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Heterogeneous Households in a Sticky-Price Model

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Abstract

This paper introduces heterogeneous households into an otherwise standard sticky-price model with industry-specific labor markets. Households differ in labor incomes and asset markets are incomplete. I show that household heterogeneity affects equilibrium dynamics nontrivially by amplifying price stickiness endogenously through wealth effects on labor supply. To quantify the importance of household heterogeneity in amplifying stickiness, I estimate and compare representative and heterogeneous household models. The quantitative exercise shows the heterogeneous household model performs better than its representative counterpart in accounting for aggregate and sectoral dynamics in the U.S., while being more consistent with empirical evidence on nominal rigidity at the aggregate and sectoral levels, thanks to the stickiness endogenously generated by the model. (*JEL C51, E13, E31, E32, E44, J20*)

Keywords: heterogeneity, price stickiness, multiple sectors, DSGE model, Bayesian estimation, real rigidities

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1 Introduction

The first generation of dynamic stochastic general equilibrium (DSGE) models, best exemplified by Kydland and Prescott (1982) and King, Plosser and Rebelo (1988), assumed away heterogeneity among households. Over the past two decades, a large body of literature has investigated how heterogeneous households can affect the aggregate dynamics of equilibrium prices and quantities.¹ However, most important works in this literature assume that prices are fully flexible, precluding consideration of the aggregate effects of household heterogeneity via nominal rigidity. To address this, I take an otherwise prototype *sticky-price* DSGE model, one of the workhorse models for the analysis of monetary policy and business cycles², and introduce heterogeneous households. Households differ because each possesses a labor skill specialized exclusively for a certain industry, and asset markets are incomplete. I show that household heterogeneity through interactions with nominal rigidity, affects the equilibrium dynamics of key macroeconomic variables quantitatively as well as qualitatively. The main finding is that household heterogeneity amplifies price stickiness endogenously³: prices adjust more slowly in response to economic shocks.

In principle, "endogenous stickiness" generated by household heterogeneity can reconcile a long-standing inconsistency between macro-model specifications and micro-level empirical evidence. At the *macro* level, sticky-price DSGE models often require a large degree of nominal rigidity to generate persistent real effects and inertial inflation. In contrast, *micro* level data indicate that nominal rigidity is weak.⁴ For example, Bils and Klenow (2004) and Nakamura and Steinsson (2008) (henceforth BK and NS respectively) document that firms update their prices less than every 2 quarters,⁵ while the estimated frequency of price changes is often greater than one year using standard sticky-price models. Whether this inconsistency will be fully resolved depends on quantitative importance of household heterogeneity.

To highlight the quantitative importance, I fit two versions of a sticky-price DSGE model, one with and one without household heterogeneity, to the U.S. time series data employing a Bayesian estimation method. I refer to these models as the heterogeneous household and representative

¹This research agenda is relatively young, but is growing rapidly, partly due to the development of faster computing machines. Important early contributions include Huggett (1993), Aiyagari (1994), Krusell and Smith (1998), and many other articles cited in the review paper by Heathcote et al.(2009).

²The standard sticky-price models (or New Keynesian models) are extensively discussed in many graduate level textbooks such as Woodford (2003), Walsh (2003), and Gali (2008), and also in an earlier paper by Goodfriend and King (1997). Aoki (2001), Benigno (2004), Clarida, Gali, and Gertler (1999, 2002), Erceg, Henderson, and Levin (2000), Benigno and Woodford (2007) and others document various issues in monetary policies in New Keynesian framework. See Christiano et al. (2005) and Smet and Wouters (2003, 2007) for leading examples of medium-scale sticky-price DSGE models. In this paper, I use the two terms, "sticky-price models" and "New Keynesian (NK) models" interchangeably.

³Endogenously amplified price stickiness is often referred to as "*real rigidity*" or "*endogenous stickiness*" in the literature to distinguish it from exogenous nominal rigidity in sticky-price models (Ball and Romer, 1990 and Chari et al., 2000).

⁴The inconsistency is perhaps summarized best by Altig et al. (2004): "*Macroeconomic and microeconomic data paint conflicting pictures of price behavior. Macroeconomic data suggest that inflation is inertial. Microeconomic data indicate that firms change prices frequently.*"

⁵This empirical finding was obtained without excluding temporary sales.

household models respectively (or the \mathcal{HH} and \mathcal{RH} models for short). The quantitative exercise shows that household heterogeneity improves the match with empirical evidence on price adjustments at the aggregate and sector levels.

As a benchmark, I first estimate the models under the assumption that every firm in the economy has the same degree of nominal rigidity (i.e. same frequency of price changes), the conventional approach in the literature. The estimation results indicate that in the \mathcal{RH} model, firms update their prices every 4.65 quarters, while in contrast, with the \mathcal{HH} model the average duration of a price contract is between 1.51 and 3.81 quarters depending on the degree of financial market friction. Therefore, in the benchmark case, the heterogeneous household model appears to match better the empirical evidence on nominal rigidity.

I subsequently relax the assumption of identical nominal rigidity and consider the more general case in which the model economy consists of multiple sectors with potentially different frequencies of price changes as in Carvalho (2006), Benigno (2004), and Aoki (2001). I do this for several reasons. First, empirical studies, such as BK and NS, show that nominal rigidities are different across sectors. Using a multiple-sector model, I can investigate whether the model-implied price adjustment frequencies match both the average or "*aggregate frequency*" and sector level frequencies suggested by the empirical studies. Second, even when focusing on the average frequency of price changes, the estimated average with multiple sectors and different frequencies is generally different from the average estimated imposing identical nominal rigidity for every firm. The empirical papers, such as BK and NS, first consider sector frequencies and then use these to calculate an *aggregate frequency*. Therefore, estimating the models with multiple sectors better mimics the estimation procedure employed in these empirical papers. Third, there is a growing interest in price dynamics with distinct aggregate and sector-specific shocks (e.g. Boivin et al., 2009). Therefore, "multiple-sector-sticky-price DSGE models" are interesting in their own right.

Allowing different degrees of nominal rigidity across sectors has non-trivial effects on the estimated *aggregate frequency*. The average duration of price contracts implied by the weighted average of the estimated sector frequencies is 1.74 quarters in the \mathcal{RH} model and 1.39 quarters in the \mathcal{HH} model. Thus with multiple sectors, even the \mathcal{RH} model is characterized by frequent price changes. This result is mainly driven by the fact that differences in nominal rigidity across sectors can themselves generate real rigidity. Nevertheless, the \mathcal{HH} model still comes closer to matching the empirical frequencies at both the aggregate and sector levels.⁶

These results are obtained because the more generalized structure of the model allows for interactions between household heterogeneity, incomplete asset markets and multiple sectors with different price stickiness. To see this, first consider household heterogeneity and incomplete asset markets. The standard sticky-price model posits monopolistic competition with differentiated goods, each good requiring a distinct labor skill for production (Woodford 2003). Labor markets are thus segmented, and households face different real wage rates and generate different labor incomes.

⁶The \mathcal{HH} model is less successful in matching the empirical evidence on price changes in the durable goods sectors. This is primarily due to the fact that the model treats all goods equally, as if they were all non-durable goods.

With a representative household model, this household heterogeneity becomes irrelevant because household consumption levels are equalized through costless trading of state-contingent assets. Consequently variations in the household's real wage do not affect that household's consumption choices. In contrast, when risk-sharing is imperfect due to asset market frictions, household's labor income (and hence the wage) affects the household's consumption levels, which in turn influences that household's incentive to supply labor through wealth effect. Furthermore, households' labor supply decisions affect firms' pricing decisions. In particular, household heterogeneity, with incomplete asset markets, endogenously amplifies price stickiness *through two types of wealth effects*: (i) a within-sector wealth effect and (ii) a cross-sector wealth effect.

The within-sector wealth effect leads to a smaller wage elasticity of each household labor supply: other things being equal, a rise in the real wage (and thus in labor income) increases household's consumption level, and consequently the household has less incentive to supply labor. The less-elastic labor supply (steeper labor supply curve) in turn makes each firm's marginal cost more sensitive to a change in the firm's price and output. Thus the within-sector wealth effect amplifies price stickiness universally for every firm in all sectors, and results in slower aggregate price adjustments. Consider a firm that contemplates lowering its price. A lower price would induce more demand for its product. However to meet the increased product demand the firm requires more labor hours, which would shift the labor demand curve to the right. This would raise the equilibrium wage rate on its own, but in addition the firm faces a steeper labor supply curve because of incomplete asset markets, which would further raise the wage rate, and thus marginal cost of the firm. Therefore, to protect its profit margin the firm *reduces its price by a smaller amount* than it would if the asset markets were perfect.

The cross-sector wealth effect creates strategic complementarity in pricing decisions across sectors. However, unlike the within-sector wealth effect, it affects sectors differently. It *decreases* price rigidity in low-frequency sectors, the sectors with a relatively high degree of nominal rigidity, and *increases* price stickiness in high-frequency sectors. Because at any given time the number of firms adjusting prices in the high-frequency sectors outnumber those in the low-frequency sectors, the high-frequency sectors dominate the process of aggregate price adjustment. Therefore the net effect of the cross-sector wealth effect is to generate slower aggregate price adjustments.

Consider an example. In response to an economy-wide positive productivity shock, a typical high-frequency sector price would decrease more than a low-frequency sector price, that is, a relative price of high-frequency to low-frequency sector decreases. This in turn leads to: a high relative demand for high-frequency sector's good; a high relative demand for high-frequency sector labor; a high relative wage in high-frequency sector; a high relative consumption level in high-frequency sector; and thus an increase in marginal cost due to wealth effect. Therefore firms in high-frequency sectors would decrease their prices by *less* than the amount they would if the asset markets were complete.⁷ The exact opposite process happens in low-frequency sectors: households, due to a

⁷If the asset markets were complete, relative consumption would be constant (since it is not affected by a change in relative price), and consequently there would be no such wealth effect.

lower consumption level, would be willing to provide more labor hours, which would further reduce marginal cost of the firms. Consequently these firms would decrease their prices by *more* than the amount they would if the asset markets were complete. Because high-frequency firms dominate aggregate price dynamics, the end result is a slower decline in aggregate price level than with complete asset markets.

It is important to note that, even though household heterogeneity at the firm level (or at industry level, as in Woodford 2003) is necessary for a within-sector wealth effect, it is not necessary for endogenous stickiness (or real rigidity). The cross-sector wealth effect continues to provide a separate channel for real rigidity when households are heterogeneous only at the sector level (i.e. labor markets are segmented at the sector level). Thus we have endogenous price stickiness even with this 'weaker' form of household heterogeneity.

This paper fits well into the growing literature on the aggregate implications of including heterogeneous households in quantitative macroeconomic models and the literature on real rigidity.⁸ This paper builds a bridge between these two active research areas by showing household heterogeneity can be a source of real rigidity, through its impact on the household labor supply, and thus can significantly affect aggregate dynamics. Some authors recently examined the consequences of various sources of real rigidity for model-implied nominal rigidity. Some important works include Altig et al. (2004), Eichenbaum and Fisher (2007), and Woodford (2005). This paper is different both because of the specific source of real rigidity in the model, and the estimation method used. I employ the likelihood-based Bayesian method,⁹ an important innovation because of the value of incorporating the entire set of predictions from the general equilibrium of the models. Instead of using the whole set of equilibrium conditions, Eichenbaum and Fisher (2007) focus on the aggregate supply equation (i.e. Phillips curve), and estimate the frequency of price changes using a generalized method of moments. Altig et al. (2004), choose the parameters that minimize the distance between the model and VAR based impulse responses. Another key difference is that this paper allows for multiple sectors with potentially different degrees of nominal rigidity, which as reported above, has non-trivial consequences for estimation.

The rest of the paper is organized as follows. In the next section, I present a simple static model to present the main theoretical results in the most explicit way. Section 3 introduces the full-blown DSGE models with Calvo-style sticky prices. The DSGE models developed here will be the basis for the quantitative exercises in following sections. Section 4 presents a generalized New Keynesian (NK) Phillips curve and details the mechanism through which household heterogeneity influences equilibrium dynamics. In section 5, I estimate the representative and heterogeneous household models and compare the two based on the estimation results match with empirical

⁸Some of the earlier works on real rigidity include Ball and Romer (1990), Kimball (1995), Basu (1995), and Bergin and Feenstra (2000). These papers have identified various sources that can amplify monetary non-neutrality. Chari et al. (2000) also have stressed importance of endogenous stickiness. They argue that sticky-price models need to amplify price stickiness endogenously to explain persistent aggregate dynamics with a reasonable degree of nominal rigidity.

⁹For recent contributions, see Smet and Wouters (2003, 2007), Rabanal and Rubio-Ramirez (2005), Lubik and Schorfheide (2005), and many other papers cited in An and Schorfheide (2006).

evidence. Section 6 summarizes the results and concludes.

2 Static Model

The key motivation for presenting the static model is that I can show my results analytically. However, the model is too simple for serious quantitative analysis. A full-blown DSGE model for that purpose is presented in the next section.¹⁰

The economy is composed of a continuum of industries indexed by $i \in [0, 1]$. Each industry i produces a distinct type of product $Y(i)$. In each industry i , there is a representative firm which I refer to as "type- i firm." The differentiated goods, $\{Y(i)\}$ are aggregated to produce final consumption good Y , through a Dixit-Stiglitz aggregator:

$$Y = \left(\int_0^1 Y(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad (1)$$

where θ is the elasticity of substitution between two products and is assumed to be greater than one. The corresponding price index, P for the final consumption good is

$$P = \left(\int_0^1 P(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \quad (2)$$

where $P(i)$ is the price of the type- i good. The optimal demand for each type of good minimizes total expenditure PY :

$$Y(i) = \left(\frac{P(i)}{P} \right)^{-\theta} Y. \quad (3)$$

Each firm has linear production technology:

$$Y(i) = H(i), \quad (4)$$

where $H(i)$ denotes type- i labor hours. Type- i labor can produce only type- i goods and cannot produce any other type of goods: labor markets are industry-specific. Type- i firm chooses its price $P(i)$ to maximize profit:

$$\Pi(i) = P(i)Y(i) - W(i)H(i),$$

subject to the demand function, (3). I let $W(i)$ denote the competitive wage rate in industry i . The optimality condition then can be obtained as

$$\frac{P(i)}{P} = \vartheta \frac{W(i)}{P}, \quad (5)$$

where $\vartheta = \theta / (\theta - 1)$ is the firm's desired mark-up.

¹⁰Those who have little interest in analytical results can skip this section and go to section 3 without loss of continuity.

For each industry i , there is a representative household which I refer to as "type- i household." Type- i household possesses a labor skill specialized exclusively for industry i . Households choose their consumption level, $C(i)$ and labor hours, $H(i)$ to maximize utility:

$$\log C(i) - H(i),$$

subject to their budget constraint which equates consumption with income:

$$PC(i) = W(i)H(i) + \Pi(i) + K(i); \quad \text{where} \quad K(i) \equiv \lambda \int_{z \neq i} P(z)Y(z)dz; \quad \lambda \in [0, 1]. \quad (6)$$

In addition to labor income $W(i)H(i)$, type- i household receives profit $\Pi(i)$ from industry i .¹¹ In the budget constraint, I include another term, $K(i)$, which is exogenously given, to denote a pre-arranged financial portfolio. Introducing $K(i)$ provides a convenient way to nest both complete and incomplete asset market economies within a single framework. Although there are no state-contingent assets, the pre-arranged financial contracts when $\lambda = 1$, guarantee that households have the same amount of income they would have if the asset markets were complete. The economy is, therefore, *effectively under complete markets* when $\lambda = 1$, and the asset markets are effectively incomplete when $\lambda < 1$. A household's first order condition is given by

$$C(i) = \frac{W(i)}{P}, \quad (7)$$

which characterizes the household's labor supply.

The model is completed by imposing the quantity equation:

$$PY = M, \quad (8)$$

where M denotes exogenous money supply. The quantity equation can be rationalized by introducing a cash-in-advance constraint which is omitted here for simplicity.

Given M , the equilibrium is characterized by allocations of quantities and prices at industry level, $\{C(i), Y(i), H(i), P(i), W(i)\}_{i \in [0,1]}$, and two aggregate variables, output and price level, $\{Y, P\}$ that satisfy the followings: (i) definitions of the aggregates, (1) and (2); (ii) firms' optimality conditions and production functions, (3), (4) and (5); (iii) households' budget constraints and optimality conditions, (6) and (7); (iv) quantity equation, (8); and (v) market clearing condition, $\int_0^1 C(i)di = Y$.

For a benchmark, I first consider an economy where prices are fully flexible in that firms observe money supply M , before setting prices. It turns out that asset market completeness (or household heterogeneity) does not affect the equilibrium outcome: there exists a unique symmetric equilibrium regardless of asset market completeness. The two main reasons for this result are

¹¹This assumption is not necessary to show the results. But it makes the exposition simpler. In the next section, I consider a more general case.

the symmetric nature of the model and the absence of idiosyncratic shocks. In the absence of idiosyncratic shocks, *ex-ante* symmetric firms choose the same price and produce the same amount of goods. Consequently, households' incomes are symmetric and there is no need for state-contingent asset markets.

Proposition 1 (*Irrelevance of heterogeneous households under flexible prices*) *If prices are flexible in response to changes in the exogenous variable, M , then there exists a unique symmetric equilibrium in which $P(i) = P$ and $Y(i) = C(i) = Y$ for all $i \in [0, 1]$. Moreover, money is neutral: the equilibrium output, Y is determined independently from M . The output and price level are explicitly given by*

$$Y = \frac{1}{\vartheta} \quad \text{and} \quad P = \vartheta M.$$

See the appendix for the proof of this (and all other propositions). In what follows I show this "irrelevance result" no longer holds under sticky prices.

Let us consider the simplest case. Suppose that the government announces a certain level of $M = \bar{M}$. Some firms, called sticky-price firms, believe the announcement and set their prices accordingly, while the other firms, called flexible-price firms, wait until they observe actual M .¹² The common price set by sticky-price firms, which believe M is equal to \bar{M} with probability one, must be $\vartheta \bar{M}$ as shown in Proposition 1. If the government indeed keeps its promise, the remaining firms also set the price to $\vartheta \bar{M}$ and the equilibrium output would be equal to ϑ^{-1} . That is, with no surprise in monetary policy, aggregate output would be equal to the flexible-price level of output.

When the government deviates from its announcement, however, money is no longer neutral. The degree of non-neutrality depends on how much the flexible-price firms would respond to a change in M . I show below that responsiveness is smaller, and thus monetary non-neutrality is larger, when asset markets are incomplete.

To see first the impact of wealth effects on a firm's pricing decision, it is useful to combine (7) with (5) and then to express type- i household consumption $C(i)$ in terms of type- i firm's price $P(i)$, employing the household budget constraint, the definition of nominal profit and the demand function for $Y(i)$:

$$\begin{aligned} P(i) &= \vartheta W(i) = \vartheta PC(i) = \vartheta \left\{ P(i)Y(i) + \lambda \int_{z \neq i} P(z)Y(z)dz \right\} \\ &= \vartheta \left\{ (1 - \lambda) P(i)Y(i) + \lambda PY \right\} = \vartheta \underbrace{\left\{ (1 - \lambda) \left(\frac{P(i)}{P} \right)^{1-\theta} PY + \lambda PY \right\}}_{PC(i)} \end{aligned} \quad (9)$$

Note that type- i firm's real marginal cost is equal to type- i household's consumption. From the second line in (9), when asset markets are incomplete (i.e. when $\lambda < 1$), a change in a firm's price

¹²This amounts to assume that the firms, which set the prices before realization of M , form a subjective probability distribution of M that places entire mass on \bar{M} after the announcement. It is may not be the most realistic or elegant way to model firms' belief. The assumption however helps us to see the main results explicitly

affects its employees' consumption, which in turn influences the firm's marginal cost (i.e. the wage $W(i)$), through the wealth effects on labor supply, in such a way as to discourage the firm from changing its price. In contrast, a household's consumption would not be affected by a firm's price if asset markets were complete,¹³ and thus wealth effects on firms' pricing decisions would not arise.

To study how the optimal price P^* , commonly chosen by flexible-price firms, responds to M , I simply replace $P(i)$ with P^* and PY with M in (9), which leads to

$$P^* = \underbrace{(1 - \lambda) \vartheta \left(\frac{P^*}{P} \right)^{1-\theta}}_{(i)} M + \underbrace{\lambda \vartheta}_{(ii)} M. \quad (10)$$

Equation (10) implicitly determines the equilibrium level of P^* given $\{M, P\}$. A response of P^* to a change in M can be characterized by the sum of the two coefficients, (i) and (ii) on M . The first coefficient (i) is however decreasing in P^* given P since $\theta > 1$, which dampens a response of P^* to a change in M . This dampening effect disappears when asset markets are effectively complete since the first coefficient becomes zero when $\lambda = 1$. Thus, with effectively complete markets, prices respond more to a change in monetary policy.

Finally the price level can be obtained by aggregating the price set by sticky-price firms, $\vartheta \bar{M}$ and the price set by flexible-price firms shown in (10), through the price index, (2). Under complete asset markets (i.e. when $\lambda = 1$), (10) implies $P^* = \vartheta M$, and the price level P_C is consequently given by

$$P_C = \left(n [\vartheta M]^{1-\theta} + (1 - n) [\vartheta \bar{M}]^{1-\theta} \right)^{\frac{1}{1-\theta}},$$

where n denotes the fraction of flexible-price firms. Under incomplete markets, however, λ is less than 1. Let us consider a special case in which $\lambda = 0$ (i.e. financial autarky). Then the price level P_{IC} under incomplete markets is

$$P_{IC} = \left(n \left[[\vartheta M]^{\frac{1}{\theta}} P_{IC}^{\frac{\theta-1}{\theta}} \right]^{1-\theta} + (1 - n) [\vartheta \bar{M}]^{1-\theta} \right)^{\frac{1}{1-\theta}}.$$

Unlike P_C , it is hard to obtain an explicit solution for P_{IC} in terms of the exogenous variables M and \bar{M} only. Nevertheless it can be shown that the price level under incomplete markets does not adjust as much as when asset markets are complete.

Proposition 2 (*Stronger Non-neutralities*) *Let P_F and Y_F denote the price level and output that would be realized when prices are completely flexible (i.e. $Y_F = 1/\vartheta$ and $P_F = \vartheta M$ as shown in Proposition 1). Also let Y_C and Y_{IC} denote aggregate output under complete markets and under incomplete markets respectively. If $M > \bar{M}$, then $P_{IC} < P_C < P_F$ and $Y_{IC} > Y_C > Y_F$, and vice versa.*

As mentioned, the "stronger non-neutralities" result is driven by smaller response of P^* to a

¹³When $\lambda = 1$, $C(i) = \vartheta Y$, as can be seen on the second line in (9).

change in M . This implies that wealth effect also has a distributional implication. Proposition 3 in particular suggests that a model with heterogeneous households and incomplete asset markets can generate stronger comovement among sectors.¹⁴

Proposition 3 (*Stronger Comovements*) *Let Y_1 and Y_2 denote a common level of output produced by sticky-price and flexible-price firms respectively. Then the difference between the two sector outputs, $|Y_1 - Y_2|$ is smaller under incomplete markets than under complete markets.*

Nakamura and Steinsson (2009) have documented that a model with intermediate inputs is more successful, relative to a standard model *without* intermediate input, in accounting for strong comovement, which is a key feature of business cycles (Lucas, 1977; Stock and Watson, 1999). Household heterogeneity plays the same role here.

The results in the propositions are not limited to the case of a monetary policy shock. For instance, the same arguments could be made for aggregate productivity shock if an exogenous productivity shock were added to the model. In the next section, I investigate *how much* the theoretical results obtained here influence aggregate and sector dynamics.

3 Sticky-Price DSGE Model

In this section, I incorporate the ideas explored in the previous section into a prototype DSGE model. Some of the model settings are the same as those in the static model. I however describe every detail for completeness.

There is a continuum of industries indexed by $i \in [0, 1]$, in which there is a representative firm referred to as "type- i firm". Firms produce differentiated goods that are aggregated into final consumption goods. Labor markets are industry-specific: a distinct labor skill is required to produce each type of good. Households are heterogeneous in labor skills: "type- i household" possesses labor skill specialized for industry i , and thus the household supplies labor to type- i firm.

The economy is divided into a finite number of mutually exclusive sectors indexed by $j \in \{1, 2, \dots, J\}$, and sectors are characterized by potentially different degrees of nominal rigidity, $\{\alpha_j\}_{j=1}^J$. I use \mathcal{I}_j to denote the set that contains the industries that belong to sector j . The length of the interval \mathcal{I}_j , denoted by n_j , gives the size of the sector.¹⁵ When the degrees of nominal rigidity are identical across sectors (i.e. $\alpha_j = \alpha \forall j$), the model becomes the standard single-sector model.

¹⁴As shown above, flexible-price firms' price P^* , due to wealth effects, does not deviate much from sticky-price firms' price. Consequently, productions, between the two groups, tend to be closer.

¹⁵Since the sectors are mutually exclusive, it must follow that $\bigcup_{j=1}^J \mathcal{I}_j = [0, 1]$, where \mathcal{I}_j , $j = 1, \dots, J$, are disjoint sets, and that $\sum_{j=1}^J n_j = 1$.

3.1 Households

Type- i household seeks to maximize a discounted sum of utilities of the form

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \Gamma_t \left[\log C_{j,t}(i) - \Xi_t^{-\varphi} \frac{H_{j,t}(i)^{1+\varphi}}{1+\varphi} \right] \right),$$

where $C_{j,t}(i)$ denotes type- i household's consumption and $H_{j,t}(i)$ denotes the hours of labor services supplied to industry i in sector j . There are two aggregate preference shocks denoted by Γ_t and Ξ_t . They serve as aggregate demand and aggregate supply shocks respectively. The parameter β is the discount factor, and φ is the inverse of the Frisch elasticity of labor supply.

The flow budget constraint of the household is given by

$$C_{j,t}(i) + \frac{B_{j,t}(i)}{P_t} + \tilde{\epsilon} \left(\frac{B_{j,t}(i)}{P_t Y_t} \right)^2 = \frac{R_{t-1} B_{j,t-1}(i)}{P_t} + \tau_t + \frac{W_{j,t}(i) H_{j,t}(i)}{P_t} + \frac{K_{j,t}(i)}{P_t}, \quad (11)$$

where τ_t denotes a government net transfer, R_t denotes the gross nominal interest, $W_{j,t}(i)$ is the competitive nominal wage rate in industry i , and P_t is the aggregate price level to be defined below.

Households do not trade state-contingent assets. Instead, they borrow and lend through trading riskless nominal bonds (IOUs). I use $B_{j,t}(i)$ to denote a household's bond holdings at time t . A convex cost, $\tilde{\epsilon} \left(\frac{B_{j,t}(i)}{P_t Y_t} \right)^2$, which I refer to as a "participation cost", is introduced in the budget constraint for two reasons. First, I wish to allow for the possibility of frictions in the private bond market.¹⁶ Heathcote and Perri (2002) considered several distinct specifications including a frictionless bond market and financial autarky. By parametrizing the bond market environment, the participation cost introduced here provides a convenient way for me to study a range of bond market conditions. Thus I interpret the "participation cost" as a reduced form that as $\tilde{\epsilon}$ increases, makes the bond market less ideal.¹⁷ If $\tilde{\epsilon}$ were close to zero, a household would borrow and lend frictionlessly against its future income to smooth consumption. Conversely, if $\tilde{\epsilon}$ were sufficiently large, households would decide not to trade bonds. Alternative ways to model asset market incompleteness would not change the main insight of this paper as long as a household's consumption depended positively on its labor income because of imperfect asset markets. Second, the cost term, when $\tilde{\epsilon} > 0$, makes the model stationary and induces a unique steady state equilibrium around which to linearize.¹⁸ I specify the participation cost in such a way that the *ex-ante* symmetric households hold zero net borrowing in the steady state.

In addition to labor income, $W_{j,t}(i)H_{j,t}(i)$, each household earns capital income, $K_{j,t}(i)$. The

¹⁶ As extensively discussed in Heaton and Lucas (1996), frictions occur even in the supposedly riskless bond markets. For instance, a substantial spread exists between lending and borrowing rates, reflecting the fact that lenders are paying monitoring costs.

¹⁷ The "participation cost" should be distinguished from a "trading cost" which might occur where there is a change in bond holding, $\Delta B_{j,t}(i)$. As noted in Heaton and Lucas, while trading cost is more important in the stock markets (for instance one has to pay commissions to a financial agent when buying or selling stocks), the participation cost is more relevant in bond markets.

¹⁸ See Schmitt-Grohe and Uribe (2003) for further discussions.

monopolistically competitive firms make positive profits which are distributed to households. It is useful to consider two extreme cases. At one extreme, type- i household owns entirely the firms in industry i , but owns zero shares in any other industry. Although this case may not be far from reality for some households, it may be too extreme to capture reality. In the other extreme case, households own the same share of every firm in the economy, so that the economy's entire profit is equally distributed among households. This case is also unrealistic. Shareholders, in the real world, often own a disproportionately larger amount of shares in the industry in which they are employed, but do own some shares in other industries. With this in mind, I specify $K_{j,t}(i)$ in such way as to allow for intermediate cases:

$$K_{j,t}(i) \equiv \chi \left(\sum_{k=1}^J \int_{\mathcal{I}_k} \Pi_{k,t}(z) dz \right) + (1 - \chi) \Pi_{j,t}(i), \quad 0 \leq \chi \leq 1,$$

where $\Pi_{k,t}(z)$ denotes nominal profit of type- z firm in sector k . The parameter, χ controls the degree of concentration of a household's portfolio in its own industry: $\chi = 1$ when a household's capital income is perfectly diversified over the different industries. The parameter χ should be different across households in principle, however a single value is assumed for simplicity. I use 1 as a benchmark value in a calibration exercise, and in a later section, the parameter χ is estimated along with the other model parameters.

The wage in each industry is fully flexible, and both the households and firms in the given industry take the wage as given. A household's first order conditions are then given by

$$1 + 2\tilde{\epsilon} \frac{B_{j,t}(i)}{(P_t Y_t)^2} = \beta R_t E_t \left[\left(\frac{\Gamma_{t+1}}{\Gamma_t} \right) \left(\frac{C_{j,t}(i)}{C_{j,t+1}(i)} \right) \left(\frac{P_t}{P_{t+1}} \right) \right], \quad (12)$$

$$\left(\frac{H_{j,t}(i)}{\Xi_t} \right)^\varphi C_{j,t}(i) = \frac{W_{j,t}(i)}{P_t}. \quad (13)$$

In contrast to the \mathcal{HH} model discussed so far, a representative household supplies every type of labor in the \mathcal{RH} model.¹⁹ The representative household maximizes the discounted sum of utilities:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \Gamma_t \left[\log C_t - \Xi_t^{-\varphi} \sum_{j=1}^J \int_{\mathcal{I}_j} \frac{H_{j,t}(i)^{1+\varphi}}{1+\varphi} di \right] \right),$$

subject to the budget constraint:

$$C_t + \frac{B_t}{P_t} = \frac{R_{t-1} B_{t-1}}{P_t} + \tau_t + \sum_{j=1}^J \int_{\mathcal{I}_j} \frac{W_{j,t}(i) H_{j,t}(i)}{P_t} di + \sum_{j=1}^J \int_{\mathcal{I}_j} \frac{\Pi_{j,t}(i)}{P_t} di.$$

¹⁹ Alternatively, the \mathcal{RH} model can have different types of households as in the \mathcal{HH} model, but with complete asset markets. The resulting equilibrium would be identical.

After imposing the market clearing condition, $C_t = Y_t$, the first order conditions are given by

$$1 = \beta(1 + i_t)E_t \left[\left(\frac{\Gamma_{t+1}}{\Gamma_t} \right) \left(\frac{Y_t}{Y_{t+1}} \right) \left(\frac{P_t}{P_{t+1}} \right) \right],$$

$$\left(\frac{H_{j,t}(i)}{\Xi_t} \right)^\varphi Y_t = \frac{W_{j,t}(i)}{P_t}. \quad (14)$$

It is helpful to compare (14) to (13) in developing an intuition on how household heterogeneity leads to a greater degree of real rigidity. In the \mathcal{HH} model, type- i household's consumption $C_{j,t}(i)$ depends positively on its labor income due to imperfect risk-sharing. Thus $C_{j,t}(i)$ depends positively on real wage, $\frac{W_{j,t}(i)}{P_t}$ as well as labor hours, $H_{j,t}(i)$, which makes the wage elasticity of labor supply smaller. As a consequence, type- i firm's marginal cost becomes more sensitive to a change in its price and production. There is no such channel in the \mathcal{RH} model since each industry is so small that the industry wage rate $\frac{W_{j,t}(i)}{P_t}$ or labor hours $H_{j,t}(i)$ do not affect directly aggregate output, Y_t .

3.2 Firms

Aggregate output, Y_t is produced by perfectly competitive firms using sector outputs, $\{Y_{j,t}\}_{j=1}^J$ with a Dixit-Stiglitz production technology:

$$Y_t = \left(\sum_{j=1}^J (n_j D_{j,t}^R)^{1/\eta} Y_{j,t}^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)}, \quad (15)$$

where η is the elasticity of substitution among sector outputs, and $D_{j,t}^R$ is given by $D_{j,t}^R \equiv D_{j,t}/D_t$, which can be interpreted as a sector-specific demand shock relative to overall strength of demand, $D_t \equiv \sum_{j=1}^J n_j D_{j,t}$. The appropriate price index for the final consumption good is the minimum cost that the firms should pay for producing one unit of the consumption good and is given by

$$P_t = \left(\sum_{j=1}^J (n_j D_{j,t}^R) P_{j,t}^{1-\eta} \right)^{1/(1-\eta)}. \quad (16)$$

Given the aggregate consumption good Y_t , and the price levels, $P_{j,t}$ and P_t , the optimal demand for a sector good would minimize total expenditure $P_t Y_t$:

$$Y_{j,t} = n_j D_{j,t}^R \left(\frac{P_{j,t}}{P_t} \right)^{-\eta} Y_t \quad \forall j. \quad (17)$$

Similar to the final consumption good, each sector good, $Y_{j,t}$ is also an aggregate of the goods

$\{Y_{j,t}(i)\}_{i \in \mathcal{I}_j}$ that are produced by the firms in sector j , and is given by

$$Y_{j,t} = \left(\left(\frac{1}{n_j} \right)^{1/\theta} \int_{\mathcal{I}_j} Y_{j,t}(i)^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)} \quad \forall j, \quad (18)$$

where θ is the elasticity of substitution between different types of goods and is larger than one. The corresponding price indexes for a sector good is given by

$$P_{j,t} = \left(\frac{1}{n_j} \int_{\mathcal{I}_j} P_{j,t}(i)^{1-\theta} di \right)^{1/(1-\theta)}, \quad \forall j. \quad (19)$$

Given $Y_{j,t}$, the optimal demand for type- i good $Y_{j,t}(i)$ would be

$$Y_{j,t}(i) = \frac{1}{n_j} \left(\frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\theta} Y_{j,t}. \quad (20)$$

Firms in industry i (i.e. type- i firm) employ labor supplied by type- i household. A firm's production function is given by

$$Y_{j,t}(i) = A_{j,t} H_{j,t}(i), \quad (21)$$

where $A_{j,t}$ is exogenous sector-specific productivity.

Prices are sticky as in Calvo (1983). A firm in sector j adjust its price with probability $1 - \alpha_j$ each period. Since only the fraction $1 - \alpha_j$ of all the prices in that sector is set anew while the remaining fraction α_j of prices is carried over from the previous period, the sector price level $P_{j,t}$ evolves as:

$$P_{j,t} = \left[\frac{1}{n_j} \int_{\mathcal{I}_j^*} P_{j,t}^*(i)^{1-\theta} di + \frac{1}{n_j} \int_{\mathcal{I}_j - \mathcal{I}_j^*} P_{j,t-1}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} = \left[\frac{1}{n_j} \int_{\mathcal{I}_j^*} P_{j,t}^*(i)^{1-\theta} di + \alpha_j P_{j,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (22)$$

where $P_{j,t}^*(i)$ is an optimal price chosen by type- i firm when $i \in \mathcal{I}_j^*$. The set $\mathcal{I}_j^* \subset \mathcal{I}_j$, with measure $n_j(1 - \alpha_j)$, is a randomly chosen subset in which firms can update their prices.

Firms that adjust their prices at time t set the optimal price, $P_{j,t}^*(i)$ maximizing expected discounted profits:

$$\max_{P_{j,t}^*(i)} E_t \sum_{k=0}^{\infty} \alpha_j^k q_{j,t,t+k}(i) \frac{\tilde{\Pi}_{j,t+k}(i)}{P_{t+k}},$$

where $q_{j,t,t+k}(i)$ is a type- i firm's real stochastic discount factor between time t and $t+k$, and $\tilde{\Pi}_{j,t+k}(i)$ is the firm's nominal profit at time $t+k$ given that the price chosen at time t is still being charged:

$$\tilde{\Pi}_{j,t+k}(i) = P_{j,t}(i) Y_{j,t+k}(i) - W_{j,t+k}(i) H_{j,t+k}(i).$$

When asset markets are incomplete and a firm is owned by more than one household, there is no obvious unique stochastic discount factor for a given firm. Each shareholder would want to use

their own stochastic discount factor in maximizing expected discounted profits. Therefore there is a conflict of interests among the shareholders. I assume that a firm maximizes the weighted average of the different objective functions of its shareholders.²⁰ Then type- i firm's discount factor can be expressed as²¹

$$q_{j,t,t+k}(i) = \beta^k \left[(1 - \chi) \left(\frac{\Gamma_t C_{j,t}(i)}{\Gamma_{t+k} C_{j,t+k}(i)} \right) + \chi \left(\sum_{l=1}^J \int_{\mathcal{I}_l} \left(\frac{\Gamma_t C_{l,t}(z)}{\Gamma_{t+k} C_{l,t+k}(z)} \right) dz \right) \right].$$

The first order condition of type- i firm is given by

$$0 = E_t \sum_{k=0}^{\infty} \alpha_j^k q_{j,t,t+k}(i) D_{j,t+k}^R Y_{t+k} \left(\frac{P_{j,t}^*(i)}{P_{j,t+k}} \right)^{-\theta} \left(\frac{P_{j,t+k}}{P_{t+k}} \right)^{-\eta} \left\{ \left(\frac{P_{j,t}^*(i)}{P_{t+k}} \right) - \left(\frac{\theta}{\theta - 1} \right) MC_{j,t+k}(i) \right\}, \quad (23)$$

where $MC_{j,t+k}(i) = \frac{W_{j,t+k}(i)}{A_{j,t+k} P_{t+k}}$ is type- i firm's real marginal cost at time $t + k$. The optimal prices chosen at time t , $\{P_{j,t}^*(i)\}_{i \in \mathcal{I}_j^*}$ that satisfy the first order condition (23) determine the equilibrium dynamics of the sector price level $P_{j,t}$ through (22). The aggregate price dynamic is then determined by aggregating these sector prices through (16).

3.3 Government

The government budget constraint is

$$\frac{B_t - R_{t-1} B_{t-1}}{P_t} + \sum_{j=1}^J \int_{\mathcal{I}_j} \tilde{\epsilon} \left(\frac{B_{j,t}(i)}{P_t Y_t} \right)^2 di = \tau_t + G_t, \quad (24)$$

where B_t is the supply of government bonds and G_t is government expenditure at time t . The government collects participation costs and redistributes them to households as a transfer.

Monetary policy is characterized by a Taylor rule:

$$R_t = \beta^{-1} R_{t-1}^{\rho_m} \left[\left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right]^{(1-\rho_m)} \exp(\mu_t). \quad (25)$$

²⁰ Alternatively one could assume that the household with the largest voting right chooses the firm's discount factor. Under this assumption, the discount factor of type- i firm would be given by $\beta^k \left(\frac{\Gamma_t C_{j,t}(i)}{\Gamma_{t+k} C_{j,t+k}(i)} \right)$. Another alternative assumption would be for the firm to use the real interest rate to discount its profits. In that case, $q_{j,t,t+k}(i)$ would no longer be firm-specific and be instead given by $q_{j,t,t+k}(i) = \prod_{l=0}^k R_{t+l}^{-1} \frac{P_{t+l+1}}{P_{t+l}}$. However, alternative choices of the discount factor do not make any difference quantitatively since only the steady state value of the discount factors enters the equilibrium conditions in the first order approximation. The steady state level of the shadow value of a dollar is identical across households and it is equal to the steady state value of the risk free real interest rate. Pescatori (2006) has also made the same argument.

²¹ Looking at type- i firm's stochastic discount factor, one might think that the firm can manipulate its discount factor by influencing $C_{j,t}(i)$. But it is not the case in this model. Just as each firm takes the industry wage as given, it also takes the discount factor as given. Recall that type- i firm and type- i household only represent respectively the infinitely many firms and households that participate in the same labor market. Therefore type- i firm's stochastic discount factor should in fact be interpreted as "industry- i stochastic discount factor."

I assume a simple Ricardian fiscal policy:

$$G_t = 0 \quad \text{and} \quad B_t = 0. \quad (26)$$

Although Ricardian fiscal policy is a nontrivial assumption for equilibrium as pointed out by Leeper (1992) and Sims (1994), I assume it for a direct comparison with many existing papers on sticky-price models as well as for simplicity.

3.4 Equilibrium

The equilibrium of the economy is characterized by quantities and prices at the micro level:

$$\left\{ \{C_{j,t}(i), Y_{j,t}(i), H_{j,t}(i), B_{j,t}(i), P_{j,t}(i), W_{j,t}(i)\}_{i,j} \}_{t=0}^{\infty}, \right.$$

outputs and prices at the sector level:

$$\left\{ \{Y_{j,t}, P_{j,t}\}_j \right\}_{t=0}^{\infty},$$

and the three aggregate variables: output, price level, and nominal interest rate:

$$\{Y_t, P_t, R_t\}_{t=0}^{\infty}$$

that satisfy the household optimality conditions, (12) and (13), the household budget constraint, (11), the firm optimality conditions, (17), (20), and (23), the government budget constraint (24), monetary and fiscal policies (25) and (26), the aggregators (15), (16), (18), and (19), and finally the market clearing conditions, $\sum_{j=1}^J \int_{\mathcal{I}_j} C_{j,t}(i) di = Y_t$ and $\sum_{j=1}^J \int_{\mathcal{I}_j} B_{j,t}(i) di = 0$, given $B_{j,-1}(i) = 0$, $\forall i \in [0, 1]$.

Let me introduce some additional notation useful for studying equilibrium dynamics. I define an aggregate hour index as the sum of hours worked by all the households in the economy:

$$H_t \equiv \sum_{j=1}^J \int_{\mathcal{I}_j} H_{j,t}(i) di.$$

I use $C_{j,t}$ to denote sectoral consumption $\int_{\mathcal{I}_j} C_{j,t}(i) di$, and $B_{j,t}$ to denote sectoral bond holding $\int_{\mathcal{I}_j} B_{j,t}(i) di$. For any generic variable X_t , I use $X_{j,t}^R(i)$ and $X_{j,t}^R$ to denote "relative" variables:

$$X_{j,t}^R(i) \equiv \frac{X_{j,t}(i)}{n_j^{-1} X_{j,t}} \quad \text{and} \quad X_{j,t}^R \equiv \frac{n_j^{-1} X_{j,t}}{X_t}.$$

I log-linearize the equilibrium conditions around a symmetric steady state to solve the model. Lowercase letters denote percentage deviation from the steady state X (i.e. $x_t \equiv \ln X_t - \ln X$), except in the case of bond holdings where they represent the deviation of nominal bond holdings

from their steady-state level $B = 0$, relative to steady state nominal output (e.g. $b_{j,t} \equiv \frac{B_{j,t} - B}{PY}$).

4 The New Keynesian Phillips Curve

The model considered here is necessarily more complicated than a standard representative-agent sticky-price DSGE model. In principle, the evolution of consumption and asset holdings across households should be computed, since these could affect aggregate equilibrium dynamics. Because of this complication, it is thus impractical to solve the model without an approximation scheme. For this reason and also for comparison with earlier studies, I use the same approximation strategy commonly employed in the NK literature. As discussed earlier, I assume time-dependent pricing as in Calvo (1983) and Yun (1996) and then I take a log-linear approximation of the model.

For the single-sector model, because of the symmetric nature of the model and the approximation technique employed, I do not need to compute the time path of households' consumption and asset holdings. Moreover the form of the equations that characterize equilibrium dynamics are identical in the \mathcal{RH} and \mathcal{HH} models. This however does not mean that household heterogeneity has no first-order effect on equilibrium aggregate dynamics, as the two models involve different mappings between the structural parameters and the reduced-form parameters in the equilibrium conditions. It turns out that household heterogeneity changes *one* reduced-form parameter: it decreases, through the within-sector wealth effect, the slope of the Phillips curve.

In the multiple-sector model, in the absence of symmetry across sectors, the evolution of the distributions of households and firms must be computed. As in the single-sector case, household heterogeneity alters the Phillips curve by decreasing the slope through the within-sector wealth effect. In addition, however, heterogeneity adds an endogenous shift term that captures the cross-sector wealth effect.

Since the Phillips curve is the key equation through which household heterogeneity influences aggregate dynamics, this section studies the generalized Phillips curve in isolation before presenting the entire set of equilibrium conditions. Below I present the Phillips curve, and in the following subsections I discuss its important properties.

4.1 Generalized Phillips Curve

Household heterogeneity complicates construction of the Phillips curve mainly because optimal prices chosen at a given time t are not identical across firms. The reason for this is that a firm's marginal cost is a function of its employees' consumption choices which depend on their bond holdings carried over from the previous period. This feature of the model creates a complication similar to one that arises when capital stocks are firm-specific. One cannot derive a Phillips curve using the conventional method, and instead must use the undetermined coefficient method suggested in Woodford (2005). I refer the interested reader to the appendix for a detailed derivation of the Phillips curve and more on the simplifying role of the assumption of time-dependent pricing. I present the main result here:

Proposition 4 *Suppose the economy consists of multiple sectors indexed by $j = 1, 2, \dots, J$. In each sector j , there is a continuum of firms whose prices are sticky as in Calvo, with the probability of price adjustment in each period is $1 - \alpha_j$. Then, the dynamics of aggregate inflation π_t can be described by the following Phillips curve:*

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa y_t + \Theta_{c,t} + \Theta_{y,t} - \zeta_t, \quad (27)$$

where $\zeta_t \equiv \sum_{j=1}^J n_j g(\alpha_j, \chi, \epsilon) \left[(1 + \varphi) a_{j,t} + \varphi \xi_t + \eta^{-1} d_{j,t}^R \right]$ is a linear combination of exogenous disturbances. The two endogenous shift terms are given by

$$\Theta_{c,t} \equiv \sum_{j=1}^J n_j g(\alpha_j, \chi, \epsilon) c_{j,t}^R, \quad \Theta_{y,t} \equiv (\varphi + \eta^{-1}) \sum_{j=1}^J n_j g(\alpha_j, \chi, \epsilon) y_{j,t}^R,$$

and the slope of the Phillips curve is given by

$$\kappa \equiv (1 + \varphi) \sum_{j=1}^J n_j g(\alpha_j, \chi, \epsilon)$$

where $g(\alpha_j, \epsilon, \chi)$ is a nonlinear decreasing function in each of α_j , ϵ and χ .

As mentioned earlier, household heterogeneity affects aggregate dynamics by changing the standard Phillips curve in two ways: (i) it decreases the slope of the Phillips curve, κ , for a given degree of nominal rigidity, $\{\alpha_j\}$, and (ii) it introduces an endogenous shift term, $\Theta_{c,t}$. The endogenous shift terms, $\Theta_{c,t}$ and $\Theta_{y,t}$, are relevant only when the economy has multiple sectors with varying price stickiness. The shift terms disappear when prices adjust with identical frequency in every sector (i.e. $\alpha_j = \alpha$ for all sectors)

4.2 Slope of the Phillips Curve

The major determinant of the slope, κ , is the function $g(\cdot)$, which is always decreasing in α . This implies that the larger the degree of nominal rigidity α , the smaller the slope of the Phillips curve. For the two models, the function $g(\cdot)$ has different forms. In the \mathcal{HH} model, for a given nominal rigidity α , the function $g(\cdot)$ and thus the slope, depend on the financial friction parameters (χ, ϵ) . However, in the \mathcal{RH} model, financial frictions play no role in determining equilibrium. Therefore, in what follows, I use $g^{\mathcal{RH}}(\alpha)$ and $g^{\mathcal{HH}}(\alpha, \chi, \epsilon)$ separately for the \mathcal{RH} and \mathcal{HH} models respectively.²²

²²Specifically, the expression for $g^{\mathcal{RH}}(\alpha)$ and $g^{\mathcal{HH}}(\alpha, \chi, \epsilon)$ are given as

$$g^{\mathcal{RH}}(\alpha) \equiv \left\{ \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \right\} \frac{1}{1 + \varphi\theta}$$

$$g^{\mathcal{HH}}(\alpha, \chi, \epsilon) \equiv \left\{ \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \right\} \left[\frac{(1 - \alpha_j\beta\delta)^2}{(1 + \varphi\theta + \psi_2)(1 - \alpha_j\beta\delta)^2 - \psi_2(1 - \alpha_j)^2\beta\delta} \right],$$

where δ and ψ_2 are some functions of (χ, ϵ) . The parameter ψ_2 measures the responsiveness of a household's relative consumption to a change in its current relative income. In the representative household model, where risk-sharing

It is tedious but straightforward to show that, for all possible values of χ and ϵ , $g^{\mathcal{HH}}(\alpha, \chi, \epsilon)$ is smaller than $g^{\mathcal{RH}}(\alpha)$ for a given α , that is, the \mathcal{HH} model is characterized by a larger degree of real rigidity.²³ Instead of showing this analytically, I present a contour map of $\frac{g^{\mathcal{HH}}(\alpha, \chi, \epsilon)}{g^{\mathcal{RH}}(\alpha)}$ in Figure 1 based on numerical calculations. In addition, the second and third panel in Figure 1 plot $g^{\mathcal{HH}}$ and $g^{\mathcal{RH}}$, with fixed $\alpha = 0.5$, while varying ϵ and χ respectively. A few observations are worth mentioning.

First, Figure 1-A shows that $g^{\mathcal{HH}} < g^{\mathcal{RH}}$ for $\epsilon > 0$ and $\chi \in [0, 1]$. Thus for a fixed degree of nominal rigidity α , the slope of the Phillips curve is smaller with heterogeneous households. This result implies that inflation responds less to economic shocks, and output deviates more from the natural level of output than in the representative household model.

Second, Figure 1-B shows $g^{\mathcal{HH}}$ is decreasing in ϵ : greater friction in bond trading increases the degree of real rigidity. This result is intuitive, significant financial frictions increase the dependence of household consumption on its labor income, and the idiosyncratic wealth effect on labor supply. Recall this wealth effect is a key determinant of real rigidity. Also note $g^{\mathcal{HH}}$ is convex in ϵ , which implies that even a small financial friction could have a substantial effect on aggregate dynamics.

Third, Figure 1-C shows $g^{\mathcal{HH}}$ is also decreasing in χ : less diversification of household capital incomes decreases real rigidity. To see this consider the extreme case of $\chi = 0$ where type- i household receives profit exclusively from type- i firm. In this case, the household's total income (the sum of labor and capital incomes) equals the industry's revenue.²⁴ Thus, in this situation, household consumption does not depend directly on its labor income, and hence the wealth effects on labor supply are weak.

4.3 Endogenous Shift Terms in the Phillips Curve

When the economy is divided into multiple sectors, in addition to decreasing the slope, household heterogeneity introduces an endogenous shift term. The Phillips curve, (27), contains two endogenous shift terms, $\Theta_{c,t}$ and $\Theta_{y,t}$, both of which contribute to slower aggregate price adjustments. The effects of $\Theta_{y,t}$ are extensively documented in Carvalho (2006) and stem from having multiple sectors. The other term $\Theta_{c,t}$, comes from adding household heterogeneity.

As documented in Carvalho (2006), when there is more than one sector, strategic complementarity in price setting can arise among sectors and thus sector prices have less tendency to deviate from the economy's average price level (i.e. aggregate price).²⁵ On the one hand, firms in "high-frequency sectors" (i.e. sectors with a small α_j) adjust their prices by a *smaller* amount than they would in the single-sector model because their pricing decision is influenced by firms in "low-frequency sectors" (i.e. sectors with a big α_j). This imposes *more rigidity* on the aggregate

is perfect, ψ_2 should be equal to 0. From the equations above, one can verify that $g^{\mathcal{RH}} = g^{\mathcal{HH}}$ when $\psi_2 = 0$. See appendix for a detailed derivation of $g^{\mathcal{HH}}$.

²³See appendix for a proof of this statement.

²⁴The household total income, in this case, is given by $W_t(i)H_t(i) + \Pi_t(i)$. But, because firm profits are $\Pi_t(i) = P_t(i)Y_t(i) - W_t(i)H_t(i)$, total income is $W_t(i)H_t(i) + P_t(i)Y_t(i) - W_t(i)H_t(i)$ and thus does not depend on the industry wage rate. The household's labor income is canceled out by the decrease in firm profits due to labor costs.

²⁵Nakamura and Steinsson (2009) have shown related results in a multi-sector menu cost model.

price level. By the same logic, however, firms in the low-frequency sectors adjust their prices by a larger amount, which provides *more flexibility* to the aggregate price level. As argued earlier, high-frequency sectors dominate the process of aggregate price adjustments. Therefore introducing multiple sectors leads to more rigid aggregate prices and inflation.

Introducing heterogeneous households reinforces this strategic complementarity through wealth effects, as described in the introduction. To understand the mechanism, it is helpful to consider a particular example. Suppose the economy is hit by a contractionary monetary shock.²⁶ Since a larger fraction of firms in high-frequency sectors adjusts their prices, a high-frequency sector price decreases *more* than a low-frequency sector price in response to the shock. Or equivalently, output from a high-frequency sector decreases *less* than output from a low-frequency sector. Moreover, *because of incomplete asset markets* consumption and output in the same sector respond in the same direction. Thus households in high-frequency sectors enjoy a relatively higher consumption level. Since households in high-frequency sectors would then have less incentive to supply labor, the marginal costs of these firms would increase. Therefore firms in high-frequency sectors do not lower their prices as much as they would have if the asset markets were complete. Although the exact opposite process happens in low-frequency sectors, high-frequency sectors, as mentioned earlier, dominate aggregate price dynamics. Hence the end result of wealth effects is a slower decline of price level.

The effects that I just mentioned are reflected by the shift terms in the Phillips curve. Note that $\Theta_{c,t}$ and $\Theta_{y,t}$, are weighted sums of the relative consumption levels $c_{j,t}^R (\equiv c_{j,t} - y_t)$ and of the relative outputs $y_{j,t}^R (\equiv y_{j,t} - y_t)$ respectively. Note, sectors are not equally weighted. Since the function $g(\alpha, \chi, \epsilon)$ is decreasing and convex in α , disproportionately larger weights are placed on the high-frequency sectors, that is, high-frequency sectors have a larger impact on a change in the aggregate price level. With a contractionary monetary shock, $c_{j,t}^R$ and $y_{j,t}^R$ are positive in high-frequency sectors and negative in low-frequency sectors. However, the total effects on $\Theta_{c,t}$ and $\Theta_{y,t}$ would be positive since high-frequency sectors dominate.

Solving the Phillips curve forward, inflation can be written as a weighted sum of expected future values of the shift terms and the output gap:

$$\pi_t = \sum_{k=0}^{\infty} \beta^k E_t [\kappa y_{t+k} + \Theta_{c,t+k} + \Theta_{y,t+k} - \zeta_{t+k}].$$

With the contractionary shock, output falls. Inflation, however, does not fall as much because $\Theta_{c,t}$ and $\Theta_{y,t}$ are expected to rise for a time after the shock. These endogenous Phillips curve shifters make the model "stickier" by making the response of the price level and of inflation more sluggish. Once again note that the mechanism discussed in this subsection does not require household heterogeneity at the industry level. Even if labor markets are segmented at the sector-level and households are homogeneous within sectors, the endogenous shift term $\Theta_{c,t}$ would still appear in the Phillips curve and thus increase price stickiness.

²⁶In the introduction, I described a positive shock to firms' productivity.

5 Estimation and Model Comparisons

This section describes the estimation procedure and evaluates the two model economies, \mathcal{RH} and \mathcal{HH} , based on the model-implied degree of nominal rigidity.

I estimate the models using Bayesian methods. A modeler's prior belief about the structural parameters ω can be incorporated into the estimation process by specifying a prior distribution $f(\omega)$ for these parameters. Given a data set \mathbf{X}^T , a likelihood function $f(\mathbf{X}^T|\omega)$ can be obtained by exploiting restrictions imposed by general equilibrium. The posterior distribution of ω , $f(\omega|\mathbf{X}^T)$ is then determined by Bayes theorem: $f(\omega|\mathbf{X}^T) = f(\mathbf{X}^T|\omega)f(\omega)/\int f(\mathbf{X}^T|\omega)f(\omega)d\omega$. Since it is impossible to obtain a solution for the posterior distribution analytically, I simulate this distribution using Markov Chain Monte Carlo methods.

I chose to evaluate the models by looking at the parameters that measure nominal rigidity for two reasons. First, comparing the nominal rigidity implied by each model is a reasonable way to *quantify* the real rigidity generated endogenously. Roughly speaking, *total* rigidity is the sum of nominal and real rigidities. A model with greater real rigidity requires less nominal rigidity to achieve the total rigidity required. Consequently it can be argued that the difference between the estimated nominal rigidities in the two models is the amount of endogenous stickiness generated by the model. The second reason is that baseline sticky-price DSGE models often require an implausibly large degree of nominal rigidity. Thus it would be interesting to see if adding household heterogeneity rectifies this problem.

Following the tradition in NK literature, I first estimate both the \mathcal{RH} and \mathcal{HH} models assuming $\alpha_j = \alpha$, that is, all firms in the economy update their prices with the same frequency. I then consider a more complicated, yet more realistic case in which nominal rigidity varies across sectors.

5.1 Single Sector

There are two ways to view the single-sector case. One can think that the economy is literally composed of one sector (i.e. $J = 1$). Or one can maintain the view that the economy consists of multiple sectors (i.e. $J > 1$), but the degrees of nominal rigidity are the same across the sectors. The two different approaches give the same log-linearized equilibrium conditions.

As mentioned in a previous section, in the single-sector case, the Phillips curve has no endogenous shift terms. Adding heterogeneous households only changes the expression for the slope. Thus, we do not need to keep track of the distributions of household consumption and wealth to compute the equilibrium dynamics of the aggregate variables. The system of equations to be estimated looks much like the standard sticky-price models:

$$y_t = E_t[y_{t+1}] - (r_t - E_t[\pi_{t+1}]) + (\gamma_t - E_t\gamma_{t+1}) \quad (28)$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa y_t - \zeta_t \quad (29)$$

$$y_t = a_t + h_t \quad (30)$$

$$r_t = \rho_m r_{t-1} + (1 - \rho_m) \{ \phi_\pi \pi_t + \phi_y y_t \} + \mu_t \quad (31)$$

The first equation (28), often called the IS equation, is obtained by integrating the log-linearized household optimality conditions over all households and then imposing a market clearing condition; (29) is the Phillips curve; the aggregate production function (30) is obtained by integrating the production functions over all firms; the last equation (31) is a log-linear approximation of the interest rate rule and it closes the model. Note that the slope of the Phillips curve is given by $\kappa = (1 + \varphi)g(\alpha, \chi, \epsilon)$. The function $g(\cdot)$ is model-specific because of the different real rigidities, but other than that, the equilibrium conditions look identical for the \mathcal{RH} and \mathcal{HH} models.

Following the convention in the DSGE literature, I assume that the two preference shocks and the technology shock follow independent AR(1) processes, while the monetary policy shock follows an i.i.d. process (since an interest rate smoothing term is already included in (31)):

$$\begin{pmatrix} \gamma_t \\ \xi_t \\ a_t \\ \mu_t \end{pmatrix} = \begin{pmatrix} \rho_\gamma & 0 & 0 & 0 \\ 0 & \rho_\xi & 0 & 0 \\ 0 & 0 & \rho_a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \gamma_{t-1} \\ \xi_{t-1} \\ a_{t-1} \\ \mu_{t-1} \end{pmatrix} + \begin{pmatrix} \sigma_\gamma & 0 & 0 & 0 \\ 0 & \sigma_\xi & 0 & 0 \\ 0 & 0 & \sigma_a & 0 \\ 0 & 0 & 0 & \sigma_\mu \end{pmatrix} \begin{pmatrix} \varepsilon_{\gamma,t} \\ \varepsilon_{\xi,t} \\ \varepsilon_{a,t} \\ \varepsilon_{\mu,t} \end{pmatrix}, \quad (32)$$

where $(\varepsilon_{\mu,t} \ \varepsilon_{a,t} \ \varepsilon_{\gamma,t} \ \varepsilon_{\xi,t})'$ is i.i.d $N(0_4, I_4)$.

The system of linear equations (28)-(32) characterizes the joint distribution of $\{\Delta y_t, \pi_t, r_t, \Delta h_t\}$. I estimated the model using time series data for the United States. I used real GDP and the GDP deflator to construct the growth rate of aggregate output and inflation, $\{\Delta y_t, \pi_t\}$.²⁷ The effective federal funds rate measures the nominal interest rate r_t . Total hours from the nonfarm business sector are used to construct the growth rate of aggregate hours. Because I have normalized the size of the model economy to one, I divided real GDP and hours worked by the total civilian non-institutional population over the age of 16. I then demeaned every time series. The data are quarterly, and the sample period runs from 1954:Q3 to 2006:Q4.

Some parameters such as $(\beta, \theta, \varphi, \chi, \epsilon)$ are not well identified unless I impose strong priors on those parameters. Throughout the paper I fix (β, θ, φ) at conventional values. I set the discount factor β to 0.99. The elasticity of substitution θ is fixed at 6 so that a firm's mark-up is 20 percent. I set φ to 1, which implies the Frisch elasticity of labor supply is 1. To my knowledge, no quantitative analysis has been done on the parameters χ and ϵ in the literature of Bayesian DSGE models, and thus there is no general agreement on what values are appropriate for sticky-price DSGE models. For the single sector model, I repeated the model estimation using several alternative values of χ and ϵ . (In the multiple sector case, I estimated these two parameters along with the other model parameters.)

The prior and posterior distributions for the remaining parameters are summarized in Table 1. The prior distributions are mostly standard. However unlike many earlier papers, I assume a flat prior on α , (instead of imposing an informative prior) because α is the key parameter of interest in

²⁷The models considered here abstract from investment decisions for simplicity and therefore, in principle I could use Personal Consumption Expenditure (PCE) as a measure of aggregate output and the PCE price index as a measure of aggregate price level. However, the estimation results are almost identical with those reported in Table 1 and 2.

this paper. The models then have complete freedom to choose any degree of nominal rigidity that provides the best model fit.

Once the posterior distribution of α is obtained, I also construct a posterior distribution of duration of price contracts D , employing the relation:

$$D = -1/\log \alpha.$$

Table 2 presents the posterior means of α and D under the \mathcal{RH} model and also under the \mathcal{HH} model with different sets of values for (χ, ϵ) .

In the \mathcal{RH} model, the posterior mean of duration is 4.65 quarters, with 3.32 and 6.80 being the lower and the upper bounds of the 95% highest posterior density region (HPD). Since the true duration is indeed less than 2 quarters, we disfavor the representative household model.

As expected, the \mathcal{HH} model performs better than the \mathcal{RH} model along this dimension. In particular, the \mathcal{HH} model with a large value of χ appears to match the empirical frequency well. With $\chi = 1$ (i.e. when household capital income is completely diversified across industries), the posterior mean is about 2.29 quarters and the 95% HPD interval is given by $[1.74Q, 3.12Q]$ when $\epsilon = 0.1$. An even smaller value of ϵ reduces implied duration substantially. When $\epsilon = 0.01$ (and $\chi = 1$), the estimated duration of price contracts in the \mathcal{HH} model is about 2.96 quarters. This result is not surprising given that the slope of Phillips curve is highly convex in ϵ (see Figure 1-B). Overall, Table 2 suggests that introducing heterogeneous households with financial frictions can potentially decrease the estimated duration of price contracts substantially, coming closer to the empirical data.

However, further analysis of appropriate values for ϵ and χ is needed. In the next section, I attempt to address this issue using the multiple-sector models. The main reason that the single sector exercise fails to provide information on (χ, ϵ) is that the two model economies are observationally equivalent. The only difference between the \mathcal{RH} and \mathcal{HH} models' equilibrium conditions is the functional form of the slope of Phillips curve κ . Since we allow α to vary freely, we are unable to identify α , χ and ϵ separately since these parameters enter only one reduced form parameter, κ . This will no longer be the case with the multiple sector models as χ and ϵ will also affect dynamics at the sector level in addition to aggregate dynamics, which provides an additional source of identification. In the single sector environment, I arbitrarily set $(\chi, \epsilon) = (1, 0.1)$ as the benchmark and, for illustrative purpose, show the posterior densities of the degree of nominal rigidity and price durations in Figure 3.

5.2 Multiple Sectors

In this exercise, I relaxed the restriction that every sector updates prices at the same frequency. I then estimate the sector "infrequencies" $\{\alpha_1, \alpha_2, \dots, \alpha_J\}$ and the implied durations $\{D_1, D_2, \dots, D_J\}$. An aggregate (or average) frequency and/or duration can then be computed based on these estimates.

As seen earlier, in the case of multiple sectors, the Phillips curve contains two endogenous shift terms $\Theta_{c,t}$ and $\Theta_{y,t}$, which makes studying equilibrium aggregate dynamics much more challenging computationally. To compute the endogenous shift terms, we should keep track of the cross-sector distributions of consumption and output, which in turn, can be computed only if we know the evolution of sector level inflation and bond holdings. Put differently, we have to know the time path of the sector variables $\{c_{j,t}^R, y_{j,t}^R, b_{j,t}^R, \pi_{j,t}\}_{j=1}^J$ and the sector weights $\{n_j\}_{j=1}^J$ to obtain the equilibrium dynamics of the aggregate variables, $\{y_t, \pi_t, r_t, h_t\}$.

It is straightforward to show the following $4 + (4 \times J)$ equations determine the equilibrium path of the aggregate variables $\{y_t, \pi_t, r_t, h_t\}$ and the sector variables $\{c_{j,t}^R, y_{j,t}^R, b_{j,t}^R, \pi_{j,t}\}_{j=1}^J$:

$$r_t = \rho_m r_{t-1} + (1 - \rho_m) \{\phi_\pi \pi_t + \phi_y y_t\} + \mu_t \quad (33)$$

$$y_t = E_t[y_{t+1}] - (r_t - E_t[\pi_{t+1}]) + (\gamma_t - E_t\gamma_{t+1}) \quad (34)$$

$$y_t = \left(\sum n_j a_{j,t} \right) + h_t \quad (35)$$

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa y_t + \left\{ \sum n_j g_j c_{j,t}^R \right\} + \left\{ (\varphi + \eta^{-1}) \sum n_j g_j y_{j,t}^R \right\} - \zeta_t \quad (36)$$

$$b_{j,t}^R = \beta^{-1} b_{j,t-1}^R + \varkappa_1 y_{j,t}^R - \varkappa_2 c_{j,t}^R - \varkappa_3 a_{j,t}^R + \varkappa_4 d_{j,t}^R \quad (37)$$

$$c_{j,t}^R = E_t c_{j,t+1}^R + 2\epsilon b_{j,t}^R, \quad (38)$$

$$y_{j,t}^R = y_{j,t-1}^R - \eta(\pi_{j,t} - \pi_t) + \Delta d_{j,t}^R, \quad (39)$$

$$\pi_{j,t} = \beta E_t \pi_{j,t+1} + g_j \left\{ (1 + \varphi) y_t + \left(\varphi + \frac{1}{\eta} \right) y_{j,t}^R + c_{j,t}^R - (1 + \varphi) a_{j,t} - \varphi \xi_t - \frac{1}{\eta} d_{j,t}^R \right\} \quad (40)$$

The first four equations (33)-(36) are identical to those in the single-sector models except for the presence of endogenous shift terms.²⁸ The remaining $(4 \times J)$ equations determine simultaneously the dynamics of aggregate as well as sector variables.

In the system of equilibrium conditions above, the \mathcal{HH} model differs from the \mathcal{RH} model in two places. First, as in the single-sector case, the slope of the Phillips curve is different. As shown before, the slope κ has a smaller value in the \mathcal{HH} model for a given nominal rigidity. Second, in the \mathcal{RH} model, (37) and (38) become irrelevant since it trivially holds that $c_{j,t}^R = b_{j,t}^R = 0$.

Similarly to aggregate shocks, the sector level shocks are assumed to follow AR(1) processes:

$$\begin{aligned} a_{j,t} &= \rho_{a,j} a_{j,t-1} + \sigma_{a,j} \varepsilon_{a,j,t}; & \varepsilon_{a,j,t} &\stackrel{i.i.d}{\sim} N(0, 1) \\ d_{j,t} &= \rho_{d,j} d_{j,t-1} + \sigma_{d,j} \varepsilon_{d,j,t}; & \varepsilon_{d,j,t} &\stackrel{i.i.d}{\sim} N(0, 1). \end{aligned}$$

Before estimating the models, I present impulse responses to a monetary shock, at some fixed parameter values, in Figure 2, to study the dynamic properties of the models. The aim is to

²⁸For a shorter notation, I use g_j to denote $g(\alpha_j, \chi, \epsilon)$. Also $\varkappa_1, \varkappa_2, \varkappa_3$ and \varkappa_4 are positive constants: $\varkappa_1 = \chi(1 + \varphi) \left(\frac{\theta-1}{\theta} \right) + (1 - \chi) \left(\frac{\eta-1}{\eta} \right)$, $\varkappa_2 = 1 - \chi \left(\frac{\theta-1}{\theta} \right)$, $\varkappa_3 = \chi(1 + \varphi) \left(\frac{\theta-1}{\theta} \right)$, and $\varkappa_4 = (1 - \chi) \frac{1}{\eta}$.

investigate the effect of introducing household heterogeneity on the dynamics of the aggregate and sector-level variables, if the two models are parameterized at the same values.²⁹ The impulse responses confirm the theoretical results summarized in Proposition 2 and 3: (i) aggregate output responds more and inflation responds less to a monetary shock in the \mathcal{HH} model than in the \mathcal{RH} model; (ii) the \mathcal{HH} model generates stronger comovements among sector outputs and sector inflation.³⁰

I estimated the models using sector-level and aggregate time series data for the United States. In addition to the aggregate time series $\{\Delta y_t, \pi_t, r_t, \Delta h_t\}$, I include in the set of observables the growth rate of sector outputs and of sector price indices $\{\Delta y_{j,t}, \pi_{j,t}\}_{j=1}^J$. I use PCE as a measure of output and the corresponding price deflator as a measure of the price index. The PCE is divided into 3 categories: durable goods, nondurable goods, and services, and these categories are disaggregated further into 13 sectors, each of which are then divided into smaller sub-sectors. Although there may be some value in estimating the models with more disaggregated variables as in Boivin et al. (2009), I chose to estimate the models with the 13 sectors for computational reasons. Table 3 presents those 13 sectors. The sector weights in the table are the expenditure weights averaged over the sample period of 1954:Q3-2006:Q4.³¹

The prior distributions are summarized in Table 4. Since no quantitative analysis has been done on the parameters χ and ϵ in the literature on Bayesian DSGE models, I chose the prior distributions for χ and ϵ based on information from the single-sector empirical exercise. The prior mode and mean for χ are set to 0.875 and 0.7 respectively. A relatively large value for χ is based on the result that a larger value of χ , in the single-sector \mathcal{HH} model, is necessary to match the empirically reasonable frequency of price adjustments. Given the large values of χ , values around 0.1 work best for ϵ in matching the empirical frequency in the single sector model. I thus set the prior mode and mean for ϵ to 0.09 and 0.1 respectively. Note that since this choice of prior distributions is based solely on the dynamic properties of the single-sector \mathcal{HH} model and the multiple-sector \mathcal{HH} model has different dynamics, similar values of χ and ϵ will not necessarily do a good job in the multiple-sector model. The AR coefficient and innovation parameters of sector shocks have the same prior distributions as those of the aggregate shocks. I set the prior mean of the cross-sector elasticity of substitution η to 1 so that final goods producers have a Cobb-Douglas production function. As in the single sector case, the key parameters of interest are the Calvo parameters. I thus assume a flat prior for each α_j . Finally, the sector weights $\{n_j\}$ follow the values in Table 3.

²⁹For this exercise, I divide the economy into 5 sectors with same size. The Calvo parameters are assigned as $\{0.1, 0.3, 0.5, 0.7, 0.9\}$. Parameterizations for the remaining parameters relevant for this exercise are summarized in the following table:

β	φ	η	χ	ϵ	ϕ_π	ϕ_y	ρ_m	σ_μ
0.99	1	1	1	0.1	1.3	0.125	0.9	0.25

Alternative sets of parameter values do not change the impulse responses qualitatively.

³⁰Impulse responses to other shocks shows the same dynamics qualitatively, and thus they are omitted.

³¹Note that the model imposes the structural relationships $y_t = \sum_{j=1}^J n_j y_{j,t}$ and $\pi_t = \sum_{j=1}^J n_j \pi_{j,t}$. This suggests that, in estimating the models, the aggregate variables y_t and π_t are redundant if I include all the sectoral counterparts in the observables. Hence I drop the two aggregate variables in actual estimation.

The posterior means and the 95% HPD of the model parameters, except the sector frequencies, are presented in Table 4. The means and 95% HPD of the Calvo parameters $\{\alpha_j\}$ and durations of price contracts that are implied by the estimated $\{\alpha_j\}$, are presented in Table 5. In addition, Table 6 and 7 compare the nominal rigidities estimated in the models to their empirical counterparts. The model-implied durations in Table 6 are constructed in the following way. For the mean duration, I first obtain the posterior distribution of $\bar{\alpha} = \sum_{j=1}^{13} n_j \alpha_j$, the weighted mean of "infrequencies" of price changes. With the posterior distribution of $\bar{\alpha}$, the posterior distribution of the mean duration \bar{D} is then obtained employing the relation $\bar{D} = -1/\log(\bar{\alpha})$. The posterior distributions of the durations in durable, non-durable, and service sectors are computed in a similar way.³² The empirical counterparts are based on BK, which are denoted by D^{BK} .³³ To be consistent, I computed the empirical durations in the same way: I first take $\bar{\alpha}$, the weighted mean of the sector "infrequencies" reported in BK, and then compute the corresponding duration by $\bar{D} = -1/\log(\bar{\alpha})$. Figure 4 is a graphical representation of Table 6, presenting the posterior densities of the model-implied durations for 3 broad sectors as well as for the whole economy along with their empirical counterparts. Table 7 reports the model-implied and empirical durations for more disaggregated sectors. BK and NS have used the consumption categories constructed by the Bureau of Labor Statistics, which do not exactly match the consumption categories in the PCE. There are however some comparable categories, and they are reported in Table 7.³⁴

Some observations from Figure 4, Table 6 and Table 7 are worth mentioning. First, allowing different degrees of nominal rigidity across sectors has non-trivial implications for estimating the frequency of price changes, even when one's interest is primarily the *aggregate frequency/duration* as in many earlier papers on real rigidity. In both the \mathcal{RH} and \mathcal{HH} models, the estimated mean durations are much smaller using multiple sectors relative to the single-sector case: they are 1.74 and 1.39 quarters respectively. Second, the \mathcal{HH} model is broadly consistent with empirical evidence of frequency of price changes not only at the aggregate but also at the *sector* level. Moreover, along this dimension, Figure 4 and Table 6 suggest that the \mathcal{HH} model performs better than the \mathcal{RH} model. However, the \mathcal{RH} model is not far worse. With multiple sectors, the \mathcal{RH} model becomes much closer to the \mathcal{HH} model in matching the empirical frequencies. It is somewhat surprising that these sticky-price DSGE models with their many strong and implausible assumptions, can match reasonably well the empirical cross-sector distribution of frequencies while fitting major U.S. time series data.³⁵

³²For instance, the posterior distribution of the durable-sector duration can be obtained by taking posterior draws of $-1/\log(\bar{\alpha})$, where $\bar{\alpha} = \frac{\sum_{j=1}^3 n_j \alpha_j}{\sum_{j=1}^3 n_j}$.

³³ D^{BK} are obtained excluding observations with item substitutions.

³⁴The empirical durations from BK and NS in Table 6 and 7 are the ones estimated *including* observations with temporary sales. There is no consensus yet whether the temporary sales should be included for macroeconomic analyses. Therefore, D^{BK} and D^{NS} in Table 6 and 7, can be a conservative criteria.

³⁵However, the models perform relatively poorly in capturing the dynamics of the durable sector with empirically plausible price stickiness. This suggests that treating household consumption behavior symmetrically for durable and non-durable goods might not be a good modeling strategy if disaggregated variables are your focus. Also, perhaps not surprisingly, as the level of disaggregation of sectors increases, the match between model-implied and empirical frequencies at the sector level becomes less accurate. Besides highly stylized nature of the models, it may be due to

There are two main reasons that the multiple-sector models do a better job than their single-sector counterparts in accounting jointly for persistent aggregate dynamics and for relatively flexible prices. The first is a statistical property of the time series data, the other comes from the theoretical properties of this sticky-price DSGE models. First of all, in general, sector level inflation is far less persistent than aggregate inflation as shown in Figure 5.³⁶ Aggregate inflation is more persistent because the idiosyncratic components in each sector's inflation rate are averaged out through aggregation. Since I include the sector level time series data in the observables in estimating the multiple-sector models, it is not so surprising that the estimated *sector level* nominal rigidities in the models are small.³⁷ Unlike sector inflation rates, however, aggregate inflation is very persistent as mentioned above. Why do the models not need large nominal rigidities to account for the persistent aggregate dynamic? As discussed in a previous section, introducing multiple sectors with different price stickiness endogenously increases the persistence of *aggregate* variables by creating a shift term in Phillips curve. Moreover introducing heterogeneous households into a multiple sector environment, amplifies this mechanism by adding another shift term. Thanks to this theoretical property of the models, there is no need for a large degree of nominal rigidity at the *sector* level to account for persistent *aggregate* dynamics. The estimation scheme used in this paper effectively takes this theoretical property into account by using the models' entire set of general equilibrium effects. Figure 6 is the theoretical counterpart of Figure 5, presenting the autocorrelation of aggregate and sector inflation rates implied by the models using estimated parameter values (the parameters are set to their posterior means). It shows that the models are able to explain the joint phenomena of persistent aggregate inflation and less persistent sector inflation.

5.3 Some Additional Observations for Multiple-Sector Models

The log marginal likelihoods of the \mathcal{RH} and \mathcal{HH} models are -8335.2 and -8291.4 respectively, which leads to the posterior odds ratio (or Bayes factor) of $\exp(43.8)$ in favor of the \mathcal{HH} model. The result suggests the heterogeneous household model is better at explaining the joint dynamics of aggregate and sector level U.S. time series data. The magnitude of the posterior odds ratio is not small statistically. However, the difference *may not be economically significant*. Investigating if the \mathcal{HH} model is systematically better in any economic sense and studying what feature of the \mathcal{HH} model is responsible for the improved fit, is beyond the scope of this paper, but offers an opportunity for future research.

The two estimated multiple-sector DSGE models have some common features worth mentioning. First, sector shocks seem to be more volatile than aggregate shocks on average. Second, many sector shocks are as persistent as the aggregate shocks. Third, the volatilities of sector demand shocks

the fact that the empirical frequencies in BK and NS are based on limited sample and hence sampling issues becomes more pronounced for highly disaggregated sectors.

³⁶The autocorrelation function of aggregate inflation is presented in a dotted black line and those of sector level inflation are presented in solid lines.

³⁷While the fact, that sector inflation rates are not persistent, might explain the overall small estimated values for the model-implied durations at the sectoral level, it does not necessarily explain why the estimated sector-by-sector model-implied durations match well the empirical counterparts.

and sector price change frequencies are positively correlated. This suggests that firms in the sectors with more volatile demand shocks tend to adjust prices more frequently. All three features may not be too surprising. On the other hand, estimation result implies firms in sectors with less volatile supply shocks tend to change prices more frequently, which is somewhat counter-intuitive. I suspect that the data fail to correctly identify sector technology shocks $a_{j,t}$ as average labor productivity since the observables do not include sector level labor data. The shocks therefore serve only as a residual in the sector Phillips curves without much economic meaning (see the equation (40)). However, since the coefficient on $a_{j,t}$ is given by $g(\alpha_j, \chi, \epsilon)(1 + \varphi)$, which is decreasing in α_j , the residual $a_{j,t}$ in a low-frequency sector has a small coefficient. To compensate for this, $a_{j,t}$ fluctuates more widely in low-frequency sectors, which leads to the negative correlation. This issue deserves further investigation.

In addition to the frequency of price changes, the financial friction parameters (χ, ϵ) , are of interest. These are potentially identifiable as they are associated with the magnitude of comovement among sectors. The parameter χ appears to be well identified. Its posterior mean is 0.12, which suggests that a smaller value of χ is needed in the multiple-sector case because heterogeneous sectors have already generated large real rigidity, and a larger value of χ would induce too much comovement among sectors. On the other hand, the data does not provide much information for ϵ ,³⁸ which suggests an additional source of identification is needed.³⁹ Having household consumption and wealth data at the sector level would solve the identification problem; another opportunity for future research.

6 Conclusion

In sum, this paper shows that relaxing the representative-household assumption in sticky-price DSGE models can affect equilibrium dynamics nontrivially by increasing the degree of real rigidity through wealth effects on labor supply. To quantify the importance of household heterogeneity in amplifying price stickiness, I estimated a representative household and a heterogeneous household model and compared them by looking at the model-implied frequency of price changes. The quantitative exercise shows that introducing household heterogeneity can improve the sticky-price models' consistency with micro-level empirical evidence on nominal rigidity.

Looking forward, an interesting study building on this paper, would investigate if the household heterogeneity introduced here could also solve another well-known puzzle: the large elasticity of aggregate labor supply vs. the small elasticity of individual labor supply. This paper has shown that idiosyncratic wealth effects lead to a smaller individual labor supply elasticity. The idiosyncratic wealth effects, however, would cancel each other out through aggregation across households. Therefore aggregate labor supply elasticity need not be identical to individual labor supply elasticity, and

³⁸The posterior mean of ϵ is a little larger than its prior mean, but the standard error in the posterior is also larger.

³⁹When I fix χ to one, ϵ is well identified as it becomes the main parameter that controls sector comovement. As expected, the estimated ϵ turns out to be very small (around at 0.001) for the same reason that estimated χ is small when χ is not fixed.

the former would be larger than the latter. If this is indeed the case, household heterogeneity would provide a single mechanism that could simultaneously resolve two important contradictions between micro and macro observations, on price rigidity and labor supply elasticity.

Boivin, Giannoni, and Mihov (2009) have documented interesting empirical evidence on the dynamics of aggregate and sector prices. While their focus is empirical, the model developed here naturally provides a structural framework for similar analysis with its multiple sectors and aggregate and sector-specific shocks. Current work is moving forward in this direction.⁴⁰

The model has a testable implication: other things being equal, the business cycles of a country with larger financial frictions should be more persistent and volatile. The model thus proposes an explanation as to why developing countries often experience more prolonged and severe business cycles than developed countries. A careful cross-country empirical analysis would be an interesting project.

Finally, the model developed here can provide a tool to study the implications of imperfect consumption insurance among heterogeneous households for optimal monetary policy. Investigating these policy and welfare implications would be another area for future research.

⁴⁰This is a joint project with Carlos Carvalho (Federal Reserve Bank of NY). Boukaze et al (2009) is another related paper along this line.

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Appendix

A Tables

Table 1: Prior and Posterior Distributions (Single-Sector)

	prior distribution	prior mean (std)	posterior mean & 95% HPD
ϕ_π	<i>Gamma</i>	1.3 (0.2)	1.5645 [1.3884, 1.7632]
ϕ_y	<i>Gamma</i>	0.125 (0.1)	0.0701 [0.0363, 0.1111]
ρ_μ	<i>Beta</i>	0.75 (0.15)	0.7595 [0.7199, 0.7945]
ρ_a	<i>Beta</i>	0.6 (0.2)	0.9150 [0.8819, 0.9437]
ρ_γ	<i>Beta</i>	0.6 (0.2)	0.9481 [0.9226, 0.9715]
ρ_ξ	<i>Beta</i>	0.6 (0.2)	0.8329 [0.7696, 0.8926]
σ_μ	<i>Inverse Gamma</i>	0.25 (0.2)	0.2677 [0.2444, 0.2933]
σ_a	<i>Inverse Gamma</i>	3 (3)	1.2995 [1.1996, 1.4078]
σ_γ	<i>Inverse Gamma</i>	3 (3)	3.3827 [2.3751, 4.9798]
σ_ξ	<i>Inverse Gamma</i>	3 (3)	11.0105 [6.2723, 16.7307]
α	<i>Uniform</i> (0, 1)	0.5 (0.25)	see Table 2

Table 2: Posterior Distribution of In-frequency and Duration (Single-Sector)

	α , In-Frequency	D , Duration (quarters)
\mathcal{RH}	0.8065 [0.7393, 0.8632]	4.65 Q [3.32, 6.80]
\mathcal{HH} ($\chi = 1$, $\epsilon = 0.1$) benchmark	0.6464 [0.5630, 0.7259]	2.29 Q [1.74, 3.12]
\mathcal{HH} ($\chi = 1$, $\epsilon = 0.01$)	0.7134	2.96 Q
\mathcal{HH} ($\chi = 1$, $\epsilon = 0.1$)	0.6464	2.29 Q
\mathcal{HH} ($\chi = 1$, $\epsilon = 1$)	0.5530	1.69 Q
\mathcal{HH} ($\chi = 1$, $\epsilon = 10$)	0.5155	1.51 Q
\mathcal{HH} ($\chi = 0$, $\epsilon = 0.01$)	0.7699	3.81 Q
\mathcal{HH} ($\chi = 0$, $\epsilon = 0.1$)	0.7555	3.56 Q
\mathcal{HH} ($\chi = 0$, $\epsilon = 1$)	0.7497	3.47 Q
\mathcal{HH} ($\chi = 0$, $\epsilon = 10$)	0.7487	3.45 Q

Table 3: Sectors and Weights

j	Sectors	Weights (n_j)
1	Motor vehicles and parts	4.91
2	Furniture and household equipment	2.52
3	Other durable goods	1.71
4	Food	18.94
5	Clothing and shoes	3.69
6	Gasoline, fuel oil, and other energy goods	4.21
7	Other nondurable goods	7.96
8	Housing	16.18
9	Household operation	5.63
10	Transportation	4.19
11	Medical care	14.37
12	Recreation	2.91
13	Other services	12.77
	Total	100%

Table 4: Prior and Posterior Distributions (Multiple Sectors)

prior distribution		prior mean (std)	\mathcal{RH} model		\mathcal{HH} model	
			posterior mean & 95% HPD		posterior mean & 95% HPD	
ϕ_π	<i>Gamma</i>	1.3 (0.2)	1.4544	[1.3248, 1.5980]	1.4446	[1.3150, 1.5929]
ϕ_y	<i>Gamma</i>	0.125 (0.1)	0.0174	[0.0062, 0.0312]	0.0148	[0.0074, 0.0241]
ρ_μ	<i>Beta</i>	0.75 (0.15)	0.6576	[0.6078, 0.7098]	0.6521	[0.6021, 0.7051]
ρ_γ	<i>Beta</i>	0.6 (0.2)	0.9753	[0.9621, 0.9848]	0.9771	[0.9638, 0.9869]
ρ_ξ	<i>Beta</i>	0.6 (0.2)	0.9908	[0.9834, 0.9975]	0.9930	[0.9861, 0.9981]
σ_μ	<i>Inverse Gamma</i>	0.25 (0.25)	0.2960	[0.2658, 0.3268]	0.3014	[0.2750, 0.3335]
σ_γ	<i>Inverse Gamma</i>	3 (3)	3.0341	[2.0664, 4.1045]	3.1918	[2.1870, 4.4487]
σ_ξ	<i>Inverse Gamma</i>	3 (3)	2.0948	[1.9111, 2.3306]	2.0631	[1.9052, 2.2456]
η	<i>Gamma</i>	1 (0.25)	0.7664	[0.6982, 0.8402]	1.3068	[1.1796, 1.4030]
χ	<i>Beta</i>	0.7 (0.2)	NA		0.1202	[0.0262, 0.1792]
ϵ	<i>Gamma</i>	0.1 (0.03)	NA		0.1464	[0.0204, 0.2859]
$\rho_{a,1}$	<i>Beta</i>	0.6 (0.2)	0.8682	[0.8155, 0.9211]	0.9156	[0.8764, 0.9540]
$\rho_{a,2}$	<i>Beta</i>	0.6 (0.2)	0.9835	[0.9674, 0.9951]	0.9881	[0.9781, 0.9972]
$\rho_{a,3}$	<i>Beta</i>	0.6 (0.2)	0.8354	[0.7665, 0.9070]	0.8732	[0.8073, 0.9341]
$\rho_{a,4}$	<i>Beta</i>	0.6 (0.2)	0.9086	[0.8731, 0.9396]	0.9311	[0.8996, 0.9599]
$\rho_{a,5}$	<i>Beta</i>	0.6 (0.2)	0.8569	[0.8076, 0.9058]	0.8263	[0.7631, 0.8864]
$\rho_{a,6}$	<i>Beta</i>	0.6 (0.2)	0.9757	[0.9552, 0.9929]	0.9821	[0.9630, 0.9947]
$\rho_{a,7}$	<i>Beta</i>	0.6 (0.2)	0.9454	[0.9109, 0.9747]	0.9627	[0.9384, 0.9862]
$\rho_{a,8}$	<i>Beta</i>	0.6 (0.2)	0.9824	[0.9583, 0.9954]	0.9886	[0.9775, 0.9966]
$\rho_{a,9}$	<i>Beta</i>	0.6 (0.2)	0.8931	[0.8343, 0.9553]	0.8891	[0.8370, 0.9380]
$\rho_{a,10}$	<i>Beta</i>	0.6 (0.2)	0.9696	[0.9403, 0.9925]	0.9733	[0.9519, 0.9927]
$\rho_{a,11}$	<i>Beta</i>	0.6 (0.2)	0.9517	[0.9271, 0.9779]	0.9588	[0.9377, 0.9793]
$\rho_{a,12}$	<i>Beta</i>	0.6 (0.2)	0.8893	[0.8351, 0.9457]	0.9112	[0.8658, 0.9690]
$\rho_{a,13}$	<i>Beta</i>	0.6 (0.2)	0.8398	[0.7831, 0.8892]	0.8543	[0.8101, 0.8949]
$\rho_{d,1}$	<i>Beta</i>	0.6 (0.2)	0.7177	[0.6573, 0.7819]	0.6668	[0.6053, 0.7241]
$\rho_{d,2}$	<i>Beta</i>	0.6 (0.2)	0.9753	[0.9556, 0.9933]	0.9608	[0.9327, 0.9855]
$\rho_{d,3}$	<i>Beta</i>	0.6 (0.2)	0.9240	[0.8856, 0.9596]	0.9275	[0.8939, 0.9588]
$\rho_{d,4}$	<i>Beta</i>	0.6 (0.2)	0.9758	[0.9537, 0.9937]	0.9834	[0.9675, 0.9959]
$\rho_{d,5}$	<i>Beta</i>	0.6 (0.2)	0.8712	[0.8156, 0.9229]	0.9799	[0.9596, 0.9965]
$\rho_{d,6}$	<i>Beta</i>	0.6 (0.2)	0.9760	[0.9561, 0.9934]	0.9828	[0.9644, 0.9961]
$\rho_{d,7}$	<i>Beta</i>	0.6 (0.2)	0.9871	[0.9763, 0.9967]	0.9862	[0.9766, 0.9964]
$\rho_{d,8}$	<i>Beta</i>	0.6 (0.2)	0.9776	[0.9556, 0.9957]	0.9751	[0.9414, 0.9939]
$\rho_{d,9}$	<i>Beta</i>	0.6 (0.2)	0.9442	[0.9127, 0.9769]	0.9619	[0.9358, 0.9854]
$\rho_{d,10}$	<i>Beta</i>	0.6 (0.2)	0.9466	[0.9018, 0.9817]	0.9367	[0.8896, 0.9766]
$\rho_{d,11}$	<i>Beta</i>	0.6 (0.2)	0.9908	[0.9816, 0.9975]	0.9865	[0.9703, 0.9967]
$\rho_{d,12}$	<i>Beta</i>	0.6 (0.2)	0.9685	[0.9478, 0.9878]	0.9649	[0.9367, 0.9861]
$\rho_{d,13}$	<i>Beta</i>	0.6 (0.2)	0.9420	[0.9103, 0.9728]	0.9040	[0.8517, 0.9543]
$\sigma_{a,1}$	<i>Inverse Gamma</i>	3 (3)	5.9218	[4.9913, 7.0129]	6.9584	[6.0268, 7.9933]
$\sigma_{a,2}$	<i>Inverse Gamma</i>	3 (3)	4.3581	[3.2713, 5.8067]	4.4731	[3.5550, 5.5924]
$\sigma_{a,3}$	<i>Inverse Gamma</i>	3 (3)	8.9397	[6.3958, 11.6060]	9.2567	[6.8493, 13.1550]
$\sigma_{a,4}$	<i>Inverse Gamma</i>	3 (3)	1.8594	[1.5873, 2.1313]	1.6962	[1.4690, 2.0037]
$\sigma_{a,5}$	<i>Inverse Gamma</i>	3 (3)	5.3915	[4.1478, 6.8178]	5.0128	[4.0896, 6.2935]
$\sigma_{a,6}$	<i>Inverse Gamma</i>	3 (3)	3.2209	[2.8756, 3.6376]	1.9048	[1.7545, 2.0786]
$\sigma_{a,7}$	<i>Inverse Gamma</i>	3 (3)	2.8096	[2.1465, 3.6233]	2.3937	[1.8842, 2.9691]
$\sigma_{a,8}$	<i>Inverse Gamma</i>	3 (3)	2.0141	[1.6857, 2.5392]	1.8656	[1.6229, 2.1155]
$\sigma_{a,9}$	<i>Inverse Gamma</i>	3 (3)	3.2699	[2.5499, 4.1379]	3.6611	[2.9248, 4.3750]
$\sigma_{a,10}$	<i>Inverse Gamma</i>	3 (3)	1.2322	[1.1138, 1.3573]	1.2955	[1.1786, 1.4270]
$\sigma_{a,11}$	<i>Inverse Gamma</i>	3 (3)	2.0037	[1.6736, 2.3966]	2.0924	[1.8156, 2.4322]
$\sigma_{a,12}$	<i>Inverse Gamma</i>	3 (3)	5.1175	[3.5623, 7.5629]	5.1796	[4.0211, 6.6388]
$\sigma_{a,13}$	<i>Inverse Gamma</i>	3 (3)	2.2275	[1.8899, 2.6483]	2.2759	[1.9704, 2.6387]
$\sigma_{d,1}$	<i>Inverse Gamma</i>	3 (3)	6.2365	[5.7256, 6.8163]	6.2837	[5.8251, 6.7771]
$\sigma_{d,2}$	<i>Inverse Gamma</i>	3 (3)	1.4525	[1.3399, 1.5678]	1.4302	[1.3262, 1.5470]
$\sigma_{d,3}$	<i>Inverse Gamma</i>	3 (3)	2.1954	[2.0034, 2.3916]	2.1920	[2.0190, 2.3753]
$\sigma_{d,4}$	<i>Inverse Gamma</i>	3 (3)	0.6440	[0.5812, 0.7078]	0.8447	[0.7507, 0.9459]
$\sigma_{d,5}$	<i>Inverse Gamma</i>	3 (3)	1.0735	[0.9873, 1.1632]	1.1080	[1.0083, 1.2177]
$\sigma_{d,6}$	<i>Inverse Gamma</i>	3 (3)	3.3325	[2.9423, 3.7816]	5.9144	[5.1244, 6.8166]
$\sigma_{d,7}$	<i>Inverse Gamma</i>	3 (3)	0.7561	[0.6785, 0.8468]	0.7867	[0.7136, 0.8699]
$\sigma_{d,8}$	<i>Inverse Gamma</i>	3 (3)	0.5444	[0.4870, 0.5987]	0.6104	[0.5427, 0.6866]
$\sigma_{d,9}$	<i>Inverse Gamma</i>	3 (3)	1.4792	[1.3667, 1.5960]	1.6268	[1.4946, 1.7637]
$\sigma_{d,10}$	<i>Inverse Gamma</i>	3 (3)	2.0892	[1.9064, 2.2694]	2.9030	[2.6296, 3.2066]
$\sigma_{d,11}$	<i>Inverse Gamma</i>	3 (3)	0.9742	[0.9004, 1.0499]	1.0492	[0.9547, 1.1435]
$\sigma_{d,12}$	<i>Inverse Gamma</i>	3 (3)	1.4949	[1.3759, 1.6245]	1.4817	[1.3552, 1.6184]
$\sigma_{d,13}$	<i>Inverse Gamma</i>	3 (3)	1.1212	[1.0135, 1.2236]	1.2046	[1.1053, 1.3013]

Table 5: Posterior Distribution of In-frequency and Duration of Price Contracts

Sectors	\mathcal{RH} model		\mathcal{HH} model	
	α_j	$D_j^{\mathcal{RH}}$	α_j	$D_j^{\mathcal{HH}}$
1. Motor vehicles and parts	0.6334 [0.6034, 0.6629]	2.23 Q [1.98, 2.51]	0.5922 [0.5620, 0.6377]	1.96 Q [1.74, 2.19]
2. Furniture and household equipment	0.8076 [0.7675, 0.8466]	5.26 Q [3.81, 7.01]	0.7449 [0.6969, 0.7939]	3.70 Q [2.86, 4.63]
3. Other durable goods	0.7712 [0.7407, 0.8012]	4.00 Q [3.42, 4.63]	0.7246 [0.6827, 0.7730]	3.31 Q [2.82, 3.88]
4. Food	0.4465 [0.3933, 0.4947]	1.26 Q [1.08, 1.46]	0.3347 [0.2880, 0.3979]	0.98 Q [0.84, 1.13]
5. Clothing and shoes	0.6795 [0.6379, 0.7229]	2.82 Q [2.30, 3.38]	0.5733 [0.5288, 0.6181]	1.98 Q [1.66, 2.34]
6. Gasoline, fuel oil, and other energy goods	0.0132 [0.0010, 0.0349]	0.23 Q [0.15, 0.31]	0.0047 [0.0004, 0.0117]	0.19 Q [0.12, 0.22]
7. Other nondurable goods	0.7065 [0.6576, 0.7496]	2.96 Q [2.45, 3.48]	0.5920 [0.5296, 0.6513]	2.09 Q [1.69, 2.56]
8. Housing	0.7408 [0.7036, 0.7748]	3.48 Q [2.86, 4.16]	0.6364 [0.5986, 0.6747]	2.42 Q [2.03, 2.84]
9. Household operation	0.5830 [0.5349, 0.6333]	1.94 Q [1.64, 2.23]	0.5181 [0.4720, 0.5639]	1.58 Q [1.36, 1.81]
10. Transportation	0.0278 [0.0106, 0.0486]	0.28 Q [0.22, 0.33]	0.0668 [0.0507, 0.0849]	0.37 Q [0.34, 0.41]
11. Medical care	0.6486 [0.6097, 0.6912]	2.39 Q [2.02, 2.78]	0.5694 [0.5278, 0.6169]	1.88 Q [1.64, 2.14]
12. Recreation	0.6981 [0.6401, 0.7486]	2.87 Q [2.32, 3.49]	0.6386 [0.5848, 0.6821]	2.33 Q [1.90, 2.85]
13. Other services	0.4475 [0.3985, 0.4941]	1.27 Q [1.11, 1.44]	0.3703 [0.3231, 0.4198]	1.06 Q [0.94, 1.19]

Table 6: Model-implied vs. Empirical durations I

	$D^{\mathcal{RH}}$	$D^{\mathcal{HH}}$	D^{BK}
Mean	1.74 Q [1.61, 1.88]	1.39 Q [1.30, 1.53]	1.24 Q
-Durable	3.01 Q [2.68, 3.31]	2.49 Q [2.24, 2.78]	1.24 Q
-Nondurable	1.38 Q [1.25, 1.53]	1.07 Q [0.98, 1.19]	1.04 Q
-Service	1.87 Q [1.71, 2.04]	1.53 Q [1.40, 1.67]	1.55 Q

Table 7: Model-implied vs. Empirical durations II

j	Sectors	$D^{\mathcal{RH}}$	$D^{\mathcal{HH}}$	D^{BK}	D^{NS}
2	Furniture and household equipment	5.26 Q [3.81, 7.01]	3.70 Q [2.86, 4.63]	1.09 Q*	1.5 Q*
4	Food	1.26 Q [1.08, 1.46]	0.98 Q [0.84, 1.13]	1.14 Q	0.81 Q**
5	Clothing and shoes	2.82 Q [2.30, 3.38]	1.98 Q [1.66, 2.34]	0.97 Q	0.93 Q
6	Gasoline, fuel oil, and other energy goods	0.23 Q [0.15, 0.31]	0.19 Q [0.12, 0.22]		0.17 Q***
10	Transportation	0.28 Q [0.22, 0.33]	0.37 Q [0.34, 0.41]	0.67 Q	1.33 Q
11	Medical care	2.39 Q [2.02, 2.78]	1.88 Q [1.64, 2.14]	3.38 Q	4.44 Q
12	Recreation	2.87 Q [2.32, 3.49]	2.33 Q [1.90, 2.85]	2.78 Q	2.26 Q

Note: *Home Furnishing, **Weighted average of processed and unprocessed food, ***Vehicle Fuel

$D^{\mathcal{RH}}$: durations of price contracts estimated in the \mathcal{RH} model.

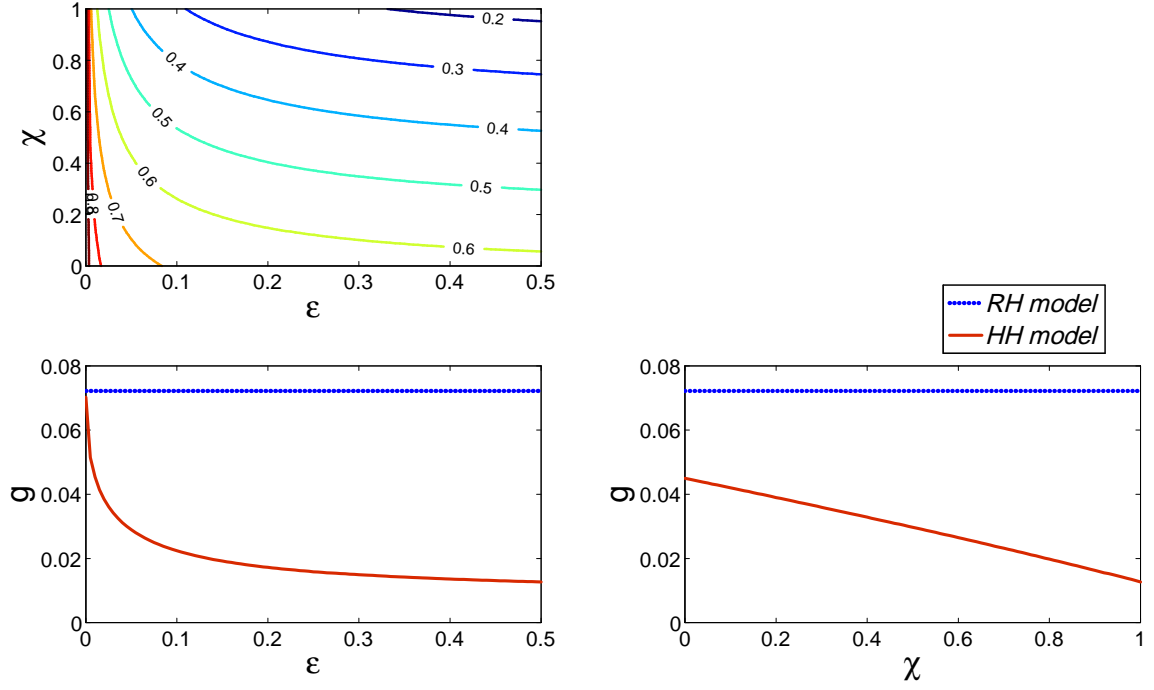
$D^{\mathcal{HH}}$: durations of price contracts estimated in the \mathcal{HH} model.

D^{BK} : durations of price contracts reported in Bills and Klenow (2004).

D^{NS} : durations of price contracts reported in Nakamura and Steinsson (2008).

B Figures

Figure 1: Comparison of slopes

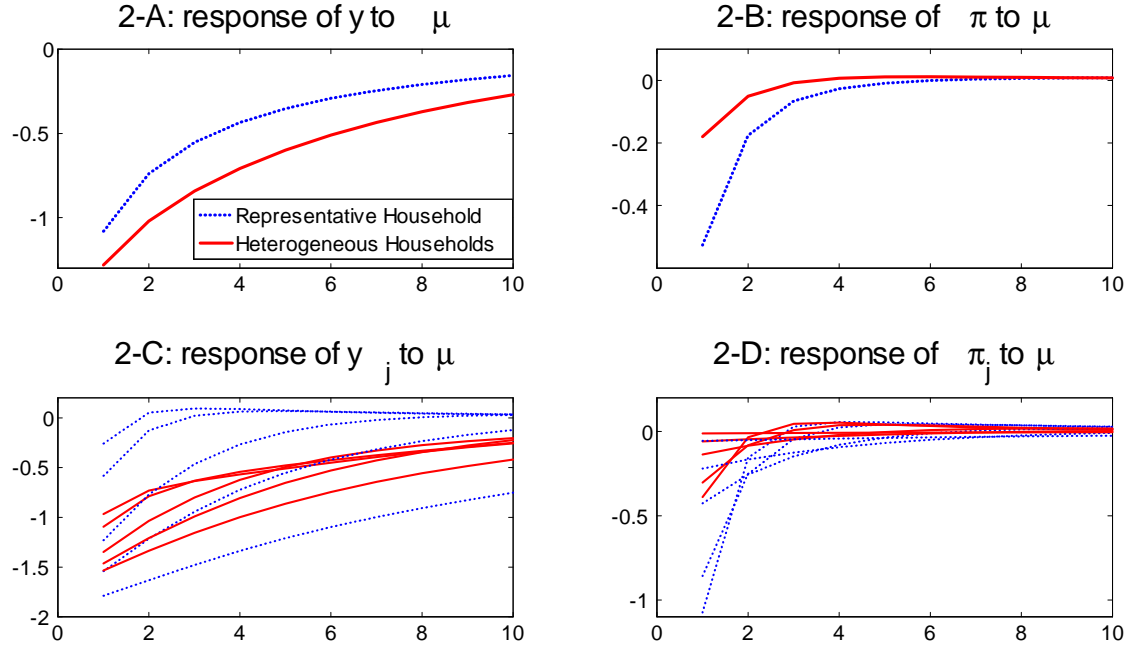


Notes: 1-A plots a contour map of $\frac{g^{\mathcal{HH}}(\alpha, \chi, \epsilon)}{g^{\mathcal{RH}}(\alpha)}$.

1-B plots $g^{\mathcal{RH}}(\alpha)$ and $g^{\mathcal{HH}}(\alpha, \chi, \epsilon)$ for $\epsilon \in (0, 0.5)$ while χ is fixed at 1.

1-C plots $g^{\mathcal{RH}}(\alpha)$ and $g^{\mathcal{HH}}(\alpha, \chi, \epsilon)$ for $\chi \in (0, 1)$ while ϵ is fixed at 0.1.

Figure 2: Impulse responses to a contractionary monetary shock



Notes: 2-A and 2-B show impulse responses of aggregate output and inflation to a monetary shock

2-C and 2-F show impulse responses of sectoral outputs and inflations to a monetary shock

Figure 3: Posterior density of α and D in single-sector economies.

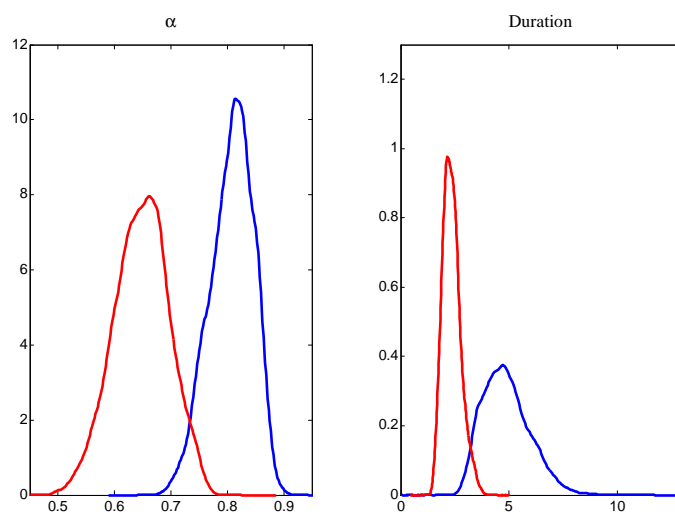


Figure 4: Posterior density of D in multiple-sector economies.

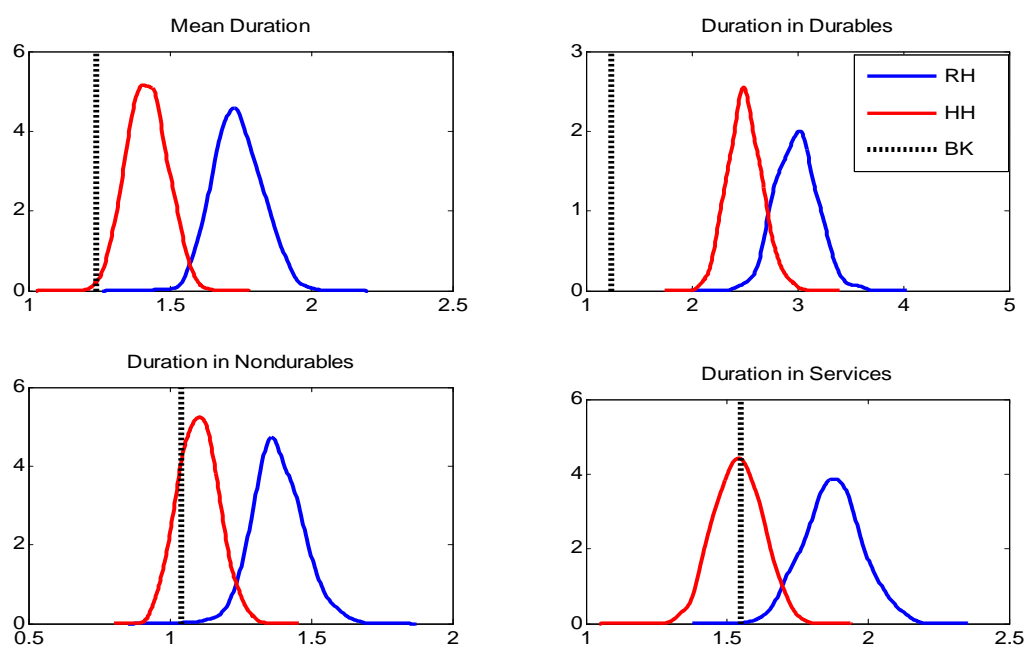
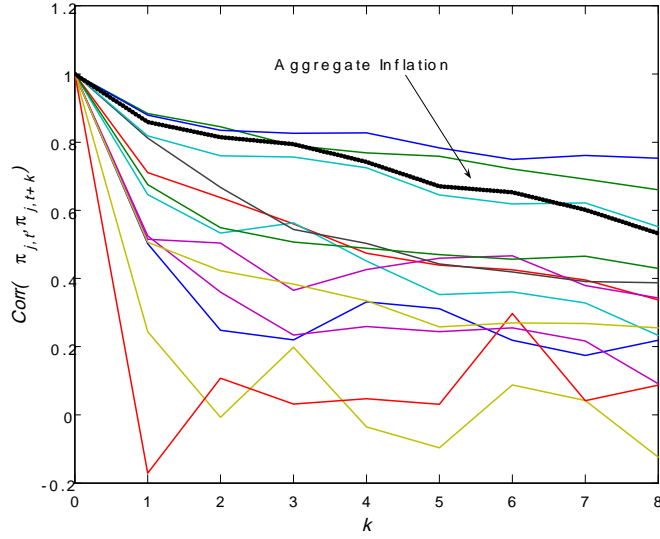
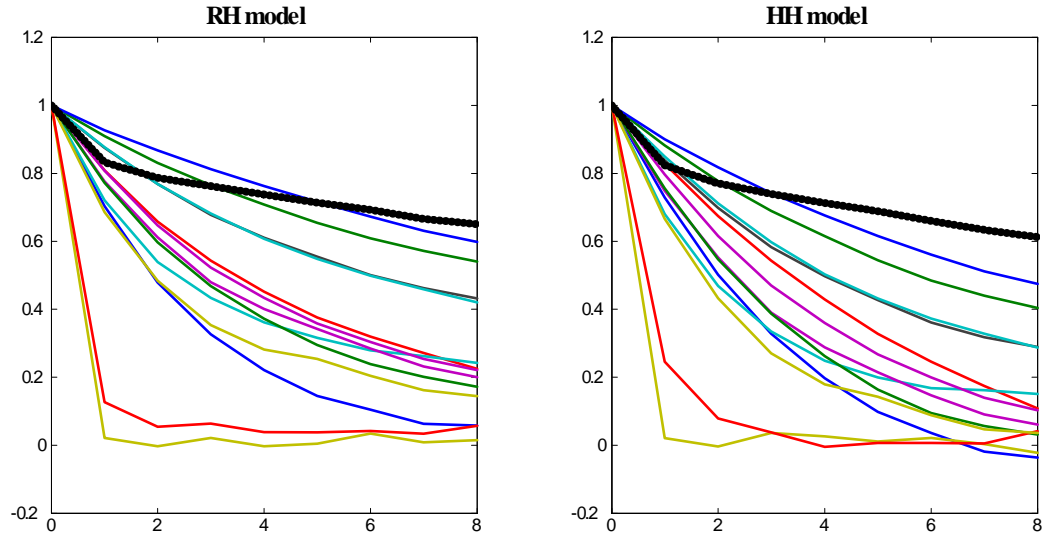


Figure 5: Autocorrelations of aggregate and sectoral inflations



Notes: autocorrelation functions on the vertical axis and lags on the horizontal axis.

Figure 6: Estimated autocorrelations of aggregate and sectoral inflations implied by the models



Notes: autocorrelation functions on the vertical axis and lags on the horizontal axis.

autocorrelation functions are plotted at the posterior mean.

C Proofs

C.1 Proof of Proposition 1

The equilibrium conditions can be reduced to

$$\frac{P(i)}{P} = \vartheta C(i), \quad (41)$$

$$C(i) = \lambda Y + (1 - \lambda) \left(\frac{P(i)}{P} \right)^{1-\theta} Y, \quad (42)$$

$$P = \left(\int_0^1 P(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \quad (43)$$

$$M = PY. \quad (44)$$

(41) is a firm's optimality condition that equates the firm's price with marginal cost multiplied by mark-up; (42) is a household budget constraint after substituting out its incomes with other optimality conditions. Combining (41) and (42) gives

$$P^R(i) = \lambda \vartheta Y + (1 - \lambda) \vartheta Y P^R(i)^{1-\theta}, \quad (45)$$

where $P^R(i) \equiv \frac{P(i)}{P}$ is firm i 's relative price. Note (45) is the same equation as (10) in section 2, and the equation should hold for all i . Thus for any arbitrary i_1 and i_2 in $[0, 1]$, it must be true that

$$P^R(i_1) - P^R(i_2) = (1 - \lambda) \vartheta Y \left\{ \left(\frac{1}{P^R(i_1)} \right)^{\theta-1} - \left(\frac{1}{P^R(i_2)} \right)^{\theta-1} \right\}. \quad (46)$$

Note that both $(1 - \lambda) \vartheta Y$ and $(\theta - 1)$ are positive. Therefore, it is not possible that either $P^R(i_1) > P^R(i_2)$ or $P^R(i_1) < P^R(i_2)$ while satisfying (46). The only case consistent with the equation (46) is $P^R(i_1) = P^R(i_2)$, and hence $P(i_1) = P(i_2)$. Then it should be that $P(i) = P, \forall i \in [0, 1]$ from (43) and that $C(i) = Y$ from (42). Finally from (41) and (44), it can be obtained that $Y = 1/\vartheta$ and $P = \vartheta M$.

C.2 Proof of Proposition 2

Let $M > \bar{M}$. If $P_{IC} \geq \vartheta M$, then we have

$$\begin{aligned} P_{IC} &= \left(n \left[[\vartheta M]^{\frac{1}{\theta}} P_{IC}^{\frac{\theta-1}{\theta}} \right]^{1-\theta} + (1-n) [\vartheta \bar{M}]^{1-\theta} \right)^{\frac{1}{1-\theta}} \\ &\leq \left(n P_{IC}^{1-\theta} + (1-n) [\vartheta \bar{M}]^{1-\theta} \right)^{\frac{1}{1-\theta}} < \left(n P_{IC}^{1-\theta} + (1-n) P_{IC}^{1-\theta} \right)^{\frac{1}{1-\theta}} \quad \left(\because \bar{M} < M \leq \frac{P_{IC}}{\vartheta} \right) \\ &= P_{IC}, \end{aligned}$$

which cannot be true. Therefore it must be that $P_{IC} < \vartheta M$. Then we have

$$\begin{aligned} P_{IC} &= \left(n \left[[\vartheta M]^{\frac{1}{\theta}} P_{IC}^{\frac{\theta-1}{\theta}} \right]^{1-\theta} + (1-n) [\vartheta \bar{M}]^{1-\theta} \right)^{\frac{1}{1-\theta}} < \left(n \left[(\vartheta M)^{\frac{1}{\theta}} [\vartheta M]^{\frac{\theta-1}{\theta}} \right]^{1-\theta} + (1-n) [\vartheta \bar{M}]^{1-\theta} \right)^{\frac{1}{1-\theta}} \\ &= \left(n [\vartheta M]^{1-\theta} + (1-n) [\vartheta \bar{M}]^{1-\theta} \right)^{\frac{1}{1-\theta}} = P_C < \left(n [\vartheta M]^{1-\theta} + (1-n) [\vartheta M]^{1-\theta} \right)^{\frac{1}{1-\theta}} = \vartheta M = P_F. \end{aligned}$$

Therefore, it has been shown that

$$P_{IC} < P_C < P_F.$$

From $Y = M/P$, it is also true that

$$Y_{IC} > Y_C > Y_F.$$

C.3 Proof of Proposition 3

The result in this proposition is a direct implication of Proposition 2. Taking log in the demand function (3), we get

$$\log Y_1 - \log Y = -\theta(\log P_1 - \log P) \quad \text{and} \quad \log Y_2 - \log Y = -\theta(\log P_2 - \log P),$$

where P_1 and P_2 are common prices, set by firms, that correspond to Y_1 and Y_2 respectively. Subtracting the second equation from the first, and then taking absolute values on both sides, we get $|\log Y_1 - \log Y_2| = \theta |\log P_1 - \log P_2|$. Since P_2 responds less to a shock under incomplete markets, $|\log P_1 - \log P_2|$ is smaller, and so is $|\log Y_1 - \log Y_2|$. Therefore $|Y_1 - Y_2|$ is also smaller under incomplete markets.

C.4 Proof of Proposition 4 (Derivation of Phillips Curve)

Since the dynamics of relative consumption and bond holding play important roles in firms' pricing decisions, I first present the household optimality conditions. Log-linearizing the household Euler equation and budget constraint, and then expressing them in terms of relative consumption, relative bond holding, and relative price yields

$$\begin{aligned} c_{j,t}^R(i) &= E_t \left[c_{j,t+1}^R(i) \right] + 2\epsilon b_{j,t}^R(i) \\ c_{j,t}^R(i) &= -\psi_1 b_{j,t}^R(i) + \beta^{-1} \psi_1 b_{j,t-1}^R(i) - \psi_2 p_{j,t}^R(i), \end{aligned}$$

where

$$\psi_1 \equiv \left[1 - \chi \left(\frac{\theta - 1}{\theta} \right) \right]^{-1} \quad \text{and} \quad \psi_2 \equiv (\theta - 1) \{ \chi(1 + \varphi) + (1 - \chi) \} \psi_1,$$

and $\epsilon \equiv \tilde{\epsilon}/PY$.⁴¹ Combining the first and the second equations, I can substitute out type- i household's relative consumption $c_{j,t}^R(i)$, which gives an equation that describes the dynamics of a household's relative bond holding given the relative price:

$$E_t \left[b_{j,t+1}^R(i) + \left(\beta^{-1} - 1 - \frac{2\epsilon}{\psi_1} \right) b_{j,t}^R(i) + \beta^{-1} b_{j,t-1}^R(i) \right] = \frac{\psi_2}{\psi_1} E_t \left[p_{j,t+1}^R(i) + p_{j,t}^R(i) \right] \quad (47)$$

Turning to firms, the log-linearized first order condition of a firm that sets its price at time t is

$$\hat{E}_t^i \sum_{k=0}^{\infty} (\alpha_j \beta)^k \{ p_{j,t+k}^*(i) - p_{t+k} \} = \hat{E}_t^i \sum_{k=0}^{\infty} (\alpha_j \beta)^k mc_{j,t+k}(i).$$

The expectation operator, \hat{E}_t^i is distinct from E_t as emphasized in Woodford (2005): \hat{E}_t^i is type- i firm's expectation at time t conditioned on its own price remaining unchanged for the entire future period from time t onwards. Because households and firms are so small in size, they cannot affect aggregate or sectoral level variables. Thus distinguishing the two expectation operators is important only for micro level variables. After substituting out relative consumption from marginal cost $mc_{j,t+k}(i)$ and then replacing $\hat{E}_t^i [p_{j,t+k}^R(i)]$ by $p_{j,t}^{*R}(i) - \sum_{s=1}^k E_t \pi_{j,t+s}$, the firm's log-linearized first order condition can be written as

$$\begin{aligned} p_{j,t}^{*R}(i) &= \left(\frac{1 - \alpha_j \beta}{1 + \varphi \theta + \psi_2} \right) \sum_{k=0}^{\infty} (\alpha_j \beta)^k E_t [V_{j,t+k}] + \sum_{k=1}^{\infty} (\alpha_j \beta)^k E_t [\pi_{j,t+k}] \\ &\quad - \psi_1 (1 - \alpha_j) \left(\frac{1 - \alpha_j \beta}{1 + \varphi \theta + \psi_2} \right) \sum_{k=0}^{\infty} (\alpha_j \beta)^k \hat{E}_t^i [b_{j,t+k}^R(i)] + \beta^{-1} \psi_1 \left(\frac{1 - \alpha_j \beta}{1 + \varphi \theta + \psi_2} \right) b_{j,t-1}^R(i), \end{aligned} \quad (48)$$

where

$$V_{j,t} \equiv (1 + \varphi) y_t + (\varphi + \eta^{-1}) y_{j,t}^R + c_{j,t}^R - (1 + \varphi) a_{j,t} - \varphi \xi_t - \eta^{-1} d_{j,t}^R$$

is the common factor across all the firms within a sector. The operator, E_t is used in the first two summations on

⁴¹ Recall that $x_{j,t}^R(i)$ denotes a percentage deviation of $X_{j,t}^R(i)$ from its steady state (which is equal to zero). Therefore it must be that $c_{j,t}^R(i) = c_{j,t}(i) - c_{j,t}$, $b_{j,t}^R(i) = b_{j,t}(i) - b_{j,t}$, and $p_{j,t}^R(i) = p_{j,t}(i) - p_{j,t}$.

the right hand side of (48) in place of \hat{E}_t^i since those terms have only aggregate and sector-level variables.

Finally, the expected value of the firm's next-period price must be a weighted average of the current price and next-period's *optimal* price:

$$E_t \left[p_{j,t+1}^R(i) \right] = \alpha_j \left[p_{j,t}^R(i) - E_t \pi_{j,t+1} \right] + (1 - \alpha_j) E_t \left[p_{j,t+1}^{*R}(i) \right]. \quad (49)$$

The three equations, (47), (48), and (49) together characterize the dynamics of micro level variables $\{b_{j,t}^R(i), p_{j,t}^R(i), p_{j,t}^{*R}(i)\}$, given the time path of the aggregate and sector level variables, $\{V_{j,t}, \pi_{j,t}\}$. The system of linear difference equations is, however, hard to solve analytically. I thus take the undetermined coefficient method as in Woodford (2005). From equation (47), I posit that the time path of relative bond holding follows

$$b_{j,t}^R(i) = \delta b_{j,t-1}^R(i) + v p_{j,t}^R(i), \quad (50)$$

where δ and v are some functions of the structural parameters. From (48) and (50), it then follows that a firm's optimal price satisfies:

$$p_{j,t}^{*R}(i) = p_{j,t}^R + \lambda b_{j,t-1}^R(i), \quad (51)$$

where λ is again a function of the parameters, and $p_{j,t}^{*R}$ denotes the common component of optimal prices of the firms who set prices anew in sector j , which is a function of the aggregate and sector variables only. If the set of parameters, $\{\lambda, \delta, v\}$ and the common component, $p_{j,t}^{*R}$ were known, one could easily construct the Phillips curve.

The first step to determine $\{\lambda, \delta, v\}$ and $p_{j,t}^{*R}$ is substituting (51) into (49) to obtain:

$$E_t \left[p_{j,t+1}^R(i) \right] = \alpha_j p_{j,t}^R(i) + \lambda(1 - \alpha_j) b_{j,t}^R(i). \quad (52)$$

Note that (50), the posited time path of bond holdings, should satisfy the difference equation (47) after $E_t \left[p_{j,t+1}^R(i) \right]$ is substituted out using (52). This is true if and only if $\{\lambda, \delta, v\}$ satisfy the following conditions:

$$v = \frac{(1 - \alpha_j) \psi_2 \delta}{\alpha_j \psi_1 \delta - \beta^{-1} \psi_1} \quad (53)$$

$$\lambda = \frac{\beta^{-1} - \alpha_j \delta}{(1 - \alpha_j) \psi_2} \left[\frac{2\epsilon}{\beta^{-1} - \delta} - \frac{\psi_1 (1 - \delta)}{\delta} \right]. \quad (54)$$

Note I have expressed λ and v as functions of δ . One more relation is needed to determine $\{\lambda, \delta, v\}$ and the firm's first order condition (48) provides that additional relation. Based on (50), $\hat{E}_t^i \left[b_{j,t+k}^R(i) \right]$ can be expressed as

$$\hat{E}_t^i \left[b_{j,t+k}^R(i) \right] = \delta \hat{E}_t^i \left[b_{j,t+k-1}^R(i) \right] + v \hat{E}_t^i \left[p_{j,t+k}^R(i) \right] = \delta \hat{E}_t^i \left[b_{j,t+k-1}^R(i) \right] + v \left[p_{j,t}^{*R}(i) - \sum_{s=1}^k E_t \pi_{j,t+s} \right],$$

which implies the following equation:

$$\sum_{k=0}^{\infty} (\alpha_j \beta)^k \hat{E}_t^i \left[b_{j,t+k}^R(i) \right] = \left(\frac{\delta}{1 - \delta \alpha_j \beta} \right) b_{j,t-1}^R(i) + \frac{v}{(1 - \alpha_j \beta) (1 - \delta \alpha_j \beta)} \left[p_{j,t}^{*R}(i) - \sum_{k=1}^{\infty} (\alpha_j \beta)^k E_t \left[\pi_{j,t+k} \right] \right].$$

Plugging this expression into the firm's first order condition, (48), I obtain:

$$\Psi p_{j,t}^{*R}(i) = \left(\frac{1 - \alpha_j \beta}{1 + \varphi \theta + \psi_2} \right) \sum_{k=0}^{\infty} (\alpha_j \beta)^k E_t \left[V_{j,t+k} \right] + \Psi \sum_{k=1}^{\infty} (\alpha_j \beta)^k E_t \left[\pi_{j,t+k} \right] + \Phi b_{j,t-1}^R(i), \quad (55)$$

where

$$\begin{aligned} \Psi &\equiv 1 - \frac{\psi_2 (1 - \alpha_j)^2 \delta}{(1 + \varphi \theta + \psi_2) (1 - \alpha_j \beta \delta) (\beta^{-1} - \alpha_j \delta)} \\ \Phi &\equiv \frac{\psi_1 (1 - \alpha_j \beta) (\beta^{-1} - \delta)}{(1 + \varphi \theta + \psi_2) (1 - \alpha_j \beta \delta)}. \end{aligned}$$

Comparing (55) and (51), one can solve for $p_{j,t}^{*R}$:

$$p_{j,t}^{*R} = \Psi^{-1} \left(\frac{1 - \alpha_j \beta}{1 + \varphi \theta + \psi_2} \right) \sum_{k=0}^{\infty} (\alpha_j \beta)^k E_t [V_{j,t+k}] + \sum_{k=1}^{\infty} (\alpha_j \beta)^k E_t [\pi_{j,t+k}], \quad (56)$$

and the coefficient λ satisfies the following equation:

$$\Psi \lambda = \Phi. \quad (57)$$

The three equations, (53), (54), and (57) jointly determine the coefficients $\{\lambda, \delta, v\}$ if a solution exists. The system of equations is nonlinear in $\{\lambda, \delta, v\}$, and thus there could be more than one solution. Following Woodford (2005), I only consider a solution that would make the joint dynamics of relative price and relative bond holdings convergent so that the means and the variances remain bounded. We can rewrite equations (50) and (52) as the following system:

$$\begin{pmatrix} E_t [p_{j,t+1}^R(i)] \\ b_{j,t}^R(i) \end{pmatrix} = \begin{pmatrix} \alpha_j + (1 - \alpha_j) \lambda v & (1 - \alpha_j) \lambda \delta \\ v & \delta \end{pmatrix} \begin{pmatrix} p_{j,t}^R(i) \\ b_{j,t-1}^R(i) \end{pmatrix}. \quad (58)$$

The system is stable if and only if the eigenvalues of the coefficient matrix are inside the unit circle.

Lemma 1 *If $\alpha_j \beta^{-1} \leq 1$, then the system (58) is stable if and only if $0 < \delta < \beta^{-1}$.*

See the following subsections for the proof of Lemma 1 and 2. Based on Lemma 1, I focus only on the values of δ on the interval $(0, \beta^{-1})$, and α_j on $(0, \beta)$ in what follows. A natural question to ask at this point might be if there exists such a $\{\lambda, \delta, v\}$ that solve (53), (54), and (57) while satisfying the stability condition, $0 < \delta < \beta^{-1}$. Lemma 2 shows that there indeed exists a unique set of $\{\lambda, \delta, v\}$ as long as ϵ is positive.

Lemma 2 *There exists a unique set of $\{\lambda, \delta, v\}$ that satisfies (53), (54), and (57), and $0 < \delta < \beta^{-1}$ if $\epsilon > 0$.*

As mentioned above, once I find the solution for $\{\lambda, \delta, v\}$, the generalized NK Phillips curve can be constructed by combining (55) which determines a firm's relative optimal price $p_{j,t}^{*R}(i)$ and (22) which determines the dynamics of the sector price level $p_{j,t}$. Log-linearizing (22) yields

$$p_{j,t} = \frac{1}{n_j} \int_{\mathcal{I}_j^*} p_{j,t}^{*R}(i) di - \alpha_j p_{j,t-1}.$$

Substituting (51) into the equation above, one obtains

$$\alpha_j \pi_{j,t} = \frac{1}{n_j} \int_{\mathcal{I}_j^*} \left(p_{j,t}^{*R} + \lambda b_{j,t-1}^R(i) \right) di,$$

implying

$$p_{j,t}^{*R} = \frac{\alpha_j}{1 - \alpha_j} \pi_{j,t}, \quad (59)$$

because $\int_{\mathcal{I}_j^*} b_{j,t-1}^R(i) di = 0$ holds due to the assumption of time-dependent pricing. Note that time-dependent pricing is a crucial assumption that allows me to avoid keeping track of distributions of household wealth. Substituting (59) into (56) gives the "sector-level Phillips curve":

$$\pi_{j,t} = \beta E_t [\pi_{j,t+1}] + g(\alpha_j) V_{j,t}, \quad (60)$$

where

$$g(\alpha_j, \epsilon, \chi) \equiv \left\{ \frac{(1 - \alpha_j \beta)(1 - \alpha_j)}{\alpha_j} \right\} \left[\frac{(1 - \alpha_j \beta \delta)^2}{(1 + \varphi \theta + \psi_2)(1 - \alpha_j \beta \delta)^2 - \psi_2(1 - \alpha_j)^2 \beta \delta} \right] \quad (61)$$

I have made explicit in (61) g 's dependence on the two financial frictions parameters, (ϵ, χ) , since δ and ψ_2 are functions of these two parameters.

To summarize the results obtained so far, the sector Phillips curve, for each sector j , is given by

$$\pi_{j,t} = \beta E_t [\pi_{j,t+1}] + g(\alpha_j, \epsilon, \chi) \left[(1 + \varphi) y_t + (\varphi + \eta^{-1}) y_{j,t}^R + c_{j,t}^R \right] - \zeta_{j,t}, \quad (62)$$

where $g(\alpha_j, \epsilon, \chi)$ is given by (61), and $\{\lambda, \delta, \nu\}$ satisfy (53), (54), (57) and $0 < \delta < \beta^{-1}$. The disturbance term $\zeta_{j,t}$ is a linear combination of exogenous shocks:

$$\zeta_{j,t} \equiv g(\alpha_j, \epsilon, \chi) \left[(1 + \varphi) a_{j,t} + \varphi \xi_t + \eta^{-1} d_{j,t}^R \right].$$

The Phillips curve for aggregate inflation π_t is consequently obtained by taking a weighted sum of sector Phillips curves:

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa y_t + \Theta_{c,t} + \Theta_{y,t} - \zeta_t,$$

where

$$\begin{aligned} \Theta_{c,t} &\equiv \sum_{j=1}^J n_j g(\alpha_j, \epsilon, \chi) c_{j,t}^R, & \Theta_{y,t} &\equiv (\varphi + \eta^{-1}) \sum_{j=1}^J n_j g(\alpha_j, \epsilon, \chi) y_{j,t}^R, \\ \kappa &\equiv (1 + \varphi) \sum_{j=1}^J n_j g(\alpha_j, \epsilon, \chi), & \zeta_t &\equiv \sum_{j=1}^J n_j \zeta_{j,t}. \end{aligned}$$

C.5 Comparison of $g^{\mathcal{HH}}(\alpha_j, \chi, \epsilon)$ to $g^{\mathcal{RH}}(\alpha_j)$:

This subsection shows that the slope of the Phillips curve is smaller in the \mathcal{HH} model than in the \mathcal{RH} model. For this purpose, I compare $g^{\mathcal{HH}}(\alpha_j, \epsilon, \chi)$ to $g^{\mathcal{RH}}(\alpha_j)$.

$$\begin{aligned} g^{\mathcal{RH}}(\alpha) &\equiv \left\{ \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \right\} \frac{1}{1 + \varphi\theta} \\ g^{\mathcal{HH}}(\alpha, \chi, \epsilon) &\equiv \left\{ \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \right\} \left[\frac{(1 - \alpha\beta\delta)^2}{(1 + \varphi\theta + \psi_2)(1 - \alpha\beta\delta)^2 - \psi_2(1 - \alpha)^2\beta\delta} \right]. \end{aligned}$$

For the sake of brevity, I omit the derivation of $g^{\mathcal{HH}}$ and refer the interested reader to Woodford (2003). Taking the ratio of $g^{\mathcal{HH}}(\alpha, \chi, \epsilon)$ to $g^{\mathcal{RH}}(\alpha)$, I obtain

$$\begin{aligned} \frac{g^{\mathcal{HH}}(\alpha, \chi, \epsilon)}{g^{\mathcal{RH}}(\alpha)} &= \frac{(1 - \alpha\beta\delta)^2(1 + \varphi\theta)}{(1 + \varphi\theta + \psi_2)(1 - \alpha\beta\delta)^2 - \psi_2(1 - \alpha)^2\beta\delta} \\ &= \frac{(1 - \alpha\beta\delta)^2(1 + \varphi\theta)}{(1 + \varphi\theta)(1 - \alpha\beta\delta)^2 + \psi_2(1 - \alpha\beta\delta)^2 - \psi_2(1 - \alpha)^2\beta\delta} \\ &= \frac{(1 - \alpha\beta\delta)^2(1 + \varphi\theta)}{(1 + \varphi\theta)(1 - \alpha\beta\delta)^2 + \psi_2 \{1 - 2\alpha\beta\delta + (\alpha\beta\delta)^2 - \beta\delta + 2\alpha\beta\delta - \alpha^2\beta\delta\}} \\ &= \frac{(1 - \alpha\beta\delta)^2(1 + \varphi\theta)}{(1 + \varphi\theta)(1 - \alpha\beta\delta)^2 + \underbrace{\psi_2 \{ (1 - \beta\delta) + \alpha^2\beta\delta(1 - \beta\delta) \}}_{>0}} < 1. \end{aligned}$$

C.6 Proof of Lemma 1

The eigenvalues of the system are the roots of the following equation:

$$f(X) = (\alpha + (1 - \alpha)\lambda\nu - X)(\delta - X) - (1 - \alpha)\lambda\delta\nu = X^2 - \{\alpha + \delta + (1 - \alpha)\lambda\nu\}X + \alpha\delta$$

The two roots are given by

$$\begin{aligned} X_1 &= 0.5 \left\{ \alpha + \delta + (1 - \alpha)\lambda v + \sqrt{(\alpha + \delta + (1 - \alpha)\lambda v)^2 - 4\alpha\delta} \right\}, \\ X_2 &= 0.5 \left\{ \alpha + \delta + (1 - \alpha)\lambda v - \sqrt{(\alpha + \delta + (1 - \alpha)\lambda v)^2 - 4\alpha\delta} \right\}. \end{aligned}$$

The term inside the root $\{(\alpha + \delta + (1 - \alpha)\lambda v)^2 - 4\alpha\delta\}$ is always positive, implying that X_1 and X_2 are two real roots with $X_1 \geq X_2$. Consequently, for the system to be stable, the following two conditions must hold:

$$(i) \ X_1 < 1 \text{ and } (ii) \ X_2 > -1.$$

(i) Note that the first condition, $X_1 < 1$, holds if and only if

$$\sqrt{(\alpha + \delta + (1 - \alpha)\lambda v)^2 - 4\alpha\delta} < 2 - \{\alpha + \delta + (1 - \alpha)\lambda v\},$$

which holds if and only if the following two conditions are met:

$$\begin{aligned} (a) \quad & : \quad (\alpha + \delta + (1 - \alpha)\lambda v)^2 - 4\alpha\delta < (2 - \{\alpha + \delta + (1 - \alpha)\lambda v\})^2 \\ (b) \quad & : \quad 2 - \{\alpha + \delta + (1 - \alpha)\lambda v\} > 0 \end{aligned}$$

(a) and (b) can be simplified as

$$\begin{aligned} (a) \quad & : \quad \alpha + \delta + (1 - \alpha)\lambda v < 1 + \alpha\delta \\ (b) \quad & : \quad \alpha + \delta + (1 - \alpha)\lambda v < 2 \end{aligned}$$

Suppose $\alpha\delta \geq 1$. Then condition (a) becomes irrelevant. And, (b) can be written as:

$$\delta < 1 + (1 - \alpha)(1 - \lambda v).$$

Suppose $\alpha\delta \leq 1$. Then condition (b) becomes irrelevant, and (a) can be written as:

$$(1 - \alpha)(\delta + \lambda v - 1) < 0,$$

which can be simplified to

$$\delta < 1 - \lambda v$$

assuming $0 < \alpha < \beta$ (which will be the case throughout the paper). In sum, the first condition, $X_1 < 1$, holds if and only if

$$\delta < 1 + (1 - \alpha)(1 - \lambda v) \quad \text{and} \quad \alpha\delta \geq 1 \tag{63}$$

or

$$\delta < 1 - \lambda v \quad \text{and} \quad \alpha\delta \leq 1 \tag{64}$$

(ii) The second condition, $X_2 > -1$, holds if and only if

$$\delta > -\left(1 + \frac{1 - \alpha}{1 + \alpha}\lambda v\right), \tag{65}$$

Note

$$\lambda v = (1 - \delta) - \frac{2\epsilon\delta}{\psi_1(\beta^{-1} - \delta)},$$

which leads to

$$1 - \lambda v = \delta + \frac{2\epsilon\delta}{\psi_1(\beta^{-1} - \delta)}$$

The condition (63) cannot be true because

$$\begin{aligned} \delta &< 1 + (1 - \alpha)(1 - \lambda v) \leq 1 + (1 - \alpha)\delta \quad (\because 1 - \lambda v \leq \delta \text{ when } \alpha\delta \geq 1) \\ &\iff \alpha\delta < 1, \end{aligned}$$

which contradicts $\alpha\delta \geq 1$. Therefore the stability conditions are summarized by (64) and (65). Consider (64) first:

$$\begin{aligned} \delta &< 1 - \lambda v = \delta + \frac{2\epsilon\delta}{\psi_1(\beta^{-1} - \delta)} \quad \text{and} \quad \alpha\delta \leq 1 \iff 0 < \frac{2\epsilon\delta}{\psi_1(\beta^{-1} - \delta)} \quad \text{and} \quad \alpha\delta \leq 1 \\ &\iff 0 < \delta < \frac{1}{\beta} \quad \text{and} \quad \delta \leq \frac{1}{\alpha} \iff 0 < \delta < \frac{1}{\beta}. \end{aligned} \quad (66)$$

Now let us consider (65). From (64), it can be shown that $\lambda v < 1 - \delta < 1$. Then we have

$$-\left(1 + \frac{1 - \alpha}{1 + \alpha}\lambda v\right) < -\left(1 + \frac{1 - \alpha}{1 + \alpha}\right) < -1 < \delta, \quad (67)$$

which shows that (64) implies (65). Therefore the inequality (66) alone gives the stability condition, and this proves Lemma 1.

C.7 Proof of Lemma 2

The system of nonlinear equations for $\{\lambda, \delta, v\}$ is given by the following three equations:

$$\left(1 - \frac{\psi_2(1 - \alpha_j)^2\delta}{(1 + \varphi\theta + \psi_2)(1 - \alpha_j\beta\delta)(\beta^{-1} - \alpha_j\delta)}\right) \frac{\beta^{-1} - \alpha_j\delta}{(1 - \alpha_j)\psi_2} \left[\frac{2\epsilon}{\beta^{-1} - \delta} - \frac{\psi_1(1 - \delta)}{\delta}\right] = \frac{\psi_1(1 - \alpha_j\beta)(\beta^{-1} - \delta)}{(1 + \varphi\theta + \psi_2)(1 - \alpha_j\beta\delta)}. \quad (68)$$

$$v = \frac{(1 - \alpha_j)\psi_2\delta}{\alpha_j\psi_1\delta - \beta^{-1}\psi_1} \quad (69)$$

$$\lambda = \frac{\beta^{-1} - \alpha_j\delta}{(1 - \alpha_j)\psi_2} \left[\frac{2\epsilon}{\beta^{-1} - \delta} - \frac{\psi_1(1 - \delta)}{\delta}\right]. \quad (70)$$

Given δ (and other parameters), λ and v are uniquely determined by (69) and (70). Therefore it remains to show if there exist δ that satisfy (68) and $0 < \delta < \frac{1}{\beta}$. Rewrite (68) as:

$$2\epsilon = \underbrace{\frac{\psi_1(1 - \delta)(\beta^{-1} - \delta)}{\delta} + \frac{\psi_1\psi_2(1 - \alpha_j\beta)(1 - \alpha_j)(\beta^{-1} - \delta)^2}{[(1 + \varphi\theta + \psi_2)(1 - \alpha_j\beta\delta)^2\beta^{-1} - \psi_2(1 - \alpha_j)^2\delta]}}_{\equiv K(\delta)}, \quad (71)$$

where $K(\delta)$ is a continuous function on $\delta \in \left(0, \frac{1}{\beta}\right)$. In the two limiting cases where $\epsilon = 0$ and $\epsilon = \infty$, $\delta = \beta^{-1}$ and $\delta = 0$ respectively satisfy (71). Moreover, it is tedious yet straightforward to show $\frac{\partial K(\delta)}{\partial \delta} < 0$ for $\delta \in \left(0, \frac{1}{\beta}\right)$, which implies that for each value of $\epsilon \in (0, \infty)$, there exists one value of δ that satisfies (71) and that δ is decreasing in ϵ with the following properties:

$$\lim_{\epsilon \rightarrow 0} \delta = \frac{1}{\beta} \quad \text{and} \quad \lim_{\epsilon \rightarrow \infty} \delta = 0.$$

This proves Lemma 2.