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Contracting over commitment vs. flexibility under asymmetric information

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Key words: Time inconsistency, self-control,
commitment, flexibility, contracts,
screening, information externalities

JEL classification: D42, D62, D82, D86,
D91, G21, G23

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Contracting over Commitment vs. Flexibility under Asymmetric Information

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Preliminary and Incomplete

February 24, 2011

Abstract

We study a dynamic model of monopolistic provision of commitment devices to sophisticated, Strotzian decision makers. We allow for unobservable heterogeneity at the contracting stage in the agents' preferences for commitment vs. flexibility. The first-best contracts under complete information allow to successfully commit to the optimal level of flexibility. Importantly, this outcome is robust to small amounts of unobservable heterogeneity. When individuals differ substantially in their self control, under asymmetric information highly time-inconsistent agents exert a positive externality on low time-inconsistent fellows. Its magnitude depends on the degree of contractual flexibility and the likelihood of facing temptation. We derive the optimal screening mechanism and characterize its distortions. We analyze the inefficiency of the monopolist's offers in terms of the induced balance between commitment and flexibility.

KEYWORDS: Time inconsistency, self-control, commitment, flexibility, contracts, screening, information externalities.

JEL CLASSIFICATION: D42, D62, D82, D86, D91, G21, G23.

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1 Introduction

Laboratory and field evidence suggests that intertemporal behavior is often time inconsistent because of temptation and lack of self control (see DellaVigna (2009) for an excellent survey). Also, individuals seem to (at least partially) understand and anticipate their self-control problems and demand commitment devices.¹ Economists have recently started to investigate whether markets (or the government) can effectively satisfy people’s demand for commitment (e.g., Bryan, Karlan and Nelson (2010)). In this paper, we study a tractable, two-period model of monopolistic provision of contracts to sophisticated, Strotzian decision makers.

The monopolist offers in the first period incentive schemes to sustain a desired consumption plan in the second period. The potential buyers of such contracts have two essential features that yield a novel model and results relative to the existing literature on contracting with dynamically inconsistent agents.

First, we introduce a trade-off between commitment and flexibility in how agents evaluate contracts. On the one hand, a Strotzian decision maker values commitment because he foresees that his valuation of consumption will change between the current and the subsequent period. On the other hand, we assume that new information that influences the optimal consumption level arrives in the second period. Thus the agents also desire flexibility.

Second, we allow the degree of self control to vary across agents.² Thus, although in our model all individuals have the same preference in the first period, the actual value each agent assigns to commitment, relative to flexibility, depends on whether she expects her self-control ability to be high or low in the second period. Furthermore, we assume that this heterogeneity is unobservable to the monopolist.

We are interested in understanding how the conflicting objectives of committing to a desired course of action (e.g., locking in a certain amount of savings to avoid splurging) and keeping enough flexibility to best respond to future information (e.g., having enough cash available for unexpected expenditures) are reconciled through the monopolist’s profit-maximizing contracts. In particular, the main purpose of this paper is to study the effect of asymmetric information about individuals’ self control on the market’s ability to successfully meet the existing demand for commitment devices.

First, we show that under complete information about the second-period preferences, the monopolist offers each agent a customized contract, depending on his degree of self control, that achieves a perfect balance between commitment and flexibility. Each individual effectively commits to implement a flexible contingent plan that is efficient according to his *current* preference.

¹Examples are savings accounts with built-in illiquidity not matched by appropriately higher interest rates, gym memberships with large up front fees but no per-visit charge, pension funds – some with tax incentives, others not –, automatic drafts from checking to investment accounts, rotating savings and credit associations (ROSCAs) and microcredit savings accounts in developing countries. (See Ashraf, Gons, Karlan and Yin, (2003), Ashraf, Karlan and Yin, (2006), DellaVigna and Malmendier (2006), Bryan, Karlan and Nelson (2010)).

²According to several experimental studies (e.g. Ashraf, Karlan and Yin, (2006)), not all individuals suffer from self-control problems and, when they do, the intensity varies across them.

Interestingly, a small amount of asymmetric information need not prevent the monopolist from optimally solving all agents' self-control problems. This is implied by the fact that, for sufficiently small heterogeneity in the intensity of time inconsistency, the first-best outcome can be achieved *also* by offering all individuals a sole, standardized contract. Next, we consider markets with strong heterogeneity in second-period preferences.

In this case asymmetric information plays an important role. Our first main result is that the individuals who are more time inconsistent exert a positive externality on the agents who are less time inconsistent. Importantly, this result is independent of the particular market structure considered here. Thus, we conjecture it should have an impact also on the performance of competitive markets for commitment devices.

Our second main contribution consists in the characterization of the inefficiencies introduced by the monopolist in order to screen the agents' self control. First, individuals with weak self control purchase a contract with an inefficiently low level of flexibility compared to the first-best complete-information benchmark. Moreover, the level of flexibility can decrease to zero as the share of the agents with weak self control shrinks or their time inconsistency becomes particularly strong. However, market exclusion never occurs. Second, the allocation achieved by the agents with strong self control may also be distorted, but in the opposite direction. When the proportion of strongly time-inconsistent individuals increases and their inferior self control becomes *less* likely to matter for their future behavior, the monopolist offers low time-inconsistent agents a contract with too much flexibility relative to the complete-information benchmark.

Our general conclusion is then the following. In a market where individuals' preferences for commitment vs. flexibility vary significantly and are unobservable, a monopolist provides an inappropriate supply of commitment devices. Individuals with strong self-control problems purchase contracts that have too little flexibility and those with weak self-control problems (including those who are time consistent) may buy contracts that involve too much flexibility. We conjecture that competitive markets may similarly underperform for the same reason.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 lays out the contracting environment and introduces the agents' preferences and information structure. Section 4 derives the first-best benchmark under complete information. In Section 5 we analyze the model under asymmetric information and present the main results of the paper. The final section concludes.

2 Related Literature

Our paper belongs to the small literature which studies the problem of contracting with time-inconsistent agents. O'Donoghue and Rabin (1999) analyze the design of an optimal incentive scheme for procrastinating agents to efficiently complete a task. DellaVigna and Malmendier (2004)'s influential work studies how firms design two-part tariffs for quasi-hyperbolic individuals to maximize profits when selling investment (leisure) goods with immediate random costs

(benefits) and deferred known benefits (costs). Their main objective is to explain why firms set per-usage prices that differ from their marginal costs, as suggested by the empirical evidence. They also show that sophisticated agents are offered a contract that maximizes the ex-ante social surplus.

We depart from DellaVigna and Malmendier (2004) at a few levels. First, they assume that all agents have the same preferences,³ but only a fraction of the population is sophisticated and correctly forecasts at the first-period contracting stage the utility at the second-period consumption stage. The other agents are partially naive in the sense of O’Donoghue and Rabin (2001). Second, they assume full information of the monopolist about agents’ naivete,⁴ the β *perceived* in the first period. Third, they study a binary decision problem, i.e., whether or not to purchase with no quantity variable, and they restrict *a priori* the class of contracts to two-part tariffs. Our complete-information analysis confirms and extends their finding that firms provide a perfect solution to agents’ self-control problems as long as they are sophisticated. However, we cast doubt on this optimistic message by showing that the result is not robust to asymmetric information about the degree of time inconsistency. Furthermore, we characterize the resulting inefficiencies in the monopolistic provision of commitment contracts.

Eliasz and Spiegler (2006) analyze a two-period environment where agents sign contracts with a monopolist in the first period to get access to a set of actions among which to choose in the second period. In their model, all individuals behave according to the utility function u in the first period which changes to v in the second period. Eliasz and Spiegler also have partial naivete, but in a different form than DellaVigna and Malmendier (2004). Their agents believe in the first period that, with some probability θ , their second-period utility function remains u and with the complement changes to v . However, the monopolist’s judgment of their likelihood of being time inconsistent is generically different from θ . The agent’s belief, θ , is his private information and affects how he evaluates any contract ex-ante, but has no impact on his behavior ex-post. In contrast, we consider a situation where today’s information is relevant in predicting tomorrow’s decisions and is unknown to the monopolist. Furthermore, in Eliasz-Spiegler’s model there is symmetric information between today’s self and the future self about the environment. So their agents care only about commitment – unless they are fully naive – and they have no desire for flexibility.

Our work is also related to Amador, Werning and Angeletos (AWA) (2006)’s paper about commitment vs. flexibility in the context of a consumption-savings problem. They derive the optimal consumption/savings policy that a benevolent planner would design for an individual with present bias, who is uncertain about his future utility from consumption. They identify necessary and sufficient conditions for minimum-savings plans to characterize the optimal policy both in a (β, δ) time-inconsistent model with sophistication and in the Gul-Pesendorfer (2001) dynamically consistent model. Again our paper differs along several dimensions. First, we look at a market environment where agents purchase incentive schemes from a profit-maximizing firm to implement their desired consumption/savings plans. By contrast, AWA are interested in what restrictions a decision maker would optimally self-impose on his future budget set. Second,

³First- and second-period preferences differ for any given agent, but coincide across agents.

⁴Jianye (2011) extends DellaVigna-Malmendier (2004)’s model by relaxing their full information assumption.

in their model there is only one individual or, equivalently, all agents face exactly the same self-control problem, whose intensity is perfectly known to the planner. Third, AWA purposefully restrict attention to non-transferable utility to rule out the possibility of any form of insurance.

Finally, Esteban, Miyagawa and Shum (2007) study the monopolist's non-linear pricing problem of designing a *single* optimal menu to offer a population of consumers who have Gul-Pesendorfer (2001) preferences and differ in their cost of self-control (i.e., their temptation utility). Their model can be seen as being essentially static and consumers only care about commitment. Esteban and Miyagawa (2005) study a similar set up, but they allow the monopolist to offer *multiple* menus. Esteban and Miyagawa (2006) looks instead at competitive markets. The crucial difference with our model is the absence of a trade-off between commitment and flexibility in all three papers, which represents an essential ingredient of our study.

3 The Model

A principal faces a population of individuals who are potentially affected by self-control problems. At $t = 0$ the principal approaches an agent randomly drawn from the population and offers him the opportunity to sign a contract for the provision of good $x \in [0, 1]$ in period $t = 1$. The principal's production cost is $c : [0, 1] \rightarrow \mathbb{R}$ with $c(0) = 0$, $c' > 0$, $c'' > 0$, $\lim_{x \rightarrow 0} c'(x) = 0$ and $\lim_{x \rightarrow 1} c'(x) = +\infty$ (this is to ensure unique interior solutions). There is no possibility to offer good x on a spot market in $t = 1$. A contract is a function $t : [0, 1] \rightarrow \mathbb{R}$ which specifies a transfer from the agent to the principal for any amount x chosen in period 1. The principal can perfectly commit to any contract which is then binding for both parties. We shall refer to period 0 as the contracting stage and period 1 as the consumption stage.

Agents are sophisticated Strotzian decision makers who may behave in a time-inconsistent manner. For each agent, we shall call self-0 the self who signs the contract and self-1 the self who chooses x from the resulting menu. Self-0's preference over (x, t) pairs is $u(x, t; \gamma) = \gamma x - t$, whereas self-1's is $v(x, t; \gamma, \beta) = \gamma \beta x - t$. We assume $\beta > 1$ denoting a systematically higher willingness to pay of self-1 for any x which determines self-0's desire for commitment. Each agent privately knows his self-control parameter β before contracting occurs. The principal does not observe β , but he believes that it is distributed according to the commonly known distribution B . The taste shock $\gamma \in \Gamma \subset \mathbb{R}_{++}$ is drawn in period 1 from the distribution G which is commonly known by the parties in period 0; it is observed only by the agent and is the reason self-0 desires flexibility. For simplicity, we are assuming that γ and β are independent. We shall refer to β as self-0's type and γ as the state of the world.

Observe that for each agent, self-0 and self-1 disagree in a systematic way about the valuation of x , but they agree that a higher γ corresponds to a higher willingness to pay.⁵ For example, both selves agree that ice cream is more enjoyable on a sunny day (high γ) than on a rainy day (low γ). However, self-0 anticipates that, after entering the store, his self-1 will be consistently more tempted to buy one independently of the weather conditions.

⁵This would not be true if, for instance, $v(x, t; \gamma, \beta) = \frac{1}{\gamma} \beta x - t$.

We start by assuming that $\Gamma = \{\gamma_1, \gamma_2\}$ with $0 < \gamma_2 < \gamma_1 < +\infty$ and $g \equiv G(\gamma_1) \in (0, 1)$. Further, $\beta \in \{\beta_l, \beta_h\}$ with $\beta_h > \beta_l > 1$ and $b \equiv B(\beta_h) \in (0, 1)$. We shall refer to selves-0 of type β_h as highly time-inconsistent agents, or HTIs for short, and to selves-0 of type β_l as low time-inconsistent individuals, or LTIs.

Given a contract t , let β_j self-1's choice in state γ_i be $x^{ji} = \arg \max_{x \in [0,1]} \gamma_i \beta_j x - t(x)$. At the contracting stage, the expected utility of self-0 of type β_j is

$$g [\gamma_1 x^{j1} - t(x^{j1})] + (1 - g) [\gamma_2 x^{j2} - t(x^{j2})],$$

if he signs contract t . Otherwise, he goes for the outside option with zero value.

The principal offers contracts in period 0 to maximize his expected profit, which is realized in period 1.

For future reference, let $x_u^i = \arg \max_{x \in [0,1]} \gamma_i x - c(x)$ be the efficient allocation in state γ_i from self-0's perspective, for $i = 1, 2$. By standard arguments, $x_u^1 > x_u^2$ given our assumptions on c and γ . Also, let $x_v^{ji} = \arg \max_{x \in [0,1]} \gamma_i \beta_j x - c(x)$ be the efficient allocation in state γ_i from β_j self-1's point of view, for $i = 1, 2$ and $j = h, l$.

4 Full Information Analysis

In this section we derive the optimal contract the principal would offer each type of self-0 if he can observe the degree of time inconsistency, β . This will provide the first-best benchmark with which to compare the market outcome under asymmetric information.

By the Revelation Principle, it is without loss of generality to look at contracts that include only two alternatives (x^1, t^1) and (x^2, t^2) among which self-1 is free to choose. Once β is observed, the principal's program is simply

$$\max_{(x^1, t^1), (x^2, t^2)} g [t^1 - c(x^1)] + (1 - g) [t^2 - c(x^2)]$$

subject to

$$\begin{aligned} IR_0 & : g [\gamma_1 x^1 - t^1] + (1 - g) [\gamma_2 x^2 - t^2] \geq 0, \\ IC_2^1 & : \gamma_1 \beta x^1 - t^1 \geq \gamma_1 \beta x^2 - t^2, \\ IC_1^2 & : \gamma_2 \beta x^2 - t^2 \geq \gamma_2 \beta x^1 - t^1. \end{aligned}$$

Note that period 1 incentive compatibility is expressed in terms of self-1's utility, whereas the participation constraint takes into account self-0's valuation.

First, IR_0 must bind at the optimum. Otherwise, the seller could raise both t^1 and t^2 by $\varepsilon > 0$ without affecting the IC constraints and obtain strictly larger profits. Then, the principal's objective becomes

$$\max_{(x^1, t^1), (x^2, t^2)} g [\gamma_1 x^1 - c(x^1)] + (1 - g) [\gamma_2 x^2 - c(x^2)]$$

subject to IC_2^1 and IC_1^2 .

If we ignore the IC constraints, $(\tilde{x}^1, \tilde{x}^2) = (x_u^1, x_u^2)$ is the unique solution to the relaxed program. To complete the argument, we need to show that there exist transfers (t^1, t^2) such that *self-1* implements such allocation. Combining the IC constraints, we get

$$(\gamma_1 - \gamma_2) [x^1 - x^2] \geq 0,$$

which implies the necessary condition $x^1 \geq x^2$. Our candidate solution $(\tilde{x}^1, \tilde{x}^2)$ satisfies it. So it remains to select transfers t^1 and t^2 in the nonempty feasible region defined by

$$\gamma_1 \beta [x_u^1 - x_u^2] \geq t^1 - t^2 \geq \gamma_2 \beta [x_u^1 - x_u^2] \quad (1)$$

$$gt^1 + (1 - g)t^2 = g\gamma_1 x_u^1 + (1 - g)\gamma_2 x_u^2 \quad (2)$$

Note that t^1 and t^2 are not uniquely pinned down because γ is discrete which creates some slack in the IC constraints in period 1. This gives the principal some freedom in choosing optimal transfers which will be relevant below.

The assumption that self-0 and self-1 agree on the ranking of γ_1 and γ_2 ensures that the self-0's state-wise efficient allocation is implementable. In contrast, suppose that self-1's utility is, for example, $\frac{1}{\gamma}\beta x - t$. In this case, self-0 and self-1 disagree not only about the systematic valuation of any x , but also about which state is the "good one". Self-1 incentive compatibility requires then that

$$(\gamma_1 - \gamma_2) [x^1 - x^2] \leq 0$$

or $x^1 \leq x^2$. Hence, (x_u^1, x_u^2) would not be implementable.

We conclude that under full information at the contracting stage, the seller offers any type of self-0 a contract that provides a perfect combination of commitment and flexibility in that the resulting allocation is ex-ante state-wise efficient. Denote these contracts by $\{\mathbf{x}_u^i, \mathbf{t}_u^i\}$ for $i = h, l$. Observe that even though \mathbf{t}_u^l may differ from \mathbf{t}_u^h , they yield the same expected profits: This is because the allocation is the same under both contracts and the expected revenue coincides with self-0's expected direct utility, which is the same for all β . Note that a similar property holds in DellaVigna and Malmendier (2004) for the profit-maximizing two-part tariff offered to sophisticated agents: Independently of β , the optimal per-usage price maximizes the ex-ante social surplus, which is entirely captured by the monopolist through the entry fee.

5 Asymmetric Information Analysis

5.1 Necessary and Sufficient Conditions for Ex-Ante Incentive Compatibility of the Full-Information Contracts

The first thing to ask when $\beta \in \{\beta_l, \beta_h\}$ is unobservable is whether the full-information contracts derived above induce self-0 agents to self select the menu corresponding to their true type. More

precisely, given any pair of contracts $\{\mathbf{x}^h, \mathbf{t}^h\}$ and $\{\mathbf{x}^l, \mathbf{t}^l\}$, we say that $\{\mathbf{x}^h, \mathbf{t}^h\}$ and $\{\mathbf{x}^l, \mathbf{t}^l\}$ are *ex-ante incentive compatible* if it is optimal for type β_h to choose the former and for type β_l to pick the latter at the contracting stage. Let \mathbf{t}_u^h and \mathbf{t}_u^l be payments associated to the full-information contracts which sustain self-1's incentive compatible selection of (x_u^1, x_u^2) .

We show that if the unobservable heterogeneity in the degree of time inconsistency, measured by the ratio β_h/β_l , is lower than the relative intensity of the ex-post preference shocks, captured by γ_1/γ_2 , then asymmetric information at the contracting stage does not interfere with the implementation of the first-best outcome.

Lemma 1 *There exists \mathbf{t}_u^h and \mathbf{t}_u^l such that the full-information contracts are ex-ante incentive compatible iff $\beta_h/\beta_l \leq \gamma_1/\gamma_2$.*

Proof. *Ex-post incentive compatibility of $\{\mathbf{x}_u^h, \mathbf{t}_u^h\}$ and $\{\mathbf{x}_u^l, \mathbf{t}_u^l\}$ requires*

$$\gamma_1\beta_h [x_u^1 - x_u^2] \geq t_u^{h1} - t_u^{h2} \geq \gamma_2\beta_h [x_u^1 - x_u^2] \quad (3)$$

$$\gamma_1\beta_l [x_u^1 - x_u^2] \geq t_u^{l1} - t_u^{l2} \geq \gamma_2\beta_l [x_u^1 - x_u^2]. \quad (4)$$

Sufficiency: it is immediate to see that if $\gamma_1\beta_l \geq \gamma_2\beta_h$, then $x_u^1 > x_u^2$ ensures that transfers \mathbf{t}^h and \mathbf{t}^l can be chosen so that (3) and (4) hold and also

$$\gamma_1\beta_h [x_u^1 - x_u^2] \geq t^{l1} - t^{l2} \geq \gamma_2\beta_h [x_u^1 - x_u^2]$$

and

$$\gamma_1\beta_l [x_u^1 - x_u^2] \geq t^{h1} - t^{h2} \geq \gamma_2\beta_l [x_u^1 - x_u^2].$$

Thus, both self-0 types expect to make the same decisions after signing any full information contract and therefore assign zero expected payoff to each of them.

Necessity: if $\gamma_2\beta_h > \gamma_1\beta_l$, for any $\{\mathbf{x}_u^h, \mathbf{t}_u^h\}$ that satisfies (3)

$$t_u^{h1} - t_u^{h2} \geq \gamma_2\beta_h [x_u^1 - x_u^2] > \gamma_1\beta_l [x_u^1 - x_u^2] > \gamma_2\beta_l [x_u^1 - x_u^2].$$

Hence, after signing $\{\mathbf{x}_u^h, \mathbf{t}_u^h\}$, a β_l self-0 will always select (x_u^2, t_u^{h2}) . Then

$$\gamma_1x_u^2 - t_u^{h2} > \gamma_1(\beta_l - 1)(x_u^1 - x_u^2) + \gamma_1x_u^1 - t_u^{h1},$$

$x_u^1 > x_u^2$ and $\beta_l > 1$ imply

$$g[\gamma_1x_u^2 - t_u^{h2}] + (1-g)[\gamma_2x_u^2 - t_u^{h2}] > g[\gamma_1x_u^1 - t_u^{h1}] + (1-g)[\gamma_2x_u^2 - t_u^{h2}] = 0.$$

We conclude that β_l self-0 would strictly prefer any $\{\mathbf{x}_u^h, \mathbf{t}_u^h\}$ to any $\{\mathbf{x}_u^l, \mathbf{t}_u^l\}$. ■

A closer look at the proof of the Lemma reveals that if $\beta_h/\beta_l \leq \gamma_1/\gamma_2$, then the principal can simply offer a unique contract that allows to choose between x_u^1 and x_u^2 with transfers appropriately designed so that the same incentive scheme works as a commitment device for both self-0 types.

Corollary 1 *If $\beta_h/\beta_l \leq \gamma_1/\gamma_2$, it is optimal for the principal to offer a unique menu $\{(x_u^1, t_u^1), (x_u^2, t_u^2)\}$ where t_u^1, t_u^2 solve*

$$\begin{aligned}\gamma_1\beta_l [x_u^1 - x_u^2] &\geq t_u^1 - t_u^2 \geq \gamma_2\beta_h [x_u^1 - x_u^2], \\ gt_u^1 + (1 - g)t_u^2 &= g\gamma_1x_u^1 + (1 - g)\gamma_2x_u^2.\end{aligned}$$

Intuitively, suppose β_l and β_h are almost equal so that both types' self-control problem is about the same. Then the incentive schemes that the principal has to provide β_l and β_h in order to implement a certain consumption plan should be sufficiently alike. Further, the slack induced by the discreteness of future taste shocks allows for some leeway in the selection of transfers. Hence, it should be – and indeed Lemma 1 shows it is – possible to find one scheme that works for both types.

In contrast, suppose that β_h is considerably larger than β_l . Consider the plan to consume a large quantity, x_u^1 say, in state γ_1 and a small quantity, x_u^2 , otherwise. For β_h the temptation to consume x_u^1 also in state γ_2 , against self-0's will, is much stronger than for β_l . So any scheme that disciplines β_h to the desired course of action has to make the large quantity sufficiently more expensive than the small one to deter overconsumption in state γ_2 . The higher β_h is, the larger the premium must be. Eventually it will exceed β_l 's willingness to pay in γ_1 for the upgrade from x_u^2 to x_u^1 . So β_l would never select the large quantity under the incentive scheme of β_h . Therefore, in order to implement the first-best allocation with β_l , the principal has to design a specific contract for β_l that differs from the one offered to β_h .

5.2 Optimal Screening of Time Inconsistency

We shall now assume that $\beta_h/\beta_l > \gamma_1/\gamma_2$ and present the main results of the paper. First, we prove the existence of a positive information externality of HTI agents in favor of LTI individuals. On the one hand, this is important because it precludes the implementation of the first-best outcome. Thus we show that unobservable heterogeneity in agents' self control can cause markets for commitment devices to underperform. On the other hand, the direction of the externality constitutes a sharp departure of our model with time inconsistency from a standard model with *time-consistent* agents where both selves' utility is $\gamma\beta x - t$: In a standard model⁶, the individuals who anticipate to have a consistently *lower* willingness to pay in the second period (β_l) exert a positive information externality on the agents with *higher* future valuations (β_h).

Second, we provide a complete analysis of how monopolistic markets for commitment contracts underperform under asymmetric information. We derive the optimal contract that the principal offers in order to screen self-0's degree of time inconsistency. We are able to characterize how the principal distorts the quantities on HTIs' *and (possibly) LTIs'* menus when trading off rent extraction and efficiency. These take the form of inefficiently low and (possibly) high

⁶Hereafter, we use the expression "standard model" to refer to a two period screening model with time-consistent agents, whose utility function is $\gamma\beta x - t$ both at the contracting and at the consumption stage, and that is otherwise identical to our model.

level of flexibility in HTIs' and LTIs' contracts, respectively. The reduction in flexibility for HTIs can be extreme: It may happen that the principal completely removes it and offers HTIs a singleton menu. However, we show that market exclusion never occurs: HTI agents always buy a menu which allows positive consumption in the second period and generates positive profits.

These predictions represent another striking difference between the present model and a standard model with time consistency. There, rents are reduced simply by lowering the quantities offered to individuals who expect a low valuation of consumption. Further, sometimes it is optimal for the principal to exclude the low types from the market.

By the Revelation Principle, it is without loss of generality to consider direct revelation mechanisms (DRMs) such that it is optimal for the agents to truthfully report β in the first period and to truthfully report γ in the second period, *conditional* on having reported truthfully β in the first period.

In our simple set up, a DRM is any array $\{\mathbf{x}, \mathbf{t}\} = \{x^{ij}, t^{ij}\}_{i=h,l}^{j=1,2}$ such that $x^{ij} \in [0, 1]$ and $t^{ij} \in \mathbb{R}$. Suppose $\{\mathbf{x}, \mathbf{t}\}$ ensures that truthful reporting of γ is optimal conditional on having truthfully reported β , i.e., for every $i = h, l$

$$IC^{i1} : \gamma_1 \beta_i x^{i1} - t^{i1} \geq \gamma_1 \beta_i x^{i2} - t^{i2}, \quad (5)$$

$$IC^{i2} : \gamma_2 \beta_i x^{i2} - t^{i2} \geq \gamma_2 \beta_i x^{i1} - t^{i1}. \quad (6)$$

On the one hand, the requirement that truthfully reporting β be optimal under such $\{\mathbf{x}, \mathbf{t}\}$ implies that the truthful strategy must be at least as good as the non-truthful strategy where self-0 misreports β and, conditional on that, optimally (mis)reports γ in the second period. On the other hand, ensuring that such best deviation is no better than the truthful strategy is sufficient to guarantee that any other sequence of lies is unprofitable. Hence, we first compute the best non-truthful strategy for each β under the assumption that (5) and (6) hold. A formal proof is given in the Appendix.

Lemma 2 *Assume $\{\mathbf{x}, \mathbf{t}\}$ satisfies (5) and (6) and $x^{i1} \neq x^{i2}$ for all i . If $\beta_h(\beta_l)$ misreports in period one, then always reporting $\gamma_1(\gamma_2)$ is the unique optimal continuation strategy.*

Intuitively, to prevent β_h self-1 from overconsuming in state γ_2 , x^{h1} must come with a significant price premium. So, if β_l chooses the contract intended for β_h , then she will deem x^{h1} overpriced and she will always go for x^{h2} . This explains the Lemma for the case of β_l misreporting β_h and hence always choosing to report γ_2 . Next, consider the contract offered to β_l . To convince β_l to pick x^{l1} in state γ_1 , x^{l1} cannot cost too much more than does x^{l2} . So if β_h selects β_l 's contract, he gives in to temptation in state γ_2 because he finds the "punishment" for overconsuming x^{l1} against self-0's will relatively mild. Thus he always reports γ_1 .

Given a DRM $\{\mathbf{x}, \mathbf{t}\}$, denote the expected payoff to self-0 of type β_i from truth-telling by

$$U_i \equiv g [\gamma_1 x^{i1} - t^{i1}] + (1 - g) [\gamma_2 x^{i2} - t^{i2}].$$

We leave implicit the dependence on $\{\mathbf{x}, \mathbf{t}\}$ to simplify notation. By Lemma 2 and the argument before it, the principal's program can be written as

$$\begin{aligned} \max_{\{\mathbf{x}, \mathbf{t}\}} \quad & b \{g [t^{h1} - c(x^{h1})] + (1 - g) [t^{h2} - c(x^{h2})]\} + \\ & + (1 - b) \{g [t^{l1} - c(x^{l1})] + (1 - g) [t^{l2} - c(x^{l2})]\} \end{aligned}$$

s.t. IC^{i1}, IC^{i2} for $i = h, l$ and

$$\begin{aligned} IR_i & : U_i \geq 0 \text{ for } i = h, l, \\ IC_h & : U_h \geq g [\gamma_1 x^{l1} - t^{l1}] + (1 - g) [\gamma_2 x^{l1} - t^{l1}], \\ IC_l & : U_l \geq g [\gamma_1 x^{h2} - t^{h2}] + (1 - g) [\gamma_2 x^{h2} - t^{h2}]. \end{aligned}$$

In order to understand self-0's incentives to misreport β , rewrite the IC constraints of self-0 of type β_i in terms of the expected utility of β_{-i} plus some number R_i – with a slight abuse of terminology, we shall call R_i rents:

$$\begin{aligned} IC_h & : U_h \geq U_l + (1 - g) [\gamma_2 x^{l1} - t^{l1} - (\gamma_2 x^{l2} - t^{l2})] = U_l + R_h, \\ IC_l & : U_l \geq U_h + g [\gamma_1 x^{h2} - t^{h2} - (\gamma_1 x^{h1} - t^{h1})] = U_h + R_l. \end{aligned}$$

In a standard model of sequential screening with time-consistent agents, β_h can have an incentive to mimic his fellow β_l and simulate a consistently low valuation of x . By doing so, β_h can behave as β_l in each state and enjoy more any resulting positive level of consumption. Thus β_h has a nonnegative rent. Instead, β_l need not be better off than β_h by pretending to have a high willingness to pay. On the one hand, he values consumption less. On the other, he ends up paying the same as β_h . Hence, β_l enjoys a nonpositive rent.⁷

In the present model with time inconsistency, rents go in the *opposite* direction. Unless β_h is offered a singleton menu so that no actual choice is left to the future self, β_l self-0 can pretend to be an HTI agent and enjoy a strictly higher expected utility from β_h 's contract than does β_h self-0. This will give rise to a positive information rent for β_l . Instead, if β_h chooses the contract for LTIs – and this contains more than one option – his expected payoff is strictly less than β_l 's valuation of the same menu. In this case, β_h is worse off than β_l if he misrepresents his self-control problem. Finally, individuals enjoy no rent when mimicking a type that receives a degenerate menu. We first state this result formally and then provide an intuitive explanation of the economics behind it.

Proposition 1 *If $\{\mathbf{x}, \mathbf{t}\}$ satisfies IC^{h1} and IC^{h2} , then $R_l \geq 0$ with strict inequality iff $x^{h1} \neq x^{h2}$. If $\{\mathbf{x}, \mathbf{t}\}$ satisfies IC^{l1} and IC^{l2} , then $R_h \leq 0$ with strict inequality iff $x^{l1} \neq x^{l2}$.*

Proof. *If $x^{i1} = x^{i2}$, then $t^{i1} = t^{i2}$ and $R_i = 0$ for all $i = h, l$. If $x^{h1} > x^{h2}$, then*

$$t^{h1} - t^{h2} \geq \gamma_2 \beta_h [x^{h1} - x^{h2}] > \gamma_1 \beta_l [x^{h1} - x^{h2}] \geq \gamma_1 [x^{h1} - x^{h2}]$$

⁷The same would be true also in the present model, had *self-1* been in charge of signing contracts right before observing γ .

and $R_l > 0$.

Using IC^{l2} and $x^{l1} > x^{l2}$, $\beta_l > 1$ implies

$$t^{l1} - t^{l2} \geq \gamma_2 \beta_l [x^{l1} - x^{l2}] > \gamma_2 [x^{l1} - x^{l2}]$$

and $R_h < 0$. ■

The economic reason for why agents with mild (or no) time inconsistency gain by mimicking those with strong self-control problems is perhaps not immediate, although it is intuitive. Observe that Proposition 1 relies only on conditional incentive compatibility. Therefore β_l 's information rents are not a consequence of the fact that β_h is exploited more than β_l by the principal due to his weaker ability to self control. They are not a consequence of profit maximization and depend on the fact that in order to implement any flexible allocation self-1 must be given the correct incentives through the contract signed by self-0.

Consider the following simple example. Suppose in period 0 β_h picks a contract with the goal of implementing the flexible choice between a large ice cream, \bar{x} , if it is sunny tomorrow (γ_1), and a small one, \underline{x} , in case it rains (γ_2). The following table reports the (positive) *utility gains* of choosing \bar{x} instead of \underline{x} for self-0 and self-1 in each state:

	Self-0	Self-1 β_l	Self-1 β_h
Sun	U	V_l	V_h
Rain	u	v_l	v_h

Each upper case letter represents a strictly larger number than the corresponding lower case letter. Suppose further that $v_h > V_l > U$, consistently with our working assumption that $\beta_h \gamma_2 > \beta_l \gamma_1 > \gamma_1$. Since β_h foresees that even if it rains he will be very tempted by \bar{x} , today he must choose a contract that makes the large ice cream considerably more expensive than the small one to carry out his plan. That is, \bar{x} must cost at least v_h more than does \underline{x} in the menu designed for β_h . Crucially, from today's perspective the price premium, say $v_h + \varepsilon < V_h$, that self-1 will inevitably pay if the sun shines, *more than* offsets the anticipated benefit U of eating $\bar{x} - \underline{x}$. Thus, the choice of \bar{x} over \underline{x} on a sunny day is perceived by *self-0* of type β_h as a net loss.

Now think about a self-0 of type β_l who chooses the menu intended for β_h . Since $V_l < v_h$, he anticipates he will not pick the large ice cream and will thus avoid paying the premium if it is sunny tomorrow. This difference in future behavior turns the perceived net loss for self-0 of type β_h into an expected net gain for self-0 of type β_l . Thus, β_l assigns an overall larger value to β_h 's menu than does β_h himself. This explains why β_l is better off than β_h if he pretends to be an HTI agent by accepting the contract designed for β_h self-0. Further, it is now transparent why β_l does not benefit from mimicking β_h in the case where the menu chosen by β_h leaves no choice to his future self with no need to incentivize self control (all entries in the table above would be zero as $\bar{x} = \underline{x}$).

A similar argument explains why β_h is worse off than β_l when pretending to be an LTI individual. Suppose now that β_l commits to the menu offering the future choice of \bar{x} or \underline{x} . On the one hand, the price premium on the large ice cream can't be higher than V_l , otherwise self-1

will never choose it on a sunny day. Hence, if in period 0 β_h picks the same price schedule as β_l , he will end up eating \bar{x} even if it rains. On the other hand, \bar{x} must cost at least v_l more than does \underline{x} . Since $v_l > u$ (recall $\beta_l > 1$), self-0 of type β_h anticipates a net loss associated to a rainy day. This explains why β_h self-0 derives a lower expected utility from the contract intended for β_l than does a self-0 of type β_l .

Finally, in a standard model with time consistency, β_h 's information rent can be erased only by excluding β_l from the market, thereby giving up any profit with the latter. By contrast, in the present environment the principal can, if necessary, wipe out the rent to the LTIs by completely removing flexibility from the menu for the HTIs. In any case, he can always serve HTI agents and extracts some surplus from them (e.g. by offering only a medium-sized ice cream). Momentarily, we will see when this is actually the optimal thing to do.

We return to the formal characterization of the profit-maximizing DRM. The following immediate Lemma, whose proof appears in the Appendix, yields a first simplification of the problem.

Lemma 3 *IR_l is redundant. IC_l and IR_h must bind at the optimum.*

Using $t^{h1} = \gamma_1 x^{h1} + \frac{1-g}{g} [\gamma_2 x^{h2} - t^{h2}]$ and IC_l binding, it follows that

$$U_l = R_l = [g\gamma_1 + (1-g)\gamma_2] x^{h2} - t^{h2},$$

and IC^{h1} and IC^{h2} are equivalent to

$$\begin{aligned} LB & : t^{h2} \geq [g\gamma_1 + (1-g)\gamma_2] x^{h2} - g\gamma_1 (\beta_h - 1) [x^{h1} - x^{h2}], \\ UB & : t^{h2} \leq [g\beta_h + (1-g)] \gamma_2 x^{h2} + g(\gamma_1 - \gamma_2 \beta_h) x^{h1}. \end{aligned}$$

Thus, the principal's program can be written as

$$\begin{aligned} \max_{\{\mathbf{x}, \mathbf{t}\}} & b \{ g [\gamma_1 x^{h1} - c(x^{h1})] + (1-g) [\gamma_2 x^{h2} - c(x^{h2})] \} + \\ & + (1-b) \{ g [\gamma_1 x^{l1} - c(x^{l1})] + (1-g) [\gamma_2 x^{l2} - c(x^{l2})] - [g\gamma_1 + (1-g)\gamma_2] x^{h2} + t^{h2} \} \end{aligned}$$

s.t. IC^{l1}, IC^{l2}, LB, UB and

$$\overline{IC}_h : R_h + R_l \leq 0.$$

It is immediate to see that UB must bind and LB is then implied by the necessary condition $x^{h1} \geq x^{h2}$. To see why the incentive compatibility constraint of β_h in state γ_2 binds, rather than in γ_1 , recall that β_l always mimics γ_2 after misreporting in the first period. So setting t^{h2} as large as possible is the first step in reducing the appeal of such deviation. Observe that the rent to β_l in terms of \mathbf{x}^h becomes

$$R_l(\mathbf{x}^h) = g[\beta_h \gamma_2 - \gamma_1] [x^{h1} - x^{h2}],$$

which is minimized if $x^{h1} = x^{h2}$ as we anticipated. We can interpret it as follows. $\gamma_1 [x^{h1} - x^{h2}]$ represents the utility loss of choosing x^{h2} over x^{h1} in state γ_1 and $\beta_h \gamma_2 [x^{h1} - x^{h2}]$ is the associated monetary savings (the avoided price premium); note the dependence on g , the likelihood

of γ_1 , which will be relevant in our characterization of the optimal DRM. Finally, the program reduces to

$$\mathcal{P} := \left\{ \begin{array}{l} \max_{\{\mathbf{x}\}, t^1, t^2} \quad b \left\{ g \left[(\gamma_1 - (\beta_h \gamma_2 - \gamma_1) \frac{1-b}{b}) x^{h1} - c(x^{h1}) \right] + \right. \\ \quad \left. + (1-g) \left[\left(\gamma_2 + (\beta_h \gamma_2 - \gamma_1) \frac{g}{1-g} \frac{1-b}{b} \right) x^{h2} - c(x^{h2}) \right] \right\} + \\ \quad + (1-b) \left\{ g [\gamma_1 x^{l1} - c(x^{l1})] + (1-g) [\gamma_2 x^{l2} - c(x^{l2})] \right\} \\ \text{s.to} \quad \quad \quad IC^{l1}, IC^{l2} \\ \quad \quad \quad \widehat{IC}_h : R_h + R_l(\mathbf{x}^h) \leq 0, \\ \quad \quad \quad M^h : x^{h1} \geq x^{h2}. \end{array} \right.$$

We are now ready to derive the optimal screening mechanism. We shall provide a characterization in terms of b , i.e., the share of HTIs in the market, to account for the usual trade-off faced by the principal between efficiency and rent extraction. We also highlight an interesting connection with the intensity of HTIs' self-control problem and the likelihood of experiencing temptation.⁸

Our intuitive explanation of why β_l and β_h self-0 gains and loses, respectively, by mimicking the other's type helps to predict the direction of optimal distortions. When β_l mimics β_h , he doesn't pay the price premium on x^{h1} that is necessary to discourage overconsumption by β_h . The closer are x^{h1} and x^{h2} , the smaller the deterrent necessary for β_h 's commitment to be effective and the less self-0 of type β_l benefits from avoiding it. Therefore, to reduce β_l 's information rent the principal has to shrink the gap between the contingent choices of β_h . However, doing so reduces the expected payoff to self-0 of type β_h from his menu and consequently its profitability.

Our first result shows that if the share of HTI agents in the market is sufficiently small, then the principal completely eliminates flexibility in his offer to β_h . In so doing, he shuts off the source of the information rent to β_l and is able to offer LTIs the full-information contract, thereby extracting from them the whole maximal surplus. To simplify notation, define $\gamma^e \equiv g\gamma_1 + (1-g)\gamma_2$, i.e., self-0's expected valuation of consumption.⁹

Proposition 2 *If $b \leq b_*$, where*

$$b_* \equiv \frac{\beta_h - \gamma_1/\gamma_2}{\beta_h - \gamma^e/\gamma_2} \in (0, 1),$$

the principal's optimal DRM is such that $\{\mathbf{x}^l, \mathbf{t}^l\} \equiv \{\mathbf{x}_u^l, \mathbf{t}_u^l\}$, $\mathbf{x}^h = (x_, x_*)$ and $\mathbf{t}^h = (t_*, t_*)$ where $c'(x_*) = \gamma^e$ and $t_* = \gamma^e x_*$.*

Proof. *See Appendix.* ■

⁸In our example with a large and a small ice cream, a time-inconsistent agent experiences the temptation to buy the large option on the menu only if it rains tomorrow.

⁹Because in this Section we are focusing on the case when $\beta_h/\beta_l > \gamma_1/\gamma_2$, we don't explicitly include this condition in the formal statement of our propositions.

The threshold b_* is increasing in β_h . Thus we can interpret the formal statement in Proposition 2 as saying: Even when the majority is β_h , i.e., strongly time inconsistent, if their self-control problem is sufficiently serious to require powerful incentive schemes, it is not in the principal's best interest to offer HTIs a flexible contract. To implement a contingent choice between a large and a small ice cream, the menu for β_h must include a large enough premium on the first option. Once again, doing so would be too costly in terms of rents to β_l .

However, contracts that give no opportunity to incorporate future information into decisions do not maximize profitability, even when optimally designed, because they disregard self-0's desire for flexibility. When the share of β_h agents exceeds the cut off b_* , the principal prefers to offer them a more lucrative flexible menu and leave some rent to β_l . It turns out that the best policy to balance profitability with β_h and rent extraction with β_l depends on the likelihood of the states. Recall that, on the one hand, β_l is better off when facing the menu intended for β_h only by avoiding in state γ_1 the price premium on the large quantity. On the other hand, β_h is worse off under the contract designed for β_l only when he gives in to temptation and chooses the large option in state γ_2 .

Our next result covers the case when β_l is sufficiently unlikely to benefit from his stronger self control under the contract of β_h – equivalently, β_h is sufficiently likely to succumb to temptation under the contract of β_l . We show that when the fraction of HTI agents exceeds b_* , β_h is allowed to choose different quantities depending on the state of the world, although the actual degree of flexibility falls short of the first-best benchmark. Instead, β_l is *always* offered an incentive scheme that sustains the full-information outcome. Only the transfers designed for β_l are adjusted, through an appropriate (uniform) discount, in order to provide the necessary level of rents. Let $\bar{g} \in (0, 1)$ be defined as

$$\bar{g} \equiv \frac{\gamma_1 \beta_l - \gamma_2}{(\beta_h - 1) \gamma_2 + (\beta_l - 1) \gamma_1}.$$

Proposition 3 *If $g \leq \bar{g}$, then for all $b > b_*$ $\hat{\mathbf{x}}^l = (x_u^1, x_u^2)$, i.e., the first-best allocation for β_l , and $\hat{\mathbf{x}}^h = (\hat{x}^{h1}, \hat{x}^{h2})$ is distorted "inwards" s.t. $x_u^1 > \hat{x}^{h1} > \hat{x}^{h2} > x_u^2$. Specifically, $\hat{\mathbf{x}}^h$ solves*

$$\begin{aligned} c'(\hat{x}^{h1}) &= \gamma_1 - (\beta_h \gamma_2 - \gamma_1) \frac{1-b}{b}, \\ c'(\hat{x}^{h2}) &= \gamma_2 + (\beta_h \gamma_2 - \gamma_1) \frac{g}{1-g} \frac{1-b}{b}. \end{aligned}$$

Moreover, $\hat{\mathbf{t}}^h$ is given by

$$\begin{aligned} \hat{t}^{h1} &= g\gamma_1 \hat{x}^{h1} + (1-g)\gamma_2 \hat{x}^{h2} + (1-g)\gamma_2 \beta_h [\hat{x}^{h1} - \hat{x}^{h2}], \\ \hat{t}^{h2} &= g\gamma_1 \hat{x}^{h1} + (1-g)\gamma_2 \hat{x}^{h2} - g\gamma_2 \beta_h [\hat{x}^{h1} - \hat{x}^{h2}], \end{aligned}$$

and $\hat{\mathbf{t}}^l$ solves

$$\begin{aligned} \gamma_1 \beta_l [x_u^1 - x_u^2] &\geq \hat{t}^{l1} - \hat{t}^{l2} \geq \gamma_2 \beta_l [x_u^1 - x_u^2], \\ g\hat{t}^{l1} + (1-g)\hat{t}^{l2} &= g\gamma_1 x_u^1 + (1-g)\gamma_2 x_u^2 - R_l(\hat{\mathbf{x}}^h). \end{aligned} \tag{7}$$

Proof. See Appendix. ■

To understand the intuition why it is feasible to sustain the first-best allocation for β_l , recall our ice-cream example and suppose tomorrow is very likely to rain (i.e. $g \approx 0$). Although β_l will avoid overpaying for the large ice cream if he picks the menu designed for β_h , the chance that he will benefit from it is small. Hence, a tiny rent should be enough to keep β_l away from mimicing β_h , e.g., via a uniform discount on β_l 's menu with no distortion in ice-cream size (cfr. the term $-R_l(\tilde{\mathbf{x}}^h)$ in equation (7)). At the same time, β_h is already sufficiently uninterested in the pricing scheme of β_l because it would lead him to give in to temptation and overpay for the large option with very high probability. So, as long as tomorrow is sufficiently unlikely to be sunny, there is no need to distort the ice-cream consumption of β_l to prevent β_h from misreporting.

Our last result is about an environment where β_l superior self control is very likely to give him an advantage over β_h and a large share of the principal's business comes from HTI individuals. As before, β_h underconsumes in state γ_1 and overconsumes in state γ_2 . Interestingly, we show that *also* the contract of β_l can contain inefficiencies – even when LTI individuals are time consistent, i.e., when $\beta_l = 1$. In particular, the menu of β_l can entail an excessively *high* level of flexibility compared to the first best: LTI agents consume too much when their valuation is high, γ_1 , and too little when low, γ_2 .¹⁰

Proposition 4 *If $g > \bar{g}$, there exists $b^*(g) \in (b_*, 1)$ such that:*

1. *If $b > b^*(g)$, $\tilde{\mathbf{x}}^l = (\tilde{x}^{l1}, \tilde{x}^{l2})$ is distorted "outwards" and $\tilde{\mathbf{x}}^h = (\tilde{x}^{h1}, \tilde{x}^{h2})$ is distorted "inwards", i.e., $\tilde{x}^{l1} > x_u^1 > \tilde{x}^{h1} > \tilde{x}^{h2} > x_u^2 > \tilde{x}^{l2}$.*

Specifically, $\tilde{\mathbf{x}}^l$ and $\tilde{\mathbf{x}}^h$ solve

$$\begin{aligned} c'(\tilde{x}^{h1}) &= \gamma_1 - (\gamma_2\beta_h - \gamma_1) \frac{1 - b + \mu}{b}, \\ c'(\tilde{x}^{h2}) &= \gamma_2 + (\gamma_2\beta_h - \gamma_1) \frac{1 - b + \mu}{b} \frac{g}{1 - g}, \\ c'(\tilde{x}^{l1}) &= \gamma_1 + (\gamma_1\beta_l - \gamma_2) \frac{\mu}{1 - b} \frac{1 - g}{g}, \\ c'(\tilde{x}^{l2}) &= \gamma_2 - (\gamma_1\beta_l - \gamma_2) \frac{\mu}{1 - b}, \end{aligned}$$

where $\mu > 0$ is the Lagrangian multiplier associated to \widehat{IC}_h . Further, we have for $\tilde{\mathbf{t}}^h$

$$\begin{aligned} \tilde{t}^{h1} &= g\gamma_1\tilde{x}^{h1} + (1 - g)\gamma_2\tilde{x}^{h2} + (1 - g)\gamma_2\beta_h [\tilde{x}^{h1} - \tilde{x}^{h2}], \\ \tilde{t}^{h2} &= g\gamma_1\tilde{x}^{h1} + (1 - g)\gamma_2\tilde{x}^{h2} - g\gamma_2\beta_h [\tilde{x}^{h1} - \tilde{x}^{h2}], \end{aligned}$$

and for $\tilde{\mathbf{t}}^l$

$$\begin{aligned} \tilde{t}^{l1} &= g\gamma_1\tilde{x}^{l1} + (1 - g)\gamma_2\tilde{x}^{l2} - R_l(\tilde{\mathbf{x}}^h) + (1 - g)\gamma_1\beta_l [\tilde{x}^{l1} - \tilde{x}^{l2}], \\ \tilde{t}^{l2} &= g\gamma_1\tilde{x}^{l1} + (1 - g)\gamma_2\tilde{x}^{l2} - R_l(\tilde{\mathbf{x}}^h) - g\gamma_1\beta_l [\tilde{x}^{l1} - \tilde{x}^{l2}]. \end{aligned}$$

¹⁰The result extends to $\beta_l = 1$ by continuity.

2. If $b_* \leq b \leq b^*(g)$ the optimal menu coincides with that in Proposition 3.

Proof. See Appendix. ■

Intuitively, as the probability g of the state γ_1 increases, β_l 's threat of choosing the menu purchased by β_h becomes more cogent for the principal. At the same time, the concerns of β_h about not resisting temptation under the contract offered to β_l vanish. So, as rain becomes less likely (i.e. $g \uparrow 1$), on the one hand, the principal must offer β_l enough discounts. On the other hand, he has to increase the price spread on β_l 's menu to counterbalance the fall in the expected net loss that β_h associates to a rainy day, if he chooses the contract of β_l .

For b close to b_* , β_h consumes almost the same quantities in both states. So a tiny price premium is enough to sustain such allocation. Consequently, discounts on the options chosen by β_l are sufficient to provide LTIs with enough rent not to mimic β_h .

As the share of profits coming from HTI individuals rises, the principal offers β_h more flexibility, boosting the rents to β_l . Eventually, the discounts offered to β_l become attractive for β_h too (formally, \widehat{IC}_h binds). The principal then has to hold down the expected payment by β_l and, at the same time, keep increasing the difference in transfers associated to his menu (cfr. the expression for \tilde{t}^l). For β_l to accept (if sunny) the necessarily larger spread in prices, at some point, the principal must make the large ice cream a little larger and the small one smaller.

Several additional comments are in order. Our results highlight an unexpected interaction between the unobservable heterogeneity in self-control and the relative likelihood of future states $\rho = \frac{g}{1-g}$ in shaping the profit-maximizing contract. From the viewpoint of self-0, the LTIs' menu sustains the state-wise efficient allocation as long as ρ is not too large. This is true independently of the composition of the market population, both in terms of the share of HTI individuals and in terms of the intensity of HTIs' self-control problem. However, if ρ rises too much, the monopolist pushes agents who are relatively more capable of self-control (even time consistent, i.e., $\beta_l = 1$) towards inefficiently high and low levels of consumption in state γ_1 and γ_2 , respectively.

As far as HTI agents are concerned, under the assumption $\beta_h/\beta_l > \gamma_1/\gamma_2$ the equilibrium allocation is never state-wise efficient – neither for self-0 nor for self-1. Further, it can vary in the permitted degree of flexibility up to its complete removal. However, no market exclusion occurs because the principal can always provide a service involving no future decision and eliminate information rents.¹¹

In general, we have shown that the HTI agents are offered a commitment device that is always worse, in terms of the sustained plan of action, than it would be in the absence of the LTI individuals. And the more so, the stronger is HTIs' behavioral bias. In addition, it may happen that also the LTI individuals commit to a plan that is suboptimal from their self-0's

¹¹This point suggests a connection with Eliaz-Spiegler (2006). With *symmetric* information between self-0 and self-1, sophisticated time inconsistent agents never exert any information externality on naive fellows because it is always feasible and optimal for the principal to offer the former a singleton menu.

perspective because of the presence of more time-inconsistent agents in the market. Notably, distortions in LTIs' contract become more likely as the chance that HTIs' inferior self control is relevant *decreases*.

Finally, suppose we could obtain accurate data about today's valuation of x across the population and states and we compared it with the consumption behavior induced by contracts agents signed at some point in the past. If we assumed that agents are time consistent and interpreted the data from the perspective of standard sequential screening models, we would find a mismatch between willingness to pay and actual quantities consumed. More specifically, there would be agents with higher valuations consuming *less* than individuals with lower willingness to pay. In particular, in our environment the order of self-1's valuations is $\gamma_1\beta_h > \gamma_2\beta_h > \gamma_1\beta_l > \gamma_2\beta_l$. However, the equilibrium allocation satisfies $x^{l1} > x^{h1} \geq x^{h2} > x^{l2}$ and $x^{h1} < x_v^{h1}$ (where x_v^{h1} corresponds to the first best for $\gamma_1\beta_h$).

6 Conclusions

We studied a simple model of monopolistic provision of commitment contracts in a market with a population of time-inconsistent, Strotzian decision makers who differ in terms of their ability to self control. We considered an environment with two periods and arrival of information in the second to incorporate the essential trade-off between commitment and flexibility.

We derived the first-best benchmark by studying the optimal contracts the principal offers when ex-ante heterogeneity in self control is observable. These contracts provide all agents with a perfect balance between commitment and flexibility in that the sustained allocation is state-wise efficient according to self-0's preference.

We analyzed the monopolist profit-maximizing contracts under asymmetric information. We found that the first-best outcome is robust to small unobservable heterogeneity. When individuals' self-control problems can be sufficiently different, we showed that more time-inconsistent agents exert an information externality on less time-inconsistent fellows. We explained its source and derived its impact on the profit-maximizing contracts and the (in)efficiency of the equilibrium allocation.

In future work we intend to consider more general distributions of taste shocks with continuous support to get a richer characterization of the degree of flexibility in the optimal menu of each type.

7 Appendix

7.1 Proof of Lemma 2

By standard arguments, IC^{i1} and IC^{i2} together imply $x^{i1} \geq x^{i2}$. Since $\beta_h > \beta_l > 1$, $\gamma_1 > \gamma_2$ and $\gamma_2\beta_h > \gamma_1\beta_l$, $x^{l1} \neq x^{l2}$ implies

$$\gamma_1\beta_h [x^{l1} - x^{l2}] > \gamma_2\beta_h [x^{l1} - x^{l2}] > \gamma_1\beta_l [x^{l1} - x^{l2}] \geq t^{l1} - t^{l2},$$

where the last inequality uses condition (5) for type β_l . It follows that β_h strictly prefers (x^{l1}, t^{l1}) to (x^{l2}, t^{l2}) independently of the realization of γ . Similarly, from (6) for type β_h , we have

$$t^{h1} - t^{h2} \geq \gamma_2\beta_h [x^{h1} - x^{h2}] > \gamma_1\beta_l [x^{h1} - x^{h2}] > \gamma_2\beta_l [x^{h1} - x^{h2}].$$

7.2 Proof of Lemma 3

The first part is immediate from $R_l \geq 0$ and $U_l \geq U_h + R_l$. If IC_l does not bind, the following is a feasible improvement on $\{\mathbf{x}, \mathbf{t}\}$. Raise t^{l1} and t^{l2} by $\varepsilon > 0$ sufficiently small so that IC_l still holds. IC^{ij} remain unchanged for all i, j as well as IR_h , R_h and R_l . IC_h is relaxed. Expected profits are strictly higher.

Using $U_l = U_h + R_l$, IC_h is equivalent to $R_h + R_l \leq 0$. Thus, if $U_h > 0$, raising all transfers by the same amount $\varepsilon > 0$ would increase profits without perturbing any remaining constraint.

7.3 Proof of Proposition 2

The pointwise solution to the relaxed version of \mathcal{P} features: For $i = 1, 2$, $x^{li} = x_u^i$ and

$$\bar{x}^{hi} = \arg \max_{x \in [0,1]} v^i x - c(x),$$

where

$$\begin{aligned} v^1 &= \gamma_1 - (\beta_h\gamma_2 - \gamma_1) \frac{1-b}{b}, \\ v^2 &= \gamma_2 + (\beta_h\gamma_2 - \gamma_1) \frac{g}{1-g} \frac{1-b}{b}. \end{aligned}$$

Since $\beta_h\gamma_2 > \gamma_1$, there exist values of b and g for which $v^1 < v^2$. In particular, $v^1 \leq v^2$ is equivalent to $b \leq b_*$, where

$$b_* \equiv \frac{\beta_h - \gamma_1/\gamma_2}{\beta_h - \gamma^e/\gamma_2} \in (0, 1).$$

Standard comparative static arguments imply $\bar{x}^{h1} \leq \bar{x}^{h2}$. Then, constraint M^h implies that the actual solution satisfies $\bar{x}^{h1} = \bar{x}^{h2} \equiv \bar{x}$ given by

$$\bar{x} = \arg \max_{x \in [0,1]} [gv^1 + (1-g)v^2] x - c(x) = \arg \max_{x \in [0,1]} \gamma^e x - c(x).$$

It remains to check that there exists t^{l1} and t^{l2} that satisfy \widehat{IC}_h , IC^{l1} and IC^{l2} . Since $x_u^1 > x_u^2$,

$$\gamma_1 \beta_l [x_u^1 - x_u^2] \geq t^{l1} - t^{l2} \geq \gamma_2 \beta_l [x_u^1 - x_u^2] \quad (8)$$

is feasible. Since $x^{h1} = x^{h2}$, \widehat{IC}_h at (x_u^1, x_u^2) becomes

$$t^{l1} - t^{l2} \geq \gamma_2 [x_u^1 - x_u^2],$$

which is satisfied because of the second inequality in (8) and $\beta_l > 1$. The formulae for the transfers come from IR_h and IC_l binding.

7.4 Proof of Proposition 3

Consider again the point-wise solution to the relaxed version of \mathcal{P} given above. Then $v^1 > v^2$ is equivalent to $b > b_*$ and $\bar{x}^{h1} > \bar{x}^{h2}$ so that M^h holds. As in proof of Proposition 2, we need to make sure that (t^{l1}, t^{l2}) can be chosen so that the remaining constraints are satisfied. We already know that (8) is feasible. \widehat{IC}^h becomes

$$(1 - g) [(t^{l1} - t^{l2}) - \gamma_2 (x_u^1 - x_u^2)] \geq g [\beta_h \gamma_2 - \gamma_1] [\bar{x}^{h1} - \bar{x}^{h2}] > 0.$$

From IC^{l2} and $\beta_l > 1$, the LHS is positive. From IC^{l1} , it is bounded above by $(\gamma_1 \beta_l - \gamma_2) (x_u^1 - x_u^2) > 0$. The next Lemma identifies the condition on g for which the relaxed solution satisfies \widehat{IC}^h for any b .

Lemma 4 *There exists $\bar{g} \in (0, 1)$ such that \widehat{IC}^h holds for every b iff $g \leq \bar{g}$.*

Proof. Let $\Delta^h(b) \equiv \bar{x}^{h1}(b) - \bar{x}^{h2}(b)$ and note that for every b the solution to the relaxed (except for M^h) program has $0 \leq \Delta^h(b) \leq x_u^1 - x_u^2$ and $\Delta^h(b) \uparrow x_u^1 - x_u^2$ as $b \rightarrow 1$. Define \bar{g} as

$$\bar{g} \equiv \frac{\gamma_1 \beta_l - \gamma_2}{(\beta_h - 1) \gamma_2 + (\beta_l - 1) \gamma_1} \in (0, 1).$$

Necessity: if $g \in (\bar{g}, 1)$, then

$$(1 - g) (\gamma_1 \beta_l - \gamma_2) (x_u^1 - x_u^2) < g (\beta_h \gamma_2 - \gamma_1) (x_u^1 - x_u^2).$$

Hence, there exists $b^*(g) < 1$ uniquely defined by the implicit condition

$$\Delta^h(b^*) = \frac{1 - g}{g} \frac{\beta_l \gamma_1 - \gamma_2}{\beta_h \gamma_2 - \gamma_1} (x_u^1 - x_u^2),$$

such that if $b > b^*(g)$

$$(1 - g) (\gamma_1 \beta_l - \gamma_2) (x_u^1 - x_u^2) < g (\beta_h \gamma_2 - \gamma_1) \Delta^h(b)$$

and \widehat{IC}^h and IC^{l1} are inconsistent at the relaxed solution.

Sufficiency: suppose $g \in (0, \bar{g})$, then for every b

$$\begin{aligned} (1-g)(\gamma_1\beta_l - \gamma_2)(x_u^1 - x_u^2) &\geq g(\beta_h\gamma_2 - \gamma_1)(x_u^1 - x_u^2) \\ &\geq g(\beta_h\gamma_2 - \gamma_1)\Delta^h(b). \end{aligned}$$

■

So if $g \leq \bar{g}$ and $b > b_*$, the optimal contract sustains the allocation $x^{l1} = x_u^1$, $x^{l2} = x_u^2$, $x^{h1} = \bar{x}^{h1} < x_u^1$ and $x^{h2} = \bar{x}^{h2} > x_u^2$ where the inequalities follow from $\gamma_1 > v_1 > v_2 > \gamma_2$. Finally, to compute $\hat{\mathbf{t}}^h$ use IC^{h2} and IR_h binding. IC_l binding provides equation (7) in the definition of $\hat{\mathbf{t}}^l$.

7.5 Proof of Proposition 4

Suppose $g > \bar{g}$. The statement for the case $b_* < b \leq b^*(g)$ follows immediately from the proof of Lemma 4. When $b > b^*(g)$, \widehat{IC}^h binds at the relaxed solution. Let $\mu \geq 0$ be the Lagrangian multiplier associated to \widehat{IC}^h . The principal's objective becomes

$$\begin{aligned} &b \{g[v^1x^{h1} - c(x^{h1})] + (1-g)[v^2x^{h2} - c(x^{h2})]\} \\ &+ (1-b) \{g[\gamma_1x^{l1} - c(x^{l1})] + (1-g)[\gamma_2x^{l2} - c(x^{l2})]\} \\ &+ \mu \{(1-g)[(t^{l1} - t^{l2}) - \gamma_2(x^{l1} - x^{l2})] - g[\beta_h\gamma_2 - \gamma_1][x^{h1} - x^{h2}]\} \end{aligned}$$

The remaining constraints are M^h ,

$$\begin{aligned} IC^{l1} &: \gamma_1\beta_l(x^{l1} - x^{l2}) \geq t^{l1} - t^{l2} \\ IC^{l2} &: t^{l1} - t^{l2} \geq \gamma_2\beta_l(x^{l1} - x^{l2}). \end{aligned}$$

Since we are looking for a solution with $\mu > 0$, it is immediate to see that IC^{l1} must bind and IC^{l2} is then implied by the necessary condition $x^{l1} \geq x^{l2}$ (M^l). Substituting $t^{l1} - t^{l2}$, we get

$$\begin{aligned} \max_{x^{l1} \geq x^{l2}, x^{h1} \geq x^{h2}} &b \{g[w^{h1}x^{h1} - c(x^{h1})] + (1-g)[w^{h2}x^{h2} - c(x^{h2})]\} + \\ &+ (1-b) \{g[w^{l1}x^{l1} - c(x^{l1})] + (1-g)[w^{l2}x^{l2} - c(x^{l2})]\} \end{aligned} ,$$

where

$$\begin{aligned} w^{h1} &= \gamma_1 - (\gamma_2\beta_h - \gamma_1) \frac{1-b+\mu}{b}, \\ w^{h2} &= \gamma_2 + (\gamma_2\beta_h - \gamma_1) \frac{1-b+\mu}{b} \frac{g}{1-g}, \\ w^{l1} &= \gamma_1 + (\gamma_1\beta_l - \gamma_2) \frac{\mu}{1-b} \frac{1-g}{g}, \\ w^{l2} &= \gamma_2 - (\gamma_1\beta_l - \gamma_2) \frac{\mu}{1-b}. \end{aligned}$$

As far as β_l is concerned, the pointwise solution yields $\tilde{x}^{l1} > x_u^1 > x_u^2 > \tilde{x}^{l2}$ so that M^l is satisfied. For type β_h one can prove by contradiction that M^h must hold strict.

Lemma 5

$$\tilde{x}^{h1} = \arg \max_{x \in [0,1]} w^{h1}x - c(x) > \arg \max_{x \in [0,1]} w^{h2}x - c(x) = \tilde{x}^{h2}.$$

Proof. If $\mu = 0$, then $w^{h1} = v^1 > v^2 = w^{h2}$ because $b > b_*$ and the result is immediate. Therefore, for M^h to bind, it must be that $\mu > 0$ sufficiently large so that $w^{h1} \leq w^{h2}$. Also, $\mu > 0$ implies that the \widehat{IC}^h constraint binds. However, if M^h forces $x^{h1} = x^{h2}$ the RHS of \widehat{IC}^h equals zero and the LHS is

$$(1 - g)(\gamma_1\beta_l - \gamma_2)(\tilde{x}^{l1} - \tilde{x}^{l2}) > 0.$$

Contradiction. ■

Finally, $w^{l1} > \gamma_1 > w^{h1}$, $w^{h2} > \gamma_2 > w^{l2}$ and the Lemma yield

$$\tilde{x}^{l1} > x_u^1 > \tilde{x}^{h1} > \tilde{x}^{h2} > x_u^2 > \tilde{x}^{l2}.$$

Use IC^{h2} , IR_h and IC_l , IC^{l1} binding to compute $\tilde{\mathbf{t}}^h$ and $\tilde{\mathbf{t}}^l$, respectively.

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