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Working Paper

Adverse selection and liquidity distortion in decentralized markets

Discussion Paper, No. 1513

Provided in Cooperation with:

Kellogg School of Management - Center for Mathematical Studies in Economics and Management Science, Northwestern University

Suggested Citation: Chang, Briana (2010) : Adverse selection and liquidity distortion in decentralized markets, Discussion Paper, No. 1513, Northwestern University, Kellogg School of Management, Center for Mathematical Studies in Economics and Management Science, Evanston, IL

This Version is available at:

<https://hdl.handle.net/10419/59654>

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Discussion Paper #1513
November 2, 2010

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Key words: Liquidity; Search frictions, Adverse selection; Uncertainty; Capital Reallocation

JEL classification: D82, G1

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Adverse Selection and Liquidity Distortion in Decentralized Markets*

Briana Chang[†]

November 2, 2010

Abstract

Why do some markets remain illiquid even when there is a positive gain from trade? In order to understand the real determinants of market liquidity in decentralized markets, we are going to analyze this question in a competitive market setting when both search frictions and adverse selection play roles. In a dynamic environment with heterogeneous sellers and buyers, we investigate the role of market frictions and how adverse selection leads to the distortion of equilibrium market liquidity. The resulting friction therefore prohibits resources from reallocating efficiently. In the application of capital reallocation, we further show that this trading friction can generate significant economic fluctuations.

Key words: Liquidity; Search frictions, Adverse selection; Uncertainty; Capital Reallocation;

1 Introduction

As the massive ongoing microeconomics restructuring and factor reallocation is crucial to aggregate performance, there is clearly a strong link between how the economy is doing and how well factors markets are functioning. Intuitively, an illiquid market hinders resource reallocation and, hence, has a negative impact on economic performance. The real question is, however, why some markets remain illiquid even when there is a positive gain from trade and why there is no market clearing. This paper aims to answer these questions within a market structure framework and provide the micro-foundation for the resulting frictions. Our framework does not only improve our understanding of the source of liquidity in a decentralized market but also allows for a richer analysis of this market

*I am indebted to Dale Mortensen for continuous support and encouragement. The paper also benefits from the discussion with Andrea Eisfeldt, Arvind Krishnamurthy, Philipp Kircher, Alessandro Pavan, Mirko Wiederholt, Simone Galperiti, and Martin Szydlowski.

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friction response to the economic environment (which is absent in the standard model assuming exogenous adjustment costs) and its resulting macroeconomic phenomena.

In decentralized markets, traders must search for the counterparty. In this environment, traders care about both the selling price as well as liquidity, which, in line with the Over-the-Counter literature, is measured by the expected search time. How fast a seller can cash his assets will depend crucially on the market liquidity, that is, how many investors are willing to buy. Meanwhile, these purchase and sales decisions are often complex, involving strategic considerations. One crucial element is that current owners tend to have private information about their assets, for example, current residents of the house, the banks who design the mortgage-backed securities, or the firms who own corporate assets, etc. This then naturally creates the problem of adverse selection. The model shows that, with the existence of adverse selection, the equilibrium liquidity will be distorted when compared to an environment with complete information. This effect is the key feature of our model. Without adverse selection, the impact of search frictions alone are modest. However, it is the informational problem prevailing in the decentralized markets that drives the market illiquidity. Compared to the earlier literature, our setting is dynamic and is designed to handle rich (continuous) type distributions of sellers as well as the competition among different buyers. This is the first paper, at least to my knowledge, explicitly characterizing this endogenous market liquidity in such an environment with adverse selection.

Our setup follows the competitive search equilibrium, where uninformed principals (buyers) post prices to attract informed agents (sellers) and agents direct their search toward their preferred market. It is well known that, given complete information, the outcome of a competitive search equilibrium put forth by Moen (1997) necessarily solves the social planner's problem. Moreover, Mortensen and Wright (2002) point out that one can think of the competitive search equilibrium outcome as if there is a third party (market maker) costlessly setting up the price and market tightness for each market. Extending this line of thought to the environment with adverse selection, this paper proposes that the equilibrium can be solved directly as a mechanism design problem subject to both sellers' and buyers' optimality constraints. The main result shows that the market with a higher quality asset will suffer a distorted market tightness when compared to the benchmark with complete information. The key intuition is that holding different quality assets results in difference preference of waiting. This is essentially the mechanism behind this paper which demonstrates an agent's type is revealed by his choice of market.

Regarding the application, we focus on the reallocation of firms' corporate assets, given that capital is one of the important factors determining aggregate productivity. As documented in the empirical literature, changes in ownerships of firms' corporate assets—for example, product lines, plants, machines, and other business units—affect productivity. More precisely, capital typically flows from less productive to more productive firms, and the productivity of acquired capital increases. Interestingly, probably counter-intuitively, at the macro level, Eisfeldt and Rampini (2006) have documented that the capital reallocation is procyclical while the cross-sectional dispersion of the productivity

is countercyclical. From this finding, they suggest that the reallocation friction should be countercyclical. This finding therefore calls for a theoretical framework to analyze and understand the macroeconomic phenomena stemming from microeconomics transactional problems. Our paper therefore provide a micro-foundation for this phenomenon. The model demonstrates that the resulting illiquidity prevents resources from reallocating efficiently, generating a drop in productivity and a slow recovery, and, therefore, results in substantial aggregate effects.

Meanwhile, in line with Bloom (2009), who provides both a model and estimations on the impact of the dispersion (uncertainty) on the aggregate economy, this paper further analyzes the relationship between equilibrium market liquidity and uncertainty, as well as the dispersion along the dynamics of the reallocation process. We, however, take a different stand on dispersion and uncertainty. Dispersion in our model will be endogenously determined. The resulting dispersion is a combination of assuming factor heterogeneity and its varied use of technology. The former is taken as given; the latter, however, is a result of the market, taking into account both suppliers' and investors' optimization problems. Uncertainty in our model, on the other hand, measures the stability of a firms' business condition. High uncertainty represents a higher rate at which a firm receives a negative shock or incurs some financial constraint so that it is optimal for it to sell its capital and exit the market. The theoretical results implies that when the economy faces a higher uncertainty, the market tightness decreases and, therefore, it is even harder for the seller to cash his assets. This market illiquidity results in a higher degree of capital mismatch, which does not only generate a drop in productivity but also creates a higher dispersion in economy. The model therefore provides a possible explanation for the co-movement of these aggregate variables documented in the empirical findings.

Section 2 introduces the basic model and characterizes the equilibrium outcome. Section 3 extends the basic model to allow for heterogenous buyers. Section 4 applies the developed method to explain firms' capital reallocation and demonstrates the aggregate effect resulting from a illiquid decentralized market.

Related Literature:

The key ingredient of our model is the endogenous market liquidity stemming from search frictions and adverse selection in the competitive decentralized trading markets. A series of papers by Duffie, Garleanu and Pedersen—Duffie et al. (2005) and Duffie et al. (2007)—are the first to introduce search in models of asset market equilibrium. They build a dynamic asset-pricing model which shows how the equilibrium properties—allocation, prices, and bid and ask prices—depend on investors' search abilities and bargaining powers. Building on their model, the emerging literature studies the effects of liquidity in search models of asset pricing.¹ Compared to this line of literature, this paper is the first to solve the equilibrium with adverse selection in OTC markets. To do that, we borrow the basic framework of the competitive search equilibrium developed in the labor search literature, put forth by Moen (1997), and Mortensen and Wright (2002); furthermore, this

¹For example, Weill (2008), Lagos and Rocheteau (2009)

paper extends the framework to the environment in which sellers have private information about their asset quality.

Theoretically, our work is closet to Guerrieri et al. (2009) , who apply the notion of competitive search equilibrium to an environment with adverse selection in a static environment. Both of our refined equilibrium concept can be seen as applications from Gale (1992) and Gale (1996), who developed the concept of Walrasian general-equilibrium in an economy with adverse selection. Our work, however, complements Guerrieri et al. (2009) in the following respects: First of all, our setup is dynamics so that we can explicitly characterize the trading delay and analyze the role of resale, both of which have important implications on macroeconomic performance. Conceptually, we also focus on the decentralized trading market, where traders care about both market price as well as liquidity, in line with OTC literature. The idea that different types of asset owners will have different preferences regarding liquidity is explicitly captured in our setup, which is also the key determinant of our equilibrium result. Second, in our general model, we allow for heterogenous buyers in the markets. This generalization then takes into account the competition among buyers as well as the diversity, which is the hallmark of economic exchange. Additionally, it allows for a rich analysis on the resulting dispersion in the economy. Finally, regarding to the technique, we use different solution method, which enables us to characterize the environment with richer (continuous) types, and we further establish that the decentralized competitive equilibrium can be solved as a mechanism design problem, subject to both sellers' and buyers optimality constraints. Another related work is Eeckhout and Kircher (2010), who study the sorting of heterogeneous agents in a competitive search trading market and also apply the similar refinement of equilibrium concept. Their work helps us establish our benchmark, when there is no adverse selection. The key difference is that we show the impact of adverse selection and its resulting inefficiency. Furthermore, we show that due to adverse selection, positive assortative matching is guaranteed by the supermodularity in matching value, which is different from the case with complete information.

Our work is also related to the long literature of adverse selection/ lemon problem in asset markets. Compared to this line of literature, in a setting of competitive search equilibrium, we obtain a separated equilibrium, implying several distinct features: 1) All markets are priced in equilibrium and open; nevertheless, some high quality asset market are close to frozen so it is hard for sellers to get rid of their assets; 2) More importantly, different dispersion of the asset quality will have a first order effect on market illiquidity. Contrary to the standard adverse selection problem, which predicts that the equilibrium outcome highly depends on the expected value of assets because of the feature of a pooling equilibrium, what matters in our framework is the dispersion of asset quality. We further show that a pooling equilibrium can not be sustained as long as buyers have freedom to post the price in decentralized markets.

The market structure we developed can be easily applied to all decentralized asset trading markets. In this paper, we focus on the market for firms' corporate assets. According to the empirical literature on this, such as, Maksimovic and Phillips (2001) and

Warusawitharana (2008), there is indeed an active market for such asset ² and these transactions improve the allocative efficiency of capital in the economy, which therefore support our story of assets reallocation. Moreover, Maksimovic and Phillips (2001) shows that the probability of a sale declines as the plant’s general productivity and the segment’s productivities increase. Namely, the higher quality assets are sold with a lower probability. The model also provides an explanation for this phenomenon.

At the macroeconomic level, Eisfeldt and Rampini (2006) first shows that this capital reallocation is procyclical while the dispersion of firms’ productivity is countercyclical. Recent research on the impact of dispersion and uncertainty also confirms this result³. In order to explain these facts, some macro-models introduce the idiosyncratic productivity shocks and capital adjustment cost into the representative agent’s optimization problem. Bloom (2009) shows that, with existence of capital adjustment costs, higher uncertainty (measured as a shock to the second moment) expands firms’ inactive regions because it increases the real-option value of waiting. This concern then slows down the reallocations from low to high productivity firms. Instead of relying on an exogenous adjustment cost, we further provides the micro-foundation for this friction. In our model, firms, who receive a negative shock, do want to exit but have a hard time finding an investor who is willing to buy their capital in the equilibrium. This idea therefore provides the explanation as to why few firms exit in bad time, as documented in Lee and Mukoyama ().

Regarding macroeconomic implications, the paper is broadly connected to the literature emphasizing the macroeconomics of restructuring. As a survey of this line of literature, Caballero (2007) is a useful reference, both for theoretical frameworks and empirical evidence. The effect of financial friction or investment adjustment cost has been widely studied in the standard macroeconomic DSGE model. Compared to this line of the literature, we look closely at the source of friction existing in the decentralized market and provide a micro-foundation in order to analyze how this friction responds to different economic environment. We show that how a combination of heterogeneous firms and the reallocation shocks can generate significant aggregate fluctuation, which produces the same pattern as if it were driven by a first moment or second moment shock.

2 A Basic Model with Homogenous Buyers

There is a continuum of sellers who own one asset with different quality indexed by $s \in S$, which is private information. Let $G(s)$ denote the measure of sellers with types weakly below $s \in S$ and assume that $S = [s_L, s_H] \subset R_+$. While holding the asset s , the seller

²Warusawitharana (2008) documented that purchases and sales of operating assets by firms generated \$162 billion for share holders over the past 20 years.

³For example, Bachmann and Bayer (2009) shows that cross-sectional standard deviation of firm-level innovations in the Solow residual, value added is robustly and significantly countercyclical; meanwhile, the cross-sectional standard deviation of firm-level investment is procyclical.

enjoys a flow payoff s but must at the same time pay a holding cost as long as the asset remains unsold. One can think of this as a simple way to model a seller's need to "cash" the asset. As explained in Duffie et al. (2007), we could imagine this holding cost to be a shadow price for ownership due to, for example, (a) low liquidity, that is, a need for cash; (b) high financing cost; (c) adverse correlation of asset returns with endowments; or (d) a relatively low personal use for the asset, for example, for certain durable consumption goods such as homes.

There is a large continuum of homogenous buyers (that is, we assume that the measure of buyers is strictly larger than sellers), who enjoy the asset, s , with a flow payoff s . In order to buy the asset, the buyer needs to enter the market to search for the seller, incurring a search cost, $k \geq 0$ for the duration of the search. All agents are infinitely lived and discount at the interest rate, r . Time is continuous. As standard, traders are subject to the random matching function. The standard assumptions on the matching function in each potential market are imposed, i.e. $m(B, \mu)$ has constant returns to scale and $m_B > 0, m(\mu, 0) = 0, m_\mu > 0, m(0, B) = 0$, where B and μ denote the measure of the buyers and sellers in each market. Hence, the market tightness of each market depends on the ratio of buyers to sellers in that market, $\theta = \frac{B}{\mu}$. Therefore, sellers meet buyers with the arrival rate $\frac{m(\mu, B)}{\mu} = m(\theta) = m(1, \frac{B}{\mu})$, which is increasing in θ , and buyers meet sellers with the meeting rate $\frac{m(\mu, B)}{B} = \frac{m(\theta)}{\theta}$, which is decreasing in θ . Particularly, throughout this paper, we assume that the matching function takes Cobb-Douglas form, $m(\mu, B) = M\mu^{1-\rho}B^\rho$.

2.1 Benchmark: Complete information

We first establish the benchmark with complete information, which is the canonical competitive search model put forth by Moen (1997). In our particular setup, buyers simply post a trading price and sellers direct their search toward their preferred market. All traders have rational expectations in the equilibrium market tightness associated with each market. Moreover, following the interpretation of Mortensen and Wright (2002), one can imagine the competitive search equilibrium as if there is a market maker who can costlessly set up a collection Θ of submarkets. Each market can be characterized by a pair (θ, p) , which is known ex ante to participants. Given the posting price and the market tightness in each market, each trader then selects the most preferred submarket in which to participate (search). With the assumption that there is perfect competition among market makers, the market maker's problem is then to maximize traders' utilities.

Sellers' and buyers' expected utilities who enter the market with the pair (θ, p) can be expressed as follows, respectively:

$$rU(s, \theta, p) = s - c + m(\theta)(p - U(\theta, s))$$

$$rU_b(s, \theta, p) = -k + \frac{m(\theta)}{\theta} \left(\frac{s}{r} - p - U_b(\theta, s) \right)$$

With the assumption that the measure of buyers are larger than that of sellers, free entry condition must hold for buyers. Hence, $U_b = 0$. With perfect information, market maker's optimization problems for each market for asset s is:

$$\begin{aligned} \max_{p,\theta} U(s) &= \max_{p,\theta} \frac{s - c + pm(\theta)}{r + m(\theta)} \\ st \quad : \quad U_b(s) &= \frac{m(\theta)(\frac{s}{r} - p) - \theta k}{r\theta + m(\theta)} = 0 \end{aligned}$$

One can easily see that θ_{FB} solves following FOC:

$$\frac{c}{k} = \frac{1}{\rho} (r\theta_{FB}^{1-\rho} + (1-\rho)\theta_{FB}) \quad (1)$$

Notice that θ_{FB} is an increasing function of the cost ratio, $\frac{c}{k}$. Namely, it is relatively easier for sellers to meet the buyer, and it takes longer for the buyer to find the seller when the holding cost is higher. Also, the first best solution is independent of the asset quality. The intuition is clear since the gain from trade is simply the holding cost, which is independent of the asset quality. The price of each asset is then: $p^{FB}(s) = \frac{s}{r} - \frac{k\theta_{FB}}{m(\theta_{FB})}$, the expected value of the asset minus the expected searching cost paid by buyers. One can easily check that IR constraint holds for all types of sellers. Obviously, first-best allocations can not be implemented in the environment with adverse selection. Facing the same market tightness, sellers always want to pretend a higher type so that they can get a higher payment.

2.2 Equilibrium with Adverse Selection

As in the complete information environment, buyers/sellers choose the price they would like to offer/accept, and we can imagine a separate market for each price. In equilibrium, one can think of there being a separate market associated with each price. Buyers and sellers congregate in the preferred market to search for the counterparty. For any posting price p , $\mu(p, s)$ is the measure of type- s who want to accept p , and $B(p)$ is the measure of buyers who want to offer p . Therefore (μ, B) describes the trading decisions of sellers and buyers.

Our equilibrium concept adopts the concept of refined Walrasian general-equilibrium approach developed in Gale (1992). Same spirit has been used in the competitive search equilibrium literature, such as Guerrieri et al. (2009)—single buyer's offer does not affect the sellers' overall expected utility and agents have rational expectations about the market tightness and the distribution of agents' types associated with each contract—and Eeckhout and Kircher (2010), where the payoff of each individual is determined only by his own decision and by the distribution of trading decisions (μ, B) in the economy. Clearly, no trade takes place at prices below zero and above $J(\bar{s})$, and we define the set of feasible prices as $P = [0, J(\bar{s})]$. To define agents' expected payoff, let the function

$\theta(\cdot) : P \rightarrow [0, \infty]$ denote the expected market tightness. For the active markets, that is, $P^* = \{p \in P | \mu(p, s) > 0 \cap B(p) > 0\}$, the market tightness is therefore well defined along with buyers' and sellers' expected payoff, which can be expressed as following:

$$\theta(p) = \frac{B(p)}{\int \mu(p, s) ds} \quad (2)$$

$$\begin{aligned} rU_b(p, \mu, \theta(p)) &= -k + \frac{m(\theta(p))}{\theta(p)} \left(\int \frac{\tilde{s}}{r} \mu(\tilde{s}|p) d\tilde{s} - p - U_b \right) \\ rU(p, \theta(p), s) &= s - c + m(\theta(p))(p - U) \end{aligned}$$

The equilibrium expected utilities of seller s at equilibrium then must satisfy:

$$V(s) = \max\left\{ \frac{s - c}{r}, \max_{p' \in P^*} U(p', \theta(p'), s) \right\}$$

We now need specify the belief out of the equilibrium path, following the spirit of refined equilibrium used in the literature. A buyer who contemplates a deviation and offers a price p which has not been posted, $p \notin P^*$, has to form a belief about the market tightness and the types he will attract. First of all, a buyer expects a positive market tightness only if there is a type of seller who is willing to trade with him. Moreover, he expects to attract the type s who is most likely to come until it is no longer profitable for them to do so. Formally, define:

$$\begin{aligned} \theta(p, s) &\equiv \inf\{\tilde{\theta} > 0 : U(p, \tilde{\theta}, s) \geq V(s)\} \\ \theta(p) &\equiv \inf_{s \in S} \theta(p, s) \end{aligned} \quad (3)$$

By convention, $\theta(p, s) = \infty$ when $U(p, \tilde{\theta}, s) \geq V(s)$ has no solution. Intuitively, we can think of $\theta(p)$ as a *lowest* market tightness for which he can find such a seller type. Now let $T(p)$ denote the set of types which are most likely to choose p :

$$T(p) = \arg \inf_{s \in S} \{\theta(p, s)\}$$

Therefore, this suggests that, given $\theta(p)$, p is optimal for every type $s \in T_p$ but not optimal for $s \notin T_p$. Hence, the buyer's assessment about $\mu(p, s)$ for any posting price p needs to satisfy the following restriction:

$$\text{For any price } p \notin P^* \text{ and type } s, \mu(p, s) = 0 \text{ if } s \notin T_p \quad (4)$$

Definition 1 *An equilibrium consists of traders' allocation (B, μ) , a function of seller's expected utilities $V(s)$, a market tightness function in each market p , $\theta(\cdot) : P \rightarrow [0, \infty]$ defined in (2) and (3), such that the following conditions hold:*

E1 (optimality for sellers): let

$$V(s) = \left\{ \frac{s - c}{r}, \max_{p \in P^*} \frac{s - c + m(\theta(p))}{r + m(\theta(p))} \right\}$$

and for any $(p, s) \in P^ \times S, \mu(p, s) > 0$ implies*

$$U(p, \theta(p), s) \geq U(p', \theta(p'), s) \quad \forall p' \in P^*$$

E2 (optimality for buyers and free-entry): for any $p \in P^$*

$$0 = U_b(p, \mu, \theta(p))$$

;and there does not exist any $p' \in P$ such that $U_b(p', \theta(p'), \mu) > 0$, where $\theta(p')$ and $\mu(p', s)$ satisfies (3) and (4).

2.3 Characterization

Following the interpretation of the market maker in our benchmark, we show that the equilibrium outcome can be characterized as the solution of a mechanism design problem which takes into account both sellers' and buyers' optimality condition. Intuitively, one can think of the market maker as the mechanism-designer who promises the price and the market tightness in each market so that sellers truthfully report their type, that is, condition *E1* has to hold. Moreover, a *feasible* mechanism must satisfy the market clear condition. In other words, the market tightness must equal the ratio of the measure of buyers who are willing to pay p to the measure of types- s sellers who are willing to accept p , which is characterized by (2). Meanwhile, given that buyers can post the price freely in the decentralized markets, any price schedule recommended by the market maker has to be optimal for buyers. Otherwise, buyers will deviate by posting price other than the ones recommended by the mechanism designer. This point is characterized by condition *E2*.

Overview of the solution: Our approach follows two steps: First, we characterize the set of feasible mechanism $\alpha \in A$, which satisfies *E1* and free-entry condition (Proposition 1). Second, we use *E2* to identify the necessary condition for which the solution to the mechanism can be decentralized in equilibrium (Lemma 1 and Lemma 2). This result enables us to pin down the unique candidate among the set of feasible mechanism A , which is a fully separating one. At the end, we show that this candidate is indeed the solution, that is, *E2* and participating constraints are all satisfied.

To find out the set of mechanism that satisfies sellers' IC constraints, we setup the problem as a mechanism design problem (of a market maker). By the revelation principle, it will be without loss of generality to focus direct revelation mechanisms. A direct mechanism is a pair (θ, p) where $\theta : S \rightarrow R_+$ is the market tightness function and a price function $p : S \rightarrow R_+$. The mechanism is interpreted as follows. A seller who reports his

type $\hat{s} \in S$ will then enter the market with the pair $(\theta(\hat{s}), p(\hat{s}))$. Hence, the value of seller s announces his type \hat{s} can then be expressed as:

$$rU(\hat{s}, s) = s - c + m(\theta(\hat{s}))(p(\theta(\hat{s})) - U(\hat{s}, s))$$

For notation simplicity, denote the price $p(\theta)$ is the price of market with the market tightness θ . The seller's optimal search problem can be rearranged as:

$$V(s) = \left\{ \frac{s-c}{r}, \max_{\hat{s}} U(\hat{s}, s) \right\} = \left\{ \frac{s-c}{r}, \max_{\hat{s}} \frac{s-c + p(\theta(\hat{s}))m(\theta(\hat{s}))}{r + m(\theta(\hat{s}))} \right\}$$

where $U(\theta(\hat{s}), s)$ is the contingent values of agent s entering the market $\theta(\hat{s})$. A seller can always choose not to participate and he will get his autarky utility, $\frac{s-c}{r}$ in that case. Notice that this is equivalent to set $m(\theta) = 0$. For convenience, one can think of not entering the market as if choosing the market where $\theta = 0$. The IC can be re-written as the requirement that $s \in \arg \max_{s' \in S} U(\theta(\hat{s}), s)$. First of all, we can prove that any mechanism which satisfies E1 can be characterized with following proposition:

Proposition 1 *The pair of function $\{\theta(\cdot), p(\cdot)\}$ satisfies sellers' optimality condition (E1) if and only if following conditions are satisfied:*

$$\frac{1}{r + m(\theta^*(s))} \text{ is non-decreasing} \quad (\text{M})$$

$$V(s) = \frac{u(s) + p^*(s) \cdot m(\theta^*(s))}{r + m(\theta^*(s))} = V(s_l) + \int_{s_l}^s U_s(\theta^*(\tilde{s}), \tilde{s}) d\tilde{s} \quad (\text{ICFOC})$$

$$V(s) \geq \frac{u(s)}{r} \quad (\text{IR})$$

Proof. Standard proof in mechanism design literature. See Appendix. In this basic model, $u(s) = s - c$. ■

From free-entry conditions, $U_b(p, \theta, \mu) = 0$, where $\mu(s, p)$ can then be found from $p(s)$, each active market (p, θ) then has to satisfy the following relation:

$$p = \int \frac{s}{r} \mu(s|p) ds - \frac{k\theta}{m(\theta)}$$

Proposition 1 and the free-entry condition then define the set of *feasible* mechanisms, A . Market clear condition is guaranteed by free-entry condition. Namely, buyers will entry until the "right" market tightness is satisfied. Moreover, let $V(\alpha, s)$ denote the expected payoff to a type- s seller under the mechanism $\alpha \equiv (p, \theta)$ Each mechanism $\alpha \in A$ is then composed of a price schedule $p^\alpha(\cdot)$, market tightness $\theta^\alpha(\cdot)$, and corresponding sellers' utilities $V^\alpha(\cdot)$. This set then includes all possible pooling as well as separating equilibrium.

Nevertheless, not all of them can be sustained in the decentralized equilibrium. A decentralized equilibrium has to satisfy buyers' optimality condition. Hence, $(p, \theta, V(s; \alpha))$ is only an equilibrium if there is no profitable deviation for buyers to open a new market p' , where the off-equilibrium belief is specified in (3) and (4), as discussed earlier. When a buyer considers to open a new market $p' \notin \text{range of } P$, they expect to only attract the type who is most likely to come, $T(p')$, as defined (4). To facilitate the analysis, we first prove following lemma:

Lemma 1 *Given any the mechanism $\alpha \in A$, which includes a price function $p^\alpha : S \rightarrow R_+$, market tightness function $\theta^\alpha : S \rightarrow R_+$, and sellers utilities $V^\alpha : S \rightarrow R_+$, for any price $p' \notin \text{range of } p$, the unique type who will come to this market p' is given,*

$$\begin{aligned} T(p') &= s^+ \cup s^- \\ \text{where } s^- &= \inf\{s \in S | p' < p^\alpha(s)\} \\ s^+ &= \sup\{s \in S | p' > p^\alpha(s)\} \end{aligned}$$

Proof. Notice that $p^\alpha(\cdot)$ is non-decreasing for $\forall \alpha \in A$.⁴ Therefore, $T(p')$ is uniquely defined⁵. Namely, the type which is most likely to come is unique. For any $p' \notin \text{range } p^\alpha$, by definition, $\theta(p', s) \equiv \inf\{\tilde{\theta} > 0 : U(p, \tilde{\theta}, s) \geq V(s; \alpha)\}$. Therefore, $\theta(p', s)$ solves: $U(p', \theta, s) - V(s; \alpha) \equiv G(p', \theta, s)$,

$$\begin{aligned} \frac{d\theta(p', s)}{ds} &= -\left(\frac{dG/ds}{dG/d\theta}\right) \propto \frac{1}{r + m(\theta^\alpha(s))} - \frac{1}{r + m(\theta(p', s))} \\ &= \begin{cases} < 0 \text{ if } p' > p^\alpha(s), (\because \theta(p', s) < \theta^\alpha(s)) \\ > 0 \text{ if } p' < p^\alpha(s), (\because \theta(p', s) > \theta^\alpha(s)) \end{cases} \end{aligned}$$

Recall that, posting a new price p' , a buyer should expect the lowest market tightness $\theta(p') = \inf_s \{\theta(p', s)\}$ and the type $T(p') = \arg \inf \{\theta(p', s)\}$. Above relation then implies that, for example, if a buyer deviates to posting a new price p' which is lower than all the existing price, so that $s^- = s_L$ and $s^+ = \emptyset$, he should attract only the lowest type, given that $\theta(p', s)$ is increasing in s and, therefore, $s_L = \arg \inf_s \theta(p, s) = T(p')$. Similar argument holds for any $p' \notin \text{range of } p^\alpha$. ■

With this condition, we can then prove following lemma:

Lemma 2 *There is no pooling submarket in equilibrium.*

⁴Suppose that $p(s) > p(s')$ for some $s < s'$ in equilibrium. Given that $\theta(s) \geq \theta(s')$ (\because monotonic condition), obviously buyers is strictly better off going the market $p(s')$. Hence, $p(s)$ will not be opened in equilibrium. Contradiction.

⁵For the case when sellers are out of the market, that is, $\theta(s) = 0$, for convenience, we define the corresponding $p(s) = \bar{P} + \varepsilon$ so that Lemma 1 is well defined in this case. Notice that, for any $\alpha \in A$, if $\theta(s) = 0$, it cannot be case that $\theta(s') > 0$ for $s' > s$. Therefore, applying Lemma 1 in this case implies that for any price p' which is higher than the existing market price, $T(p') = s^*$, where s^* is the marginal type who is indifferent to staying the market, where $V(s^*) = \frac{u(s)}{r}$.

Proof. See Appendix. Intuitively, a buyer can post a new price p' which is only slightly higher than the original pooling price. In that case, he only pays a little bit more but gets the best type in the original pooling for sure (as implied from lemma 1), which therefore generates a profitable deviation. ■

Lemma 2 then allows us to focus on a fully separating equilibrium. In each market, (θ, p, s) , the price schedule then has to satisfy:

$$p(s) = \frac{s}{r} - \frac{k\theta(s)}{m(\theta(s))} \quad (5)$$

Substituting this payment schedule into (ICFOC) :

$$V(s) = \frac{s - c + \left(\frac{s}{r} - \frac{k\theta}{m(\theta)}\right)m(\theta^*(s))}{r + m(\theta^*(s))} = V(s_l) + \int_{s_l}^s U_s(\theta^*(\tilde{s}), \tilde{s})d\tilde{s}$$

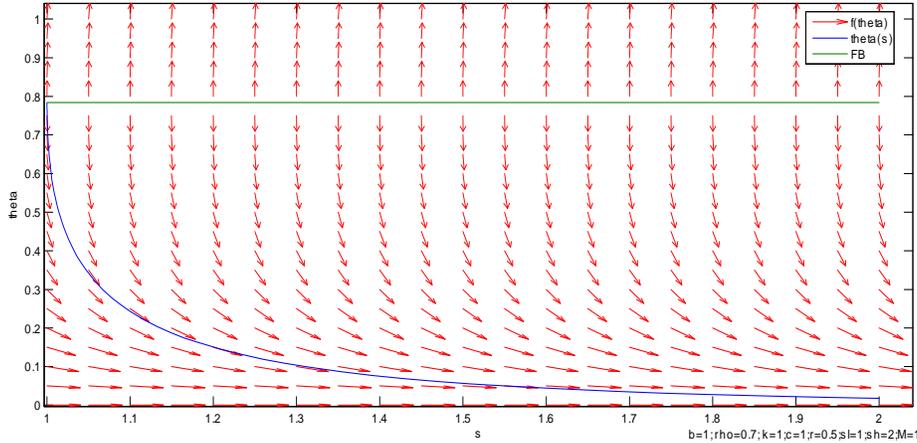
One can then get differential equation of $\theta^*(s)$:

$$\left[c + \frac{k}{\rho}((\rho - 1)\theta - \frac{r\theta}{m(\theta)}) \right] \frac{d\theta}{ds} = -\frac{\theta}{\rho r}(r + m(\theta)) \quad (6)$$

Therefore, the market tightness function $\theta^*(\cdot)$ has to be the solution of (6) subject to the monotonic condition, (M) in order to satisfy the incentive compatible constraints and free-entry condition. Left hand side of (6) is monotonically decreasing in θ and reaches zero at θ^{FB} . Therefore, for any initial condition $\theta_0 > \theta^{FB}$, the solution will be explosive and violate the monotonic solution. (6) is a separable nonlinear first-order differential equation with a family solution form:

$$s = C + \int \frac{1}{f(\theta)} d\theta$$

where $f(\theta) = \frac{-\frac{\theta}{\rho r}(r + m(\theta))}{\left[c + \frac{k}{\rho}((\rho - 1)\theta - \frac{r\theta}{m(\theta)}) \right]}$. One can understand the qualitative properties of the solutions by constructing a simple phase diagram, as shown in the figure below. Observing that for any $\theta \in (0, \theta^{FB})$, a) $f(\theta) < 0$; b) $f'(\theta) > 0$ and c) $\lim_{\theta \rightarrow 0} f(\theta) = 0$, one can then easily characterize the solution in the phase diagram.



Phase Diagram

With the following initial condition⁶, we are able to pin down the unique solution which satisfies both sellers' and buyers' optimality constraints.

Lemma 3 *The lowest type has to achieve his first-best utility. That is, the initial condition is:*

$$\theta(s_L) = \theta^{FB}(s_L)^- \quad (7)$$

Proof. See Appendix for detail. The intuition is clear: a downward distorted market tightness is to preventing a lower-type from mimicking a higher-type. Therefore, it should be clear that there is no reason to distort θ for the lowest type. ■

The mechanism can be summarized as following. Because of the asymmetric information, sellers will then face a lower meeting rate, $\theta^*(s) < \theta^{FB}$ for all s but get a higher transfer $p^*(s) = \frac{s}{r} - \frac{k\theta^*}{m(\theta^*)} > p^{FB}(s)$. There will be also less buyers participating the market, who needs to pay a higher price but with relatively high meeting rate. To note that, this result holds for any arbitrary distribution of sellers. Traders' participation and therefore, the trading volume, which is governed by the meeting rate, is endogenously determined. Also, we can easily check that IR constraint holds for all sellers⁷ and, indeed, buyers will not deviate by a opening market p' other than those which are already open. The argument is following: First, note that the price function is continuous. Denote (p_L, p_H) as

⁶For the purpose of avoiding the technique difficulties raised at the singular point $\theta^{FB}(s_L)$, the initial condition of the above solution satisfies $\theta(s_L) = \theta^{FB}(s_L) - \varepsilon$, in order to guarantee the uniqueness of the solution. One can easily show that welfare loss is negligible in this case.

⁷Define $G(s) = V(s) - \frac{s-c}{r}$ and $\lim_{s \rightarrow \infty} G(s) = 0$, given $\lim_{s \rightarrow \infty} \theta(s) = 0$. From (ICFOC) :

$$\frac{dg(s)}{ds} = \frac{1}{r + m(\theta^*(s))} - \frac{1}{r} < 0 \text{ for all } \theta(s) > 0$$

Hence $V(s) > \frac{s-c}{r}$ for $\forall s < \infty$.

the lower bound and the upper bound support of function $p(s)$ constructed above. From Lemma 1, if buyers post the price $p' > p^H$, he will only attract the highest type. One can easily show that it involves more distortion and hence not profitable. Similarly, if posting $p' < p^H$, buyer will attract the lowest type and buyers' utilities will decrease due to the distortion. Namely, it confirms that no profitable deviation exists for buyers.

From above discussions, the equilibrium can be summarized as followings:

Proposition 2 *Given $S = [s_L, s_H]$, a pair of $(p(s), \theta(s))$ that satisfies (5), (6), (7) solves the market maker's constrained incentive-efficient problem. Moreover, this solution is the unique decentralized equilibrium where traders' allocations (μ_t, B_t) satisfy:*

$$\begin{aligned}\mu_t(p(s'), s) &= \begin{cases} g_t(s) & \text{if } s' = s \\ 0 & \text{otherwise} \end{cases} \\ B_t(p) &= \begin{cases} \mu_t(p, s) \cdot \theta(s) & \text{if } \mu_t(p, s) > 0 \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

where $g_t(s)$ is the measure of sellers' type s at t

3 A General Model with Heterogenous Buyers

The goal of this section is to study a more general economic environment where the gain from trade is generated by different personal valuation of the asset. To do that, we are going to relax two assumptions made in the basic model. First of all, in the basic model, the gain from trade, sellers' holding cost c , is constant across types. This is just a simple way to model the gain from trade. More generally, the gain from trade can depend on the type of traded assets, which is a function of asset s . This assumption can be easily relaxed and the equilibrium can be solved as before but with a more complicated differential equation. In general, the flow payoff of the asset s to the seller is now given by $u(s)$, with the following assumption, $u'(s) > 0, u''(s) < 0$.

More importantly, we now allow for heterogenous buyers in the markets. Many decentralized markets have this feature. Understanding the trading pattern is crucial since it determines the allocation and therefore welfare. In particular, in the factor market, the resource allocation determines aggregate productivity. For example, different companies might have different technology to utilize the assets (machine or capital). Productivity of the assets is determined by assets allocation, which is mainly governed by both the pattern of trade and the equilibrium liquidity. How long does it take for an asset to change to a better hand? And, moreover, to which type of firms is the asset going to reallocate? The contribution of this section is therefore twofold: 1) to analyze the pattern of trade, taking into account the competition among heterogenous buyers, when both search friction and adverse selection play roles; 2) to characterize the trading price and endogenous liquidity as a result of equilibrium outcome.

Consider the extended model with two buyer types, $j \in \{h, l\}$. The flow payoff of an asset owned by buyer j and bought from seller s is given by $h(a^j, s)$, where $a^h > a^l$, and h is a strictly positive function $q : R_+^2 \rightarrow R_{++}$, where $h_a \geq 0$ and $h_s \geq 0$. For example, the productivity can simply be the product of firms' technology and assets quality, $a^j s$, where $a^h > a^l$, which is the simple functional form widely used for a macroeconomic model with heterogenous firms. Therefore, we will use this as an illustrative example in the later section.

Buyers' types are unobservable. It is clear, however, that this assumption does not play a role, since sellers only care about the resulting trading price and market liquidity, making buyers' type irrelevant from sellers' perspectives. The expected value of the seller in the market (p, θ) is:

$$rV(p, \theta, s) = u(s) + m(\theta)(p - V(p, \theta, s))$$

The contingent value of the buyer j searching in the market (p, θ) can then be expressed as follows:

$$rU_b^j(p, \mu, B, a^j) = -k + \frac{m(\theta(p))}{\theta(p)} \left(\int \frac{h(a^j, \tilde{s})}{r} \mu(\tilde{s}|p) d\tilde{s} - p - U_b^j \right)$$

3.1 Benchmark: Complete Information

In the basic model, the buyer's outside option is normalized to zero. In general, the buyers' outside option can also be different across types, now given by ϕ^j . As before, in equilibrium, buyers will enter markets until they are indifferent. The equilibrium outcome can be thought of as a competitive market maker who promises traders (p, θ) in each market. The solution has to solve following the optimization problem:

$$V(s) = \max_{j, p, \theta} \left\{ \frac{u(s) + m(\theta)p}{r + m(\theta)} : U_b^j(p, \mu, \theta, b^j) = \phi^j \right\}$$

After some algebra, the optimization problem can be rewritten as:

$$V(s) = \frac{u(s)}{r} + \max_{j, \theta} \frac{\frac{1}{r} m(\theta) (h(a^j, s) - u(s) - r\phi^j) - (k + r\phi^j)\theta}{r + m(\theta)} \quad (8)$$

$$= \frac{u(s)}{r} + \max_j \frac{m(\theta^*(j, s))g(j, s) - (k + r\phi^j)(\theta^*(j, s))}{r + m(\theta^*(j, s))} \quad (9)$$

$$\text{where } \theta^{FB}(j, s) = \arg \max_{\theta} \frac{m(\theta)R(j, s) - \theta}{r + m(\theta)} \quad (10)$$

Define $g(j, s) = \frac{1}{r} \{ h(a^j, s) - u(s) - r\phi^j \}$, representing the social surplus from the trade and $R(j, s) \equiv \frac{g(j, s)}{k + r\phi^j}$; $\theta^{FB}(j, s) = \arg \max_{\theta} \frac{m(\theta)R(j, s) - \theta}{r + m(\theta)}$ and solves the following FOC:

$$rR(j, s) = \left(\frac{r + m(\theta) - \theta m'(\theta)}{m'(\theta)} \right)$$

- (A1) : $g(j, s) = h(a^j, s) - u(s) - r\phi(j) > 0$ and $g_s(s, j) \geq 0$ for $\forall s \in S$;
(A2) : $\phi^h > \phi^l$;
(A3) : $h_s(s, h) - h_s(s, l) > 0$

Discussion of the assumptions: Assumption (A1) simply guarantees that there is a gain from trade and, moreover, this gain (weakly) increases with asset quality. Observing from (8), $V(s)$ increases with a^j and decreases with ϕ^j . Obviously, if $\phi^h < (\leq) \phi^l$, sellers then always obtain higher value if trading with the higher type buyer and one can easily show that, facing resulting (p, θ) , lower type buyer will not enter the market. In that case, the environment can be trivially solved just like as with homogenous buyers. Hence (A2) is needed to make the environment relevant. One can interpret $\phi^h > \phi^l$ as firms who have better technology but also have a higher outside option. Moreover, as mentioned in Shleifer and Vishny (1992), buyers from the same sector with sellers tend to have better use of the capital. However, when sellers want to sell because of facing a negative sector demand shock, a buyer from the same sector tends to be financially constrained as well. Hence, a buyer from the same sector needs to be compensated more to enter the market. Therefore, ϕ_j also can be interpreted as the ease with which the buyer j can obtain funding. Given that buyers essentially compete with each other in the market, in our framework, the advantage of a lower type buyer is his funding liquidity.

As pointed out in Shi (2001) and Eeckhout and Kircher (2010), the complementarity in the production function $h_{as}(a^j, s) > 0$ is not enough to guarantee the positive sorting. The reason is that if a seller trades with a higher type buyer, he has to suffer a longer waiting time. Given that social surplus increases with s , leaving a good asset in a bad hand decreases total output. As a result, when the complementarity is small, it is more efficient to match high-type seller with a low-type buyer by promising him a tight market, that is, a higher utilization. Positive sorting, however, is guaranteed if the complementarity is large enough.⁸ In the rest of the paper, we first focus on the environment when positive sorting behavior is obtained under complete information, that is, for $s' > s$, $V^{FB}(h, s) - V^{FB}(l, s) > 0 \implies V^{FB}(h, s') - V^{FB}(l, s') > 0$. In the later section, we will discuss how this assumption can be relaxed and the possible generalization. In fact, we further show that in the environment with adverse selection, positive sorting behavior is guaranteed under (A3).

Given the condition for the positive assortative matching, one can now provide the condition for which both type of buyers are *active* in the market, that is, $V^{FB}(l, s_L) > V^{FB}(h, s_L)$ and $V^{FB}(l, s_H) < V^{FB}(h, s_H)$. If any of these condition fails, the environment is reduced to the case with homogenous buyers so we can solve as before. Therefore, the environment is only interesting when both types of buyers have their advantage to staying in the market. Intuitively, an efficient solution will maximize the gain from trade and minimize agents' searching cost. Higher type can generate higher matching value but at the mean time, his actual searching cost, $k + r\phi^h$, is higher due to his higher outside option.

⁸Eeckhout and Kircher (2010) provides the condition in the static setting, where positive assortative matching obtains when complementarities in match value outweigh the elasticity of substitution of the search technology.

Notice that higher types' advantage increases with assets quality, $g_s(s, h) - g_s(s, l) \geq 0$, this means that given any assuming parameters, we can always find a corresponding range of the assets $s \in [s_L, s_H]$ such that both types remain active in the markets. Those will be the relevant environment we want to investigate in the following section. In the later discussion, we also discuss the solution to the case when the range of the assets does not satisfy $V^{FB}(l, s_L) > V^{FB}(h, s_L)$.

To solve the equilibrium with complete information, we can easily apply the same concept as before, that is, the solution to which can be easily solved as a market maker's problem, as developed in Mortensen and Wright (2002) and Eeckhout and Kircher (2010). The market maker now does not only need to find the optimal market tightness for each type but as well as the optimal trading pattern, $j^*(s)$. The corresponding utility function $V(s)$, a price function $p(s)$, and a market tightness function $\theta(s)$ need to satisfy:

$$\begin{aligned}
V^{FB}(s) &= \max_j \{V^{FB}(j, s)\} & (11) \\
\text{where } V^{FB}(s, j) &= \max_{\theta} (k + r\phi^j) \frac{m(\theta)R(j, s) - \theta}{r + m(\theta)} \\
\theta^{FB}(j, s) &= \arg \max_{\theta} \frac{m(\theta)R(j, s) - \theta}{r + m(\theta)} \\
p(j, s) &= \frac{h(a^j, s)}{r} - \frac{(k + r\phi^j)\theta(j, s)}{m(\theta(j, s))} - \phi^j \\
j^*(s) &= \arg \max_j \{V(j, s)\} \\
\theta^{FB}(s) &= \theta^{FB}(j^*(s), s); p^{FB}(s) = p(j^*(s), s)
\end{aligned}$$

Hence, in the decentralized markets, the distribution of traders' trading decisions $(\mu_t(p, s), B_t(p, j))$ satisfies:

$$\begin{aligned}
\mu_t(p(s'), s) &= \begin{cases} g_t(s) & \text{if } s' = s \\ 0 & \text{otherwise} \end{cases} \\
B_t(p, j) &= \begin{cases} \mu_t(p, s) \cdot \theta(j, s) & \text{if } j = j^*(s) \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

where $g_t(s)$ is the measure of sellers' type s at t . Notice that one can easily see that facing $\{V, p, \theta\}$, buyers j will not enter the market where $j^*(s) \neq j$. That is, $U(p(i, s), \theta(i, s), a^j) < \phi^j$ for $i \neq j$.

Define s^{FB} as the marginal type who is indifferent to trading with high type and low type buyers in equilibrium with complete information. That is, he is indifferent to the combination $(p(s^{FB}, h), \theta(s^{FB}, h))$ and $(p(s^{FB}, l), \theta(s^{FB}, l))$. Hence, s^{FB} solves

$$V^{FB}(s, h) = V^{FB}(s, l)$$

In the equilibrium, for a seller with assets $s < s^{FB}$, he will only trade with a lower type buyer in the equilibrium and vice versa for sellers with assets $s > s^{FB}$ ⁹.

⁹It is clear from (11) that $R(h, s^{FB}) < R(l, s^{FB})$, given $\phi^h > \phi^l$. Therefore, $\theta(h, s^{FB}) < \theta(l, s^{FB})$ and

3.2 Competitive Search Equilibrium with Adverse Selection

The refined equilibrium concept is the same. Redefine B as the measure of buyers j entering the market p , where $B : P \times \{h, l\} \rightarrow R$. For the active markets, the market tightness is therefore well defined along with buyers' and sellers' expected payoff, which can be expressed as follows:

$$\theta(p) = \frac{\sum_j B(p, j)}{\int \mu(p, s) ds} \quad (12)$$

$$\begin{aligned} rU_b(p, \mu, \theta(p), a^j) &= -k + \frac{m(\theta(p))}{\theta(p)} \left(\int \frac{h(a^j, \tilde{s})}{r} \mu(\tilde{s}|p) d\tilde{s} - p - U_b \right) \\ rU(p, \theta(p), s) &= u(s) + m(\theta(p))(p - U) \end{aligned}$$

Definition 2 *An equilibrium consists of an allocation of traders (μ, B) , a function of sellers expected utilities $V(s)$, and a market tightness function in each market p , $\theta(\cdot) : P \rightarrow [0, \infty]$ defined in (12) and (3), such that the following conditions hold:*

E1 (optimality for sellers): let

$$V(s) = \max_{p \in P^*} \frac{u(s) + pm(\theta(p))}{r + m(\theta(p))}$$

and for any $(p, s) \in P^ \times S$, $\mu(p, s) > 0$ implies*

$$U(p, \theta(p), s) \geq U(p', \theta(p'), s) \quad \forall p' \in P^*$$

E2 (optimality for buyers and free-entry): for any $p \in P$

$$U_b(p, \mu, \theta(p), a^j) \leq \phi^j$$

with equality if $p \in P^$ and $B(p, j) > 0$.*

Clearly, IC constraints for sellers are the same as before, that is, Proposition 1 still holds. The only difference is that we need to make sure the buyers' optimality condition will hold for both types. In particular, facing the price and market tightness recommended by the market maker, a buyer will benefit neither from going to the markets which belong to the other buyers, nor from opening a market which has not been open. The mechanism can be interpreted as follows: given $(p(s), \theta(s))$, a seller reports his type \hat{s} optimally; meanwhile, $j^*(s)$ denotes the sorting pattern recommended by the market maker, who recommends buyers $j^*(s)$ post the price $p(s)$, that is, entering the market $(p(s), \theta(s))$.

$p(h, s^{FB}) < p(l, s^{FB})$. Namely, there will be two separating markets for the asset s^{FB} . These two markets are different from the trading price and the liquidity, between which the seller s^{FB} is indiscriminate. High type buyers will pay more for the good with shorter waiting time in one market and, vice versa for the low type buyers in the other market.

The sets of types who trade with the lower type buyer, $\Omega_L = \{s : j^*(s) = l\}$, and of those who trade with the high type, $\Omega_H = \{s : j^*(s) = h\}$, are disjointed and satisfy $\Omega_L \cup \Omega_H \equiv S$. Then, define s^* as the marginal type $j^*(s^*) = \{l, h\}$. Obviously, some lessons learned from the basic model are still applied: there is no submarket involving pooling and, hence, we can focus on the full separation on the sellers' sides. From buyers' view points, each market can therefore be characterized as a pair of (p, θ, s) . Given (p, θ, s) , buyers will choose to go to the preferred markets and expect to trade with seller s .

Moreover, once we identify the set of sellers who trade with buyers j , Ω_j , the market tightness can be solved as in the case in which there is only one type of buyer j . Given Ω_j , one can show that $\theta(s; j)$ is the solution to the following differential equation:

$$\left[(h(a^j, s) - u(s) - r\phi^j) + \frac{k + r\phi^j}{\rho} \left((\rho - 1)\theta - \frac{r\theta}{m(\theta)} \right) \right] \frac{d\theta}{ds} = -(r + m(\theta)) \cdot \frac{\theta}{\rho} \frac{h_s(a^j, s)}{r} \quad (13)$$

Although the differential equation is more complicated than the basic case, one can understand the qualitative properties in the phase diagram as before. First of all, LHS decreases with θ and approaches zero as $\theta \rightarrow \theta^{FB}(s)$ and increases with s given $g_s(s, j) > 0$. This then implies that any for any initial condition $\theta^0 \leq \theta^{FB}(s_L)$, $\frac{d\theta(s; j)}{ds} < 0$ for $\forall s > s_L$. On the other hand, RHS depends on $h_s(a^j, s)$. The larger $h_s(a^j, s)$ implies the sharper decreases in θ . This allows us to analyze the shape of $\theta^*(s)$. As before, the corresponding price schedule $p(j, s)$ is then pinned down with the free entry condition:

$$p(s, j) = \frac{h(a^j, s)}{r} - \frac{(k + r\phi^j)\theta(s; j)}{m(\theta(s; j))} - \phi_j \quad (14)$$

Notice that solutions can be easily characterized once we have the initial condition for $\theta(s; j)$. Therefore, the key remaining task is essentially finding out the set Ω_j , that is, the marginal type s^* and identifying the initial condition $\{\theta_L^0, \theta_H^0\}$, which gives $\theta(s_L; j) = \theta_L^0$ and $\theta(s^*; j) = \theta_H^0$. For notation convenience, let $p^j(s), \theta^j(s)$ denote the price and the market tightness in the market with buyer type j . In equilibrium, it must be the case that the buyer j will not enter the market where $j^*(s) \neq j$. Hence, following constraints must be satisfied:

$$\begin{aligned} U_b(p^h, \theta^h, s, a^l) &< \phi^l \text{ for } j^*(s) = h \\ U_b(p^l, \theta^l, s, a^h) &< \phi^h \text{ for } j^*(s) = l \end{aligned}$$

To facilitate the analysis, define $\tilde{\theta}(s)$ to solve the following:

$$\begin{aligned} \phi^l &= U_b(p^h, \theta, s, a^l) \\ &= U_b(p^h, \theta, s, a^h) - \frac{q(\theta)}{r + q(\theta)} \left(\frac{h(a^h, s) - h(a^l, s)}{r} \right) \\ &= \phi^h - \frac{q(\theta)}{r + q(\theta)} \left(\frac{h(a^h, s) - h(a^l, s)}{r} \right) \end{aligned} \quad (15)$$

where $q(\theta) = \frac{m(\theta)}{\theta}$. Given that $h(a^h, s) - h(a^l, s)$ increases with s , $\tilde{\theta}(s)$ increases with s . This function then plays an important role in determining buyers' incentive constraint. Entering the high-type buyers' markets, the difference in utilities gain is characterized by the second term, $\frac{q(\theta)}{r+q(\theta)} \left(\frac{h(a^h, s) - h(a^l, s)}{r} \right)$, which captures low types' disadvantage. The impact of this disadvantage is higher when the expected waiting time for buyers is shorter, that is, for the higher $q(\theta)$ and hence the lower θ . As a result, for any $\theta < \tilde{\theta}(s)$, the low type will not mimic high type to enter the market. Similarly, when a high-type buyer contemplates entering a low-type market, he will only enter when $\theta < \tilde{\theta}(s)$ so that his advantage is high enough to compensate¹⁰. Hence, we can conclude the following claim:

Claim 1 *In equilibrium, the market (p, θ, s) attracts high-type buyers but not low-type buyers if $\theta < \tilde{\theta}(s)$; similarly, the market (p, θ, s) attracts low-type buyers but not high-type buyers if $\theta > \tilde{\theta}(s)$.*

Denote the function $\theta_j^{FB}(s), V_j^{FB}(s)$ as the market tightness and sellers' utility, respectively, when trading with buyer j with *complete information*. We next prove that the equilibrium can be characterized by following proposition.

Proposition 3 *The unique solution to the mechanism is a market tightness function $\theta : S \rightarrow R_+$, a price schedule $P : S \rightarrow R_+$, a marginal type s^* , a pair of initial condition $\{\theta_L^0, \theta_H^0\}$, where:*

$$\begin{aligned} \theta^*(s) &= \left\{ \begin{array}{l} \theta(s, l; \theta_L^0), \text{ for } s \leq s^* \\ \theta(s, h; \theta_H^0), \text{ for } s \geq s^* \end{array} \right\}; p^*(s) = \left\{ \begin{array}{l} p(s, l), \text{ for } s \leq s^* \\ p(s, h), \text{ for } s \geq s^* \end{array} \right\} \\ V^*(s) &= V_L^{FB}(s_L) + \int_{s_L}^s \frac{u'(\tilde{s})}{r + m(\theta^*(\tilde{s}))} d\tilde{s} \end{aligned}$$

where $\theta(s, j; \theta_j^0)$ is the solution to (13) with the initial condition: $\theta(s_L, l) = \theta_L^0, \theta(s^*, h) = \theta_H^0$, and corresponding $p(j, s)$ is defined in (14).

a) The initial condition θ_l^0 :

$$\theta_l^0 = \theta_l^{FB}(s_L)$$

b) The marginal types:

$$s^* = \left\{ \begin{array}{l} s^A, \text{ if } \tilde{\theta}(s^A) \geq \theta_H^{FB}(s) \\ s^B, \text{ if } \tilde{\theta}(s^A) < \theta_H^{FB}(s) \end{array} \right\}$$

¹⁰One can show that the utility of a high-type buyer entering a low-type market is: $U_b(p^l, \theta, s, a^h) = \phi^l + \frac{q(\theta)}{r+q(\theta)} \frac{h(a^h, s) - h(a^l, s)}{r}$, which is bigger than ϕ^h iff $\theta < \tilde{\theta}(s)$.

where (s^A, s^B) is the unique¹¹ solution to the following equation:

$$\begin{aligned} s^A & : V(l, s) = V_L^{FB}(s_L) + \int_{s_L}^s \frac{u'(\tilde{s})}{r + m(\theta^*(\tilde{s}, l; \theta_L^0))} d\tilde{s} = V_H^{FB}(s) \\ s^B & : \tilde{\theta}(s) = \theta(s, l; \theta_L^0) \end{aligned}$$

c) The initial conditions θ_H^0 :

$$\theta_H^0 = \begin{cases} \theta_H^{FB}(s^*), & \text{if } s^* = s^A \\ \tilde{\theta}(s^*), & \text{if } s^* = s^B \end{cases}$$

Proof. See Appendix. ■

As explained earlier, once we can separate the buyers from different markets, we can apply the method for homogenous buyers separately. Therefore, the equilibrium solution is expected to be a combination of two. However, it has to be combined in a particular way so that traders' optimality conditions hold. In appendix, we prove that the constructed solution above is the unique solution. As expected, this solution is also the unique outcome of decentralized equilibrium and can be decentralized as follows:

Proposition 4 *The solution to the mechanism $(\theta(s), p(s), s^*)$ is the outcome of decentralized equilibrium, where traders' allocations $\mu_t(s, p)$ and $B(p, j)$, sorting function $J^*(s)$ satisfies:*

$$\begin{aligned} \mu_t(p(s'), s) & = \begin{cases} g_t(s) & \text{if } s' = s \\ 0 & \text{otherwise} \end{cases} \\ B_t(p, j) & = \begin{cases} \mu_t(p, s) \cdot \theta(s) & \text{if } \mu_t(p, s) > 0 \text{ and } J^*(s) = j \\ 0 & \text{otherwise} \end{cases} \\ J^*(s) & = h \text{ for } \forall s > s^*; J^*(s) = l \text{ for } \forall s < s^* \end{aligned}$$

where $g_t(s)$ is the measure of sellers' type s at t .

3.3 Extension: Resale

When a firm thinks about buying the capital, he will need to take the business condition into account. If the environment is rather uncertain—for example, the financial source or the deploying technology is rather unstable—it will obviously decrease the firm's expected profit of the investment. When the firm receives a negative shock, he will then need to disinvest, that's, he will then naturally become the seller in the market. Clearly, taking this situation into account, the firm's expected profit will also be a function of the resale value. To capture this idea, this section extends our main model to allow for the

¹¹Observe that (s^A, s^B) is unique (and all smaller than s^{FB}). Notice that, $V(l, s)$, $V_H^{FB}(s)$, $\tilde{\theta}(s)$, $\theta_H^{FB}(s)$ are all well defined and monotonically increases in s and $\theta(s, l; \theta_L^0)$ is strictly decreasing in s . Given that $\tilde{\theta}(s_L) \leq \theta_L^{FB}(s_L)$ under the assumption $V_L^{FB}(s_L) > V_H^{FB}(s_L) \implies s^B$ always exists and is unique.

resale. To be precise, a firms' flow value of the capital decreases when hit by the negative shock and, in that case, he naturally becomes the seller in the market, which arrives at the Poisson arrival rate δ .

$$rJ(s) = h(a_j, s) + \delta(V(s) - J(s))$$

where $V(s)$ is the expected value of the seller, as before. Obviously, the expression of a buyers' expected value searching in the market is the same as before and it has to equal to its outside option ϕ_j because of free-entry condition. All methods developed in our main model remain valid. As before, in order to solve for the equilibrium, we need derive following relations defined from the complete information case:

$$\begin{aligned} V^{FB}(s, j) &= \max_{\theta} \frac{r + \delta}{r} \left(\frac{u(s)}{r + \delta} + \frac{m(\theta) \left(\frac{h(a_j, s) - u(s)}{r + \delta} - \phi_j \right) - k\theta}{r + \delta + m(\theta)} \right) \\ \theta_{\delta}^{FB}(j, s) &= \arg \max V^{FB}(s, j) \end{aligned}$$

and the differential equation $\theta(s, j)$ and $p(s, j)$ derived from the sellers' IC constraints, given that he trades with the buyer j :

$$\left[(h(a^j, s) - u(s) - (r + \delta)\phi^j) + \frac{k + r\phi^j}{\rho} ((\rho - 1)\theta - \frac{(r + \delta)\theta}{m(\theta)}) \right] \frac{d\theta}{ds} \quad (16)$$

$$= - \left(\frac{r + \delta + m(\theta)}{r + \delta} \right) \cdot \frac{\theta}{\rho} \left(h_s(a_j, s) + \frac{\delta u'(s)}{r + m(\theta)} \right) \quad (17)$$

$$p(s, j) = \frac{h(j, s)}{r + \delta} + \frac{u(s)}{r + \delta} + \frac{m(\theta) \left(\frac{h(a_j, s) - u(s)}{r + \delta} - \phi_j \right) - k\theta}{r + \delta + m(\theta)} - \frac{(k + r\phi_j)\theta}{m(\theta)} - \phi_j \quad (18)$$

the modification to (15), $\tilde{\theta}(s)$ now solves:

$$\phi_h - \phi_l = \frac{q(\theta)}{r + q(\theta)} \left(\frac{h(a_h, s) - h(a_l, s)}{r + \delta} \right) \quad (19)$$

One can easily check that the previous version is simply the case when $\delta = 0$. Obviously, the same approach can be applied. The only difference is that we now have a different differential equation and, different first best solution, different initial condition and different marginal type.

Notice that the first best solution $\theta_{\delta}^{FB}(s)$ has to satisfy:

$$(r + \delta)R(j, s) = \frac{h(j, s) - u(s) - (r + \delta)\phi^j}{k + r\phi_j} = \frac{1}{\rho} \left((r + \delta)\theta_{FB, \delta}^{1-\rho} + (1 - \rho)\theta_{FB, \delta} \right) \quad (20)$$

To note that $\theta_{FB, \delta}$ is decreasing in δ . The intuition is clear since no buyers would want to enter the market if they need to sell it again soon. Since the trading surplus

is decreasing in δ , the higher δ , the less entry and hence the lower equilibrium market tightness.

Let $g(s)$ denote the total measure asset supply s . Given the solution $(p^*(s), \theta^*(s))$, the steady state can then be found :

$$g(s) = H(s) + \mu(s)$$

$$\frac{m(\theta(s))}{\theta(s)} B_{ss}(s) = \delta H_{ss}(s) = m(\theta_s) \mu_{ss}(s) \quad (21)$$

where $B(s)$ is the measure of buyers in the market s . $H(s)$ and $\mu(s)$ denote the measure of the owner and the seller with asset s , respectively. The variable X_{ss} denote the measure of X at the steady state. First equality of (21) follows since the inflow of the owner with asset s equals the outflow. The second equality describes the inflow to the seller s equal to the outflow, simply leaving the market. The steady state ratio, $\frac{\mu_s}{H_s}$ is then pinned down by $\frac{\delta}{m(\theta_s)}$. The measure of $H_{ss}(s)$ and $\mu_{ss}(s)$ at the steady state then equal $\frac{m(\theta_s)}{\delta+m(\theta_s)}g(s)$ and $\frac{\delta}{\delta+m(\theta_s)}g(s)$, respectively.

3.4 Remarks on Efficiency

Does this decentralized equilibrium outcome necessarily solve the centralized planner's problem? The answer is obvious from our solution method. As explained earlier, among the set of feasible mechanisms A defined from Proposition 1, the decentralized outcome is the one satisfying the buyers' optimality constraint, $E2$. Namely, given that buyers have the freedom to post new prices in the decentralized market, $E2$ is the additional constraint compared to the social planner's problem. This logic clearly implies that a social planner can always (weakly) do better than the market. In fact, in our basic model, a full pooling equilibrium always achieves the first best welfare level as long as it is sustainable. The main reason is that the first best solution of market tightness is independent of types. A pooling equilibrium, simply subsidizing some at the expense of others, therefore does not incur any distortions as long as participating constraints of the highest types are satisfied. To be precise, a full pooling equilibrium maximizes social welfare as in the environment under complete information if and only if 1) the gain from trade is independent of types and 2) the IR constraint of the highest type is satisfied.

The above point then leads us to the next question: is the decentralized equilibrium outcome Pareto efficient? The answer can also be understood from our basic model. First of all, notice that the outcome of separating equilibrium does not depend on the distribution of types. On the other hand, traders' utilities in any kind of pooling equilibrium will obviously depend on the distribution. Intuitively, the highest type in a pooling equilibrium suffers a lower price because the market is only willing to pay the expected value of the asset. Nevertheless, in a separating equilibrium, he enjoys a much higher trading price but must suffer a long waiting time, decreasing his expected utilities. One extreme case

would be that a full pooling equilibrium drives the highest type out of the market, which can happen when there are too many bad assets; therefore, the highest type is obviously better off in the separating equilibrium. In this particular case, the separating equilibrium is not Pareto ranked by the full pooling equilibrium¹². Another extreme case would be that average quality is high enough so that even the highest type is better off in a full pooling equilibrium. The separating equilibrium then is Pareto ranked by the full pooling equilibrium. Hence, the answer to the efficiency properties of equilibrium will depend on the distribution assumed, which is actually a straightforward task once some particular distribution is given. This point then explains why the competitive search equilibrium is Pareto inefficient for some parameter values, as shown in Guerrieri et al. (2009). The important lesson is that the equilibrium outcome is not necessarily constrained Pareto efficient and the main reason is that pooling cannot be sustained even it is desirable due to the competitive nature of markets. And the distortions in market tightness resulting from separating equilibrium are rather costly.

3.5 Remarks on Sorting Behavior

Shi (2001) and Eeckhout and Kircher (2010) have shown that the complementarity in production is not enough to guarantee positive assortative matching (PAM) in an environment with complete information. The intuition is that, given that the social surplus increases with types, it could be optimal to match high-type seller with a low-type buyer by promising him a tight market, that is, a higher utilization. The above intuition still holds in our framework with complete information. However, with adverse selection, we prove that the supermodularity of the matching value necessarily induces PAM in the equilibrium.

Proposition 5 *In the competitive search equilibrium with adverse selection, the equilibrium trading pattern $j^*(s)$ satisfies PAM, that is, for $s' > s$, $j^*(s) = h \implies j^*(s') = h$ under the assumption $h_s(a^h, s) - h_s(a^l, s) > 0$,*

Proof. Suppose Not. There exists $s' > s$ such that $j^*(s) = h$ and $j^*(s') = l$. According to Claim 1, the equilibrium market tightness must satisfy: $\theta^*(s) \leq \tilde{\theta}(s)$ and $\theta^*(s') \geq \tilde{\theta}(s')$. Moreover, from the monotonic condition, (M), $\theta^*(s) \geq \theta^*(s')$. The above relation then implies $\tilde{\theta}(s) \geq \tilde{\theta}(s')$. This is a contradiction to the fact that $\tilde{\theta}(s)$ is strictly increasing with s under the assumption $h_s(a^h, s) - h_s(a^l, s) > 0$. (Recall $\tilde{\theta}(s)$ solves $\phi^l = \phi^h - \frac{q(\theta)}{r+q(\theta)} \left(\frac{h(a^h, s) - h(a^l, s)}{r} \right)$) ■

To understand this result, recall that the reason as to why a higher type can be better off when trading a low-type buyer is that he can be compensated by a higher utilization. That is, given that a lower type buyer is more willing to wait, it could be optimal for a high type seller to choose to trade with a lower type buyer, enjoying a lower gain from trade but a tighter market compared to trading with a high type buyer. Hence, contingent

¹²However, depending on distribution, it could be ranked by a semi-pooling equilibrium.

on negative assortative matching (NAM), a high type seller must be compensated with a higher market tightness compared to low type sellers. This situation, however, can not be sustained in an environment with adverse selection, as it violates the monotonic condition. Namely, it is not incentive compatible for the sellers. Notice that in an environment with complete information, a high type seller prefer to trader faster as his gain from trade is higher. Nevertheless, with adverse selection, when all sellers are facing the same market price schedule and market tightness, a high type seller becomes the one who is more patient in the sense that he will prefer the combination of a higher price and a lower market tightness as contrary to a low type seller. This implies that it would be optimal to match a high type seller with a buyer who is more willing to offer a higher price and less willing to wait. Obviously, a high type buyer is more willing to do this. Hence, a lower type buyer no longer has his advantage to trade with a high type seller as in the case with complete information.

Our solution developed earlier starts with the environment with PAM and $V^{FB}(l, s_L) > V^{FB}(h, s_L)$. However, according to the above Proposition, one should expect that those conditions can be relaxed. First of all, suppose $V^{FB}(h, s_L) > V^{FB}(l, s_L)$, so it is clear that $j^*(s_L) = h$ from Lemma 3 and, clearly, from the above Proposition, $j^*(s) = h$ for $\forall s \in S$. Hence, we can simply solve the model as if there are only high-type buyers in the market, regardless of positive or negative assortative matching under complete information. Suppose that we are now in the environment with NAM, that is, for $s' > s$, $V^{FB}(l, s) - V^{FB}(h, s) > 0 \implies V^{FB}(l, s') - V^{FB}(h, s') > 0$ and $V(l, s_L) > V(h, s_L)$, implying that only low type buyers are active in the case with complete information. Although we do not provide the formal solution for this case, our conjecture tells us that the solution should take similar pattern as the developed method. And, depending on the range of S , it could be the case that $j^*(s) = h$ for some $s' > s$. The above argument shows that adverse selection essentially makes a higher type buyer more likely to stay the market compared to the case with complete information. Notice that this phenomenon can be understood for our main results as well, given that the marginal type decreases in the case of adverse selection, that is, $s^* < s^{FB}$. Hence, more sellers end up trading with high-type buyers with adverse selection.

This result might seem at first counter-intuitive but, in fact, it is simply the flip side of market illiquidity. Adverse selection creates a downward distortion of market liquidity, that is, a low ratio of buyers over sellers in the market. This distortion makes it hard for a seller to find a buyer; on the other hand, it also makes it easier for a buyer to find a seller, shortening a buyer's wait time and expected search cost. Given that high-type buyers like to secure trade with high probability and they are willing to pay for this, the environment effectively makes a high type buyer more competitive, compared to low-type buyers.

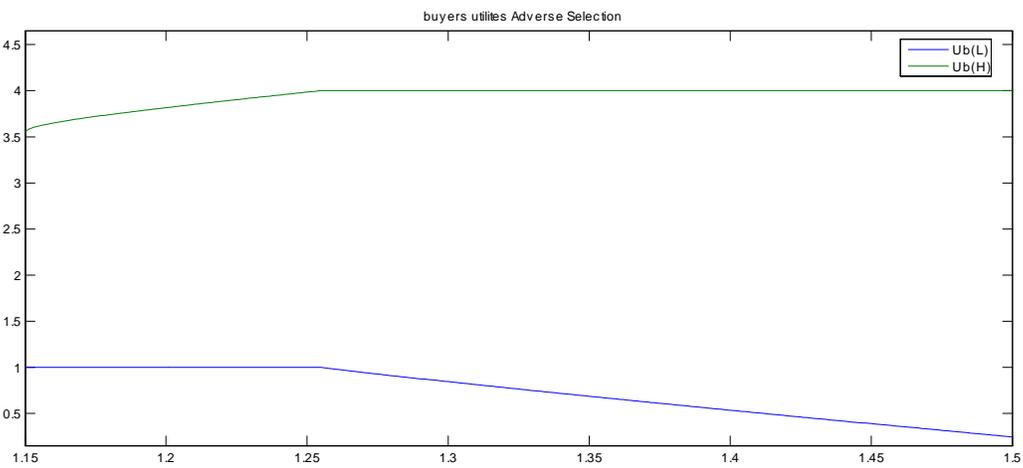
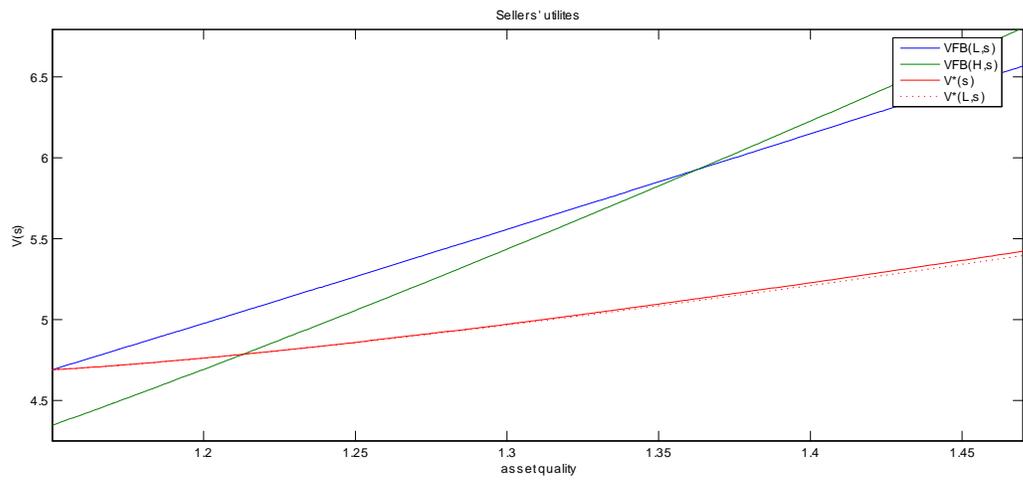
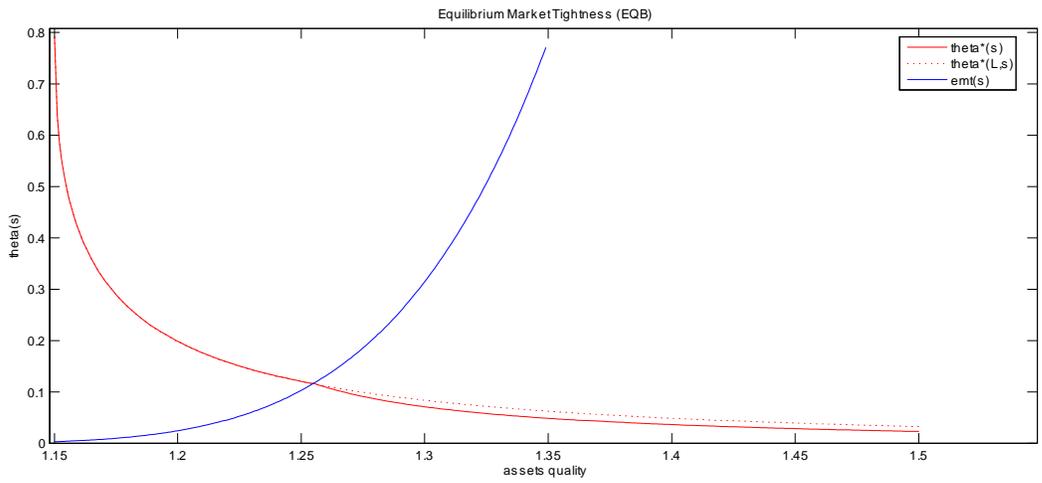
4 Macroeconomic Implication

This section investigates the aggregate performance as a result of market imperfection developed in our model. The key characteristic of our model is the market liquidity, which will be endogenously determined in the equilibrium. A highly illiquid market will prevent resources from allocating efficiently and, therefore, has a negative impact on aggregate performance. To illustrate our result, we now specify the function governing traders' payoff. The flow value of the capital is simply the product of the capital quality s and its use of technology a_j , that is, $h(a_j, s) = a_j s$. More productive firms will be the natural buyers. In the economy, there are two profitable technologies $j \in \{H, L\}$, where $a_H > a_L$, and the owner of the technology j has the outside option ϕ_j , where $\phi_H > \phi_L$. Firms who received a negative shock, at the arrival rate of δ , became unproductive and only produced a flow payoff $a_o s$, where $a_o < a_L < a_H$, that is, $u(s) = a_o s$. As explained earlier in our extension of resale, the parameter δ then measures the downward uncertainty in the economy facing by firms. In the later discussion, we show the impact of uncertainty on the market liquidity and aggregate performance.

4.1 An illustrative solution

Applying the result in the previous section, the following graph shows the numerical solution to the equilibrium market liquidity $\theta^*(s)$, sellers' utilities $V(s)$ and buyers utilities in each market¹³. Notice that this is an example of the case when $\tilde{\theta}(s^A) < \theta_H^{FB}(s^A)$ and therefore, the marginal type solves $\tilde{\theta}(s) = \theta(l, s)$, that is, the intersection of the the dotted line and the blue line. The second graph shows the sellers' utilities, which are represented by the red line. The blue and the green line represent sellers' utility when traded with the low type buyer and the high type buyer, respectively, in the case of complete information. Hence, without adverse selection, sellers' utilities will be the upper envelope of these two functions. As expected, sellers' utilities decrease (except for the lowest type) as a result of liquidity distortion. The last figure confirms the fact that a high type buyer will not enter the market $(s, p(s), \theta(s))$ for $s < s^m$ and a low type buyer will not enter the market $(s, p(s), \theta(s))$ for $s > s^m$.

¹³With following parameters value: $\rho = 0.8, k = 1, r = 0.3, S = [1.15, 1.5], \delta = 0.05; a^l = 2, \phi^l = 1, a^h = 3, \phi^h = 4$.



4.2 The Impact of Uncertainty Shocks

We first illustrate the impact of an uncertainty shock. As shown in the earlier analysis, the equilibrium market tightness is a function of uncertainty. Intuitively, a higher uncertainty will reduce investors' willingness to invest and hence decrease the market liquidity and therefore creates more resource mismatch, resulting a drop in aggregate TFP. One can see this formally from a simple comparative statics on δ .

Proposition 6 *In equilibrium, market Liquidity $\theta^*(s; \delta)$ decreases with uncertainty.*

Proof. See appendix. ■

Let $H_t^j(s)$ represent the measure of capital s owned by the firm with technology j and let $\mu_t(s)$ denote the measure of sellers who owned capital s . The aggregate productivity of the capital can then be defined as follows:

$$\bar{A}_t = \int \{a_o s \mu_t(s) + a_l s H_t^l(s) + a_h s H_t^h(s)\} ds \quad (22)$$

Given δ , we can easily pin down the steady state of $\mu_{ss}(s)$ and $H_{ss}^j(s)$. Clearly, \bar{A}_{ss} decreases with δ since the proportion of capital s in goods hands, $\frac{m(\theta^*(s; \delta))}{\delta + m(\theta^*(s; \delta))}$, decreases with uncertainty δ . Therefore, the aggregate TFP is endogenously determined and is a decreasing function of uncertainty. In the meantime, the market liquidity is low when uncertainty is high, as emphasized in the above proposition. The above result then provides a microfoundation for the co-existence of high uncertainty, high dispersion, and an illiquid market when the economy performs poorly.

Furthermore, Eisfeldt and Rampini (2006) have documented that the cross-sectional dispersion of the productivity is countercyclical while the capital reallocation is procyclical. From this finding, they suggest that the reallocation friction should be countercyclical. Our model does provide the explanation as to why the reallocation frictions, which are mainly governed by the equilibrium market liquidity, are higher in bad times. The net effect on capital reallocation, however, is ambiguous. To see this, the total amount of reallocation at the steady state can be expressed as:

$$\frac{\delta}{\delta + m(\theta^*(s; \delta))} \cdot m(\theta^*(s; \delta))$$

The first part represents the measure of assets in bad hands, which increases with uncertainty; the second part represents how liquid the market is, which decreases with uncertainty. Obviously, the second effect dominates if $|\frac{d\theta(s; \delta)}{d\delta}|$ is large enough, therefore resulting in procyclical reallocation.

The above point highlights the impact of uncertainty on market liquidity. Moreover, market liquidity is provided by investors (buyers), and, in reality, their ability to do so depends on the availability of funding. If it is more costly for investors to borrow, their

willingness of entry decreases and, therefore, must be compensated more in the trading market. This idea can be captured in our model with the parameter ϕ . In our setup, δ essentially measures the downward uncertainty of firms' business conditions. A higher δ implies a higher probability that a firm receives a negative shock. Hence, taking this into account, one should expect that a higher uncertainty also creates a negative effect on buyers' funding ability. Although this idea can be explicitly modeled, we take this effect as given for now, that is, assuming ϕ^j increases in δ . In our framework, one can easily see that equilibrium market tightness decreases in ϕ ; therefore, this additional effect then increases the impact of uncertainty on market liquidity, amplifying $|\frac{d\theta(s;\delta)}{d\delta}|$. Adding this feature in our framework, one can then understand why more reallocations take place in good times, that is, the period with low uncertainty.

Notice that TFP as well as dispersion is endogenously determined and essentially driven by the reallocation process. Without adverse selection, above results still hold qualitatively; nevertheless, the impact of search friction is modest. With adverse selection, the market liquidity is then distorted and, therefore, reallocation frictions can generate a substantial effect on aggregate performance, as illustrated in the following section.

4.2.1 Dynamics with a transitory shock

Thanks to the tractability of the Poisson arrival rate, one can easily track the dynamic path of the reallocation. Suppose the economy was in a steady state and suddenly received a *one time* liquidity shock so that $\lambda > 0$ *additional* measures of owners (firms) suddenly suffer a permanent negative technology shock or liquidity shock, and, therefore, it is optimal for them to sell their capital and exit the market. Obviously, since it is a transitory shock, $\theta^*(s)$, $p^*(s)$, and $j^*(s)$ remain the same. Given the equilibrium market tightness function $\theta^*(s)$, the law of motion of $H_t^j(s)$ and $\mu_t(s)$ can then be expressed as:

$$\begin{aligned} d\mu_t(s) &= -m(\theta^*(s))\mu_t(s) + 1(s) \cdot \delta H_t^l(s) + (1 - 1(s)) \cdot \delta H_t^h(s) \\ dH_t^l(s) &= 1(s)m(\theta^*(s))\mu_t(s) - \delta H_t^l(s) \\ dH_t^h(s) &= (1 - 1(s))m(\theta^*(s)) - \delta H_t^h(s) \end{aligned} \quad (23)$$

where $1(s)$ is the indicator function defined as:

$$1(s) = \begin{cases} 1 & \text{if } j^*(s) = l \\ 0 & \text{if } j^*(s) = h \end{cases}$$

The solution can be solved easily,

$$\begin{aligned} \mu_t(s) &= [\mu_o(s) - \mu_{ss}] \cdot e^{-(m(\theta^*(s))+\delta) \cdot t} + \mu_{ss}(s) \\ H_t^l(s) &= \begin{cases} -[\mu_o(s) - \mu_{ss}(s)] \cdot e^{-(m(\theta^*(s))+\delta) \cdot t} + H_{ss}(s) & \text{if } j^*(s) = l \\ 0 & \text{otherwise} \end{cases} \\ H_t^h(s) &= \begin{cases} -[\mu_o(s) - \mu_{ss}(s)] \cdot e^{-(m(\theta^*(s))+\delta) \cdot t} + H_{ss}(s) & \text{if } j^*(s) = h \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

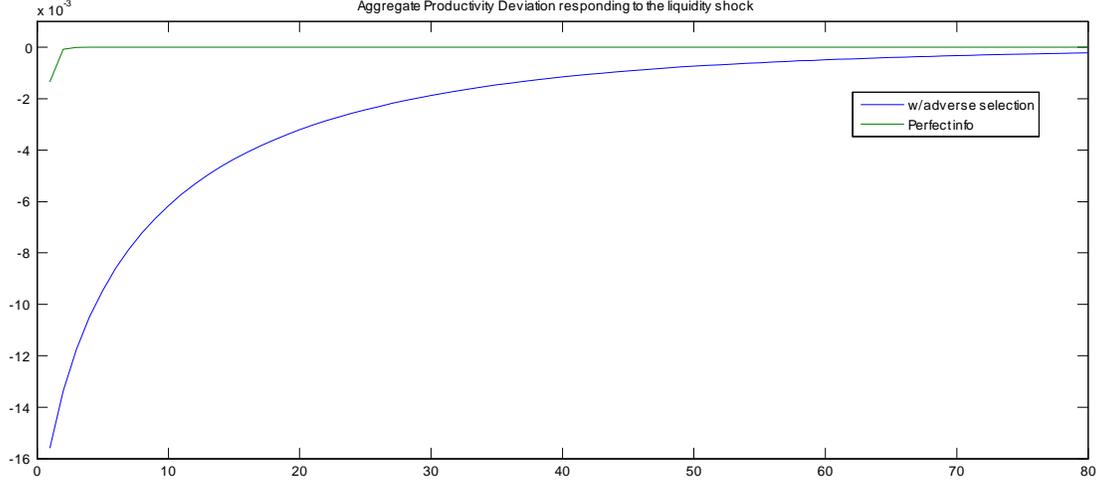
where $\mu_0(s)$ is the initial measure of sellers when a shock hit. Solving $\mu_t(s)$ and $H_t^j(s)$ then gives us an endogenous TFP, according to (22), as well as dispersion along the path. The dispersion of the capital s equals $\frac{\mu_t(s)}{g(s)} \cdot (1 - \frac{\mu_t(s)}{g(s)}) \cdot (a_{j^*(s)} - a_0)^2$. Therefore, the dispersion of capital productivity in this economy is then given by:

$$\bar{\sigma}_t = \int_{s_L}^{s_H} \frac{\mu_t(s)}{g(s)} \cdot (1 - \frac{\mu_t(s)}{g(s)}) \cdot (a_{j^*(s)} - a_0)^2 s^2 g(s) ds \quad (24)$$

Assuming $\frac{\mu_t(s)}{g(s)} < \frac{1}{2}$, that is, when λ and δ are small enough to ensure that most of capital is still in good hands. Therefore, dispersion first increases when the uncertainty shock hits and decreases over time as $\frac{\mu_t(s)}{g(s)}$ decreases and assets reallocate to better hands. This uncertainty then creates a decrease in TFP and an increase in dispersion, which provides an explanation as to why, as shown in the data, uncertainty is positively correlated with dispersion in the economy, and dispersion increases when the economy is bad.

From (22), one can see that a drop in aggregate productivity can simply result from a severe mismatch. Because of the trading frictions, it takes time for the economy to reallocate the resource and for aggregate productivity to recover. This paper therefore provides an alternative mechanism to first-moment shocks for generating recessions. Recessions could simply be periods of high resource mismatch without an aggregate shock. Notice that in our model, a better technology is always available. The question is how fast the resource can be reallocated to the better hands. The above impacts exist whenever the search friction is considered. One new feature of this model is the effect of adverse selection. As illustrated in the figure below, adverse selection creates a much more persistent drop in aggregate productivity, compared to a case with complete information. In the case of complete information, higher-quality assets will have a higher market tightness since their gain from trade is larger. On the other hand, with adverse selection, a higher-quality asset will suffer a longer trading delay resulting from the distortion of the market tightness. This fact also matches the micro evidence put forth by Maksimovic and Phillips (2001), who show that firms are more likely to sell their least efficient plants and that the rate of plant sales is lower for more efficient segments.

As illustrated in the figure below, the economy takes a longer time to reallocate better capitals to the more efficient technology, which makes the drop in productivity much more persistent. Moreover, observing from (24), we can see that dispersion is also higher in the economy than in the one with complete information. The reason mainly comes from the fact that there is a higher ratio of assets in bad hands, especially for the better-quality ones.



4.2.2 Dynamics with a permanent shock

A permanent shock will change the equilibrium market tightness, price, and trading pattern. Luckily, the dynamics remain tractable, given that all of these key equilibrium elements are forward looking so that they are all jump variables, adjusting to the shock right away. The law of motion of ownership will now change according to the new equilibrium, $\theta^*(s; \delta')$, $p^*(s; \delta')$, $j^*(s; \delta')$. Supposing $\delta' = \delta_h > \delta_0$, the only additional complication compared to the previous case is that the increase in uncertainty also changes the sorting pattern. In particular, the marginal type $s^*(\delta)$ increases with δ (PROOF). This then again emphasizes the impact of uncertainty. Not only does it decrease the willingness of entry and therefore the market liquidity, it also leads more sellers to trade with the buyer who has a lower-quality technology, resulting in a further decrease in aggregate productivity.¹⁴ Hence, a higher buyer will no longer enter the market for capital $s < s^*(\delta_h)$. The law of motion of ownership of capital s , for $s^*(\delta_0) < s < s^*(\delta_h)$ will instead follow:

$$d\mu_t(s) = -m(\theta^*(s))\mu_t(s) + \delta H_t^l(s) + \delta H_t^h(s) \quad (25)$$

$$dH_t^l = m(\theta^*(s))\mu_t(s) - \delta H_t^l(s) \quad (26)$$

$$dH_t^h(s) = -\delta H_t^h(s)$$

Obviously, the law of motion for capital for $s < s^*(\delta_0)$ and $s > s^*(\delta_h)$ still satisfies (23) so the previous solution can be applied easily. Given the initial condition $\mu_0(s) = \mu_{ss}(s; \delta_0)$, we will then be able to solve for $H_t^j(s)$ and $\mu_t(s)$; therefore, the dynamic of both aggregate productivity and dispersion in the economy is endogenously given. (Graph to be added)

¹⁴This change decreases the aggregate productivity. However, it is optimal in respect to welfare, taking into account traders' outside options.

4.3 The Dispersion of Asset Quality

Notice that our theoretical result holds for any distribution of asset quality. Moreover, contrary to the standard set up of the lemon problem, there are several distinct features of the developed model: 1) All markets are priced in equilibrium and open; nevertheless, some high quality asset market are close to *frozen* so it's hard for sellers to get rid of their assets; 2) More importantly, different dispersion of the asset quality will have a first order effect on market illiquidity.

The model predicts that the higher the dispersion, the more illiquid the market is. The change in dispersion therefore has a significant aggregate effect: an increase in the dispersion of asset quality will result in a higher distortion in aggregate market liquidity, which therefore generates a drop in productivity, meanwhile, amplifying the aggregate dispersion, which is endogenously determined by the combination of asset quality as well as its use of technology. Hence, even without any first order change in TFP, the dispersion itself in our model can create a significant effect on aggregate performance. The resulting aggregate fluctuation also matches the stylized fact documented in the empirical literature: aggregate dispersion is high in bad times. Meanwhile there is less ongoing reallocation.

4.4 Discussions

In reality, the gain from trade depends on many factors. The example shown in this section emphasizes the fact that firms have different technology (or management ability) to utilize the capital. Our general theoretical result, however, is rich enough to capture other perspectives. For example, as emphasized in Caballero (2007), specificity—which holds that a factor is specific with respect to a given production arrangement, and, therefore, its value would diminish if used outside this arrangement—plays an important role regarding reallocation. This concept can be easily nested in our model. For example, the buyer production function can instead take the form of $h(a^j, us)$, where $0 < u < 1$ and u can be interpreted as the depreciation of capital value when it changes hands. Obviously, the higher the depreciation, the more illiquid the market, that is, the larger the reallocation friction. Meanwhile, our theoretical result also implies that buyers' outside option has a direct impact on the market liquidity. Interpreting the a buyer's outside option as his funding liquidity, the market liquidity obviously decreases with buyers' funding liquidity. In sum, our theoretical framework can easily capture other factors which have been emphasized in the literature, and this allows for a richer analysis of the equilibrium market liquidity.

5 Conclusion

(To be added)

6 Appendix

6.1 Omitted Proof

(A) Proof of Proposition 1:

Proof. One can observe that $U(\theta, s)$ satisfies following properties: that 1) $U_2(\theta, s)$ exists; 2) has an integrable bound: $\sup_{s \in S} |U_2(\theta, s)| \leq \frac{M}{r}$ for all s , where $M = u'(s_L)$ for; 3) $U(\theta, \cdot)$ is absolutely continuous (as a function of s) for all θ ; 4) $\theta^*(s)$ is nonempty. Following the mechanism literature, (see Milgrom and Shannon (1994)), let

$$V(s) = \max_{\hat{s}} U(\theta(\hat{s}), s) = \max_{\hat{s}} \frac{u(s) + p(\theta(\hat{s}))m(\theta(\hat{s}))}{r + m(\theta(\hat{s}))}$$

then any selection $\theta(s)$ from $\theta^*(s) \in \arg \max_{\theta'} U(\theta', s)$,

$$V(s) = V(s_l) + \int_{s_l}^s U_s(\theta^*(\tilde{s}), \tilde{s}) d\tilde{s}$$

Namely, (*ICFOC*) is the necessary condition for any IC contract. To prove the sufficiency, define function: $x = q(\theta) = \frac{1}{r+m(\theta)}$ and $q^{-1}(x) = \theta$. Also, since $\theta > 0$, $0 < x \leq \frac{1}{r}$. One can then easily see $U(x, s)$ satisfies the strict single crossing difference property under the assumption $u'(s) > 0$. For any $x' > x$ and $s' > s$:

$$U(x', s') - U(x', s) + U(x, s) - U(x, s') = x'(u(s') - u(s)) - x(u(s') - u(s)) > 0$$

Therefore, $U(x', s') - U(x, s') > U(x', s) - U(x, s)$. Given that $U(x, s)$ satisfies SSCD condition, then any non-decreasing $x(s)$ combined with (*ICFOC*) are also sufficient conditions for the achievable outcome. Hence, $x(s) = \frac{1}{r+m(\theta^*(s))}$ has to solve subject to the non-decreasing constraint. Namely, the market tightness function $\theta^*(\cdot)$ has to be non-increasing. ■

B) Proof of Lemma 2: No pooling

Proof. Suppose Not: There exists a subset of sellers $s \in S' = [s_1, s_2] \subset S$ are in the same market $(p_\alpha, \theta_\alpha)$. From the free entry condition,

$$p_\alpha = \frac{E[s|s \in S']}{r} - \frac{k\theta_\alpha}{m(\theta_\alpha)}$$

and denote $V^\alpha(s_2) = V(p_\alpha, \theta_\alpha, s_2)$ as the utilities of the highest type seller in the market, and define the pair (p_2, θ_2) solves:

$$\left\{ \begin{array}{l} p_2 = \frac{s_2}{r} - \frac{k(\theta_2)}{m(\theta_2)} \\ V(p_2, \theta_2, s_2) = V^\alpha(s_2) \end{array} \right\}$$

Given (p_2, θ_2) solves above relations and $\frac{E[s|s \in S']}{r} < \frac{s_2}{r}$, there exists $p' = p_2 + \varepsilon$ such that $p_\alpha < p' < p_2$ and θ' which solve $V(p', \theta', s_2) = V^\alpha(s_2)$. From lemma 1, $T(p') = s_2$.

Namely, if a buyer deviates to posting p' , only the highest type in the original pooling market will come. And, as explained, the expected market tightness is defined from (3), that is, $\theta' = \theta(p')$. Obviously, $U_b(p', \theta', s_2) > U_b(p_2, \theta_2, s_2) = 0$. Contradiction. ■

C) Proof of Lemma 3: the lowest type always receives his first best utility.

Proof. Suppose not, pick any initial condition $\theta'_0 \in (0, \theta^{FB}(s_L))$ and denote its corresponding market tightness as $\theta'(s; \theta'_0)$ and price schedule $p'(s)$. One can easily show that there exists $\tilde{p} = p'(s_L) - \varepsilon$ and, from Lemma 1, $T(\tilde{p}) = s_L$. That is, a buyer can open a new market with lower price and expect the lowest type to come. Due to the violation of the tangent condition at $(p'(s_L), \theta'(s_L))$ when $\theta'(s_L) \neq \theta^{FB}(s_L)$ and $V'(s_L) < V^{FB}(s_L)$, buyers' utility can be improved, $U(\tilde{p}, \theta(\tilde{p}), s_L) > \phi^L$. Contradiction. ■

D) Proof of Heterogenous buyers

Before proving the constructed solution is indeed the solution, we first prove that the following claim holds:

Claim 2 $\forall \theta' < \theta$, if $V(s', p, \theta) - V(s', p', \theta') \geq 0$ then $V(s, p, \theta) - V(s, p', \theta') > 0$ for $\forall s' > s$

Proof. $V(s, p, \theta) - V(s, p', \theta') = \{V(s', p, \theta) + \frac{u(s)-u(s')}{r+m(\theta)}\} - \{V(s', p', \theta') + \frac{u(s)-u(s')}{r+m(\theta')}\} \geq (u(s') - u(s))(\frac{1}{r+m(\theta')} - \frac{1}{r+m(\theta)}) > 0$

Proof. a) Sellers' optimality: NTS: Given $(p(s), \theta(s))$, $s = \arg \max_{\hat{s}} V(s, p(\hat{s}), \theta(\hat{s}))$. First of all, we need to show that monotonic condition holds. The solution $\theta(s)$ is essentially the combination of $\theta(s; l)$ and $\theta(s; h)$, which are both non-increasing as long as the initial condition θ_j^0 is smaller than $\theta_j^{FB}(s_0)$. Therefore, $\theta(s)$ is also non-increasing as long as $\theta_H^0 \leq \theta(s^*, l)$, which holds by construction. From proposition 1, it is clear that facing $(p(s, l), \theta(s, l))$, IC holds for sellers $s < s^*$, and, similarly, given $(p(s, h), \theta(s, h))$, IC holds for sellers $s \geq s^*$. What is left to prove is that those sellers will not benefit from entering the markets $\{p(s'), \theta(s')\}$ for $\forall s' \geq s^*$. Clearly, given that $V(s^*, p(s^*, l), \theta(s^*, l)) = V(s^*, p(s^*, h), \theta(s^*, h)) > V(s^*, p(s', h), \theta(s', h))$ and $\theta(s') < \theta(s^*)$ for all $s' \geq s^*$, from claim 2:

$$V(s, p(s), \theta(s)) > V(s, p(s'), \theta(s')) \text{ for } \forall s' > s$$

■ ■

Similarly, one can use the fact that $V(s^*, p(s^*, h), \theta(s^*, h)) \geq V(s^*, p(s), \theta(s))$ and $\theta(s) < \theta(s')$ for all $s \leq s^*$ to prove $V(s', p(s'), \theta(s')) > V(s', p(s), \theta(s))$ for $\forall s < s^*$

b) Buyers' optimality: In order to make sure there is no profitable deviation for buyers, following two conditions much hold: b-1) $U_b(p(s), \theta(s), s, a^j) < \phi^j$ if $j^*(s) \neq j$. Namely, given the markets which are already open, buyers will not enter the market to trade with seller s , if $j^*(s) \neq j$. Note that this is an additional condition we need to prove with heterogenous buyers; b-2) There does not exist $p' \notin \text{support } P$, such that $U_b(p', \theta(p'), s', a^j) < \phi^j$, where $\theta(p')$ is defined as (3) and $s' \in T_s$ is the type of a seller who is most likely to come. That is, the buyer will not benefit from posting a price p' which is not recommended by the market maker.

b-1) First of all, we show that low type of buyers will not enter the market with sellers s , where $j^*(s) = h$. Observe that by construction, $\theta(s; h, \theta_H^0) < \tilde{\theta}(s)$ for $\forall s > s^*$. This is true as long as $\theta_H^0 \leq \tilde{\theta}(s^*)$, given that $\theta(s; h, \theta_H^0)$ decreases with s and $\tilde{\theta}(s)$ increases with s . The condition is satisfied since, by construction, if $\tilde{\theta}(s^A) \geq \theta_H^{FB}(s^A)$, $\theta_H^0 = \theta_H^{FB}(s^A) \leq \tilde{\theta}(s^A)$; otherwise, $\theta_H^0 = \tilde{\theta}(s^B)$. Hence, from claim 1, low type will not enter the market with the pair $\{p^*(s), \theta^*(s)\}$ for $\forall s > s^*$. Similarly, by construction, $\theta(s; l, \theta_L^0) \geq \theta(s)$ for $\forall s \leq s^*$. Hence, high-type buyers will not enter the the market with the pair $\{p^*(s), \theta^*(s)\}$ for $\forall s < s^*$

b-2) Let $\{\bar{p}, \underline{p}\}$ as the upper bound and the lower bound of the support constructed price schedule P . Apply Lemma 1, for any new posting $p' > \bar{p}$, $T_s(p') = s_H$ and $p' < \underline{p}$, $T_s(p') = s_L$. Obviously, the lower type will not benefit from opening $p' < \underline{p}$ since (p^*, θ^*) is the first-best solution. The higher type obviously will not benefit from attracting the lowest type seller, given $V^{FB}(s, h) < V^{FB}(s, l)$ and, therefore, $U_b(p', \theta', s_L; V^{FB}(s_L, l)) < \phi^h$. Similarly, as before, neither the high type will benefit from posting $p' > \bar{p}$ to attract the highest type with a even higher distortion of market liquidity, nor the lower type buyer. Notice that $p(\cdot)$ has a jump discontinuity at s^* when $s^* = s^A$. In particular, there are two markets opened for the seller s^* , and, among two of them, sellers are indifferent. Given any $p' \in (p(s^*, l), p(s^*, h))$, $T(p') = s^*$ according to Lemma 1. Given that the type who is most likely come is s^* , it is clear that low type buyer will not raise the price $p' > p(s^*, l)$ to attract the same seller. Nor the high type will benefit from posting $p' < p(s^*, h)$ since the pair of $p(s^*, h)$ and $\theta(s^*, l) = \theta_H^{FB}(s)$ has already maximized the joint surplus (first best).

Proof. Uniqueness: From Lemma 3, we know that $\theta_L^0 = \theta_L^{FB}(s)$. Hence, to prove the uniqueness, we essentially need to show that marginal types s^* constructed above and its corresponding θ_H^0 is unique. First of all, by construction, $V_H^{FB}(s) < V(s, l)$ for $\forall s < s^A$, it is clear that a high-type buyer will not enter the market for $s < s^A$, given that the highest utilities he can offer to the seller is lower than the one offered by a low type buyer. Also, from Claim 1, s^* can not be larger than s^B in the equilibrium otherwise there exists $s < s^*$ such that $\theta(s, l) < \tilde{\theta}(s)$, which implies that it is profitable for a higher type to enter this market. Hence, the only possible range is $[s^A, s^B]$. Given that there are two different cases, depending on the relation between $\tilde{\theta}(s^A)$ and $\theta_H^{FB}(s^A)$, we will prove the uniqueness separately for each case. Before that, we first prove formally that s^A is unique. Notice that s^A is the intersection of $V_H^{FB}(s)$ and $V(s, l)$. Therefore, the unique of s^A is obtained as long as $V_H^{FB}(s) - V(s, l)$ satisfies single crossing condition. As a result of the following inequalities, we can conclude that $V_H^{FB}(h, s') - V(l, s') > V_H^{FB}(h, s) - V(l, s)$ for any $s' > s$

$$\begin{aligned} & V_H^{FB}(s') - (V_L^{FB}(s) - V(l, s)) - V(l, s') \\ & > V_H^{FB}(s') - (V_L^{FB}(s') - V(l, s')) - V(l, s') \\ & > V_H^{FB}(s) - (V_L^{FB}(s) - V(l, s)) - V(l, s) \end{aligned}$$

The first inequality follows from the fact that $V_L^{FB}(s') - V(l, s') > V_L^{FB}(s) - V(l, s)$, that is, the utility of a high type seller decreases more than the one of a low type, resulting

from a higher distortion¹⁵. The second inequality follows from the condition of PAM, that's, $V_H^{FB}(s') - V_L^{FB}(s') > V_H^{FB}(s) - V_L^{FB}(s)$. Moreover, from the discussion of b-2), we can conclude the following claim: (Claim 3) If there is a discontinuity in $\theta^*(\cdot)$ at s^* , which necessarily induces a discontinuity in $p(\cdot)$, it has to be the case that $\theta_H^0 = \theta_H^{FB}(s^*)$. Namely, θ_H^0 must equal its first best value $\theta_H^{FB}(s^*)$ when $\theta_H^0 \neq \theta(s^*, l)$; otherwise, there is a profitable deviation for a high type buyer, who will deviate by posting a new price $p' = p(\theta_H^0) - \varepsilon \notin \text{range of } P$ to attract s^* .

(CASE 1) $s^* = s^A$ when $\tilde{\theta}(s^A) \geq \theta_H^{FB}(s^A) : \tilde{\theta}(s^A) \geq \theta_H^{FB}(s^A)$ immediately implies that $\theta(s^A, l) \geq \tilde{\theta}(s^A) \geq \theta_H^{FB}(s^A)$ ¹⁶ and the equality holds iff $\theta(s^A) = \theta_H^{FB}(s^A)$. Pick $s_m \in (s^A, s^B]$ as the marginal type. By definition, the marginal type must be indifferent among two markets, that is, indifferent between $(\theta(s_m, l), p(s_m, l))$ and $(\theta_H^0, p(s_m, h))$. Given that $s_m > s^A \implies V_H^{FB}(s_m) > V(s_m, l)$, It has to be case that $\theta_H^0 < \theta_H^{FB}(s_m)$, that is, there must be a downward distortion¹⁷ in market tightness otherwise s_m is strictly better off going to the market with a high type buyer. According to Claim 3, this can not be sustained in equilibrium if there is a discontinuous in $\theta^*(\cdot)$. Namely, the only possible case is that $\theta_H^0 = \theta(s_m, l) < \theta_H^{FB}(s_m)$. However, given that $\theta(s^A, l) \geq \theta_H^{FB}(s^A)$, $\theta(s_m, l)$ decreases with s and $\theta_H^{FB}(s)$ increases with s , $\theta(s_m, l) < \theta_H^{FB}(s_m) \implies \exists s' \in (s^A, s_m)$ such that $\theta(s', l) = \theta_H^{FB}(s') > \tilde{\theta}(s')$. The above relation implies that $V(p^l(s'), \theta(s', l)) > V_H^{FB}(s')$, which contradicts with the fact that $V_H^{FB}(h, s') - V(l, s') > 0$ for $\forall s' > s^A$. Therefore, we show that $s^* = s^A$ is the unique solution when $\tilde{\theta}(s^A) \geq \theta_H^{FB}(s^A)$.

(CASE 2) $s^* = s^B$ when $\tilde{\theta}(s^A) < \theta_H^{FB}(s^A)$. First of all, $\tilde{\theta}(s^A) < \theta_H^{FB}(s^A)$ implies that $\theta_H^{FB}(s^A) > \theta(s^A, l) > \tilde{\theta}(s^A)$. Same as before, the only possible range for s_m is $[s^A, s^B]$. Given that $\tilde{\theta}(s)$ and $\theta_H^{FB}(s)$ increase with s , while $\theta(s, l)$ decreases with s and $\theta_H^{FB}(s^A) > \theta(s^A, l) > \tilde{\theta}(s^A)$, it has to be the case that $\theta_H^{FB}(s_m) > \theta(s_m, l) > \tilde{\theta}(s_m)$ for $\forall s_m \in [s^A, s^B]$. Moreover, according to Claim 1, the initial condition θ_H^0 has to smaller $\tilde{\theta}(s_m)$. Therefore, $\theta(s_m, l) > \tilde{\theta}(s_m) \geq \theta_H^0$, which necessarily results in a discontinuity of $\theta^*(\cdot)$ at s_m . The resulting discontinuity and $\theta_H^0 < \theta_H^{FB}(s_m)$ violates Claim 3. Contradictions. The above argument also confirms why only s^B and $\theta_H^0 = \theta(s^B, l) = \tilde{\theta}(s^B)$ is the unique solution in this case, guaranteeing the continuity of $\theta^*(s)$. ■

¹⁵One can show that $\theta_L^{FB}(s) - \theta^*(s, l)$ increases with s , given that $\theta_L^{FB}(s)$ increases with s while $\theta^*(s, l)$ decreases with s .

¹⁶Note that $\theta(s^A, l)$ is the intersection of a low-type buyer's utility at ϕ^l and the utility of a seller s^A with the level of $V_H^{FB}(s^A)$. Given that $\tilde{\theta}(s^A) \geq \theta_H^{FB}(s^A)$ and the tangent condition of $V_H^{FB}(s^A)$ and the utility of a high-type buyer is satisfied, $\theta(s^A, l)$ must be larger than $\tilde{\theta}(s^A)$. This is because that, by construction, the utility curve of $U_b(s, p, \theta) = \phi^l$ lies below $U_b(s, p, \theta) = \phi^h$ for any $\theta < \tilde{\theta}(s)$ and above $U_b(s, p, \theta) = \phi^h$ for any $\theta > \tilde{\theta}(s)$.

¹⁷Clearly, it has to be downward distortion instead of upward one since the monotonic condition from sellers' IC won't hold for any $\theta_H^0 > \theta_H^{FB}(s)$.

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