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Fund Managers—Why the Best Might Be the Worst: On the Evolutionary Vigor of Risk-Seeking Behavior

Björn-Christopher Witte
University of Bamberg

Abstract This article explores the influence of competitive conditions on the evolutionary fitness of different risk preferences. As a practical example, the professional competition between fund managers is considered. To explore how different settings of competition parameters, the exclusion rate and the exclusion interval, affect individual investment behavior, an evolutionary model is developed. Using a simple genetic algorithm, two attributes of virtual fund managers evolve: the share of capital invested in a risky asset and the amount of excessive risk accepted, where a positive value of the latter parameter points to an inefficient investment portfolio. The simulation experiments illustrate that the influence of competitive conditions on investment behavior and attitudes towards risk is significant. What is alarming is that intense competitive pressure generates risk-seeking behavior and undermines the predominance of the most skilled. In these conditions, evolution does not necessarily select managers with efficient portfolios. These results underline the institutional need for the creation of a competitive framework in which risk-taking does not provide an evolutionary advantage per se, and indicate measures on how to achieve this.

JEL C73, D81, G11, G24
Keywords Risk preferences; competition; genetic programming; fund managers; portfolio theory

Correspondence Björn-Christopher Witte; Department of Economics, University of Bamberg, Feldkirchenstrasse 21, D-96045 Bamberg, Germany.
Email: bjoern-christopher.witte@uni-bamberg.de

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1 Introduction

Evolutionary selection leads to the predominance of the fittest. However, it is not always clear from the very beginning what the fittest behavior will be. This also applies to the field of economics.

Of course, economics does not refer to evolution in the original sense of the reproduction of the superior and genetic progress between generations. Rather, we assume that selection is conducted by market competition rewarding the best with economic wealth and driving the worst into bankruptcy. To give an example, Milton Friedman (1953, p.23) stated that profit maximization is an "appropriate summary" of the conditions of survival, delivering an exemplar for the assumption that competition would favor agents who behave optimally, or rationally, from the angle of economic theory. Today, there is a myriad of studies, many using modeling techniques, which provide strong evidence that this assumption is sometimes misleading. Schaffer (1989), for instance, shows that Friedman's proposition is not valid, if deviating from profit maximization is less harmful for the firm considered than for its contenders.

The present study is related to the class of evolutionary models in which agents are represented by financial traders and ways of behavior are typically trading strategies. To provide a brief overview, models in this field can be classified into two groups. The first one has been described by Hommes (2001) as *adaptive belief systems* (ABS). ABS formed by boundedly rational traders having different expectations about the future. Traders select strategies with reference to the utility generated by the rules in the past. Typically, utility is measured by the realized net or risk-adjusted profits produced. Evolution is reflected in the change of fractions of beliefs or strategies in the population of traders. Normally, these strategies are either based on technical or fundamental analysis. One of the most important insights provided by ABS is that under certain conditions, apparently irrational noise traders are able to survive evolutionary competition and attain similar profits as fundamentalists, as demonstrated by Brock and Hommes (1998), DeLong et al. (1990, 1991), and Hommes (2001).

Whereas ABS use to be simulation models, a second group of studies follows a purely analytical approach. Taking a more Darwinian perspective, these studies

evaluate the evolutionary fitness of a strategy in terms of its ability to survive market selection in the long run. In Blume and Easley (1992), for instance, this ability is determined by the expected growth rates of the wealth share. Based on this criterion, the authors develop a general equilibrium model of a dynamically complete asset market, in which prices are formed endogenously and the set of strategies available is constant. It is found that, contrary to the common belief, the market does not necessarily select investors with rational expectations. Evstigneev et al. (2002) confirm this result for incomplete markets, and Amir et al. (2005) explore its conditions for general strategies. In contrast, Sandroni (2000) illustrates that rational expectations do prevail if the intertemporal discount factor is equal among agents. The heterogeneity of findings is dissolved by Blume and Easley (2006) who show that for any Pareto-optimal allocation, the selection for or against rational expectations is determined entirely by discount factors and beliefs.

The present article deals with one aspect of behavior which has been vastly ignored in the field of studies described above: differences in risk preferences.[1] Risk preferences are relevant if decisions are to be made under uncertainty, that is, when agents know merely the probabilities of the possible consequences of an action. A large body of experimental evidence has led to the notion that human beings behave in a risk-averse manner (at least when the situation of decision making is about the realization of monetary gains), that is, they prefer to get a definite payoff towards a lottery with an expectation equal to this payoff (Allais 1953; Arrow 1971; Kahneman and Tversky 1979). Economic modelers adopt this finding by using CARA (Constant Absolute Risk Aversion) or CRRA (Constant Relative Risk Aversion) utility functions. Others assume agents to be risk neutral, meaning that they simply seek to maximize expected payoffs. Risk-seeking behavior, however, is usually excluded from consideration.

With regards to the subprime crisis, the exclusion of risk-seeing behavior is not easy to justify. Hoping for supreme returns, professional investors deposited huge

[1] The interest of such questions, however, is noted relatively early. In his eponymous study on "some elementary selection processes in economics", M.J. Farrell (1970) develops two abstract probabilistic models of an evolutionary asset market and finds that "a large group of inept speculator will always be present". Having regard to future research, he remarks: "Also interesting, […], is to allow the variance of the probability distribution of the outcome of the gamble to vary independently from expected return and so permit a comparison of the effects of selection on risk-seekers and risk-averters."

amounts of capital in assets whose fundamental value was far exceeded, and appeared not to avoid risk at all. How can that be? The finding that human beings behave in a risk-averse manner depends on the condition that the individuals' goal is to maximize their own utility. However, when being in competition with each other, the criterion of selection is not necessarily individual utility but the payoff per se may be crucial for survival. As a result, risk-seeking behavior should not be excluded ex ante, and we should assume that it is adopted whenever it provides a competitive advantage.

Previous research has already delivered several insights on the emergence of different risk-preferences in evolutionary environments. In one of the earliest contributions, Rubin and Paul Il (1979) present a simple, static model in which the goal of individuals is to achieve some threshold income, which is interpreted as the creation of offspring. Rubin and Paul Il argue that, if the functional dependency between evolutionary fitness and individual income is non-linear, maximizing the probability to meet the threshold can lead to a different behavior than maximizing expected income. In particular, if the population is already dense, youngsters are inclined to accept gambles, which might even be unfair.

Robson (1992) assumes individual utility to directly depend on wealth (in a concave fashion) as well as on status, that is, the rank of one individual relative to others (in a convex fashion). Like in Rubin and Paul Il (1979), this implies a non-linear utility function like the convex-concave-convex function discussed in Friedman and Savage (1948). Robson shows that in this setup, individuals may purchase insurance and gamble at the same time. This effect appears if increasing wealth leads to a greater boost of status than losing wealth causes a drawback.

Robson (1996) elaborates another refinement of Rubin and Paul Il (1979) with polygyny. In his model, the threshold is derived from the behavior of females choosing men depending on their wealth. As multiple men are chosen, thresholds arise repeatedly, which produces discontinuities in the fitness function of men. Although the payoff function of men implies risk-aversion, it is shown that risk-taking behavior – the acceptance of fair bets – may emerge with arbitrary extension.

The observation that risk-taking behavior may improve individual chances to survive has coined the term of "gambling for redemption". Malone (2011) sets this problem in the context of the fear of unemployment. In his model, he assumes a politician who is faced with the possibility of a sovereign default. It is shown that

under some conditions politicians are inclined to gamble by initiating policies which increase the variance of outcomes but may decrease the outcome expectation. The conditions are: (i) Default increases the probability of a job loss, and (ii) the rents from the job rise with the output of policies whereas in case of a default a minimum rent is paid which is independent from the magnitude of the default. In combination, (i) and (ii) create a convex utility-function, similar to Robson (1992, 1996).

Gambling for redemption has also been tackled by financial research. Kareken and Wallace (1978) or Diamond and Dybvig (1986), for example, argue that deposit guarantees generate incentives for risk-taking behavior of one asset-owner, which is potentially harmful for other share-owners. Again, this result is obtained by a convexity of the individual payoff-function.

The emergence of risk-taking behavior in the models above stems from the fact that achieving some extra income may produce a jump in utility, which is survival or reproduction, whereas there is "little to lose". In this sense, taking risks per se is rational under some conditions. In contrast, Dekel and Scotchmer (1992) illustrate that evolutionary competition does not necessarily favor rational agents. Their model follows a game-theoretical approach, and evolution is based on the well-known replicator dynamics. The analysis shows that the evolutionary outcome is sensitive to what is inheritable. In particular, if players can only inherit pure strategies, strategies that are never a best reply can persist.[2] We may conclude that rationality might not be a necessary condition for the survival of risk-seeking behavior.

In the present study, the evolutionary fitness of different risk-preferences is investigated by agent-based modeling. Essentially, this leads to a highly dynamic model. A difference to most analytical models is that the threshold income emerges model-endogenously. This allows a complex interplay between the individual behavior towards risk and the outcome needed to survive. One goal is to check if individuals behave as predicted by traditional models, or if the agent-based approach can uncover other relevant phenomena.

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[2] Jones (2001) points out that in reality, irrational behavior might also be a temporal phenomenon, which is "the product of a mismatch between the environment in which the brain evolved and the environment in which the brain now must operate".

The article is divided into two major parts. The first part (section two) represents an abstract, theoretical reflection about the evolutionary potential of risk-seeking behavior. In the second part (section three), we present a practical example of a competitive scenario in which risk-preferences play a central role: the competition among fund managers. To explore this scenario, we construct an evolutionary model, in which agents differ in terms of the risk-return profile of their asset portfolio. Competitive pressure is created by exclusions of the worst performing managers. The discards are then replaced by newcomers, whose investment behavior results from a genetic algorithm. Competition parameters are the interval in which exclusions take place and the share of agents to be excluded. Specific research questions are:

- Will evolutionary competition always lead to the prevalence of agents with efficient portfolios?
- Which portfolios will be fittest under different settings of the competition parameters?

Our experiments show that agents tend to build conservative but efficient portfolios if the exclusion rate is low. Agents' risk-aversion becomes less if the exclusion rate rises, and/or if the exclusion interval is prolonged. Notably, if the exclusion rate is high and the exclusion interval is low, agents completely deviate from risk-averse behavior but take great risks, although the additional risk does not improve the expected return of their portfolios. Under these competitive conditions, even agents with inefficient portfolios can survive. These results are alarming as they suggest that intense competitive pressure triggers the acceptance of excessive risk and undermines the prevalence of the most skilled.

2 A theoretical view on the evolutionary advantage of risk-seeking behavior

The rational choice between several actions implies the evaluation of the consequences of the alternatives. In reality, however, individuals usually do not know which consequence an action will have but estimate probabilities about the likelihood of a certain outcome – the decision has to be made "under uncertainty". Decisions under uncertainty are frequent in economic contexts. Calculating the

profitability of a potential investment, for example, requires information about the future development of revenues, interest-rates, prices etc. Yet, as respective predictions may be subject to error, the present value of the investment cannot be identified definitely. Nevertheless, existing information can be used to establish a set of possible outcomes and to assign probabilities to each of them. If the set of outcomes is continuous (instead of discrete), the probability distribution over this set is often assumed to follow a normal distribution. The latter applies, for instance, for the future return of asset portfolios (Markowitz 1952, 1959).[3]

Figure 1 shows the probability density functions of two normal distributions (profile 1 and 2). The curves may be interpreted as the payoff profiles of two ways of behavior, or more specifically, of two asset portfolios. The profiles differ in terms of the expected payoff, μ, and the payoff variance, σ^2. Obviously, economic theory would predict that individuals strongly prefer profile 1 towards 2, because 1 affords the better expected payoff ($\mu_1 > \mu_2$) at the lower variance ($\sigma_1^2 < \sigma_2^2$), and subjects are commonly assumed to be risk-averse or risk-neutral. Adopting the latter assumption, portfolio theory (Markowitz 1952, 1959) distinguishes between efficient and inefficient portfolios. Profile 2 represent an inefficient portfolio, defined as any composition of assets for which there exists an alternative one which offers a better expected payoff without implying a greater payoff variance (here: profile 1).

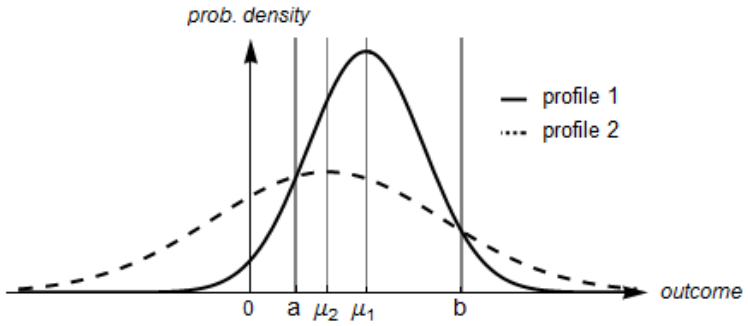

Figure 1: Probability density functions of two alternative behaviors

[3] Although this is a simplification because financial dynamics reveal power-law behavior which leads to heavier tails of the return distribution than predicted by the normal distribution [15].

From the angle of economic theory, the attractiveness of profile 1 towards 2 seems, therefore, to be evident. From the perspective of evolutionary game theory (Weibull 1997), however, ranking the alternatives is more ambiguous. The reason is that in this theory, ways of behavior are not evaluated through the individual utility produced but through their ability to generate an outcome which assures survival. The degree to which a certain behavior improves survivability is called evolutionary fitness. The credo is that the fittest strategies will tend to dominate in the long-run as rivals die out sooner or later. Therefore, if individuals are in competition with each other, the fittest strategy must be established.

Figure 1 indicates that this strategy cannot be identified uniquely. Rather, the evolutionary fitness of both profiles depends on the precise payoff needed to survive and, thus, on competitive conditions. This argument can be illustrated formally as follows: With k being the threshold outcome needed to survive, the evolutionary fitness, F, of some behavior α, can be written as:

$$F_\alpha(k) = \int_{x=k}^{\infty} PDF_\alpha\,(x) \tag{1}$$

Based on this principle, some behavior α is fitter than any alternative behavior β if and only if α offers a higher probability to attain an outcome greater than k. Formally:

$$\alpha \gg \beta \leftrightarrow F_\alpha(k) > F_\beta(k) \tag{2}$$

where "≫" should be read as "dominates", in the sense of "fitter as".

For any k with $k < a$, it is easy to see that $\int_{-\infty}^{x=k} PDF_1\,(x) < \int_{-\infty}^{x=k} PDF_2\,(x)$ and, thus, $\int_{x=k}^{+\infty} PDF_1\,(x) > \int_{x=k}^{+\infty} PDF_2\,(x)$. In other words, whenever preventing relatively bad outcomes (equal or smaller than a) is sufficient for survival, profile 1 strictly dominates 2.

In contrast, for any $k > b$, it can be seen that $\int_{x=k}^{+\infty} PDF_1\,(x) < \int_{x=k}^{+\infty} PDF_2\,(x)$. Accordingly, if survival requires large outcomes, profile 2 provides the better fitness.

The considerations above are kept as simple as possible but convey an important message: A strategy which is worse than another with regards to its expected outcome but equipped with larger outcome variance may still be prevalent in a competitive environment if competitive conditions are appropriate. Proposition 1 takes this insight onto a more general level.

Proposition 1: *Assume two behaviors, 1 and 2, whose outcomes are distributed normally with means* u_1 *and* u_2 *and variances* σ_1^2 *and* σ_2^2*. Furthermore, assume that* $u_1 > u_2$ *with* $\{u_1, u_2\} \in]-\infty; +\infty[$ *and* $\sigma_1^2 < \sigma_2^2$*. Then, for any setting of* $\{u_1, u_2, \sigma_1^2, \sigma_2^2\}$ *complying with the above conditions, there is some threshold payoff k for which behavior 2 dominates 1 according to definition (2). (Proof in appendix)*

Proposition 1 has a meaningful implication for the selection among agents. Why? Maximizing outcome expectation at a given outcome variance requires knowledge, skill, or other abilities. Portfolio theory, for example, teaches the theoretical knowledge to maximize expected return on investment at a given risk through intelligent composition of assets. To increase outcome variance at a given expectation, however, such abilities are not essential. Taking risks, respectively gambling, is sufficient. Against this background, proposition 1 can be read as follows. Under the assumption that undertaking risks increases the probability of attaining extreme outcomes, any less capable group can succeed in competition with a higher capable group if it undertakes enough risk and if the outcome needed to survive is sufficiently high. In respective scenarios, risk-seeking behavior per se provides evolutionary fitness.

The evolutionary model presented in the next section aims at sharpening our understanding about how such competitive scenarios may arise in practice, and how the outcome needed to survive can result endogenously from competitive conditions. The insights gained in this section will be helpful to interpret the simulation results obtained.

3 Example: An abstract model of the competition among fund managers

This section highlights an evolutionary model of the professional competition among fund managers. This practical example is appropriate as it fulfills two important conditions: (i) Agents compete with each other through the outcome generated by their behavior, and (ii) risk-preferences are a crucial determinant of this behavior. Section 3.1 introduces the model, Section 3.2 describes the setup of the simulation experiments, and Section 3.3 presents the simulation results.

3.1 The model

The competition among fund managers has been sketched recently by N.N. Thaleb (2001). The author, himself a fund manager for several years, indicates that fluctuation in the profession is large. Professional status is highly dependent on the profitability of the managers' investments and changes quickly. In particular, unsuccessful managers are dismissed rapidly, no matter if their failing was due to chance or lack of skill. Empirical studies confirming and extending these insights include Chevalier and Ellison (1999a,b), Golec (1996), and Khorana (1996). Chevalier and Ellison (1999a) and Golec (1996), for instance, report that differences of the performance and the risk involved between mutual funds can be attributed to the behavior and different characteristics of the individual managers. Khorana (1996) shows that manager performance, which is either measured in terms of the adjusted asset growth rates or in terms of fund portfolio returns, is indeed used as selection criterion since "there is an inverse relation between the managerial performance and fund performance". Finally, Chevalier and Ellison (1999b) delivers evidence that managers react to the resulting competitive pressure by adjusting their investment behavior, and more specifically, their attitude towards risk. For example, young managers are found to hold more conservative portfolios since they are more likely to be replaced when performing badly.

In the following, a model which replicates the real competition in a stylized fashion will be introduced. As its principle, the approach rests on agent-based modeling. Agent-based modeling is the reproduction of complex systems through the formulation of specific assumptions about the behavior of agents and their interaction. The collective volumes by Rosser (2009) and Tesfatsion and Judd (2006) demonstrate the potential of this method for the analysis of social and financial systems. Note that our model is not meant to be a precise reproduction of reality but to uncover emergent phenomena on a general level and in a tractable framework. For a better overview, the framework will be described in three parts: (i) the investment logic, (ii) the selection mechanism, and (iii) the genetic algorithm, with (ii) and (iii) representing characteristic elements of an evolutionary model.

(i) Investment Logic:

The investment logic can be summarized by the following rules:

(1) Each agent *i* disposes of some amount of capital at some period *t*, denoted by $C_{i,t}$. $C_{i,t}$ is invested into two classes of assets: a riskless and a risky one.

(2) Assets of the riskless class offer a constant, real return, μ_L. The real return produced by risky assets follows a normal distribution with mean μ_R and variance σ_R^2, with $\mu_R > \mu_L$, and $\sigma_R^2 > 0$.

(3) The share of capital invested in the risky asset by agent *i* is expressed by s_i. s_i is a mirror of *i*'s degree of risk-aversion: A lower s_i, reflects greater risk-aversion, as *i* is willing to forego more payoff on average for the purpose of a lower likelihood of extremely bad payoffs.

(4) Furthermore, agents can undertake additional risk, denoted by δ. The additional risk of the portfolio of agent *i*, δ_i, increases the variance of the portfolio without improving the expected return.

(5) Portfolios pay-off in every period t, and the pay-off generated is reinvested instantly. Consequently, the evolution of capital results as:

$$C_{i,t+1} = (1 + g_{i,t})C_{i,t},\ with\ g_{i,t} \in N(\mu_i, \sigma_i^2)\ , \tag{3}$$

where $g_{i,t}$ stands for the real return of *i*'s portfolio in period *t*; μ_i is the expected return and σ_i^2 the variance of *i*'s portfolio. Returns $g_{i,t}$ are assumed to be independent between agents.

The investment logic defined above is fairly simple but allows a large range of portfolios. In accordance with portfolio theory, the portfolio of some agent *i* can be described by its expected payoff, μ_i, and the payoff variance, σ_i^2. In our model, both values are determined by the two attributes of the respective agents: the share of capital invested in the risky asset, s_i, and the amount of additional risk taken, δ_i. We get:

$$\mu_i = s_i * \mu_R + (1 - s_i)\mu_L \tag{4}$$

and

$$\sigma_i^2 = s_i * \sigma_R^2 + \delta_i \tag{5}$$

Figure 2 illustrates the resulting set of possible portfolios in the common form of a risk-return diagram (Brealey and Myers 2003).

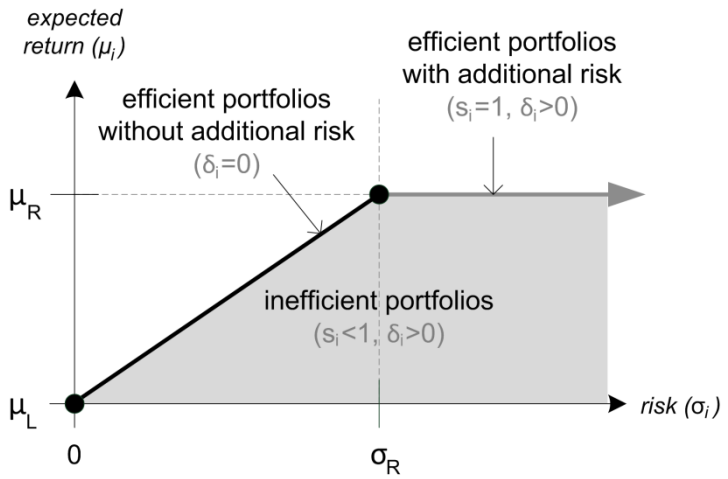

Figure 2: Risk-return diagram of portfolios possible.

The diagram shows that, depending on the choice of s_i and δ_i, efficient as well as inefficient portfolios may occur. If $\delta_i = 0$ (black line), an efficient portfolio results because, given the respective payoff variance, it is not possible to achieve a better return expectation. For any $\delta_i > 0$ and $s_i < 1$ (gray area), the portfolio is inefficient since a higher expected return could be attained at equal risk by increasing s_i. This is not possible if $\delta_i > 0$ and $s_i = 1$ (gray line); hence, the portfolio is efficient. In principle, portfolios can be any in the gray area and on its borders (including the infinite, omitted range following on the right which corresponds to a further rise of δ_i).

Let us conclude by a brief comment on the variable δ_i. Through δ_i, the model setup implies that the variance of the portfolio return can be raised arbitrarily. This may appear to be an unrealistic assumption. However, achieving greater outcome variance in reality requires little. Placing the raw investment payoff on bets with an expected return of zero is sufficient, technically. The analogue of the financial world would be investments in speculative assets with the chance for extreme returns but relatively poor expectation (e.g. so-called Junk Bonds). Choosing such assets, or respectively a positive value of δ_i, can be interpreted as risk-seeking behavior. On the other hand, if $\delta_i > 0$ implies an inefficient portfolio, a positive δ_i can be read as lack of skill: The agent could attain a better expectation at equal risk

but she has simply not learned the appropriate investment behavior. In other words, we interpret "skill" as the ability to build an efficient portfolio.

(ii) Selection Mechanism:

The selection mechanism, as one of the principal elements of an evolutionary model, is essential for the evolution of the population. For this purpose, the selection alternatives (here: different investment behaviors represented by the agents using them) are evaluated by a defined criterion, setting them in competition with each other. The better an alternative fulfills the target, the higher the likelihood of spreading instead of dying out.

In the present model, selections are undertaken by an external entity, which can be interpreted as the employing company. The rules of selection are:

(6) In constant intervals of time, the worst agents are excluded. The length of this interval is specified by the parameter v. Another parameter, r, specifies the precise percentile of agents to be excluded.

(7) The selection criterion is the return achieved by agents i in the v periods preceding to t, denoted by $g_{i,t,v}$. Due to (3), this return is mirrored by the relative change of individual investment capital. $g_{i,t,v}$ can thus be written as:

$$g_{i,t,v} = (C_{i,t} - C_{i,t-v})/C_{i,t-v} \quad (6)$$

With *v* and *r*, the selection mechanism described above has two parameters, where *v* represents the "exclusion interval" and *r* the "exclusion rate". Each parameter has a distinct function for the competition among agents. The exclusion rate has a direct effect on competitive pressure. The higher *r* is, the more agents are excluded, and the greater the outcome needed to survive. The exclusion interval, on the other hand, constitutes agents' "probation period". The higher *v* is, the more time agents have to produce investment returns.

Of course, we do not believe that promotions and exclusions in reality follow the rigorous mechanism described above. Instead, *v* and *r* should be regarded as a stylized representation of the probation time and the performance needed to be promoted. This design accounts for our general goal to keep the model simple and tractable.

(iii) The Genetic Algorithm:

By specifying the set of agents to be excluded, the selection mechanism determines the outflow from the population. In contrast, the genetic algorithm relates to the inflow by establishing the character of agents entering the population.[4]

Genetic algorithms, originated by J.H. Holland (1975), are learning methods which mimic the biological process of evolution. A genetic algorithm creates "candidate solutions" from a defined set of building blocks, which can be interpreted as genes. The construction of candidates is based on two genetic operators: crossover and mutation. Crossover is combining two candidate solutions, the "parents", to create a new one. Then, some of the genes of the new code are modified slightly, which is denoted as mutation. Typically, the resulting candidate solutions are not preselected by a defined fitness criterion but prove their fitness in competition with each other.

The technique has considerable advantages. The modeler must not define an initial set of candidates to be evaluated but merely their building blocks. Therefore, less previous knowledge about possible solutions is required. Furthermore, the search space usually becomes extremely large. Thus, the solution finally found is likely to beat the best one in the limited set of alternatives the modeler would propose himself (with the restriction that the set of potential solutions is limited by construction rules and building blocks).

Due to these features, genetic algorithms have proven to be a successful tool in many economic models – Safarzyńska and van den Bergh (2010) review some of them with focus on the modeling techniques used. Goals of application are in particular the derivation of optimal rules of trading and investment, as conducted by Lensberg (1999), Lensberg and Schenk-Hoppe (2007), Letteau (1997), Neely et al. (1997) and Potvin et al. (2004). Hauser and Kaempff (2011) represent a recent contribution from the Journal of Evolutionary Economics. A study which is related to the present one quite closely is Letteau (1997). Like in the present study, Letteau's agents build a portfolio by choosing the share of capital, *s*, to be invested

[4] It is also common to interpret the selection mechanism as a part of the genetic algorithm. From that point of view, (iii) is equivalent to the "reproduction mechanism". However, we do not adopt this view for the present model, because our selection mechanism could be implemented without genetic programming, and thus, does not contribute to the genetic feature of the model.

in a risky asset; to find the optimal value, s^*, the genetic algorithm is used. However, in contrast to the present study, the focus is not on professional investors and their competition, but agents seek to maximize their own utility function. The latter follows the standard CARA design ($U(w) = -\exp(-\gamma * w)$), with γ being the risk-aversion coefficient and w the investment payoff), which excludes risk-seeking behavior ex ante. The goal of the study is to explore if evolution will lead to the optimal solution s^* – the one which generates the greatest utility on average. (Analytically, s^* is easy to derive: $s^* = (\mu_R - p_R)/\gamma\sigma_R^2$, with p_R being the price of the risky asset). The author finds that this result is actually reached if simulation time is long enough. For shorter horizons, however, the solutions tend to exceed s^*. The reason for the latter finding is quite interesting: In the short term, agents holding large shares of risky assets can be lucky and achieve great utility. This creates a bias towards greater solutions for s than s^*. The influence of chance tends to decrease if the period under observation is longer. In a different fashion, this phenomenon will also be observable in the present study.

The implementation of the genetic algorithm in our model is relatively easy because agents are characterized by two attributes only: the share of capital invested in the risky asset, s_i, and the amount of additional risk undertaken, δ_i. The algorithm can be outlined as follows:

(8) The excluded agents are replaced by "newcomers". The investment behavior of the newcomers is determined by a genetic algorithm.
(9) Each newcomer has two genetic parents, who are randomly chosen survivors of the present selection.
(10) Initially, each of the two attributes of the newcomer is adopted identically from a different parent. This leads to a temporary setting of attributes.
(11) The final setting is reached by a slight, random alteration of one randomly chosen attribute of the temporary setting.

In the algorithm described above, rule (10) reflects crossover, and (11) mutation. Together with (9), (10) guarantees that an investment behavior is more likely to spread, the more successful it has proven to be for the survival of competition. Rule (11) effects that virtually any investment behavior, that is, any combination of values of s_i and δ_i can emerge. In other words, any of the portfolios marked in figure 2 is actually possible. Regarding the world of fund managers, "crossover" stands for a combination of existing investment strategies,

e.g. considering different rules for the selection of assets simultaneously. On the other hand, "mutation" refers to the invention of completely new selection rules.[5] In the following, the evolutionary model is used to explore two principal research questions:

- Will evolutionary competition always lead to the prevalence of agents with efficient portfolios? Transferred to the model world: Will there be any agents with $\delta_i > 0$ and $s_i < 1$ at the simulation end?
- Which investment behavior is most successful in competition under different competitive conditions? Transferred to the model world: Which values of δ_i and s_i will have emerged under different settings of competition parameters, *v* and *r*?

To answer these questions, simulation experiments are conducted, whose setup will be described in the following section.

3.2 Simulation Setup

The model calibration consists in the setting of six parameters: the three asset parameters, μ_R, μ_L and σ_R^2; the two competition parameters, *v* and *r*; and the size of the population, *N*. According to rule (2) in Section 3.2, the asset parameters are subject to the restrictions $\mu_R > \mu_L$, and $\sigma_R^2 > 0$. Complying with these restrictions, we set $\mu_R = 3\%$, $\mu_L = 0\%$, and $\sigma_R = 5\%$. Judged by empirical data, this setting can be regarded as rather conservative. For example, in the time from 1950 to 2009, the inflation adjusted total return per year produced by US large company stocks (S&P 500) was 9.68% on average with a standard deviation of 17.40%.[6] Thus, the risky asset in our model still has a less risky profile than respective equivalents in practice.[7] The competition parameters, *v* and *r*, represent the

[5] Of course, the Darwinian approach of evolution described here is not an accurate representation of the evolution of strategies in reality. This is evident by the fact that fund managers do not use to have two fund manager parents. What is important for our purpose is that the algorithm is capable of uncovering the fittest behavior.

[6] Data Sources: Ibbotson Associates (Nominal Total Return); Bureau of Labor Statistics (Inflation Rates).

[7] The reason not to adopt empirical data is that the empirical values vary largely relative to the time frame considered, as economic conditions are continuously changing. For example, between 1990

independent variables. The exclusion interval v is incremented from 1 to 20 period(s), while the exclusion rate r is altered from 0.05 to 0.95 in steps of 0.05. Each combination of v and r signifies a different setup of competition, which gives a total of 380 scenarios to be simulated. Finally, N is set to 10,000. The large number of agents makes sure that the result of evolution can be reliably attributed to a systematic advantage of the respective investment behavior. The simulation runs terminate either if convergence is reached such that no significant change of agent attributes occurs anymore, or, in case of no convergence, if the path of evolution is evident (the latter will sometimes occur with regard to δ_i, which tends to rise continuously under specific competitive conditions). The initial setting of portfolio parameters is $s_i = \delta_i = 0.5, \forall i$, which corresponds to an inefficient portfolio. Agents thus have to learn an efficient investment behavior.

3.3 Results

Figure 3 depicts the results of the simulation experiments in an illustrative two-dimensional scheme. The upper lattice shows the share of capital invested in the risky asset, s_i, the bottom one the risk undertaken additionally, δ_i. Each square represents a particular setup of competition, as specified by the setting of exclusion interval v and exclusion rate r. Different gray-levels indicate the corresponding value of the dependent variables at the simulation end. The general rule is: the darker the square, the greater the value of the respective variable. In particular, white can be read as "0". With regard to s_i, black stands for "1" (by definition s_i can never exceed 1). Regarding δ_i, black stands for extremely large values. The respective squares correspond to setups in which convergence is not reached. Hence, the values would continue to rise if the simulation went on. The lattices can be divided into five numbered areas. Each area represents a characteristic asset portfolio.

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and 2009, the respective return is merely 2.79% with a variance of 19.52%. In conclusion, the empirical data are of little use for a reliable estimation of the risk-return profile of today's risky assets. Therefore we choose an artificial setting that appears to be realistic but conservative in so far as risky assets with comparable profiles are likely to exist in reality.

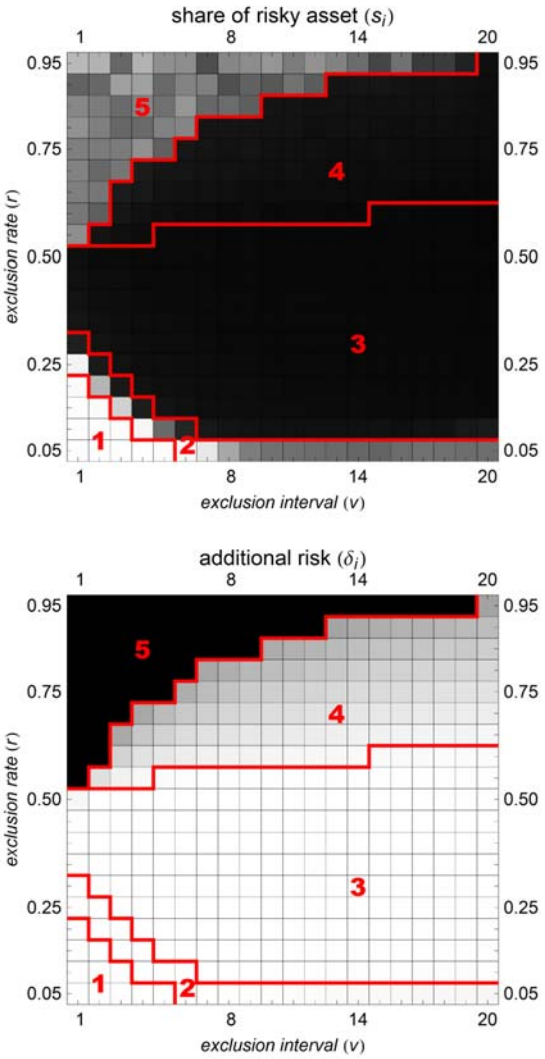

Figure 3: Values of portfolio parameters at simulation end for variable settings of competition parameters. Greater values indicated by darker gray tone.

Area 1: In this area, the exclusion rate r is relatively low but the exclusion interval v is short, that is, exclusions take place relatively often. With these competitive conditions, evolution leads to the emergence of a portfolio which consists entirely of riskless assets ($s_i = 0$) and does not involve any additional risk ($\delta_i = 0$). As explained in Section 3.1, this configuration belongs to the class of efficient portfolios. Related to the level of agents, the respective portfolio corresponds to investors embodying maximum risk-aversion and skill, with the latter being interpreted as the ability to find an efficient composition of assets. The reason for the emergence of maximally conservative but efficient investment behavior lies in the specific requirements for survival. Due to the low exclusion rate, the prevention of extremely bad outcomes is sufficient for not being discarded. Furthermore, due to the short exclusion interval, random effects have hardly leveled out before returns are evaluated. As a result, undertaking risks is perilous, as risks raise the likelihood of attaining insufficient outcomes.

Area 2: The portfolios built in area 2 differ relative to area 1 in the parameter s_i. Investors continue to avoid additional risk ($\delta_i = 0$) but now spend a positive fraction of capital on the risky asset ($0 < s_i < 1$). Comparing the competitive conditions of area 1 and 2 reveals that this difference can be due to two reasons: a slight rise of the exclusion rate (i), or of the exclusion interval (ii). The causal paths from (i) and (ii) to the increase of s_i are quite distinct. By (i), the return needed to survive is enhanced. Pure risk prevention does not imply the best probability of achieving the respective outcomes because the amount of return expectation foregone becomes too great. Investing some capital in the risky asset increases the return expectation and, thus, improves survivability. Accordingly, a mixture of risky and riskless assets provides the best evolutionary fitness. By (ii), the same result is obtained through an extension of probation time. If the exclusion interval is raised, return expectation carries more weight because random effects tend to level out, leading to an evolutionary boost of agents with a positive fraction of risky assets. In other words, undertaking risk becomes less perilous for being eliminated if probation time increases. Note that the respective portfolios remain efficient and investors still behave in a risk-averse manner, however, to a smaller degree than in area 1.

Area 3: In area 3, the exclusion rate and the exclusion interval have increased further. As a result, the tendencies described above disembogue into an extreme. The fittest strategy is now simply to maximize return expectation as reflected by

the fact that portfolios are composed entirely of risky assets ($s_i = 1$). This behavior reveals that risk-prevention does not provide any competitive advantage anymore. This result can be produced by a sufficient rise of any competition parameter; r or v. If r is raised sufficiently, any sacrifice of return expectation decreases evolutionary fitness because the relatively great outcomes needed are less likely to be attained. For a sufficient rise of v, any sacrifice of return expectation is harmful because return expectation tends to become the only criterion for survival.

The investment behavior in area 3 corresponds to risk-neutrality. Risk-neutrality implies that risks are not regarded as being a "value per se", which can be verified by the fact that agents do not undertake any risk if it does not enhance return expectation ($\delta_i = 0$). Apparently, undertaking additional risk deteriorates evolutionary fitness. The reason is that the exclusion rate is still low enough so that preventing extremely bad outcomes remains paramount towards seeking extremely good ones. Additional risk, hence, is not needed but rather destructive as it raises the probability of performing badly.

Area 4: Area 4 differs from area 3 in the very inclination of agents to undertake additional risk, which now converges at some positive value ($0 < \delta_i < +\infty$). This behavior is risk-seeking because agents accept risks even if the latter do not contribute to the maximization of return expectation. In other words, risk is regarded as a "value per se". The dominance of risk-seeking behavior is due to another rise of r, which has led to the transition of a critical level: 0.5 or 50%. To understand the phenomenon, assume that $v = 1$ and that the population consists of two groups: The first group obtains a sure periodical return of μ percent. The second group gets the same return μ but additionally plays a fair lottery which gives them a 50% chance for the return $\mu + \sigma$, respectively $\mu - \sigma$ in case of losing. It is easy to see that for both groups, the expected periodical return ($E[g_{i,t,1}]$) is equal to μ. If the population is sufficiently large, μ is also the median return; on average, half the population will achieve a return greater than μ whereas the other half falls below. However, if more than half of the population is excluded at selection periods ($r > 0.5$), the median return is not sufficient to proceed. This signifies a heavy competitive disadvantage for the first group because, in contrast to the second one, they never achieve a return greater than μ. The same occurs in the simulation. By choosing $\delta_i = 0$, agents accumulate probability mass of their return distribution near their return expectation. This is disadvantageous as the

expected return does not suffice to survive. Similar agents with $\delta_i > 0$ have greater fitness as more probability mass lies above the return needed. In conclusion, risk per se generates evolutionary fitness.

Still, the explanations above do not capture all mechanisms driving the simulation results. For example, they suggest that whenever $r > 0.5$, $\delta_i > 0$, which, as shown by figure 3, is not true. Another fact left to clarify is the convergence of δ_i on a specific value. Let us focus the latter phenomenon first. Basically, the convergence of δ_i on δ^* indicates that any agents with $\delta_i > \delta^*$ is less fit than an agent with $\delta_i = \delta^*$. The degree of risk which provides a competitive advantage, hence, is limited. To understand the cause, reconsider the second group mentioned above, which plays the lottery, and assume that $v = 2$, which implies the selection criterion to be $g_{i,t,2}$. Then, on average, half of the agents win in one period and lose in the other. The resulting return $g_{i,t,2}$ is $(1 + \mu + \sigma)(1 + \mu - \sigma) - 1$. It is easy to show that for any μ, this value decreases continuously with greater $|\sigma|$. In other words, agents who are moderately lucky achieve lower returns $g_{i,t,2}$ (and thus are less likely to survive) if the variance of their periodical returns is greater. The same happens in the model. Agents with $\delta_i > \delta^*$ require more luck than agents with $\delta_i = \delta^*$ to obtain the same return $g_{i,t,v}$. This effect is absent if $v = 1$ but increases for v being incremented above. As a result, agents chose lower values of δ_i , the greater v. This also explains why for some $r > 0.5$, $\delta_i = 0$. In the model, investing in the risky asset alone can generate the optimal return variance even without undertaking additional risk.

Area 5: The results in area 5 differ sharply from all previous ones in several aspects. Technically, the competitive conditions in this area are the only ones which do not disembogue in a convergence of portfolio parameters. In particular, the tendency of δ_i to rise does not settle down. Simultaneously, s_i fluctuates in a range below 1; an optimal weight of risky assets does not emerge. The chaotic pattern of different gray-levels confirms that the precise value of s_i at the simulation end is quite random. Because $s_i < 1$ and $\delta_i > 0$, the portfolios built are not efficient. In contrast to all previous ones, the competitive conditions in area 5 do not provoke the predominance of agents able or willing to choose an efficient composition of assets.

The peculiarity of these results shows that the competition in area 5 follows a logic which is very distinct. Apparently, skill is not an essential property to survive. Seeking risks, however, becomes vitally important. Any agent improves

her evolutionary fitness by raising δ_i. These results are caused by the particular setting of competition parameters. Due to the great value of r and relatively low v, agents require large profits in a short span of time. To increase their chance for such profits, they need to seek risks. However, investing in an equally risky manner than everyone else does not produce a competitive advantage. As a result, agents force each other into taking greater and greater risks in the attempt to excel their rivals. The counter effect described in area 4 – greater risks decreases the returns $g_{i,t,v}$ if being moderately lucky – is not relevant here because survival is only possible if very lucky.

Agents following these incentives build portfolios which enable them to attain huge profits. The actual return of these high-risk portfolios depends largely on chance. In contrast, differences in periodical return expectation, as determined by the setting of s_i, merely account for a minor part of actual returns. Hence, skill is of little importance. These findings mirror the insights from the theoretical analysis in chapter 2: Since the outcome needed to survive is relatively great, risk-seekers possess greater evolutionary fitness than risk-averters, even though the outcome expectation of risk-seekers is worse. As a result, the competition between agents leads to the predominance of risk-seekers who have been lucky, but who are not necessarily skilled.

Summary: The simulation results show that the fittest risk-preference is dependent significantly on competitive conditions. In this context, two competition parameters have proven to be decisive: the exclusion rate and the exclusion interval. The following relationships could be found: The greater the exclusion rate, that is, competitive pressure, the greater the advantage from risk-seeking behavior relative to risk prevention. Furthermore, the attractiveness of risky assets rises with greater exclusion interval, representing probation time, as chance tends to level out in the long run. Via these relationships, the setting of competition parameters can lead to various scenarios: risk-aversion, risk-neutrality, risk-seeking behavior and any risk preference in between can be the fittest behavior.

In addition, the simulation results reflect that the more risk involved by portfolios, the more actual returns are shaped by randomness and the less by the return expectation. As a result, skill is less important for survival, the more risks agents take. This logic can lead to an extreme scenario in which competition does not select the most skilled because skill is almost entirely blurred by chance. This

scenario tends to arise if competitive pressure is intense and the great returns needed to survive must be produced in a short span of time.

Of course, the above insights raise the question about the actual competitive conditions in the fund manager profession. Analyzing empirical data from 1992 to 1994, Chevalier and Ellison (1999b) find that fear of being dismissed motivates young managers to choose low-risk portfolios. Against the background of our insights, this behavior indicates that competitive pressure was in the lower range in the time of data gathering. Future research should check if competitive conditions have changed today, which might contribute to the explanation of the attractiveness of high-risk investments.

3.4 Outlook

The analysis above has shown that certain competitive conditions lead to the emergence of risk-loving agents. It is an interesting question how this behavior of individuals affects the behavior of the respective macro system. Financial markets represent such a macro system whose dynamic is driven by individual traders and their interaction.

A model that touches this question is the Santa Fe Artificial Stock Market (SFI market) (summary by LeBaron et al. 1999). As in our study, agents in the SFI market can choose between a risky or riskless asset. Another common feature of the model framework is that the population of agents evolves via a genetic algorithm. The simulation experiments concentrate on two different settings of the learning rate which turns out to be a crucial parameter for the behavior of agents and for the dynamics of the market. A fast learning rate can be compared to a low exclusion interval (the parameter v in our model) because strategies have to prove their value in a short period of time. It is found that with a fast learning rate, agents tend to rely on technical instead of fundamental trading strategies. Technical strategies aim at returns beyond fundamental payoffs and can be compared with risk-seeking behavior. The interaction of these agents leads to a destabilization of the market in terms of greater excess returns and fatter tails of the return distribution. In this fashion, the article of LeBaron (1999) gives an idea about the connection between competitive pressure, the behavior of agents, and market dynamics. Future work could concentrate on this connection in more detail. For example, it would be interesting to expand our model to include a financial market

which is driven by the interaction of our agents, while feeding back to the profit of their strategies. This might produce a broader picture of the mechanisms driving real world scenarios.

4 Conclusion

The present article explores the relationship between various competitive conditions and the evolutionary fitness of different risk preferences. The analysis is conducted theoretically as well as by an evolutionary multi-agent model that reproduces an exemplary empirical case in a stylized fashion: the competition among fund managers. This case reflects the problem very well because the risk preference of managers is a central determinant for the composition of their asset portfolio whose return managers compete about. To simulate how fund managers develop their strategies, a genetic algorithm is applied, which ensures that virtually any combination of portfolio parameters can emerge as the fittest.

On the whole, we observe that competitive environments have great influence on risk preferences and investment behavior. Hereby, two variables play a central role. On the one hand, there is the exclusion rate. A high exclusion rate means high competitive pressure and creates the need for large outcomes. This makes risk-seeking behavior attractive because the probability of high outcomes is increased. On the other hand, the time conceded to agents to attain the outcomes is crucial as random effects tend to even out in the long-run. All in all, these mechanisms can cause the emergence of risk-aversion in any degree but also risk-seeking behavior.

The latter fact gives raise to warnings. Under competitive conditions that favor risk-seeking behavior, the importance of skill (the ability to build an efficient portfolio) decreases because actual outcomes are determined mostly by chance. In the extreme case, survivors are not the most skilled but simply the luckiest risk-seekers. The simulation model reflects this phenomenon by showing that, in the respective competitive conditions, agents with inefficient portfolios can survive, provided that they choose high risk investments. Such scenarios are dangerous on any economic level, not only with reference to the competition between fund managers, because of two reasons. First, because the predominance of the most capable is undermined, and second, because the risky behavior of agents implies unforeseeable outcomes which run counter to economic stability. An example is

the competition between banks. Banks that are in heavy competition with each other have greater incentives to invest in a risk-seeking way. As a consequence, they become more vulnerable to economic crises so that the economic breakdown is reinforced.

To prevent these scenarios, our study yields two generic recommendations. First, competitive pressure must be held strictly below some critical level from which risks per se generate a competitive advantage. In the model, this is achieved by setting the exclusion rate to a low or moderate level. Second, 'probation time' should be sufficiently long such that skill instead of chance shapes individual outcomes. Of course, these recommendations need to be adapted to the particular case of application.

By showing that competitive conditions can lead to the emergence of risk-aversion or risk-taking behavior in various degrees, our model combines insight from previous research. Szpiro (1997), for example, uses a genetic algorithm to explain risk-aversion. On the other hand, Rubin and Paul Il (1979), Robson (1992, 1996) and other contributions mentioned before, focus on risk-taking behavior. A nice feature of our model might be its ability to reproduce both scenarios in a relatively simple and illustrative agent-based framework that demonstrates the importance of competitive conditions.

We believe that several mechanisms that drive the result of our simulations can be condensed algebraically. This is going to be a focal point for future research.

Appendix
Proof of Proposition 1:

To prove proposition 1, we have to show that the PDF of profile 2 is strictly greater than the one of profile 1 for $x \to +\infty$. Since by assumption both profiles represent normal distributions, the inequality to prove can be expressed as follows:

$$\lim_{x\to+\infty}\left(\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{(x+\mu_2)^2}{2\sigma_2^2}} > \frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(x+\mu_1)^2}{2\sigma_1^2}}\right)$$

s.c.:

(i) $\mu_2 = \mu_1 + \Delta_\mu$ with $\Delta_\mu \in\ ]-\infty;+\infty[$

(ii) $\sigma_2 = \sigma_1 + \Delta_\sigma$ with $\Delta_\sigma > 0$ and $\neg(\sigma_1 \to 0)$

To start, replace μ_2 and σ_2 by its correspondences. Logarithmize, and simplify the result. This gives:

$\lim_{x\to+\infty}\left(\frac{(x-\mu_1+\Delta_\mu)^2}{2\sigma_1^2+4\sigma_1\Delta_\sigma+2\Delta_\sigma^2} < \mathrm{Z}\right)$ with $\mathrm{Z} = \frac{(x-\mu_1)^2}{2\sigma_1^2} + \mathrm{Log}[\sigma_1] - \mathrm{Log}[\sigma_1 + \Delta_\mu]$.

Next apply the equivalence $\frac{a}{b+c} = \frac{a}{b} - \frac{ac}{b(b+c)}$ setting $a = (x - \mu_1 + \Delta_\mu)^2$, $b = 2\sigma_1^2$, and $c = 4\sigma_1\Delta_\sigma + 2\Delta_\sigma^2$. This gives:

$$\lim_{x\to+\infty}\left(\frac{a}{b} - \frac{ac}{b(b+c)} < \mathrm{Z}\right)$$

Expanding a in the first term and rearranging the result leads to:

$$\lim_{x\to+\infty}\left(\frac{(x-\mu_1)^2}{b} + \frac{\Delta_\mu^2 - 2\Delta_\mu\mu_1 + 2\Delta_\mu x}{2\sigma_1^2} - \frac{ac}{b(b+c)} < \mathrm{Z}\right)$$

Expanding Z and subtracting $\frac{(x-\mu_1)^2}{b}$ from both sides finally yields:

$$\frac{\Delta_\mu^2 - 2\Delta_\mu\mu_1 + 2\Delta_\mu x}{2\sigma_1^2} - \frac{ac}{b(b+c)} < \mathrm{Log}[\sigma_1] - \mathrm{Log}[\sigma_1 + \Delta_\sigma]$$

It can been seen that the instance of x with the highest power is included in a with x^2. The limit of the left side as x approaches $+\infty$ is thus determined by the term $-\frac{ac}{b(b+c)}$ whose limit is $-\infty$. Since due to (ii), the limit of the right side is greater than $-\infty$, the inequality is true.

q.e.d.

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