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Targeted pricing and customer data sharing among rivals

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Targeted Pricing and Customer Data Sharing Among Rivals

Nicola Jentzsch, Geza Sapi, Irina Suleymanova

July 2012
Targeted Pricing and Customer Data Sharing Among Rivals*

Nicola Jentzsch† Geza Sapi‡ Irina Suleymanova§

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Abstract

It is increasingly observable that competitors in different industries share customer data, which can be used for targeted pricing. We propose a modified Hotelling model with two-dimensional consumer heterogeneity to analyze the incentives for such sharing and its ensuing welfare effects. We show that these incentives depend on the type of customer data and on consumer heterogeneity in the strength of brand preferences. Only data on consumer transportation cost parameters is shared. The incentives to do so are stronger if consumers are relatively homogeneous. Customer data sharing is most likely to be detrimental to consumer surplus, while the effect on social welfare can be positive.

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1 Introduction

Recent advances in information technologies allow firms to collect, analyze and share detailed information about customers and to use it for targeted advertising and pricing. For example, in the airline industry there is a widespread exchange of passenger data over Computer Reservation Systems (CRSs), among which are Amadeus, Sabre, Mercator and WorldSpan.1 CRSs allow to share the so-called Passenger Name Records (PNRs) among competing airlines, travel agencies and other affiliated firms. PNRs contain personal data on passengers such as name and address, age, gender, loyalty program membership and booking details including previous travels.2 While passenger data sharing is justified by quality assurance, airlines have some discretion on what data they share.3 The International Air Transport Association (IATA) reports that PNR data can be used to “segment passengers, observe their behavioral patterns, and reach out to them with relevant, targeted promotions” (IATA, 2010).

Another example is the telecommunications industry, where firms collect and process vast amounts of personal data on customers. In the U.S. such data are known as Customer Proprietary Network Information (CPNI). These CPNI entail the customer’s identity data as well as time, date, destination and duration of calls, and comprise information on any purchases that appear on the phone bill.4 Firms can use this information for customer retention analysis and targeted promotions. In 1999, the Federal Communications Commission (FCC) allowed “all telephone companies to use CPNI in their efforts to ‘win back’ customers lost to competitors, reasoning that ‘winback’ campaigns are good for competition and consistent with the Act.” (FCC, 1999). Customer information are also shared with marketing firms, such as Acxiom Corp., in the U.S. (Notaras, 1998). Data collected in marketing databases can potentially be used by competitors (Liu and Serfes, 2006, Footnote 2). In 2007 the opt-out framework for sharing CPNI with joint venture partners or independent contractors for marketing purposes

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1Some CRSs were formerly owned by airlines. Driver (1999, p. 136) notes that airlines owning CRSs “used the data stored to analyze the customer specific data of their rivals using that information for strategic and tactical advantage – this was one of the forms of abuse Virgin brought about against British Airways in the infamous ‘dirty tricks’ case.”

2An example of such PNRs from WorldSpan is provided at: http://globallearningcenter.wspan.com/learningcenter/pdfs/pfg/sections/pfg_onlineversion_pnr.pdf.

3For example, Worldspan indicates that Delta (unlike other airlines) did not share frequent flyer data within the system (see http://globallearningcenter.wspan.com/emealearningcenter/PDFs/Student%20Workbooks/210/1101%20PNR%20Lesson.pdf).

4CPNI are defined in the U.S. Telecommunications Act of 1996, see also 47 USC § 222 - Privacy of customer information (http://www.law.cornell.edu/uscode/text/47/222).
was replaced by a stricter regime, which requires an opt-in consent from a customer (FCC, 2007, p. 22).  

Information sharing in several industries has initiated heated debates between consumer privacy advocates, business groups, competition authorities and other regulators. At this stage economic theory is lagging behind in providing answers to several important questions: What type of customer data (allowing targeted pricing) would a firm share with a competitor and what type of data would it keep private? Under which conditions is customer data sharing likely to take place? What welfare effects arise from such sharing?

To answer these questions, we analyze the incentives of competing firms to share customer data that enable targeted price offers. We augment the standard Hotelling model by introducing two-dimensional consumer heterogeneity and assume that consumers differ both in their brand preferences and transportation cost parameters (flexibilities). Firms may have access to two types of data on customer preferences, reflecting their brand preferences and transportation cost parameters. Customer data allow firms to engage in competitive first- and third-degree price discrimination. This novel approach allows several new insights. First, we show that information sharing incentives depend on the type of customer data, more precisely, only data on the flexibility of consumers are shared. Second, the incentives to share such data depend on consumer heterogeneity in flexibility. Firms have stronger incentives to share data, when consumers are relatively homogeneous in their reaction to price changes. Third, customer data sharing is most likely to be detrimental to consumer surplus, while the effect on social welfare can be positive.

The main intuition for our results is as follows. The sharing of data on consumer brand preferences leads to an unambiguous negative effect on joint profits due to a strong competition effect. If a firm can distinguish its own loyal consumers from that of the rival, it targets aggressively the rival’s customer base. This negative competition effect always outweighs the positive rent-extraction effect emerging from the better targeting opportunities of a firm with initially less customer data. When data on consumer transportation cost parameters are shared,

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5 The FCC notes that “the black market for CPNI has grown exponentially” due to unauthorized access to CPNI by data brokers (FCC, 2007, p. 22). Those data brokers sell customer data to third parties, which can be also a firm’s rivals. For an example of such a data broker, Intelligent e-Commerce Inc., see http://www.bestpeoplesearch.com.

6 See, for example, “Your Phone Company is Selling Your Personal Data,” CNNMoney, November 1, 2011 (http://money.cnn.com/2011/11/01/technology/verizon_att_sprint_tmobile_privacy/).
the effect on joint profits is ambiguous. Prices increase for consumers with strong brand preferences, but they decrease for consumers with weak brand preferences. It depends on consumer heterogeneity in flexibility, how this ambiguity is resolved. When consumers are relatively differentiated, the firm without data on consumer flexibility targets only its loyal consumers having high transportation cost parameters. However, when consumers are relatively homogeneous, it also targets its loyal consumers having the lowest transportation cost parameters. Hence, before data sharing the price(s) of the firm without data on transportation cost parameters is (are) relatively high in the former case and relatively low in the latter case. As a result, before data sharing profits tend to be higher when consumers are relatively differentiated in flexibility, which decreases the incentives to share customer data.

Our model shows that sharing of customer data (used for price discrimination) can be profitable for competing firms. We conclude that data sharing between rivals should receive more attention by the competition authorities as it is most likely to have negative effect on consumer surplus. When competition authorities scrutinize customer data sharing agreements between rivals, price discrimination considerations should be taken into account. For example, the recently revised horizontal guidelines of the European Commission were extended by a chapter on information exchange.\textsuperscript{7} This chapter, however, does not explicitly mention competitive price discrimination as a possible theory of harm.

The rest of the article is organized as follows. Section 2 reviews the related literature. The model is presented in Section 3. In Section 4 we analyze the effects of price discrimination based on two types of customer data with regards to firm profits. In Section 5 we investigate the incentives to share customer data and its welfare implications. In Section 6 we check the robustness of our results. Finally, Section 7 concludes.

\section{Related Literature}

Borenstein (1985) analyzes the profit effects of competitive price discrimination when consumer preferences are heterogeneous along different dimensions. He assumes that consumers are characterized by different reservation prices and transportation cost parameters. Borenstein’s simulation results show that price discrimination based on either reservation prices or transportation cost parameters is profitable. This gives rise to the conjecture that firms may have an incentive

\textsuperscript{7}For this legislation, see http://ec.europa.eu/competition/antitrust/legislation/horizontal.html.
to share data on consumer transportation cost parameters. Our results confirm this conjecture, adding that incentives for data sharing are stronger, if consumers are relatively homogeneous in flexibility.

In a recent article, Esteves (2009) proposes a model where consumers differ in their preferences both for a brand name and a product. However, in her analysis consumers are homogeneous in transportation costs (both in the brand and product differentiation dimensions). Esteves shows that price discrimination can boost profits only when firms have information about the location of consumers in the less differentiated dimension and remain ignorant about their other preferences.

Shaffer and Zhang (2002) analyze profit effects of one-to-one promotions enabled by the use of data on consumer brand loyalties when firms are asymmetric in customer bases. Using the authors’ results one can show that only the firm with a smaller customer base will share its data with the rival, and only if the firms are sufficiently asymmetric. In our analysis customer data include two types of information on a consumer: her brand preference and transportation cost parameter. In Shaffer and Zhang those two types of data are aggregated in consumer’s brand loyalty. We show that, when the two types of data are available separately, customer data are shared even when firms are symmetric.

Chen et al. (2001) directly address the problem of customer data sharing among rivals. They analyze a model, where a firm can classify a given consumer only with a less-than-perfect accuracy as its own loyal customer or switcher. The authors show that the firm with a lower level of targetability can sell its information to the rival even when both firms have the same number of loyal customers. However, when firms are symmetric and information is perfectly accurate, no data sharing takes place. We show, in contrast, that sharing of customer data can also be profitable, when customer data are perfectly accurate and firms are symmetric. This difference to the results of Chen et al. is driven by the fact that in their analysis consumers are highly differentiated in flexibility. While the switching costs of the loyal customers are prohibitive, switchers can change brands costlessly. Our analysis reveals that the incentives to share data on customer flexibility are stronger when consumers are relatively homogenous along this dimension.

Liu and Serfes (2006) consider a two-period model where each firm obtains information on brand loyalties of those consumers who bought from it in the first period. Customer data can
be sold between firms and it may be used for price discrimination in the second period. Here, information sharing is profitable only if the firms are sufficiently asymmetric in their customer bases. We show that information sharing is also possible when firms are symmetric, provided data on customer brand preferences and transportation cost parameters are available separately.

3 The Model

We consider two differentiated firms, A and B, each selling a variety of the same product, while competing in prices. Firms are situated at the two ends of a Hotelling line of a unit length with firm A located at point 0 and firm B at point 1. Every consumer is characterized by an address \( x \in [0,1] \) corresponding to her brand preference for the ideal product. If a consumer buys from a firm that does not provide the ideal product she incurs linear transportation costs proportional to the distance to the firm. Now we depart from the standard Hotelling setup by introducing heterogeneity in consumer transportation costs per unit distance, which we denote by \( t \in [\underline{t}, \bar{t}] \), with \( \underline{t} \geq 0, \bar{t} > 0 \) and \( \bar{t} > \underline{t} \). The mass of consumers is normalized to unity and every consumer is uniquely described by a pair \((t, x)\). With \( t \) and \( x \) being uniformly and independently distributed, we have the following density functions: 

\[
    f_t = 1/(\bar{t} - \underline{t}), \quad f_x = 1, \quad f_{t,x} = 1/(\bar{t} - \underline{t}).
\]

We consider two important limiting cases with respect to the heterogeneity of consumers in flexibility measured by the parameter \( k(t, \bar{t}) := \bar{t}/\underline{t} \), which represent two versions of our model. In the first version we call consumers relatively differentiated and assume that \( \underline{t} = 0 \), in which case

\[
    \lim_{\underline{t} \to 0} k(t, \bar{t}) = \infty
\]

such that the ratio of the highest to the lowest transportation cost parameters approaches infinity. In the second version we label consumers as relatively homogeneous and assume that \( \bar{t} > 0 \) and \( k(t, \bar{t}) \leq 2 \) such that the ratio of the largest to the lowest transportation cost parameters is very small. One could proxy the level of consumer heterogeneity in flexibility through the switching rate in a market. A low switching rate then indicates a high level of heterogeneity such that only consumers with relatively low transportation costs switch brands.

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\(^8\) Apart from a few exceptions (Borenstein, 1985; Shaffer and Zhang, 1995; Chen et al., 2001; Armstrong et al., 2006) most of the works on price discrimination assume that consumers are heterogeneous only in brand preferences, and not in transportation cost parameters. There are also several articles, which consider consumer heterogeneity in brand loyalty, which combines both a consumer’s brand preference and transportation cost parameter (Bester and Petrakis, 1994; Shaffer and Zhang, 2000; Shaffer and Zhang, 2002; Liu and Serfes, 2006).

\(^9\) The ratio of the largest to the lowest transportation cost parameters approaches infinity in two cases: i) if \( \underline{t} = 0 \), ii) if \( \bar{t} > 0 \) and \( \bar{t} \to \infty \). In our analysis we only consider the first case. However, all our results on firms’ incentives to share customer data and its welfare implications are also valid for the second case. The proof is available from the authors upon request.
Switching rates vary across markets. For example, the switching rate for a current bank account was around 9 percent in 2007-08 across EU-countries compared to 25 percent for a car insurance (European Commission, 2009, p. 10).

The utility of a consumer \((t, x)\) from buying at firm \(i \in \{A, B\}\) at price \(p_i\) is

\[
U_i(p_i, t, x) = v - t|x - x_i| - p_i,
\]

where \(v > 0\) is the basic utility from buying the ideal product at the price of zero (large enough such the market is always covered in equilibrium), \(x_i\) denotes firm \(i\)’s location with \(x_A = 0\) and \(x_B = 1\). A consumer buys from the firm delivering higher utility. If a condition

\[
t(1 - 2x) + p_B > p_A
\]

holds, then firm \(A\) provides a strictly higher utility.\(^{10}\) We say that a consumer \((t, x)\) is on \(i\)’s turf if her utility from buying at firm \(i\) is higher than at firm \(j\) when prices are equal.\(^{11}\) The turf of firm \(A\) (\(B\)) is given by consumers with addresses \(x < 1/2\) \((x > 1/2)\).

Depending on the available customer data firms can adopt different pricing strategies. If a firm has information on both consumer locations and flexibility, it can offer an individual price to each consumer. With information on either consumer locations or flexibility a firm can differentiate among groups of consumers. Without customer data a firm must set a uniform price. Marginal costs are assumed to be zero. Firms set prices \(p_i(t, x)\) to maximize their profits,

\[
\Pi_i = \int \int_{X_i, T_i} p_i(t, x) f_i dt dx,
\]

with \(X_i\) and \(T_i\) denoting the domains of addresses and transportation cost parameters of consumers who buy from firm \(i\). Next we explain the way firms may hold and share customer data and describe the game played.

**Customer Data and the Game.** Let \(X\) and \(T\) be the two sets containing information on brand

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\(^{10}\)We follow Liu and Serfes (2006) and use two tie-breaking rules. Assume that both firms offer equal utilities. First, in this case a consumer chooses the firm closer in the brand preference dimension if both firms hold same data on consumers (if \(x = 1/2\), then the consumer visits firm \(A\)). Second, if one firm has more data, then a consumer chooses the firm with more data.

\(^{11}\)Instead of the term “turf” we will also use the terms “customer base” and “loyal customers.”
preferences and transportation cost parameters of all consumers in the market, respectively. We refer to X and T as datasets.\footnote{We assume that datasets T and X include data on the preferences of all consumers in the market, such that firms may hold data on the same consumers. This assumption is a good approximation that describes well some real-life situations. First, it applies to newly liberalized network industries (such as telecommunications and airline industries) where incumbents have data on a large share of customers while entrants hold no proprietary data. Second, in traditional industries such as retail where sophisticated discount and targeting programs are in place, significant overlaps exist in the customer pools of competing firms. For instance, in UK 42 percent of Tesco clubcard holders had a loyalty card of competing Sainsbury’s in 2000 (Addley, 2000). As Tesco is perceived to have a technological advantage in the ability to analyze customer data for targeted offers, it is likely to end up with more data on the preferences of joint customers compared to the rival (Davis, 2007). In Section 6 we provide a robustness check where we assume that each firm may hold data only on share of all consumers, such that firms never hold data on the same consumers. We show that all our main results remain unchanged.}

We define the union of datasets that firm i holds as firm i’s information set and denote it by I_i. Each firm may hold information either only on brand preferences (I_i = X), only on transportation cost parameters (I_i = T), complete information on consumer preferences (I_i = X \cup T), or no information (I_i = \emptyset). To simplify the notation, we write I_i = XT to denote the case where firm i has complete information on consumers.

We use the term information scenario to describe the datasets held by both firms in a pricing game and denote it by \{I_A, I_B\}. The superscript \Pi^XT|XT denotes the functions and variables in the information scenario \{I_A, I_B\}. For example, \Pi^XT|XT denotes the profit function of firm A when both firms have full information on consumers: \{I_A, I_B\} = \{XT, XT\}. We refer to the cases where I_A = I_B as symmetric information scenarios. Cases where I_A \neq I_B are referred to as asymmetric information scenarios. For the remainder of this article we assume that firm A (B) is the firm with a larger (smaller) information set in asymmetric information scenarios. We also assume that firms can exchange datasets X and/or T only in their entirety.

Take the aforementioned example of airlines and PNR data. Brand preference of a customer would correspond to the ‘ideal’ airline, indicated by the membership in the frequent flyer program of an airline or frequent bookings made there. This information is readable from a customer’s PNR. Transportation cost parameter of a customer shows how responsive the passenger is to marketing campaigns, i.e., how much it would take to lure her away from rival offers. It is more difficult to lure a customer away, when she holds a loyalty status at the airline she frequently travels with compared to a switcher who is not member of such a program and used different airlines to travel the same route in the past. Moreover, a higher loyalty status (Silver, Gold, Platinum) would indicate a higher transportation cost parameter. In fact, information on loyalty
is used by airlines to charge higher prices to loyal travellers.\textsuperscript{13}

Empirical economic research often distinguishes between consumer brand preferences and the strength thereof. Significant research has been devoted to deriving individual-level parameter estimates for consumer preferences and sensitivity to different marketing variables (such as price and advertising).\textsuperscript{14} These two dimensions correspond to information in datasets $X$ and $T$ in our model. To build these datasets firms may typically use three main sources: external surveys, information on consumer purchase behavior and demographic data. The PNRs and CPNI rely strongly on the latter two sources.

We proceed with the description of the game. It unfolds as follows.

**Stage 1** (Data sharing). Firm $A$ decides whether and which dataset(s) to offer to the rival for sale. After a dataset is shared (sold), it becomes available to both firms.

**Stage 2** (Competition). First, firms choose their regular prices. Second, targeted at consumer groups, firms discount regular prices. Third, firms decide on individual discounts from the group prices.

We assume that information sharing takes place in Stage 1 if joint profits strictly increase. To illustrate the timing of the pricing game in Stage 2 we consider the information scenario $\{XT, X\}$. First, firms choose their regular prices. Then they decide on group discounts from the regular prices for each brand preference. Finally, firm $A$ issues individual discounts from its group prices depending on a consumer’s flexibility.\textsuperscript{15} The assumed timing is consistent with a large part of the literature on competitive price discrimination, where firms determine their targeted offers after setting uniform prices (e.g., Thisse and Vives, 1988; Shaffer and Zhang, 1995, 2002; and Liu and Serfes, 2004, 2006). This timing corresponds to the observation that it is easier to adjust prices that are targeted at smaller groups of consumers than uniform prices.

\textsuperscript{13}In Germany, anecdotal evidence shows that some airlines charge more for tickets paid for by frequent flyer miles compared to normal tickets. This practice has sparked a controversial discussion (see Warnholtz, 2011).

\textsuperscript{14}For instance, Allenby and Rossi (1999) use a continuous model of consumer heterogeneity to estimate the distribution of consumer brand preferences and price sensitivities based on a scanner panel dataset of ketchup purchases. Gupta and Chintagunta (1994) show how demographic characteristics (income and household size) can be used for market segmentation where each consumer group is characterized by a specific combination of brand preference and sensitivity to different marketing variables (including prices). Horsky, Misra and Nelson (2006) show how the inclusion of survey-based customer preference measures improves the estimates of customer preferences derived from scanner panel data on customer purchases.

\textsuperscript{15}Note that the timing in Stage 2 is equivalent to the following: i) in symmetric information scenarios both firms choose all the prices simultaneously, and ii) in asymmetric information scenarios the firm with a smaller information set chooses all its prices (price) first and the other firm follows.
or prices targeted at larger groups. Moreover, if all prices are chosen simultaneously, a Nash equilibrium in pure strategies does not always exist if one of the firms holds full customer data. Resorting to sequential moves is a standard way to tackle this problem. In Section 6 we check the robustness of our results with regards to the timing of pricing decisions and analyze data sharing incentives under the assumption that firms move sequentially in all information scenarios and show that all our main results remain intact.

4 Customer Data and Price Discrimination

To solve the pricing game we seek for a Nash equilibrium in symmetric information scenarios and for a subgame-perfect Nash equilibrium in asymmetric information scenarios. We restrict our attention to pure strategies. Proposition 1 characterizes the equilibria.

**Proposition 1.** Equilibrium prices and profits in each information scenario are as stated in Tables 1 and 2, respectively.

**Proof.** See Appendix.

Our first observation relates to the value of customer data: The firm with more customer data realizes higher profits in all information scenarios. Although firms are initially symmetric in their customer bases, more data allow a firm to better target consumers and capture a larger market share.

Note, next, that in equilibrium firms use all the available customer data for price discrimination. All equilibrium prices in Table 1 are functions of the data firms hold. In symmetric information scenarios a firm’s best-response function specifies the profit-maximizing price to any given price of the competitor. In this case, the only effect of not using all the available customer data is a decrease in the degree of freedom in pricing. The same is true for the firm with more customer data in asymmetric information scenarios, as it moves after observing the competitor’s

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16 Precisely, under the assumption that all the prices are chosen simultaneously, no Nash equilibrium in pure strategies exists in the information scenarios \{XT, Ø\} and \{XT, T\}, irrespectively of consumer heterogeneity in flexibility. In the information scenario \{XT, X\} there is no Nash equilibrium in pure strategies when consumers are relatively differentiated. When consumers are relatively homogeneous, there is a unique equilibrium that corresponds to the subgame-perfect Nash equilibrium derived under the timing in Stage 2. The proof is available from the authors upon request, here we only sketch out the intuition for the scenario \{XT, Ø\}. In this scenario the optimal strategy of firm A is to make consumers indifferent between buying from both firms whenever it can do so with a positive price. Firm B has then an incentive to deviate from any positive price to capture all the consumers served by firm A. Hence, there can be no equilibrium in which firm B charges a positive price. However, in equilibrium firm B cannot charge the price of zero either. Indeed, by increasing its price slightly firm B could serve some consumers on its turf and realize positive profits.
Table 1: Equilibrium Prices in Different Information Scenarios

<table>
<thead>
<tr>
<th>( I_A )</th>
<th>( I_B )</th>
<th>( p^*_A )</th>
<th>( p^*_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relatively Differentiated Consumers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>X</td>
<td>( \frac{2t}{1 - 2x} / 3, x \leq 1/2 )</td>
<td>( \frac{t}{1 - 2x} / 3, x \leq 1/2 )</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>( \frac{7}{2} (2x - 1) / 3, x &gt; 1/2 )</td>
<td>( 2t (2x - 1) / 3, x &gt; 1/2 )</td>
</tr>
<tr>
<td>XT</td>
<td>XT</td>
<td>( \begin{cases} \frac{t (1 - 2x)}{2}, x \leq 1/2 \ 0, x &gt; 1/2 \end{cases} )</td>
<td>( \begin{cases} 0, x \leq 1/2 \ t (2x - 1), x &gt; 1/2 \end{cases} )</td>
</tr>
<tr>
<td>X</td>
<td>( \emptyset )</td>
<td>( \begin{cases} \frac{t (1.47 - 2x)}{2}, 0.5 \leq x \leq 0.27 \ 0.47t \end{cases} )</td>
<td>( 0.47t )</td>
</tr>
<tr>
<td>T</td>
<td>( \emptyset )</td>
<td>( \begin{cases} \frac{0.85t - t}{2}, t \leq 0.28t \ \frac{(0.85t + t)}{2}, t \geq 0.28t \end{cases} )</td>
<td>( 0.85t )</td>
</tr>
<tr>
<td>XT</td>
<td>( \emptyset )</td>
<td>( \max {0, 0.28t + (1 - 2x)} )</td>
<td>( 0.28t )</td>
</tr>
<tr>
<td>Relatively Homogeneous Consumers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>X</td>
<td>( \frac{t (1 - 2x)}{2}, x \leq 1/2 )</td>
<td>( \begin{cases} \frac{t (1 - 2x)}{2}, x \leq 1/2 \ 0, x &gt; 1/2 \end{cases} )</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>( \begin{cases} \frac{t (1 - 2x)}{2}, x \leq 1/2 \ 0, x &gt; 1/2 \end{cases} )</td>
<td>( \begin{cases} 0, x \leq 1/2 \ t (2x - 1), x &gt; 1/2 \end{cases} )</td>
</tr>
<tr>
<td>XT</td>
<td>XT</td>
<td>( \frac{\tilde{H}(t, \tilde{t}) + t (1 - 2x)}{2}, x \leq \frac{1}{2} )</td>
<td>( \tilde{H}(t, \tilde{t}) )</td>
</tr>
<tr>
<td>X</td>
<td>( \emptyset )</td>
<td>( \begin{cases} \frac{\tilde{H}(t, \tilde{t}) - \tilde{t} (2x - 1)}{2}, \frac{1}{2} \leq x \leq \frac{1}{2} + \frac{\tilde{H}(t, \tilde{t})}{2 t (2 - t)} \ \frac{\tilde{H}(t, \tilde{t}) - \tilde{t} (2x - 1)}{2}, \frac{1}{2} \leq x \leq \frac{1}{2} + \frac{\tilde{H}(t, \tilde{t})}{2 t (2 - t)} \end{cases} )</td>
<td>( \tilde{H}(t, \tilde{t}) )</td>
</tr>
<tr>
<td>T</td>
<td>( \emptyset )</td>
<td>( \frac{t}{2} + 3 \tilde{H}(t, \tilde{t}) / 4 )</td>
<td>( 3 \tilde{H}(t, \tilde{t}) / 2 )</td>
</tr>
<tr>
<td>XT</td>
<td>( \emptyset )</td>
<td>( \max {0, \tilde{H}(t, \tilde{t}) / 2 + (1 - 2x)} )</td>
<td>( \tilde{H}(t, \tilde{t}) / 2 )</td>
</tr>
<tr>
<td>XT</td>
<td>X</td>
<td>( \tilde{t} (1 - 2x), x \leq 1/2 )</td>
<td>( \begin{cases} 0, x \leq 1/2 \ \tilde{t} (2x - 1), x &gt; 1/2 \end{cases} )</td>
</tr>
<tr>
<td>XT</td>
<td>T</td>
<td>( \max {0, t / 2 + t (1 - 2x)} )</td>
<td>( t / 2 )</td>
</tr>
</tbody>
</table>

\( \tilde{H}(t, \tilde{t}) = (\tilde{t} - \tilde{t}) / \ln(\tilde{t} / \tilde{t}), \quad \tilde{H}(t, \tilde{t}) = (\tilde{t} - \tilde{t}) / \ln(2 \tilde{t} / \tilde{t} - 1) \)
price(s). The firm with a smaller information set then also maximizes its profit by using all the available customer data.

| $I_A$ | $I_B$ | $\Pi_{A|B}^{I_A}$ | $\Pi_{A|B}^{I_B}$ | $\Pi_{A|B}^{I_A}$ | $\Pi_{A|B}^{I_B}$ |
|-------|-------|--------------------|--------------------|--------------------|--------------------|
| $X$   | $X$   | $t/8$              | $t/8$              | $L/4$              | $L/4$              |
| $T$   | $T$   | $t/4$              | $t/4$              | $A(t,\bar{t})/2$  | $A(t,\bar{t})/2$  |
| $XT$  | $XT$  | $\bar{t}/8$       | $\bar{t}/8$       | $A(t,\bar{t})/4$  | $A(t,\bar{t})/4$  |
| $X$   | $\emptyset$ | $0.32\bar{t}$     | $0.12\bar{t}$     | $5H(t,\bar{t})/8 + t/4$ | $H(t,\bar{t})/4$  |
| $T$   | $\emptyset$ | $0.53\bar{t}$     | $0.23\bar{t}$     | $21H(t,\bar{t})/32 + A(t,\bar{t})/8$ | $9H(t,\bar{t})/16$ |
| $XT$  | $X$   | $5\bar{t}/32$     | $\bar{t}/16$     | $A(t,\bar{t})/4$  | $L/4$              |
| $XT$  | $T$   | $9\bar{t}/32$     | $\bar{t}/16$     | $9A(t,\bar{t})/16$ | $A(t,\bar{t})/8$  |

Table 2: Equilibrium Profits in Different Information Scenarios

Pay-to-stay and pay-to-switch strategies. We now explain how the overall flexibility of consumers determines the pricing strategies when firms can discriminate based on consumer brand preferences. Shaffer and Zhang (2000) distinguish between pay-to-switch and pay-to-stay strategies where a firm charges a lower price to the rival’s (its own) customer base in the former (latter) case. We adopt this terminology and say that a firm uses the pay-to-switch strategy if the lowest price a firm charges on its own turf is (weakly) higher than the highest price a firm charges on the rival’s turf. In all the other cases firms are said to follow the pay-to-stay strategy. The comparison of prices in Table 1 reveals that in information scenarios, where at least one of the firms holds dataset $X$, both firms adopt only the pay-to-switch strategy, provided consumers are relatively homogeneous. In this case, firms try to attract the rival’s loyal customers by offering lower prices than the prices they charge to the own customer base.

When the overall differences in consumer flexibility are large, firms may also adopt the pay-to-stay strategy. This is the case in the information scenarios $\{X, X\}$ and $\{XT, X\}$. In the former case both firms resort to the pay-to-stay strategy. The less loyal consumers of firm $A$ ($B$) with addresses $1/4 < x < 1/2$ ($1/2 < x < 3/4$) can be offered a price which is lower than a price charged to some consumer on the rival’s turf.\(^{17}\) In the information scenario $\{XT, X\}$ firm $B$ follows the pay-to-switch strategy, while firm $A$ uses the pay-to-stay strategy. If $t < \bar{t}/2$, a consumer from firm $A$’s turf may be charged a price by firm $A$, which is lower than a price charged to some consumer from firm $B$’s turf.

\(^{17}\)Precisely, a consumer from firm $A$’s turf with an address $x_A > 1/4$ is charged by firm $A$ a lower price than a consumer with brand preference $x_B$ from firm $B$’s turf if $2x_A > 3/2 - x_B$. The case of firm $B$ is symmetric.
Scenario \(\{X, X\}\) is useful to illustrate how differences in overall consumer flexibility alter pricing strategies. When consumers are relatively homogeneous in flexibility, both firms resort to the pay-to-switch strategy. For any price of the competitor a firm aims to serve all the consumers on its own turf. This “protective” pricing strategy of a firm on its own turf induces the rival to price very aggressively for these consumers and reduce the price to zero. When in turn consumers are relatively differentiated, both firms follow the pay-to-stay strategy. On its own turf, a firm wishes to serve all consumers with a given brand preference only if the rival’s price is sufficiently high. Then both firms can charge higher prices to some customers on the competitor’s turf compared to some of their own loyal customers, because they can attract the former even with a relatively high price.

In Shafer and Zhang (2000) a price-differentiating firm follows the pay-to-stay strategy only if firms’ customer bases are sufficiently different in the maximal customer loyalties. We in turn show that a pay-to-stay strategy can be profitable also when customer bases are symmetric with respect to their maximal customer loyalties. The necessary condition is that consumers are relatively differentiated in flexibility, as stated.

**Best-response symmetry and best-response asymmetry.** Next we analyze the profitability of price discrimination based on different types of customer data. We resort to the concepts of best-response symmetry and best-response asymmetry introduced by Corts (1998).\(^{18}\) According to Corts, the necessary condition for price discrimination to have an unambiguous effect on the equilibrium profits is best-response asymmetry. On the other hand, best-response symmetry is a sufficient condition for price discrimination to have an ambiguous effect on firms’ profits. Formally, if for any price of the rival a firm charges a higher price to one consumer group than to the other, the same relation should hold for the other firm if best-response symmetry applies. In all other cases best-response asymmetry holds.

Since the concepts of best-response symmetry and asymmetry require the comparison of the best-response functions, they can only be applied to symmetric information scenarios. In the

\(^{18}\)Those concepts turn out to be very useful for understanding the effects of price discrimination based on different types of customer data and predicting the incentives to share customer data. Nevertheless, some remarks are necessary. First, in his analysis Corts (1998) considers the case of two firms and two groups of consumers. This is different in our analysis, where the availability of either dataset \(T\) or dataset \(X\) allows to discriminate among infinitely many groups of consumers. However, Corts notes that the restriction to two groups and two firms is not essential for his results. Second, Corts analyzes the effects of price discrimination compared to the case of uniform pricing. This is different in our analysis where both with and without information sharing there is price discrimination (by at least one of the firms).
information scenarios \{X, X\} and \{XT, XT\}, where both firms have access to dataset \(X\), best-response asymmetry holds. Consider the information scenario \{X, X\} for the case of relatively homogeneous consumers. The best-response functions for \(x \leq 1/2\) are

\[
p^X_{A|X}(p_B, x|x, x) \leq 1/2) = \begin{cases} 
[p_B + \bar{t}(1 - 2x)]/2, & p_B < \bar{t}(1 - 2x) \\
p_B, & p_B \geq \bar{t}(1 - 2x)
\end{cases}
\]

\[
p^X_{B|X}(p_A, x|x, x) \leq 1/2) = \begin{cases} 
[p_A - t(1 - 2x)]/2, & p_A < 2\bar{t}(1 - 2x) \\
p_A - \bar{t}(1 - 2x), & p_A \geq 2\bar{t}(1 - 2x).
\end{cases}
\]

Assume that for some price \(p\) and some brand preferences \(x_1, x_2 \leq 1/2\) it holds \(p^X_{A|X}(p, x|x_1) > 1/2\) > \(p^X_{A|X}(p, x|x_2 \leq 1/2)\), then it must be that \(x_1 < x_2 \leq 1/2\) and \(p < \bar{t}(1 - 2x_1)\). The latter implies that \(p^X_{B|X}(p, x|x_1 \leq 1/2) < p^X_{B|X}(p, x|x_2 \leq 1/2)\), such that best-response asymmetry holds. For a given price of the rival, every firm charges a lower (higher) price to a group of consumers who prefer the rival’s (own) product more. As a result, both firms approach the two consumer groups with different brand preferences asymmetrically.

Consider now the information scenario \{XT, XT\}, where the best-response functions are

\[
p^{XT}_{A|XT}(p_B, t, x) = \begin{cases} 
p_B + t(1 - 2x), & x \leq 1/2 \\
\max\{0, p_B + t(1 - 2x) - \epsilon\}, \epsilon > 0 & x > 1/2
\end{cases}
\]

\[
p^{XT}_{B|XT}(p_A, t, x) = \begin{cases} 
\max\{0, p_A - t(1 - 2x) - \epsilon\}, \epsilon > 0 & x \leq 1/2 \\
p_A - t(1 - 2x), & x > 1/2.
\end{cases}
\]

When firms have full data on consumer preferences, they target every consumer individually. If for some price \(p\) and some consumers \((t_1, x_1)\) and \((t_2, x_2)\) we have that \(p^{XT}_{A|XT}(p, t_1, x_1) > p^{XT}_{A|XT}(p, t_2, x_2)\), then \(t_1(1-2x_1) > t_2(1-2x_2)\) must hold. It then follows that \(p^{XT}_{B|XT}(p, t_1, x_1) < p^{XT}_{B|XT}(p, t_2, x_2)\), provided that \(p > t_2(1-2x_2)\), such that best-response asymmetry again applies. For a given price of the rival, every firm prices more aggressively a consumer who prefers the rival’s product more.

Due to best-response asymmetry joint profits are among the lowest in the information scenarios \{X, X\} and \{XT, XT\}.\textsuperscript{10} Aggressive pricing of the competitor on a firm’s turf forces the

\textsuperscript{10}Similarly, Liu and Serfes (2004) show that for any level of precision of the information on consumer brand preferences, firms are worse-off when they can discriminate. The same result is also derived in Thisse and Vives (1988).
firm to set low prices to its customer base, too. When consumers are relatively differentiated in flexibility, the ranking of the three lowest joint profits is $\Pi_{A+B}^{X|X} < \Pi_{A+B}^{X|T} = \Pi_{A+B}^{XT|X}$. If consumers are relatively homogenous, then the ranking is $\Pi_{A+B}^{X|X} < \Pi_{A+B}^{X|T} < \Pi_{A+B}^{XT|XT}$.

The information scenario $\{T, T\}$ is characterized by the best-response symmetry. Indeed, best-response functions take the form

$$p_{i|T}^{T|T}(p_j, t) = \begin{cases} (p_j + t)/2, & p_j < 3t \\ p_j - t, & p_j \geq 3t. \end{cases}$$

Assume that for some price $p$ and two consumer groups with flexibilities $t_1$ and $t_2$, we have $p_A^{T|T}(p, t_1) > p_A^{T|T}(p, t_2)$. This implies that $t_1 > t_2$ if $p < 3t$ and $t_1 < t_2$ if $p \geq 3t$. In both cases we have $p_B^{T|T}(p, t_1) > p_B^{T|T}(p, t_2)$, such that best-response symmetry holds. For a given price of the competitor, any firm sets a lower (higher) price to the group of consumers with a weaker (stronger) brand preference. Hence, both firms are symmetric in their approach to consumer groups with different flexibilities. The ability of firms to discriminate based on the dataset $T$ can enhance profits. The ranking of the two highest joint profits is $\Pi_{A+B}^{T|T} < \Pi_{A+B}^{T|\varnothing}$ both in the case of relatively homogeneous and relatively differentiated consumers.

The concepts of best-response symmetry and asymmetry explain the profit effects of price discrimination based on consumer brand preferences and flexibility. They also allow to predict incentives to share customer data. Based on the above analysis we can conjecture that the sharing of dataset $X$ is likely to be unprofitable, whereas the sharing of dataset $T$ can be profitable. In the next section we show that the unambiguous profit effect of sharing dataset $X$ is negative such that it is never shared. The profit effect of sharing dataset $T$, on the other hand, is ambiguous. This ambiguity is resolved in a way that it is more likely to be positive when consumers are relatively homogeneous in flexibility.

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20 Distinguishing between the strategies pay-to-switch and pay-to-stay allows to understand firms’ equilibrium prices when they are able to discriminate based on consumer brand preferences. However, it does not provide any guidance for predicting firms’ incentives to share customer data. For instance, in the scenario $\{X, T, \varnothing\}$ firm $A$ uses the pay-to-switch strategy irrespectively of consumer heterogeneity. One could then conjecture that firm $A$’s incentives to share dataset $T$ in that scenario should not depend on consumer heterogeneity. However, as we show in the next section, they do. The reason is that distinguishing between the strategies pay-to-switch and pay-to-stay does not say anything about the general level of prices in equilibrium. In equilibrium firm $A$ anchors its prices on that of firm $B$, which is $0.28t$ when consumers are relatively differentiated and $H(t, T)/2$ when consumers are relatively homogeneous. It is the difference in those prices, driven by the overall consumer flexibility, that is responsible for the difference in firm $A$’s incentives to share dataset $T$ in the scenario $\{X, T, \varnothing\}$.
5 Customer Data Sharing and Its Welfare Effects

We now analyze the incentives of a firm with a larger information set to share its customer data with the competitor. The dataset(s) with information on brand preferences ($X$) and/or flexibility ($T$) may be sold to the rival. The following proposition summarizes our results.

**Proposition 2.** The incentives to share customer data depend on consumer heterogeneity in flexibility:

i) When consumers are relatively differentiated, a firm with full information on consumer preferences shares its data on customer flexibility with the competitor, provided the latter holds data on customer brand preferences.

ii) When consumers are relatively homogeneous, a firm with full information on consumer preferences shares its data on customer flexibility with the competitor if the latter either has no customer data or holds data on customer brand preferences.

**Proof.** See Appendix.

| I_A | I_B | $\Pi^I_A|I_B$ | $\Pi^I_B|I_B$ | $\Pi_A+\Pi_B$ | $I_A$ | $I_B$ | $\Pi^I_A|I_B$ | $\Pi^I_B|I_B$ | $\Pi_A+\Pi_B$ | Share? |
|-----|-----|-------------|-------------|--------------|-------|-------|-------------|-------------|--------------|-------|
| X   | φ   | .32         | .12         | .44          | X     | X     | .14         | .14         | .28          | No    |
| T   | φ   | .53         | .23         | .76          | T     | T     | .25         | .25         | .50          | No    |
| XT  | φ   | .32         | .05         | .37          | XT    | X     | .16         | .06         | .22          | No    |
| XT  | φ   | .32         | .05         | .37          | XT    | T     | .28         | .06         | .34          | No    |
| XT  | X   | .16         | .06         | .22          | XT    | XT    | .13         | .13         | .26          | Yes   |
| XT  | T   | .28         | .06         | .34          | XT    | XT    | .13         | .13         | .26          | No    |

Table 3 shows how information sharing alters joint profits using the examples with $\bar{t} = 1$ for relatively differentiated and $\bar{t} = 1$ and $\bar{t} = 2$ for relatively homogeneous consumers.

As known from the literature on competitive price discrimination, two main effects determine incentives to share customer data. First, customer data sharing improves the ability of firm $B$...
to extract rents from consumers. We refer to this as the rent-extraction effect. Data sharing may also lead to a more symmetric distribution of consumers between the firms such that more consumers buy from their most preferred firm (firm $B$). If the latter happens, then the overall consumer transportation costs decrease, thus allowing firm $B$ to extract even more rents from consumers. Second, data sharing influences also competition between the firms, to which we refer as the competition effect. Having additional data, firm $B$ may change its equilibrium price(s) charged to consumers served by firm $A$. Due to strategic complementarity between firms’ prices, this has a negative (positive) effect on firm $A$’s prices if the prices of firm $B$ tend to decrease (increase).

As we conjectured in the previous section, dataset $X$ is never shared. According to Corts (1998), best-response asymmetry is a necessary condition for price discrimination to have an unambiguous effect on equilibrium profits. We show that when data on consumer brand preferences are shared, the profit effect is unambiguously negative. In that case the negative competition effect always outweighs the positive rent-extraction effect, independently of consumer heterogeneity in flexibility.

Dataset $X$ can be potentially shared in the information scenarios $\{X, \emptyset\}$, $\{XT, \emptyset\}$ and $\{XT, T\}$. As an example, we consider the effect of dataset $X$ sharing in the scenario $\{X, \emptyset\}$ for the case of relatively differentiated consumers. Following the (possible) sharing of dataset $X$, firm $B$ targets consumers on firm $A$’s turf more aggressively as it can now set different prices for its loyal consumers and that of the rival. Aggressive pricing by firm $B$ in turn puts pressure on firm $A$ to set lower prices on its own turf, too. The intensified competition results in lower joint profits. Table 1 shows that before information sharing, in scenario $\{X, \emptyset\}$, firm $B$ sets a uniform price of $0.47\bar{t}$ to all consumers. Following the sharing of dataset $X$, firm $B$ sets a lower price of $\bar{t}(1 - 2x)/3$ for a consumer on firm $A$’s turf with an address $x$. This reduces firm $A$’s equilibrium price on its own turf from $\bar{t}(0.73 - x)$ to $2\bar{t}(1 - 2x)/3$ for consumers with addresses $0 \leq x < 0.27$ and from $0.47\bar{t}$ to $2\bar{t}(1 - 2x)/3$ for consumers with addresses $0.27 \leq x \leq 0.5$. A similar logic applies in the case of relatively homogeneous consumers and in the other information scenarios where dataset $X$ can be shared.

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21 The result that dataset $X$ is not shared in the scenario $\{X, \emptyset\}$ can be also derived from the analysis of Liu and Serfes (2004). They show that when the quality of information is perfect, the profit of the price-discriminating firm approaches $9t/16$ (where $t$ is the transportation cost parameter) when its rival has no customer data. If both firms have customer data, each firm’s profit approaches $t/4$, such that a firm with data on customer brand preferences would not share it with the rival.
Based on the analysis of the profitability of price discrimination using dataset \( T \), our second conjecture was that its sharing can be profitable. This also turns out to be true. Proposition 2 shows that the incentives to share dataset \( T \) depend on overall consumer flexibility such that sharing is more likely when consumers are relatively homogeneous in flexibility.\(^\text{22}\) According to Corts (1988), best-response symmetry is a sufficient condition for price discrimination to lead to higher prices for one group of consumers and lower prices for the other group of consumers. As a result, the effect of price discrimination on profits is ambiguous. Our analysis resolves this ambiguity by showing that the profit effect of sharing dataset \( T \) is likely to be positive (negative) if consumers are relatively homogeneous (differentiated) in flexibility.

We now explain how the interplay between the rent-extraction and competition effects shapes incentives to share dataset \( T \). We focus on the information scenarios, where data sharing takes place: \( \{XT, X\} \) and \( \{XT, \varnothing\} \). In the scenario \( \{XT, X\} \) firm \( A \) can undercut any price of firm \( B \) on its turf such that firm \( B \) offers the price of zero. The sharing of dataset \( T \) does not change the equilibrium price of firm \( B \) on firm \( A \)'s turf such that the competition effect is absent in this case irrespectively of consumer heterogeneity in flexibility. When consumers are relatively differentiated, firm \( A \) loses consumers on firm \( B \)'s turf and its profits decrease following data sharing. However, the rent-extraction effect is strong enough such that joint profits increase. When consumers are relatively homogeneous, firm \( A \)'s profits do not change such that dataset \( T \) can be shared without transfers, too. We conclude that data sharing incentives in the scenario \( \{XT, X\} \) are stronger when consumers are relatively homogeneous.

In the information scenario \( \{XT, \varnothing\} \) firm \( A \) shares dataset \( T \) with the rival only when consumers are relatively homogeneous in flexibility. In that case the uniform price of firm \( B \) is relatively low as it aims to serve even the most flexible consumers if they have an address very close to firm \( B \)'s location (see Figure 1). Following data sharing firm \( B \) is able to discriminate among consumers based on flexibility and its price increases to more than a half of all consumers (all those with \( t > H(t, \bar{t}) \)). This in turn allows firm \( A \) to charge a higher price to those consumers too. As a result, firm \( A \)'s profits increase following data sharing due to a positive competition effect. The profits of firm \( B \) increase due to a positive rent-extraction effect.

The effect of data sharing is different when consumers are relatively differentiated in flexi-

\( ^{22} \text{It also follows from Proposition 2 that a firm shares dataset } T \text{ with the rival only if the former also holds dataset } X. \text{ In the next section we show, however, that this result is not robust with respect to the timing of pricing decisions. Dataset } T \text{ is also shared in the scenario } \{T, \varnothing\}, \text{ if in the scenario } \{T, T\} \text{ firms also move sequentially, provided that consumers are relatively homogeneous in flexibility.} \)
bility. In that case the uniform price of firm $B$ is relatively high. That price is tailored to serve only the least flexible consumers located close to firm $B$ (see Figure 1). Serving the most flexible consumers would require the price of zero. After data sharing firm $B$ reduces its price to more than a half of all consumers (those with $t < 0.56\bar{t}$). This forces firm $A$ to reduce its prices to those consumers too, which constitutes a negative competition effect. This effect turns out to be stronger than the positive rent-extraction effect: The decrease in firm $A$’s profit is larger than the increase in the rival’s profit. As a result, data sharing does not take place.

We can now summarize our main results on the sharing of different types of customer data. First, dataset $X$ is never shared. The strong competition effect unambiguously leads to a decrease in joint profits. Second, the incentives to share dataset $T$ depend on consumer heterogeneity in flexibility. After dataset $T$ is shared, prices increase to some consumers and decrease to the others with an ambiguous effect on profits. This ambiguity is resolved in a way that the competition effect is likely to be positive when consumers are relatively homogeneous in flexibility.

**Figure 1: Demand Regions with Relatively Differentiated and Homogeneous Consumers**

![Demand Regions with Relatively Differentiated and Homogeneous Consumers](image)

- **Relatively Differentiated Consumers**
  - Before Sharing: $I_A = X \cup T$, $I_B = \emptyset$
  - After Sharing: $I_A = X \cup T$, $I_B = T$

- **Relatively Homogeneous Consumers**
  - Before Sharing: $I_A = X \cup T$, $I_B = \emptyset$
  - After Sharing: $I_A = X \cup T$, $I_B = T$

**Welfare effects of customer data sharing.** The welfare implications of customer data sharing are summarized in Proposition 3.

**Proposition 3.** Welfare implications of customer data sharing depend on consumer heterogeneity in flexibility:

i) With relatively differentiated consumers, information sharing is neutral to consumer surplus and enhances social welfare.
ii) With relatively homogeneous consumers, information sharing always decreases consumer surplus and social welfare either decreases or remains unchanged.

**Proof.** See Appendix.

Proposition 3 highlights the importance of consumer heterogeneity in flexibility for predicting the welfare effects of customer data sharing. When consumers are relatively differentiated, information sharing is Pareto-optimal. It increases joint profits and leaves consumer surplus unchanged. However, customer data sharing is likely to be detrimental to both consumers and social welfare if consumers are relatively homogeneous in their flexibility.

In our setup social welfare can be suboptimal only due to the misallocation of consumers, which occurs if consumers do not visit their most preferred firms. When dataset $T$ is shared in the scenario $\{XT, X\}$ it can be either beneficial or neutral to social welfare, because in the resulting scenario $\{XT, XT\}$ every consumer visits her most preferred firm. Thus, social welfare is maximized. Data sharing increases social welfare, if the initial distribution of consumers between the firms is asymmetric. This happens when consumers are relatively differentiated, in which case firm $B$ tailors its price to target only the least flexible consumers on its turf such that the consumers with low transportation cost parameters are served by firm $A$. Following customer data sharing the least flexible consumers on firm $B$’s turf lose, because firm $B$ uses its new dataset $T$ to extract more rents from them. However, the most flexible consumers on firm $B$’s turf gain because they are served by their most preferred firm. These two effects cancel each other out rendering information sharing neutral to consumer surplus. In case of relatively homogeneous consumers their initial distribution is symmetric and data sharing does not change social welfare. In that case information sharing (implying higher joint profits) harms consumers.

With relatively homogeneous consumers, customer data sharing also takes place in the scenario $\{XT, \emptyset\}$, in which case social welfare decreases. After data sharing firm $B$ increases its price to consumers with strong brand preferences such that some of those consumers who prefer firm $B$ switch to firm $A$ and their transportation costs increase. At the same time firm $B$ reduces its price to consumers with weak brand preferences, and some of those consumers who prefer firm $B$ switch to their most preferred firm and their transportation costs decrease. The former

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23 The result that dataset $T$ sharing is Pareto-optimal in the scenario $\{XT, X\}$ when consumers are relatively differentiated, depends on the assumption that $\ell = 0$. When $\ell > 0$, $\ell \to \infty$ and $\lim_{\ell \to \infty} k(\ell, \ell) = \infty$, consumer surplus decreases when dataset $T$ is shared. However, social welfare again increases as stated in Proposition 3. The proof is available from the authors upon request.
effect turns out to be stronger as consumers in the former group have stronger brand preferences, such that the overall transportation costs increase and social welfare decreases. Since industry profits increase following customer data sharing, consumer surplus gets smaller.

Summarizing the welfare effects of customer data sharing, we conclude that it is most likely to harm consumers, while the effect on social welfare can be positive.

6 Robustness Check

Timing. We now check the robustness of our results with respect to the timing of the pricing decisions. In those cases where following data sharing an asymmetric information scenario is replaced by a symmetric one, firms’ profits change not only due to a change in the available customer data, but also due to a change in the timing of firms’ moves. To eliminate the latter effect, we modify the timing of moves in Stage 2 of the game and invoke Assumption 1.

Assumption 1. In Stage 2 of the game firm B chooses all its prices first and firm A follows in all the information scenarios (symmetric and asymmetric).

The following proposition characterizes the subgame-perfect Nash equilibria in the symmetric information scenarios \{X, X\}, \{T, T\} and \{XT, XT\} under Assumption 1.24

Proposition 4. Equilibrium prices and profits in the symmetric information scenarios under Assumption 1 are as stated in Tables 4 and 5, respectively.

Proof. See Appendix.

Comparing the equilibrium prices and profits in symmetric information scenarios with simultaneous and sequential moves we conclude that both the equilibrium prices and profits are (weakly) higher under Assumption 1. When firms move sequentially, firm B takes into account the positive strategic effect of its higher prices on firm A’s prices. This effect (weakly) increases both firms’ equilibrium prices and profits compared to the case when firms move simultaneously. However, in the scenario \{XT, XT\} the subgame-perfect Nash equilibrium and Nash equilibrium fully coincide. Hence, under Assumption 1 the incentives to share customer data may only change in the scenarios \{X, \emptyset\} and \{T, \emptyset\}. The following proposition relates to the customer data sharing incentives and associated welfare implications under Assumption 1.

24To derive the incentives for data sharing we compare joint profits before and after sharing. In this case, the choice of a firm making the first move in the symmetric information scenarios is irrelevant for our results.
Table 4: Equilibrium Prices in Symmetric Information Scenarios with Sequential Moves

<table>
<thead>
<tr>
<th>$I_A$</th>
<th>$I_B$</th>
<th>$p_A$</th>
<th>$p_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relatively Differentiated Consumers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X$</td>
<td>$X$</td>
<td>$3\bar{t}(1 - 2x)/4, x \leq 1/2$</td>
<td>$\bar{t}(1 - 2x)/2, x \leq 1/2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{t}(2x - 1)/2, x &gt; 1/2$</td>
<td>$\bar{t}(2x - 1), x &gt; 1/2$</td>
</tr>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$5\bar{t}/4$</td>
<td>$3\bar{t}/2$</td>
</tr>
<tr>
<td>$XT$</td>
<td>$XT$</td>
<td>$t(1 - 2x), x \leq 1/2$</td>
<td>$0, x \leq 1/2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0, x &gt; 1/2$</td>
<td>$t(2x - 1), x &gt; 1/2$</td>
</tr>
</tbody>
</table>

Relatively Homogeneous Consumers

| $X$ | $X$ | $t(1 - 2x), x \leq 1/2$ | $0, x \leq 1/2$ |
| | | $0, x > 1/2$ | $t(2x - 1), x > 1/2$ |
| | if $k \leq 3/2$ | | |
| | | $t(1 - 2x), x \leq 1/2$ | $0, x \leq 1/2$ |
| | | $0, x > 1/2$ | $t(2x - 1), x > 1/2$ |
| | if $k > 3/2$ | | |
| $T$ | $T$ | $(2\bar{t} - 3\bar{t})(2x - 1)/4, x > 1/2$ | $(2\bar{t} - \bar{t})(2x - 1)/2, x > 1/2$ |
| | | $5\bar{t}/4$ | $3\bar{t}/2$ |
| $XT$ | $XT$ | $t(1 - 2x), x \leq 1/2$ | $0, x \leq 1/2$ |
| | | $0, x > 1/2$ | $t(2x - 1), x > 1/2$ |

Proposition 5. Under Assumption 1 dataset $T$ is shared in the scenario $\{T, \emptyset\}$ if consumers are relatively homogeneous in flexibility. In that case information sharing reduces consumer surplus, while social welfare increases. In all the other information scenarios the incentives to share customer data and its welfare implications are as stated in Proposition 2.

Proof. See Appendix.

Proposition 5 shows that customer data are shared in an additional information scenario, which is $\{T, \emptyset\}$, due to the change in the timing of firms’ moves in symmetric information scenarios. With sequential moves, in symmetric information scenarios $\{X, X\}$ and $\{T, T\}$ firms realize (weakly) higher profits than with simultaneous moves, which should strengthen the incentives to share customer data. Nevertheless, in the scenario $\{X, \emptyset\}$ dataset $X$ is not shared, just as under the original timing. The negative competition effect still outweighs the positive rent-extraction effect. However, dataset $T$ is shared now also in the scenario $\{T, \emptyset\}$, provided that consumers are relatively homogeneous in flexibility.\textsuperscript{25} Again, stronger incentives to share dataset

\textsuperscript{25}In the example considered by Armstrong et al. (2006) where two consumer groups have different transportation cost parameters, customer data on consumer transportation costs are shared. In that example the uniform price of the firm with initially no data is “low,” such that data sharing is profitable. Our analysis shows that this uniform price depends on consumer heterogeneity in flexibility. It is “low” when consumers are relatively homogeneous and “high” when consumers are relatively differentiated. In the latter case data on consumer transportation cost parameters are not shared.
T with relatively homogeneous consumers are driven by the pricing strategy of firm B. In that case firm B tailors its uniform price to serve all consumers with the locations close to its address regardless of the flexibility. Consequently, the equilibrium price of firm B is low. When consumers are relatively differentiated, the uniform price of firm B is high and it only serves the least flexible consumers with addresses close to its own location. As the equilibrium prices of firm A increase with the uniform price of firm B, profits tend to be higher when consumers are relatively differentiated. As a result, the incentives to share customer data are higher with relatively homogeneous consumers.

Table 5: Equilibrium Profits in Symmetric Information Scenarios with Sequential Moves

<table>
<thead>
<tr>
<th>IA</th>
<th>IB</th>
<th>Π_A(IA,B)</th>
<th>Π_B(IA,B)</th>
<th>Π_A(IB,B)</th>
<th>Π_B(IB,B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relatively Differentiated Consumers</td>
<td>Relatively Homogeneous Consumers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>X</td>
<td>13t/64</td>
<td>10t/64</td>
<td>k ≤ 3/2</td>
<td>t/4</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>25t/64</td>
<td>9t/32</td>
<td>k &gt; 3/2</td>
<td>(\frac{t(4k^2+4k-7)}{64(k-1)})</td>
</tr>
<tr>
<td>XT</td>
<td>XT</td>
<td>t/8</td>
<td>t/8</td>
<td>(A(\bar{t}, \bar{t})/4)</td>
<td>(A(\bar{t}, \bar{t})/4)</td>
</tr>
</tbody>
</table>

The sharing of dataset T in the scenario \(\{T, \varnothing\}\) is beneficial to social welfare. Following data sharing, firm B increases its price to less flexible consumers (those with \(t > H(\bar{t}, \bar{t})\)) and charges a lower price to more flexible consumers (with \(t < H(\bar{t}, \bar{t})\)). Some of the former consumers switch from their most preferred firm (firm B), leading to an increase in the overall transportation costs. Some of the latter consumers switch to their most preferred firm (firm B), thereby decreasing the overall transportation costs. The former effect turns out to be weaker. The reason is that although the masses of the two consumer groups are same, in the latter group there are relatively more consumers with a large change in the distance they have to travel. Also, the largest change in the distance a consumer faces in each group is larger in the latter group (\(3H(\bar{t}, \bar{t})/(4t) - 1/2\) compared to \(1/4\)). Although overall consumer transportation costs decrease, higher payments to the firms make consumers worse-off.

Proposition 5 shows that our main results remain unchanged when the effect of timing of pricing decisions is taken into account. First, dataset X is never shared. Second, the incentives to share dataset T are stronger when consumers are relatively homogeneous in flexibility. Third, customer data sharing tends to be detrimental to consumer surplus while the effect on social
welfare can be positive.

**Customer Data.** We now check the robustness of our results with respect to the assumption that datasets contain information on preferences of all consumers in the market. Here, in contrast, we assume that a dataset comprises information on only a share of all consumers. Precisely, we assume that firm A may hold dataset $X_\alpha (T_\alpha)$ with data on brand preferences (transportation cost parameters) of consumers with addresses $x \leq \alpha$, where $0 \leq \alpha \leq 1$. Firm $B$ then may have information on consumers with addresses $x > \alpha$, such that firms never have data on the same consumers. Dataset $X (T)$ in our main analysis coincides with $X_1 (T_1)$.

We analyze the incentives of firm $A$ to share dataset $X_\alpha$ and/or $T_\alpha$ with the rival for different values of $\alpha > 0$ (the analysis for firm $B$ is symmetric). It is straightforward that dataset $X_\alpha$ will not be shared for any $\alpha$. We showed in our main analysis that dataset $X$ is not shared due to the negative competition effect, which always outweighs the positive rent-extraction effect. When $\alpha < 1$, then there is less scope for a rent-extraction effect as dataset $X_\alpha$ contains data on brand preferences of consumers which are less loyal to firm $B$ compared to the case $\alpha = 1$, such that the competition effect will again dominate. Dataset $T_\alpha$ can be shared in the information scenarios where firm $A$ holds only dataset $T_\alpha$ or both datasets ($X_\alpha$ and $T_\alpha$). We denote those scenarios as $\{T_\alpha, \emptyset_\alpha\}$ and $\{X_\alpha T_\alpha, \emptyset_\alpha\}$, respectively, such that we only specify which data firms have on consumers with addresses $x \leq \alpha$. Following data sharing we have scenarios $\{T_\alpha, T_\alpha\}$ and $\{X_\alpha T_\alpha, T_\alpha\}$, respectively. We analyze firm $A$’s incentives under Assumption 1 as under that assumption, as we showed, there is more scope for data sharing. The following proposition states firms’ equilibrium prices and profits in the region $x \leq \alpha$ in the relevant information scenarios.

**Proposition 6.** *Equilibrium prices and profits in the region $x \leq \alpha$ in the information scenarios $\{T_\alpha, \emptyset_\alpha\}$, $\{X_\alpha T_\alpha, \emptyset_\alpha\}$, $\{T_\alpha, T_\alpha\}$ and $\{X_\alpha T_\alpha, T_\alpha\}$ under Assumption 1 are as stated in Tables 6 and 7, respectively.*

**Proof.** See Appendix.

Table 6 shows that in all the information scenarios the equilibrium price of firm $B$ decreases with $\alpha$. Firm $B$ is forced to charge a lower price, because with a decrease in $\alpha$ the average transportation cost of buying from $B$ among consumers with addresses $x \leq \alpha$ gets larger. When $\alpha$ becomes very small, firm $B$ cannot do better than charging the price of zero. This critical value

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26 Implicit here is the assumption that each firm may have information only on its most loyal consumers. This information can be gained through analyzing data on consumer purchases.
In the information scenario Proposition 7.

...rm A to its turf for any price of the rival. In the following proposition we summarize the incentives of

monopolization strategy requires only a small reduction from the price targeted at consumers

of $\alpha$ is $1/4$ when firm A holds only dataset $T_\alpha$. In that case for any price of the rival and any
given $t$ firm A wants to serve all consumers with brand preferences $x \leq \alpha$. When $\alpha$ is small, this
monopolization strategy requires only a small reduction from the price targeted at consumers
with addresses close to $x = 0$. When firm A holds both datasets, $T_\alpha$ and $X_\alpha$, the critical value
of $\alpha$ is $1/2$. Having full data on consumer preferences, firm A serves all the consumers belonging
to its turf for any price of the rival. In the following proposition we summarize the incentives of
firm A to share dataset $T_\alpha$ with the rival for different values of $\alpha$.

**Proposition 7.** In the information scenario $\{T_\alpha, \varnothing_\alpha\}$ ($\{X_\alpha T_\alpha, \varnothing_\alpha\}$) firm A shares dataset

$T_\alpha$ provided $\alpha > 1/4$ ($\alpha > 1/2$). Data sharing takes place only if consumers are relatively
homogeneous in flexibility. The welfare effects of data sharing in the region $x \leq \alpha$ are as follows:

i) When $T_\alpha$ is shared in $\{T_\alpha, \varnothing_\alpha\}$, social welfare increases. There exists $\alpha(k) > 1/4$ such that
consumer surplus increases (decreases) if $\alpha < \alpha(k)$ ($\alpha > \alpha(k)$) and does not change if $\alpha = \alpha(k)$.

ii) When $T_\alpha$ is shared in $\{X_\alpha T_\alpha, \varnothing_\alpha\}$, both consumer surplus and social welfare decrease.

**Proof.** See Appendix.

Proposition 7 shows that dataset $T_\alpha$ is shared only when consumers are relatively homogeneous.
In other words, the results that dataset $T$ is shared in the information scenarios $\{XT, \varnothing\}$ and
$\{T, \varnothing\}$ only with relatively homogeneous consumers (stated in Propositions 2 and 5, respectively)
do not depend on the assumption that datasets $X$ and $T$ contain information on all consumers in the market. The same intuition as above applies here. When consumers are relatively homogeneous, firm $B$ prefers to serve consumers with all transportation cost parameters if they have an address close to $x = \alpha$. This low price compared to the case of relatively differentiated consumers forces firm $A$ to set low prices too. As a result, profits before data sharing tend to be lower with relatively homogeneous consumers, which increases the incentives to share dataset $T_A$ in that case. However, no data sharing takes place when datasets $X_A$ and $T_A$ contain information only on consumers which are very loyal to firm $A$. In that case both with and without dataset $T_A$ firm $B$ cannot do better than charging the price of zero in equilibrium, such that data sharing does not change joint profits.

In Liu and Serfes (2006) a firm shares its customer data with the rival only if it also contains information on the loyal consumers of the latter. We get the same result when firm $A$ has data on both brand preferences and transportation cost parameters of consumers. However, in the scenario $\{T_A, \varnothing_A\}$ firm $A$ shares data only on its own loyal consumers whenever $1/4 < \alpha < 1/2$, provided consumers are relatively homogeneous. This is profitable because the sharing of data on transportation cost parameters in that case does not lead to a more aggressive pricing of the rival to a firm’s loyal consumers, which is always the case in Liu and Serfes.

Concerning the welfare effects of data sharing we get one difference compared to the previous results, which is that consumers benefit from data sharing in the information scenario $\{T_A, \varnothing_A\}$

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### Table 7: Equilibrium Profits on the Region $x < \alpha$

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\alpha$</th>
<th>$\Pi_A$</th>
<th>$\Pi_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relatively Differentiated Consumers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>${T_A, \varnothing_A}$</td>
<td>$&gt;1/4$</td>
<td>$\bar{t} (0.42\alpha^2 + 0.07\alpha + 0.02)$</td>
<td>$\bar{t} (0.41\alpha^2 - 0.2\alpha + 0.03)$</td>
</tr>
<tr>
<td>${X_AT_A, \varnothing_A}$</td>
<td>$&gt;1/2$</td>
<td>$\bar{t} (0.12\alpha^2 + 0.2\alpha - 0.005)$</td>
<td>$0.05\bar{t} (2\alpha - 1)^2$</td>
</tr>
<tr>
<td>Relatively Homogeneous Consumers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>${T_A, \varnothing_A}$</td>
<td>$&gt;1/4$</td>
<td>$(16\alpha^2 + 8\alpha - 3)H(\bar{t}, \bar{t})/32 + A(\bar{t}, \bar{t})/8$</td>
<td>$(4\alpha - 1)^2 H(\bar{t}, \bar{t})/16$</td>
</tr>
<tr>
<td>${X_AT_A, \varnothing_A}$</td>
<td>$&gt;1/2$</td>
<td>$(2\alpha - 1)(2\alpha + 3)H(\bar{t}, \bar{t})/16 + A(\bar{t}, \bar{t})/4$</td>
<td>$(2\alpha - 1)^2 H(\bar{t}, \bar{t})/8$</td>
</tr>
<tr>
<td>Both Versions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>${T_A, T_A}$</td>
<td>$&gt;1/4$</td>
<td>$(4\alpha + 1)^2 A(\bar{t}, \bar{t})/32$</td>
<td>$(4\alpha - 1)^2 A(\bar{t}, \bar{t})/16$</td>
</tr>
<tr>
<td>${X_AT_A, T_A}$</td>
<td>$&gt;1/2$</td>
<td>$(2\alpha + 1)^2 A(\bar{t}, \bar{t})/16$</td>
<td>$(2\alpha - 1)^2 A(\bar{t}, \bar{t})/8$</td>
</tr>
<tr>
<td>${T_A, \varnothing_A}$</td>
<td>$\leq 1/4$</td>
<td>$\alpha (1 - 2\alpha) A(\bar{t}, \bar{t})$</td>
<td>$0$</td>
</tr>
<tr>
<td>${T_A, T_A}$</td>
<td>$\leq 1/4$</td>
<td>$\alpha (1 - 2\alpha) A(\bar{t}, \bar{t})$</td>
<td>$0$</td>
</tr>
<tr>
<td>${X_AT_A, \varnothing_A}$</td>
<td>$\leq 1/2$</td>
<td>$\alpha (1 - \alpha) A(\bar{t}, \bar{t})$</td>
<td>$0$</td>
</tr>
<tr>
<td>${X_AT_A, T_A}$</td>
<td>$\leq 1/2$</td>
<td>$\alpha (1 - \alpha) A(\bar{t}, \bar{t})$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

$A(\bar{t}, \bar{t}) = (\bar{t} - \bar{t})/2$, $H(\bar{t}, \bar{t}) = (\bar{t} - \bar{t})/\ln(\bar{t}/\bar{t})$
when $\alpha$ is small enough. In that case the overall decrease in transportation costs responsible for the increase in social welfare compensates higher payments to the firms.

We conclude that our main results remain unchanged when datasets contain information only on a share of all consumers in the market, provided that the share is large enough. First, dataset $X$ is never shared. Second, the incentives to share dataset $T$ are stronger when consumers are relatively homogeneous in flexibility. Third, customer data sharing tends to be detrimental to consumer surplus, while the effect on social welfare can be positive.

7 Conclusions

It is increasingly observable that competitors in different industries exchange profiles of their customers with each other. These activities have raised the suspicion of both consumer advocates as well as regulatory authorities. In this article, we present a modified Hotelling model, where consumers are heterogeneous with respect to both their brand preferences and transportation cost parameters. We allow firms to hold data on consumer preferences, which can be used for first- and third-degree price discrimination. We analyze firms’ incentives to share the available customer data with the rival.

The novelty of our approach to model customer data sharing is that we distinguish between two datasets, which encompass brand preferences and transportation cost parameters of consumers. We find that only information on transportation cost parameters is shared. The incentives to do so are stronger when consumers are relatively homogeneous in flexibility.

Our results highlight that the evaluation of agreements involving customer data sharing between rivals depends on the welfare standard adopted by a competition authority. Competition authorities aiming at maximizing consumer surplus should regard customer data sharing agreements critically. Even without taking into account other potentially problematic issues such as privacy and collusion (which can be facilitated through the intensified information flows between the rivals), our results advocate scepticism towards the claims that consumers overall could benefit from such agreements. However, under a social welfare standard customer data sharing may be beneficial.
Appendix

Definitions and Notation. Before we proceed with the proofs, we introduce some definitions and notation. Let \( t^c(p_A,p_B,x) \) denote the transportation cost parameters of the indifferent consumers with brand preference \( x \) for given prices \( p_A \) and \( p_B \): \( t^c(\cdot) := (p_B - p_A)/(2x - 1) \). It holds that \( U_A(p_A,t^c(\cdot),x) = U_B(p_B,t^c(\cdot),x) \). For given \( p_A, p_B \) and \( x \) we have \( \Pr \{ t \geq t^c(\cdot) \} = 0 \) if \( t^c(\cdot) > \bar{t} \), \( \Pr \{ t \geq t^c \} = f_t[\bar{t} - t^c(\cdot)] \) if \( t \leq t^c(\cdot) \leq \bar{t} \) and \( \Pr \{ t \geq t^c \} = 1 \) if \( t^c(\cdot) < t \). As equilibrium strategies may differ on the intervals \( x < 1/2 \) and \( x > 1/2 \), it is useful to distinguish between \( t^c := t^c(\cdot \mid x < 1/2) \) and \( \bar{t}^c := t^c(\cdot \mid x > 1/2) \), respectively.

Similarly, let \( x^c(p_A,p_B,t) \) denote the brand preference of indifferent consumers with transportation cost parameter \( t \) for given prices \( p_A \) and \( p_B \): \( x^c(\cdot) := 1/2 - (p_A - p_B)/2t \). It holds that \( U_A(p_A,t,x^c(\cdot)) = U_B(p_B,t,x^c(\cdot)) \). For given \( p_A, p_B \) and \( t \) we have \( \Pr \{ x \geq x^c(\cdot) \} = 0 \) if \( x^c(\cdot) > 1 \), \( \Pr \{ x \geq x^c(\cdot) \} = 1 - x^c(\cdot) \) if \( 0 \leq x^c(\cdot) \leq 1 \) and \( \Pr \{ x \geq x^c(\cdot) \} = 1 \) if \( x^c(\cdot) < 0 \). Let \( \bar{x}(p_A,p_B,\bar{t}) \) and \( \bar{x}(p_A,p_B,\bar{t}) \) denote the brand preferences of the indifferent consumers for given prices \( p_A \) and \( p_B \) with the lowest and highest transportation cost parameters, respectively.

Formally, \( t^c(p_A,p_B,\bar{x}) = \bar{t} \) and \( t^c(p_A,p_B,x) = \bar{t} \).

We introduce \( A(t,\bar{t}) := (\bar{t} + t)/2 \) and \( H(t,\bar{t}) := (\bar{t} - t)/\ln(\bar{t}/t) \) to denote the arithmetic and the harmonic mean of the transportation cost parameters when \( t > 0 \), respectively. Note that for any \( t, \bar{t} \) it holds that \( A(t,\bar{t}) > H(t,\bar{t}) \). We also introduce \( H(t,\bar{t}) := (\bar{t} - t)/\ln[(2\bar{t} - t)/\bar{t}] \).

For the sake of simplicity we will write \( k \) instead of \( k(t,\bar{t}) = \bar{t}/t \).

We will omit the notation of information scenarios for best-response functions and equilibrium prices, which should be clear from the context.

Proof of Proposition 1. We derive equilibrium prices and profits in different information scenarios. We start with symmetric information scenarios.

Claim 1. Let \( \bar{t} = 0 \). Consider the information scenario \( \{X,X\} \). In equilibrium firm \( i \) charges \( p^*_i(x) = 2\bar{t}|1 - 2x|/3 \) on its own turf and \( p^*_i(x) = \bar{t}|1 - 2x|/3 \) on the competitor’s turf. Firm \( i \) serves consumers with \( t \geq \bar{t}/3 \) on its own turf and consumers with \( t < \bar{t}/3 \) on the competitor’s turf and realizes the profit \( \Pi_i^X = \bar{t}/8 \).

Proof of Claim 1. As firms are symmetric, we only analyze pricing strategies on firm \( A' \) turf, \( x < 1/2 \). A consumer in this region chooses firm \( A \) if \( t \geq t^c \). Both firms treat the consumer transportation cost parameter as a random variable and maximize their expected profits for a given
Claim 2. Let $x: E \left[ \Pi^X_A \mid x < 1/2 \right] = p_A \Pr \{ t \geq t^c (\cdot) \}$ and $E \left[ \Pi^X_B \mid x < 1/2 \right] = p_B \Pr \{ t < t^c (\cdot) \}$. Solving the corresponding maximization problems yields equilibrium prices $p^*_A(x) = 2t(1 - 2x)/3$ and $p^*_B(x) = \bar{t}(1 - 2x)/3$ for $x < 1/2$. Consider now $x = 1/2$. It follows from the first tie-breaking rule that $E \left[ \Pi^X_B \mid x = 1/2 \right] = 0$, whenever $p_B \geq p_A$. Firm $B$ will always undercut firm $A$ if $p^*_A(1/2) > 0$, hence, it must be that $p^*_A(1/2) = p^*_B(1/2) = 0$. The equilibrium prices yield $t^c = \bar{t}/3$ and $\bar{t} = t^c$ (due to symmetry). To compute firm $A$’s equilibrium profits, we sum up the revenues across the demand regions: $\Pi^X_A = \int \int \left[ f_i(1 - 2x)/3 \right] dtdx + \int_{1/2}^{1} t^c \int [f_i(2x - 1)/3] dtdx = 5A(\bar{t}, \bar{t})/18$. Since firms are symmetric, $\Pi^X_A = \Pi^X_B$. This completes the proof of Claim 1.

Claim 2. Let $i > 0$ and $k \leq 2$. Consider the information scenario $\{X, X\}$. In equilibrium firm $i$ charges $p^*_i(x) = \hat{t} \mid 1 - 2x \mid$ on its own turf and $p^*_i(x) = 0$ on the competitor’s turf. Every firm serves all consumers on its own turf and realizes the profit $\Pi^X_i = \hat{t}/4$.

Proof of Claim 2. As firms are symmetric, we only analyze firms’ pricing strategies on firm $A$’ turf. A consumer in this region chooses firm $A$ if $t \geq t^c (\cdot)$. Both firms treat consumer transportation cost as a random variable and maximize their expected profits for a given $x$: $E \left[ \Pi^X_A \mid x < 1/2 \right] = p_A \Pr \{ t \geq t^c (\cdot) \}$ and $E \left[ \Pi^X_B \mid x < 1/2 \right] = p_B \Pr \{ t < t^c (\cdot) \}$. Solving the corresponding maximization problems yields equilibrium prices $p^*_A(x) = \hat{t}(1 - 2x)$ and $p^*_B(x) = 0$ for $x < 1/2$. Consider now $x = 1/2$. It follows from the first tie-breaking rule that $E \left[ \Pi^X_B \mid x = 1/2 \right] = 0$, whenever $p_B \geq p_A$. Firm $B$ will always undercut firm $A$ if $p^*_A(1/2) > 0$, hence, it must hold that $p^*_A(1/2) = p^*_B(1/2) = 0$. On its turf firm $A$ serves all consumers. Equilibrium profits are $\Pi^X_A = \int \int [f_i(1 - 2x)] dtdx = \hat{t}/4 = \Pi^X_B$. This completes the proof of Claim 2.

Claim 3. Consider the information scenario $\{T, T\}$. In equilibrium firm $i$ charges $p^*_i(t) = t$ and serves all consumers on its own turf. Firms realize profits $\Pi^T_i = A(t, \bar{t})/2$.

Proof of Claim 3. Both firms treat consumer brand preference as a random variable and maximize their expected profits for a given $t$: $E \left[ \Pi^T_A \mid t \right] = p_A \Pr \{ x \leq x^c (\cdot) \}$ and $E \left[ \Pi^T_B \mid t \right] = p_B \Pr \{ x > x^c (\cdot) \}$, which yields $p^*_A(t) = p^*_B(t) = t$ and $x^c = 1/2$. Equilibrium profits are $\Pi^T_A = \int \int (f_i(t)) dtdx = A(\bar{t}, \bar{t})/2 = \Pi^T_B$. This completes the proof of Claim 3.

Claim 4. Consider the information scenario $\{XT, XT\}$. In equilibrium firm $i$ charges $p^*_i(t, x) = \hat{t} \mid 1 - 2x \mid$.
\( t|1 - 2x| \) on its own turf and \( p^*_A(t, x) = 0 \) on the competitor’s turf, and serves all consumers on its own turf. Firms realize profits \( \Pi^{XT}_{\xi} = A(t, T)/4 \).

**Proof of Claim 4.** As firms are symmetric, we only consider pricing decisions in the region \( x \in [0, 1/2] \). Under the first tie-breaking rule, an indifferent consumer with \( x \leq 1/2 \) buys from firm \( A \). As firm \( A \) has a cost advantage on its turf, its best response to any \( p_B \) is to render consumers indifferent by setting \( p_A(p_B, t, x) = p_B(t, x) + t(1 - 2x) \geq 0 \). Firm \( B \)’s best response is to undercut firm \( A \)'s price by setting \( p_B(p_A, t, x) = p_A(t, x) - t(1 - 2x) - \varepsilon \) whenever it is feasible (i.e., \( p_A(t, x) - t(1 - 2x) > 0 \), with \( \varepsilon > 0 \). Otherwise, firm \( B \) sets \( p_B = 0 \). Those best responses yield \( p_A^*(t, x) = t(1 - 2x) \) and \( p_B^*(t, x) = 0 \). Firm \( A \)'s profit is

\[
\Pi^{XT}_{A} = \int_0^{1/2} \frac{1}{t} \int f(t(1 - 2x)) dtdx = A(t, T)/4.
\]

Due to symmetry, \( \Pi^{XT}_{A} = \Pi^{XT}_{B} \). This completes the proof of Claim 4.

We now turn to the asymmetric information scenarios.

**Claim 5.** Let \( t = 0 \). Consider the information scenario \( \{X, \emptyset\} \). If \( x < 1/2 - p_B^*/(2T) \), then in equilibrium firm \( A \) charges \( p^*_A(x) = \frac{t(1 - 2x) + p^*_B}{2} \) and serves consumers with \( t/2 - p_B^*/[2(1 - 2x)] \leq t \leq T \). If \( 1/2 - p_B^*/(2T) \leq x \leq 1/2 \), then firm \( A \) charges \( p^*_A(x) = p^*_B \) and serves consumers with \( t \leq T \). If \( 1/2 < x < 1/2 + p_B^*/(4T) \), then firm \( A \) charges \( p^*_A(x) = p_B^* - T(2x - 1) \) and serves consumers with \( t \leq T \). If \( x \geq 1/2 + p_B^*/(4T) \), then firm \( A \) sets \( p_A^*(x) = p_B^*/2 \) and serves consumers with \( t \leq p_B^*/[2(2x - 1)] \). Firm \( B \) charges \( p_B^* = 0.47T \). Firms realize profits \( \Pi^{X, \emptyset}_{A} = 0.32T \) and \( \Pi^{X, \emptyset}_{B} = 0.12T \).

**Proof of Claim 5.** We start from the second stage where firm \( A \) moves. On its own turf firm \( A \) maximizes the expected profit \( E \left[ \Pi^{X, \emptyset}_{A} \mid x < 1/2 \right] = \Pr \{ t \geq t^c(\cdot) \} p_A \), which yields the equilibrium strategy \( p_A(p_B, x) = \frac{\tilde{t}(1 - 2x) + p_B}{2} \) if \( 0 \leq p_B < \tilde{t}(1 - 2x) \) and \( p_A(p_B, x) = p_B \) if \( p_B \geq \tilde{t}(1 - 2x) \). We also get \( p_A(p_B, 1/2) = p_B \). These equilibrium strategies yield \( t^c(p_B, x) = \tilde{t}/2 - p_B/[2(1 - 2x)] \). Solving \( t^c(p_B, x) = t \) we get \( x(p_B) = 1/2 - p_B/(2T) \). If \( x \leq x(p_B) \), firm \( A \) captures consumers with \( t \geq t^c(p_B, x) \), while it gets all consumers if \( x(p_B) < x < 1/2 \). On the competitor’s turf firm \( A \) maximizes the expected profit \( E \left[ \Pi^{X, \emptyset}_{A} \mid x > 1/2 \right] = \Pr \{ t < t^c(\cdot) \} p_A \), which yields the equilibrium strategies \( p_A(p_B, x) = p_B - \tilde{t}(2x - 1) \) if \( p_B \geq \tilde{t}(2x - 1) \) and \( p_A(p_B, x) = p_B/2 \) if \( 0 \leq p_B < 2\tilde{t}(2x - 1) \). These equilibrium strategies give \( \tilde{t}(x, p_B) = p_B/[2(2x - 1)] \). Solving \( \tilde{t}(x, p_B) = t \) we get \( x(p_B) = 1/2 + p_B/(4T) \). If \( 1/2 < x < x(p_B) \), then firm \( A \) gets all consumers, while it captures consumers with \( t < t^c(p_B, x) \) if \( x \geq x(p_B) \). Given
firm A’s equilibrium strategies, firm B’s profit is \( \Pi_B^{X^\varnothing} = \int_0^\infty \int_0^t (f(t)p_B) dt dx + \int_0^t (f(t)p_B) dt dx. \) Maximization of the latter profit yields \( p_B^* = 0.47\bar{t} < \bar{t}, \) which implies that, indeed, \( 0 < x(p_B^*) < 1/2 \) and \( 1/2 < \bar{x}(p_B^*) < 1. \) Firm A’s profit is computed as \( \Pi_A^{X^\varnothing} = \int_0^\infty \int_0^t [f(t)(1-2x) + p_B^*/2] dt dx + \int_0^t [f(t)(1-2x) + p_B^*/2] dt dx. \) Firms realize profits \( \Pi_A^{X^\varnothing} = 0.32\bar{t} \) and \( \Pi_B^{X^\varnothing} = 0.12\bar{t}. \) This completes the proof of Claim 5.

Claim 6. Let \( \bar{t} > 0 \) and \( k \leq 2. \) Consider the information scenario \( \{X, \varnothing\}. \) In equilibrium, on its own turf firm A charges \( p_A^*(x) = p_B + \bar{t}(1-2x) \) and serves all consumers. If \( 1/2 < x < 1/2 + p_B^*/[2(2\bar{t} - \bar{t})], \) then firm A sets \( p_A^*(x) = p_B - \bar{t}(2x - 1) \) and serves all consumers. If \( 1/2 + p_B^*/[2(2\bar{t} - \bar{t})] \leq x \leq 1/2 + p_B^*/(2\bar{t}), \) then firm A sets \( p_A^*(x) = [p_B - \bar{t}(2x - 1)]/2 \) and serves consumers with \( t \leq \bar{t}/2 + p_B^*/[2(2x - 1)]. \) If \( x > 1/2 + p_B^*/(2\bar{t}), \) then firm A sets \( p_A^*(x) = 0 \) and serves no consumers. Firm B sets \( p_B^* = \bar{H}(\bar{t}, \bar{t}). \) Firms realize profits \( \Pi_A^{X^\varnothing} = 5\bar{H}(\bar{t}, \bar{t})/8 + \bar{t}/4 \) and \( \Pi_B^{X^\varnothing} = \bar{H}(\bar{t}, \bar{t})/4. \)

Proof of Claim 6. We start from the second stage where firm A moves. On its own turf firm A maximizes the expected profit \( E\left[\Pi_A^{X^\varnothing} \mid x < 1/2\right] = \Pr\{t \geq t^c(\cdot)\}p_A, \) which yields the equilibrium strategy \( p_A(p_B, x) = p_B + \bar{t}(1-2x). \) We also get \( p_A(p_B, 1/2) = p_B. \) Firm A captures all consumers on its own turf. On the competitor’s turf firm A maximizes the expected profit \( E\left[\Pi_A^{X^\varnothing} \mid x > 1/2\right] = \Pr\{t < t^c(\cdot)\}p_A, \) which yields the equilibrium strategies \( p_A(p_B, x) = p_B - \bar{t}(2x - 1) \) if \( p_B \geq (2\bar{t} - \bar{t})(2x - 1), \) \( p_A(p_B, x) = [p_B - \bar{t}(2x - 1)]/2 \) if \( \bar{t}(2x - 1) < p_B < (2\bar{t} - \bar{t})(2x - 1) \) and \( p_A(p_B) = 0 \) if \( p_B \leq \bar{t}(2x - 1). \) These equilibrium strategies give \( t^c(p_B, x) = \bar{t}/2 + p_B^*/[2(2x - 1)]. \) Solving \( t^c(p_B, x) = \bar{t} \) we get \( \bar{x}(p_B) = 1/2 + p_B^*/[2(2\bar{t} - \bar{t})], \) while \( t^c(p_B, x) = \bar{t} \) yields \( \bar{x}(p_B) = 1/2 + p_B^*/(2\bar{t}). \) If \( 1/2 < x < \bar{x}(p_B), \) then firm A captures all consumers; if \( \bar{x}(p_B) \leq x \leq \bar{x}(p_B), \) then firm A serves consumers with \( t \leq t^c(x, p_B); \) finally, firm A does not get any consumers if \( x > \bar{x}(p_B). \) Given firm A’s equilibrium strategies, firm B’s profit is \( \Pi_B^{X^\varnothing} = \int_0^\infty \int_0^{t^c} (f(t)p_B) dt dx. \) Maximization with respect to \( p_B \) yields \( p_B^* = \bar{H}(\bar{t}, \bar{t}). \) Under the constraint \( 1 < k \leq 2 \) it holds that \( \bar{H}(\bar{t}, \bar{t}) < \bar{t}, \) hence, indeed, \( 1/2 < \bar{x}(p_B^*) \leq \bar{x}(p_B^*) < 1. \) Firm A’s profits are computed as \( \Pi_A^{X^\varnothing} = \int_0^\infty \int_0^{t^c} [f(t)(1-2x)] dt dx + \int_0^t [f(t)(1-2x)] dt dx + \int_0^\infty \int_0^{t^c} [f(t)(1-2x)] dt dx + \int_0^t [f(t)(1-2x)] dt dx. \) Firms realize profits \( \Pi_A^{X^\varnothing} = 5\bar{H}(\bar{t}, \bar{t})/8 + \bar{t}/4 \) and \( \Pi_B^{X^\varnothing} = \bar{H}(\bar{t}, \bar{t})/4. \) This completes the proof of Claim 6.
Claim 7. Consider the information scenario \(\{T, \varnothing\}\). If \(t = 0\), then in equilibrium firm A sets \(p_A^*(t) = p_B - t\) and serves all consumers if \(t \leq p_B^* / 3\), if \(t > p_B^* / 3\), then it charges \(p_A^*(t) = (p_B^* + t) / 2\) and serves consumers with \(x \leq 1/4 + p_B^* / (4t)\). Firm B sets \(p_B^* \approx 0.85T\). Firms realize profits \(\Pi_A^{T, \varnothing} \approx 0.53T\) and \(\Pi_B^{T, \varnothing} \approx 0.23T\). If \(t > 0\) and \(k \leq 2\), then in equilibrium firms set prices \(p_A^*(t) = (t + p_B^* / 2)\) and \(p_B^* = 3H(t, T) / 2\). Firm A serves all consumers if \(x \leq 1/4 + p_B^* / (4T)\), serves consumers with \(t \leq p_B^* / (4x - 1)\) if \(1/4 + p_B^* / (4T) < x \leq 1/4 + p_B^* / (4T)\) and serves no consumers when \(x > 1/4 + p_B^* / (4T)\). Equilibrium profits are \(\Pi_A^{T, \varnothing} = 21H(t, T) / 32 + A(t, T) / 8\) and \(\Pi_B^{T, \varnothing} = 9H(t, T) / 16\).

Proof of Claim 7. We start from the second stage where firm A moves. Firm A maximizes its expected profit \(E[\Pi_A^{T, \varnothing} | e] = p_A \Pr \{x \leq x^e(\cdot)\}\) for any \(t\), which yields firm A’s equilibrium strategies as \(p_A(p_B, t) = (p_B + t) / 2\) if \(p_B < 3t\) and \(p_A(p_B, t) = p_B - t\) if \(p_B \geq 3t\), from where we get \(t^c(p_B, x) = p_B / (4x - 1)\). Assume that \(t > 0\) and \(k \leq 2\). Solving \(t^c(p_B, x) = \tilde{t}\) and \(t^c(p_B, x) = \tilde{t}\), we get \(T(p_B) = 1 / 4 + p_B / (4T)\) and \(\tilde{x}(p_B) = 1 / 4 + p_B / (4T)\). Depending on the relation between \(T(p_B)\) and \(\tilde{x}(p_B)\) and one two cases are possible in equilibrium: \(T(p_B) \geq 1\) if \(3t \leq p_B < 3\tilde{t}\) and \(\tilde{x}(p_B) < 1\) if \(p_B^* < 3\tilde{t}\). We show that \(3t \leq p_B < 3\tilde{t}\) does not emerge in equilibrium. Assume that \(3t \leq p_B < 3\tilde{t}\). Firm B chooses its price to maximize the profit \(\Pi_B^{T, \varnothing} = \int_0^T \int_0^T (f_B) dtdx\). The optimal price \(p_B\) solves the equation \(p_B [1 + \ln(9)] - \tilde{T} - 2p_B \ln(p_B / \tilde{T}) = 0\). There is no analytical solution to this problem, the value \(p_B = 0.85T\) is, however, a good numerical approximation which fulfills the second-order condition. Note that \(0.85T < 3\tilde{t}\) given \(1 < k \leq 2\), hence, \(3t \leq p_B^* < 3\tilde{t}\) cannot hold in equilibrium. Assume further that \(p_B^* < 3\tilde{t}\). Firm B maximizes the profit \(\Pi_B^{T, \varnothing} = \int_0^T \int_0^T (f_B) dtdx\), which yields the optimal price \(p_B = 3H(t, \tilde{T}) / 2\). Under the constraint \(1 < k \leq 2\) it holds that \(3H(t, \tilde{T}) / 2 < 3t\), hence, \(p_B^* = 3H(t, \tilde{T}) / 2\). Firm A’ profits are computed as \(\Pi_A^{T, \varnothing} = \int_0^T \int_0^T [f_A(p_B^* + t) / 2] dtdx + \int_0^T \int_0^T [f_A(p_B^* + t) / 2] dtdx\). Equilibrium profits are \(\Pi_A^{T, \varnothing} = 21H(t, \tilde{T}) / 32 + A(t, \tilde{T}) / 8\) and \(\Pi_B^{T, \varnothing} = 9H(t, \tilde{T}) / 16\). Consider now \(t = 0\). Maximization of \(\Pi_B^{T, \varnothing} = \int_0^T \int_0^T (f_B) dtdx\) yields \(p_B^* \approx 0.85T\). Firm A’ profits are computed as \(\Pi_A^{T, \varnothing} = \int_0^T \int_0^T [f_A(p_B^* - t)] dtdx + \int_0^T \int_0^T [f_A(p_B^* + t) / 2] dtdx + \int_0^T \int_0^T [f_A(p_B^* + t) / 2] dtdx\). Firms realize profits \(\Pi_A^{T, \varnothing} \approx 0.53T\) and \(\Pi_B^{T, \varnothing} \approx 0.23T\). This completes the proof of Claim 7.

Claim 8. Consider the information scenario \(\{\mathcal{X}, \varnothing\}\). If \(t = 0\), then in equilibrium firms set prices \(p_A^*(t, x) = \max\{p_B^* + t(1 - 2x), 0\}\) and \(p_B^* \approx 0.28T\). If \(x \leq 1/2 + p_B^* / (2T)\), firm A serves all
consumers; if \( x > 1/2 + p_B^*/(2\tilde{t}) \), firm \( A \) serves consumers with \( t \leq p_B^*/(2x - 1) \). Equilibrium profits are \( \Pi_A^{XT|\varnothing} \approx 0.32\tilde{t} \) and \( \Pi_B^{XT|\varnothing} \approx 0.05\tilde{t} \). If \( \tilde{t} > 0 \) and \( k \leq 2 \), in equilibrium firms set prices \( p_A^*(t, x) = \max\{p_A^* + t(1 - 2x), 0\} \) and \( p_B^* = H(t, \tilde{t})/2 \). If \( x \leq 1/2 + p_B^*/(2\tilde{t}) \) firm \( A \) serves all consumers; if \( 1/2 + p_B^*/(2\tilde{t}) < x \leq 1/2 + p_B^*/(2\tilde{t}) \) firm \( A \) serves consumers with \( t \leq p_B^*/(2x - 1) \); if \( x > 1/2 + p_B^*/(2\tilde{t}) \) firm \( A \) serves no consumers. Equilibrium profits are \( \Pi_A^{XT|\varnothing} = 5H(t, \tilde{t})/16 + A(t, \tilde{t})/4 \) and \( \Pi_B^{XT|\varnothing} = H(t, \tilde{t})/8 \).

Proof of Claim 8. Consider first \( \tilde{t} > 0 \) and \( k \leq 2 \). We start from the second stage where firm \( A \) maximizes its profit given \( p_B \). Firm \( A \)'s equilibrium strategy is \( p_A(p_B, t, x) = \max\{0, t(1 - 2x) + p_B\} \), which yields \( t^c(p_B, x) = p_B/(2x - 1) \), \( \pi(p_B) = 1/2 + p_B/(2\tilde{t}) \) and \( \varnothing(p_B) = 1/2 + p_B/(2\tilde{t}) \). Depending on the relation between \( \varnothing(p_B) \) and 1 two cases are possible in equilibrium: \( \varnothing(p_B) \leq 1 \) if \( p_B \leq \tilde{t} \) and \( \varnothing(p_B) > 1 \) if \( \tilde{t} < p_B < \tilde{t} \). We show first that \( \tilde{t} < p_B < \tilde{t} \) cannot characterize firm \( B \)'s equilibrium price. Assume that \( \tilde{t} < p_B < \tilde{t} \). Firm \( B \) sets \( p_B \) to maximize the profit \( \Pi_B^{XT|\varnothing} = \int_{\tilde{t}}^{\tilde{t}} \int_{\varnothing}^{\tilde{t}} (f_t p_B) dt dx \). The optimal price \( p_B \) solves the equation \( p_B \left[ 2 \ln \left( \frac{p_B}{\tilde{t}} \right) - 1 \right] + \tilde{t} = 0 \). There is no analytical solution to this problem, the value \( p_B = 0.28\tilde{t} \) is, however, a good numerical approximation, which fulfills the second-order condition. Note that \( 0.28\tilde{t} < t \) given \( 1 < k \leq 2 \), hence, \( \tilde{t} < p_B < \tilde{t} \) is not possible in equilibrium. We show next that in equilibrium \( p_B^* \leq \tilde{t} \).

Assume this is the case. Firm \( B \) sets \( p_B \) to maximize the profit \( \Pi_B^{XT|\varnothing} = \int_{\tilde{t}}^{\tilde{t}} \int_{\varnothing}^{\tilde{t}} (f_t p_B) dt dx + \int_{\varnothing}^{\tilde{t}} \int_{\varnothing}^{\tilde{t}} (f_t p_B) dt dx = \left[ p_B(\tilde{t} - \tilde{t} - p_B \ln(\tilde{t}/\tilde{t})) \right] / \left[ 2(\tilde{t} - \tilde{t}) \right] \), which yields \( p_B = H(t, \tilde{t})/2 \). Under the constraint \( 1 < k \leq 2 \) it holds that \( H(t, \tilde{t})/2 < \tilde{t} \) hence, \( p_B^* = H(t, \tilde{t})/2 \). Firm \( A \)'s profits are computed as \( \Pi_A^{XT|\varnothing} = \int_{\tilde{t}}^{\tilde{t}} \int_{\varnothing}^{\tilde{t}} \left[ f_t \left( p_B^* + t(1 - 2x) \right) \right] dx dt + \int_{\tilde{t}}^{\tilde{t}} \int_{\varnothing}^{\tilde{t}} \left[ f_t \left( p_B^* + t(1 - 2x) \right) \right] dx dt \). Equilibrium profits are \( \Pi_A^{XT|\varnothing} = 5H(t, \tilde{t})/16 + A(t, \tilde{t})/4 \) and \( \Pi_B^{XT|\varnothing} = H(t, \tilde{t})/8 \). Consider finally \( \tilde{t} = 0 \), in which case \( \Pi_B^{XT|\varnothing} = \int_{\tilde{t}}^{\tilde{t}} \int_{\varnothing}^{\tilde{t}} (f_t p_B) dt dx \) and \( p_B^* \approx 0.28\tilde{t} \). Firm \( A \)'s profits are computed as \( \Pi_A^{XT|\varnothing} \approx 0.32\tilde{t} \) and \( \Pi_B^{XT|\varnothing} \approx 0.05\tilde{t} \). This completes the proof of Claim 8.

Claim 9. Consider the information scenario \( \{XT, X\} \). In equilibrium firm \( A \) sets \( p_A^*(t, x) = t(1 - 2x) \) if \( x \leq 1/2 \) and \( p_A^*(t, x) = (2x - 1) \max\{0, \tilde{t}/2 - t\} \) if \( x > 1/2 \). Firm \( B \) sets \( p_B^*(x) = 0 \) if \( x \leq 1/2 \) and \( p_B^*(x) = (2x - 1) \bar{m} \) if \( x > 1/2 \) and serves consumers with \( x > 1/2 \) and \( \bar{t} > \bar{m} \), where \( \bar{m} = \max\{\bar{t}/2, \bar{t}\} \). Firms realize profits \( \Pi_A^{XT|X} = 5\tilde{t}/32 \) and \( \Pi_B^{XT|X} = \tilde{t}/16 \) if \( \tilde{t} = 0 \) and \( \Pi_A^{XT|X} = A(t, \tilde{t})/4 \) and \( \Pi_B^{XT|X} = \tilde{t}/4 \) if \( \tilde{t} > 0 \) and \( k \leq 2 \).
Proof of Claim 9. Firm $B$ treats $t$ as a random variable and maximizes its expected profits given firm $A$’s equilibrium strategy separately in the regions $x \leq 1/2$ and $x > 1/2$. In the region $x \leq 1/2$ firm $A$ makes any consumer indifferent for any $p_B(x)$, hence, $p_B^*(x) = 0$ for $x \leq 1/2$. In the region $x > 1/2$ firm $A$ makes a consumer indifferent as long as it can set a non-negative price, which is the case if $t(2x - 1) \leq p_B(x)$. Firm $B$’s expected profit in the region $x > 1/2$ is $E \left[ \Pi^X_B | x > 1/2 \right] = p_B \Pr \{t(2x - 1) > p_B | x > 1/2 \}$. Maximization of the latter profit yields the optimal price of firm $B$: $p_B^*(x) = \bar{t}(x - 1/2)$ if $\bar{t} > 2t_x$ and $p_B^*(x) = t_x(2x - 1)$ if $\bar{t} \leq 2t_x$. If $\bar{t} = 0$, then $p_B^*(x) = \bar{t}(x - 1/2)$, which yields $t^c = \bar{t}/2$ and firm $B$ serves consumers with $t > t^c$ on its turf. Firms realize profits $\Pi^X_A = \int \int [f(1 - 2x)] \, dx \, dt + \int_{1/2}^{1/2} \int_{0}^{t_x} \frac{7}{2} \left[ (t - 2t_x)(x - 1/2) \right] dx \, dt = 5t_x/32$ and $\Pi^X_B = \int_{1/2}^{1/2} \int_{0}^{\bar{t}} \frac{7}{16} \left[ f(t)(x - 1/2) \right] dx \, dt = t_x/16$. If $\bar{t} > 0$ and $k \leq 2$, then $p_B^*(x) = \bar{t}(2x - 1)$ and firm $B$ serves all consumers on its turf. Firms realize profits $\Pi^X_A = \int_{0}^{t_x} \frac{7}{2} \left[ f(t)(1 - 2x) \right] dx = A(t_x, \bar{t})/4$ and $\Pi^X_B = \int_{1/2}^{1/2} \frac{7}{4} \left[ f(t)(2x - 1) \right] dx = t_x/4$. This completes the proof of Claim 9.

Claim 10. Consider the information scenario $\{XT, T\}$. In equilibrium firm $A$ sets $p_A^*(t, x) = \max \{t/2 + t(1 - 2x), 0 \}$ and serves consumers with $x \leq 3/4$. Firm $B$ sets $p_B^*(t) = t/2$. Firms realize profits $\Pi^X_A = 9A(t_x, \bar{t})/16$ and $\Pi^X_B = A(t_x, \bar{t})/8$.

Proof of Claim 10. The equilibrium strategy of firm $A$ is $p_A(p_B, t, x) = \max \{p_B + t(1 - 2x), 0 \}$. Firm $B$ treats $x$ as a random variable and maximizes its expected profit given firm $A$’s equilibrium strategy for any $t$: $E \left[ \Pi^X_B(t) \right] = p_B \Pr \{t(2x - 1) > p_B \}$. Solving the maximization problem for $p_B$ yields $p_B^*(t) = t/2$, which gives $p_A^*(t, x) = \max \{t/2 + t(1 - 2x), 0 \}$, such that $t/2 + t(1 - 2x)$ is positive whenever $x < x^c = 3/4$. Firms $A$ and $B$ realize profits $\Pi^X_A = \int_{0}^{t_x} \frac{7}{2} \left[ f(t/2 + t(1 - 2x)) \right] dx = 9A(t_x, \bar{t})/16$ and $\Pi^X_B = \int_{x^c}^{t_x} \frac{7}{2} \left[ f(t/2) \right] dx = A(t_x, \bar{t})/8$, respectively. This completes the proof of Claim 10.

Q.E.D.

Proof of Proposition 2. The comparison of joint profits in the case of relatively differentiated consumers is straightforward and shows that only dataset $T$ is shared, in the information scenario $\{XT, X\}$. We now turn to the case of relatively homogeneous consumers. Many comparisons are straightforward using $H(t_x, \bar{t}) < A(t_x, \bar{t})$. We only consider the non-trivial cases. Let $\Pi^{X=I}_{A+B}$ denote the sum of profits in the scenario $\{I_A, I_B\}$. We first show that dataset $X$ is not
shared in the scenario \( \{XT, \varnothing\} \). By substituting \( k \) into \( \Pi_{A+B}^{XT|\varnothing} - \Pi_{A+B}^{XT|X} \) and rearranging we get
\[
16 \ln k(\Pi_{A+B}^{XT|\varnothing} - \Pi_{A+B}^{XT|X})/t = 7(k - 1) - 4 \ln k.
\]
The RHS of the latter equation increases on the interval \( 1 < k \leq 2 \) and approaches zero when \( k \to 1 \), hence, \( \Pi_{A+B}^{XT|\varnothing} > \Pi_{A+B}^{XT|X} \). We next show that both datasets together are not shared in this information scenario either. Substituting \( k \) into \( \Pi_{A+B}^{XT|\varnothing} - \Pi_{A+B}^{XT|XT} \) and rearranging yields
\[
16 \ln k(\Pi_{A+B}^{XT|\varnothing} - \Pi_{A+B}^{XT|XT})/t = 7(k - 1) - 2(k + 1) \ln k.
\]
The second derivative of the RHS of the latter equation is negative on the interval \( 1 < k \leq 2 \) and the first derivative is positive at the point \( k = 2 \), hence, the RHS increases on the whole interval. Note, finally, that the RHS approaches zero when \( k \to 1 \), hence, \( \Pi_{A+B}^{XT|\varnothing} > \Pi_{A+B}^{XT|XT} \).

There is no information sharing in the scenario \( \{T, \varnothing\} \). By substituting \( k \) into \( \Pi_{A+B}^{T|\varnothing} - \Pi_{A+B}^{T|T} \) and rearranging we get
\[
32 \ln k(\Pi_{A+B}^{T|\varnothing} - \Pi_{A+B}^{T|T})/t = 39(k - 1) - 14(k + 1) \ln k.
\]
The second derivative of the RHS of the latter equation is negative on the interval \( 1 < k \leq 2 \) and the first derivative is positive at the point \( k = 2 \), hence, the RHS increases on the whole interval. Note, finally, that the RHS approaches zero when \( k \to 1 \), it follows that \( \Pi_{A+B}^{T|\varnothing} > \Pi_{A+B}^{T|T} \). Finally, we show that dataset \( X \) is not shared in the information scenario \( \{X, \varnothing\} \). Substituting \( k \) into \( \Pi_{A+B}^{X|\varnothing} - \Pi_{A+B}^{X|X} \) and rearranging yields
\[
8 \ln(2k - 1)(\Pi_{A+B}^{X|\varnothing} - \Pi_{A+B}^{X|X})/t = 7(k - 1) - 2 \ln(2k - 1).
\]
The derivative of the RHS of the latter equation is positive on the interval \( 1 < k \leq 2 \). Moreover, the RHS approaches zero when \( k \to 1 \), hence, it takes only positive values and \( \Pi_{A+B}^{X|\varnothing} > \Pi_{A+B}^{X|X} \). Q.E.D.

**Proof of Proposition 3.** Consider first \( t = 0 \). Consumer surplus in the information scenario \( \{XT, X\} \) is
\[
CS_{XT|X} = \int_0^{1/2} \int_0^1 [U_A(p_A^c(t, x), t, x)f_i] dt dx + \int_{1/2}^{1} \int_0^1 [U_A(p_A^c(t, x), t, x)f_i] dt dx + \int_0^{1/2} \int_0^1 [U_B(p_B^c(t, x), t, x)f_i] dt dx = v - 3\bar{t}/8.
\]
In the information scenario \( \{XT, XT\} \) consumer surplus is
\[
CS_{XT|XT} = \int_0^{1/2} \int_0^1 [U_A(p_A^c(t, x), t, x)f_i] dt dx + \int_{1/2}^{1} \int_0^1 [U_B(p_B^c(t, x), t, x)f_i] dt dx = v - 3A(t, \bar{t})/4.
\]
We conclude that \( CS_{XT|X} = CS_{XT|XT} \). Social welfare follows immediately from adding up profits and consumer surplus: \( SW_{XT|X} = v - 5\bar{t}/32 < SW_{XT|XT} = v - \bar{t}/8 \).

Consider now \( t > 0 \) and \( k \leq 2 \). Consumer surplus in the information scenario \( \{XT, \varnothing\} \) is
\[
CS_{XT|\varnothing} = \int_0^{1/2} \int_0^1 [U_A(p_A^c(t, x), t, x)f_i] dt dx + \int_{1/2}^{1} \int_0^1 [U_A(p_A^c(t, x), t, x)f_i] dt dx + \int_0^{1/2} \int_0^1 [U_B(p_B^c(t, x), t, x)f_i] dt dx + \int_{1/2}^{1} \int_0^1 [U_B(p_B^c(t, x), t, x)f_i] dt dx = v - [A(t, \bar{t}) + H(t, \bar{t})]/2 \quad \text{and social welfare is} \quad SW_{XT|\varnothing} = v - [4A(t, \bar{t}) + H(t, \bar{t})]/16.
\]
Consumer surplus in the information scenario \( \{XT, T\} \) is
\[
CS_{XT|T} = \int_0^{1/2} \int_0^1 [U_A(p_A^c(t, x), t, x)f_i] dt dx + \int_{1/2}^{1} \int_0^1 [U_B(p_B^c(t, x), t, x)f_i] dt dx = v - A(t, \bar{t}) \quad \text{and social welfare is} \quad SW_{XT|T} = v - A(t, \bar{t}).
\]
\[ SW^{XT|T} = v - 5A(L, \bar{t})/16. \] Straightforward comparisons yield that \( CS^{XT|\varnothing} > CS^{XT|T} \) and \( SW^{XT|\varnothing} > SW^{XT|T} \).

Consumers enjoy \( CS^{XT|X} = \int_0^{1/2} \int [U_A(p_A^*(t, x), t, x)f_t] dtdx + \int_0^{1/2} \int [U_B(p_B^*(x), t, x)f_t] dtdx = v - (\bar{t} + 3\bar{t})/8 \) in the information scenario \( \{XT, X\} \). We use \( CS^{I_A|I_B} = SW^{I_A|I_B} - \Pi_A^{I_A|I_B} - \Pi_B^{I_A|I_B} \) to derive consumer surplus \( CS^{XT|XT} = v - 3A(L, \bar{t})/4 \). We get \( CS^{XT|X} > CS^{XT|XT} \). As in the information scenarios \( \{XT, X\} \) and \( \{XT, XT\} \) every firm serves only consumers on its own turf, it follows that \( SW^{XT|X} = SW^{XT|XT} \). Q.E.D.

**Proof of Proposition 4.** We derive equilibrium prices and profits in the symmetric information scenarios \( \{T, T\}, \{X, X\} \) and \( \{XT, XT\} \) under Assumption 1.

**Claim 1.** Consider the information scenario \( \{T, T\} \) and assume that firm B moves first, while firm A follows. Firms set prices \( p_B^*(t) = 3t/2 \) and \( p_A^*(t) = 5t/4 \). Firm A serves all consumers with \( x < 5/8 \). Firms realize profits \( \Pi_A^{T|T} = 25A(L, \bar{t})/32 \) and \( \Pi_B^{T|T} = 9A(L, \bar{t})/16 \).

**Proof of Claim 1.** We start from the second stage where firm A moves. Firm A takes \( p_B \) as given and maximizes its expected profit \( E \left[ \Pi_A^{T|T} | t \right] = p_A \Pr \{ x < x^c(\cdot) \} \), which yields firm A’s equilibrium strategy as \( p_A(p_B, t) = (t + p_B)/2 \) if \( p_B < 3t \) and \( p_A(p_B) = p_B - t \) if \( p_B \geq 3t \). In the latter case firm A serves all consumers and firm B gets zero profit. It is straightforward then that firm B does not choose \( p_B \geq 3t \). Assume that firm B chooses \( p_B < 3t \) for any \( t \), which yields \( x^c(p_B, t) = 1/4 + p_B/(4t) \). Given \( p_B < 3t \) it holds that \( 0 < x^c(p_B, t) < 1 \), such that firm B’s expected profit is \( E \left[ \Pi_B^{T|T} | t \right] = p_B \Pr \{ x \geq x^c(\cdot) \} \). Maximization of the latter profit yields \( p_B^*(t) = 3t/2 \), such that \( p_B^*(t) < 3t \) holds. Firm A’s price is then \( p_{A}^*(t) = 5t/4 \), which yields \( x^c = 5/8 \). Firms realize profits \( \Pi_A^{T|T} = \int_0^{1/2} \int [f_t(5t/4)] dtdx = 25A(L, \bar{t})/32 \) and \( \Pi_B^{T|T} = \int_0^{1/2} \int [f_t(3t/2)] dtdx = 9A(L, \bar{t})/16 \). This completes the proof of Claim 1.

**Claim 2.** Let \( L = 0 \). Consider the information scenario \( \{X, X\} \) and assume that firm B moves first, while firm A follows. Firm A sets \( p_A^*(x) = 3\bar{t}(1 - 2x)/4 \) on its own turf and \( p_A^*(x) = \bar{t}(2x - 1)/2 \) on the competitor’s turf. Firm B sets \( p_B^*(x) = \bar{t}(2x - 1) \) on its own turf and \( p_B^*(x) = \bar{t}(1 - 2x)/2 \) on the competitor’s turf. Firm A serves all consumers with \( t \geq \bar{t}/4 \) on its own turf and all consumers with \( t < \bar{t}/2 \) on firm B’s turf. Firms realize profits \( \Pi_A^{X|X} = 13\bar{t}/64 \) and \( \Pi_B^{X|X} = 10\bar{t}/64 \).

**Proof of Claim 2.** We start from the second stage where firm A moves. Firm A takes \( p_B \) as
given and maximizes its expected profit for given $x$. Consider first $x < 1/2$, in which case firm $A$’s profit is $E \left[ \Pi_A^{X|X} | x < 1/2 \right] = p_A \Pr \{ t \geq t^c(\cdot) \}$. Firm $A$’s equilibrium strategy takes the form $p_A(p_B, x) = \{ \bar{t}(1-2x) + p_B \}/2$ and $p_A(p_B, x) = \bar{t}(1-2x)) = p_B$. Assume that firm $B$ chooses $p_B < \bar{t}(1-2x)$ for any $x < 1/2$, which yields $t^c(p_B, x) = \left( \bar{t}(1-2x) - p_B \right)/[2(1-2x)]$. Given $p_B < \bar{t}(1-2x)$, it holds that $0 < t^c(p_B, x) = \bar{t}$ and firm $B$’s expected profit for given $x$ is $E \left[ \Pi_B^{X|X} | x < 1/2 \right] = p_B \Pr \{ t < t^c(\cdot) \}$, which yields $p_B^*(x) = \bar{t}(1-2x))/2 < \bar{t}(1-2x)$. Firm $A$’s equilibrium price is then $p_A^*(x) = 3\bar{t}(1-2x)/4$ and $t^c = \bar{t}/4$. Consider now $x > 1/2$, in which case firm $A$’s profit takes the form: $E \left[ \Pi_A^{X|X} | x > 1/2 \right] = p_A \Pr \{ t < t^c(\cdot) \}$. Firm $A$’s equilibrium strategy is then: $p_A(p_B, x) = 2\bar{t}(1-2x)) = p_B/2$ and $p_A(p_B|x) = 2\bar{t}(1-2x)) = p_B-\bar{t}(1-2x)$. Assume that $p_B < 2\bar{t}(1-2x)$ for any $x$, which yields $t^c(p_B, x) = p_B/[2(2x-1)]$. Given $p_B < 2\bar{t}(1-2x)$, it holds that $0 < t^c(\cdot) < \bar{t}$. Firm $B$’s expected profit is then $E \left[ \Pi_B^{X|X} | x > 1/2 \right] = p_B \Pr \{ t \geq \bar{t}(\cdot) \}$, which yields the optimal price $p^*_B(x) = \bar{t}(2x-1) < 2\bar{t}(1-2x)$. Firm $A$’s price is $p^*_A(x) = \bar{t}(2x-1)/2$ and $t^c = \bar{t}/2$. If $x = 1/2$, then $p^*_A(x) = p^*_B(x) = 0$. Firms realize profits $\Pi_A^{X|X} = \int_{0}^{1/2} \int_{0}^{t^c} \left[ f_I(3\bar{t}(1-2x))/4 \right] dt dx = \frac{1}{12} \bar{t}^2$ and $\Pi_B^{X|X} = \int_{0}^{1/2} \int_{0}^{t^c} \left[ f_I(\bar{t}(1-2x)/2) \right] dt dx = 10\bar{t}/64$. This completes the proof of Claim 2.

Claim 3. Let $\xi > 0$ and $k \leq 2$. Consider the information scenario $\{X, X\}$ and assume that firm $B$ moves first, while firm $A$ follows. On firm $A$’s turf firms set prices $p^*_A(x) = \xi(1-2x)$ and $p^*_B(x) = 0$, where firm $A$ serves all consumers. On firm $B$’s turf firms’ prices depend on $k$:

i) If $k \leq 3/2$, then $p^*_A(x) = 0$ and $p^*_B(x) = \xi(2x-1)$. Firm $B$ serves all consumers on its turf.

Profits are $\Pi_A^{X|X} = \Pi_B^{X|X} = \xi/4$.

ii) If $k > 3/2$, then $p^*_A(x) = (2\xi - 3\xi)(2x-1)/4$ and $p^*_B(x) = (2\xi - \xi)(2x-1)/2$. Firm $B$ serves all consumers on its turf with $t \geq (2\xi + \xi)/4$. Profits are $\Pi_A^{X|X} = \xi(4k^2 + 4k - 7)/[64(k-1)]$ and $\Pi_B^{X|X} = \xi(2k-1)^2/[32(k-1)]$.

Proof of Claim 3. Consider the second stage where firm $A$ moves. Firm $A$ chooses its price for a given $x$ and given $p_B$. Consider first the interval $x < 1/2$. Maximization of firm $A$’s profit $E \left[ \Pi_A^{X|X} | x < 1/2 \right] = p_A \Pr \{ t \geq t^c(\cdot) \}$ yields the equilibrium strategy $p_A(p_B, x) = p_B + \xi(1-2x)$. The optimal price of firm $B$ is $p^*_B(x) = 0$. In equilibrium firm $B$ does not serve any consumers on firm $A$’s turf. We now turn to firm $B$’s turf and start from firm $A$’s equilibrium strategy given firm $B$’s price. Firm $A$ maximizes the profit $E \left[ \Pi_A^{X|X} | x > 1/2 \right] = p_A \Pr \{ t < t^c(\cdot) \}$, which yields
firm A’s equilibrium strategy as \( p_A(p_B, x) = 0 \) if \( p_B \leq \ell(2x - 1) \), \( p_A(p_B, x) = [p_B - \ell(2x - 1)]/2 \) if \( \ell(2x - 1) < p_B < (2\ell - \ell)(2x - 1) \) and \( p_A(p_B, x) = p_B - \ell(2x - 1) \) if \( p_B \geq (2\ell - \ell)(2x - 1) \). Firm B’s optimal price depends on \( k \). If \( k \leq 3/2 \), then \( p_B^*(x) = \ell(2x - 1) \), which yields \( t^* = 7 \). Profits are \( \Pi_A^{X|X} = \Pi_B^{X|X} = \frac{1}{1/2 \ell} \int_0^\ell \int [f_i t(1 - 2x)] dt \, dx = \ell/4 \). If \( k > 3/2 \), then \( p_B^*(x) = (2\ell - \ell)(2x - 1)/2 \), which yields \( t^* = (2\ell + \ell)/4 \). Finally, if \( x = 1/2 \), then \( p_A^*(x) = p_B^*(x) = 0 \). Profits are \( \Pi_A^{X|X} = \frac{1}{1/2 \ell} \int_0^\ell \int [f_i t(1 - 2x)] dt \, dx + \frac{1}{1/2 \ell} \int_0^\ell \int [f_i (2\ell - 3\ell)(2x - 1)/4] dt \, dx = \ell (4k^2 + 4k - 7) / [64(k - 1)] \) and

\[
\Pi_B^{X|X} = \frac{1}{1/2 \ell} \int_0^\ell \int [f_i (2\ell - \ell)(2x - 1)/2] dt \, dx = \ell (2k - 1)^2 / [32(k - 1)].
\]

This completes the proof of Claim 3.

**Claim 4.** Consider the information scenario \( \{XT, XT\} \) and assume that firm B moves first, while firm A follows. On its own turf firm i sets \( p_i^*(t, x) = t[1 - 2x] \) and on the competitor’s turf \( p_i^*(t, x) = 0 \). Every firm serves consumers on its own turf. Firms realize profits \( \Pi_A^{XT|XT} = \Pi_B^{XT|XT} = A(\ell, \ell)/4 \).

**Proof of Claim 4.** We start from the second stage where firm A moves. Consider first \( x > 1/2 \). The equilibrium strategy of firm A is \( p_A(p_B, t, x) = \max \{ p_B - t(2x - 1) - \epsilon, 0 \} \), where \( \epsilon > 0 \). If for a consumer \((x, t)\) firm B chooses \( p_B(x, t) > t(2x - 1) \), then firm A will undercut and gain that consumer. Hence, firm B must choose \( p_B^*(x, t) = t(2x - 1) \) and \( p_A^*(x, t) = 0 \). Consider next \( x \leq 1/2 \). The equilibrium strategy of firm A is \( p_A(p_B, t, x) = \max \{ p_B - t(2x - 1), 0 \} \). Firm B can only win a consumer \((x, t)\) if it sets \( p_B(x, t) < t(2x - 1) \leq 0 \), which is not possible, hence, \( p_B^*(x, t) = 0 \) and \( p_A^*(x, t) = t(1 - 2x) \). Firms’ profits are \( \Pi_A^{XT|XT} = \frac{1}{1/2 \ell} \int_0^\ell \int [f_i t(1 - 2x)] dt \, dx = A(\ell, \ell)/4 = \Pi_B^{XT|XT} \). This completes the proof of Claim 4.

Q.E.D.

**Proof of Proposition 5.** The comparison of joint profits in the case of relatively differentiated consumers is straightforward and shows that only dataset \( T \) is shared, in the information scenario \( \{XT, X\} \). We now turn to the case of relatively homogeneous consumers. With sequential moves profits only change in the information scenarios \( \{X, X\} \) and \( \{T, T\} \). Straightforward comparisons show that \( \Pi_A^{T|T} > \Pi_A^{T|\emptyset} \), such that dataset \( T \) is shared. We next show that dataset \( X \) is not shared in the information scenario \( \{X, \emptyset\} \) if \( k > 3/2 \). By substituting \( k \) into \( \Pi_A^{X|\emptyset} - \Pi_A^{X|X} \) and rearranging we get \( 64(k - 1) \ln(2k - 1) \left( \Pi_A^{X|\emptyset} - \Pi_A^{X|X} \right) / \ell = 56(k - 1)^2 - (12k^2 - 20k + 11) \ln(2k - 1) \). Successive differentiation of the RHS of the latter expression shows
that it takes only positive values on the interval \(3/2 < k \leq 2\), which implies that \(\Pi_{A+B}^{X|\varnothing} > \Pi_{A+B}^{X|\varnothing}\).

We finally consider how dataset \(T\) sharing in the scenario \(\{T, \varnothing\}\) changes consumer surplus and social welfare. Consumer surplus can be computed as \(CS^{T|\varnothing} = \iiint_{x^c} [U_A(p_A^*(t), t, x)f_t] dxdt + \iiint_{x^c} [U_B(p_B^*(t), t, x)f_t] dxdt = v - 7A(t, \bar{t})/16 - 75H(t, \bar{t})/64\). In the scenario \(\{T, T\}\) consumer surplus can be calculated as \(CS^{T|T} = \iiint_{x^c} [U_A(p_A^*(t), t, x)f_t] dxdt + \iiint_{x^c} [U_B(p_B^*(t), t, x)f_t] dxdt = v - 103A(t, \bar{t})/64\). Straightforward comparison yields \(CS^{T|\varnothing} > CS^{T|T}\). Adding up consumer surplus and profits we get \(SW^{T|\varnothing} = v + 3H(t, \bar{t})/64 - 5A(t, \bar{t})/16\) and \(SW^{T|T} = v - 17A(t, \bar{t})/64\), from where we conclude that \(SW^{T|\varnothing} < SW^{T|T}\). Q.E.D.

**Proof of Proposition 6.** We derive equilibrium prices and profits in different information scenarios in the region \(x \leq \alpha\) under Assumption 1.

**Claim 1.** Consider the information scenario \(\{T_0, \varnothing_\alpha\}\) and assume \(\alpha > 1/4\). If \(\bar{t} = 0\), then in equilibrium firm \(A\) sets \(p_A^*(t) = p_B^* - t(2\alpha - 1)\) if \(t \leq 0.28\bar{t}\) and serves all consumers, if \(t > 0.28\bar{t}\), then it sets \(p_A^*(t) = (p_B^* + t)/2\) and serves consumers with \(x \leq 1/4 + p_B^*/(4t)\). Firm \(B\) charges \(p_B^* = 0.28\bar{t}(4\alpha - 1)\). Firms realize profits \(\Pi_A^{T_0|\varnothing_\alpha} = \bar{t}(0.42\alpha^2 + 0.07\alpha + 0.02)\) and \(\Pi_B^{T_0|\varnothing_\alpha} = \bar{t}(0.41\alpha^2 - 0.2\alpha + 0.03)\). If \(\bar{t} > 0\) and \(k \leq 2\), then in equilibrium firms charge \(p_A^*(t) = (p_B^* + t)/2\) and \(p_B^* = (4\alpha - 1)H(t, \bar{t})/2\). For any \(t\) firm \(A\) serves consumers with \(x \leq 1/4 + p_B^*/(4t)\). Firms realize profits \(\Pi_A^{T_0|\varnothing_\alpha} = (16\alpha^2 + 8\alpha - 3)H(t, \bar{t})/32 + A(t, \bar{t})/8\) and \(\Pi_B^{T_0|\varnothing_\alpha} = (4\alpha - 1)^2H(t, \bar{t})/16\).

**Proof of Claim 1.** We start from the second stage where firm \(A\) moves. Maximization of firm \(A\)'s profit for a given \(t\) yields the equilibrium strategies \(p_A(p_B, t) = (p_B + t)/2\) if \(p_B < t(4\alpha - 1)\) and \(p_A(p_B, t) = p_B - t(2\alpha - 1)\) if \(p_B \geq t(4\alpha - 1)\), from where we get \(x^c(p_B, x) = p_B/(4x - 1)\) and \(x^c(p_B, t) = 1/4 + p_B/(4t)\). Assume that \(\bar{t} > 0\) and \(k \leq 2\). Depending on the relation between \(x^c(p_B^*, \bar{t})\) and \(\alpha\) two cases are possible in equilibrium: \(x^c(p_B^*, \bar{t}) \leq \alpha\) and \(x^c(p_B^*, \bar{t}) > \alpha\). We show that only the former case holds. Assume, in contrast, that \(x^c(p_B^*, \bar{t}) > \alpha\). Firm \(B\) chooses its price to maximize the profit \(\Pi_B^{T_0|\varnothing_\alpha} = \iiint_{x^c} [f_t p_B] dxdt\). The optimal price solves \(p_B [2\ln (p_B/\bar{t}(4\alpha - 1))] - 1] + \bar{t}(4\alpha - 1) = 0\). There is no analytical solution to this problem, the value \(p_B = 0.28\bar{t}(4\alpha - 1)\) is, however, a good numerical approximation which fulfills the second-order condition. The inequality \(x^c(0.28\bar{t}(4\alpha - 1), \bar{t}) > \alpha\) requires that \(k > 3.6\), which contradicts the assumption \(k \leq 2\). Hence, \(x^c(p_B^*, \bar{t}) > \alpha\) is not possible in equilib-
rium. Assume further that $p_B^*$ satisfies $x^c(t, p_B^*) \leq \alpha$. The optimal price of firm $B$ maximizes the profit $\Pi_B^{T_t|\varnothing_\alpha} = \int_0^\alpha \int f_t(p_B) \, dx dt$, which yields $p_B = (4\alpha - 1) H(t, \tilde{t})/2$. The inequality $x^c((4\alpha - 1) H(t, \tilde{t})/2, t) \leq \alpha$ requires that $2\ln k - k + 1 \geq 0$, which is true under the assumption $k \leq 2$. Hence, $p_B^* = (4\alpha - 1) H(t, \tilde{t})/2$. Firm $A$’s profits are computed as $\Pi_A^{T_t|\varnothing_\alpha} = \int_0^\alpha \int [f_t(p_B^* + t)/2] \, dx dt$. Firms’ profits are $\Pi_A^{T_t|\varnothing_\alpha} = (16\alpha^2 + 8\alpha - 3) H(t, \tilde{t})/32 + A(t, \tilde{t})/8$ and $\Pi_B^{T_t|\varnothing_\alpha} = (4\alpha - 1)^2 H(t, \tilde{t})/16$. Consider now $t = 0$. Maximization of $\Pi_B^{T_t|\varnothing_\alpha} = \int_0^\alpha \int (f_t p_B) \, dx dt$ yields $p_B^* \approx 0.28\tilde{t}(4\alpha - 1)$. Firm $A$’s profits are computed as $\Pi_A^{T_t|\varnothing_\alpha} = \int_0^\alpha \int [f_t (p_B^* - t(2\alpha - 1))]/dx dt + \int_0^\alpha \int [f_t (p_B^* + t)/2] \, dx dt$. Firms realize profits $\Pi_A^{T_t|\varnothing_\alpha} \approx \tilde{t}(0.42\alpha^2 + 0.07\alpha + 0.02)$ and $\Pi_B^{T_t|\varnothing_\alpha} \approx \tilde{t}(0.41\alpha^2 - 0.2\alpha + 0.03)$. This completes the proof of the Claim 1.

**Claim 2.** Consider the information scenario $\{T_\alpha, \varnothing_\alpha\}$ and assume $\alpha \leq 1/4$. In equilibrium firms charge prices $p_A^*(t) = t(1 - 2\alpha)$ and $p_B^* = 0$. Firm $A$ serves all consumers with $x \leq \alpha$. Firms realize profits $\Pi_A^{T_t|\varnothing_\alpha} = \alpha(1 - 2\alpha)A(t, \tilde{t})$ and $\Pi_B^{T_t|\varnothing_\alpha} = 0$.

**Proof of Claim 2.** We start from the second stage where firm $A$ moves. Maximization of firm $A$’s profit for a given $t$ yields the equilibrium strategy $p_A(p_B, t) = p_B - t(2\alpha - 1)$. For any $p_B$ firm $A$ serves all consumers in the region $x \leq \alpha$, such that firm $B$ cannot do better than charging $p_B^* = 0$. The profits of firm $A$ are computed as $\Pi_A^{T_t|\varnothing_\alpha} = \int_0^\alpha \int [f_t (1 - 2\alpha)] \, dx dt = \alpha(1 - 2\alpha)A(t, \tilde{t})$. This completes the proof of Claim 2.

**Claim 3.** Consider the information scenario $\{X_\alpha T_\alpha, \varnothing_\alpha\}$ and assume $\alpha > 1/2$. If $t = 0$, then in equilibrium firms charge prices $p_A^*(t, x) = \max\{0, t(1 - 2x) + p_B\}$ and $p_B^* \approx 0.28\tilde{t}(2\alpha - 1)$. If $t \leq p_B^*/(2\alpha - 1)$, then firm $A$ serves all consumers with $x \leq \alpha$. If $t > p_B^*/(2\alpha - 1)$, then firm $A$ serves consumers with $x \leq 1/2 + p_B^*/(2t)$. Firms realize profits $\Pi_A^{X_\alpha T_\alpha|\varnothing_\alpha} \approx \tilde{t}(0.12\alpha^2 + 0.2\alpha - 0.005)$ and $\Pi_B^{X_\alpha T_\alpha|\varnothing_\alpha} \approx 0.05\tilde{t}(2\alpha - 1)^2$. If $\tilde{t} > 0$ and $k \leq 2$, then $p_A^*(t, x) = \max\{0, t(1 - 2x) + p_B\}$ and $p_B^* = (2\alpha - 1) H(t, \tilde{t})/2$. For any $t$ firm $A$ serves consumers with $x \leq 1/2 + p_B^*/(2t)$. Firms realize profits $\Pi_A^{X_\alpha T_\alpha|\varnothing_\alpha} = (2\alpha - 1)(2\alpha + 3) H(t, \tilde{t})/16 + A(t, \tilde{t})/4$ and $\Pi_B^{X_\alpha T_\alpha|\varnothing_\alpha} = (2\alpha - 1)^2 H(t, \tilde{t})/8$.

**Proof of Claim 3.** We start from the second stage where firm $A$ moves and maximizes its profit given $p_B$. Firm $A$’s equilibrium strategy is $p_A(p_B, t, x) = \max\{0, t(1 - 2x) + p_B\}$, which yields $t^c(p_B, x) = p_B/(2x - 1)$ and $x^c(p_B, t) = 1/2 + p_B/(2t)$. Assume $\tilde{t} > 0$ and $k \leq 2$. Depending
on the relation between $x^c(p^*_B, t)$ and $\alpha$ two cases are possible in equilibrium: $x^c(p^*_B, t) > \alpha$ and $x^c(p^*_B, t) \leq \alpha$. We show that only the latter case applies. Assume, in contrast, that $x^c(p^*_B, t) > \alpha$. The optimal price of firm $B$ maximizes the profit $\Pi_B^{X^\alpha T_\alpha |\sigma_\alpha} = \int_0^T \alpha \int_{t' \in (p_B, \alpha) x^c} f(t)p_B \, dt \, dx$ and solves $\bar{t}(2\alpha - 1) - p_B - 2p_B \ln [\bar{t}(2\alpha - 1)/p_B] = 0$. There is no analytical solution to the latter equation, however, the value $p_B = 0.28\bar{t}(2\alpha - 1)$ yields a good numerical approximation, which fulfills the second-order condition. The inequality $x^c(p^*_B, t) > \alpha$ requires that $k > 3.6$, which contradicts the assumption $k \leq 2$. Hence, $x^c(p^*_B, t) > \alpha$ is not possible in equilibrium. Assume further that $p^*_B$ satisfies $x^c(p^*_B, t) \leq \alpha$. The optimal price of firm $B$ maximizes the profit $\Pi_B^{X^\alpha T_\alpha |\sigma_\alpha} = \int_0^\bar{t} \int_0^\alpha \int_{t' \in (p_B, \alpha) x^c} f(t)p_B \, dt \, dx \, dt$, which yields $p_B = (2\alpha - 1) H(\bar{t}, \bar{t})/2$. The inequality $x^c((2\alpha - 1) H(\bar{t}, \bar{t})/2, t) \leq \alpha$ requires that $k - 1 - 2\ln k < 0$, which is fulfilled for $k \leq 2$. Hence, $p_B^* = (2\alpha - 1) H(\bar{t}, \bar{t})/2$. Firm $A$’s profits are computed as $\Pi_A^{X^\alpha T_\alpha |\sigma_\alpha} = \int_0^\bar{t} \alpha \int_0^\alpha \int_{t' \in (p_B, \alpha) x^c} f(t)(p^*_B + t(1 - 2x)) \, dt \, dx \, dt$. Firms’ profits are $\Pi_A^{X^\alpha T_\alpha |\sigma_\alpha} = (2\alpha - 1)(2\alpha + 3) H(\bar{t}, \bar{t})/16 + A(\bar{t}, \bar{t})/4$ and $\Pi_B^{X^\alpha T_\alpha |\sigma_\alpha} = (2\alpha - 1)^2 H(\bar{t}, \bar{t})/8$. Consider finally $\bar{t} = 0$. Firm $B$ maximizes the profit $\Pi_B^{X^\alpha T_\alpha |\sigma_\alpha} = \int_0^\alpha \int_0^\alpha \int_{t' \in (p_B, \alpha) x^c} f(t)(p_B^* + t(1 - 2x)) \, dt \, dx \, dt$, which yields $p_B^* \approx 0.28\bar{t}(2\alpha - 1)$. Firm $A$’s profits are computed as $\Pi_A^{X^\alpha T_\alpha |\sigma_\alpha} = \int_0^\bar{t} \int_0^\alpha \int_{t' \in (p_B, \alpha) x^c} f(t)(p^*_B + t(1 - 2x)) \, dt \, dx \, dt$. Firms’ profits are $\Pi_A^{X^\alpha T_\alpha |\sigma_\alpha} \approx 7(0.12\alpha^2 + 0.2\alpha - 0.005)$ and $\Pi_B^{X^\alpha T_\alpha |\sigma_\alpha} \approx 0.05\bar{t}(2\alpha - 1)^2$. This completes the proof of Claim 3.

Claim 4. Consider the information scenario $\{X^\alpha T_\alpha, \sigma_\alpha\}$ and assume $\alpha \leq 1/2$. In equilibrium firms charge prices $p_A^*(t, x) = t(1 - 2x)$ and $p_B^* = 0$. Firm $A$ serves all consumers with $x \leq \alpha$. Firms realize profits $\Pi_A^{X^\alpha T_\alpha |\sigma_\alpha} = \alpha(1 - \alpha) A(\bar{t}, \bar{t})$ and $\Pi_B^{X^\alpha T_\alpha |\sigma_\alpha} = 0$.

Proof of Claim 4. We start from the second stage where firm $A$ moves. For any $p_B$ the equilibrium strategy of firm $A$ is $p_A(p_B, t, x) = t(1 - 2x) + p_B$, such that firm $A$ serves all consumers in the region $x \leq \alpha$. Then firm $B$ cannot do better than charging $p_B^* = 0$. The profits of firm $A$ are computed as $\Pi_A^{X^\alpha T_\alpha |\sigma_\alpha} = \int_0^\alpha \int_0^\alpha \int_{t' \in (p_B, \alpha) x^c} f(t)(1 - 2x) \, dt \, dx \, dt = \alpha(1 - \alpha) A(\bar{t}, \bar{t})$. This completes the proof of Claim 4.

Claim 5. Consider the information scenario $\{T_\alpha, T_\alpha\}$ and assume $\alpha > 1/4$. In equilibrium firms charge prices $p_A^*(t) = t(4\alpha + 1)/4$ and $p_B^*(t) = t(4\alpha - 1)/2$. Firm $A$ serves all consumers with $x < \alpha/2 + 1/8$. Firms realize profits $\Pi_A^{T_\alpha |\sigma_\alpha} = (4\alpha + 1)^2 A(\bar{t}, \bar{t})/32$ and $\Pi_B^{T_\alpha |\sigma_\alpha} = (4\alpha - 1)^2 A(\bar{t}, \bar{t})/16$. 

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Proof of Claim 5. We start from the second stage where firm $A$ moves. Maximization of firm $A$'s profit for a given $t$ yields the equilibrium strategies $p_A(p_B, t) = (p_B + t)/2$ if $p_B < t(4\alpha - 1)$ and $p_A(p_B, t) = p_B - t(2\alpha - 1)$ if $p_B \geq t(4\alpha - 1)$. Assume that for any $t$ firm $B$ chooses $p_B$ such that $p_B < t(4\alpha - 1)$ and $x^c(t, p_B) = 1/4 + p_B/(4t)$. Maximization of firm $B$'s profit for a given $t$ yields $p_B^*(t) = t(4\alpha - 1)/2$, such that $p_B^*(t) < t(4\alpha - 1)$ indeed holds. Firm $A$'s price is $p_A^*(t) = t(4\alpha + 1)/4$ and $x^c = \alpha/2 + 1/8$. Firms' profits are computed as $\Pi_A^{T_\alpha|T_\alpha} = \int \int [ft(4\alpha + 1)/4] \, dxdt$ and $\Pi_B^{T_\alpha|T_\alpha} = \int \int [ft(4\alpha - 1)/2] \, dxdt$. Firms realize profits $\Pi_A^{T_\alpha|T_\alpha} = (4\alpha + 1)^2 A(t, \bar{t})/32$ and $\Pi_B^{T_\alpha|T_\alpha} = (4\alpha - 1)^2 A(t, \bar{t})/16$. This completes the proof of Claim 5.

Claim 6. Consider the information scenario $\{T_\alpha, T_\alpha\}$ and assume $\alpha \leq 1/4$. In equilibrium firms charge prices $p_A^*(t) = t(1 - 2\alpha)$ and $p_B^*(t) = 0$. Firm $A$ serves all consumers with $x \leq \alpha$. Firms realize profits $\Pi_A^{X_\alpha T_\alpha|T_\alpha} = \alpha(1 - 2\alpha)A(t, \bar{t})$ and $\Pi_B^{X_\alpha T_\alpha|T_\alpha} = 0$.

Proof of Claim 6. The proof is analogous to that of Claim 2.

Claim 7. Consider the information scenario $\{X_\alpha T_\alpha, T_\alpha\}$ and assume $\alpha > 1/2$. In equilibrium firms charge prices $p_A^*(t, x) = \max \{0, t(1 - 2x) + p_B^*(t)\}$ and $p_B^*(t) = t(2\alpha - 1)/2$. Firm $A$ serves consumers with $x \leq (2\alpha + 1)/4$. Firms realize profits $\Pi_A^{X_\alpha T_\alpha|T_\alpha} = (2\alpha + 1)^2 A(t, \bar{t})/16$ and $\Pi_B^{X_\alpha T_\alpha|T_\alpha} = (2\alpha - 1)^2 A(t, \bar{t})/8$.

Proof of Claim 7. We start from the second stage where firm $A$ moves given $p_B$. The equilibrium strategy of firm $A$ is $p_A(p_B, t, x) = \max \{0, t(1 - 2x) + p_B\}$. Maximization of firm $B$'s profit for a given $t$ yields the equilibrium price $p_B^*(t) = t(2\alpha - 1)/2$ and $x^c = (2\alpha + 1)/4$. Firms' profits are computed as $\Pi_A^{X_\alpha T_\alpha|T_\alpha} = \int \int [ft((1 - 2x) + (2\alpha - 1)/2)] \, dxdt$ and $\Pi_B^{X_\alpha T_\alpha|T_\alpha} = \int \int [ft(2\alpha - 1)/2] \, dxdt$. Firms realize profits $\Pi_A^{X_\alpha T_\alpha|T_\alpha} = (2\alpha + 1)^2 A(t, \bar{t})/16$ and $\Pi_B^{X_\alpha T_\alpha|T_\alpha} = (2\alpha - 1)^2 A(t, \bar{t})/8$. This completes the proof of Claim 7.

Claim 8. Consider the information scenario $\{X_\alpha T_\alpha, T_\alpha\}$ and assume $\alpha \leq 1/2$. In equilibrium firms charge prices $p_A^*(x, t) = t(1 - 2x)$ and $p_B^*(t) = 0$. Firm $A$ serves all consumers with $x \leq \alpha$. Firms realize profits $\Pi_A^{X_\alpha T_\alpha|T_\alpha} = \alpha(1 - \alpha)A(t, \bar{t})$ and $\Pi_B^{X_\alpha T_\alpha|T_\alpha} = 0$.

Proof of Claim 8. The proof is analogous to that of Claim 4.

Q.E.D.
Proof of Proposition 7. Most of the comparisons of firms’ joint profits are straightforward. Here we only consider the scenario \{X_\alpha T_\alpha , \emptyset \alpha \} for relatively differentiated consumers. We have

$$\Pi_{A+B}^X - \Pi_{A+B}^{X_\alpha T_\alpha} = 0.055\alpha^2 - 0.125\alpha + 0.049 = 0.055(\alpha - 1.77)(\alpha - 0.5),$$

which is negative for any \( \alpha > 1/2 \), such that dataset \( T_\alpha \) is not shared in \{X_\alpha T_\alpha , \emptyset \alpha \}. We next turn to the welfare effects of data sharing. In all the relevant information scenarios social welfare in the region \( x \leq \alpha \) can be computed as

$$SW = \alpha v - \int_0^\infty \int_0^x f(tx) dx dt - \int_0^\infty \int_0^\infty f(t(1-x)) dx dt.$$ 

We start from the scenario \{X_\alpha T_\alpha , \emptyset \alpha \}. We get

$$SW^{X_\alpha T_\alpha} = \alpha v + (2\alpha^2 - 4\alpha + 1) A(t, \bar{t})/4 - (2\alpha - 1)^2 H(t, \bar{t})/16$$

and

$$SW^{X_\alpha T_\alpha|T_\alpha} = \alpha v + (4\alpha^2 - 12\alpha + 3) A(t, \bar{t})/16.$$ 

Straightforward comparison shows that \( SW^{X_\alpha T_\alpha} > SW^{X_\alpha T_\alpha|T_\alpha} \) for any \( \alpha \). As firms’ joint profits increase following data sharing, consumer surplus decreases. We now consider the scenario \{T_\alpha , \emptyset \alpha \}. We get

$$SW^{T_\alpha} = \alpha v + \frac{1}{16} (8\alpha^2 - 16\alpha + 3) A(t, \bar{t})/16 - \frac{1}{24} (16\alpha^2 - 24\alpha + 5) H(t, \bar{t})/64$$

and

$$SW^{T_\alpha|T_\alpha} = \alpha v + \frac{1}{16} (16\alpha^2 - 40\alpha + 7) A(t, \bar{t})/64.$$ 

Straightforward comparison shows that \( SW^{T_\alpha} > SW^{T_\alpha|T_\alpha} \) for any \( \alpha > 1/4 \). Subtracting firms’ joint profits from social welfare we get

$$CS^{T_\alpha} = \alpha v - 7\frac{A(t, \bar{t})}{16} - \frac{96\alpha^2 - 16\alpha - 5}{24} H(t, \bar{t})/64$$

and

$$CS^{T_\alpha|T_\alpha} = \alpha v - \frac{24\alpha^2 - 24\alpha - 1}{24} A(t, \bar{t})/64.$$ 

We introduce the function

$$f(k, \alpha) := 64 \frac{(CS^{T_\alpha}) - (CS^{T_\alpha|T_\alpha})}{t},$$

such that \( f(k, 1) > 0 \) and

$$\lim_{\alpha \to 1/4} f(k, \alpha) < 0 \text{ for any } k \leq 2.$$ 

Note that for any \( k \leq 2 \), \( f(k, \alpha) \) is a continuous monotone function in \( \alpha \).

$$\partial f(k, \alpha)/\partial \alpha = (160\alpha + 24) A(t, \bar{t})/t - (192\alpha - 16) H(t, \bar{t})/t > 0 \text{ holds for any } k \leq 2 \text{ and any } 1/4 < \alpha \leq 1.$$ 

Hence, there exists \( \alpha(k) \) such that \( f(k, \alpha(k)) = 0 \). Then

$$f(k, \alpha) < 0 \text{ (} f(k, \alpha) > 0 \text{) if } \alpha < \alpha(k) \text{ (} \alpha > \alpha(k) \text{). Q.E.D.}$$
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