[
\frac{d}{dp}\left[\frac{\frac{1}{2}p(V_1 - L) - (K - L)}{p + (1 - p)(1 - q)}\right] = \frac{\frac{1}{2}(1 - q)(V_1 - L) + q(K - L)}{[p + (1 - p)(1 - q)]^2} > 0.
\]
Within Region 3,
\[
\frac{d}{dp}\left[\frac{\frac{1}{2}p(V_r - L) - (K - L)}{p + (1 - p)(1 - q)}\right] = \frac{\frac{1}{2}(1 - q)(V_r - L) + q(K - L)}{[p + (1 - p)(1 - q)]^2} > 0.
\]
Thus, the level of short-term vesting plus diversion of funds decreases within Regions 2 and 3.

**Proof of Proposition 4.** From the proof of Proposition 2, \( p > p_{3,4} \) is equivalent to \( W_T(R_3) > W_T(R_4) \). Conversely, let \( W_T(R_4) > W_T(R_3) \) be referred to by \( p < p_{4,3} \), where \( p_{4,3} \equiv p_{3,4} \). Solving \( p > p_{1,2}, p > p_{2,3} \) and \( p < p_{4,3} \) for \( q \) gives the inequality \( q > q_{i,j} \), where \((i, j) \in \{(1, 2), (2, 3), (4, 3)\}\), such that
\[
q > q_{1,2} = \frac{K - \frac{1}{2}p(V_r + \nu - L)}{(1 - p)L},
q > q_{2,3} = \frac{k - \frac{1}{2}(1 + p)}{(1 - p)(k - \frac{1}{2})},
q > q_{4,3} = \frac{\frac{1}{2}p(V_r - L) - (K - L) - M}{\frac{1}{2}p(V_r - L) - (K - L) - (1 - p)M}.
\]
For any \( p \) and for Region pairs \((i, j) \in \{(1, 2), (2, 3), (4, 3)\}\) defined by Proposition 2, any point \((p, q^i)\) in Region \( i \) must be such that \( q^i > q_{i,j}(p) \) and any point \((p, q^j)\) in Region \( j \) must be such that \( q^j \leq q_{i,j}(p) \), hence \( q^j < q^i \). Thus, for a fixed \( p \), as \( q \) decreases, the equilibrium region changes from Region \( i \) to \( j \). For such a change of regions, Regions 2, 3, and 4 have VC control and Region 1 has E control, so E control decreases weakly among regions. Within regions there is no change. Regions 1 and 2 have effort-intensive projects and \( e^* = 1 \) while Regions 3 and 4 do not, so the effort-intensive project and the level of effort decrease weakly among regions. Within regions, there is no change. Region 4 has separation while Regions 1, 2 and 3 have pooling, so there is weakly a change from a separating equilibrium to a pooling equilibrium. Within regions there is no change.

The level of incentives for each region is the value of choosing the effort-intensive project as follows. \( R_1: \)
\[
W_T(R_1, i) - \kappa(1) - W_T(R_1, r) = (1 - k)\nu.
\]
$R_2$: 

$$W_T(R_2, i) - \kappa(1) - W_T(R_2, r) = \left[ \frac{p + \frac{1}{2}(1-p)(1-q)}{p + (1-p)(1-q)} - k \right] \nu \{p, q: p \leq |p_{1,2}|, p > p_{2,3} > 0, p > p_{2,4} > 0 \} \in (0, (1-k)\nu).$$

$R_3$: 

$$W_T(R_3, i) - \kappa(1) - W_T(R_3, r) = \left[ \frac{p + \frac{1}{2}(1-p)(1-q)}{p + (1-p)(1-q)} - k \right] \nu \{p, q: p \leq |p_{1,2}|, p \leq |p_{2,3}|, p > p_{3,4} > 0 \} \leq 0.$$ 

$R_4$: 

$$W_T(R_4, i) - \kappa(1) - W_T(R_4, r) = -k\nu \leq 0.$$ 

Thus, the level of incentives decreases with decreases in $q$ from Region $i$ to $j$ for Region pairs $(i, j) \in \{(1, 2), (2, 3), (4, 3)\}$. Within the regions of $R_1$, $R_3$, and $R_4$ there is no change. Within the region of $R_2$, 

$$\frac{d}{dq} \left\{ \left[ \frac{p + \frac{1}{2}(1-p)(1-q)}{p + (1-p)(1-q)} - k \right] \nu \right\} = \frac{\frac{1}{2} p(1-p)\nu}{[p + (1-p)(1-q)]^2} > 0,$$

so the level of incentives decreases. 

Diversion of funds in $R_1$ is $L$ and zero for other regions, $W_1(R_1) = 0$. Comparing diversion of funds plus short-term vesting across regions, again holding $p$ constant, 

$$L + W_1(R_1) = L$$

$$> W_1(R_2) = \frac{\frac{1}{2} p(V_1 - L) - (K - L)}{p + (1-p)(1-q)} \nu \{p, q: p \leq |p_{1,2}|, p > p_{2,3} > 0, p > p_{2,4} > 0 \}$$

$$> W_1(R_3) = \frac{\frac{1}{2} p(V_r - L) - (K - L)}{p + (1-p)(1-q)} \nu \{p, q: p \leq |p_{1,3}|, p \leq |p_{2,3}|, p > p_{3,4} > 0 \},$$

$$W_1(R_4) = \frac{M}{1-q}$$

$$> W_1(R_3) = \frac{\frac{1}{2} p(V_r - L) - (K - L)}{p + (1-p)(1-q)} \nu \{p, q: p \leq |p_{1,3}|, p \leq |p_{2,3}|, p > p_{3,4} > 0 \},$$

so the amount of short-term vesting plus diversion of funds decreases with decreases in $q$ from Region $i$ to $j$ for Region pairs $(i, j) \in \{(1, 2), (2, 3), (4, 3)\}$. Short-term vesting plus diversion
of funds is constant within Region 1. Within Region 2,
\[
\frac{d}{dq} \left[ \frac{\frac{1}{2}p(V_1 - L) - (K - L)}{p + (1 - p)(1 - q)} \right] = \frac{(1-p)[\frac{1}{2}p(V_1 - L) - (K - L)]}{[p + (1-p)(1-q)]^2} > 0.
\]

Within Region 3,
\[
\frac{d}{dq} \left[ \frac{\frac{1}{2}p(V_r - L) - (K - L)}{p + (1 - p)(1 - q)} \right] = \frac{(1-p)[\frac{1}{2}p(V_r - L) - (K - L)]}{p + (1-p)(1-q)]^2} > 0.
\]

Within Region 4,
\[
\frac{d}{dq} \left[ \frac{M}{1 - q} \right] = \frac{M}{(1-q)^2} > 0.
\]

Thus, the level of short-term vesting plus diversion of funds decreases within Regions 2, 3 and 4. ■

**Proof of Proposition 5.** E’s level of incentives to choose the effort-intensive project under E control is
\[
W_T(R_1, i) - \kappa(1) - W_T(R_1, r) = (1-k)\nu
\]
\[
> 0
\]
for all \(p < 1, k < 1\), so he always chooses it. E’s level of incentives to choose the effort-intensive project under E control is
\[
\xi = W_T(R_2, i) - \kappa(1) - W_T(R_3, r)
\]
\[
= \left[ \frac{p + \frac{1}{2}(1-p)(1-q)}{p + (1-p)(1-q)} - k \right] \nu.
\]

Setting this equal to zero and solving for \(k\) gives
\[
\kappa(p') = \frac{p' + \frac{1}{2}(1-p')(1-q)}{p' + (1-p')(1-q)}.
\]

For \(k > \kappa(p')\), \(\xi|_{k>\kappa(p')} < 0\), so the effort-intensive project is not chosen. ■
References


