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Abstract

In a 1999 paper, Freeman proposes a model in which discount window lending and open market operations have different outcomes—an important development because in most of the literature the results of these policy tools are indistinguishable. Freeman's conclusion that the central bank should absorb losses related to default to provide risk-sharing goes against the concern that central banks should limit their exposure to credit risk. We extend Freeman's model by introducing moral hazard. With moral hazard, the central bank should avoid absorbing losses, contrary to Freeman's argument. However, we show that the outcomes of discount window lending and open market operations can still be distinguished in this new framework. The optimal policy would be for the central bank to make a restricted number of creditors compete for funds. By restricting the number of agents, the central bank can limit the moral hazard problem. And by making agents compete with each other, the central bank can exploit market information that reveals the state of the economy.

Key words: payment systems, money, open market operations, discount window

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1 Introduction and Motivation

Most central banks have at their disposition two tools to provide liquidity to depository institutions. Central banks can offer collateralized loans at an announced interest rate, as does the Federal Reserve with the discount window and the Bank of Canada with the Bank rate. Central banks can also buy or sell assets through open market operations. This is typically done by auctioning securities to a small number of banks, called primary dealers in the U.S. and Canada.[1] The primary dealers in turn distribute the funds to the market.

In most monetary models these two tools are indistinguishable, making it difficult to discuss or evaluate policy. A notable exception is Freeman (1999). Freeman shows that policies resembling discount window lending and open market operations have different implications in terms of risk-sharing. The model shows that liquidity provision policy by the central bank is beneficial to the extent that the central bank absorbs losses related to default. This conclusion stands in stark contrast to most policy discussions concerning liquidity provision policies that focus on efforts to avoid exposing the central bank to credit risk.[2]

In this paper, we extend Freeman's framework to allow for moral hazard. We show that the central bank should avoid being exposed to risk because creditor's incentives to monitor their loans are reduced when the central bank is likely to suffer losses. However, policies resembling discount window lending and open market operations can be distinguished in our framework. We find that central banks should let a restricted group of agents compete for central bank liquidity; as is typically done in open market operations. We also argue that the discount window may have benefits in particular situations where it should be widely accessible. This distinction does not arise in Freeman's work.

[1]For the rest of the paper we will refer these banks simply as primary dealers

[2]See for example Coleman (2002) or Madigan and Nelson (2002).

In our model, the economy can be in a good state, in which case no loans default, or in a bad state, in which case a fraction of loans default. Creditors can reduce the probability of default of the loans they make by exerting costly effort. Other creditors can observe the quality of a loan but the central bank cannot.[3]

The central bank can provide incentives for creditors to monitor their loans if it restricts the set of creditors who can interact with the central bank. The intuition is that it is costly for creditors to deviate from the optimal level of effort if they acquire funds from other creditors; who can evaluate the quality of the loan they hold. The fewer creditors have access to the central bank, the higher the probability that a deviation will be costly. In turn these costly deviations cause creditors to increase their monitoring and reduce the effect of an aggregate default.

The central bank can learn the state of the economy if it makes creditors compete for funds. The intuition is that if creditors have to compete for funds, they bid up the price at which they borrow until the expected value of the loan is exactly equal to the value of the collateral. This is not possible with a policy that proposes funds at a fixed rate.

A policy resembling open market operations has both desirable features and allows the central bank to provide liquidity without distorting incentives to monitor.[4] Our model thus suggests that central bank should prefer this kind of policy. However, we also argue that in some special situations, a policy of lending funds at a fixed interest rate may be desirable. After the terrorist

[3]The assumption that the central bank cannot observe loan quality is without loss of generality. The weaker assumption that the central bank has a less precise signal of loan quality then the creditors would complicate the math below but deliver the same qualitative results.

[4]We should note that the key point we are making is that the central bank allows the operation of the policy to depend on the information in the market. In the Canadian case the market sensitive policy is conducted at a fixed price but the implementation of it is based on market conditions.

attacks of September 11, destruction of telecommunication equipment made it impossible for the interbank market to operate properly for a few days, for example. In such a case, it is desirable for agents to have access to the central bank directly through the discount window. Another case is where the central bank must forecast the amount of liquidity needed in the market in this case the market insensitive acts as cap on the discount rate and helps ensure creditors against large forecasting mistakes on the part of the central bank.

In this paper we take the decision of an individual to default as exogenous since we want to focus on the moral hazard problems of monitoring creditors. However, the *size* of an aggregate default is endogenous in our model. In a related paper Mills (2006) examines a model with strategic default of debtors and looks at the distortions of collateralized central bank lending and discounted central bank lending. Our work differs from Mills (2006) in that we focus on creditors as the monitors of debtors and examine how central bank policies give creditors the right incentives to monitor.

Also in a survey paper Zhou (2000) conjectures on the effect that endogenous timing of arrivals at the central island would have on the different effects the policy instruments would have in Freeman (1999) model. While the underlying timing of arrivals to the central island is exogenous in our model the endogenous monitoring choice makes arrival time effectively endogenous and confirms her conjecture that the policy prescriptions of Freeman (1999) overturned.

Our paper is based in the payment economics literature that was started by Freeman and Tabellini (1998) and Freeman (1996). In addition to the previously mentioned works above examples from this literature are: Green (1999) who uses this framework to examine public versus private provision of liquidity; Fujiki (2003) who extends the framework to an international setting; Chapman (2007) who looks at optimal international policy coordination; and Mills (2004) who uses a mechanism approach to the examine the key assumptions in this class of model.

Our paper is also related to recent work in the Lender of Last Resort (LOLR) Literature. Specifically our model has some of the same intuition as Freixas, Parigi, and Rochet (2004); who find that the form that LOLR facilities should take depends on the what incentive effects a particular policy has on the effort devoted to loan portfolio monitoring.

The paper is structured as follows. We develop the model with laissez-fair intervention in section 2. We then examine the equilibrium of this model in section 4 and find the amount of monitoring and the welfare in this equilibrium. In section 5 we introduce the two types of policy mechanisms we wish to study and we then find the equilibria under these two different policy mechanisms and compare them to the base case of a laissez-fair policy. We sketch some extensions to the model in section 6 and finally we conclude in section 7.

2 The model

Our framework is an extension of Freeman (1999)'s model of aggregate credit risk in a payment system. Consider an economy on an archipelago around a central island. The chain of islands consists of a large number of island pairs. Each island pair consists of an island inhabited by a large number of two-period lived agent we call debtors and an island inhabited by a large number of two-period lived agent we call creditors. All agents are endowed with a type specific good in their first period of life.

2.1 Debtors

The debtor's problem is identical to the problem in Freeman (1999). A debtor would like to consume when young both their own good as well as that of the creditor. A generation t debtor values different consumption plans according to the utility function

$$v_d(d_t^d) + v_c(c_t^d) + \theta(1 - e_t)\eta v_d(d_{t+1}^d), \tag{1}$$

where d_t^d is the amount of their own debtor good they consume, c_t^c is the amount of creditor good they consume, θ is the probability of a default shock, $(1-e_t)\eta$ is the probability that a debtor defaults on her loan, and d_{t+1}^d is the amount of the debtor good a defaulting debtor consumes when old. The functions $v_d(\cdot)$, $v_c(\cdot)$, and $v_d(\cdot)$ have strictly positive first derivatives and strictly negative second derivatives.

A debtor's probability of default depends on the monitoring effort, e_t, exerted by the debtor's trading partner. We assume that $(1-e_t)\eta$ also denotes the fraction of debtors that default, if all creditors choose the same effort. Hence $(1-e_t)\eta$ can be thought of as the size of the default shock. Debtors know the choice of effort by creditors when they trade.

The budget constraints, and relevant market clearing conditions, are:

$$p_t = p_t d_t^d + m_t, \tag{2}$$

$$m_t = h_t, \tag{3}$$

$$c_t^d = h_t \pi_t, \tag{4}$$

$$p_{t+1} d_{t+1}^d = m_t. \tag{5}$$

Let p_t denote the dollar price of a good on debtor island at t, or the price level. Also, m_t is the amount of money acquired by a debtor, h_t is the nominal value of the debtor's debt, and π_t is the price in creditor island goods at t of a promise to pay 1 on the central island at $t+1$.

Equation (2) says that a debtor splits his unit endowment between consumption and sale to creditors in exchange for money. Equation (3) states that a debtor must have enough money to repay his debt. Equation (4) states that the amount of creditor good consumed by a debtor is equal to the real value of his debt. Equation (5) states that a debtor will consume debtor goods when old if he does not have to repay his debt in the central island.

Using equations (2) through (5), the utility function of a debtor can be written as

$$v_c\left(c_t^d\right)+v_d\left(d_t^d\right)+\theta(1-e_t)\eta v_d\left(d_{t+1}^d\right)=v_c\left(m_t\pi_t\right)+v_d\left(1-\frac{m_t}{p_t}\right)+\theta(1-e_t)\eta v_d\left(\frac{m_t}{p_{t+1}}\right). \tag{6}$$

Debtors choose m_t to maximize this expression. The first order condition is

$$v_c'\left(m_t\pi_t\right)\pi_t-\frac{v_d'\left(1-\frac{m_t}{p_t}\right)}{p_t}+\theta(1-e_t)\eta\frac{v_d'\left(\frac{m_t}{p_{t+1}}\right)}{p_{t+1}}=0. \tag{7}$$

2.2 Creditors

The creditors have the same endowments and utility functions as in Freeman (1999). They wish to consume their own goods when young and wish to consume debtor goods when old. There exists a probability $(1-\alpha)$ that the creditor will experience a taste shock and wish to consume early. α is realized at the beginning of an agent's second period in life and is private information. The utility function of a creditor is of the form

$$u_c(c_t^c)+\alpha u_d(d_{t+1}^c)+(1-\alpha)u_d(\tilde{d}_{t+1}^c), \tag{8}$$

where c_t^c is the amount of their own creditor good consumed, d_{t+1}^c is the amount of debtor good consumed by the late-leaving creditor and $\tilde{d}_{t+1}^c$ is the amount of debtor good consumed by the early-leaving creditor. The functions $u_c(\cdot)$ and $u_d(\cdot)$ are have strictly positive first derivatives and strictly negative second derivatives.

The budget constraint of young creditors is the same as in Freeman (1999),

$$1 = c_t^c + l_t \pi_t, \tag{9}$$

where l_t is the nominal value at t of a creditor's loans to debtors, and π_t is the price in creditor island goods at t of a promise to pay \$1 on the central island at $t+1$. This constraint says that young creditors split their endowment between consumption and sale to debtors in exchange for an IOU.

2.3 Loan Monitoring

A creditor can invest effort in monitoring the loan he extends to the debtor. This monitoring helps reduce the idiosyncratic risk involved in the event of a default. The disutility of effort from monitoring is denoted by $\Phi(e_t)$ where Φ is an increasing concave function and $\Phi(0)$ equals zero. Effort linearly decreases the idiosyncratic risk involved in a given loan; for example a loan where the creditor has invested e_t will default with probability $(1 - e_t)\eta$. All creditors can verify the amount of monitoring inherent in a loan but the central bank cannot.

We should note that loan monitoring and the fact that the central bank does not observe effort are conditional on the structure of the banking system. Member banks of the payment system are typically very low risk institutions and the central bank can observe this low risk nature. What the central bank cannot observe is what risk reducing actions a bank is taking at the margin.

2.4 Goods Trading

The pattern of trade in our model is similar to Freeman (1999). Young debtors travel to the creditor islands and trade a loan for creditor goods. When they return to their own island a old creditor arrives and trades outside money for some debtor goods. We assume that there is no financial market on which young creditors can trade loans among each other. Creditors have an opportunity to trade loans in their second period of life.

There exists two risks inherent in the creditor extending payment credit to the debtor: First, the debtor may default on the loan and, second, the debtor may not be able to settle the loan until after the creditor has a taste shock to consume early.[5] Given these two shocks and equation (9) we can write a generation t creditor's value for choices of monitoring and loans in his first period of life as

$$u_c(1 - \pi_t l_t) - \Phi(e_t) + E\left[V(l_t, e_t)\right], \tag{10}$$

where l_t is the amount of loans extended, e_t is the effort put into monitoring the loans that have been given, and $V(l_t, e_t)$ is the value of loans and monitoring next period when the creditor is old with the expectation being taken over the two risks mentioned above.

The value function of loans and monitoring $V(l_t, e_t)$ is the utility that the creditor receives when old after the shocks he is subject to are realized, and the expectation is over the probability of being in the given state. Since the return on the monitored loans depends on the intervention policy of the central bank the value function will be discussed further below.

3 The Secondary Loan Market

When creditors and debtors are old they must travel to the central island to settle the payment obligations from the previous period's transactions. All creditors arrive at the central island but only a fraction λ of debtors arrive. Once a debtor arrives his payment debt is settled and the corresponding creditor

[5] This latter coordination problem has been extensively studied in the payment economics literature, see for example Martin (2004)

receives l_t in money. Once this first round of settlement has taken place an individual creditor receives two pieces of information: First, whether she has received a liquidity shock and must consume. Second, whether an aggregate default shock has occurred. If a default shock occurs, it is not known which uncleared debt contracts will be defaulted on until the remaining debtors reach the central island. Hence at the start of a settlement period there are four ex ante types of creditors depending on whether they received a liquidity shock and whether the debt they hold cleared. After the shocks have been revealed, the secondary debt market opens and an individual may trade a portion of his debt for either some debt from another creditor or for outside money.

3.1 No Default Shock

Let ρ_{t+1} denote the rate of discount, relative to its face value, at which unsettled debt trades in the secondary market.[6] Assume for the moment that a default shock has not occurred. Early-leaving creditors whose debt has been settled do not trade in the market. Similarly, late-leaving creditors with unsettled debt do not trade since they can wait for the full value of their debt. Early-leaving creditors who have uncleared debt would like to sell their debt in exchange for money. Late-leaving creditors who have settled their debt can buy this debt and wait until it gets settled. The per capita amount of debt bought by those later-leaving creditors is denoted by q_{t+1}

Therefore the supply of money for debt will equal at most the aggregate amount of money that late-leaving creditors receive from cleared debt; this equals $\alpha\lambda l_t$. The demand for money in the secondary market equals the aggregate supply of uncleared loans held by early-leaving creditors; this equals $(1-\alpha)(1-\lambda)l_t$. There is a liquidity constraint when demand is greater then

[6]We use the terms secondary market and discount market interchangeably.

the maximum supply; that is when

$$(1-\alpha)(1-\lambda)l_t > \alpha\lambda l_t. \tag{11}$$

When the liquidity constraint is binding, the discount rate decreases below one to clear the market. Formally,

$$\rho_{t+1} = \frac{\alpha\lambda}{(1-\alpha)(1-\lambda)} < 1. \tag{12}$$

Equation (12) comes from the fact that supply $(1-\alpha)(1-\lambda)l_t$ of loans in the discount market must equal face value of uncleared loans demanded (q_{t+1}) and that the discounted value of loans a creditor buys $\rho_{t+1}q_{t+1}$ is constrained by the amount of money they bring to the discount market $\alpha\lambda l_t$.

We can summarize the return obtained by creditors as follows:

- A settled early-leaver receives a return of one.
- An unsettled early-leaver receives a return of ρ_{t+1}.
- A settled late-leaver receives a return of $1/\rho_{t+1}$.
- An unsettled late-leaver receives a return of one.

3.2 Default Shock

Now assume that there is a default shock and that all creditors have exerted the same effort in monitoring. Recall that the size of the shock and the level of effort is common knowledge to all creditors. We assume the condition that $1-\eta$ is greater then λ which makes the early settlement of debt uninformative. In this case settled early-leavers have a return of one, as before. Unsettled late-leavers are holding loans that may default with probability

$$\frac{(1-e_t)\eta}{(1-\lambda)}. \tag{13}$$

Since the creditors are risk-averse, they prefer to diversify this risk and receive a certain return of

$$r(e) = 1 - \frac{(1-e_t)\eta}{(1-\lambda)}. \tag{14}$$

Late-leavers can achieve this diversification by trading unsettled debt with each other. As a result, late-leavers hold a riskless diversified portfolio of unsettled debt. Late-leaving creditors with settled debt use their money to buy the unsettled debt of early-leaving creditors. The price at which they buy the debt is given by equation (12) if the liquidity constraint binds, or by the expected return of the debt, equation (14) if the liquidity constraint does not bind. Once late-leaving creditors have bought the early-leaver's debt they have an incentive to perfectly diversify their holdings of the debt and therefore receive a certain return of $r(e)/\rho_{t+1}$.

Therefore in the case of a default shock, and if the liquidity constraint binds, the returns for the different types of creditors are:

- A settled early-leaver receives a return of one.
- A unsettled early-leaver receives a return of ρ_{t+1}.
- A settled late-leaver receives a return of $r(e)$.
- A unsettled late-leaver receives a return of $\frac{r(e)}{\rho_{t+1}}$.

After the discount market is closed, the early-leavers consume debtor goods on the debtor islands, unsettled debt contracts are either settled or revealed to have defaulted, and the late-leavers consume debtor goods on the debtor islands. Both early and late leavers trade money for debtor goods at the price p_{t+1}; determined to equate the supply and demand of money.

4 Equilibrium

We define an equilibrium in the usual manner; agents have rational expectations and solve their individual decision problems given prices, and prices clear the various markets given the optimal behavior of the agents. We formalize this in the following definition

Definition 1 (Equilibrium)**.** *An equilibrium is defined as a sequence of prices* $\{\rho_t, \pi_t, p_t\}_{t=0}^{\infty}$*, vectors of agent specific quantities by agents in different states* $\{e_t, \mathbf{d}_t^c, \mathbf{d}_t^d, \mathbf{c}_t^d, \mathbf{c}_t^c\}_{t=0}^{\infty}$*, and a sequence of per period money stocks* $\{M\}_{t=0}^{\infty}$ *such that markets clear; and both creditors and debtors solve their respective optimization problems.*

We will focus on stationary and symmetric equilibria. A stationary symmetric equilibrium is defined as an equilibrium where all quantities and prices are constant for all periods and agents of the same type choose similar actions.

The key equations we want to focus on in an equilibrium are the first order conditions of the young creditor. In a stationary equilibrium these first-order conditions are:

$$u_c'(1 - \pi l) = E[V_1(l, e)], \tag{15}$$

$$\Phi'(e) = E[V_2(l, e)], \tag{16}$$

where V_i is the derivative of the value function with respect to its i the argument. The RHS of equation (15) is the sum of derivatives of the creditors utility function for debtor goods weighted by the probability of the state and the return on the loan in that state, where effort determines what is the expected return to a loan.

The choice of effort and loan supply is determined by the intersection of these two equations in (e, l) space. Since the set of possible equilibrium values is a closed convex set it follows that at least one equilibrium must exist.

5 Central Bank Intervention

Assume that there exists an infinite-lived agent on the central island which we will label as the monetary policy authority or central bank. We assume that the central bank has the ability to costlessly print money and can enforce all contracts on the central island. The central bank's objective function is to implement an allocation where the liquidity constraint in the secondary loan market is relaxed while avoiding bailing out any creditors through money injections.[7]

5.1 The Benchmark Allocation

Since we are interested in the liquidity provision policy of the central bank, we want to compare the allocation achieved when the liquidity constraint binds with the allocation obtained when the liquidity constraint does not bind. The analysis of section 3 implies that if the liquidity constraint does not bind and there is no default shock, all creditors receive a return of one on their loans. When the liquidity constraint does not bind and there is a default shock, creditors whose debt are settled early receive a return of one on their loan, regardless of whether they must consume early or late. Creditors whose debt is not settled early receive a return of $r(e_t)$. It follows that

$$E\left[V(l_t, e_t)\right] = (1-\theta)u_d\left(l_t\right) + \theta\lambda u_d\left(l_t\right) + \theta(1-\lambda)u_d\left(l_t r(e_t)\right). \qquad (17)$$

[7]This is optimal in situations where moral hazard is sufficiently sever.

The first order conditions (15) thus become

$$u_c'(1-\pi l) = [1-\theta(1-\lambda)]\, u_d'(l_t) + \theta\lambda r(e_t) u_d'(l_t) + u_d'(l_t r(e_t)), \quad (18)$$

$$\Phi'(e) = \theta(1-\lambda) r'(e_t) l_t u_d'(r(e_t) l_t). \quad (19)$$

We denote the quantities associated with this allocation with asterisks.

5.2 Laissez-Fair Equilibrium

In a laissez-fair equilibrium the central bank is inactive and all agents know that the liquidity constrain will bind. In this case the discount factor is a constant given by equation (12) and we will denote this constant discount factor by $\bar{\rho}$. For the discount factor to be binding in all states of the world it is bounded from above by the case of no effort in monitoring loans,

$$\bar{\rho} < \frac{(1-\eta)(1-\lambda)}{\eta + (1-\eta)(1-\lambda)}. \quad (20)$$

In the case when equation (20) is binding then a given unsettled loan will trade at the discount $r\bar{h}o$ when traded for money regardless of the amount of effort that has been devoted to monitoring the loan. We can then write the expected value of a loan as, this assumes the individual creditor,

$$\begin{aligned} E\left[V(l_t, e_t]\right. = (1-\theta)\Bigg\{ & [\lambda(1-\alpha) + (1-\lambda)\alpha]\, u_d\left(\frac{l_t}{p_{t+1}}\right) \\ & + (1-\lambda)(1-\alpha) u_d\left(\frac{\bar{\rho} l_t}{p_{t+1}}\right) + \lambda\alpha u_d\left(\frac{l_t}{\bar{\rho} p_{t+1}}\right) \Bigg\} \\ + \theta\Bigg\{ & \lambda(1-\alpha) u_d\left(\frac{l_t}{p_{t+1}}\right) + (1-\lambda)(1-\alpha) u_d\left(\frac{\bar{\rho} l_t}{p_{t+1}}\right) \\ & + \lambda\alpha u_d\left(\frac{r(e_t) l_t}{\bar{\rho} p_{t+1}}\right) + (1-\lambda)\alpha u_d\left(\frac{r(e_t) l_t}{p_{t+1}}\right) \Bigg\}. \quad (21) \end{aligned}$$

We can use equation (21) to derive the first order condition of creditors' problem with respect to effort.

$$\Phi'(e_t) = \theta\alpha\lambda\frac{r'(e_t)l_t}{\bar{\rho}p_{t+1}}u_d'\left(\frac{r(e_t)l_t}{\bar{\rho}p_{t+1}}\right) + \theta\alpha(1-\lambda)\frac{r'(e_t)l_t}{p_{t+1}}u_d'\left(\frac{r(e_t)l_t}{p_{t+1}}\right). \tag{22}$$

Assume that $e_t = e^*$ and $l_t = l^*$. We can subtract equation (19) from equation (22) to get

$$\theta\alpha\lambda\frac{r'(e^*)l^*}{\bar{\rho}p_{t+1}}u_d'\left(\frac{r(e^*)l^*}{\bar{\rho}p_{t+1}}\right) = (1-\alpha)\theta\alpha(1-\lambda)\frac{r'(e^*)l^*}{p_{t+1}}u_d'\left(\frac{r(e^*)l^*}{p_{t+1}}\right). \tag{23}$$

Using the definition of $\bar{\rho}$, we can simplify this expression to $u_d'\left(\frac{r(e^*)l^*}{\bar{\rho}p_{t+1}}\right) = u_d'\left(\frac{r(e^*)l^*}{p_{t+1}}\right)$. Since this equality holds if and only if $\bar{\rho} = 1$, then it must be the case that the allocation under laissez faire is different from the benchmark allocation. Indeed, since $\bar{\rho} < 1$, then $r(e_t)l_t < r(e^*)l^*$ under laissez faire.

5.3 Liquidity Provision by the Central Bank

In this section we consider different ways in which the central bank can relax the liquidity constraint. First, we show that if the central bank can perfectly enforce the loans it makes, then it can achieve the benchmark allocation by making loans at an interest rate of zero. The more interesting case is when the central bank is unable to enforce loans perfectly, as in Freeman (1999). In this case, we compare a policy of providing liquidity at an interest rate fixed ex-ante with a policy under which a subset of agents can compete for central bank reserves.

5.3.1 Perfect Enforcement

If the central bank is able to enforce the financial transactions it makes perfectly, then it can achieve the benchmark allocation by lending at a net interest rate of zero. By perfect enforcement, we mean that the central bank is able to make loans only to late-leavers and can force these late-leavers to repay their debt before they leave the central island. Hence, with perfect enforcement the central bank does not need to worry about agents defaulting of their loans, regardless of the quality of the collateral they offer. By lending enough funds, the central bank can eliminate the liquidity constraint and, by lending at a net interest rate of zero, the central bank makes no profits from its lending activity. Since agents cannot default on their central bank loans, their incentive to exert effort is the same as under the benchmark allocation. Hence they choose $e_t = e^*$ and $l_t = l^*$.

5.3.2 Market Insensitive Policy

For the remainder of the paper, we assume that the central bank is not able to perfectly enforce financial transaction. With limited enforcement, agent will default on their debt if the collateral they have pledged is worth less than the debt. Similarly, agents will renege on repurchase agreements.

We define a market insensitive policy as one where the central bank sets a discount rate ρ^{CB} prior to the discount market opening. The central bank can therefore not condition its trades with creditors on any information that it learns during the discount market. The central bank implements the market insensitive lending policy by setting a discount rate ρ^{CB} at which it buys uncleared loans. To show that a market insensitive policy cannot achieve the benchmark allocation, we again assume that $e_t = e^*$ and $l_t = l^*$ and establish a contradiction. In that case, the central bank would need to set $\rho^{CB} = 1$, when there is no default shock, and $\rho^{CB} = 1 - \frac{(1-e^*)\eta}{(1-\lambda)}$ when there is a default shock. Since the price must be set ex-ante, the central bank cannot set the correct price in all cases. If it chooses $\rho^{CB} = 1$ and there is a liquidity shock, all agents will ask for a central bank loan and default on that loan. If the central bank sets $\rho^{CB} = 1 - \frac{(1-e^*)\eta}{(1-\lambda)}$ and there is no default shock, then the central bank does not loosen the liquidity constraint completely.

It is also worth noting that if agents have free access to the central bank, either agents choose no effort or they do not borrow from the central bank. The value of a diversified loan portfolio in a default situation and with no effort involved will be equal to

$$\rho^{no} = \frac{(1-\lambda)(1-\eta)}{\eta + (1-\lambda)(1-\eta)}. \tag{24}$$

If ρ^{CB} is less then ρ^{no} then it follows that no creditor will wish to transact with the central bank and the market insensitive policy will be inactive. If ρ^{CB} is greater then ρ^{no} then creditors with unsettled debt prefer to acquire money from the central bank rather than from other creditors since the central bank pays more than the expected return on the debt. Moreover, since the price offered by the central bank is independent of the effort chosen by creditors, all creditors prefer to exert no effort.

5.3.3 Market Sensitive Policy

In this section, we consider a different liquidity provision policy by the central bank. We assume that the central bank chooses a subset of creditors at random at the beginning of the settlement period. We call the creditors in that subset "dealers" and let S denote the size of the subset. Only dealers are allowed to obtain liquidity from the central bank. However, dealers can serve as intermediaries between other creditors and the central bank.

The subset of creditors who are designated dealers compete against each other when attempting to obtain liquidity. Assume for a moment that the equilibrium level of effort is e. In this case the dealers are willing to buy loans from the non-dealers at a discount equal to $\rho(e)$. They compete against each other on terms of price in a Bertrand competition.

The dealers can in turn offer these loans as collateral for loans of liquidity from the central bank at a rate ρ^{CB} which depends on the effort reports of the dealers. Therefore, banks make a profit per unit of loans equal to

$$r(e) + [\rho^{CB} - \rho(e)]. \tag{25}$$

where $r(e)$ is the return on loans based on the true effort. Since dealers observe effort they will not want to pay more then $\rho(e)$ for a given loan and since dealers are competing for loans any offer less then $\rho(e)$ will be bid up to $\rho(e)$ by the other dealers; at this price each dealer buys an equal share of loans from the non-dealers.

Too buy these loans they must simultaneously offer them for collateral with the central bank in a Bertrand price competition where the central bank makes loans at the highest interest rate (lowest discount rate). We show that the equilibrium discount rate for these central bank loans is $\rho(e)$; this can be seen from the per unit profit given in (25). At this equilibrium a given dealer can deviate by either raising or lowering the discount rate they offer the central bank. If the deviating dealer raises his bid above $\rho(e)$ then the central bank will not make any loans too them in preference to the other dealers. If the deviating dealer lowers his bid below $\rho(e)$ then he will receive all the liquidity he wants from the central bank but will make losses since the dealer is receiving an interest rate below that which he pays to the non-dealers.

We show that if the subset of dealers is sufficiently small, then there exists and equilibrium such that all creditors provide the level of effort $e_t = e^*$. To do that, we assume that all creditors choose effort $e_t = e^*$ and consider a creditor's incentive to deviate.

The zero profit condition above implies that dealers will offer to pay an interest rate on a loan at the central bank exactly equal to the expected value of the collateral they offer. Hence, the interest rate on central bank loans will be $\rho^{CB} = 1$ when there is no default shock and $\rho^{CB} = \frac{[1-(1-e^*)\eta](1-\lambda)}{(1-e^*)\eta+[1-(1-e^*)\eta](1-\lambda)}$ when there is a default shock.

If the deviating agent is chosen to be a dealer, he can borrow from the central bank at the rate ρ^{CB} despite having chosen a suboptimal effort level e'. The benefit from the deviation is then $\Phi(e^*) - \Phi(e')$. A deviation is costly if the creditor's loan is not settled early and the creditor is not chosen to be a dealer, which happens with probability $1 - S(1-\lambda)$. In that case, the creditor must find a dealer who will rediscount the note at the central bank. Since dealers can perfectly observe the quality of the loan, the deviating creditor obtains less consumption. Let $E[V(l', e')]$ denote the creditor's expected value function when effort e' is chosen.

We can write the payoff from deviating as

$$\Phi(e^*) - \Phi(e') - [1 - S(1-\lambda)] \{E[V(l^*, e^*)] - E[V(l', e')]\}.$$

Taking the derivative of this expression with respect to e' and setting it equal to zero yields $\Phi'(e') = [1 - S(1-\lambda)]\, E[V_2(l', e')]$.

This expression is similar to equation (14) with the added term $[1 - S(1-\lambda)]$. The creditor who considers deviating will not do so if $S = 0$. Hence, if $S \to 0$, then there is an equilibrium such that all creditors exert the efficient level of effort while otherwise this equilibrium does not exist.[8]

Restricting the fraction of agents who can trade with the central bank also limits the incentives to deviate in the case of the market insensitive policy. However, as we have already shown, the central bank is unable to consistently choose the right cost for liquidity in that case. Below, we argue that in special

[8]Note that if the effort took value on a discreet set with no converging subsequence, then we could find $\underline{S} > 0$ such that agents choose the efficient level of effort whenever $S < \underline{S}$.

situations where the discount window plays an important role, it is important that it be widely accessible.

The fact that there are creditors on both sides of the discount market is the key feature that allows the central bank to inject liquidity into the payment system while also providing the proper incentives for creditors. Suppose for example that instead of the market sensitive above the central bank provides liquidity by first observing a small sample (which we will denote as S) of market transactions and then setting a discount rate ρ^o, based on this sample, to transact with the rest of the market. A creditor knows, in this case, that if they are in the set S then they cannot misrepresent their effort, if they transact in the set $1 - S$ then they can misrepresent their loans by selling them to the central bank at ρ^o. Since presumably only observing a small subset is feasible, the risk to the creditors of misrepresenting their effort is small and correspondingly the effort used in monitoring loans will be small.

5.3.4 Discussion

As in Freeman (1999), the policy resembling open market operations in our model differ from typical central bank practice. Central banks usually repurchase treasury securities in open market operations rather than loans. Neither model is rich enough to capture all the institutional details but they can nevertheless provide interesting insights. As in Freeman (1999), we find that a market sensitive policy, resembling in some ways an open market operation, should be preferred by the central bank. The reasons are different, however, and our model also provides new conclusions such as the need to restrict access to the central bank in the case of market sensitive policies.

The market insensitive policy captures the principle features of the discount window in the US system or the upper and lower bands of the channel in the Canadian system. These prices are set periodically and are reviewed at a much lower frequency then the frequency of market interactions; without much loss of generality we consider them fixed at the beginning of time.

Similarly, the market sensitive policy captures some important features of open market operations, which are an important tool in the implementation of monetary policy in most industrialized countries. The central bank interacts with one set of banks when it conducts policy and these banks in turn interact with other banks in the interbank market. In the U.S. and Canada these banks are called primary dealers.

While in our model dealers are chosen randomly, the primary dealers that can bid for funds from the Federal Reserve and the members of the LVTS in Canada do not change frequently. In our model, agents cannot establish reputations with the central bank. For that reason, randomness is useful to mitigate moral hazard. In practice, primary dealers are well known institutions and the Federal Reserve or the Bank of Canada have other ways to mitigate moral hazard problems with these institutions. Nevertheless, the importance of restricting direct access to the central bank is due to similar frictions in each case. As an alternative, we could have assumed that the central bank can enforce loans it makes to a subset of banks, but that it cannot enforce the loans it makes to other banks. The former kind of bank could be chosen to be dealers. Under that assumption, dealers would not be chosen randomly.

6 A Role for the Discount Window

The preceding analysis implies that an open-market operation will dominate discount lending by relaxing a liquidity shock but not letting default risk be transferred from the creditors to the central bank. This then leads one to wonder under what conditions a discount window would be an optimal policy? In this section we argue that a discount window can be useful if the central bank makes mistakes in evaluating the amount of liquidity it needs to inject. Also, in rare situations the interbank market may not function properly. The event following the terrorist attacks of September 11, 2001, provide an example.

6.1 Random Liquidity Needs

The amount of liquidity needed in the interbank market can fluctuate for reasons that are out of the control of the central banks. These "autonomous factors," such as the funding needs of the Treasury, must be estimated and accommodated when the central bank does open market operations.[9] If the central bank makes a mistake and provides too little liquidity, the interest rate will increase, as in our model.

If a discount window is available, the interest rate will be capped by the discount window rate. Indeed, if the market rate were to exceed the discount window rate, banks should prefer to borrow from the discount window. If mistakes by the central bank are rare, the availability of the discount window will not affect incentives to monitor very much.

If a role for the discount window is to protect banks against these kind of random shocks, it is desirable that it be widely accessible. Otherwise, one could be concerned that the restricted set of banks who have access to the discount window would exploit their position.

One way to approximate such shocks in our model would be to assume that α is random. If α turns out to be smaller than the central bank expected, the interest rate will rise because the liquidity constraint (12) will bind. The discount window rate should be set high enough that creditors will not choose to access the discount window when there is a default shock but only when α is small.

[9] In the Canadian case these fiscal agency operations are done by the Bank of Canada, but the need for forecasting is essentially the same.

6.2 Market Collapse

The description by Lacker (2004) of the events on September 11th 2001 gives another example of when discount window lending plays an important role. In this case a large scale shock caused a temporary disruption in the inter-bank market which made trade difficult or impossible. Open-market operations where impossible due to a the lack of a functioning market. In this case the blunt instrument of discount window lending allowed the Federal Reserve to relax the effects this shock had on market participants.

In the context of our model we could think of a September 11th situation as one where α_t is an iid random variable with two outcomes. In one state it takes some finite value α and in the second state, which we term a market collapse, it takes the value of one. We further assume that the second state has some arbitrarily small probability of occurring and is transitory in nature.

If the market collapses, there are no late-leaving creditors, and there is no discount market since no one wants to buy the unsettled loans. The lack of a market then rules out the use of a market sensitive policy which requires a functioning discount market. In contrast, the discount window operates outside of the market and can be used to help relax the liquidity constraint.

Assume that the ex ante probability distribution for the liquidity parameter α_{t+1} is a random variable where with probability ϵ the market breakdown happens (α_{t+1} equals one) otherwise with probability $1-\epsilon$ the parameter α_{t+1} is some α. Taking ϵ to be arbitrarily small it follows that the first order conditions of the creditors will, in the limit, approach those of the model above. In this economy, setting a market insensitive rate less than the discount rate associated to the no monitoring case will make the policy inactive as described in section 5.3.2. In the case of an ex post market breakdown (α_{t+1} equals one) the discount window becomes the active tool of intervention and in this equilibrium the discount rate becomes ρ^{no} defined by the non-market collapse parameter

α. This policy then partially relaxes the liquidity constraint, and partially improves the risk-sharing for the creditors exposed to the market breakdown.[10]

In a market breakdown situation the results are quantitatively similar to the description of events by Lacker (2004) as well as the discussion by Williamson (2004). In a 9/11 situation discount lending will drastically increase to help the settlement of funds. One key difference between our model and that of Williamson (2004) is that the central bank in our model must ensure that an excessively loose discount window policy will not have incentive effects in a normally functioning economy which restricts the amount of liquidity provision in a market breakdown.

7 Conclusion

In this paper we extend the model of Freeman (1999) to include moral hazard. This is important because moral hazard is a major aspect of policy discussions concerning the provision of liquidity by central banks. When moral hazard is a concern, it is no longer optimal for the central bank to absorb losses associated with default, as suggested by Freeman.

We show how a policy resembling open market operations allows the central bank to provide liquidity without reducing creditors incentives to monitor. Restricting the fraction of creditors who have access to the central bank increases the probability that agents will have to obtain funds from other creditors. Since creditors are better informed about the quality of loans than is the central bank, this provides more incentive for creditors to make sure they hold loans of high quality. By letting creditors compete for funds, the central bank can extract information about the state of the economy to which creditors have access.

[10] These results depend on the fact that the market breakdown is transitory in nature; which implies that stationary equilibrium and central bank policy have not changed for future discount markets.

In addition this type of policy provides proper incentives to creditors to monitor their loans efficiently and hence reduce the aggregate default size when compared to a policy consisting only of a discount window policy.

We also argue that in particular situations it may be desirable for banks to have direct access to the central bank so that liquidity can be obtained without relying on the market. For example, after September 11, 2001, the interbank market could not operate because of physical destruction to telecommunication infrastructures. It was therefore important for banks to have direct access to the central bank.

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