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Direct democracy, political delegation, and responsibility substitution

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"Direct Democracy, Political Delegation, and Responsibility Substitution"

Key words: Direct Democracy, Initiative, Referendum, Political Agency, Delegation

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Direct Democracy, Political Delegation, 
and Responsibility Substitution*

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Abstract

Can direct democracy provisions improve welfare over pure representative democracy? This paper studies how such provisions affect politicians’ incentives and selection. While direct democracy allows citizens to correct politicians’ mistakes, it also reduces the incentives of elected representatives to search for good policies. This responsibility substitution reduces citizens’ ability to screen competent politicians, when elections are the only means to address political agency problems. A lower cost of direct democracy induces a negative spiral on politicians incentives, which we characterize by a disincentive multiplier. As a consequence, introducing initiatives or lowering their cost can reduce voters’ expected utility. Moreover, when elections perform well in selecting politicians and provide incentives, this indirect welfare reducing effect is stronger.

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1 Introduction

Direct democracy provisions, such as citizen initiatives, proposals, and referenda, play an important role in regimes that are otherwise based on representative democracy. In the United States, for instance, 377 initiatives have been proposed at state level between 1990 and 1999 and 371 between 2000 and 2008.\(^1\) In Europe, referenda have recently been used to decide on treaty adoption by members of the European Union. Whether such direct democracy provisions improve welfare is an ongoing debate.\(^2\)

In this paper, we put aside the well-known “tyranny of the majority” argument associated with direct democracy,\(^3\) and focus instead on another essential, yet unexplored, implication: the responsibility substitution between citizens and their representative politicians. The paper studies how responsibility substitution modifies the political agency problem between citizens and elected politicians, politicians’ behavior, and citizens’ welfare. How severe is this substitution effect?

Our analysis identifies three natural conditions which, taken together, imply a potentially severe substitution effect. The first is that politician competence affects multiple aspects of policy making, some which may be substituted for by direct democracy, and some which may not. This means, in particular, that while citizens can correct some of the mistakes that politicians make, they cannot fully compensate the shortcomings of an incompetent politician. The second, standard, assumption is that the more involved a politician is on some issue, and the better citizens can assess his competence. This assumption is reminiscent of the standard single-crossing condition in signaling models, whereby the impact of a politician’s effort on the quality of policy making is increasing in his competence. The specific condition, however, is not a standard single-crossing condition.\(^4\) Finally we assume that, while they have aligned preferences, citizens are unable to commit to (or coordinate on) a punishment strategy for a low-performing politician. In our model, citizens reelect the incumbent if and only if their expected utility is higher with him than with his challengers, and do not take his past actions into account beyond that comparison.

Taken together, these assumptions imply that the substitution effect can largely offset the apparent benefit of direct democracy provisions, and can even decrease citizens’ welfare. More disturbingly, this welfare reducing indirect effect is stronger when elections are more successful in providing

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1 See Gerber and Matsusaka (2009).
2 An example is the recent proposal to reform California’s initiative process, considered by many as partly responsible for the state’s budget crisis. Another is the attempt to add initiative and referendum provisions to the constitution of Connecticut in the 2008 proposed Constitutional Convention.
3 A famous exposition of this theory, due to Madison, is in the *Federalist Papers* (1788). The idea can be traced back as far as Plato’s Republic (Book VIII).
4 See Assumption 13.
incentives and screening politicians. Understanding this result requires some model description. The model considers two dimensions of policy making. One dimension can be amended by direct democracy at some cost, while the other cannot. The first period starts with an incumbent politician who exerts privately-observed effort to improve the expected quality of his (binary) decision making along the first dimension, and whose competence affects both the cost of this effort and the quality of his decision making along the second dimension. At the end of the first period, citizens get a signal about the quality of the incumbent’s policy along the first dimension. Based on their posterior about the incumbent’s competence, they decide whether to reelect him over his challengers. If the incumbent is reelected, his past efforts carry over to the second and last period of the model.

Now suppose that, at some cost, citizens can amend the politician’s decisions along the first dimension. This means that they care relatively less about the role of politicians on the first dimension and relatively more about their role on the second dimension. Hence, they care less about the incumbent’s past efforts as they do about his competence per se. Consequently, their posterior about incumbent competence must be higher, in order for them to reelect the incumbent (that is the “reelection posterior threshold”). This posterior is built on the politician’s equilibrium effort, which depends on his type. In equilibrium, a politician risks not being reelected only if he makes a mistake, and citizens’ posterior conditional on a mistake determines their indifference condition for reelection. For this posterior to be higher, the politician must have less incentive to make effort. Indeed, as one increases the politician’s incentive for searching a good policy, there is more separation across competence types, which means that a mistake makes it more likely that the politician was incompetent. However, less search on the part of the politician means, first, that mistakes are more likely to occur, which involves costly correction by citizens and, second, that citizens are less able to screen competent types. At the extreme, if citizens care only about the second dimension (zero cost of direct democracy), the incumbent makes no effort at all. As a result, citizens get no signal whatsoever about the incumbent’s competence, which destroys their ability to use reelection for screening competent politicians.

Moreover, having politicians that provide less search also makes them less attractive, other things equal, relative to their challengers. This again implies that the reelection posterior threshold must be higher, which triggers less selection and less search, and so forth. This spiral is summarized by a disincentive multiplier, which describes how a small cost reduction in direct democracy can have an amplified disincentive on politician’s effort and selection. The size of his amplifying effect is increasing in the initial level of search. The implication is that, while direct democracy can help ‘fixing’ a poorly performing representative democracy, it will also prevent citizens from enjoying the benefits of an improved political process.
This informal description of the mechanisms at play shows that cheap citizen initiatives can worsen political agency problems and citizen’s welfare. Such effects cannot occur if any of the three key assumptions is relaxed. With commitment, for example, citizens could enforce a large range of search level, independently of the cost of initiatives, and would only enjoy the benefit from direct democracy. Similarly, if politician abilities were not correlated across policy dimensions, the reduced ability to screen politicians would only affect those dimensions that can be directly chosen by citizens. Finally, the relation between competence and signaling is also key.

Existing theoretical work has viewed direct democracy as an alternative regime to representative democracy, rather than a specific set of institutions potentially featured in the latter. A few papers study direct democracy in the context of a representative regime and, to our knowledge, only one – Matsusaka and McCarty (2001) – analyzes how the presence of these institutions affects the policies chosen by elected representatives. Their analysis focuses on static agency problems and ignores the connection between political delegation and information acquisition, which is the source of our main results. More generally, we are unaware of any work exploring how the presence of direct democracy affects simultaneously voters’ behavior, political selection, and the overall quality of policy-making.

The paper is organized as follows. Section 2 introduces the model and the key assumptions of the paper. Section 3 analyzes the equilibria of the model and their connection to contracting. Section 4 exposes the disincentive multiplier implied by direct democracy. Section 5 studies the impact on citizens welfare of direct democracy. Section 6 considers the impact of the cost of direct democracy provisions on their frequency, as well as other comparative statics. Section 7 contains the literature review, Section 8 discusses the robustness of our assumptions, and Section 9 concludes. Longer proofs are relegated to an appendix.

2 Model

This section presents a simple model that contains the minimal structure needed to capture the concepts previously introduced and present the results in the cleanest possible way.

Overview. The model has two periods. In the first period, citizens face an incumbent politician of privately-known competence whose only goal is to be reelected for a second term. Before the election, the incumbent chooses some costly effort to learn a state \( \tilde{h} \in \{h, h'\} \) that affects citizens’ utility in the first period. Upon observing a signal about the state, the incumbent chooses either the default policy, \( x \), or a reform, \( x' \). Citizens then observe the policy and the state and decide, based

\[\text{Examples include Maskin and Tirole (2004), Persson and Tabellini (1994), and Loeper (2008).} \]
on these observations, whether to reelect the incumbent or to elect a challenger. In the second and final period, a new independently distributed state of the world arises, which the elected politician makes no additional effort to learn. However, if the incumbent is reelected, his past efforts are valuable also in the second period, which we simply model as a second-period signal with the same accuracy as in the first period. The politician then chooses some policy in \( \{x, x'\} \). Finally, citizens observe the state of the second period and the policy and decide, if needed, to amend it at some cost \( \mu \).

The state of nature \( \tilde{h} \) at each period is independently drawn from the following distribution:

\[
\tilde{h} \sim \begin{cases} 
  h & 1 - \rho \\
  h' & \rho 
\end{cases}
\]

\( \tilde{h} \) describes all elements determining how the policy \( x \) or \( x' \) affects citizens; examples include the cost of providing a public good or the transaction cost associated with a certain redistribution scheme. We assume that when \( \tilde{h} = h \), it is optimal for voters to have \( x \) implemented, while they prefer \( x' \) when \( \tilde{h} = h' \).

A politician (either incumbent or challenger) is competent \( (\theta = C) \) with probability \( 1 - q \) and incompetent \( (\theta = N) \) with probability \( q \). Incumbent and challengers have independently distributed types.

The incumbent has an outside option \( O_\theta \), which depends on his competence \( \theta \). The incumbent decides whether or not to run for reelection, trading-off the expected payoff of running for reelection (net of effort cost) and the outside option. If he runs for reelection \( (e = 1) \), the incumbent may perform a costly search to determine \( \tilde{h} \). More specifically, a search effort \( s \) costs the incumbent \( T_\theta(s) \) and reveals state \( h' \) with probability \( s \), while revealing nothing otherwise. Thus, the incumbent knows that \( \tilde{h} = h' \) if revelation occurred, and uses Bayesian updating to assess \( \tilde{h} \) conditional on no revelation.

If the incumbent is reelected, he receives a rent \( R \) in the second-period. Otherwise, a challenger whose type distribution is identical to the incumbent’s is elected.

\(^6\)One could model this “learning by doing” with a less informative signal in the second period, without affecting the results.

\(^7\)This choice of an asymmetric search signal structure simplifies the analysis and can be modified without affecting the main results of the paper. The choice of this asymmetric signal structure is, as we will see later, motivated by making the model simpler. It follows, among others, Bénabou and Tirole (2006), who employ a similar idea in the different context of a model of endogenous belief formation about society’s fairness. A similar signal structure is also employed in Tirole (2009).
If, instead, the incumbent chooses the outside option \((e = 0)\), he gets a payoff of \(O_\theta\), the policy is left at the default level \(x\) and the challenger becomes the incumbent in period 2 via an uncontested election.

The cost function \(T_\theta(s)\) is increasing, continuous, and convex in \(s\), vanishes at 0, and is such that \(T'_\theta(0) = 0\) and \(T'_\theta(1) \geq \rho R\) for both types \(\theta\). Moreover, we assume that the competent type has a higher marginal productivity of search effort:

**Assumption 1 (Single Crossing)**

\[
T'_C(s) > T'_N(s) \forall \ s > 0
\]

The next assumption states that the differential in responsiveness across types increases with the strength of the electoral incentives:

**Assumption 2 (Discerning Power of Electoral Incentives)**

\[
\kappa(z) = \frac{T''_C(T^{-1}_C(z))}{T''_N(T^{-1}_N(z))} \text{ is weakly decreasing in } z.
\]

This assumption implies that as the marginal gain from search effort (the electoral incentive) increases, the ratio of search intensities of competent and incompetent types increases. Therefore, by inducing higher search from both types, voters are able to make a better inference on the type of the incumbent. The assumptions imply that, when both types are searching, \(C\) is not only more productive, but also more responsive than \(N\) to the electoral incentive. Next, we assume that the outside option is higher for the \(C\)-type, normalizing to zero the one of the \(N\)-type. We also require that, at least in some circumstances (depending on voters behavior), the \(C\)-type might actually prefer \(O_C\) over running for reelection.

**Assumption 3 (Outside Option)** \(O_N = 0\) and

\[
O_C > \max_{s \in [0,1]} R(1 - \rho) + R\rho s - T_C(s) \tag{1}
\]

This outside option ensures uniqueness of an equilibrium with positive search and rules out the implausible first-best equilibrium where politicians make maximal effort. \(O_C\) can be interpreted as a market premium for political competence in the private sector. The second part of the assumption states that a competent incumbent prefers his outside option over a regime where voters reelect the

\[\text{Precisely, it guarantees uniqueness of a “natural” equilibrium with positive search. See Section 3.}\]
incumbent only when he chooses the optimal policy. This imposes an upper bound on the pressure that voters can put on the incumbent.\footnote{Different versions of Assumption have been employed in the literature on political selection. Mattozzi and Merlo (2008), Messner and Polborn (2004), and Caselli and Morelli (2004), among others, assume that productivity in a political job is positively correlated with productivity in a private sector job.}

Finally, we introduce an assumption that acts as a “political linkage” across periods.

**Assumption 4 (Learning by doing)** In period 2, a reelected politician who previously exerted effort $s$ is able to discover the state $h'$ with probability $s$ at no additional cost.

This assumption is needed to guarantee the existence of an equilibrium with strictly positive effort in the first period.\footnote{To see why, intuitively, suppose on the contrary that past effort has no impact on the second period. Then, voters reelect the incumbent if and only if their posterior is higher than the challenger’s expected competence. If the incumbent puts strictly positive effort, any mistake makes voters strictly more pessimistic about the incumbent’s type and prompts them to surely elect a challenger. In such case, a competent incumbent prefers his outside option, and the equilibrium unravels.} It captures the fact that, if the incumbent exerts effort today, he will accumulate experience and additional competence that can be used in the future. The results derived in the paper are easily shown to hold with a weaker version of this assumption in which only a fraction of the previous search effort is exerted at no cost in Period 2. Alternatively, the assumption can be replaced by the introduction of correlation\footnote{What is important is that exerting effort in period 1 gives the incumbent a superior information over the challenger for the policy-making in period 2. This information advantage is the reason why voters care about past effort when choosing the future policy-maker.} across periods between the draws of $\tilde{h}$.

This set of assumptions implies that when the challenger is in office in period 2, $s = 0$. It also implies that a politician who exerted $s = s^*$ in Period 1 and gets reelected, $s = s^*$ will be exerted in Period 2.

### 2.1 Voters

Without initiatives, voters have the same utility function at each Period $t \in \{1, 2\}$: If policy $\tilde{x}$ is chosen at Period $t$ and the state of the world for that period is $\tilde{h}$, their utility for that period is

$$U_t = U(\tilde{x}; \tilde{h}) + I_{\{\theta = C\}} M.$$  

Their total payoff $U = U_1 + U_2$.\footnote{One could introduce a discount factor across periods without affecting the results.} $M$ is the “valence” of a competent politician (with zero valence for an incompetent one), and represents how competence affects policy dimensions other than the one which may be directly
affected by direct democracy. For example, $M$ may be the expected relative payoff that a $C$-type provides voters by better handling unforeseen contingencies (for example, a terroristic attack or a financial crisis) with respect to a $N$-type. Since competence is unobserved, voters are unable to perceive $M$ until period 2.

The payoff from default policy $x$ is normalized to zero for all states of nature, $U(x; h) = U(x; h') = 0$, with the net benefit of a reform being captured by $U(x'; h') = \Delta > 0$ and $U(x'; h) = -\Delta' < 0$. Equivalently, $\Delta$ is the relative loss of not implementing a reform when it is needed (political inertia), and $\Delta'$ is the loss associated with an unnecessary reform. We assume that reforms are ex ante not desirable:

$$\rho \Delta < (1 - \rho)\Delta'.$$

In particular, it is efficient to choose $x'$ only if a signal about state $h'$ has been observed.

If the incumbent chooses to run for office, citizens make a reelection decision after observing the state $\tilde{h}$ and the implemented policy. Voters’ reelection strategy consists of a reelection probability $f(\tilde{h}, \tilde{x})$ for each pair $(\tilde{h}, \tilde{x})$ of state and policy.

In Period 2, the elected politician, possibly after observing a signal about the new state of nature, chooses a second policy. A newly elected politician makes no effort ($s = 0$) and chooses the default policy $x$. In contrast, if the incumbent politician is reelected, he learns the new state of the world $h'$ with probability $s_\theta$, which is his effort level in the first period. Thus, the incumbent’s past effort carries over to the second period, resulting in the same signal accuracy that depends on that past effort, and hence on the equilibrium search level of the first period. This implies that a reelected incumbent will provide higher policy quality if he is competent.

### 2.2 Initiatives

In existing economic models, representative democracy has been compared to direct democracy in its purest form, namely a situation in which citizens are directly choosing policies without the presence of elected politicians. It is quite difficult to find, in modern societies, examples of communities where decisions are always made via special elections and political representation is absent. In our model, elected representatives are present and exert sovereignty, but the policy can be amended by citizens via ballot initiatives.

Citizens can propose an initiative in Period 2. An initiative is the process of calling a special...
election in which voters have the possibility of amending the default policy \((x)\) with a reform \((x')\). Proposing an initiative has a per capita cost of \(\mu\), which all voters should pay. An initiative is proposed (and accepted) if and only if \(\mu \leq \Delta\).

In our model, “adding citizen initiatives” is tantamount to reducing the cost \(\mu\) of initiatives below \(\Delta\).

In representative democracy \((\mu \geq \Delta)\), if a politician of type \(\theta\) who chose effort \(s_\theta\) gets reelected, voters’ expected utility in Period 2 equals \(U^r(\theta) = s_\theta \Delta \rho + I_{\theta=C} M\). If a challenger is elected, voters’ expected utility in Period 2 equals \(U^r_c = (1-q) M\). With direct democracy \((\mu < \Delta)\), the expected second-period utility associated to a \(\theta\)-type is \(U^d(\theta) = s_\theta \Delta \rho + (1-s_\theta)(\Delta - \mu) \rho + I_{\theta=C} M\). If a challenger is elected, the expected utility for voters is given by \(U^d_c = (1-q) M + \rho(\Delta - \mu)\). The assumption that \(\rho \Delta < (1-q) M\) implies that

\[
U^r(C) > U^r_c > U^r(N) \tag{3}
\]

and

\[
U^d(C) > U^d_c > U^d(N) \tag{4}
\]

for all levels of \(s_C\) and \(s_N\). These two last relations are important: they imply that if voters knew for sure that the incumbent was competent, they would strictly prefer to reelect him, no matter how little effort he put in the past, while if they knew for sure that the incumbent was incompetent, they would strictly prefer to oust him, no matter how much effort he put in the first period.

A lower initiatives cost makes voters relatively less concerned about search and relatively more concerned about political selection, keeping the payoff \(M\) constant.

In the remaining of the paper, we assume that

\[\mu \leq \Delta\]

\(\mu = \Delta\) corresponds to pure representative democracy (since there is no gain in amending policy), while \(\mu < \Delta\) implies that citizens strictly benefit (ex post) from amending the policy.

### 3 Equilibrium and Political Contract

In this model, the “natural policy” is to implement the reform if and only if the state \(h'\) is revealed. Citizens’s ideal would be for politicians to put maximal search effort (i.e., \(s = 1\)), followed the
natural policy. In the present setting, there are three possible kinds of equilibria. There always exists a “no search” equilibrium, in which the incumbent puts no effort, hence revealing nothing about his type, and is reelected by citizens, who are indifferent between him and his challenger. This equilibrium is suboptimal, whenever an equilibrium with positive search exists.

The most natural type of equilibrium, when it exists, is one in which citizens surely reelect the incumbent whenever he got the policy right, i.e., chose \( x \) when the state was \( h \) and \( x' \) when the state was \( h' \), and punish him when he made a mistake. Assuming for now that the incumbent follows the natural policy, the only mistake that happens in equilibrium is to choose the default option \( x \) when the state was \( h' \). Punishment takes the form of a lower reelection probability conditional on a mistake happening. In equilibrium, that reelection probability must be such that it induces just the right amount of effort from both incumbent types so that, conditional on the mistake happening, voters are indeed indifferent between reelecting the incumbent or not. Such type of equilibrium is characterized by a single number, the probability \( f \) of reelection conditional on observing \((h', x)\).

Finally, there exists a “mirror equilibrium” in which the politician does the exact opposite of what his signal tells him to do, i.e., chooses \( x' \) if he does not learn the state, and \( x \) if \( h' \) is revealed. That equilibrium is clearly suboptimal for voters. However, because that behavior does reveal the incumbent’s type, and because citizens highly value competence and lack commitment power, they will reelect the incumbent with high enough prior despite this suboptimal policy.

We focus on the natural kind of equilibrium, which maximizes welfare within the class of all self-enforcing contracts that citizens, who lack commitment, can propose to politicians.

In the rest of this section, we show that there exists exactly one positive-search equilibrium of this kind, provided that the gain from searching, \( \Delta \) is high enough. We characterize this equilibrium, and also verify that it is indeed welfare maximizing within the class of self-enforcing contracts.

### 3.1 Reelection probabilities

Let \( \bar{p} \) denote the probability threshold for incumbent competence, given search efforts \( s_C, s_N \) and conditional on observing mistake \((h', x)\), above which voters reelect the incumbent. This competence posterior threshold depends on search efforts because those efforts carry over to the second period (learning-by-doing). The next proposition characterizes this threshold.

**Proposition 1** After observing \((\bar{h}, \bar{x})\), voters strictly prefer to reelect the incumbent if and only if their actual posterior \( p(\bar{h}, \bar{x}) \) satisfies

\[
p(\bar{h}, \bar{x}) > \bar{p}^r = \frac{(1 - q)M - s_N \mu \rho}{M + \mu \rho (s_C(\sigma V) - s_N(\sigma V))},
\]
and are indifferent across all reelection probabilities if \( p(\tilde{h}, \tilde{x}) = \bar{p}' \).

Proof. Voters’ expected second-period utility from reelecting the incumbent, given posterior \( p \)

\[
p[M + p s_C \Delta + \rho (1 - s_C) (\Delta - \mu)] + (1 - p) [p s_N \Delta + \rho (1 - s_N) (\Delta - \mu)],
\]

while their expected second-period utility with a challenger is \( (1 - q) M \). Comparing these expressions yields the threshold. ■

3.2 Incumbent’s Choice

The incumbent’s strategy in the first period consists of the following dimensions:

- Whether to run for reelection \( e \in \{0, 1\} \). We allow randomization and let \( \lambda = Pr[e = 1] \).
- Search effort \( s \in [0, 1] \)
- Policy \( y \) upon receiving no signal.
- Policy \( y' \) upon learning that the state of the world is \( h' \).

An incumbent of type \( \theta \) chooses his strategy so as to solve

\[
\max_{e,s,y,y'} e \left\{ R \left[ (1 - \rho) f(y, h) + p s f(y', h') + (1 - s) \rho f(y, h') \right] - T_\theta(s) \right\} + (1 - e) O_\theta
\]

The incompetent type always runs for reelection, since his outside option is zero. In any equilibrium with positive search, the competent type must also run with positive probability \( \lambda > 0 \). Otherwise, voters would know the incumbent’s incompetence conditional on his running for reelection, and would not reelect him, independently of his equilibrium search level (see (B)). Thus the running type would optimally choose zero effort.

Convexity of the cost function imply that the optimal search effort is characterized by

\[
s_\theta = T_\theta' \left( \left( f(h', y'_\theta) - f(h', y_\theta) \right) R \rho \right) .
\]

In particular, \( s_\theta \) is independent from \( f(h, x) \) and \( f(h, x') \).

3.3 Optimal Self-Enforcing Political Contract

We show that implementing “natural policy” where the incumbent implements the reform if and only if he learns the state \( h' \), is optimal from voters’ viewpoint, and that an optimal implementation
is to surely reelect the incumbent if he does not make any mistake, randomize reelection if he chose the default option when reform was needed, and punish him with certain ousting if he made the other type of mistake.

Suppose that an equilibrium with positive search exists, with \( f(h, x) = f(h', x') = 1 \) \( f(h, x') = 0 \) and \( f(h', x) \in [0, 1) \). We will see that there exists exactly one such equilibrium, provided that \( \mu \) is high enough. That equilibrium clearly dominates any no-search equilibrium from voters’ viewpoint.

We now show that this equilibrium is optimal among all possible equilibria inducing positive search. Thus, consider any equilibrium that induces some positive search.

First, consider voters’ reelection strategy conditional on observing \( h \). In that case, the incumbent has surely not received any signal about \( h' \). In that situation, therefore, the incompetent type can always mimic the competent type, so no type revelation can occur. Given that no type separation can occur, the best voters can hope for is for the incumbent to choose the optimal strategy conditional on this information set, which is to take action \( x \) (since that action is ex ante optimal, and a fortiori optimal conditional on not receiving any signal that the state is \( h' \)). Moreover, voters want to reelect the incumbent, other things equal, because i) their posterior about his competence is the same as their belief about challengers’ competence, and ii) the incumbent has the strict advantage, compared to challengers, of having performed positive effort, which carries over to the second period. Therefore, an optimal strategy is to set \( f(h, x) = 1 \) and \( f(h, x') = 0 \), as it achieves exactly this.\(^{16}\)

Now consider the case where the state is \( h' \). In that case, ignoring separating considerations, voters want the incumbent to choose the reform \( x' \) if he learned the state. Thus, abstracting for now from the adverse selection problem, voters would want the incumbent to choose \( x' \) if and only if he received the signal. We now take into account adverse selection. In any equilibrium with positive search, the competent type searches more than the incompetent type, since it is less costly for him to do so. Therefore, implementing the natural policy not only is optimal from voters’ myopic viewpoint, but also increases the probability of selecting a more competent type. Thus, setting \( f(h', x') = 1 \) is optimal. The remaining probability \( f(h', x) \) is pinned down by equilibrium conditions. The case where no positive search equilibrium exists is included, by setting \( f(h', x) = 1 \), in which case the incumbent is always reelected if he chooses the default action \( x \).

We have shown the following.

**Proposition 2 (Natural Positive-Search Equilibrium)** Any welfare maximizing self-enforcing

\(^{16}\)Setting \( f(h, x') \) small enough would also yield an optimal strategy, provided that the incumbent follows the “natural policy,” since \( (h, x') \) does not arise on the equilibrium path.
political contract can be implemented by setting \( f(h, x) = f(h', x') = 1 \) and \( f(h, x') = 0 \). The remaining reelection probability \( f = f(h', x) \in [0, 1] \) is determined by equilibrium conditions.

For natural equilibria, equilibrium search then equals

\[
s_\theta(f) := T_\theta^{-1}([1 - f] \rho R].
\]

(5)

Convexity of \( T \) implies that \( s_\theta(f) \) is decreasing in \( f \). The single crossing property of \( T_\theta(\cdot) \) implies that \( s_C(f) \geq s_N(f) \) for all \( f \).

In our setting, there may exist two kinds of natural equilibria. We call “interior equilibrium” one in which the competent type surely runs for reelection \( \lambda = 1 \), and “constrained equilibrium” one in which the outside option of the incumbent type is binding, and where he runs for reelection with probability \( \lambda < 1 \) and takes his outside option with the remaining probability. The incumbent’s outside option prevents equilibria where a mistake is punished by certain ousting.

**Proposition 3 (Reelection and Search Bounds)** The competent type runs for reelection only if \( f(h', x) \geq f \), where \( f \) solves

\[
T_C(s_C(f)) + O_C = R(1 - \rho) + Rps_C(f) + R(1 - s_C(f))f
\]

(6)

Consequently,

\[
s_N \leq \bar{s}_N \text{ and } s_C \leq \bar{s}_C,
\]

where \( \bar{s}_N = s_N(f) \) and \( \bar{s}_C = s_C(f) \).

**Proof.** The expected payoff for a competent incumbent who runs for reelection and chooses effort \( s \) is, assuming that \( f(h, x) = f(h', x') = 1 \),

\[
R(1 - \rho) + Rps + R(1 - s)f - T_C(s).
\]

The objective is increasing in \( f \) and submodular in \( s \) and \( f \), implying that the optimal search level is decreasing in \( f \) and that the maximum is increasing in \( f \). Running for reelection is optimal if and only if the maximum is higher than \( O_C \). Therefore, running for reelection is optimal if and only if \( f \geq f \). The bound also holds if \( f(h, x) \) and/or \( f(h', x') \) are strictly less than one, as it reduces the value of effort even more. The rest of the proposition follows from the previous analysis. ■

**Corollary 1** In any equilibrium with positive search, \( f(h', x) \geq f \).

**Proof.** Suppose that \( f(h', x) < f \). Then, the competent incumbent does not run for reelection. This implies that an incumbent who runs for reelection immediately reveals himself as incompetent. However, in that case voters do not reelect him, which is a direct consequence of (4). ■

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Thus, voters cannot use a “tough” strategy, where the mistake \((h', x)\) is punished by a very likely removal from office, as this would create a lemons problem where only incompetent types would seek office for a second term and, hence, immediately reveal themselves as incompetent.

We now show that there exists at most one (natural) positive-search equilibrium. Let \(\phi_1 = \frac{\mu \rho}{M(1-q)}\), \(\phi_2 = \frac{\mu \rho}{Mq}\), and define

\[
\mu_1 = \left[ \frac{\rho}{Mq} + \frac{\rho}{M(1-q)} \frac{T''_N(0)}{T''_N(0)} \right]^{-1} \left[ 1 - \frac{T''_N(0)}{T''_N(0)} \right],
\]

\[
\mu_2 = \left[ 1 - \frac{\bar{s}_N}{\bar{s}_C} \right] \left[ \frac{\rho}{Mq} (1 - \bar{s}_C) + \frac{\bar{s}_N}{\bar{s}_C} \frac{\rho}{M(1-q)} (1 - \bar{s}_N) \right]^{-1}.
\]

**Proposition 4 (Equilibrium)** If \(\mu \leq \mu_1\), there is no equilibrium with positive search. If \(\mu_1 < \mu \leq \mu_2\), there is a unique equilibrium with positive search. Both types surely run for reelection, and the reelection probability \(f^*\) conditional on \((h', x)\) solves

\[
s_N(f^*)[1 + \phi_1] - s_N^2(f^*)\phi_1 = s_C(f^*)[1 - \phi_2] + s_C^2(f^*)\phi_2.
\]

If \(\mu > \mu_2\), there is a unique equilibrium with positive search. The competent type runs for reelection with probability

\[
\lambda = \left[ 1 - \bar{s}_N[1 + \phi_1 + \phi_1\bar{s}_N^2] \right] \left[ 1 - \bar{s}_C[1 - \phi_2 - \phi_2\bar{s}_C^2] \right]^{-1}
\]

and the reelection probability conditional on \((h', x)\) is \(f\).

The proof is based on the following lemma.

**Lemma 1** For fixed parameter values, there exists at most one equilibrium with \(\lambda = 1\) (competent type enters surely) and at most one equilibrium with \(\lambda \in (0, 1)\). Furthermore, these equilibria cannot coexist.

**Proof.** If \(\lambda = 1\), equilibrium search solves

\[
\frac{(1-q)(1-s_C)}{(1-q)(1-s_C) + q(1-s_N)} = \frac{(1-q)M - \mu \rho s_N}{M + \mu \rho (s_C - s_N)}.
\]

which can be reexpressed as

\[
a(f) = b(f),
\]

where

\[
a(f) = s_N(f)[1 + \phi_1] - s_N^2(f)\phi_1
\]

and

\[
b(f) = s_C(f)[1 - \phi_2] + s_C^2(f)\phi_2.
\]
If \( \lambda \in (0, 1) \), equilibrium search solves
\[
\frac{(1-q)(1-s_C)\lambda}{(1-q)\lambda(1-s_C)+q(1-s_N)} = \bar{p},
\]
which becomes
\[
1 - \bar{s}_N(1+\phi_1) + \bar{s}_N^2\phi_1 = \lambda \left\{ 1 - \bar{s}_C(1-\phi_2) - \bar{s}_C^2\phi_2 \right\} . \tag{12}
\]
Since (12) entirely pins down \( \lambda \), there can exist at most one constrained equilibrium. Since each side of (12) is positive, this implies that
\[
a(f) > b(f) \tag{13}
\]
It is easy to show that \( a \) is decreasing in \( f \), and that \( b \) is either everywhere decreasing in \( f \) or first decreasing and then increasing.
An equilibrium \( f \) is a root of the function \( a - b \). We show that the function \( a - b \) is strictly quasiconcave (i.e., strictly increasing, then strictly decreasing). Let \( f_b \) denote the (possibly, equal to 1) value of \( f \) such that \( b \) is decreasing on \([0, f_b]\) and increasing on \([f_b, 1]\). Since \( a \) is decreasing, the function \( a - b \) is decreasing on \([f_b, 1]\). To conclude the proof of quasiconcavity, it suffices to show that if \( a - b \) is decreasing on any value \( f < f_b \), then it is decreasing on \([f, 1]\). For \( f < f_b \), we have \( a'(f), b'(f) \) negative, and
\[
\frac{a'(f)}{b'(f)} = \kappa((1-f)pR) \frac{(1+\phi_1) - 2s_N(f)\phi_1}{1 - \phi_2 + 2s_C(f)\phi_2},
\]
which is increasing in \( f \), since \( \kappa \) is decreasing by assumption and \( 1 - \phi_2 + 2s_C(f)\phi_2 \) is positive (since \( b' \) is negative). Now suppose that \( (a - b)'(\hat{f}) \leq 0 \) for any \( \hat{f} < f_b \). Then, \( a'/b'(\hat{f}) > 1 \) and hence \( a'/b'(f) > 1 \) and \( (a - b)'(f) < 0 \) for all \( f \in (\hat{f}, f_b) \). In particular, given that \( a(1) - b(1) = 0 \), the function \( a - b \) can only cross the \( x \)-axis once, and it does so from below. This shows that there exists at most one level of \( f \) for which (11) holds and, therefore, at most one interior equilibrium. Now suppose that \( a(f) > b(f) \). If \( a(\bar{f}) = b(\bar{f}) \) for some \( \bar{f} \in (f, 1) \), strict quasi-concavity implies that \( a(f) < b(f) \) for all \( f > \bar{f} \), contradicting the fact that \( a(1) = b(1) \). This shows that (11) and (13) cannot hold together.

**Proof of Proposition 4.** Lemma (1) shows that exactly three cases are possible: i) no equilibrium with positive search, ii) an equilibrium with \( \lambda = 1 \), and iii) an equilibrium with \( \lambda < 1 \). There remains to identify which of these cases holds, depending on parameter values. Existence of an

\footnote{Indeed, the left-hand side may be rewritten as \( (1 - \bar{s}_N)(1 - \phi_1\bar{s}_N) \), which is positive, with a similar factorization for the right-hand side.}

\footnote{Seen as a function of \( s_N \), \( a \) is a quadratic concave polynomial that reaches its peak at \( (1 + \phi_1)/(2\phi_1) > 1 \). Therefore, \( a \) is increasing in \( s_N \) on \([0, 1]\) and, therefore, decreasing in \( f \). Similarly, \( b \) is a quadratic convex polynomial in \( s_C \), whose minimizer \( s_b \) is always strictly less than 1, and is negative if \( \phi_2 < 1 \). Therefore there exists \( f_b \) such that \( s_C(f) \) is in the increasing range of \( b \) if and only if \( f > f_b \).}
interior equilibrium obtains if and only if \((11)\) is satisfied for some \(f \in (f, 1)\). Since \(a(1) = b(1) = 0\) and \(a'(f)/b'(f)\) is increasing in \(f\), a necessary condition the existence of \(f < 1\) such that \(a(f) = b(f)\) is that \(a'(1)/b'(1) \geq 1\). Otherwise, \(b'(f) < a'(f) < 0\) for all \(f\) (recalling that \(a, b\) are decreasing) and, therefore, \(a(f) > b(f)\) for all \(f < 1\). Using that \(a'(1)/b'(1) = \kappa(0)(1 + \phi_1)/(1 - \phi_2)\), we get the condition \(\mu \geq \mu_1\).

An equilibrium with \(\lambda < 1\) exists if and only if \((13)\) holds. Using the definitions of \(\phi_1\) and \(\phi_2\), this is equivalent to \(\mu > \mu_2\). In conclusion, if \(\mu > \mu_2\) there exists a constrained equilibrium. If \(\mu < \mu_2\), then \((12)\) implies (using again positivity of both sides) that \(a(f) < b(f)\). If \(\mu > \mu_1\), we have \(a'(1)/b'(1) > 1\) and, more precisely \(a'(1) < b'(1) < 0\). Since \(a(1) = b(1)\), this implies that \(a(f) > b(f)\) in a left-neighborhood of 1. Combined with the fact that \(a(f) < b(f)\), we conclude by continuity of the functions \(a, b\) that there exists some \(f \in (f, 1)\) such that \(a(f) = b(f)\), which shows the existence of an equilibrium with \(\lambda = 1\), whenever \(\mu > \mu_1\) and \(\mu < \mu_2\). To conclude, we show that \(\mu_1 < \mu_2\). This is immediate: suppose that \(\mu = \mu_2\). Then, as we saw, \(a(f) = b(f)\). This implies that \(\mu > \mu_1\), since we also saw that in the opposite case \(a(f) > b(f)\) for all \(f < 1\).

\[\blacksquare\]

4 Selection and Disincentive Multiplier

How well does the election filter out incompetent politicians? The incumbent risks not being reelected if he made a mistake. The only kind of mistake that may occur in equilibrium is \((h', x)\): the incumbent chooses the default policy when the reform is needed. Ideally, voters would like such mistake to reveal, as much as possible, an incompetent type. The \textit{separating power} of the electoral process is its ability to weed out only incompetent politicians. This is measured by the posterior probability \(p(h', x)\) that the incumbent be competent conditional on observing \((h', x)\): the lower that posterior, and the more powerful the electoral process. The next proposition shows that this posterior is decreasing with electoral incentives: the lower the probability \(f\) of reelection conditional on the mistake \((h', x)\), and the lower the probability of weedng out a competent type.

Let \(p(f) = \frac{(1-q)(1-s_{C(f)})}{(1-q)(1-s_{C(f)}) + q(1-s_{N(f)})}\) denote the probability that the incumbent is competent, conditional on \((h', x)\). The proof of the next proposition in the Appendix.

PROPOSITION 5 \ i) \(p(f)\) is increasing in \(f\).

\[i)\) \(s_{C(f)} - s_{N(f)}\) is decreasing in \(f\).

The equilibrium condition for the interior equilibrium is that \(f\) solves

\[
\frac{(1-q)}{(1-q) + q\tau(f)} = \frac{(1-q)M - \mu ps_{N(f)}}{M + \mu p(s_{C(f)} - s_{N(f)})},
\]

(14)
where
\[ \tau(f) = \frac{1 - s_N(f)}{1 - s_C(f)} > 1 \]
and \( s_C, s_N \) are the equilibrium search levels of both types of incumbent given reelection disincentive \( f \). Let \( f(\mu) \) denote the solution of (11), which we have shown to be unique.

The left-hand side of (14) is the posterior probability that the incumbent be competent, conditional on making a mistake. It is decreasing in \( \tau \).

The right-hand side is the posterior level at which voters are indifferent between reelecting the incumbent and electing a challenger, given i) equilibrium search levels \( s_N(f) \) and \( s_C(f) \) that the incumbent has chosen depending on his type, and ii) the cost \( \mu \) of correcting mistakes in the second period. The right-hand side is decreasing in \( \mu \).

What is the effect of decreasing the cost \( \mu \) of direct democracy? A lower cost of direct democracy makes voters more demanding, in terms of the posterior, to reelect the incumbent, because they care more about competence per se and less about the value of past effort (since they can amend mistakes at lower cost). Other things equal, thus, \( \tau \) must decrease: the equilibrium posterior conditional on a mistake must increase, in order to keep voters indifferent. We have shown in part i) of Proposition 5 that \( \tau(f) \) is decreasing in \( f \). Therefore, \( f \) must increase in equilibrium. The intuition is that, to achieve a higher posterior conditional on a mistake, the incumbent must have searched less, which is possible only if the search disincentive \( f \) is higher.

From part ii) of Proposition 5 a higher \( f \) implies that \( s_N(f) \) and \( s_C(f) - s_N(f) \) decrease. Other things equal, this increases the right-hand side of (14), which must again be offset by a lower \( \tau \), hence a higher \( f \), and so forth.

Therefore, the impact of a lower direct democracy cost has a spiraling negative effect on electoral incentives.

Figure 1 illustrates the equilibrium reelection probability for two values of \( \mu \) associated with an interior equilibrium. The solid lines are obtained from a value of \( \mu \) twice as large as the one generating the dotted lines. As argued earlier, as the cost of direct democracy decreases, the responsibility substitution makes voters more willing to reelect an incumbent who mistakenly chose the status quo policy over the reform.

5 Welfare, Direct Democracy, and Political Agency

This section studies the impact of direct democracy on citizens’ welfare. Starting from representative democracy, i.e., the case \( \mu = \Delta \), we study how citizens’ ex ante expected utility changes
as $\mu$ decreases. As we know from the previous analysis, there are two cases to consider: when representative democracy is in an interior equilibrium ($\mu_1 < \Delta \leq \mu_2$) and when representative democracy is in a constrained equilibrium ($\Delta > \mu_2$).\footnote{When $\Delta < \mu_1$, the incumbent makes no search effort. Therefore, direct democracy only has a direct, beneficial impact.}

Let $EU(\mu)$ denote the expected utilities under direct democracy when the cost is $\mu$. If $EU(\Delta) \geq EU(\mu)$ it means that direct democracy at cost $\mu$ reduces citizens’ ex ante welfare, compare to pure representative democracy.

For the following proposition, let $\lambda(\Delta)$ denote the equilibrium participation probability of a competent incumbent (see \cite{10}) and let $s(\theta)$ denote equilibrium search under pure representative democracy. Moreover, call the possibility of direct democracy with $\mu = 0$ costless amendment.

**Proposition 6** Costless amendments reduce citizens welfare in the following cases:
i) If \( qs_N + (1 - q)s_C > \frac{1}{2} \).

ii) If \( q\bar{s}_N + (1 - q)\lambda(\Delta)\bar{s}_C > \frac{1 + q(1 - \lambda(\Delta))}{2\gamma_1} \).

The proposition means that, when expected equilibrium search is high enough, costless direct democracy reduces ex ante welfare. The result is driven by the responsibility substitution effect. Decreasing the cost of direct democracy has two effects on welfare: a direct effect on the cost associated with the lack of a reform, which occurs with probability (1-expected search) and goes from \( \Delta \) to \( \mu \), and an indirect effect on equilibrium search, through responsibility substitution. High initial search makes the direct effect small, while allowing the responsibility substitution effect to operate over a larger portion of the domain. This latter fact implies that direct democracy is likely to improve citizens’ welfare only when the initial quality of the political process is low. As politicians’ search technologies improve, or voters become able to credibly commit to tougher punishment strategies, then direct democracy prevents these improvements to result in higher equilibrium search and better selection because the responsibility substitution becomes stronger.

In a constrained equilibrium, decreasing \( \mu \) slightly below \( \Delta \) makes voters slightly less hurt by the lack of a reform vis-à-vis political selection. This implies that the reelection posterior threshold should increase, which means that \( \lambda \) must go up. In other words, there is a “selection complementarity” that makes the \( C \)-type more likely to run for reelection in equilibrium. Therefore, as long as \( \mu \leq \mu_2 \), the intuition suggests that initiatives should be beneficial. As \( \mu \) goes below \( \mu_2 \), \( \lambda = 1 \) and equilibrium search will start decreasing, as the equilibrium will become interior. As a result, if the initial \( \lambda \) is close enough to one, costless initiatives will be welfare reducing as average search will be large enough to offset the direct effect.

Proposition 6 has compared two extremes: no direct democracy provision vs. costless direct democracy provision. The next proposition shows a stronger result. It provides conditions under which citizens welfare is increasing in \( \mu \) over the entire domain \([\mu_1, \mu_2]\).

**Proposition 7** When \( \kappa \) is constant and \( \Delta \leq \mu_2 \), \( EU(\mu) \) is strictly increasing on \((\mu_1, \Delta]\).

A parameterized family of cost functions where \( \kappa \) is constant is the following one: \( T_\theta(s) = \alpha(\theta)s^\beta \), with \( \alpha > 0 \) is decreasing in competence, and where \( \beta > 1 \) guarantees convexity of the cost function. This family includes, for example, quadratic cost functions, scaled by a competence factor. That additional assumption acts as a regularity condition on the derivative of the welfare function with respect to the cost and allows to establish a uniform monotonicity result that is otherwise not guaranteed when \( \kappa \) is highly responsive to the electoral incentives.

\(^{20}\) that is \( \Delta \) is close enough to \( \mu_2 \)
In our model, then, initiatives improve welfare only in the following circumstances: when search in representative democracy was already low, when the separating power of elections is exceptionally high, and when elections have become completely unsuccessful in both selecting politicians and providing them with incentives to search.

The situation in which $\mu < \mu_1$ very interesting on an empirical level, for two reasons: first, the real-world analog of $\mu$ can be lowered up to a certain level: together with legal \[ \text{signature requirements for petition, geographical representation requirements} \]

21 costs, chosen by the constitution, there are technological costs associated with proposing an initiative (legislative knowledge required to write a bill, mobilizing voters, organizing a special election) that cannot be arbitrarily lowered. Second, and more generally, $\mu < \mu_1$ implies that voters can basically exert direct policy making at a lower cost than using representative democracy, since the informational effort exerted by politicians has basically a negative social value. Therefore, representative democracy itself is unnecessary in this case. Additionally, a very rapidly decreasing $\kappa(\mu)$ is also relatively unrealistic, since more and more phases of the policymaking process have become largely delegated to bureaucracies, thereby making the signal extraction problem relatively more sophisticated.

Figures 2 and 3 plot $EU(\mu)$ in the two types of initial equilibria for the simple quadratic cost function case.\[ \text{All figures are obtained using a quadratic cost function for each type, with } T_C = .6T_N. \]

To summarize, when there is a complementarity between voters’ payoffs and improved selection (constrained equilibrium) initiatives can be beneficial if the cost of proposing them preserves these complementarities.

When there is a substitutability between voters’ payoffs and effort (constrained equilibrium) initiatives reduce equilibrium search; this effect is likely to be welfare decreasing and is stronger when the initial equilibrium search is higher. This result is somewhat surprising as, in our setting, initiatives are ex–post welfare improving. In this model initiatives are immune from their standard drawbacks (tyranny of the majority, interest groups). Nevertheless, initiatives can lead, owing to the disincentive multiplier, to a significant reduction of politicians incentives for effort and, consequently, of citizens’ ability to select competent politicians.

6 Comparative Statics: Frequency of Initiatives, Optimality of Reforms, and Politician Competence

In this section we show how various comparative static of the model can be used to suggest that empirical evidence is consistent with the interior equilibrium, that changes in various parameters

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21 signature requirements for petition, geographical representation requirements

22 All figures are obtained using a quadratic cost function for each type, with $T_C = .6T_N$. 

20
might change the welfare effect of direct democracy, and that initiatives are more likely to be welfare reducing when the expected competence of politicians improves (i.e. goes from low to moderate).

6.1 Frequency of initiatives

Our model yields predictions concerning the probability of observing initiatives, or in an empirical context, about the frequency at which they occur. As it turns out, the two types of equilibria studied in earlier sections, interior and constrained, yield different predictions about the frequency of initiatives, which provides a way to distinguish them. When the equilibrium is constrained, a lower cost $\mu$ results in a higher probability of the competent type running for reelection. Therefore, we would conjecture that the presence of complementarities might decrease the frequency of initiatives precisely because the need of using them is lower. When the equilibrium is interior, one would suspect that lowering $\mu$ worsens both search effort and political selection through the responsibility substitution, which translates into an increased need of using direct democracy. On the other hand, lowering $\mu$ also decreases the probability that a challenger will be elected, thereby decreasing the chances of observing initiatives.

The question basically becomes empirically distinguishing between interior and constrained equilibrium. Although this is not an empirical paper, the comparative static of the model can help

Figure 2: Welfare from Direct Democracy as a function of $\mu$: Interior Equilibrium case.
Figure 3: Welfare from Direct Democracy as a function of $\mu$: Constrained Equilibrium case.

connecting its insights to what happened in the last century to direct democracy institutions in United States, and possibly suggest an interpretation for these facts.

The following proposition confirms this intuition. In our model the ex ante probability of observing an initiative can be interpreted as the expected frequency.

**Proposition 8**

i) The expected frequency in an interior equilibrium is constant in $[0, \mu_1]$ and, if expected search is responsive enough to $f$, decreasing in $(\mu_1, \Delta]$.

ii) The expected frequency in a constrained equilibrium is increasing.

In the appendix, we give a precise meaning to the condition that expected search, that is $qs_N(f) + (1 - q)s_C(f)$, must satisfy. We also show that for a simple quadratic cost functions the condition holds. Proposition 8 implies that an inverse relationship between cost and frequency can only be compatible with an interior equilibrium with positive search, a situation in which reducing $\mu$ decreases voters’ expected utility.

The massive increase in the frequency of initiatives that has occurred in the last 3 decades in United States can be interpreted in light of this finding. The reason is that all evidence available suggests that the cost of proposing an initiative in the last 3 decades has also fallen. On one hand, improvements in IT have decreased the "technological" component of $\mu$, in a period in which, especially in western states (California, Oregon), the number of initiatives has been steadily increasing. A thorough discussion of this fact is beyond the scope of this paper, but several authors
have highlighted this mechanism (see, for example, Matsusaka (2005)). On the other hand, the
data suggest that in the last few decades the "legal" component of the cost of initiatives also seems
to be negatively related to the frequency of initiatives. This relationship is featured in several other
theoretical contributions and, more important, is supported by empirical evidence. One of the
predictions of the model in Matsusaka and McCarty (2004) is that the cost (defined and interpreted
as $\mu$ here) of proposing initiatives should be negatively related to their frequency. In their empirical
analysis the authors show, using data from 22 states between 1953 to 1993, that the frequency of
initiatives is negatively related to two key variables related to $\mu$: the signature requirement for the
petition$^{23}$ and a dummy indicating whether the state law requires some geographic dispersion in
the signatures$^{24}$. The empirical findings in Gerber (1996) also go in the same direction: in states in
which popular opinion and existing policies on parental consent requirement for teenage abortion
are divergent, having a higher signature requirements significantly lowers the likelihood of having
an initiative on the subject.

6.2 Higher probability of a reform being optimal

In this subsection we analyze the effect on the equilibrium of a change in $\rho$, the probability of a
reform being optimal. A higher $\rho$ makes, everything else equal, voters to care relatively more about
policies. Therefore, we would conjecture that increasing $\rho$ would make complementarities more
likely to arise. This intuition is only partially true. The reason is that $\rho$ also impacts the strength
of the electoral incentive that the incumbent faces: increasing $\rho$ makes both types more responsive
to changes in the search disincentive $f$. This might lead to non trivial consequences on the welfare
comparison. The following proposition describes the impact of $\rho$ more precisely.

**Proposition 9** i) An increase in $\rho$ in an interior equilibrium increases search through two cha-

nels: a lower equilibrium $f$ and a higher $\frac{d\sigma(f)}{df} \theta \in C, N$. This second effect remains even when the
equilibrium is constrained (through lower $\mu_2$).

ii) Both $\mu_1$ and $\mu_2$ are decreasing in $\rho$.

This proposition basically states that an increase of the probability of a reform being optimal
has two effects: it makes the policy component of voters’ payoff relatively higher, thereby making
constrained equilibria more likely. On the other hand, it also makes the incumbent ceteris paribus
more responsive to electoral incentives. This second effect increases the set of parameters for which
interior equilibria are observed.

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$^{23}$In most cases signatures must be a fraction of the votes cast in the previous elections or a fraction of registered
voters

$^{24}$For example, each county must have at least some percentage of registered voters or past votes cast
As a result, an exogenous change in $\rho$ can change the welfare ranking between direct and representative democracy. If, for a given set of parameters, direct democracy is welfare improving because it allows to exploit complementarities, a higher $\rho$ might make politicians too responsive to electoral incentives, thereby triggering responsibility substitution.

### 6.3 Higher expected competence

In this section we examine the effect of a change in $q$, the ex ante probability of a politician to be incompetent. Inspecting the condition defining $f^*$ and $\lambda$ leads to conclude that $q$ has an \textit{a priori} ambiguous effect on both. To see this more clearly, consider Figure 4 which displays the relationship between $q$ and $\mu_1, \mu_2$, the threshold values for $\mu$ that define the three equilibrium regions of the model. We can interpret $\mu_1$ as the smallest cost of initiative for which search matters to voters and $\mu_2$ as the largest cost of initiative for which responsibility substitution arises.

Search is maximized when $\mu = \mu_2$ (that is when $f = f^*$ and $\lambda = 1$) while it is zero for $\mu \leq \mu_1$. Therefore, fixing a value for $\mu$ (say, low enough to be below $\mu_1$ for some $q$) and letting $q$ vary,

\[\text{Figure 4: Equilibrium regions for } q\]

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25 The picture is also based on the assumption that $\frac{(1-\bar{s}_N)\bar{s}_N}{(1-\bar{s}_C)\bar{s}_C} < \kappa(0)$, which is guaranteed when $\bar{s}_N > .5$ or when $\kappa$ is a constant, and implies that $\mu_1$'s peak is to the right of $\mu_2$'s.
one can see that there must be a non-monotonic relationship between $q$ and search. For $q$ high enough the equilibrium is constrained. That means that further lowering $q$ does not change search, but only the probability of entry of the competent type because of the selection complementarity, until the curve $\mu_2$ is crossed. Then responsibility substitution starts driving search down, until the latter reaches zero, as $\mu_1$ is crossed. Eventually, while $q$ keeps decreasing, $\mu_1$ is crossed again and search starts increasing, as the responsibility substitution gets weaker. This dynamic lasts until $\mu_2$ is crossed again, and search, after reaching a peak, remains constant and the probability that the competent type enters decreases.

Given that, let’s now consider how the effect how introducing direct democracy (or lowering $\mu$) changes as $q$ varies. When $q$ is very close to $\hat{q}$ the range of $\mu$ in which the equilibrium remains constrained is very large, which means that initiatives are beneficial unless $\mu$ is very low. As $q$ gets larger though, the ranges of the interior and constrained equilibria get larger, thereby making initiatives more likely to generate responsibility substitution effects. Finally, as $q$ approaches one, the range of the constrained equilibrium starts increasing again. In short, responsibility substitution is more likely to operate for moderate/low values of $q$ rather than very high or very low.

The following proposition formalizes the graphical intuition of Figure 4.

**Proposition 10** Both $\mu_1$ and $\mu_2$ are strictly quasiconcave in $q$ with an interior maximizer.

**Proof.** The thresholds can be expressed as

$$
\mu_1 = \frac{M}{\rho} \left[ 1 - \kappa(0) \right] \left[ \frac{1}{q} + \frac{\kappa(0)}{1 - q} \right]^{-1}
$$

$$
\mu_2 = \frac{M}{\rho} \left[ 1 - \bar{s}_N \bar{s}_C \right] \left[ \frac{1 - \bar{s}_C}{q} + \frac{\bar{s}_N}{\bar{s}_C} \frac{1 - \bar{s}_N}{1 - q} \right]^{-1}
$$

their denominators are strictly quasiconvex in the relevant domain: therefore, they are both decreasing between 0 and their unique interior minimizer, and then increasing from there onwards. Moreover, these denominators are also strictly positive in the whole domain of $q$. Since the reciprocal of a strictly quasiconvex function that is positive on a subset of $\mathbb{R}^n$ is strictly quasiconcave on that subdomain, then $\mu_1$ and $\mu_2$ are both strictly quasiconcave functions, and their maximizer must be the minimizer of their denominators. 

The latter proposition implies that, as expected politicians’ competence goes from very low to moderate, initiatives are more likely to be welfare decreasing, This result further supports the idea that direct democracy is more likely to generate responsibility substitution when the political environment is generally more favorable to citizens (that is, when on average politicians become more competent).
This analysis can suggest a mechanism for the losing popularity of direct democracy over time. When initiatives were introduced at the state level (mostly in western states), politicians were often perceived as weak and controlled by large corporations. It is reasonable to suspect that politicians’ average competence has increased over the last century, because of better accountability mechanism and improvements in the resources that politicians can access to deliver good policies to voters. To turn such a vague intuition into a serious claim, one would need to support it with a strong empirical analysis, which is beyond the scope of the paper. Nevertheless, we believe this comparative static result points at a potential, an interesting, channel through which initiatives might have gradually lost popularity as their beneficial effect over the last decades has weakened.

7 Related literature

Our model has a similar structure to the one of Besley and Smart (2007). In their public finance model, voters use elections to discipline the incumbent and select “honest” politicians for the second period. In our model, politicians differ in their ability to acquire information and to provide good policies. Information is endogenously acquired, rather than determined at the outset.

More broadly, our results contribute to the contracting literature on information acquisition. Crémer and Kahlil (1992), Crémer at al (1998-(1)) and Crémer and al. (1998-(2)) study different versions of a principal-agent model in which the agent’s information is endogenous and depends on the contract proposed by the principal. Our paper focuses on the particular form of contracts likely to arise in the context of political delegation, which features limited commitment and amendment ability. Our contribution to this literature also includes a new condition (Assumption 2), different from the standard single crossing condition, under which an adverse shock on incentives (in our context, a lower cost of initiatives) is magnified by equilibrium effects.

Our welfare result is also related to Gilligan and Krehbiel (1987). Comparing the allocation of power between a legislative body and a committee, they study the choice between the Restrictive Amendment procedure and the Unrestrictive Procedure. Their model includes an information–acquisition task, which is better performed when the legislature commits itself not to amend the proposal of the committee, thereby raising the value of information. There is no adverse selection.

In the present comparison of direct democracy and representative democracy, Persson and Tabellini (1994) show how electing a representative can help solving the credibility problem associated with

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26 This reasoning applies especially to complex, multidimensional issues where voters are likely not to be able to use initiatives, which in our model are captured by the payoff $M$.

27 An honest politician sets a high tax rate only when the cost of providing the public good is high, which is its private information, and never diverts public funds when the cost is low.
dynamic capital taxation. In Loeper (2007) and Redoano and Scharf (2004), the two procedures are compared in a setting with multiple districts and inter-jurisdictional externalities. In Redoano and Scharf (2004) the representative serves as a commitment device for voters in different districts to credibly commit to a compromise policy. As a result, representative democracy is more likely to deliver policy centralization than referenda. In Loeper (2007), direct democracy is immune from strategic delegation problems, which makes it Pareto superior to representative democracy. Maskin and Tirole (2004) find that, when voters are uncertain enough about the optimal policy, direct democracy is dominated by representative democracy. By contrast, our results do not rely on any informational disadvantage of direct democracy compared to representative democracy. Besley and Coate (2001) investigate the role of initiatives as “unbundling” institutions. Initiatives allow citizens to reduce the number of issues that are bundled in the electoral competition through candidates, thereby making the outcome likely to be closer to their preferences. This result holds even in absence of any type of agency problem between voters and politicians. Boehmke and Patty (2007) look at how proposed initiatives can play a role of informational cues: by the fact that a certain measure was not enacted by the legislature and became an initiative, voters can infer that the size and relevance of the set of losers of that measure is significant, and use this information to update their beliefs.

Finally, there is a large body of literature in political science on direct democracy institutions in United States. Matsusaka (1992) and Matsusaka (2005) develop a theory, close to Boehmke and Patty (2007), of the optimal allocation of policies between legislature and direct democracy. His main finding, supported by empirical evidence, is that when a policy creates a substantial set of losers, direct democracy is more likely (and more effective) in tackling these issues, while elected representative will prefer to adopt measures that are likely to benefit most of the population. For this reason, direct democracy institutions are welfare enhancing. Matsusaka and McCarty (2001) is closest to our paper. In that model, direct democracy and representative democracy may coexist. Direct democracy introduces competition in the legislative process by giving amendment power to an interest group over the policy chosen by and elected representative. Initiatives are usually beneficial and for that not to be the case, several conditions are needed: having the incumbent aligned to voters but highly uncertain about voters’ preferences and an extreme interest group which poses a significant threat to the elected politician. In contrast, our results provides a rationale for having positive costs, like participation quorums, associated with the proposal of an initiative or a referendum.
8 Robustness

This section briefly discusses which assumptions of the model can be relaxed without altering the results. Although we not provide formal proofs, most of the extensions are straightforward.

**Discount factor.** If voters discount second-period utility by a factor $\beta < 1$, this does not affect their posterior threshold, which is only based on second-period utility. If politicians discount their second-period payoff, this is equivalent to reducing their rent from being reelected, and so this does not affect qualitatively the analysis. Overall, discounting second-period payoffs reduces equilibrium search but does not qualitatively affect the impact on welfare of direct democracy.

**Initiatives in both periods.** Allowing initiatives in the first period, has no impact on the equilibrium: it does not affect voters’ threshold and politicians reelection incentives. However, it does mitigate the impact of second-period direct democracy on politician’s first-period effort.

**Symmetric direct democracy.** If citizens can cancel a reform (i.e., move from $x'$ to $x$) this does not affect the analysis, given the signal structure, since in any natural equilibrium politicians only implement the reform if it is optimal to do so. (Mathematically, $(h, x')$ does not arise in equilibrium).

**Symmetric signal structure.** We considered the case where the incumbent’s signal about the state has a symmetric structure: given effort $s$, $\Pr[Learn h' \mid \tilde{h} = h', s] = \Pr[Learn h \mid \tilde{h} = h, s] = \frac{1 + s}{2}$. Such model makes the analysis less tractable but does not affect the qualitative nature of the results.

**Less extreme forms of learning-by-doing.** Assuming that a reelected incumbent inherits only a (possibly, type–dependent) fraction $\delta_\theta$ of his first–period search effort does not affect the results.

9 Conclusion

Direct democracy creates a responsibility substitution between citizens and politicians, and affects the agency problem between the two groups. The current expansion of direct democracy provisions, in the United States and elsewhere, requires a careful analysis of such effects. The present paper shows how citizens’ lack of commitment power, combined with adverse selection and moral hazard, can significantly reduce the benefits of direct democracy, and may even have an overall negative impact on citizens’ welfare. Moreover, the negative impact is more likely to arise when elections alone are relatively successful in addressing moral hazard and adverse selection. As a consequence, while initiatives can partially improve the performance of a ‘broken’ pure representative democracy,
they also prevent citizens to benefit from improvements in the political system. These phenomena arise even when citizens have identical preferences, and in the absence of interest groups.

In our simple setting, where actions, states, and competence types are binary, we identified a novel condition on cost functions, different from the single-crossing property, under which electoral incentives and citizens ability to filter out competent politicians are monotonically related: stronger electoral incentives reduce the probability that citizens oust a competent politician. This condition has further implications. First, it guarantees uniqueness of a “natural equilibrium,” i.e., one in which conditional on his information, an office-motivated politician chooses the welfare maximizing policy. Second, and perhaps more importantly, that condition generates a vicious circle: as explained in Section 4, a lower cost of direct democracy raises voters’ posterior threshold to reelect the incumbent, which can only be sustained, in equilibrium, by weaker electoral incentives (this is where the condition enters), and lower search, which raises voters’ posterior threshold further, and so forth.

In the parameter range where initiatives improve welfare, cheaper initiatives reduces, paradoxically, the *ex ante* probability of observing them. Instead, when initiatives are welfare decreasing, the relationship is either reversed or absent. This implication is empirically testable, and, as argued in Section 6, most existing evidence available in United States in the last three decades (as shown in various empirical contributions) suggests that various proxies for that cost (signature requirement and geographic distribution requirements) and the frequency of initiatives are inversely related.

Our model abstracts from conflicts among voters and from the presence of interest groups. Earlier models have shown that direct democracy provisions significantly affect the organization of interest groups and their influence on policymaking. An interesting extension is to study the impact of the responsibility substitution emphasized here on interest groups organization and influence. It has been argued in the literature that direct democracy provisions can allow interest groups to bring more competition in the policymaking process, and thereby improve the policy outcome. It is an unclear question whether this result will be mirrored in a framework where information is endogenous; our model may offer a useful framework for investigating this issue.
Appendix

Proof of Proposition 5

i) Equivalently, we need to show that the ratio \( \frac{1-s_N(f)}{1-s_C(f)} \) is decreasing in \( f \). Since \( T'_C(s) < T'_N(s) \) for all \( s \) and \( T'_C(0) = T'_N(0) = 0 \), we necessarily have \( T''_C(0) < T''_N(0) \), or \( \kappa(0) < 1 \). Since \( \kappa \) is decreasing by assumption, this implies that \( \kappa \) is everywhere less than 1. Since \( s_\theta(f) = T'^{-1}((1-f)R\rho) \), we have, by differentiation, \( \kappa((1-f)R\rho) = \frac{s'_N(f)}{s'_C(f)} \). Since \( \kappa \) is less than 1 and \( s'_\theta < 0 \), we conclude that for all \( f \)

\[
s'_C(f) < s'_N(f).
\]

Intuitively, reducing electoral incentives (i.e., increasing \( f \)) has a worse impact on the competent type than on the incompetent one. The derivative of the ratio \( \frac{1-s_N(f)}{1-s_C(f)} \) has the same sign as

\[
-s'_N(f)(1-s_C(f)) + (1-s_N(f))s'_C(f).
\]

As observed earlier, \( 1-s_N(f) > 1-s_C(f) \). Combined with \( s'_C(f) < s'_N(f) < 0 \), this implies that this sign is negative.

ii) Since \( s_\theta(1) = 0 \) for \( \theta = \{N, C\} \), we have

\[
s_\theta(f) = \int_f^1 (-s'_\theta(\tilde{f}))d\tilde{f}
\]

and therefore

\[
s_C(f) - s_N(f) = \int_f^1 (1-\kappa((1-\tilde{f})R\rho))(-s'_C(\tilde{f}))d\tilde{f},
\]

using that \( \kappa((1-f)R\rho) = \frac{s'_N(f)}{s'_C(f)} \) for all \( f \). Since \( \kappa \) is everywhere less than 1 and \( s'_C \) is negative, the integrand in (15) is positive, which shows that \( s_C(f) - s_N(f) \) increases with the interval \([f, 1]\) of integration and, therefore, decreases in \( f \).

Proof of Proposition 6

i) The difference between \( EU(\Delta) \) and \( EU(\mu) \) in an interior equilibrium is given by

\[
D^I(\mu) = 2\Delta \rho [qs_N(\Delta) + (1-q)s_C(\Delta)] + 2M(1-q)(\Delta-\mu)\rho - (\mu+\Delta)\rho [qs_N(\mu) + (1-q)s_C(\mu)] - 2M(1-q).
\]

and the condition follows from computing that difference for \( \mu = 0 \).

\[\text{28We assume away the knife-edge case } \kappa(0) = 1.\]
ii) The difference between $EU(\Delta)$ and $EU(\mu)$ in a constrained equilibrium is given by

$$EU(\Delta) - EU(\mu) = \begin{cases} 
2[q\bar{s}_N + (1 - q)\lambda\bar{s}_C]\Delta \rho + q(1 - q)M(\lambda - \lambda(\mu)) + \\
-(\Delta - \mu)\rho - [q\bar{s}_N + (1 - q)\lambda(\mu)\bar{s}_C]\rangle [\mu + \Delta] \rho & \text{if } \mu \geq \mu_2 \\
2[[q\bar{s}_N + (1 - q)\lambda\bar{s}_C]\Delta \rho] - q(1 - q)M(1 - \lambda) + \\
-(\Delta - \mu)\rho - (\Delta + \mu)[pq\bar{s}_N(\mu) + (1 - q)s_C(\mu)] & \text{if } \mu < \mu_2 
\end{cases}$$

and the condition follows from computing that difference for $\mu = 0$.

**Proof of Proposition [7]**

As long as equilibrium search is positive, the derivative of the expected utility with respect to $\mu$ is given by

$$\frac{d}{d\mu}EU(\mu) = -[q(1 - s_N) + (1 - q)(1 - s_C)] + (\mu + \Delta)\frac{df^*}{d\mu}[qs_N' + (1 - q)s_C']$$

where $s'_\theta = \frac{\partial s(\mu)}{\partial \theta}$ < 0 and $\frac{df^*}{d\mu} < 0$.

This derivative has two components: the first, $-[q(1 - s_N) + (1 - q)(1 - s_C)]$ is negative while the second, $(\mu + \Delta)\frac{df^*}{d\mu}[qs_N' + (1 - q)s_C']$, is positive.

**Step 1** Since

$$(\mu + \Delta)\frac{df^*}{d\mu}[qs_N' + (1 - q)s_C'] > 2\mu\frac{df^*}{d\mu}[qs_N' + (1 - q)s_C']$$

if one proves that

$$2\mu\frac{df^*}{d\mu}[qs_N' + (1 - q)s_C'] - [q(1 - s_N) + (1 - q)(1 - s_C)] > 0$$

then one has also proved [17].

**Step 2** By applying the implicit function theorem to the equation defining $f^*$ we obtain

$$\mu\frac{df^*}{d\mu} = \frac{(1 - s_N)s_N\phi_1 + (1 - s_C)s_C\phi_2}{s_N'[2s_N\phi_1 - \phi_1 - 1] + s_C[2s_C\phi_2 - \phi_2 + 1]}$$

Therefore [17] is equivalent to

$$\frac{qs_N' + (1 - q)s_C'}{s_N'[2s_N\phi_1 - \phi_1 - 1] + s_C[2s_C\phi_2 - \phi_2 + 1]} > \frac{[q(1 - s_N) + (1 - q)(1 - s_C)]}{2(1 - s_N)s_N\phi_1 + 2(1 - s_C)s_C\phi_2}$$

(18)

to simplify the task, let’s use the following notation: $\frac{(1 - s_N)}{(1 - s_C)} = \tau(\mu)$, $\frac{s_N'}{s_C} = \kappa((1 - f(\mu))\rho R) = \hat{\kappa}(\mu)$, $\nu(\mu) = \frac{s_N'}{s_C}$ Re-arranging yields

$$\frac{1}{2s_C} \frac{1 - q + \hat{\kappa} - 1 - \hat{\kappa}}{\frac{1 - q}{q}} - \frac{1 - q}{q} \frac{(1 - \nu)(\tau - \hat{\kappa})}{(\nu + \frac{1 - \nu}{q})} \geq 0.$$
The condition then is equivalent to
\[
\frac{1}{2s_C} \left[ \frac{1-q}{q} \kappa - \frac{1-\kappa}{\phi_1} \right] - \frac{1-q}{q} \frac{(1-\nu)(\tau-\kappa)}{\left(\tau + \frac{1-q}{q}\right)} \geq 0
\]
That is
\[
[\phi_2 + \phi_1 \kappa - 1 + \kappa] - 2s_C \phi_2 \frac{(1-\nu)(\tau-\kappa)}{\left(\tau + \frac{1-q}{q}\right)} \geq 0
\] (19)

**Step 3** If \(\kappa\) is a constant, then \(\kappa = \nu\) for all \(\mu\). Rearranging the inequality \(\mu \geq \mu_1\), we obtain that \(\kappa \geq 1/2\). Therefore, (18) simplifies to
\[
\frac{q\kappa + 1-q}{q\kappa^2 + 1-q} \geq \frac{q\tau(\mu) + 1-q}{2q\kappa \tau(\mu) + 2(1-q)}
\]
the right hand side of the former is then smaller than
\[
\frac{q\tau(\mu) + 1-q}{2q\kappa^2 \tau(\mu) + \kappa(1-q)} < 1.
\]
As a result, the condition holds, since \(RHS > 1 > LHS\).

**Proof of Proposition 8**

i) The first part follows from the fact that below \(\mu_1\) search stays constant at zero, since \(P_I = \rho^2\). In a constrained equilibrium, instead, the ex ante probability of observing an initiative is given by \(F_I(\mu) = P(1 - f(\mu))\rho + (1 - P(1 - f(\mu)))\rho\), where \(P = [q(1 - s_N(\mu)) + (1 - q)(1 - s_C(\mu))]\rho\). Therefore
\[
\frac{d}{d\mu} F_I(\mu) = \rho[P'(1 + (1 - 2P)(1 - f)) - f'P(1 - P)]
\]
where, using the same notation a before (recall that \(f' = \frac{d}{d\mu} f(\mu) < 0\) and \(s_\theta' = \frac{d}{d\theta} s_\theta\)) we have \(P' = \frac{d}{d\mu} P = \rho [-qs_{N}' - (1 - q)s_{C}']f' < 0\). To get \(\frac{d}{d\mu} F_I(\mu) < 0\) we then need
\[
[-qs_{N}' - (1 - q)s_{C}'] > \frac{P(1 - P)}{1 + (1 - 2P)(1 - f)}
\] (20)
which, depending on the shape of \(T\) and the value of \(f\) can hold or not.

**Example in which (18) holds**

For the standard quadratic case, \(T_C(s) = R\rho \frac{s^2}{2}\), \(T_N(s) = R\rho s^2\) then we can prove that, since \(\rho f < P < \frac{1}{2}\rho(1 + f)\)
\[
[-qs_{N}' - (1 - q)s_{C}'] = 1 - \frac{q}{2} > \frac{1}{2(1 + \rho(1 + f)(1 - \rho f))} > \frac{P(1 - P)}{1 + (1 - 2P)(1 - f)}
\]
ii) If the equilibrium is constrained, the only equilibrium object which depends on $\mu$ is $\lambda(\mu)$. We also know that $\lambda(\mu)$ is decreasing in $\mu$. Therefore, we can simply look at the case in which the incumbent is competent, since the behavior of the other type is independent of $\mu$.

Given that the type is $C$, we have:

$$F^C(\mu) = [1 - \lambda(\mu)]\rho + \lambda(\mu)(1 - \bar{s}_C)\rho(1 - f) + (1 - \lambda(\mu)(1 - \bar{s}_C)(1 - f)\rho)(1 - \bar{s}_C)\rho$$

taking its derivative yields $\frac{d}{d\mu}P_{DD}^2 = -\lambda'(\mu)\rho[1 - (1 - \bar{s}_C)\rho(1 - f) + (1 - \bar{s}_C)^2(1 - f)\rho] > 0$.

The rest of the analysis ($\mu \leq \mu_2$) for this case is identical to part i).

**Proof of Proposition 9**

i) $s_\theta(f; \rho)$ is increasing in $\rho$ from (5) and convexity of the cost function.

The equilibrium reelection probability $f^*$ is the unique solution of the equation

$$F(f, \rho) = s_N(f)[1 + \phi_1(\rho)] - s_N^2(f)\phi_1(\rho) - s_C(f)[1 - \phi_2(\rho)] - s_C^2(f)\phi_2(\rho) = 0$$

The proof of Lemma 1 implies that $F(\cdot, \rho)$ has the single crossing property in $f$, being negative below $f^*$ and positive above. Moreover,

$$\frac{\partial F}{\partial \rho}(f^*, \rho) = s_N(f^*)(1 - s_N^2(f^*))\frac{\Delta}{M(1 - q)} + s_C(f^*)(1 - s_C(f^*))\frac{\Delta}{Mq} > 0$$

which shows that $F$ is increasing in $\rho$. Therefore, $f^*(\rho)$ is decreasing in $\rho$.

ii) Simple inspection shows that $\mu_1$ is decreasing in $\rho$. To see that $\mu$ also decreases $\mu_2$, we first apply the implicit function theorem to (6), substitute the first-order condition defining $s_C$, and obtain that

$$\frac{\partial f}{\partial \rho} = \frac{(1 - \bar{s}_C - (1 - \bar{s}_C)f)}{\rho(1 - \bar{s}_C)f} > 0.$$ 

This implies that $\bar{s}_N$ and $\bar{s}_C$ are decreasing in $\rho$, while $\frac{\bar{s}_N}{\bar{s}_C}$ is increasing in $\rho$. Therefore, the numerator entering the definition of $\mu_2$, $1 - \frac{\bar{s}_N}{\bar{s}_C}$, decreases in $\rho$, while the denominator

$$\left[ \frac{\rho}{Mq}(1 - \bar{s}_C) + \frac{\bar{s}_N}{\bar{s}_C} M(1 - q)(1 - \bar{s}_N) \right]$$

increases in $\rho$, concluding the proof.
References


