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Heterogeneous Skills and Homogeneous Land: Segmentation
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Abstract

This paper analyzes the effect of skill heterogeneity on regional patterns of production and housing in the presence of pecuniary externalities within a general-equilibrium framework, assuming monopolistic competition in intermediate goods markets. It shows that the interplay of heterogeneous skills and comparatively homogeneous land demand triggers skill segmentation and agglomeration. The core region that is more attractive to high skilled workers has a disproportionately large share of production at all levels of the supply chain. The paper extensively discusses welfare increasing tax policies. This paper also briefly studies how trade in intermediate goods and endogenous land demand affect segmentation and agglomeration.

JEL Classification: R12, R13, R14, H22.

Keywords: Skill heterogeneity, land use, sorting, agglomeration.

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1 Introduction

At least since the pioneering work of Marshall (1890) it has been widely accepted that skills and agglomeration are positively related. Knowledge spillovers and the local availability of specific skills act as centripetal forces. Many empirical studies have confirmed that average productivity is increasing in local market size and population density.¹ The city size elasticity of productivity ranges from 3-8% (Rosenthal and Strange, 2004). Productivity in large cities is considerably higher than that found outside cities (Glaeser and Mare, 2001) and there is a substantial wage premium associated with the largest cities and metropolitan areas (see, for Asia and France, respectively, Fujita and Thisse, 2002; Combes et al., 2008). However, recent studies have shown that externalities and location advantages are not the only factors contributing to the urban wage premium; quality-selection processes also play a role (Lee, 2010; Fu and Ross, 2007; Combes et al., 2008; Baum-Snow and Pavan, 2012). Large cities attract more talented people (Berry and Glaeser, 2005; Bacolod et al., 2009; Lee, 2010). Although a disproportionately large fraction of people from the bottom of the skill distribution live in large metropolitan areas, average skill levels in these areas are higher than in small cities (Bacolod et al., 2009; Combes et al., 2012). Selection occurs because agglomeration economies are stronger for better-educated individuals (Wheeler, 2001; Glaeser and Resseger, 2010) and individuals with better cognitive and people skills (Bacolod et al., 2009). Furthermore, the skill level is higher in urban areas because human capital accumulates more quickly (Glaeser and Mare, 2001; Glaeser and Resseger, 2010). Urban work experience is especially valuable for white-collar workers (Gould, 2007) and in large cities (Baum-Snow and Pavan, 2012).

While the effect of firm heterogeneity on trade and foreign direct investment has been extensively studied,² heterogeneous workers have attracted less interest in the theoretical

¹There is a substantial number of empirical studies on the relationship between size/density and productivity/wages including Sveikauskas (1975), Ciccone and Hall (1996), Wheeler (2001), Wheaton and Lewis (2002), Syverson (2004), Wheeler (2006), Glaeser and Resseger (2010). For surveys, see Rosenthal and Strange (2004) and, more recently, Strange (2009).

²See, among others, Melitz (2003), Nocke (2006), Helpman et al. (2004), Baldwin and Okubo (2006), Bernard et al. (2007), Behrens and Robert-Nicoud (2008), Melitz and Ottaviano (2008), and Mion and Naticchioni (2009).

regional economics literature on migration and interregional trade despite the overwhelming evidence on spatial sorting of heterogeneous workers. A notable exception is Mori and Turrini (2005) who showed that within a standard new economic geography (NEG) framework skill heterogeneity causes spatial sorting and agglomeration if communication costs have skill-specific effects whereas trade integration also increases the degree of agglomeration. Theoretical work on skill heterogeneity needs to account for the full spectrum of market forces analyzed within the NEG framework. It is especially valuable to analyze the interplay between the scarcity of land, the sorting of individuals, and agglomeration at the regional level. Following von Thünen (1826), in the urban economics literature, the relationship between heterogeneity, transport costs, land demand and sorting has long been recognized. In a monocentric city model, income classes self-segregate. Depending on the income elasticity of land demand and the commuting costs, either high-income earners or low-income earners prefer to live closer to the city center (see, among others, Wheaton, 1976; Hartwick et al., 1976; Fujita, 1989). At the regional level, the effect of land scarcity on dispersion and agglomeration has been analyzed, but its relationship with sorting has not. Within an NEG framework, Helpman (1998) showed that using land for housing purposes may reverse the agglomeration trend over the course of trade integration. By specifying a model that combines congestion with the pecuniary externalities considered by Krugman (1991), Pflüger and Südekum (2008) found a hump-shaped location pattern that implies that declining transport costs first induce an agglomeration and then a redistribution of mobile agents. Recently, Pflüger and Tabuchi (2010) confirmed this result using a model that allows for land to be used for both production and housing.

This paper aims to provide a general-equilibrium-based theory of sorting and agglomeration on a regional scale that explains two stylized facts in particular, namely, the existence of agglomerations despite congestion and the positive correlation between size and skill levels, wages, and rents. Therefore, we compile a two-region general-equilibrium model in which land is used for both production and housing. To allow for the benefits of size, following Fujita and Hamaguchi (2001), we introduce input-output linkages between firms.³ The model includes economies of scale in the intermediate goods sector, benefits of variety

³Vertical input-output linkages between firms are considered a major explanation for the endogeneity of market size (see Venables, 1996; Krugman and Venables, 1995; Puga, 1999; Ottaviano and Thisse, 2004).

in intermediate products, and interregional trade costs for transporting intermediate goods that are prohibitively high in the baseline version of the model. Intermediate trade costs consist of transport costs, compliance costs associated with legal regulations, costs related to just-in-time production at the final goods level, and so forth. Competition at the intermediate goods level is assumed to be monopolistic.⁴ As in Mori and Turrini (2005), we include observable skill heterogeneity in the model by considering a continuum of workers with ex-ante different skills. Assuming perfect mobility, we analyze whether segmentation and agglomeration result from individual desires and market forces.

Our main results can be summarized as follows:

First, the interplay of observable skill heterogeneity and (almost) homogeneous land demand may trigger skill segmentation and agglomeration and, hence, an asymmetric equilibrium. Highly skilled individuals live in wealthier regions where the aggregate human capital and aggregate output are higher. Household mobility implies that wages and the prices of non-tradable goods, especially land, are positively correlated across regions. Facing the choice between a high-wage region with high land prices and a low-wage region with low land prices, highly skilled workers prefer the former and low-skilled workers the latter. This finding is unambiguous when individual demand for land is independent of income (and the underlying preference orderings are the same for all workers). However, except when the income elasticity of demand for land is too high, highly skilled individuals prefer the high-wage region. This finding confirms the empirical results on sorting noted above. Furthermore, we demonstrate that skill heterogeneity combined with market-size based agglomeration forces cause an agglomeration of economic activities despite competition for land. On the whole, our model contributes to a more complete explanation of the amazing persistence of core-periphery patterns despite congestion and various forms of equalization policies. In an appendix, we also show that the main properties of the equilib-

⁴Rather than modeling pecuniary externalities, we could have considered other centripetal forces related to human capital such as knowledge spillovers (for a comprehensive survey on urban agglomeration economies, see Duranton and Puga, 2004). However, there is some evidence that pecuniary externalities are a more prevalent source of agglomeration than knowledge spillovers (see Ellison et al., 2010). In addition, knowledge spillovers are strictly spatially bounded (see, e.g., Baldwin et al., 2008) presumably excluding larger regions as units of analysis (for a survey on human capital externalities, see Moretti, 2004).

rium do not change when a skill-independent attachment to home that induces imperfect sorting is introduced.

Second, as a result of pecuniary externalities, the laissez-faire equilibrium is typically inefficient. Using a simple non-linear tax characterized by residence-based tax rates, residence-based lump-sum transfers, and a land tax, a social planner could implement Pareto improvements. Furthermore, potential Pareto improvements could be achieved by even simpler policies, including non-linear wage taxes/subsidies. Because cities with high productivity, high wages, and high land rents bear more of the burden of labor income taxes, federal taxation redistributes toward peripheral regions. From the general-equilibrium trade model perspective, there is no rationale for redistribution to regions that are inefficient in the traded-good sector (see Albouy, 2009); in the presence of pecuniary externalities federal taxation may enhance total welfare by reducing over-agglomeration.

Third, the properties of the segmented equilibrium match further stylized facts highlighted by recent empirical studies. First, the core region wage premium is positively correlated with individual productivity, but individual ability does not fully explain the urban wage premium (Glaeser and Mare, 2001). Moreover, the input intensity is larger in the core than in the periphery. Following Holmes (1999), a larger value of purchased inputs as a percentage of the output value could be interpreted as indicating greater vertical disintegration. In our model, the ratio of input value to output value is constant, but quantity ratios differ across regions. Producers in the core use intermediate goods rather intensively. Furthermore, for the highly skilled, there is not only a nominal core wage premium but also a real urban wage premium. While Glaeser and Mare (2001) did not observe a real wage premium, Yankow (2006) – arguing that price indices based on a fixed bundle of consumption goods overstate the true cost of living – uncovers a real urban wage premium (for a recent overview, see Heuermann et al., 2010). Finally, consistent with recent findings (see Baum-Snow and Pavan, 2011), the model predicts a positive monotonic relationship between wage inequality and city size if the overall skill distribution is skewed to the right. Moreover, the upper parts of the distributions of skills look similar to the the distributions of worker effects shown by Combes et al. (2012) provided that there is attachment to home and that skills follow a truncated log-normal distribution.

Fourth, while a reduction in intermediate trade costs increases the number of varieties

available in both regions and reduces the production costs of final goods, it also reduces the skill advantage of the core and thus alleviates regional differences. However, neither segmentation nor agglomeration fully disappears, provided there are some, albeit small, trade costs. In contrast to the basic core-periphery model of Krugman (1991), but as in Helpman's (1998) model, trade integration weakens agglomeration.

Our model is in the spirit of Roback (1982) because wage differentials compensate workers for differences in land rents, but in contrast to their model, the labor force is heterogeneous. In the segmented equilibrium, only the critical type is indifferent between both locations, and more and less productive workers each strictly prefer one region. There are no amenities in the basic model but attachment to home – analyzed as an extension in an appendix – could be considered a type specific amenity. Attachment to home is a strong dispersion force (see Tabuchi and Thisse, 2002), as a generally decreasing willingness to leave the home region for higher pecuniary utility alleviates agglomeration by limiting the impact of a regional wage gap on migration flows.

The model is related to Black (1999), who specified up a multi-region model with knowledge spillovers embedded in the production function, two skill levels, and land use for housing. The fundamental force triggering segregation in his model is the same as in our model. Higher wages in core regions will only offset higher living costs for the highly skilled, not for the low-skilled. The segregation process in his model also requires that average productivity is increasing along with an increase in average skills. However, his model differs from our model in important aspects. First, agglomeration externalities are not the result of market forces, they are simply imposed. Second, there are only two skill levels, and the number of regions is endogenous (and larger than two). Assuming an endogenous number of regions, is useful for a very long-run analysis of cities, but is less suitable for the analysis of countries. Third, in Black's model, there is a continuum of free-mobility equilibria, making predictions and policy analysis difficult. Accordingly, interregional redistribution policies may be allocatively neutral.

Mori and Turrini (2005) also established segmentation and agglomeration in a regional model with skill heterogeneity,⁵ but in their model, where differentiated worker-sellers

⁵Abdel-Rahman (1998) also identifies segmentation in a two-skill-type, two-sector model with commuting where public infrastructure is the basic agglomeration force.

produce consumer goods, land is absent. Uniform communication costs associated with interregional trade that fall more heavily on low-skill suppliers are the driving force behind segmentation. In their model, the most skilled also live in wealthier locations in which aggregate human capital is higher. In contrast to the result presented in this paper, compared with the periphery, the cost of living and local individual demand are lower rather than higher in the agglomeration. The cost of living is not determined by the scarcity of land but by the set of available varieties. Nevertheless, the more skilled also sort into the agglomeration, because they face lower trade costs and, therefore, depend less upon local markets. Both Mori and Turrini's (2005) mechanism and the mechanism introduced in this paper induce sorting and agglomeration, but the former is more related to Krugman (1991) while the latter is more in the spirit of von Thünen (1826) and Helpman (1998). While there is overwhelming evidence that land prices and house prices are higher in more densely populated areas (see, among others, Roback, 1982; Tabuchi, 2001), evidence on the effect of city size on the total cost of living and the cost of living minus land use prices is less clear. While several studies identified a positive correlation between city size and cost of living (DuMond et al., 1999; Tabuchi, 2001), more recently, using a large micro data set and controlling for store and shopping effects, Handbury and Weinstein (2011) found lower variety adjusted prices in large cities. Most likely, whether the total cost of living decrease or increase with size depends on the amount of trade costs at the final goods level, which are not modeled in this paper. For high trade costs, the model of Mori and Turrini (2005) is a powerful explanation of the effects of skill heterogeneity; for low trade costs, the model in this paper appears to be a useful framework.

Using a similar model of input-output linkages and urban costs, very recently Behrens et al. (2010) have shown that large cities are more productive than small cities because of sorting, selection, and agglomeration externalities. Although the model of Behrens et al. (2010) is richer insofar as it also considers occupational choice, land use for production is considered in this paper, but not in their model. When land use for production is taken into consideration, population has two opposing effects on wages. On the one hand, an increase in population size – holding the skill distribution constant – increases productivity via a larger set of available intermediate goods; on the other hand, population growth increases land prices and, thus, reduces land use for production and, due to the complementarity

of land and labor, productivity. However, this paper shows that, although land use for production is a strong dispersion force (see also Pflüger and Tabuchi, 2010), sorting, in combination with agglomeration externalities, induces agglomeration.

The paper is organized as follows. The next section develops the basic economic model and discusses existence, stability, and optimality of short- and long-run equilibria. Section 3 extends and modifies the basic model so as to study the effects on segmentation and agglomeration of trade in intermediate goods and endogenous land demand. Section 4 concludes the paper.

2 The basic model

We consider a country comprised of two ex-ante identical regions, $i = C, P$. Each region is endowed with L square miles of land. The total mass of individuals living in the country is denoted N , $N > 0$. Each person is assumed to own an identical share of land and total land rents, R , are assumed to be equally distributed among all citizens. Each person inelastically supplies one unit of labor in their region of residence. However, workers are heterogeneous in skills. In contrast to Mori and Turrini (2005), skills are purely quantitative, essentially capturing productivity. The effective labor supply of a worker of type s is simply s , and gross wage income is sw_i , $i = C, P$, where w_i is the wage rate for a unit of normalized labor. The shift parameter s could be considered the inverse of labor requirements. Skills are distributed according to a continuous distribution function $F(s)$ with an associated density function $f(s)$ and support $[\underline{s}, \bar{s}]$, where $\bar{s} > \underline{s} > 0$. Hence, the total stock of skills-weighted labor, termed human capital, S , is as follows

$$S = N \int_{\underline{s}}^{\bar{s}} sf(s)ds; \quad (1)$$

the total stock of skills-weighted labor in region i is denoted S_i , with $S_i \geq 0$ and $S_C + S_P = S$. The total population in region i is denoted N_i , where $N_i \geq 0$ and $N_C + N_P = N$.

Land in each region is assumed to be as a straight line, with one unit of land at each location. Production takes place in the central business district (CBD) near the center, whereas workers – living outside the CBD – commute to the CBD. Because we assume that workers inelastically consume one unit of land and that there is a single final good with a

price normalized at unity, the indirect utility of workers of type s in region i can be written in terms of consumption, that is, as income minus land use expenses and commuting costs:

$$U_i(s) = w_i s + \frac{R}{N} - \tilde{r}_i, \quad i = C, P, \quad (2)$$

where \tilde{r}_i indicates land use expenses including commuting costs.

Workers are perfectly mobile. They maximize utility through their residential decisions. However, we begin our analysis by considering the short-run equilibrium for a given inter-regional allocation of workers and then, in a second step, analyze the long-run equilibrium where mobility across regions is fully taken into consideration.

2.1 Short-run equilibrium

There are two types of goods, a final good, X_i , and a continuum of differentiated intermediate goods, $q_i(j)$. The final good is produced using human capital, S_{xi} , land, L_{xi} , and the intermediate aggregate, I_i . Following Ethier (1982), Abdel-Rahman and Fujita (1990), and Fujita and Hamaguchi (2001), the production function in region i , $i = C, P$, is

$$X_i = S_{xi}^\eta I_i^\mu L_{xi}^{1-\mu-\eta}, \quad \text{where } I_i = \left[\int_0^{n_i} q_i(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}. \quad (3)$$

The substitution elasticity of intermediate goods is indicated by σ , with $\sigma > 1$. The number of varieties, n_i , human capital, S_{xi} , and land use, L_{xi} , are endogenously determined. Because the production function exhibits constant returns to scale, the number of firms and output per firm are indeterminate. Without loss of generality, we proceed as if the total output in every region is produced by a single representative firm that behaves competitively. The final good is intra- and inter-regionally traded at no cost; its price is normalized at unity. In the basic model, intermediate goods are non-tradable; their prices may vary by variety and region. Furthermore, we let r_i , the price of land, $p_i(j)$, the price of the intermediate good j , and P_i , the CES-price index of intermediate goods, with

$$P_i = \left[\int_0^{n_i} p_i(j)^{-(\sigma-1)} dj \right]^{-\frac{1}{\sigma-1}}, \quad i = C, P. \quad (4)$$

The representative firm selects its inputs to maximize profits, $\Pi_i = X_i - w_i S_{xi} - r_i L_{xi} - P_i I_i$. As a result, the demand functions for human capital, land, and intermediate goods in region

$i, i = C, P$, are

$$\begin{aligned} S_{xi} &= X_i \frac{r_i^{1-\mu-\eta} P_i^\mu}{w_i^{1-\eta}} \left(\frac{\eta}{1-\mu-\eta} \right)^{1-\mu-\eta} \left(\frac{\eta}{\mu} \right)^\mu, \\ L_{xi} &= X_i \frac{w_i^\eta P_i^\mu}{r_i^{\mu+\eta}} \left(\frac{1-\mu-\eta}{\eta} \right)^\eta \left(\frac{1-\mu-\eta}{\mu} \right)^\mu, \\ q_i(j) &= X_i \frac{r_i^{1-\mu-\eta} w_i^\eta}{P_i^{1-\mu}} \left(\frac{\mu}{1-\mu-\eta} \right)^{1-\mu-\eta} \left(\frac{\mu}{\eta} \right)^\eta \left(\frac{P_i^\sigma}{p_i^\sigma} \right). \end{aligned} \quad (5)$$

Hence, per-unit costs are

$$c_i = \psi w_i^\eta P_i^\mu r_i^{1-\mu-\eta}, \quad i = C, P, \quad (6)$$

with $\psi = \mu^{-\mu} \eta^{-\eta} (1-\mu-\eta)^{-(1-\mu-\eta)}$. At equilibrium, per-unit costs are equal to the price of the final good. Hence, an interior final goods market equilibrium requires identical per-unit costs in both regions:

$$c_C = c_P = 1. \quad (7)$$

Using Equation (7), at the final goods market equilibrium, demand functions given by Equations (5) can be written as

$$\begin{aligned} S_{xi} &= \eta \frac{X_i}{w_i}, \quad L_{xi} = (1-\mu-\eta) \frac{X_i}{r_i}, \\ q_i(j) &= \mu \frac{X_i P_i^{\sigma-1}}{p_i^\sigma(j)}, \quad \text{implying } I_i = \mu \frac{X_i}{P_i}. \end{aligned} \quad (8)$$

The ratio of the value of inputs to the value of output is constant: $P_i I_i / X_i = \mu$. However, the lower is the price index P_i , the larger the input intensity $I_i / X_i = \mu / P_i$ is.

Each intermediate good is produced using only human capital.⁶ For $q_i(j)$ units of intermediate goods, $S_{qi}(j) = A + a q_i(j)$ units of human capital are required. The parameter A represents the fixed labor requirement, a the per-unit variable labor requirement. If firms can differentiate their goods without cost, at equilibrium each good will be produced by only one firm. Because there is a continuum of varieties, the intermediate sector is

⁶A natural but fairly complex extension would be to include land use in intermediate goods production. If each firm uses one unit of land, the zero-profit condition would require a higher production level in response to an increase in local land prices implying fewer varieties. However, a higher wage would have the opposite effect. To predict the effect on the equilibrium, a full analysis of the systemic repercussions would be necessary, which would require a somewhat simpler baseline model.

characterized by monopolistic competition à la Dixit and Stiglitz (1977), where all firms act as if their behavior does not affect the price level. As a consequence, each firm maximizes its profits

$$\pi_i(j) = p_i(j)q_i(j) - w_i [A + aq_i(j)], \quad i = C, P, \quad (9)$$

based on the final goods sector's demand function, Equation (8), by mark-up pricing, where the mark up is the same for all intermediate goods suppliers:

$$p_i := p_i(j) = \left(\frac{\sigma}{\sigma - 1} \right) w_i a, \quad i = C, P. \quad (10)$$

Furthermore, free market entry enforces zero profits, which, along with the mark-up price policy, implies that all intermediate goods suppliers produce the same quantity:

$$q_i(j) = q := \frac{(\sigma - 1)A}{a}. \quad (11)$$

The higher the substitution elasticity is, the lower the price is, but the greater is the quantity. An increase in fixed costs and a decrease in variable costs raise the equilibrium quantity.

Taking the final goods sector's demand, Equation (8), and the intermediate goods sector's zero-profit condition, Equation (11), into account, the intermediate goods market equilibrium determines – depending on the number of varieties – the regional final good production:

$$X_i = \frac{\sigma A w_i n_i}{\mu}, \quad i = C, P. \quad (12)$$

Higher wages, higher fixed costs, higher substitutability, and more varieties increase the regional final goods production.

The number of varieties in a region is related to the stock of human capital available in the region, S_i . A labor market equilibrium requires

$$S_{x_i} + \int_0^{n_i} [A + aq_i(j)] dj = S_i, \quad i = C, P. \quad (13)$$

Using the labor-market-equilibrium condition, Equation (13), the demand functions of the final goods sector, Equation (8), the intermediate goods sector's zero-profit condition, Equation (11), and the formula for regional production, Equation (12), the number of

varieties can be written as

$$n_i = \frac{S_i}{\sigma A \left(1 + \frac{\eta}{\mu}\right)}, \quad i = C, P. \quad (14)$$

The number of varieties is proportional to the stock of human capital. Sorting may only increase productivity through its effects on aggregate skills and the range of varieties. Because all intermediate firms are homogeneous, the quality of the matching of skills and firms is not an issue.

Production occurs in the CBD near the center, and workers live to both the right and the left of the CBD. The land market is perfectly competitive. Because the intermediate sector does not use land, the size of the CBD is calculated using the final good sector's demand for land: $2\hat{\rho}_i = (1 - \mu - \eta)X_i/r_i$, where $\hat{\rho}_i$ indicates the border of the CBD in region i . For simplicity, we omit the transportation of goods and people within the CBD. Commuting costs per mile, k , with $\bar{k} > k > 0$, are constant. \bar{k} is a finite number that is sufficiently small to ensure that commuting does not exhaust the society's resources. The rent in the CBD is r_i , beginning at $\hat{\rho}_i$, it declines monotonically towards the edge of the inhabited area, $\bar{\rho}_i$. Hence, the land rent schedule in the city can be written as a function of the distance ρ from the center:

$$r_i(\rho) = \min\{r_i - k(\rho - \hat{\rho}_i), r_i\}, \quad i = C, P. \quad (15)$$

Because workers are perfectly mobile within the region, each worker pays for land use plus commuting in total $\tilde{r}_i = r_i$. We assume that each region is large enough that the land rent at the edge of the inhabited area is equal to the exogenously given opportunity cost of land, $r_A = 0$: $r_i - k(\bar{\rho}_i - \hat{\rho}_i) = 0$. Furthermore, because everyone must live somewhere, the inhabited area matches the population: $2(\bar{\rho}_i - \hat{\rho}_i) = N_i$. Hence, the land rent in the CBD is

$$r_i = \frac{kN_i}{2}, \quad i = C, P. \quad (16)$$

As land is abundant but commuting is costly, population size determines the price of land that the producer encounters. Taking both the final goods sector's demand for land and the land rent in the CBD into account, the aggregate land rent can be determined:

$$R = 2 \sum_{i=C,P} \left[r_i \hat{\rho}_i + \frac{r_i(\bar{\rho}_i - \hat{\rho}_i)}{2} \right] = (1 - \mu - \eta)(X_C + X_P) + \frac{k(N_C^2 + N_P^2)}{4}. \quad (17)$$

Using the definitions of the intermediate goods price index, Equation (4), the equilibrium price, Equation (10), the land rent, Equation (16), and the equilibrium number of varieties, Equation (14), the international final goods market equilibrium condition, Equation (7), can be written as

$$1 = w_i^{\mu+\eta} (kN_i)^{1-\mu-\eta} \kappa S_i^{\frac{\mu}{1-\sigma}}, \quad i = C, P, \quad (18)$$

where

$$\kappa = \left(\frac{1}{2}\right)^{1-\mu-\eta} \psi \left(\frac{a\sigma}{\sigma-1}\right)^\mu \left[A\sigma \left(1 + \frac{\eta}{\mu}\right)\right]^{\frac{\mu}{\sigma-1}}. \quad (19)$$

Hence, equilibrium wages can be written as

$$w_i = \left[(kN_i)^{1-\mu-\eta} \kappa S_i^{\frac{\mu}{1-\sigma}}\right]^{-\frac{1}{\mu+\eta}}, \quad i = C, P. \quad (20)$$

Ceteris paribus, the regional wage increases as skills increase or population declines. On the one hand, this is due to constant returns to scale at the final-goods stage. On the other hand, this is because population and land rents are positively correlated, but skills and the intermediate goods price index are negatively correlated. Lower wages compensate for higher land rents and a higher intermediate goods price index.

We can now define the *short-run equilibrium*, for any given interregional allocation of workers and skills, as a set of prices, namely, wages, final goods prices (normalized to unity), intermediate goods prices, and land-rent schedules, such that intermediate goods suppliers and final goods suppliers maximize profits and workers maximize utility through consumption and the choice of a residence within the region. Additionally, supply is equal to demand in all markets: land markets, labor markets, intermediate goods markets, and the final goods market. Hence, the short-run equilibrium is determined by equations (10), (16), (15), and (20). Together with the regional final goods production, Equation (12), these equations also determine aggregate land rent and, therefore, worker utility. A feasible interregional allocation of population and skills, (N_C, S_C) is defined as

$$N \int_{\underline{s}}^{s_1} s f(s) ds \leq S_C \leq N \int_{s_2}^{\bar{s}} s f(s) ds,$$

where s_1 and s_2 are determined by

$$\int_{\underline{s}}^{s_1} f(s) ds = \int_{s_2}^{\bar{s}} f(s) ds = N_C/N.$$

The upper bound of aggregate skills in the core region is given by the level of skills that is achieved if the citizens of the two regions belong to different connected subsets of the set of skills, that is, the maximum skill level in the periphery region does not exceed the minimum skill level in the core. The lower bound is defined accordingly.

Proposition 1 *For any feasible interregional allocation of population and skills, there exists a commuting-cost-upper bound \bar{k} such that a unique short-run equilibrium exists.*

Proof. (i) $N_i = 0$ implies $S_i = 0$ and vice versa. For $N_i = S_i = 0$ there is no production and no land use in region i . Because labor is an essential input, firms cannot enter. (ii) For $N_i > 0$ and $S_i > 0$, w_i, r_i, n_i, X_i , and p_i are strictly positive. Labor and goods markets are in equilibrium. (iii) Using equations (12), (14), (17), and (20), \bar{k} is calculated, such that \bar{k} ensures $w_i s + R/N - r_i \geq 0$ for all s and i . For any given feasible allocation of population and skills with $S_C, S_P, N_C, N_P > 0$, w_i and X_i go to infinity if k goes to zero. Hence, there exists a small k such that $U_i(s) > 0$ for all s and $i = C, P$. As a consequence, for a sufficiently small \bar{k} , there is a land market equilibrium where all individuals have a residence. (iv) Because N_i and S_i uniquely determine $w_i, r_i, n_i, X_i, p_i, P_i$, for $i = C, P$, and R and $U_i(s)$, the short-run equilibrium is unique. \square

2.2 Long-run equilibrium: existence and stability

In the long-run, workers are perfectly mobile across borders and choose in which region to live so as to maximize utility.⁷ A *long-run equilibrium* is a short-run equilibrium with the additional property that no worker can increase utility by migrating to the other region, taking into account that individual agents are unable to change wages, prices, and rents. In the long-run equilibrium, for all types $s \in [\underline{s}, \bar{s}]$, it holds that $U_i(s) \geq U_j(s)$, $j \neq i$, if the worker with skill level s lives in region i . The long-run equilibrium utility is $U(s) = \max\{U_C(s), U_P(s)\}$.

⁷It is assumed that there are no mobility costs. In particular, skills and mobility costs are not considered to be negatively correlated. Essentially, this assumption is made to obtain a tractable model. However, in the section on attachment to home (see appendix), some kind of immobility will be introduced.

Obviously, a *symmetric long-run equilibrium* always exists. If the number of workers and aggregate skills are the same in both regions, i.e., if $N_C = N_P$ and $S_C = S_P$, wages, prices, rents, and, therefore, utility for every type of worker are the same in both regions. All markets are in equilibrium, and there is no incentive to migrate. A special case is the *perfectly* symmetric long-run equilibrium where half of each type of workers live in region C and the other half in region P . Regions would be identical not only ex-ante, but also ex-post.

In addition to the symmetric equilibria, asymmetric equilibria may exist. Of particular interest are equilibria with perfect sorting. In a *segmented long-run equilibrium*, all high-skilled workers live in one region and all low-skilled workers in the other. More precisely, a segmented long-run equilibrium is a long-run equilibrium where a critical level \hat{s} , with $\underline{s} \leq \hat{s} \leq \bar{s}$, exists such that all workers of type s with $\bar{s} \geq s > \hat{s}$ and no worker of type s with $\underline{s} \leq s < \hat{s}$ lives in region i , i.e., $U_i(s) > U_j(s)$ for all $s > \hat{s}$ and $U_i(s) < U_j(s)$ for all $s < \hat{s}$, with $j \neq i$. Without loss of generality, we focus on those segmented equilibria where $i = C$ and $j = P$: skilled workers live in region C , labeled the core, and unskilled workers live in region P , called the periphery. Hence, with perfect sorting

$$S_C = N \int_{\hat{s}}^{\bar{s}} s f(s) ds, \quad S_P = S - S_C, \quad N_C = N \int_{\hat{s}}^{\bar{s}} f(s) ds, \quad \text{and} \quad N_P = N - N_C. \quad (21)$$

With partial agglomeration, where $\underline{s} < \hat{s} < \bar{s}$, a segmented long-run equilibrium is characterized by \hat{s} such that

$$U_C(\hat{s}) = U_P(\hat{s}). \quad (22)$$

Segmentation occurs because variation in land expenses is the same for all workers, but labor income differences depend on skills. $U_C(s) - U_P(s)$ is increasing in s if and only if $w_C > w_P$. Provided that wages in the core exceed wages in the periphery and rents in the core are sufficiently high, it is obvious that no worker has an incentive to migrate. High-skilled workers are better off in the core, and low-skilled workers prefer to live in the periphery.

The properties of a segmented long-run equilibrium with partial agglomeration can be described in detail. To induce perfect sorting, wages in the core must be higher than in the periphery: $w_P < w_C$. To compensate workers in the periphery for lower wages, land

prices in the periphery's CBD also need to be lower: $r_P < r_C$. Taking the relationship between population and land prices, i.e., Equation (16), into account, lower land prices imply that the periphery is not only poor in skills but also less densely populated than the core: $N_P < N_C$. Because the average worker in the periphery has less human capital than the average worker in the core, the periphery's smaller population size leads to less aggregate skills, i.e., $S_P < S_C$.⁸ Because the size of the intermediate goods market is chiefly determined by labor supply, equation (14) makes it evident that lower aggregate human capital results in a smaller variety of intermediate goods: $n_P < n_C$. Along with lower wages, the smaller variety of intermediate goods according to Equation (12) leads to a smaller final good sector in the periphery: $X_P < X_C$. This is true, even though, due to the mark-up pricing rule (10), intermediate goods prices are comparatively low in the periphery: $p_P < p_C$. According to Equation (6), per-unit costs are the same in both regions because a larger set of varieties compensate the final-good producer in the core for higher wages, higher land rents, and higher intermediate goods prices. As a consequence, $P_C < P_P$. The scarcity of land and heterogeneity of workers not only lead to segmentation, but also to agglomeration. The core is more productive and larger. However, because land in the periphery is useful for final good production and housing, agglomeration is only partial. Finally, in the more densely populated region, the range of wages is larger than in the periphery. For a right-skewed overall skill distribution, the coefficient of variation is also higher. Income inequality is positively correlated with size (see Baum-Snow and Pavan, 2011).

With full agglomeration, the utility in the core is higher than in the periphery for all workers: $U_C(s) \geq U_P(s)$, for all s . This requires that the wage in the periphery has an upper bound if the population size goes to zero. From the wage equation (20), it is obvious that the wage in region $i = C, P$ adjusts within a certain range:

$$\begin{aligned} & \left(\kappa k^{1-\eta-\mu} \underline{s}^{\frac{\mu}{1-\sigma}} \right)^{-\frac{1}{\mu+\eta}} N_i^{-\left(1-\mu-\eta+\frac{\mu}{1-\sigma}\right)\left(\frac{1}{\mu+\eta}\right)} \\ & \leq w_i \leq \left(\kappa k^{1-\eta-\mu} \bar{s}^{\frac{\mu}{1-\sigma}} \right)^{-\frac{1}{\mu+\eta}} N_i^{-\left(1-\mu-\eta+\frac{\mu}{1-\sigma}\right)\left(\frac{1}{\mu+\eta}\right)}. \end{aligned} \quad (23)$$

⁸That the core region is larger in terms of both population and aggregate skills, is essentially a result of the land-market model. Comparatively small modifications of the land-market component of the model may allow for a core with a smaller population.

Hence, to ensure that wages do not go to infinity if the population size goes to zero, $1 - \eta - \mu \leq \frac{\mu}{\sigma-1}$ has to hold. This condition requires a low partial output elasticity of land, a high partial output elasticity of the intermediate composite good, and a low elasticity of substitution of intermediate varieties. Provided that wages are zero in an unsettled region in which rents are automatically zero, full agglomeration is an equilibrium if even the least skilled does not benefit from migration to the periphery, i.e., $U_C(\underline{s}) - U_P(\underline{s}) = w_C \underline{s} - kN/2 \geq 0$. Hence, together with the wage equation (20), two existence conditions can be derived (see Proposition 2). On the one hand, to guarantee that wages do not explode in the depopulated periphery, the substitution elasticity has to be sufficiently low. On the other hand, to ensure that even the least skilled citizen of the core would not prefer to live from rent income alone in the periphery, commuting costs per mile must be sufficiently low. The lower bound for commuting costs is increasing in the average skill level.

Interestingly, asymmetric equilibria without perfect sorting do not exist (see Proposition 2). Whenever $w_C \neq w_P$, the utility differential $U_C(s) - U_P(s)$ varies consistently with s . $U'_C(s) - U'_P(s) > 0$ ensures sorting.

In addition to the existence of equilibria, stability is a major issue in spatial equilibrium analysis. Following Mori and Turrini (2005), stability can be analyzed assuming a myopic and smooth adjustment process. To analyze stability, a region-specific density function $f_i(s)$ is defined, where $f(s) \equiv f_C(s) + f_P(s)$. A simple, smooth adjustment process is considered where the direction and speed of adjustment are determined by the utility differential and the distribution of workers:⁹

$$\dot{f}_C(s) = -\dot{f}_P(s) = \Phi [U_C(s) - U_P(s), f_C(s), f_P(s)], \quad (24)$$

where $\Phi = 0$ if (a) $U_C(s) \geq U_P(s)$ and $f_P(s) = 0$, and (b) $U_C(s) \leq U_P(s)$ and $f_C(s) = 0$. Furthermore, Φ is weakly increasing in $U_C(s) - U_P(s)$ and strictly positive if and only if $U_C(s) > U_P(s)$, $f_C(s) \geq 0$, and $f_P(s) > 0$. Finally, we assume that the response of migration flows to utility differences is sufficiently strong such that $d\dot{S}_C/dS_C \geq 0$, $d\dot{N}_C/dS_C \geq 0$, $d\dot{S}_C/dN_C \leq 0$, and $d\dot{N}_C/dN_C \leq 0$, where $\dot{N}_C = N \int_{\underline{s}}^{\bar{s}} \dot{f}_C(s) ds$ and $\dot{S}_C = N \int_{\underline{s}}^{\bar{s}} s \dot{f}_C(s) ds$.

⁹The dot indicates the time derivative.

boundary, the utility differential varies smoothly according to

$$\begin{aligned} \frac{d(U_C(\hat{s}(N_C)) - U_P(\hat{s}(N_C)))}{dN_C} &= \hat{s} \left(\frac{dw_C}{dS_C} \frac{dS_C}{dN_C} + \frac{dw_C}{dN_C} + \frac{dw_P}{dS_P} \frac{dS_P}{dN_P} + \frac{dw_P}{dN_P} \right) \\ &\quad + (w_C - w_P) \frac{d\hat{s}}{dN_C} - k, \end{aligned} \quad (25)$$

where $dS_C/dN_C = dS_P/dN_P = \hat{s}$. The curve DEB , where the wage differential is zero, i.e., $w_C = w_P$, must cross the symmetric equilibrium E and is unambiguously upward sloping because

$$\left. \frac{dS_C}{dN_C} \right|_{w_C=w_P} = - \frac{\frac{dw_C}{dN_C} + \frac{dw_P}{dN_P}}{\frac{dw_C}{dS_C} + \frac{dw_P}{dS_P}} > 0. \quad (26)$$

At segment EA , where $N_C = N_P$ and $S_C > S_P$, for all workers, wage and utility are higher in region C than in region P . If the same is true at F , a segmented equilibrium with full agglomeration exists that is by definition stable. If, for some workers, the utility differential $U_C - U_P$ is negative at F , then along the upper boundary the curve $U_C(\hat{s}(N_C)) - U_P(\hat{s}(N_C))$ crosses the zero line somewhere between A , where $N_C = N/2$, and F , where $N_C = N$, at least once. Hence, a segmented long-run equilibrium with partial agglomeration exists. For at least one equilibrium, $U_C(\hat{s}(N_C)) - U_P(\hat{s}(N_C))$ is declining in N_C , a necessary precondition for stability. Therefore, at least one of these equilibria is stable (see Proposition 2).

Using Figure 1, it is also possible to provide a description of the dynamics. In area $CEBF$ (in Figure 1.b: CEB), where $w_C \leq w_P$ and $N_C \geq N_P$, every individual prefers to live in region P (at E only weakly): $\dot{N}_C < 0$ and $\dot{S}_C < 0$. Similarly, in $AEDG$ (in Figure 1.b: AED), $\dot{N}_C > 0$ and $\dot{S}_C > 0$. At line segment EA above E and, by continuity, in small neighborhoods of this line segment, inhabitants of region P intend to migrate to region C : $\dot{N}_C > 0$ and $\dot{S}_C > 0$. Similarly, at EC below E , $\dot{N}_C < 0$ and $\dot{S}_C < 0$. Furthermore, if $w_C > w_P$, the higher skilled have a stronger preference for region C than the lower skilled. Hence, on the one hand, $\dot{S}_C < 0$ implies $\dot{N}_C < 0$; on the other hand, $\dot{N}_C > 0$ implies $\dot{S}_C > 0$. Similarly, if $w_C < w_P$, $\dot{S}_C > 0$ implies $\dot{N}_C > 0$ and $\dot{N}_C < 0$ implies $\dot{S}_C < 0$.

Heavily relying on the smoothness of the utility differential and the adjustment process, Proposition 2 makes conclusive statements on the existence, the stability, and the properties of the long-run equilibria:

Proposition 2 (i) *A symmetric long-run equilibrium always exists and is unstable.*

(ii) *At least one stable, segmented long-run equilibrium with either partial or full agglomeration exists.*

(iii) *A full-agglomeration long-run equilibrium exists if and only if*

$$\sigma \leq \frac{1 - \eta}{1 - \eta - \mu} \quad \text{and} \quad (27)$$

$$k \leq \frac{(2\underline{s})^{\eta+\mu} S^{\frac{\mu}{\sigma-1}}}{\kappa N}. \quad (28)$$

(iv) *An asymmetric equilibrium without perfect sorting never exists.*

(v) *A cycle does not exist.*

(vi) *If a segmented long-run equilibrium with partial agglomeration exists, wages, land rents, and intermediate goods prices are higher in the core than in the periphery. The core has higher aggregate skills, population, intermediate goods production, and final goods production than the peripheral region.*

Proof. (i/ii/iii) On existence of equilibria, see above.

Regarding the stability of the segmented long-run equilibrium with partial agglomeration H and the smallest population size possible (see Figure 1.a): while at EA and AH $\dot{S}_C > 0$ and $\dot{N}_C > 0$ hold, at EB , $\dot{S}_C < 0$ and $\dot{N}_C < 0$. Because $d\dot{S}_C/dN_C \leq 0$, there exists a path from E to H (called Ω), where $\dot{S}_C = 0$ and $\dot{N}_C < 0$ ($\dot{S}_C = 0$ and $\dot{N}_C > 0$ is impossible because $w_C > w_P$; $\dot{S}_C = \dot{N}_C = 0$ without perfect sorting is also impossible, see below). Hence, all paths beginning in the area between segments EA/AH and path Ω , where $S_C > 0$, eventually end at equilibrium H . Furthermore, paths beginning in an arbitrarily small neighborhood to the right of H and path Ω must lead to H or path Ω . Hence, E is (locally) unstable and H is (locally) stable.

Regarding the stability of a segmented long-run equilibrium with full agglomeration F (see Figure 1.b): the equilibrium is by definition stable if $U_C(\underline{s}) - U_P(\underline{s}) > 0$ holds for $N_C = N$. However, for the same reason as above, if $U_C(\underline{s}) = U_P(\underline{s})$ and $d[U_C(\hat{s}) - U_P(\hat{s})]/d\hat{s} < 0$ at F , a stable segmented equilibrium with partial agglomeration exists.

(iv) (a) If $N_C = N_P$, $r_C = r_P$ holds. If $w_C = w_P$, Equation (20) implies $S_C = S_P$. The equilibrium would be symmetric. If $w_C \neq w_P$, $U'_C(s) - U'_P(s) \neq 0$ holds and perfect sorting would occur. (b) If $N_C > N_P$, $r_C > r_P$ holds. $w_C \leq w_P$ would imply $U_P > U_C$

for all s which implies $N_C = 0$ and, therefore, contradicts $N_C > N_P$. $w_C > w_P$ implies $U'_C(s) - U'_P(s) = (w_C - w_P) > 0$ and, thus, perfect sorting.

(v) Assuming that $w_C < w_P$ at $N_C = N$, the following reasoning refers to Figure 1.a, but the argument still holds if $w_C > w_P$ at $N_C = N$. By contradiction: suppose a cycle exists. Obviously, a complete cycle cannot be located entirely in $CEBF$ or in $AEDG$. The cycle must be (partially) located in area ABE and/or ECD . Taking smoothness into account, in ABE this circle must include one N_C - S_C combination such that $\dot{N}_C > 0 \wedge \dot{S}_C = 0$ (this N_C - S_C combination cannot be located on EB where $\dot{N}_C < 0 \wedge \dot{S}_C < 0$). This contradicts $w_C > w_P$. Similar reasoning applies to the area ECD .

(vi) See above. □

The perfectly symmetric equilibrium is unstable if the adjustment is assumed to be myopic, that is, if workers move smoothly to the region where utility is highest. For example, if for some reason the composition of skills changed so that aggregate skills in region C are higher than aggregate skills in region P , but the regions are still identical in terms of pure numbers, normalized wages in region C will exceed those in region P , but land prices in CBD will be identical. Thus, high-skilled workers would face the strongest incentive to migrate to region C , reinforcing the initial difference in skills, and ultimately resulting in full segmentation. The symmetry-breaking result in Mori and Turrini's (2005) model is driven by the interaction of skill heterogeneity and communication costs; here, however, symmetry may be broken by the interplay of skill heterogeneity and land demand. Labor income depends on skills, but land demand does not. The crucial point is not that land demand is fully independent of skills but that an increase in skills has a stronger effect on labor income than on land demand.

The impact of changes in commuting costs and production costs on a stable, segmented, partial agglomeration equilibrium can be derived using the long-run equilibrium condition, Equation (22). An increase in either commuting costs per mile, k , variable costs a , or fixed costs, A , reduces population size and aggregate skills in the core region. An increase in commuting costs increases land price, reduces land use in production, and, therefore, reduces labor productivity in both regions. Because the wage in the large region is higher than in the small region, the negative effect on the core's wage prevails. Direct effects on

land prices are outweighed by wage effects. Higher variable production costs reduce the wage via the positive effect on intermediate goods prices while higher fixed costs reduce the wage through their negative effect on the number of varieties.

To gain insights when analytical solutions are unavailable, we simulate the basic model and extended versions numerically. To this end, we assume a uniform distribution: $f(s) = 1/(\bar{s} - \underline{s})$ ¹⁰ The model parameters in our model are $k = 0.5$, $N = 20$, $L = 25$, $a = 0.1$, and $A = 0.05$. We conduct several sensitivity analyses, particularly regarding σ , μ , and η .

2.3 Long-run equilibrium: welfare

Usually, welfare analysis in the NEG framework focuses on the comparison of stable equilibria with dispersion and agglomeration. Welfare is calculated for both a classical utilitarian approach (see, e.g., Baldwin et al., 2003) and a more general welfare function (see, e.g., Charlot et al., 2006; Ottaviano and Robert-Nicoud, 2006) to rank the equilibria. Alternatively Charlot et al. (2006) and Ottaviano and Robert-Nicoud (2006) considered real and potential Pareto improvements. This paper will focus on a utilitarian welfare function and real and potential Pareto improvements.¹¹

In the framework presented here, full and partial agglomeration equilibria may only co-exist for a small set of parameters. Simulations suggest that a unique equilibrium is the normal case. Moreover, complete dispersion is never a stable equilibrium. Therefore, the main issue is not whether full dispersion or full agglomeration is superior but is rather to determine the optimum degree of agglomeration. Due to pecuniary externalities, long-run equilibria are presumably spatially inefficient. Assuming that the government cannot determine quantities and locations directly, any improvement requires an instrument that induces a change in the spatial allocation of workers. Dependent on the welfare measure, additional instruments that redistribute gains across the population are necessary. To simplify the notation, only reforms that establish a perfect-sorting segmented equilibrium

¹⁰Simulations are available from the author on request. Simulations using the distribution $f(s) = 1/[s(\ln \bar{s} - \ln \underline{s})]$ lead to qualitatively similar results.

¹¹Due to the complex nature of the model, for alternative measures such as CES-welfare functions or Rawlsian welfare functions, general statements could not be obtained.

with either partial or full agglomeration will be considered.¹²

Integrating utility, defined in Equation (2), over the entire population results in aggregate utilitarian welfare, i.e., labor income plus land rents minus commuting costs,

$$W = w_C S_C + w_P S_P + R - r_C N_C - r_P N_P \quad (29)$$

which simplifies to

$$W = \frac{w_C S_C + w_P S_P}{\mu + \eta} - \frac{r_C N_C + r_P N_P}{2}, \quad (30)$$

when the properties of short-term equilibria are fully taken into consideration. A policy $\Pi = (t_C, t_P, T_C, T_P, \tau)$ is defined as a simple non-linear tax schedule characterized by residence-based tax rates t_i , $t_i < 1$, lump-sum transfers T_i , and a land tax, τ , implying $U_i(s) = (1 - t_i)w_i s + (1 - \tau)R/N + T_i - r_i$, $i = C, P$. A policy Π is considered feasible if the budget constraint $\sum_{i=C,P} (t_i w_i S_i - T_i N_i) + \tau R = 0$ is satisfied. Hence, at least one instrument is fixed by the budget constraint. A feasible policy Π is welfare increasing relative to the laissez-faire $\bar{\Pi} = (0, 0, 0, 0, 0)$ if $W_\Pi > W_{\bar{\Pi}}$ holds for long-run equilibria.¹³ It is a Pareto improvement if $U^\Pi(s) > U^{\bar{\Pi}}(s)$ for all $s \in [\underline{s}, \bar{s}]$. From the utilitarian point of view, taxes and transfers only matter in that they induce migration; given the allocation of workers, aggregate welfare is not affected by taxes and transfers. For Pareto improvements, the redistributive function of taxes and transfers for a given allocation of workers is also crucial.

Interesting special cases are pure interregional lump-sum transfers, i.e., $t_C = t_P = \tau = 0$, a non-linear federal tax with lump-sum redistribution, i.e., $t_C = t_P = t$, $T_C = T_P = T$, and $\tau = 0$, and regional non-linear taxes without explicit interregional redistribution, i.e., $T_i = t_i w_i S_i / N_i$ for $i = C, P$ with $\tau = 0$. Both pure interregional transfers and a federal tax redistribute across regions. The federal tax has a highly unequal geographical burden (see Albouy, 2009), it redistributes from high-wage-high-rent regions to low-wage-low-rent regions. Obviously, the federal tax not only redistributes across regions but also affects the interregional allocation of workers and production. It changes the critical skill level

¹²The (unstable) symmetric long-run equilibrium does not maximize utilitarian welfare. Although congestion and commuting costs are minimized with symmetric population size, pecuniary externalities are not fully exploited.

¹³If there are multiple long-run equilibria, this comparison refers to specific equilibria.

in the segmented long-run equilibrium. From the migration equilibrium equation (22), it immediately follows that federal taxation increases the critical skill level, provided the long-run equilibrium is stable, as

$$\frac{\partial [U_C(\hat{s}) - U_P(\hat{s})]}{\partial t} = (w_P - w_C)\hat{s} < 0. \quad (31)$$

Hence, federal taxation reduces regional differences in population, aggregate skills, and production. The periphery grows at the expense of the core. Pure interregional lump-sum transfers from the core to the periphery have a similar effect on the relative size of both regions.

Two types of reforms have to be considered: small-scale and large-scale reforms where the former changes the population distribution only marginally while the latter alters the type of equilibrium. The results indicate that for some parameters, a marginal welfare improvement requires convergence, but full agglomeration is the global optimum. The reason for this is that aggregate welfare is not concave.

We first consider small reforms. From a utilitarian perspective, a segmented partial-agglomeration equilibrium exhibits (locally) over-agglomeration if – taking equation (21) into consideration – $dW/dN_C < 0$. Hence, the introduction of a non-linear federal tax increases aggregate utilitarian welfare if it induces a change in the critical skill level \hat{s} that improves the population and skill distribution. Beginning with a segmented partial-agglomeration equilibrium, a small increase in the core's population changes aggregate welfare, W , according to:

$$\begin{aligned} \left. \frac{dW}{dN_C} \right|_{U_C(\hat{s})=U_P(\hat{s})} &= \frac{1}{\mu + \eta} \left(S_C \frac{dw_C}{dN_C} - S_P \frac{dw_P}{dN_P} + w_C \frac{dS_C}{dN_C} - w_P \frac{dS_P}{dN_P} \right) \\ &\quad - \frac{1}{2} \left(r_C - r_P + N_C \frac{dr_C}{dN_C} - N_P \frac{dr_P}{dN_P} \right). \end{aligned} \quad (32)$$

Using $dS_C/dN_C = \hat{s}$, $r_i = kN_i/2$ and $U_C(\hat{s}) = U_P(\hat{s})$, this can be written as

$$\begin{aligned} \left. \frac{dW}{dN_C} \right|_{U_C(\hat{s})=U_P(\hat{s})} &= \frac{\hat{s}(w_C - w_P)}{(\sigma - 1)(\eta + \mu)^2} [\mu - (\sigma - 1)(1 - \eta - \mu)^2] \\ &\quad - \left[\left(\frac{S_C}{N_C} - \hat{s} \right) w_C - \left(\frac{S_P}{N_P} - \hat{s} \right) w_P \right] \frac{1 - \eta - \mu}{(\eta + \mu)^2}. \end{aligned} \quad (33)$$

While the second row is unambiguously negative, the first row is only negative if the substitution elasticity is quite large, i.e., if $\sigma > 1 + \mu/(1 - \eta - \mu)^2$. In particular, if a

full agglomeration equilibrium exists, the first row is clearly positive. Simulations suggest that over-agglomeration is quite common but under-agglomeration is not frequent, though indeed possible.

The full-agglomeration equilibrium maximizes utilitarian welfare only if it maximizes labor income, $w_C S_C + w_P S_P$, as it also maximizes commuting costs. For example, full agglomeration is optimal if regional labor income, $w_i S_i$, is increasing and strictly convex in the population and if the positive effect of an increase in population size on labor income exceeds the negative effect on commuting costs. The latter effect vanishes if commuting costs per unit of distance, k , go to zero.

As noted above, for some parameters, the laissez-faire long-run equilibrium is locally over-agglomerated but globally under-agglomerated. For a different parameter set, over-agglomeration is global. For example, for a uniform distribution of skills with $\underline{s} = 6$, $\bar{s} = 8$, and $\sigma = 1.5$, over-agglomeration of the long-run equilibrium $N_C = 10.5168$ is only local for $\mu = \eta = 0.15$ but is global for the equilibrium $N_C = 11.3856$ for $\mu = \eta = 0.2$ (see Figure 2 (a) and (b)). The skill distribution, the relative size of the partial output elasticities in the final goods sector and the elasticity of substitution are crucial for these findings, but in a highly non-monotonic way. A closer look at the total income reveals the possible absence of monotonicity. Population changes regional labor income according to

$$\frac{d(w_i S_i)}{dN_i} = w_i \left[\frac{S_i}{N_i} \left(1 - \frac{1}{\eta + \mu} \right) + \hat{s} \left(1 + \frac{\mu}{(\sigma - 1)(\eta + \mu)} \right) \right], \quad i = C, P. \quad (34)$$

While the right term in curved brackets is positive, the left term is negative. The larger σ is, the smaller the positive term is relative to the negative. An increase in η increases both terms in absolute terms, but an increase in μ increases the entire term in squared brackets. Because the positive term is the same for the core and the periphery but the negative term is larger for the core than for the periphery, the effect of migration from the periphery to the core depends on the difference in wages and may change the sign when the population distribution changes.

If, for perfect sorting, the total welfare is either monotonically increasing or monotonically decreasing in the core's population size over the interval $[N/2, N]$, the analysis of the boundaries reveals the welfare properties of the equilibrium. To this end, ΔW is defined as the difference between aggregate welfare for an even division of the population

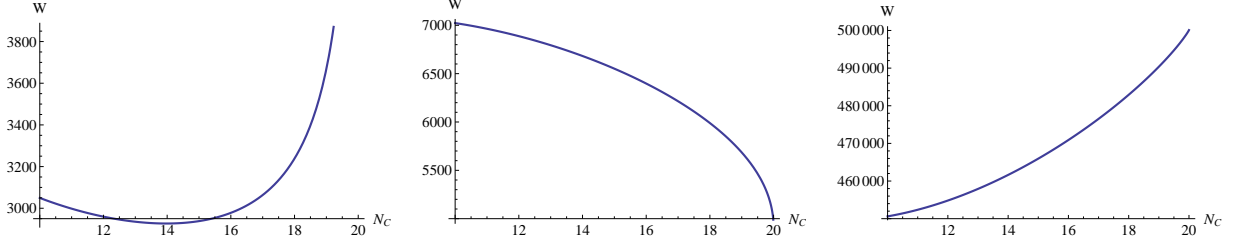


Figure 2: (a), (b), (c): The core's size and welfare

with perfect sorting, i.e., for $(N_C, S_C) = (N/2, S_C(N/2))$, and aggregate welfare for full agglomeration, i.e., for $(N_C, S_C) = (N, S)$. Depending on the skill distribution, $S_C(N/2)$ lies in the interval $(S/2, S)$. The results reveal that ΔW is an increasing function of the skill level $S_C(N/2)$ bounded by

$$\lim_{S_C(N/2) \rightarrow S/2} \Delta W = k \left[\frac{S \left(2^{\frac{1-\eta-\mu}{\eta+\mu} - \frac{\mu}{(\sigma-1)(\eta+\mu)}} - 1 \right) w_C^N/k}{\eta + \mu} + \frac{N^2}{4} \right], \quad (35)$$

$$\lim_{S_C(N/2) \rightarrow S} \Delta W = k \left[\frac{S \left(2^{\frac{1-\eta-\mu}{\eta+\mu}} - 1 \right) w_C^N/k}{\eta + \mu} + \frac{N^2}{4} \right], \quad (36)$$

where w_C^N denotes the core's wage for full agglomeration. While the upper bound is unambiguously positive, the lower bound may be negative if $\sigma < (1 - \eta)/(1 - \eta - \mu)$. Hence, if Equation (27) is not satisfied, welfare is highest for $(N_C, S_C) = (N/2, S_C(N/2))$, provided that, for perfect sorting, total welfare is a monotonic function of the core region's population size. Because $(N_C, S_C) = (N/2, S_C(N/2))$ is never a laissez-faire long-run equilibrium, $\sigma \geq (1 - \eta)/(1 - \eta - \mu)$ implies over-agglomeration. Otherwise, under-agglomeration or efficient full agglomeration is possible. For example, for the same uniform distribution of skills as in the previous example, where symmetry in population size is welfare maximizing for $\mu = \eta = 0.2$ and $\sigma = 1.5$ (see Figure 2 (b)), full agglomeration is efficient for $\mu = \eta = 0.2$ and $\sigma = 1.3$ (see Figure 2 (c)).

To analyze (real) Pareto improvements, Figure 3 is used. In the laissez-faire partial-agglomeration long-run equilibrium $\bar{\Pi}$, utility is an increasing convex function of the skill level with a kink at the critical skill level $\hat{s} = s_0$, with $\underline{s} < s_0 < \bar{s}$. Note that Π is welfare increasing if it is a Pareto improvement. Suppose that some policy Π_0 exists that increases

instruments are needed to realize Pareto improvements. Whether tax rates should differ across regions and whether explicit interregional transfers have to be a part of the tax-transfer package is an open question. Consider, for example, a small agglomeration-reducing, welfare-increasing policy when σ is rather large. If the reform reduced the wage in the periphery, low productivity individuals in the periphery would require compensation from the core region.

In contrast to NEG models, tax policies that increase aggregate welfare by changing the critical skill level, \hat{s} , are always potential Pareto improvements (see on NEG models, Charlot et al., 2006; Ottaviano and Robert-Nicoud, 2006). The reason is that the utility is equal to income minus land rents at the CBD, that all individuals face the same final goods prices, and that output, intermediate goods prices, the number of varieties, land prices and total land rent are completely determined by the distribution of population and skills. To see the equivalence of aggregate welfare increases and potential Pareto improvements, consider the introduction of a non-linear federal income tax characterized by t and T , and suppose that this tax increases aggregate welfare and induces the critical skill level s_1 . The parameters $N_i, S_i, n_i, X_i, L_i, r_i$, and R are completely determined for $i = C, P$. While the Kaldor test requires that the gainers can compensate losers for the change, given the prices of the post-reform equilibrium, the Hicks test requires compensation of the gainers by the losers for forgoing the reform, given the prices of the initial equilibrium (Kaldor, 1939; Hicks, 1940). Suppose that compensation payments depend on skills but not on the residence choice: $\Gamma(s)$, with $\int_{\underline{s}}^{\bar{s}} \Gamma(s) f(s) ds = 0$. The reform passes the Kaldor test if a function $\Gamma(s)$ exists such that

$$(1 - t)w_i^1(s)s - (R^1 + T)/N - r_i^1 + \Gamma(s) \geq U^0(s), \quad s \in [\underline{s}, \bar{s}], \quad (37)$$

where the initial equilibrium is indicated by the superscript 0, the post-reform equilibrium by the superscript 1, and the residence region in the new equilibrium by the subscript i . Skill-dependent compensation payments affect neither residence choices nor individual land demand; they change individual final good demand but not aggregate final goods demand. The function $\Gamma(s)$ obviously exists if the tax reform increases aggregate welfare. By the same argument, the Hicks test is passed if $w_j^0(s)s - R^0/N - r_j^0 + \Gamma(s) \leq U^1(s)$, $s \in [\underline{s}, \bar{s}]$, where j indicates the residence region in the initial equilibrium. Again, for a welfare

increasing federal tax, the function $\Gamma(s)$ exists. Therefore we have:

Proposition 4 *Suppose that a welfare-increasing tax policy exists that establishes a perfect-sorting segmented equilibrium with either partial or full agglomeration. Because this reform passes the Kaldor and the Hicks tests, it is a potential Pareto improvement.*

Finally, note that compensation payments must not depend on the residence choice, as residence-dependent payments change the critical skill level and thus production.

3 Modifications and extensions

In this section, the basic model is modified and extended in a number of ways to check its robustness. We separately analyze trade in intermediate goods and endogenous land demand.

3.1 Trade in intermediate goods

Previously, trade in final goods has been considered to be frictionless, but prohibitively high barriers have foreclosed the possibility of trade in intermediate goods. In this subsection, we examine a more realistic situation and allow trade in intermediate goods. For this purpose, we assume iceberg costs $\tau - 1$ for trade in intermediate goods, with $\tau > 1$. Intermediate goods suppliers export part of their output, albeit at higher costs. The intermediate goods quantity and price indices in region i are

$$I_i = \left[\int_0^{n_i} q_{ii}(j)^{\frac{\sigma-1}{\sigma}} dj + \int_0^{n_k} q_{ik}(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad (38)$$

$$P_i = \left[\int_0^{n_i} p_i(j)^{-(\sigma-1)} dj + \int_0^{n_k} \phi p_k(j)^{-(\sigma-1)} dj \right]^{-\frac{1}{\sigma-1}}, \quad (39)$$

for $k \neq i$, where $q_{ik}(j)$ denotes region i 's demand for quantity j produced in region k , and $\phi = \tau^{1-\sigma}$ indicates "trade freeness", with $0 \leq \phi < 1$. Profit maximization of final goods producing firms determine the demand for local and foreign intermediate goods

$$q_{ii}(j) = \mu \frac{X_i P_i^{\sigma-1}}{p_i^\sigma} \quad \text{and} \quad q_{ik}(j) = \mu \frac{X_i P_i^{\sigma-1}}{(\tau p_k)^\sigma}, k \neq i. \quad (40)$$

Intermediate goods suppliers in region i have demand

$$q_i(j) = q_{ii}(j) + \tau q_{ki}(j), \quad k \neq i. \quad (41)$$

However, as is well known, mill pricing is optimal and the mark-up rule is the same as without trade, i.e. as determined by (10). Furthermore, trade also leaves individual intermediate goods quantities per firm unaltered; Equation (11) still holds. However, as the supply of intermediate goods meets the demand from both regions, the intermediate goods market equilibrium conditions become

$$\frac{(\sigma - 1)A}{a} = \mu \frac{X_i P_i^{\sigma-1}}{p_i^\sigma} + \phi \mu \frac{X_k P_k^{\sigma-1}}{p_i^\sigma}, \quad k \neq i. \quad (42)$$

Substituting for prices and price indices, yields the final goods demand

$$X_i = \frac{A\sigma (w_i^\sigma - \phi w_k^\sigma) (n_i w_i^{1-\sigma} + \phi n_k w_k^{1-\sigma})}{\mu(1 - \phi^2)}, \quad k \neq i. \quad (43)$$

This type of equilibrium, where both regions produce intermediate and final goods, requires a moderate wage differential, i.e., $\phi < (w_C/w_P)^\sigma < 1/\phi$. Sufficiently high trade costs are required for this type of equilibrium to exist.¹⁴ Because final goods demand is related to wages and quantities in both regions, the number of varieties cannot be explicitly calculated:

$$n_i = \frac{S_i - \frac{\eta X_i}{w_i}}{\sigma A}. \quad (44)$$

What can be said, though, is that, in contrast to separated markets for intermediate goods, the number of varieties is not simply proportional to regional aggregate skills. The zero-profit conditions of final goods producers can be written as

$$\kappa k^{1-\eta-\mu} \left[\left(1 + \frac{\eta}{\mu} \right) A\sigma \right]^{\frac{\mu}{1-\sigma}} N_i^{1-\mu-\eta} w_i^\eta (n_i w_i^{1-\sigma} + \phi n_k w_k^{1-\sigma})^{\frac{\mu}{1-\sigma}} = 1, \quad k \neq i. \quad (45)$$

Finally, it should be stressed that trade in intermediate goods has no direct impact on land markets.

As in the case without trade in intermediate goods, a symmetric long-run equilibrium always exists, and a segmented long-run equilibrium with partial agglomeration exists

¹⁴We disregard corner equilibria where final goods production or intermediate goods production are concentrated in one region.

for a certain range of parameters. Unfortunately, equations (44) and (45) do not allow for explicit solutions for the number of varieties and regional wages, and we are thus not able to fully characterize short-run and long-run equilibria analytically. However, the segmented long-run equilibria for both the case with and without trade in intermediate goods are very similar. If an interior segmented equilibrium with partial agglomeration exists, to induce perfect sorting, the wage in the core needs to be higher than the wage in the periphery. From the same arguments as in the basic model, the wage differential implies a rent differential and differences in population and aggregate skills. Wages and land rents in the CBD of the core region exceed those in the periphery, the core is more densely populated, and the core has a larger stock of human capital: $w_C > w_P$, $r_C > r_P$, $N_C > N_P$, and $S_C > S_P$. Furthermore, from Equation (45), it follows that $n_C w_C^{1-\sigma} + \phi n_P w_P^{1-\sigma} > n_P w_P^{1-\sigma} + \phi n_C w_C^{1-\sigma}$, which results in $n_C > n_P$. Together with Equation (43), it also implies $X_C > X_P$. Hence, regardless of whether trade in intermediate goods is feasible, the core produces more final and intermediate goods despite the intermediate goods mill prices being higher in the core than in the periphery, i.e., $p_C > p_P$. The following proposition summarizes these findings.

Proposition 5 *If a segmented long-run equilibrium with partial agglomeration exists, even with trade in intermediate goods, the core will be larger than the periphery in terms of aggregate skills, population, and production. Wages, land rents, and intermediate goods prices will also be higher.*

Trade effects can be analyzed numerically. Using the benchmark distributions and parameters introduced in the previous section with $\sigma = 4$, $\mu = 0.4$ and $\eta = 0.3$, an interior segmented equilibrium with partial agglomeration exists at least for $0 \leq \phi < 0.9$. An increase in ϕ increases wages in both regions but reduces the wage differential, while diminishing the interregional differences in aggregate skills, population, land rents, and production. Moreover, the efficiency gain from the reduction in interregional trade barriers causes an increase in final good production in both regions.

Opening interregional markets for intermediate goods increases the number of varieties available in both regions, which, in turn, lowers the unit costs of final goods. Production becomes more efficient, and income and utility increase. However, trade integration

at the intermediate goods level reduces the core's skill advantage, as the periphery is no longer cut off from the variety of intermediate goods produced in the core. Regional differences diminish, but they do not disappear. Although trade integration slightly dampens agglomeration, even if trade were nearly free, agglomeration forces would continue to be substantial. Dispersion forces are always too weak to overcome the strong forces that cause segmentation and, as a result, agglomeration. Skill heterogeneity combined with far-reaching land-demand homogeneity generates segmentation, regardless of whether there is trade in intermediate goods.

3.2 Endogenous land demand

Because the interplay between resident land demand and human capital supply is the basic force behind segmentation, we test the robustness of our results with respect to endogenous individual land demand. For this purpose, we assume a limited area of land and disregard commuting costs. When all land L is in use, the final goods suppliers' demand for land, along with population size and lot size, determines land prices in the CBD. The utility is Cobb-Douglas defined on the final good consumption and, as a proxy for housing, lot size. The indirect utility is as follows:

$$U_i(s) = \frac{w_i s + \frac{R}{N}}{r_i^{1-\alpha}}, \quad i = C, P, \quad (46)$$

where $1 - \alpha$ is the weight of land, with $0 < \alpha < 1$. Cobb-Douglas utility implies the land-demand function $L_i = (1 - \alpha) [(w_i s + R/N)/r_i]$ and exhibits unit income elasticity. From regional land-market equilibrium conditions,

$$L = (1 - \mu - \eta) \frac{X_i}{r_i} + (1 - \alpha) \frac{w_i S_i + N_i \frac{R}{N}}{r_i}, \quad i = C, P, \quad (47)$$

and the definition of aggregate land rent $R = (r_C + r_P)L$, land prices and aggregate land rent can be determined:

$$\begin{aligned} r_i &= \frac{1}{\alpha L(N_i + N_k)} \{M_i(1 - \mu - \eta)(N_i + \alpha N_k) + (1 - \alpha) [M_k(1 - \mu - \eta)N_i \\ &\quad + S_i(N_i + \alpha N_k)w_i + N_i S_k(1 - \alpha)w_k]\}, \quad i = C, P, k \neq i, \\ R &= \frac{1}{\alpha} \sum_{i=C,P} [(1 - \mu - \eta)X_i + (1 - \alpha)w_i S_i]. \end{aligned} \quad (48)$$

Increases in population and production increases the price of land. The price of land in the CBD is no longer simply proportional to population. Because equal land sharing redistributes income from the high-rent region to the low-rent region, the aggregate land rent is larger than the respective expenditure share of land calculated for the gross labor income. Intermediate goods markets and labor markets are not directly affected by consumers' endogenous land demand, but the zero-profit conditions at the final goods level are different from those conditions in the basic model. Defining the following,

$$\bar{\kappa} = \psi \left(\frac{a\sigma}{\sigma - 1} \right)^\mu \left(\frac{1}{\alpha(\mu + \eta)} \right)^{1-\mu-\eta} \left(\frac{\mu}{A\sigma(\mu + \eta)} \right)^{\frac{\mu}{1-\sigma}}, \quad (49)$$

the final goods market equilibrium condition, Equation (7), can be written as

$$\begin{aligned} 1 = & \bar{\kappa} S_i^{\frac{\mu}{1-\sigma}} w_i^{\mu+\eta} \left(\frac{1}{LN} \right)^{1-\mu-\eta} (S_i \{ [1 - \alpha(\mu + \eta)] (N_i + \alpha N_k) \} w_i \\ & + (1 - \alpha) N_i S_k [1 - \alpha(\mu + \eta)] w_k)^{1-\mu-\eta} \quad i = C, P, k \neq i. \end{aligned} \quad (50)$$

Non-linearity makes explicit solutions for regional wages impossible. However, numerical simulations produce results that are qualitatively similar to those employing exogenous land use by residents. Segmentation is the likely outcome of worker migration, accompanied by the agglomeration of skills and production, though the core region is sparsely populated relative to the peripheral region. The reason for the low population density in the core is that individual land demand is an increasing function of income and that the land area for final goods production is larger than in the periphery. If either commuting in regions with an endogenous total area or housing production were included in the model, the inverse relationship between aggregate skills and population density would disappear. However, endogenous land demand probably reduces the gap in size between the core and the periphery.

Segmentation occurs even though high-skilled workers buy larger lots than do low-skilled workers. To clarify the relationship between the demand for land and the preference for the high-wage-high-rent region, suppose that there is a continuum of regions and (indirect) utility that could be written as $V[sw, r(w)]$, where $r(w)$ captures the empirical cross-regional relationship between wages and rents, with $dr(w)/dw > 0$. For the critical

skill type, using Roy's identity,

$$\frac{dV[sw, r(w)]}{dw} = \frac{\partial V[sw, r(w)]}{\partial y} s + \frac{\partial V[sw, r(w)]}{\partial r} \frac{dr(w)}{dw} = \frac{\partial V[sw, r(w)]}{\partial y} \left[s - l \frac{dr(w)}{dw} \right] \quad (51)$$

is zero, where y is the individual income and l is the individual land demand. Assuming a negative second derivative with respect to w , the more highly skilled prefer a higher wage if

$$\left. \frac{d^2 V[sw, r(w)]}{dw ds} \right|_{\frac{dV}{dw}=0} = \frac{\partial V[sw, r(w)]}{\partial y} \left[1 - \frac{dl(s)}{ds} \frac{dr(w)}{dw} \right] > 0. \quad (52)$$

Hence, if

$$\frac{dl(s)}{ds} \frac{s}{l} < 1, \quad (53)$$

that is, if the skill elasticity of the demand for land is lower than 1, skills and preferred wages are positively correlated. Taking rent income into account, the Cobb-Douglas utility meets this requirement.

4 Concluding remarks

This paper analyzed regional patterns of production and housing in the presence of pecuniary externalities within a general-equilibrium framework with monopolistic competition in intermediate goods markets. First, it showed that the interplay of heterogeneous skills and comparatively homogeneous land demand triggers skill segmentation and agglomeration. The core region, being more attractive to high-skilled workers, gains a disproportionately large share of production at all levels of the supply chain. Second, the paper showed that due to pecuniary externalities, the long-run equilibrium is inefficient. In general, Pareto improvements could be achieved with relatively simple tax policies. It was demonstrated that federal taxation that automatically redistributes toward the periphery might be welfare enhancing by averting over-agglomeration. Any aggregate welfare-increasing policy is also a potential Pareto improvement. Third, the paper discussed whether a reduction in intermediate trade costs weakens agglomeration by narrowing the interregional wage gap.

For clarity of exposition, the model was constructed as a two-region model, but it could be easily extended to a multi-region model. Segregation and agglomeration would

still be triggered by wage gaps and land rent differences. In the basic model without trade in intermediate goods, all major results would hold. A perfectly symmetric equilibrium would also exist. At the segmented long run-equilibrium, regions could be ranked according to aggregate skills, population size, output, wages, and land rents. Migration would lead to a strictly monotonic relationship between average skills and size in terms of industry and population.

Certain limitations of the model suggest avenues for future research. Additional agglomeration forces, such as knowledge spillovers, could be integrated into the model. The inclusion of imperfect competition in the final goods sector, thus inducing a market-access effect and a cost-of living effect, would increase our understanding of the forces of segmentation and agglomeration. Finally, it would be worthwhile to take land use at the intermediate stage of production into account.

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Appendix: attachment to home

In this appendix, we confirm the dispersive nature of attachment to home identified by Tabuchi and Thisse (2002). A generally decreasing willingness to leave the home region for higher pecuniary utility alleviates agglomeration by limiting the impact of a regional wage gap on migration flows. A second dimension of heterogeneity that is not perfectly correlated with skills allows for imperfect sorting.

In the basic model, all workers are perfectly mobile. In this subsection, we allow for one important source of immobility, namely attachment to the home region. Following Wellisch (2000) and others, we assume that workers differ in their psychic attachment to their home regions and the psychic utility component is additively separable. Types are two-dimensional: there is a skill dimension, s , and a regional preference dimension, ν , uniformly distributed over $[0, 1]$. For each (s, ν) type, preferences are given by

$$V(s, \nu) = \begin{cases} U_C(s) + \beta(1 - \nu) & \text{if she lives in } C, \\ U_P(s) + \beta\nu & \text{if she lives in } P, \end{cases} \quad (54)$$

where $\beta \geq 0$. For each skill type, the pecuniary utility $U_i(s)$ is the same. Workers with a small ν have a preference for living in region C ; workers with a large ν prefer region P . β measures the degree of heterogeneity in tastes for a region and effectively indicates the degree of household mobility. If $\beta = 0$, the worker is perfectly mobile, while $\beta > 0$ means imperfect mobility. If $\beta \rightarrow \infty$, the worker becomes perfectly immobile. For simplification, skills and attachment to the core region are uncorrelated.

For each type, a critical spatial preference could be calculated as follows:

$$\hat{\nu}(s) = \max \left\{ \min \left\{ \frac{1}{2} + \frac{U_C(s) - U_P(s)}{2\beta}, 1 \right\}, 0 \right\}. \quad (55)$$

Similar to the basic model, an (almost) perfectly symmetric equilibrium always exists where for every skill level s , all workers with $\nu < 0.5$ live in the core and all workers with $\nu > 0.5$ live in the periphery. Regions only differ in the spatial preferences of their constituents, but are otherwise identical. If again myopic adjustment is assumed, by the same reasoning as in the basic model, the symmetric equilibrium is unstable.

For some parameters, a (stable) asymmetric equilibrium with full agglomeration also exists. If $U_C(s) > U_P(s) + \beta$ for all s , provided that $N_C = N$ and $S_C = S$, no worker

would have an incentive to migrate to the periphery. As stated in the previous section, this requires that wages and population are positively correlated.

At a long-run, weakly segmented equilibrium with attachment to home and partial agglomeration, all workers of type (s, ν) live in region C if $\nu < \hat{\nu}(s)$ and in region P if $\nu > \hat{\nu}(s)$. The core's population and aggregate skill at the long-run equilibrium is given by

$$N_C = N \int_{\underline{s}}^{\bar{s}} \hat{\nu}(s) f(s) ds \quad \text{and} \quad S_C = N \int_{\underline{s}}^{\bar{s}} s \hat{\nu}(s) f(s) ds. \quad (56)$$

For $w_C > w_P$, the critical spatial parameter $\hat{\nu}$ is a monotonically non-decreasing function of the skill level, as the pecuniary utility differential $U_C(s) - U_P(s)$ is increasing in s . The fraction of highly-skilled workers living in the core is large compared to the number of low-skilled workers living there. If attachment to home is rather minor, segmentation is only present within skill groups at an interval around some critical skill level (see Figure 4).¹⁵ Within this group, some workers live in the core and others in the periphery. By contrast, all workers with rather high skill levels live in the core, and all workers with quite low skills reside in the periphery. That the long-run equilibrium with attachment to home does not exhibit perfect stratification of skill levels is fairly consistent with empirical observations.

The weakly segmented equilibria with attachment to home differ from those without attachment to home. Any segmented equilibrium without home attachment cannot be an equilibrium if there is home attachment, as some workers with skills very close to \hat{s} would have an incentive to migrate to their preferred regions. The domain of 'feasible' allocations presented in the main text shrinks if home attachment is introduced. Beginning at full mobility, an increase in β leads to the immigration of low-skilled workers and the emigration of high-skilled workers in the core. The most likely outcome is a decrease in aggregate skills in the core and an increase of the same size in the periphery. Simulations show an increasing degree of equalization.¹⁶ As a result, wages in the core shrink, while wages in the periphery rise – provided that the implied changes in population do not overcompensate for

¹⁵In the numerical simulations, a uniform distribution is assumed. The model parameters are as follows: $k = 0.5$, $N = 30$, $L = 25$, $a = 0.1$, $A = 0.05$, $\underline{s} = 1$, $\bar{s} = 8$, $\mu = 0.4$, $\eta = 0.3$, $\sigma = 6$, and $\beta = 0.05$.

¹⁶For $\sigma = 6$, increasing β from 10^{-5} to 0.1, the core shrinks and the periphery grows at an increasing rate.

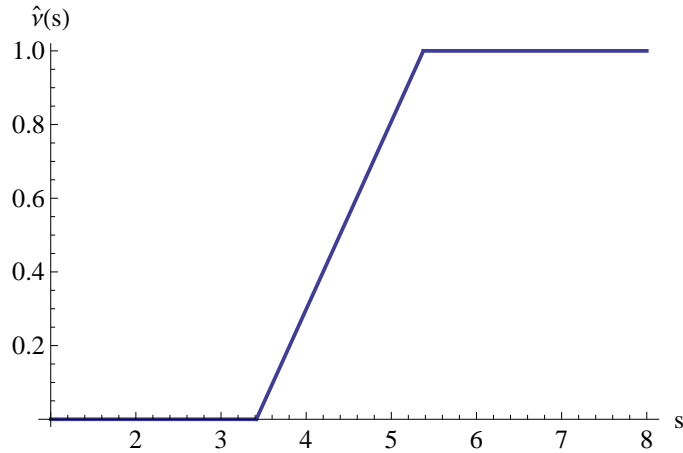


Figure 4: Imperfect sorting: skills and residence choices

the skill effects. Agglomeration forces become weaker, and regional differences gradually disappear.¹⁷ In other words, increasing mobility amplifies agglomeration.

If $\beta \rightarrow 0$, the weakly segmented long-run equilibrium converges to the long-run segmented equilibrium of the basic model. If, however, $\beta \rightarrow \infty$, only the perfectly symmetric equilibrium of the basic model remains, as $\hat{v}(s) \rightarrow 1/2$ for all skill levels.

If attachment to home and skills were negatively correlated, i.e., if $\beta'(s) < 0$, low-skilled workers would be more evenly distributed, but the highly skilled would be skewed toward the core. Compared to homogenous attachment to home, a negative correlation of skills and immobility should enlarge the core.

Interestingly, if skills follow a truncated log-normal distribution with $\underline{s} = 1$, $\bar{s} = 8$, $f_C(\underline{s}) = 0.4 * f(\bar{s})$, and $f_C(\bar{s}) = 0.9 * f(\bar{s})$, probability distribution functions in the core and in the periphery are shown in Figure 5. The upper parts of the probability distribution functions look similar to the distributions of worker effects shown by Combes et al. (2012). In contrast to the results of Combes et al. (2012), low-skilled workers are not over-represented in the denser region, because the model of this paper does not capture

¹⁷Because both changes in population size and changes in aggregate skills affect the pecuniary utility differential and, therefore, the critical spatial preference level for all types, it cannot be ruled out analytically that for some levels of immobility a small increase in the attachment to home strengthens agglomeration forces. However, simulations have consistently shown monotonic changes.

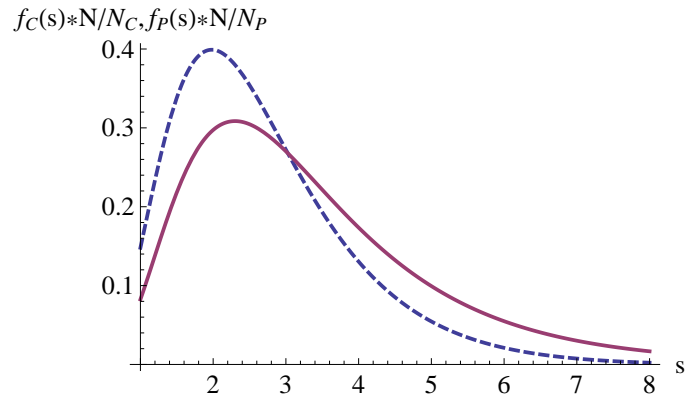


Figure 5: Distribution of skills in the core and in the periphery

any low-skill-specific benefit of denser regions.

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