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The difference between the equal division value and the equal surplus
division value

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UNIVERSITÄT LEIPZIG

Wirtschaftswissenschaftliche Fakultät
Faculty of Economics and Management Science

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Nullifying vs. dummifying players or
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Abstract

We provide new characterizations of the equal surplus division value and the equal division value as well as of the class of their convex mixtures. This way, the difference between the Shapley value, the equal division value, and the equal surplus division value is pinpointed to one axiom. Moreover, we shed light on solidarity principles embodied in the equal division value and in the equal surplus division values.

Key Words: Solidarity, nullifying player, equal division value, equal surplus division value, desirability

JEL code: C71, D60

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1. INTRODUCTION

The class of convex mixtures of the equal surplus division value (*ES-value*) and the equal division value (*ED-value*) for cooperative games with transferable utility (*TU-games*) is particularly interesting in the context of the allocation of an indivisible good via auctions. Graham, Marshall and Richard (1990) derive a TU-game from the auction setup, which captures the consequences of bidder coalitions. The allocations resulting from the application of the Shapley value to this TU-game are not envy-free. van den Brink (2007) shows that the envy-free and Pareto-efficient allocations coincide with the payoffs prescribed by the convex mixtures of the ES-value and the ED-value. Hence, one might be interested in the properties that actually drive these concepts.

van den Brink (2007) suggests a surprising characterization of the ED-value that employs the *nullifying player property* instead of the null player property in the standard characterization of the Shapley value (Shapley, 1953), which also includes efficiency, additivity, and the equal treatment property. A player is called *nullifying* if his presence in a coalition prevents production, i.e., if his presence renders a coalition's worth zero. The nullifying player property requires a nullifying player to obtain a zero payoff. This way, the difference between the Shapley value and the ED-value can be pinpointed to just one axiom.

van den Brink (2007) also provides a characterization of the ES-value, which deviates from his characterization of the ED-value above by restricting the nullifying player property to zero-normalized games and adding the invariance property. We feel that this characterization has a minor drawback. For, one would like to have that the difference of the solution concepts manifest itself in a single axiom.

We provide a characterization of the ES-value that replaces the null player property in the standard characterization of the Shapley value with the *dummifying player property*. A player is called *dummifying* if his presence in a coalition prevents cooperation, so that the players in this coalition fall back to their singleton productivities. The dummifying player property then requires a dummifying player to obtain his singleton worth as payoff. Further, another two axioms employed by van den Brink (2007) in order to characterize the ED-value are modified in order to characterize the ES-value.

Next, we turn to a feature that is shared by the nullifying player property and the dummifying player property. Both axioms deal with a player who neutralizes

productivity or gains from cooperation. Alternatively, one could look at the effects on the players' payoffs if one player's productivity or his ability to create gains from cooperation *is* neutralized, i.e., if he is *nullified* (becomes a null player) or if he is *dummified* (becomes a dummy player).

Clearly, the ED-value changes the outcomes of all players by the same amount whenever a player is nullified or dummified. Hence, the ED-value satisfies the *nullified player property* (*dummified player property*), which requires that whenever a player is nullified (dummified) the payoffs of all players are changing in the same direction.

We show that replacing the null player property in the standard characterization of the Shapley value by the nullified player property yields a new characterization of the ED-value. At first glance, this seems to be surprising since Chun and Park (2012) employ a similar property, *population solidarity*, in order to characterize the ES-value. Whenever a number of players leaves the game, population solidarity requires that the payoffs of all players remaining in the game are changing in the same direction. In contrast to our nullified player property, their population solidarity axiom involves varying player sets.

While the ES-value violates the nullified player property, it turns out to satisfy the *dummified player property*, which demands that whenever a player is dummified it is not possible that one player gains while another one loses. Apart from the ES-value and the ED-value, there exists a multitude of other solution concepts that satisfy the dummified player property as well as efficiency, additivity, and the equal treatment property. Among them, we find the class of convex mixtures of the ES-value and the ED-value. We give characterizations of this class and of the ES-value employing the dummified player property.

This paper is organized as follows. Further definitions and notations are given in Section 2. Sections 3 and 4 provide new characterizations of the ES-value and the ED-value, respectively. In Section 5, we explore implications of the dummified player property.

2. BASIC DEFINITIONS AND NOTATION

A **(TU-)game** is a pair (N, v) consisting of a non-empty and finite set of players N and a **coalition function** $v \in \mathbb{V}(N) := \{f : 2^N \rightarrow \mathbb{R} \mid f(\emptyset) = 0\}$. Since we work within a fixed player set, we frequently drop the player set as an argument. In particular, we address $v \in \mathbb{V}$ as a game. Subsets of N are called **coalitions**;

$v(S)$ is called the worth of coalition S . For $v, w \in \mathbb{V}$, $\alpha \in \mathbb{R}$, the coalition functions $v + w \in \mathbb{V}$ and $\alpha \cdot v \in \mathbb{V}$ are given by $(v + w)(S) = v(S) + w(S)$ and $(\alpha \cdot v)(S) = \alpha \cdot v(S)$ for all $S \subseteq N$. For $T \subseteq N$, $T \neq \emptyset$, the game $e_T \in \mathbb{V}$, $e_T(S) = 1$ if $T = S$ and $e_T(S) = 0$ for $T \neq S$, is called a **standard game**. The **null game** $\mathbf{0} \in \mathbb{V}$ is given by $\mathbf{0}(S) = 0$ for a all $S \subseteq N$. For $T \subseteq N$, $T \neq \emptyset$, the game $u_T \in \mathbb{V}$, $u_T(S) = 1$ if $T \subseteq S$ and $u_T(S) = 0$ for $T \not\subseteq S$, is called a **unanimity game**. Any $v \in \mathbb{V}$ can be uniquely represented by unanimity games,

$$v = \sum_{T \subseteq N: T \neq \emptyset} \lambda_T(v) \cdot u_T, \quad \lambda_T(v) := \sum_{S \subseteq T: S \neq \emptyset} (-1)^{|T|-|S|} \cdot v(S). \quad (1)$$

For $x \in \mathbb{R}^N$, the **modular game** $m_x \in \mathbb{V}$ is given by $m_x(S) := \sum_{i \in S} x_i$. A game $v \in \mathbb{V}$ is called **zero-normalized** if $v(\{i\}) = 0$ for all $i \in N$; for $v \in \mathbb{V}$, the **associated zero-normalized game** $v^0 \in \mathbb{V}$ is given by $v^0(S) := v(S) - \sum_{i \in S} v(\{i\})$ for all $S \subseteq N$. A game $v \in \mathbb{V}$ is called **monotonic** if $v(S) \geq v(T)$ for all $S, T \subseteq N$ such that $T \subseteq S$. Player $i \in N$ is called a **dummy player** in $v \in \mathbb{V}$ if $v(S \cup \{i\}) - v(S) = v(\{i\})$ for all $S \subseteq N \setminus i$; player $i \in N$ is called a **null player** in $v \in \mathbb{V}$ if $v(S \cup \{i\}) = v(S)$ for all $S \subseteq N \setminus \{i\}$. Players $i, j \in N$ are called **symmetric** in $v \in \mathbb{V}$ if $v(S \cup \{i\}) = v(S \cup \{j\})$ for all $S \subseteq N \setminus \{i, j\}$.

A **value** on N is an function φ that assigns a payoff vector $\varphi(v) \in \mathbb{R}^N$ to any $v \in \mathbb{V}$. The **equal division value (ED-value)** is given by

$$\text{ED}_i(v) := \frac{v(N)}{|N|}, \quad \text{for all } i \in N. \quad (2)$$

The **equal surplus division value (ES-value)** (Driessen and Funaki, 1991) is given by

$$\text{ES}_i(v) := v(\{i\}) + \frac{v(N) - \sum_{j \in N} v(\{j\})}{|N|}, \quad \text{for all } i \in N. \quad (3)$$

Later on, we employ the following standard axioms for values on N .

Efficiency, E. For all $v \in \mathbb{V}$, $\sum_{i \in N} \varphi_i(v) = v(N)$.

Null player, N. For all $v \in \mathbb{V}$ and every $i \in N$, who is a null player in v , $\varphi_i(v) = 0$.

Additivity, A. For all $v, w \in \mathbb{V}$, $\varphi(v + w) = \varphi(v) + \varphi(w)$.

Linearity, A. For all $v, w \in \mathbb{V}$ and $\alpha \in \mathbb{R}$, $\varphi(v + w) = \varphi(v) + \varphi(w)$ and $\varphi(\alpha \cdot v) = \alpha \cdot \varphi(v)$.

Equal treatment, ET. For all $v \in \mathbb{V}$ and $i, j \in N$, who are symmetric in v , $\varphi_i(v) = \varphi_j(v)$.

Symmetry, S. For all $v \in \mathbb{V}$, $i \in N$, and all bijections $\pi : N \rightarrow N$, $\varphi_{\pi(i)}(v \circ \pi^{-1}) = \varphi_i(v)$, where $v \circ \pi^{-1} \in \mathbb{V}$ is given by $v \circ \pi^{-1}(S) = v(\pi^{-1}(S))$, $S \subseteq N$.

Invariance, I. For all $v \in \mathbb{V}$, $\alpha \in \mathbb{R}$, and $b \in \mathbb{R}^N$, $\varphi(\alpha \cdot v + m_b) = \alpha \cdot \varphi(v) + b$.

3. DUMMIFYING PLAYERS

The standard characterization of the Shapley value (Shapley, 1953) employs efficiency, additivity, the equal treatment property, and the null player property. The null player property indicates the fact that the Shapley value particularly reflects a player's own productivity. Within the null player property, van den Brink (2007) replaces null players by nullifying players. A player $i \in N$ is **nullifying** in $v \in \mathbb{V}$ if $v(S) = 0$ for all $S \subseteq N$ such that $i \in S$. This yields the following axiom.

Nullifying player, Ng. For all $v \in \mathbb{V}$ and $i \in N$ such that i is nullifying in v , we have $\varphi_i(v) = 0$.

According to the nullifying player property, a zero payoff is assigned to nullifying players, i.e., to players whose presence renders a coalition's worth zero. van den Brink (2007) justifies this axiom with a bargaining argument. Replacing the null player property by the nullifying player property, he obtains a new characterization of the ED-value.

Theorem 1 (van den Brink 2007). *The ED-value is the unique value that satisfies efficiency, additivity, the equal treatment property, and the nullifying player property.*

van den Brink (2007, Theorem 4.1) also provides a characterization of the ES-value, which deviates from his characterization of the ED-value above by restricting the nullifying player property to zero-normalized games and adding the invariance property. However, we feel that making use of the restricted nullifying property somewhat blurs the characteristic property of the equal surplus division rule.

A nullifying player does not only obstruct cooperation within any coalition containing him, but also neutralizes the productive potential of such a coalition. Dropping the latter feature of a nullifying player leads to the notion of a dummifying player, i.e., a player whose presence rules out any cooperation but does not neutralize the stand-alone productivities of the players in his coalition. Formally, a player $i \in N$ is **dummifying** in $v \in \mathbb{V}$ if $v(S) = \sum_{j \in S} v(\{j\})$ for all $S \subseteq N$ such that $i \in S$. Analogously to the nullifying player property one obtains the dummifying player property below.

Dummifying player, Dg. For all $v \in \mathbb{V}$ and $i \in N$ such that i is dummifying in v , we have $\varphi_i(v) = v(\{i\})$.

Replacing the null player property in the standard characterization of the Shapley value by the dummifying player property, one obtains a characterization of the ES-value.¹

Theorem 2. *The ES-value is the unique value that satisfies efficiency, additivity, the equal treatment property, and the dummifying player property.*

Proof. By (3), it is clear that ES obeys **E**, **A**, and **ET**. Let $i \in N$ be dummifying in $v \in \mathbb{V}$. Hence, $v(N) = \sum_{j \in N} v(\{j\})$. By (3), we have $\text{ES}_i(v) = v(\{i\})$. Thus, ES meets **Dg**.

Now, let φ be a value on N that satisfies **E**, **A**, **ET**, and **Dg**. Clearly, **Dg** implies the restriction of **Ng** to zero-normalized games. In view of (the proof of) van den Brink (2007, Theorem 3.1), it suffices to show that **A** and **Dg** imply $\varphi(v + m_b) = \varphi(v) + \varphi(m_b)$ for all $v \in \mathbb{V}$ and $b \in \mathbb{R}^N$. By **A**, we have $\varphi(v + m_b) = \varphi(v) + \varphi(m_b)$. Since all players are dummifying in m_b , **Dg** implies $\varphi(m_b) = b$ and we are done. \square

van den Brink (2007) suggests another two axioms in order to characterize the ED-value. The first axiom, coalitional standard equivalence² is related to a version of Chun's (1989) coalitional strategic equivalence³. The second one, coalitional monotonicity⁴, is related to Young's (1985) strong monotonicity⁵. In coalitional standard equivalence, we replace nullifying players by dummifying players and obtain coalitional surplus equivalence below. In order to accommodate to the ES-value, we further modify coalitional monotonicity yielding coalitional surplus monotonicity below.

Coalitional surplus equivalence, CSE. For all $v, w \in \mathbb{V}$ and $i \in N$ such that i is dummifying in w , we have $\varphi_i(v + w) = \varphi_i(v) + w(\{i\})$.

¹One easily checks that all characterizations in this paper are non-redundant.

²A solution φ satisfies coalitional standard equivalence if for all $v, w \in \mathbb{V}$ and $i \in N$ such that i is a nullifying player in w , we have $\varphi_i(v + w) = \varphi_i(v)$.

³A solution φ satisfies coalitional strategic equivalence if for all $v, w \in \mathbb{V}$ and $i \in N$ such that i is a null player in w , we have $\varphi_i(v + w) = \varphi_i(v)$.

⁴A solution φ satisfies coalitional monotonicity if for all $v, w \in \mathbb{V}$ and $i \in N$ such that $v(S) \geq w(S)$ for all $S \subseteq N$, $S \ni i$, we have $\varphi_i(v) \geq \varphi_i(w)$.

⁵A solution φ satisfies strong monotonicity if for all $v, w \in \mathbb{V}$ and $i \in N$ such that $v(S \cup \{i\}) - v(S) \geq w(S \cup \{i\}) - w(S)$ for all $S \subseteq N \setminus \{i\}$, we have $\varphi_i(v) \geq \varphi_i(w)$.

Coalitional surplus monotonicity, CSM. For all $v, w \in \mathbb{V}$ and $i \in N$ such that $v^0(S) \geq w^0(S)$ for all $S \subseteq N$, $S \ni i$, we have $\varphi_i(v) - v(\{i\}) \geq \varphi_i(w) - w(\{i\})$.

Replacing coalitional standard equivalence in van den Brink's (2007) characterization of the ED-value by coalitional surplus equivalence, one obtains a characterization of the ES-value.

Theorem 3. *The ES-value is the unique value that satisfies efficiency, the equal treatment property, and either (i) coalitional surplus equivalence or (ii) coalitional surplus monotonicity.*

Proof. By (3), it is clear that ES obeys **E**, **ET**, **CSE**, and **CSM**. Now, let φ be a value on N that satisfies **E**, **ET**, and **CSE**.

For all $v \in \mathbb{V}$, all players are dummifying in $v - v^0$. By **CSE**, $\varphi_i(v) = \varphi_i(v^0) + v(\{i\})$ for all $i \in N$. Therefore, it suffices to restrict attention to the class of zero-normalized games. For this class, **CSE** becomes coalitional standard equivalence. Moreover, the proof of van den Brink (2007, Theorem 3.2) works within the class of zero-normalized games. Hence, $\varphi = \text{ES}$ for zero-normalized games, what establishes uniqueness for (i). Part (ii) follows from the observation that **CSM** implies **CSE**. \square

In analogy to van den Brink (2007, Theorem 3.3), we establish that the equal treatment property can be relaxed into the weak equal treatment property in the second part of the above theorem.

Weak equal treatment, ET^- . For all $v \in \mathbb{V}$ such that all players are pairwise symmetric in v , we have $\varphi_i(v) = \varphi_j(v)$ for all $i, j \in N$.

Theorem 4. *The ES-value is the unique value that satisfies efficiency, the equal treatment property, and coalitional surplus monotonicity.*

Proof. By (3), it is clear that ES obeys **E**, **ET**⁻, and **CSM**. Now, let φ be a value on N that satisfies **E**, **ET**⁻, and **CSM**.

We adapt the proof of van den Brink (2007, Theorem 3.3). For $v \in N$ and $i \in N$, define $v^* \in \mathbb{V}$ by

$$v^*(S) = \begin{cases} v^0(N), & S = N, \\ \min_{T \subseteq N} v^0(T), & S \subsetneq N, |S| > 1, \\ 0, & S \subsetneq N, |S| \leq 1. \end{cases}$$

By **E** and **ET**⁻, $\varphi_j(v^*) = v^0(N) / |N|$ for all $j \in N$. Since $v^0(S) \geq (v^*)^0(S)$ for all $S \subseteq N$, **CSM** implies

$$\varphi_i(v) - v(\{i\}) \geq \varphi_i(v^*) - v^*(\{i\}) = v^0(N) / |N|$$

for arbitrary $i \in N$. Hence, $\varphi_i(v) \geq \text{ES}_i(v)$ for all $i \in N$. By **E**, we have $\varphi = \text{ES}$. \square

The theorems in this section show that the equal surplus division rule treats dummifying players as the Shapley value treats dummy players, while the equal division rule handles nullifying players as the Shapley value treats null players.

4. NULLIFIED PLAYERS

Chun and Park (2012) employ population solidarity, in order to characterize the ES-value. This axiom refers to a situation where a number of players leaves the game and with them their productivity. Population solidarity then requires that the payoffs of all players remaining in the game are changing in the same direction. However, this axiom cannot be employed in order to characterize the ES-value on a fixed player set, i.e., when these player do not leave the game but stay as null players. Indeed, the ES-value fails the nullified player property.

Nullified player, Nd. For all $v \in \mathbb{V}$, and all $i, j \in N$, $\varphi_i(v) \geq \varphi_i(v^{\text{Ni}})$ implies $\varphi_j(v) \geq \varphi_j(v^{\text{Ni}})$, where $v^{\text{Ni}} \in \mathbb{V}$ is given by

$$v^{\text{Ni}}(S) = v(S \setminus \{i\}) \quad \text{for all } S \subseteq N. \quad (4)$$

This axiom refers to a situation where some player i loses his entire productivity, i.e., player i becomes a null player, while the worths that can be achieved without him remain unaffected. In such a scenario, the axiom imposes some sort of solidarity among the players. In particular, it prevents that another player gains, while player i suffers losses.

Other than the ES-value, the ED-value clearly meets the nullified player property. In fact, together with additivity, efficiency and the equal treatment property, the nullified player property already characterizes the ED-value.

Theorem 5. *The ED-value is the unique TU-value that satisfies efficiency, additivity, the equal treatment property, and the nullified player property.*

Proof. It is easy to check that the ED-value satisfies **E**, **A**, **ET**, and **Nd**. Let now φ satisfy **E**, **A**, **ET**, and **Nd**. First, we show that

$$\varphi_i(v) - \varphi_i(v^{\text{Ni}}) = \varphi_j(v) - \varphi_j(v^{\text{Ni}}) \quad (5)$$

for all $v \in \mathbb{V}$ and all $i, j \in N$. Suppose there are $v \in \mathbb{V}$ and $i, j \in N$ such that $\varphi_i(v) - \varphi_i(v^{\mathbf{N}i}) \neq \varphi_j(v) - \varphi_j(v^{\mathbf{N}i})$. By **A**, we are allowed to restrict attention to the case

$$\eta_i := \varphi_i(v) - \varphi_i(v^{\mathbf{N}i}) > \varphi_j(v) - \varphi_j(v^{\mathbf{N}i}) =: \eta_j.$$

Then, there exists some $\xi \in \mathbb{R}$ such that $\eta_i > \xi > \eta_j$. Define $w := v - \xi \cdot |N| \cdot u_N$. By **E**, **A**, and **ET**, we have $\varphi_i(w) = \varphi_i(v) - \xi$ and $\varphi_j(w) = \varphi_j(v) - \xi$. Further, by (4), $w^{\mathbf{N}i} = v^{\mathbf{N}i}$. Hence, we obtain

$$\varphi_i(w) - \varphi_i(w^{\mathbf{N}i}) = \varphi_i(v) - \varphi_i(v^{\mathbf{N}i}) - \xi = \eta_i - \xi > 0$$

but

$$\varphi_j(w) - \varphi_j(w^{\mathbf{N}i}) = \varphi_j(v) - \varphi_j(v^{\mathbf{N}i}) - \xi = \eta_j - \xi < 0,$$

a contradiction to **Nd**. This establishes (5) for all $v \in \mathbb{V}$ and all $i, j \in N$.

Let now $\lambda \in \mathbb{R}$ and $T \subseteq N$, $T \neq \emptyset$. For $i \in T$, we have $(\lambda \cdot u_T)^{\mathbf{N}i} = \mathbf{0}$. By **E** and **ET**, $\varphi_j(\mathbf{0}) = 0$ for all $j \in N$. Together with (5), we infer $\varphi_j(\lambda \cdot u_T) = \varphi_k(\lambda \cdot u_T)$ for all $j, k \in N$. Then, **E** implies $\varphi_j(\lambda \cdot u_T) = \lambda/|N|$ for all $j \in N$. Finally, **A** gives $\varphi = \text{ED}$. \square

5. DUMMIFIED PLAYERS

While the ES-value does not exhibit the kind of strong solidarity embodied in the nullified player property, it satisfies another solidarity requirement, the dummified player property.

Dummified player, Dd. For all $v \in \mathbb{V}$, and all $i, j \in N$, $\varphi_i(v) \geq \varphi_i(v^{\mathbf{D}i})$ implies $\varphi_j(v) \geq \varphi_j(v^{\mathbf{D}i})$, where $v^{\mathbf{D}i} \in \mathbb{V}$ is given by

$$v^{\mathbf{D}i}(S) = \begin{cases} v(S \setminus \{i\}) + v(\{i\}), & i \in S, \\ v(S), & i \in N \setminus S, \end{cases} \quad \text{for all } S \subseteq N. \quad (6)$$

Compare this axiom with the nullified player property. While the nullified player property deals with players who not only lose their ability to cooperate but also lose their entire productivity, the dummified player property considers players who just lose their ability to cooperate, i.e., who become dummy players. In such a situation, the dummified player property rules out that one player gains while another player loses.

Clearly, the ED-value also satisfies the dummified player property. Hence, replacing the null player property in the standard characterization of the Shapley value

by the dummified player property does not give a characterization of the ES-value. A closer inspection of the dummified player property gives hints for why this is the case: A crucial difference between the nullified and the dummified player property is that the latter is silent whenever there are no gains from cooperation, i.e., for modular games. This gap can be bridged by the modularity axiom below.

Modularity, M. For all $b \in \mathbb{R}^N$, $\varphi(m_b) = b$.

This property states that whenever there are no gains from cooperation, the player remain with their stand-alone worths. Note that virtually all single-point solutions for TU-games except the ED-value satisfy this property.

Theorem 6. *The ES-value is the unique value that satisfies efficiency, additivity, the equal treatment property, the dummified player property, and modularity.*

Proof. It is clear that ES obeys **E**, **A**, **ET**, and **M**. Moreover, we have $ES_i(v) - ES_j(v) = ES_i(v^{\mathbf{D}^i}) - ES_j(v^{\mathbf{D}^i})$ for all $v \in \mathbb{V}$ and $i, j \in N$, i.e., ES satisfies **Dd**.

Let now φ satisfy **E**, **A**, **ET**, and **Dd**. For every $v \in \mathbb{V}$, set $m^v := v - v^0$. Since m^v is modular, **M** entails $\varphi_i(m^v) = v(\{i\})$. Moreover, **Dd** implies **Nd** on the class of zero-normalized games. Since the proof of Theorem 5 works within this class, we have $\varphi(v^0) = ED(v^0)$ for all $v \in \mathbb{V}$. Thus by **A**, we have $\varphi(v) = ED(v^0) + v(\{i\}) = ES(v)$. \square

Now we turn to the question which solution concepts apart from the ES-value and the ED-value are obtained if we no longer require modularity. Strengthening additivity into linearity, we obtain a first answer to this question.

Proposition 7. *A value φ satisfies efficiency, linearity, the equal treatment property, and the dummified players property if and only if there is some $\alpha \in \mathbb{R}$ such that $\varphi = E^\alpha$ where*

$$E^\alpha := \alpha \cdot ES + (1 - \alpha) \cdot ED. \quad (7)$$

Proof. It is clear that E^α , $\alpha \in \mathbb{R}$ obeys **E**, **L**, and **ET**. Moreover, we have $E_i^\alpha(v) - E_j^\alpha(v) = E_i^\alpha(v^{\mathbf{D}^i}) - E_j^\alpha(v^{\mathbf{D}^i})$ for all $v \in \mathbb{V}$ and $i, j \in N$, i.e., E^α satisfies **Dd**.

Let now φ satisfy **E**, **L**, **ET**, and **Dd**. For $|N| = 1$, **E** implies $\varphi = E^\alpha$ for any α . Now let $|N| > 1$. Since $(v - \xi \cdot |N| \cdot u_N)^{\mathbf{D}^i} = v^{\mathbf{D}^i}$ for $\xi \in \mathbb{R}$, an argument as in the proof of Theorem 5 yields

$$\varphi_i(v) - \varphi_i(v^{\mathbf{D}^i}) = \varphi_j(v) - \varphi_j(v^{\mathbf{D}^i}) \quad (8)$$

for all $v \in \mathbb{V}$ and all $i, j \in N$.

Next, fix some $i, j \in N, i \neq j$. Define $\alpha := 1 - |N| \cdot \varphi_j(u_{\{i\}})$. Malawski (2008, Theorem 2) shows that **E**, **L**, and **ET** imply **S**. Hence, α is well-defined, i.e., it does not depend on the choice of i and j .

We proceed by showing $\varphi = E^\alpha$. By **L**, it suffices to consider unanimity games, i.e., to show $\varphi(u_T) = E^\alpha(u_T)$ for all $T \subseteq N, T \neq \emptyset$. For $T = \{i\}, i \in N$. By definition of α , we have $\varphi_j(u_{\{i\}}) = (1 - \alpha) / |N| = E_j^\alpha(u_{\{i\}})$ for $j \in N \setminus \{i\}$. Hence by **E**, $\varphi_i(u_{\{i\}}) = 1 - (|N| - 1) / |N| \cdot (1 - \alpha) = E_i^\alpha(u_{\{i\}})$. Let now $|T| > 1$. For $i \in T$, we have $(u_T)^{D^i} = \mathbf{0}$. By **E** and **ET**, $\varphi_j(\mathbf{0}) = 0$ for all $j \in N$. Together with (8), we infer $\varphi_j(u_T) = \varphi_k(u_T)$ for all $j, k \in N$. Then, **E** implies $\varphi_j(u_T) = 1 / |N| = E^\alpha(u_T)$ for all $j \in N$. \square

The answer given in the previous proposition is not fully satisfactory. In particular, for $\alpha < 0$, the values E^α may not satisfy a desirable property as desirability (Maschler and Peleg, 1966) below.

Desirability, D. For all $v \in \mathbb{V}$ and $i, j \in N$ such that $v(K \cup \{i\}) \geq v(K \cup \{j\})$ for all $K \subseteq N \setminus \{i, j\}$, we have $\varphi_i(v) \geq \varphi_j(v)$.⁶

Moreover, Kalai and Samet (1987) suggest the positivity axiom in order to characterize the weighted Shapley values. Positivity requires non-negative payoffs in monotonic games. Within the present context, a weakening of this property seems to be appropriate since the ES-value fails positivity: For $N = \{1, 2, 3\}$, we have $ES_3(u_{\{1\}} + u_{\{2\}} - u_{\{1,2\}}) = -1/3$. Note that this game is not weakly cohesive. A game $v \in \mathbb{V}$ is called **weakly cohesive** if $v(N) \geq \sum_{i \in N} v(\{i\})$. Restricting positivity to weakly cohesive games leads to the following property.

Positivity for weakly cohesive games, PWC. For all $v \in \mathbb{V}$ that are monotonic and weakly cohesive, we have $\varphi_i(v) \geq 0$ all $i \in N$.

Positivity for weakly cohesive games reflects the notion that no player needs to obtain a negative outcome whenever players are productive and the pie to be distributed is large enough. Yet, for $\alpha > 1$, the values E^α may fail this property: For $N = \{1, 2\}$, we have $E_2^2(u_{\{1\}}) = -1/2$ while $u_{\{1\}}$ is weakly cohesive and monotonic.

The following theorem identifies those parameters α for which both of the properties mentioned above are met. Note that these parameters describe exactly the

⁶Desirability is also known as *local monotonicity* (e.g. Levinský and Silársky, 2004) or *fair treatment* (e.g. Radzik and Driessen, 2012).

class of solutions that van den Brink (2007, Proposition 5.1) identifies to be interesting in the context of the allocation of an indivisible good as is indicated in the introduction.

Theorem 8. *A value φ satisfies efficiency, additivity, the desirability, the dummified players property, and positivity for weakly cohesive games if and only if there is some $\alpha \in [0, 1]$ such that $\varphi = E^\alpha$.*

Proof. According to Proposition 7, E^α satisfies **E**, **A**, and **Dd**. Since $E_i^\alpha(v) - E_j^\alpha(v) = \alpha \cdot (v(\{i\}) - v(\{j\}))$, E^α satisfies **D** for $\alpha \geq 0$. Moreover, for monotonic and weakly cohesive v , we have

$$E_i^\alpha(v) = \alpha \cdot \left(\frac{v(N) - \sum_{j \in N} v(\{j\})}{|N|} \right) + (1 - \alpha) \cdot \frac{v(N)}{|N|} + \alpha \cdot v(\{i\}) \geq 0$$

for $\alpha \in [0, 1]$ and $i \in N$, i.e., E^α satisfies **PWC**.

Now let φ obey **E**, **A**, **D**, **Dd** and **PWC**. For $|N| = 1$, E^α does not depend on α . Let now $|N| > 1$. According to Casajus and Huettner (2012, Lemma 3), **E**, **A**, and **D** imply **L**. By Proposition 7, there exists some $\alpha \in \mathbb{R}$ such that $\varphi = E^\alpha$. Since $E_i^\alpha(u_{\{i\}}) - E_j^\alpha(u_{\{i\}}) = \alpha$ for $i, j \in N$, $i \neq j$, $\alpha < 0$ contradicts **D**. And since $E_j^\alpha(u_{\{i\}}) = (1 - \alpha) / |N|$ for $i, j \in N$, $i \neq j$, $\alpha > 1$ contradicts **PWC**. \square

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