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Monetary Policy with Heterogeneous Households and Financial Frictions

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Abstract

This paper presents and estimates a sticky-price model with heterogenous households and financial frictions. Frictions in state-contingent asset markets lead to imperfect risk-sharing among households with idiosyncratic labor incomes. I study the impacts of the introduced financial frictions on optimal monetary policy by documenting implications for the central bank's objective function, the equation that characterizes inflation-output gap trade-offs, targeting rules, interest rate rules, and welfare of the economy. Employing the estimated model, the paper argues that the central bank should place a stronger emphasis on stabilizing inflation than it has, and failing to do so can generate nontrivial welfare costs.

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1 Introduction

Financial frictions can lead to imperfect risk-sharing among households with idiosyncratic labor incomes. Does this matter for monetary policy? If yes, how? Through what mechanism does household heterogeneity affect optimal monetary policy and the targeting rules of a central bank? This paper attempts to provide an answer to these questions using a standard sticky-price macroeconomic model (also known as a New Keynesian (NK) model), one of the workhorse models for the analysis of monetary policy and business cycles.

Frictions in financial markets are often considered a major source of inefficient market outcomes. In particular, in the presence of financial frictions households are not able to fully insure against idiosyncratic shocks to labor income, which leads to inefficient consumption allocation across households. To the best of my knowledge, however, the literature on optimal monetary policy in the sticky-price framework has paid little attention to this particular implication of financial frictions. Most of the literature either assumes perfect risk-sharing among households, or equivalently relies on the representative-household abstraction. Whether the representative-household abstraction is a good approximation in studying aggregate dynamics and government policies depends on the questions being asked. This paper documents how introducing frictions in asset markets affects equilibrium consumption allocation, the monetary transmission mechanism, and the central bank policy objectives. I then examine if monetary policies well-designed under the special assumption of a representative household (or perfect asset markets) work in a more general environment in which household heterogeneity becomes relevant. I summarize the main results in the subsequent paragraphs.

First, this paper demonstrates that the familiar policy prescription, often referred to as "flexible inflation targeting", that attempts to stabilize two aggregate indices, inflation and the output gap, continues to characterize optimal monetary policy.[1] The central bank does not need to pay attention to any other aggregate/disaggregate variables or construct indices to address the inefficiency caused by financial frictions. However, eliminating financial frictions is *not* a good simplification for evaluating monetary policies. This paper argues that in the presence of financial frictions, stabilizing inflation provides the additional benefit of reducing undesired consumption dispersion. Therefore central banks should adopt "stronger" inflation targeting. In sum, while the central bank should continue to target inflation and the output gap, it should place a larger relative weight on inflation stabilization.

However, in an environment with financial frictions, the 'cost' of stabilizing inflation increases as well as the benefit. Financial frictions amplify price stickiness endogenously, reduc-

[1] Inflation and output gap are the only variables that enter the central bank's loss function. The term "flexible" inflation targeting was first introduced in Svensson (1999). According to Svensson (2009), most inflation-targeting central banks (if not all) currently conduct flexible inflation targeting rather than "strict" inflation targeting that puts zero weight on output gap stabilization.

ing the slope of the NK Phillips curve for a given degree of nominal rigidity. As a result, under financial frictions the central bank faces a less favorable inflation-output gap tradeoff than in an environment without these frictions. In adjusting the inflation rate, the central bank would have to tolerate a larger deviation of output from its efficient level. In other words, the 'cost' of inflation stabilization is higher. Thus "stronger" inflation targeting under financial frictions does not necessarily result in inflation actually being more stable than it would be in an environment without financial frictions.

More generally, financial frictions generates both macroeconomic and microeconomic inefficiencies. In a model with staggered price setting, unstable inflation leads to undesired dispersion of output across firms. This leads to variability in household's labor income, and when household risk-sharing is imperfect, inefficient dispersion in household consumption. Thus inflation stability becomes more desired relative to the case of perfect risk-sharing. Besides generating the microeconomic inefficiency in the allocation of household consumption, financial frictions aggravate inefficiency in macroeconomic dynamics, generating larger deviations of output from its efficient level.[2] In other words, financial frictions adversely affect not only the second moment (i.e. cross-section dispersion of micro variables) but also the first moment (i.e. the average of micro variables, *a.k.a.* aggregate variables) of the equilibrium.

Therefore the central bank must consider the magnitude of these effects in the design of optimal monetary policy. Policymakers should stabilize inflation more aggressively only when the extra benefit of stabilizing inflation exceeds the extra cost, or equivalently when the second-moment effect (consumption dispersion) dominates the first-moment effect (deviation from the efficient aggregate output level) in its welfare consequences.

The question of which effect in fact dominates is quantitative in nature and the answer depends on specific values of model parameters. To address this issue, I estimate the model using Bayesian methods and let the data choose parameter combinations that are most likely to explain U.S. time series data. Based on estimation results, this paper reaches several conclusions. First, the extra benefit of stabilizing inflation is likely to exceed the cost, and therefore the central bank should place a stronger emphasis on stabilizing inflation. Second, taking the inefficiency from financial frictions into account in formulating either an optimal targeting rule or interest rate rule is crucial for the welfare of households. The central bank's failing to do so can lead to non-trivial welfare loss.

The paper is organized as follows. After discussing the related literature briefly, I develop the model in Section 2 Section 3 defines equilibrium and presents equations that characterize

[2]Lee (2009) shows, in a simlar but different model setting, that the economy with imperfect risk-sharing is characterized by a grater degree of real rigidities so that the aggregate output, after either nominal or real shocks hit the economy, would deviate from the efficient level of output by a larger amount and for a longer period of time. This paper shows the same result although the type of financial frictions assumed is different.

the equilibrium dynamics of key aggregate variables. Section 4 derives the loss function of the central bank that maximizes household welfare and presents some theoretical results on optimal monetary policy. Section 5 estimates the model and discusses the quantitative implications of financial frictions for optimal monetary policy and the welfare costs associated with some alternative policy rules. Section 6 summarizes the results and concludes.

Related Literature I adopt a *basic* NK model[3] as a framework for two main reasons. First, NK models have recently become the workhorse for monetary policy analysis, and often serve as the basis for large-scale models developed at central banks. Second, the basic model is simple enough to show the main results analytically. In this regard, I deliberately avoid using more elaborate sticky-price models[4] .

Recently, a number of papers have extended NK models to allow various forms of financial market imperfections. An earlier contribution, Bernanke et al. (1999), continues to serve as a reference model in the literature. They showed that financial frictions amplifies the real impact of a policy shock through financial-accelerator mechanism. This paper proposes another mechanism through which financial frictions affect real sectors, showing less insured household consumption distorts household incentive to supply labor hours, which increases price rigidities and thus amplifies business cycle fluctuations.

More recently, Christiano et al. (2007), Gertler and Karadi (2009), and Curdia and Woodford (2009a and 2009b) have presented models that feature both nominal rigidities and financial frictions. These studies assumed away heterogeneous households with idiosyncratic labor incomes thus precluding any consideration of the policy implications of frictions in transferring resources among households. In contrast, this paper focuses on the somewhat more traditional issues associated with incomplete markets and heterogeneous households that have been studied extensively in flexible-price macroeconomic models, along the line of Huggett (1993), Aiyagari (1994) and Krusell and Smith (1998).[5] Many of the papers mentioned above are more focused on understanding the recent financial crisis from both a positive and normative perspective.[6]

Curdia and Woodford (2009a) is particularly related to this paper, showing a similar result: "flexible inflation targeting" continues to serve as optimal monetary policy (at least

[3]The standard sticky-price models are extensively discussed in many graduate level textbooks such as Woodford (2003), Walsh (2003), and Gali (2008). Goodfriend and King (1997), Aoki (2001), Benigno (2004), Clarida, Gali, and Gertler (1999, 2002), Erceg, Henderson, and Levin (2000), Benigno and Woodford (2007) and others document various issues in monetary policies in New Keynesian framework.

[4]See Christiano et al. (2005) and Smet and Wouters (2003, 2007) for leading examples of medium-scale sticky-price DSGE models

[5]Also see Heathcote et al.(2009) for a review of the literature.

[6]Also see Gertler and Kiyotaki (2009) and Reis (2009b) for a review of U.S. monetary policy in response to the recent crisis.

approximately) even with certain financial frictions. Curdia and Woodford (2009a) constructs household heterogeneity by positing two types of households with different time preferences for consumption, one is patient, the other is impatient.[7] As a consequence, households have an incentive to transfer their resources although their labor incomes are identical. In contrast, this paper assumes households have the same preferences and thus identical impatience to consume, but they produce differentiated goods, which leads to idiosyncratic labor incomes. This paper and Curdia and Woodford (2009a) complement each other by showing the robustness of flexible inflation targeting to different forms of household heterogeneity and financial frictions. However, since Curdia and Woodford's model excludes the channel through which relative price distortion leads to relative consumption distortion, it shows no extra benefit of stable inflation, and hence there is no motivation for the central bank to place additional importance on inflation stabilization beyond that suggested by the basic NK model. In contrast, the analysis in this paper provides a compelling argument for "stronger" inflation targeting.

2 Model

This section describes the model economy. The model is similar to the basic NK model with industry-specific labor markets in Woodford (2003). The only deviation from the basic model is the existence of a cost of transferring resources among households as in Schulhofer-Wohl (2007). As a result, households are not able to insure their income risks perfectly. The model nests the basic NK model as a special case, which makes it possible to compare the complete and incomplete market economies within a single framework.

2.1 Households

There is a continuum of industries indexed by $i \in [0,1]$, each of which produces a different type of good. Each industry i has a representative firm, called type-i firm. Each type of good requires a distinct labor skill to be produced and thus labor markets are industry-specific. In each industry i, there is a representative household called type-i household. Type-i household possesses a labor skill specialized exclusively for industry i, and thus supplies labor service to type-i firm.

Type-i household maximizes the following discounted expected utility function:

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t(i)^{1-\sigma}-1}{1-\sigma}-\frac{H_t(i)^{1+\varphi}}{1+\varphi}\right]\right),$$

[7] Iacoviello (2005) also considered two types of households along the same line.

where $C_t(i)$ denotes type-i household's consumption, $H_t(i)$ denotes the hours of labor services supplied to industry i, β is the discount factor, φ is the inverse of the Frisch elasticity of labor supply and σ is the coefficient of relative risk aversion. The parameters σ and φ are non-negative and $\beta \in (0,1)$.

A household's dynamic budget constraint is given by

$$P_t C_t(i) + E_t\left[Q_{t,t+1} B_{t+1}(i)\right] + P_t \underbrace{\phi \Phi\left(C_t(i), X_t(i)\right)}_{\text{real transaction cost}} = B_t(i) + W_t(i) H_t(i) + \Pi_t - P_t T_t,$$

where P_t denotes aggregate price level, $W_t(i)$ is the competitive nominal wage rate in industry i, T_t is a lump-sum tax and Π_t is the aggregate profit of the economy, $\Pi_t = \int_0^1 \Pi_t(i) di$. A household's *total* income at time t is the sum of *labor* income $W_t(i)H_t(i)$ and *capital* income Π_t. I let $X_t(i)$ denote type-i household's *total* after-tax real income at time t:

$$X_t(i) \equiv \frac{W_t(i) H_t(i) + \Pi_t - P_t T_t}{P_t}.$$

Unlike labor income, capital income and taxes are not idiosyncratic. An implicit simplifying assumption is that every household holds the same mutual fund so that the economy's total profit is equally distributed among households, and that the government collects the same amount of lump-sum tax from each household. Consequently, the income differential between any two households is entirely due to a difference in labor income. This simplifying assumption does not affect the main results of this paper.[8]

Households can trade nominal securities with arbitrary patterns of state-contingent payoffs. In the budget constraint, $B(i)$ denotes type-i household's holding of one period state-contingent nominal securities and $Q_{t,t+1}$ is a stochastic discount factor. At time t, a household makes its portfolio decision. Households completely specify the desired revenue for each possible state ($B_{t+1}(i)$) taking the market prices for the state-contingent payoffs as given. Thus $B_{t+1}(i)$ is a random variable that can have different values depending on the state realized at time $t+1$.

Making consumption different from income is costly. If a household's consumption $C_t(i)$ is different from its income $X_t(i)$ then the cost is the amount $\phi \Phi\left(C(i), X(i)\right)$ of consumption good. I assume $\phi \geq 0$. An important special case arises when $\phi = 0$. The model presented here is then the same as the basic NK model. Following Schulhofer-Wohl (2007), I let the

[8] Following the convention in the NK literature on optimal monetary policy, I later introduce an employment subsidy, which leads to zero steady state value of ptofit Π. Consequently the profit have no first order effects on a household's total income anyway. While introducing an employment subsidy leads to a cleaner expression of objective function of the central bank, the main results of this paper do not depend on this particular assumption.

transaction cost function $\Phi(\cdot)$ have the following form:

$$\Phi\left(C,X\right)=\frac{C}{2}\left(\log\frac{C}{X}\right)^{2}.$$

Note that any convex functions would lead us to the same results.

There are other ways to introduce incomplete asset markets in macroeconomic models. Among others, one of the most standard approaches is to assume there exists only one financial security, a short-term riskless bond. In contrast, this model has a full set of state-contingent assets. The transaction cost, however, causes households to insure their income risks by a lesser amount than they otherwise would, which leads to less ideal risk-sharing. Therefore, although the asset markets are complete *in a nominal sense*, they are *effectively incomplete*.

This approach has both advantages and disadvantages. As noted in Schulhofer-Wohl (2007), the model of complete markets with a transaction cost may not really correspond to any real-word institution. Moreover, the approach does not investigate the exact nature of the transaction cost. While this would be interesting, it is not the focus of this paper. In addition, reality suggests a multiplicity of "risk-sharing" and "risk-sharing-preventing" mechanisms. However using a single device allows the paper to focus on the research question of interest. In addition, the current approach provides a straightforward way to include the basic NK model as a special case within a single framework. Most important, an alternative asset market institution would not change the main insights of this paper as long as the asset market characterization caused a household's relative consumption to move in the same direction as its relative income.

A household's optimality conditions are

$$\beta\frac{P_{t}C_{t}(i)^{\sigma}\left\{1+\phi\Phi_{C}(C_{t}(i),X_{t}(i))\right\}}{P_{t+1}C_{t+1}(i)^{\sigma}\left\{1+\phi\Phi_{C}(C_{t+1}(i),X_{t+1}(i))\right\}}=Q_{t.t+1}, \tag{1}$$

$$H_{t}(i)^{\varphi}C_{t}(i)^{\sigma}\frac{\left\{1+\phi\Phi_{C}(C_{t}(i),X_{t}(i))\right\}}{\left\{1-\phi\Phi_{X}(C_{t}(i),X_{t}(i))\right\}}=\frac{W_{t}(i)}{P_{t}}, \tag{2}$$

where Φ_C and Φ_X are the partial derivatives of $\Phi\left(\cdot\right)$ with respect to the level of consumption and income:

$$\Phi_{C}\equiv\frac{\partial\Phi}{\partial C}=\left(\log\frac{C_{t}(i)}{X_{t}(i)}\right)+\frac{1}{2}\left(\log\frac{C_{t}(i)}{X_{t}(i)}\right)^{2},$$

$$\Phi_{X}\equiv\frac{\partial\Phi}{\partial X}=-\frac{C_{t}(i)}{X_{t}(i)}\left(\log\frac{C_{t}(i)}{X_{t}(i)}\right).$$

The gross nominal interest rate R_t is determined by $R_{t}^{-1}=E_{t}\left[Q_{t,t+1}\right]$ because R_{t}^{-1} is the price of a portfolio in which $B_{t+1}(i)=1$ for every state of the economy at time $t+1$. The equation

$R_t^{-1} = E_t\left[Q_{t,t+1}\right]$ and (1) together yield a consumption Euler equation

$$R_t^{-1} = \beta E_t\left[\frac{P_t C_t(i)^{\sigma}\left\{1+\phi\Phi_C(C_t(i),X_t(i))\right\}}{P_{t+1}C_{t+1}(i)^{\sigma}\left\{1+\phi\Phi_C(C_{t+1}(i),X_{t+1}(i))\right\}}\right]. \tag{3}$$

It is straightforward to show that, with no financial frictions ($\phi = 0$), the economy is characterized by perfect consumption insurance. Using a normalizing assumption on the distribution of households' initial wealth, one obtains from (1) that

$$C_t(i) = C_t(j) = Y_t, \quad \forall i,j \in [0,1], \tag{4}$$

which should hold for every time period t and also for every possible state of the economy. Note Y_t denotes aggregate output. In this case, (2) can be rearranged into the familiar expression:

$$H_t(i)^{\varphi} Y_t^{\sigma} = \frac{W_t(i)}{P_t}. \tag{5}$$

Equation (4) characterizes the efficient consumption distribution realized in the special case of no financial frictions. In general, however, a household's relative consumption depends positively on its relative income, and consequently cross-household consumption distribution departs from the efficient one specified in (4).

It is helpful to compare (2) to (5) in developing an intuition for the first-moment effect of financial frictions. Financial frictions affect aggregate dynamics by amplifying price stickiness. In the case of imperfect risk-sharing, a household's consumption $C_t(i)$ depends positively on labor income, and thus on the real wage $\frac{W_t(i)}{P_t}$ and labor hours $H_t(i)$. As a household's labor income increases, its consumption level also rises, and consequently the household has less incentive to supply labor.[9] Thus the wage elasticity of a household's labor supply is *smaller* due to a *wealth effect*. Prices thus adjust more slowly as a direct consequence of a less elastic labor supply. In contrast, there is no such wealth effect in the case of perfect risk-sharing because households can completely insure against income risk, smoothing their consumption path.

As an example, suppose type-i firm considers lowering its price because a positive technology shock has decreased its marginal cost. Lowering the price in turn leads to a higher

[9] Note that the marginal rate of substitution,

$$MRS_t(i) \equiv H_t(i)^{\varphi} C_t(i)^{\sigma} \frac{\left\{1+\phi\Phi_C(C_t(i),X_t(i))\right\}}{\left\{1-\phi\Phi_X(C_t(i),X_t(i))\right\}}\Xi_t$$

is an increasing function of $C_t(i)$. And $C_t(i)$ is an increasing function of $X_t(i)$ and thus of $H_t(i)$ and $W_t(i)/P_t$. Since $MRS_t(i) = W_t(i)/P_t$ in equilibrium, it can be easily shown that the supply of labor hours $H_t(i)$ is less sensitive to the real wage $W_t(i)/P_t$.

demand for type-i firm's good; a higher demand for type-i labor (i.e. the labor demand curve shifts out); an increase in the wage rate for type-i labor and thus type-i firm's marginal cost. As the later increase in marginal cost will offset the initial decrease in marginal cost due to the shock, the firm's incentive to lower its price diminishes. However, the later increase in marginal cost (higher wage rate) is bigger when the labor supply is inelastic, and thus, when there are financial frictions, type-i firm's price will not adjust as much as it would otherwise. Since every firm experiences this, the aggregate price level adjusts more slowly in response to a shock. This is a classic example of "real rigidities".[10] As will be seen later, this first-moment effect is captured by a flatter Phillips curve.

The equilibrium conditions can be log-linearized around the symmetric non-stochastic steady state. The log-linear approximations of (1), (2) and (3) take the form

$$c_t(i) = c_{t+1}(i) + \frac{1}{\sigma+\phi}\left(q_{t,t+1} + \pi_{t+1}\right) + \frac{\phi}{\sigma+\phi}\left(x_t(i) - x_{t+1}(i)\right), \tag{6}$$

$$w_t(i) - p_t = \varphi h_t(i) + \sigma c_t(i), \tag{7}$$

$$c_t(i) = E_t c_{t+1}(i) - \frac{1}{\sigma+\phi}\left(r_t - E_t\pi_{t+1}\right) + \frac{\phi}{\sigma+\phi}E_t\left(x_t(i) - x_{t+1}(i)\right), \tag{8}$$

where I use lowercase letters to denote percentage deviations from the steady state.[11]

From (6), one can derive an analytical expression for a household's consumption as a function of its idiosyncratic income and aggregate output.

Proposition 1 ***(Financial Frictions and Imperfect Risk-Sharing)*** *Up to the first order approximation, a typical household's consumption function can be expressed as a weighted sum of the household's idiosyncratic income and aggregate income,*

$$c_t(i) = \frac{\omega}{1+\omega}x_t(i) + \left(1 - \frac{\omega}{1+\omega}\right)y_t, \tag{9}$$

where y_t is aggregate output (which is equal to aggregate consumption c_t in equilibrium), and the parameter, ω is the ratio of transaction cost to risk aversion,

$$\omega \equiv \phi/\sigma.$$

The proof of this (and all other results) is in the appendix. An alternative way to write

[10] To my knowledge, Ball and Romer (1990) were among the first to introduce this terminology. Lee (2009) has also argued that an incomplete market economy is characterized by larger degrees of real rigidities than its complete market counterpart.

[11] For example,

$$c_t(i) \equiv \log C_t(i) - \log C,$$

where C is the common steady state level of consumption of households.

(9) is:

$$c_t^R(i) = \frac{\omega}{1+\omega} x_t^R(i). \tag{10}$$

The variables with superscript R, $c_t^R(i)$ and $x_t^R(i)$, denote respectively $c_t(i) - y_t$ and $x_t(i) - y_t$: type-i household's consumption and after-tax real income relative to aggregate output. The equation (10) indicates that a household's relative consumption moves in the same direction with its relative income as long as ϕ (and thus ω) is positive. When $\phi = 0$, the model is characterized by perfect risk-sharing (i.e. $c_t^R(i) = 0$) and consequently becomes identical to the basic NK model with a representative household.

Using micro level data, Schulhofer-Wohl (2007) estimated ω, the ratio of transaction cost to risk aversion, and showed that a reasonable value of ω should be in the range of 0.117-0.205 if heterogeneous preferences among households were taken into account. If households are assumed to have identical preferences as in this paper, the estimated ω can be as large as 0.54. In this paper, I use 0.2 as a benchmark value for ω,[12] but also consider other values in the range of [0, 0.35]. In the benchmark case, a one percent increase in income raises consumption by 0.167 percent.[13]

2.2 Firms

This subsection describes the supply side of the economy. As mentioned earlier, there is a continuum of industries indexed by $i \in [0, 1]$, each of which has a representative firm called type-i firm that produces a distinct type of good $Y_t(i)$. The final good, Y_t, which is consumed by households, is produced by perfectly competitive firms using the differentiated goods, $\{Y_t(i)\}_{i\in[0,1]}$ with a Dixit-Stiglitz production technology:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where θ denotes elasticity of substitution and is assumed to be larger than one. The corresponding price index P_t for the final consumption good is

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}},$$

[12] Schulhofer-Wohl (2007) estimates ω using two different definitions of "New Household". Roughly, 0.2 is a point estimate of ω under one of the two definitions. I refer the interested readers to Schulhofer-Wohl (2007) for a detailed discussion of the estimation.

[13] Households in the model are *ex-ante* identical in their expected incomes. For an illustration, let's assume households earn and consume \$10,000 per quarter on average (and in the steady state), which is a roughly consistent figure for many developed countries including the US. If a shock raises a household's income unexpectedly by 100%, and thus the household earns extra \$10,000, the benchmark case suggests it would spend an extra \$1,670 for consumption and save the remaining amount \$8330.

where $P_t(i)$ is the price of type-i good. The optimal demand for each type of good is then given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t. \tag{11}$$

Type-i firm's production function is:

$$Y_t(i) = A_t H_t(i), \tag{12}$$

where A_t denotes the level of economy-wide productivity.

As in Calvo (1983) and Yun (1996), firms adjust their prices with probability $1-\alpha$ each period. Consequently, the price level P_t evolves as:

$$P_t = \left[\int_{\mathcal{I}^*} P_t^*(i)^{1-\theta} di + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{13}$$

where $\mathcal{I}^* \subset [0,1]$, with size of $1-\alpha$, is a randomly chosen subset in which firms update their prices and $P_t^*(i)$ is an optimal price chosen by firm i where $i \in \mathcal{I}^*$. A firm that re-optimizes at time t chooses $P_t^*(i)$ to maximize its expected discounted profit:

$$\max_{P_t^*(i)} E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left\{ \underbrace{P_t^*(i) Y_{t+k}(i) - (1-\tau_{t+k}) W_{t+k}(i) H_{t+k}(i)}_{\equiv \Pi_{t+k}(i)} \right\}.$$

At time $t+k$, the government provides each firm with an employment subsidy $\tau_{t+k} W_{t+k}(i) H_{t+k}(i)$, with a time-varying rate of τ_{t+k}. I assume the steady state level of τ_t is $\frac{1}{\theta}$. The presence of an employment subsidy makes the equilibrium output in the steady state efficient. The subsidies are financed through the lumpsum taxes collected from households.[14] The first order condition is

$$E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left(\frac{P_t^*(i)}{P_{t+k}}\right)^{-\theta} Y_{t+k} \left\{ P_t^*(i) - S_{t+k} \frac{W_{t+k}(i)}{A_{t+k}} \right\} = 0, \tag{14}$$

where $S_{t+k} \equiv \left(\frac{\theta}{\theta-1}\right)(1-\tau_{t+k})$ denotes a stochastic mark-up.

In sticky-price models, there is no need to introduce idiosyncratic shocks to induce heterogeneity in household incomes. When prices are not flexible, aggregate shocks induce idiosyncratic shocks because price adjustments are not synchronized across firms.[15] The price dis-

[14] The assumption that the government provides an employment subsidy removes monopolistic distortions at the steady state, but it is not necessary for the results of this paper. The assumption is convenient, especially when one uses the model for welfare analysis. In addition the presence of the subsidy makes the algebra easier and leads to a cleaner expression for the loss function of the central bank. But the main results would be essentially unchanged if this assumption were dropped.

[15] Introducing idiosyncratic shocks would not change the main insights of this paper laid out in the intro-

persion generates income dispersion, which then leads to consumption dispersion (the second-moment effect of financial frictions). Heterogeneous labor incomes and consumptions across households then induce the wealth effects on labor supply discussed above. Aggregate dynamics are in turn affected through the amplified price stickiness delivered by the wealth effects (the first-moment effect).

Loglinearizing (13) and (14), I can obtain the generalized NK Phillips curve that accounts for household heterogeneity and financial frictions, whose effects on aggregate dynamics are entirely captured by the reduced slope.

Proposition 2 ***(Financial Frictions and Output-Inflation Tradeoffs)*** *Consider the heterogeneous household sticky-price model with imperfect risk-sharing described in this paper. Aggregate output and inflation must satisfy a Phillips curve (or an aggregate supply curve) of the form*

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \mu_t,$$

where

$$\kappa \equiv \left\{ \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \right\} \left\{ \frac{\sigma+\varphi}{1+\theta(\varphi+\sigma\gamma)} \right\},$$
$$\gamma \equiv \frac{1+\varphi}{\frac{\theta-1}{\theta}\frac{1+\omega}{\omega} - \sigma},$$

and $x_t = y_t - y_t^E$ denotes the output gap, where y_t^E is the efficient level of output that would arise in the absence of nominal rigidities, financial frictions and monopolistic distortions.[16] *The reduced-form of the Phillips curve therefore remains the same as in the basic NK model. However, given other parameters (especially a given degree of nominal rigidities, α), the slope of the Phillips curve κ gets smaller as the degree of financial frictions ϕ (captured in ω) gets larger, as long as $\omega < \frac{1}{\left(\frac{\theta}{\theta-1}\right)\sigma-1}$ (or equivalently $\phi < \frac{\sigma}{\left(\frac{\theta}{\theta-1}\right)\sigma-1}$).*

The proof is outlined in the appendix. The residual term $\mu_t \equiv \kappa(\sigma+\varphi)^{-1} s_t$ is proportional to the stochastic mark-up, and is often called a "cost-push shock" in the literature. The inequality $\omega < \frac{1}{\left(\frac{\theta}{\theta-1}\right)\sigma-1}$ is a condition that makes γ positive and thus makes the slope, κ smaller. If this inequality does not hold, the Phillips curve gets steeper. Intuitively, when the transaction cost is too large, wealth effects are so large that the slope of the labor supply curve becomes *negative*: households supply fewer labor hours as the real wage increases. Since I view this case as rather unusual, I will focus only on the case in which the transaction cost

duction section, as consumption dispersion would still be expressed in terms of current and past inflation. See Lee (2009) for a model similar to the current model, but with sectoral shocks.

[16] The efficient level of output is given by $y_t^E = \left(\frac{1+\varphi}{\sigma+\varphi}\right) a_t$, which is a well-known expression in the literature. Therefore I omit detailed derivation and refer the interested readers to Woodford (2003) or Gali (2008).

is non-negative but not too large; i.e. $0 \leq \omega < \frac{1}{(\frac{\theta}{\theta-1})\sigma-1}$. Note that the inequality is necessary only when $\sigma > \frac{\theta-1}{\theta}$ since γ is *always* positive, regardless of ω, when $\sigma \leq \frac{\theta-1}{\theta}$.

In the special case in which $\omega = 0$, the slope of the Phillips curve would be the same as in the basic NK model with complete markets (or a representative household). Another special case arises when households are risk-neutral (i.e. $\sigma = 0$). In that case, the slope would be unaffected even if $\omega > 0$, The reason is that when $\sigma = 0$, household consumption decisions do not affect the marginal rate of substitution and thus the wealth effect on labor supply does not arise. This suggests that the degree of risk aversion is another key factor that determines how much financial frictions matter in this model.

Except for these two special cases, financial frictions generally influence equilibrium aggregate dynamics by making the short-run Phillips curve flatter. For example, if $\sigma = 3$, and $\omega = 0.2$, the slope is only about one fourth of the slope under perfect consumption insurance (see Figure 1).[17] If a higher value of either σ or ω were used, then the slope would become even smaller. This suggests that the first-moment effect of financial frictions can be substantial. Figure 1 plots the slope of the Phillips curve for $\omega \in [0, 0.35]$ and $\sigma = 3$.

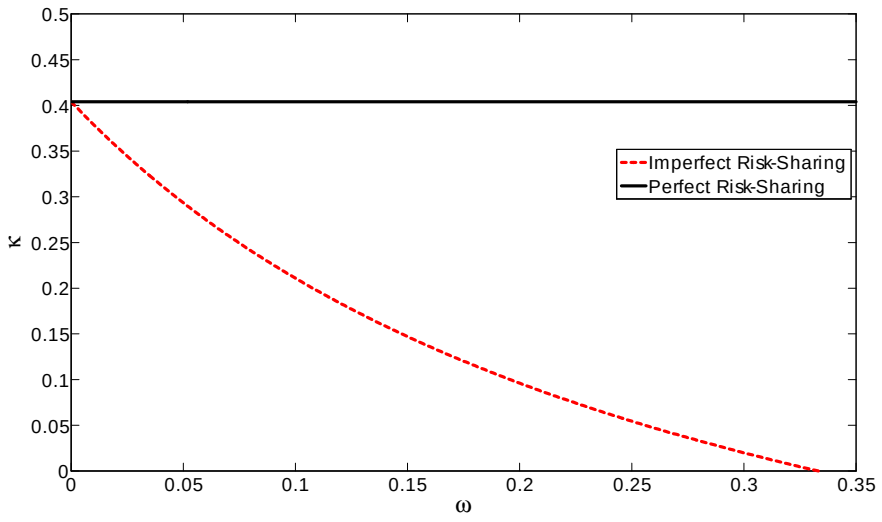

Figure 1: Slope of Phillips Curve

[17] For Figures 1-3, I set β to be 0.99, φ to be 1, θ to be 4, and α to be 0.5.

2.3 Government

Assuming the government does not issue the state-contingent assets, the government budget constraint is given by

$$P_t G_t + \tau_t \int_0^1 W_t(i) H_t(i) di = P_t T_t + \int_0^1 \phi \Phi \left(C_t(i), X_t(i) \right) di,$$

where G_t is government expenditure; $P_t T_t$ is the lump-sum tax revenues collected from households; $\tau_t \int_0^1 W_t(i) H_t(i) di$ is the sum of the employment subsidies given to firms; $\int_0^1 \phi \Phi \left(C_t(i), X_t(i) \right) di$ is the sum of the transactions costs which, I assume, the government collects. For simplicity, I assume $G_t = 0$ throughout this paper.

3 Equilibrium

3.1 Sticky-Price Equilibrium: Definition

Equilibrium is characterized by the prices and the allocation of quantities that satisfy the household optimality conditions and budget constraints, the firm optimality conditions, the government budget constraint, monetary policy (to be specified later) and finally the market clearing conditions:

$$\int_0^1 C_t(i) di = Y_t \quad \text{and} \quad \int_0^1 B_t(i) di = 0$$

for every time t and every state of the economy. The first market clearing condition is a resource constraint which can be derived by integrating the households' budget constraints and the budget constraint of the government. The second equation is the market clearing condition for each state-contingent asset.

3.2 Approximation of Equilibrium Conditions

I solve the model and analyze policy implications by log-linearizing the equilibrium conditions. In the basic NK model, which is characterized by a representative household (and/or complete asset markets), it is well known that only three equilibrium conditions, often referred to as the IS curve, Phillips Curve, and monetary policy rule, determine equilibrium dynamics of the three key aggregate variables: the output gap, inflation and the nominal interest rate $\{x_t, \pi_t, r_t\}$, although there are infinitely many other equilibrium conditions because the model has a continuum of firms. The assumption of time dependent pricing, together with the symmetric nature of the model, plays a key role in reducing the number of state variables required to study aggregate variables. Researchers therefore do not need to consider other

equilibrium conditions unless they want to know the equilibrium dynamics of disaggregate quantities and/or prices.

For the same reasons, the three equilibrium conditions continue to characterize equilibrium dynamics of $\{x_t, \pi_t, r_t\}$ even after introducing a continuum of heterogeneous households. As shown earlier, financial frictions effect on aggregate dynamics is captured entirely by an adjusted slope of the Phillips curve. Consequently, there is no need to keep track of the cross-sectional distribution of households' consumption and asset holdings.

Specifically, the following two equations,

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1}\right) + \delta_t, \tag{15}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \mu_t, \tag{16}$$

and a characterization of monetary policy (to be discussed below) will characterize the equilibrium dynamics of $\{x_t, \pi_t, r_t\}$. The IS curve, can be derived by integrating the Euler equations (8) across households. The Phillips curve has been introduced in Proposition 2. Finally, δ_t in the IS curve is an exogenous term that is proportional to the efficient real interest rate rr_t^E and is given by

$$\delta_t = \frac{1}{\sigma} rr_t^E = \sigma E_t\left[\Delta y_{t+1}^E\right] = \sigma\left(\frac{1+\varphi}{\sigma+\varphi}\right) E_t\left[\Delta a_{t+1}\right].$$

4 Monetary Policy

4.1 The Central Bank's Loss Function

To study the implications of financial frictions for optimal monetary policy, I assume that the central bank maximizes the sum of the household intertemporal utilities:

$$E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \left(\frac{C_t(i)^{1-\sigma} - 1}{1-\sigma} - \frac{H_t(i)^{1+\varphi}}{1+\varphi}\right) di, \tag{17}$$

I then derive a quadratic loss function that is the second order approximation of (17), following the method presented in Rotemberg and Woodford (1997) and Woodford (2003), in order to compare the findings of this paper to the results of earlier papers that have studied optimal monetary policy in sticky-price models with a representative household.

It is well known that, in the basic NK model, the output gap and inflation enter the central bank loss function: the welfare of the economy depends negatively on the volatility of the output gap and inflation. Stabilizing inflation is especially important because an unstable aggregate price level leads to inefficient price and production dispersions.

If the complete market assumption (or representative household assumption) is relaxed, the inefficient price (production) dispersions lead to inefficient consumption dispersion: over the business cycles, the cross-sectional distribution of household consumption deviates from the efficient distribution that would arise if households were able to insure fully against their income risks. Therefore, the introduced financial frictions provide an additional case for stable inflation. The following proposition indeed shows that a measure of inefficient consumption dispersion can be entirely summarized by fluctuations of current and past inflation levels.

Proposition 3 ***(Consumption Dispersion and Inflation Fluctuations)*** *Up to the second order approximation, cross-sectional dispersion of current consumption is given by a weighted sum of quadratic terms in current and past inflation:*

$$\frac{1}{(\theta\gamma)^2} Var_i\left[c_t(i)\right] = \sum_{s=0}^{t} \alpha^{t-s} \left(\frac{\alpha}{1-\alpha}\right) \pi_s^2 + \alpha^{t+1} Var_i\left[p_{-1}(i)\right] + O\left(\|\xi\|^3\right),$$

where $Var_i[\cdot]$ *is the cross-sectional variance and* $O(\|\xi\|^3)$ *denotes all relevant terms that are of third or higher order.*

The result in the proposition above suggests that the central bank should place an even larger weight on inflation stabilization unless it believes that asset markets are functioning perfectly. But how much larger should the weight be? The following proposition shows that the relative weight on inflation stabilization is a particular function of the degree of financial frictions ω.

Proposition 4 ***(Financial Frictions and Loss Function)*** *The discounted sum of utilities of households (17) is given by*

$$E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \left(\frac{C_t(i)^{1-\sigma}-1}{1-\sigma} - \frac{H_t(i)^{1+\varphi}}{1+\varphi}\right) di = -\frac{\Omega}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t + t.i.p + O\left(\|\xi\|^3\right)$$

where

$$L_t = \pi_t^2 + \lambda x_t^2,$$

$$\lambda = \frac{(1-\alpha)(1-\alpha\beta)(\sigma+\varphi)}{\alpha\left\{1+\varphi\theta+\theta\sigma\gamma^2\right\}} \frac{1}{\theta},$$

t.i.p stands for the terms independent of monetary policy, and Ω *is a positive constant.*

The effect of financial frictions is captured by the term γ^2 in the denominator of the relative weight on the output gap λ. Since the parameter $\gamma \equiv \frac{1+\varphi}{\frac{\theta-1}{\theta}\frac{1+\omega}{\omega}-\sigma}$ is increasing in the measure of financial frictions ω, a larger ω makes λ smaller, which implies a greater relative weight

on inflation stabilization. Figure 2 plots the optimal relative weight on inflation, λ^{-1}, for $\omega \in [0, 0.3]$ and $\sigma = 3$, showing λ^{-1} increases at an accelerating rate as ω increases.

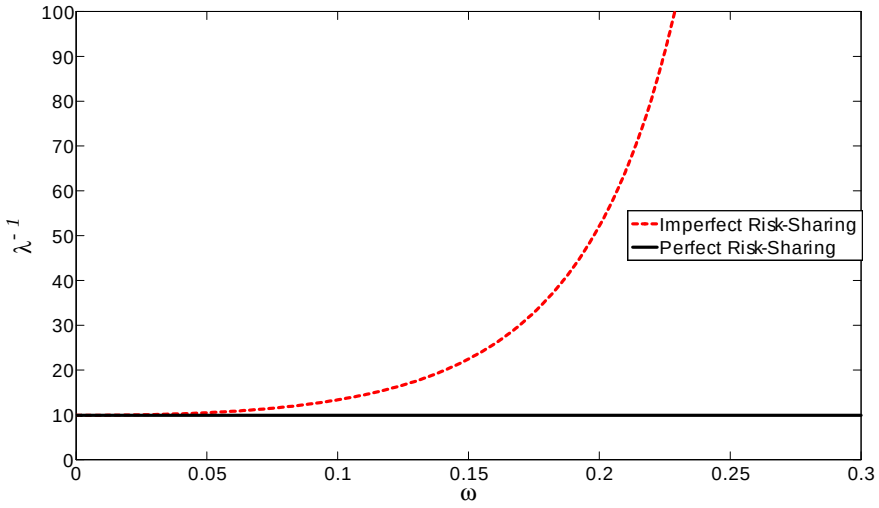

Figure 2: Relative Weight on Inflation

4.2 The Impact of Financial Frictions on Optimal Monetary Policy

In this section, I analyze optimal monetary policy under discretion. Another possibility is to assume the central bank can commit to a policy plan with full credibility. For brevity, I omit a discussion of optimal monetary policy under commitment because doing so provides no additional insight and all the theoretical results obtained under discretion also apply to the case of commitment.

The central bank takes the public's expectation as given and minimizes the loss function period by period. More specifically the central bank picks the pairing of inflation and output gap that minimizes

$$\pi_t^2 + \lambda x_t^2,$$

and that satisfies the constraint

$$\pi_t = \kappa x_t + \xi_t,$$

taking the term $\xi_t \equiv \beta E_t \pi_{t+1} + u_t$ as given.

The first order condition that characterizes the solution to the problem provides the inflation-targeting central bank with its "optimal targeting rule":

$$\pi_t^* = -\frac{\lambda}{\kappa} x_t^*, \tag{18}$$

where π_t^* and x_t^* denote the equilibrium inflation rate and output gap under the optimal policy. The targeting rule serves as a guideline for monetary policy: the central bank adjusts its instrument until the target variables satisfy the optimal relation specified in the rule (18). Another useful way to understand optimal monetary policy is to write the targeting rule (18) as

$$\frac{std(\pi_t^*)}{std(x_t^*)} = ORV \equiv \frac{\lambda}{\kappa}, \tag{19}$$

which is a weaker condition than (18) since (19) is implied by (18), but not vice versa. ORV stands for "*optimal relative volatility*": the relative volatility of inflation versus the output gap realized under optimal monetary policy, which is given by *the ratio of the cost (or price) to the benefit of stabilizing inflation*, $\frac{1/\kappa}{1/\lambda} = \frac{\lambda}{\kappa}$.[18] In this interpretation, the central bank adjusts its instrument until the relative volatility of inflation becomes equal to the cost-benefit ratio.

We can see that the introduced financial frictions are relevant for the optimal targeting rule only to the extent that they affect ORV, that is, only when they disproportionately affect either the benefit or the cost of stabilizing inflations. In addition, the targeting rule in practice can be implemented through interest rate rules. Therefore any impact of financial frictions on ORV would also affect interest rate rules by changing the parameters that measure the responsiveness of interest rates to inflation and/or output gap movements. In the remainder of this paper, I thus study implications of financial frictions for optimal monetary policy by mainly focusing on their impact on ORV.

Do financial frictions affect the *optimal relative volatility* of equilibrium inflation? Yes. However, they do not necessarily decrease it. Based on Proposition 4, one might conjecture that, relative to the case of perfect risk-sharing, the realized time path of inflation $\{\pi_t^*\}$ would be more stable given $\{x_t^*\}$. However, the cost of stabilizing inflation also increases with financial frictions because the central bank now faces a less favorable inflation-output gap tradeoffs as shown in Proposition 2. The central bank thus might choose to have a more volatile path of π_t^*. Therefore it is not obvious that the targeting rule (18) (or its implication (19)) facilitates more stable inflation relative to the case of perfect risk-sharing.

Whether the introduced financial frictions decrease ORV depends on specific values of the model parameters, especially the degree of financial frictions ω. An analytical expression for ORV, in fact, can be obtained:

$$ORV = \frac{\lambda}{\kappa} = \left(\frac{1+\theta\varphi+\theta\sigma\gamma}{1+\theta\varphi+\theta\sigma\gamma^2}\right)\frac{1}{\theta}. \tag{20}$$

[18] The relative price of inflation to output gap is $\frac{1}{\kappa}$ from the Phillips curve. On the other hand, $\frac{1}{\lambda}$ measures the relative benefit of inflation stabilization versus the output gap stabilization.

Under the special case of perfect risk-sharing (i.e. $\omega = \gamma = 0$), it is given by

$$ORV_{\omega=0} = \frac{1}{\theta}, \tag{21}$$

where $ORV_{\omega=0}$ denotes the value of ORV when $\omega = 0$. By comparing (20) to (21), one can see that ORV is smaller than $ORV_{\omega=0}$ if and only if $\gamma > 1$ and vice versa. The first-moment effect (the cost) increases linearly as financial frictions increase (see the term $\theta\sigma\gamma$ in the numerator of ORV) while the second-moment effect (the benefit) increases exponentially (see the term $\theta\sigma\gamma^2$ in the denominator of ORV). Therefore, the first-moment effect of financial frictions on welfare dominates the second-moment effect when the degree of financial frictions is small, but the second-moment effect becomes more dominant as the degree of financial frictions gets larger. Consequently, the central bank finds it optimal to stabilize inflation more aggressively than it would under no financial frictions only if the degree of financial frictions is sufficiently large. The following proposition presents the exact threshold.

Proposition 5 ***(Cost vs. Benefit of Inflation Stabilization)*** *The extra benefit, associated with financial frictions, of stabilizing inflation exceeds the extra cost if and only if the degree of financial frictions is sufficiently large. Specifically, the relative volatility of inflation versus the output gap under optimal monetary policy should satisfy the following properties:*

$$ORV \lessgtr ORV_{\omega=0} \Longleftrightarrow \omega \gtrless \frac{\theta - 1}{\theta\left(\sigma + \varphi\right) + 1}.$$

Besides the parameter ω, the impact of the introduced financial frictions on ORV crucially depends on the measure of risk aversion σ. The reason is that both the first and second-moment effects are amplified as σ gets larger for a given degree of financial frictions ω. As discussed in a previous section, the first-moment effect is amplified because the wealth effect on labor supply becomes more important as σ gets larger. Meanwhile, greater risk-aversion (a higher value of σ) amplifies the second-moment effect because more risk-averse households dislike uninsured consumption more. Thus aggregating for total welfare, increased dispersion reduces households' welfare even more as risk-aversion rises.

Figure 3 shows ORV for some alternative values of σ and ω and confirms my arguments above that (i) ORV is smaller than $ORV_{\omega=0}$ only if ω is sufficiently large and (ii) a larger value of σ amplifies the effects on ORV. Note that, for certain parameter combinations that satisfy $\omega \simeq \frac{\theta-1}{\theta(\sigma+\varphi)+1}$, the first and the second moment effects completely or nearly cancel each other out, or equivalently the introduced financial frictions proportionately increases the benefit and cost of stabilizing inflation. In those cases, conducting monetary policy ignoring the effects of financial frictions causes little problem because doing so yields the identical end-result. However, for other parameter values, asset market frictions can make a big difference.

Consider the case in which households are relatively more risk-averse ($\sigma = 4$). In this case, even with a reasonable degree of financial market frictions, the extra benefit of stabilizing inflation is so large that the central bank would focus only on inflation volatility, completely ignoring output gap fluctuations.

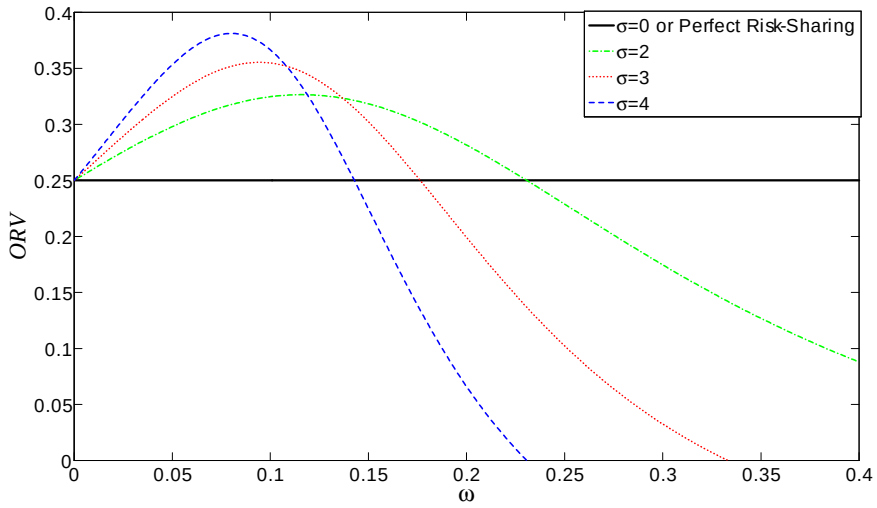

Figure 3: Optimal Relative Volatility

5 Quantitative Analysis

I have so far presented a simple sticky-price model and discussed the main theoretical results. The previous section illustrated that the impact of financial frictions on the optimal targeting rule depends on the values of some model parameters. Thus in this section, I take the model to the U.S. data, and estimate the model parameters using Bayesian methods. I then use the estimated model to answer several questions of interest.

5.1 Closing the Model

In theory, the central bank seeks to choose its desired level of inflation and the output gap that satisfy the optimal targeting rule (18) at each point in time. However, in practice, it can neither set these variables directly nor satisfy (18) in every time period. For the purpose of model estimation, I follow the convention in the Bayesian DSGE literature and assume the following: the central bank sets interest rates through a Taylor rule

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) \{\phi_\pi \pi_t + \phi_x x_t\} + \sigma_r \varepsilon_{r,t}, \tag{22}$$

hoping that setting its instrument according to this rule generates the equilibrium time path of $\{x_t, \pi_t\}$ close to the optimal path $\{x_t^*, \pi_t^*\}$ specified in (18).[19] The monetary shock $\varepsilon_{r,t}$ is a standard normal random variable.

I assume that the disturbances in the IS and Phillips curves follow independent AR(1) processes:

$$\begin{pmatrix} \delta_t \\ \mu_t \end{pmatrix} = \begin{pmatrix} \rho_\delta & 0 \\ 0 & \rho_\mu \end{pmatrix} \begin{pmatrix} \delta_{t-1} \\ \mu_{t-1} \end{pmatrix} + \begin{pmatrix} \sigma_\delta & 0 \\ 0 & \sigma_\mu \end{pmatrix} \begin{pmatrix} \varepsilon_{\delta,t} \\ \varepsilon_{\mu,t} \end{pmatrix},$$

where each of the innovations $\{\varepsilon_{\delta,t}, \varepsilon_{\mu,t}\}$ is normally distributed with standard deviation 1.

5.2 The Data, Likelihood Function and Bayes Theorem

The three equations (15), (16) and (22) characterize the equilibrium values of the output gap, inflation, and interest rates $\mathbf{X}_t = \{x_t, \pi_t, r_t\}$ as a function of the exogenous variables. I use the standard solution method for linear rational expectation models (Sims, 2001) and the Kalman filter to evaluate the likelihood of $\mathbf{X}^T = \{\mathbf{X}_t\}_{t=0}^T$. I use HP-filter-detrended quarterly US GDP as a measure of the output gap, and its price deflator as a measure of price levels. The effective US federal funds rate measures the nominal interest rate. Since the size of the model economy has been normalized to one, I divide GDP by the total civilian non-institutional population over the age of 16. I demean inflation and the interest rate. I assume the model parameters, including the standard deviation of the shocks, are constant over time. As a consequence, I focus on the period starting right after Alan Greenspan became the chairman of the Federal Reserve, from 1986:Q3 to 2007:Q4. This period is characterized by moderate macroeconomic volatility with no major structural breaks.[20]

The model has fourteen parameters, $\lambda = \{\beta, \varphi, \theta, \sigma, \alpha, \omega, \phi_\pi, \phi_y, \rho_r, \rho_\delta, \rho_\mu, \sigma_r, \sigma_\delta, \sigma_\mu\}$. I estimate the model taking the Bayesian full-information approach that exploits restrictions imposed by general equilibrium of the model economy. I first assign my prior uncertainty to the structural parameters by specifying a prior distribution $f(\lambda)$. Given the data set $\mathbf{X}^T$, the model gives the likelihood function $f(\mathbf{X}^T|\lambda)$. Then the posterior distribution of λ is determined by Bayes theorem: $f(\lambda|\mathbf{X}^T) = f(\mathbf{X}^T|\lambda)f(\lambda)/\int f(\mathbf{X}^T|\lambda)f(\lambda)d\lambda$. I then simulate the posterior distribution by the Markov Chain Monte Carlo method.

[19] One can show that there exists an interest rule that guarantees the optimal path $\{x_t^*, \pi_t^*\}$. In this case, the coefficient on inflation ϕ_π and on output gap $\phi_x$in the interest rate rule are some functions of the model parameters. In this paper, I treat ϕ_π and ϕ_x as free parameters and thus give the interest rule more flexibility to fit the data, following the convention in the Bayesian DSGE literature.

[20] Reis (2009) has considered the same smaple period based on the same logic. The estimation results were unchanged when I extended the sample period until 2008.

5.3 The Priors

For most parameters, the prior distribution follows the convention in the literature of Bayesian DSGE estimation. The prior distributions are summarized in the first four columns in Table 1. Due to identification issues, I impose dogmatic priors on three parameters, β, φ, and θ. I fix the discount factor β to be 0.99 and the elasticity of labor supply φ to be 1, which are common values in the literature. I set $\theta = 4$ based on two arguments. First, it is roughly consistent with estimates of elasticity of demand based on micro-level data.[21] Second, as briefly mentioned above, although the central bank may not be able to satisfy the optimal targeting rule at every point in time, I parameterized θ so that the conduct of monetary policy is *roughly* consistent with the prescription provided by the targeting rule *under no financial frictions*. The relative standard deviation of inflation to output gap is about 0.25 during the sample period (i.e. $\frac{std(\pi_t)}{std(x_t)} = 0.25$ for 1986:Q3 to 2007:Q4). Moreover, in the absence of financial frictions, the *optimal relative volatility* implied by the targeting rule is given by

$$\frac{std(\pi_t^*)}{std(x_t^*)} = ORV_{\omega=0} = \frac{1}{\theta}.$$

By setting $\theta = 4$, I therefore implicitly assume that, while the central bank has been failing to incorporate financial frictions into policy decisions, the central bank's monetary policy has been successful in generating the equilibrium paths of inflation and the output gap that satisfy at least the weaker form of the targeting rule without financial frictions:

$$\frac{std(\pi_t^*)}{std(x_t^*)} = ORV_{\omega=0} = \frac{1}{\theta} = 0.25 = \frac{std(\pi_t)}{std(x_t)}.$$

The conclusions in the following sections are robust to some alternative choices of (φ, θ).[22]

Even with fixed (β, φ, θ), identification issues arise for α and ω because only the slope of the Phillips curve contains the measure of nominal rigidities α and the measure of financial market frictions ω. But I chose not to fix these because it may be interesting to see how the introduced financial frictions affect the estimate of the measure of nominal rigidities α. I instead specify somewhat informative priors for α and ω based on information external to the data used to estimate this model. The infrequency of price changes α, follows a beta distribution with support of [0,1]. Both the prior mean and mode are 0.5 and standard deviation is 0.15, which implies a 95% probability region of [0.2, 0.8]. This choice of prior is based on a recent empirical study of the frequency of price changes (Bils and Klenow 2004). The choice of prior density

[21] Nakamura and Steinsson (2009) used the same value based on estimates from the industrial organization and international trade literatures, for example, Berry et al. (1995).

[22] I have estimated the model using some different values of φ in the range of 1-2 and of θ in the range of 4-8. These alternative parameterizations do not change my arguments in the following sections.

for the degree of financial frictions is based on the estimates in Schulhofer-Wohl (2007). The mean of ω is set to 0.2, with standard deviation of 0.1. The prior mode of ω is only 0.14. I assume a small friction as a priori to be conservative, but I do not exclude completely the possibility of larger or smaller ω.

The coefficient of relative risk aversion, σ has a somewhat diffuse prior, which reflects the wide variety of estimates for this parameter. I use a prior mean of 3 and a standard deviation of 1. Researchers often use relatively small values in the range of 1-3. But much larger values are also employed in the literature. In addition the estimated σ in a NK framework is often quite large. For example, Rabanal and Rubio-Ramirez (2005) report that the estimate of σ is in the range of 4.5-8.3, although the model and the data employed in that paper are somewhat different from those in this paper. This observation leads me to specify a somewhat diffuse prior for σ.

Finally, the priors for the exogenous processes and the interest rate rule parameters are quite standard.

5.4 The Posteriors

Next to the summary of priors, the last two columns of Table 1 show some key moments of the posterior distributions. Figure 4 shows the prior and posterior densities. In most cases, the estimates are in line with the previous studies that estimate basic NK models, and the data appears to be informative for the parameters.

The estimate of the coefficient of relative risk aversion is relatively large, but in a reasonable range. The exogenous shock processes are somewhat persistent, which indicates that the model still does not have a sufficient internal propagation mechanism. This is not surprising given the stylized nature of the model even with the added financial friction. I deliberately abstracted from features such as habit persistence in household expenditure and past inflation indexation in firms price setting, elements often included in the medium-scale NK DSGE models, in an effort to make the model as simple and transparent as possible.

The posterior mean of α is 0.61 which is smaller than the typical estimates in the Bayesian DSGE literature. This value implies that the average duration of price contracts is about 2 quarters, which is more consistent with the recent evidence in Bils and Klenow (2004).[23] They report that the median duration of prices is between 4 and 6 months. The small estimate of α is due to the presence of asset market frictions. Since imperfect risk-sharing due to frictions generates real rigidities, the model does not require an implausibly large degree of nominal rigidities to explain persistent aggregate dynamics. When I assume no financial frictions and

[23] The duration is computed as $-\frac{1}{\log \alpha}$.

hence impose a restriction $\omega = 0$, the posterior mean of α is 0.83. This estimate implies that prices change every 5.37 quarters, which is less consistent with the micro evidence.

For the asset market friction parameter ω, I obtain a posterior mean of 0.23 with standard deviation of 0.09. The model does not require implausibly large financial frictions to capture aggregate dynamics. This macro estimate is in line with the micro-evidence in Schulhofer-Wohl (2007).

Table 1: Priors and Posteriors

	Prior	**Prior Mode**	**Prior Mean (Std)**	**Posterior Mean (Std)**	**90% HPD**
σ	Gamma	2.67	3 (1)	3.42 (0.99)	[2.23, 4.74]
ϕ_x	Gamma	0.045	0.125 (0.1)	0.65 (0.11)	[0.51, 0.80]
ϕ_π	Gamma	1.47	1.5 (0.2)	1.45 (0.16)	[1.24, 1.66]
ρ_r	Beta	0.67	0.6 (0.2)	0.87 (0.02)	[0.84, 0.89]
ρ_δ	Beta	0.67	0.6 (0.2)	0.83 (0.04)	[0.78, 0.88]
ρ_μ	Beta	0.67	0.6 (0.2)	0.55 (0.06)	[0.46, 0.61]
σ_δ (%)	Inv. Gamma	1	1 (0.5)	0.18 (0.03)	[0.14, 0.22]
σ_μ (%)	Inv. Gamma	1	1 (0.5)	0.14 (0.02)	[0.11, 0.16]
σ_r (%)	Inv. Gamma	0.125	0.25 (0.25)	0.11 (0.01)	[0.10, 0.13]
α	Beta	0.5	0.5 (0.15)	0.61 (0.12)	[0.43, 0.76]
ω	Inv. Gamma	0.14	0.2 (0.1)	0.23 (0.09)	[0.13, 0.35]

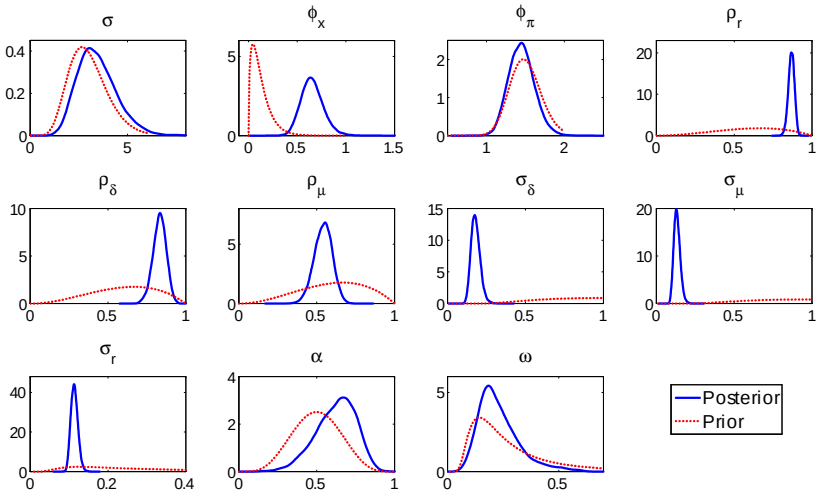

Figure 4: Posterior Densities

5.5 The Impact of Financial Frictions on Targeting Rule and ORV

Recall that the main question that the quantitative exercise intends to address is whether introducing financial frictions will change the optimal targeting rule. To answer to this question, I construct the posterior distribution of ORV and compare it to $ORV_{\omega=0}$.

Table 2 summarizes some key characteristics of the posterior distribution of ORV, and Figure 5 draws its density. The main observations are the following. The case that $ORV < ORV_{\omega=0}$ is more likely than the alternative. Specifically, I can obtain $\Pr(ORV < ORV_{\omega=0}) = 0.83$ and $\Pr(ORV > ORV_{\omega=0}) = 0.17$ from the posterior distribution of ORV. Therefore, the introduced financial frictions are likely to increase the benefit of stabilizing inflation by more than they raise the cost. The point estimates of ORV, such as mean and median, are smaller. For instance, the posterior mean of ORV is 0.1243 which is less than $ORV_{\omega=0} = 0.25$. These results suggest that the central bank should put a stronger emphasis on stabilizing inflation. If the central bank correctly identified the potential inefficiency from financial frictions, it would have followed the targeting rule $\pi_t^* = -0.1243x_t^*$ instead of $\pi_t^* = -0.25x_t^*$ resulting in a time path for inflation and the output gap satisfying $\frac{std(\pi_t)}{std(x_t)} = 0.1243$, not $\frac{std(\pi_t)}{std(x_t)} = 0.25$. In sum, I argue that the central bank has been under-stabilizing inflation, due to its failure to consider financial frictions and its consequence for risk-sharing among heterogeneous households. And, this may have led to suboptimal equilibrium dynamics.[24]

Table 2: A Summary of Posterior Distribution of ORV

Mean	Median	Standard Deviation	90% HPD
0.1243	0.0895	0.1056	[0.0083, 0.3339]

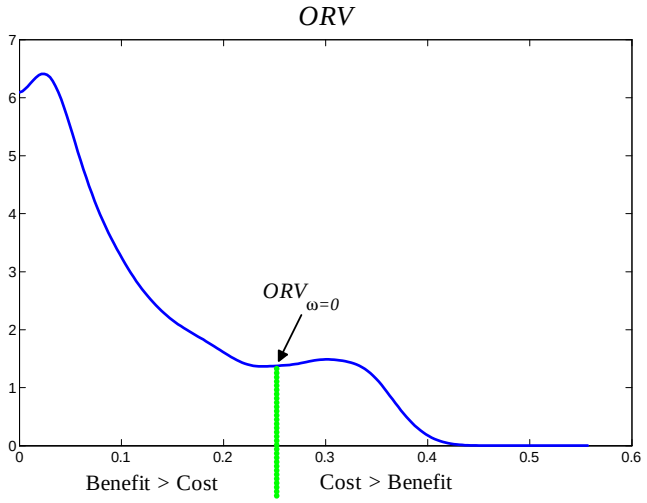

Figure 5: Posterior Density of ORV

[24] On the other hand, a 90% probability interval for ORV still contains $ORV_{\omega=0}$. If I were to make a more conservative conclusion and conduct a hypothesis test in the classical sense, I would not be able to reject the hypothesis that $ORV = ORV_{\omega=0}$. Therefore, a researcher could reach a different conclusion, depending on how he/she interprets the estimation result.

5.6 Welfare Costs

Let us now turn to examining the welfare costs resulting from suboptimal equilibrium dynamics. Does it really matter quantitatively for the welfare of the economy if the central bank sticks to the *suboptimal targeting rule* $\pi_t^* = -0.25x_t^*$ (which does not take into account financial frictions) rather than the *optimal targeting rule* $\pi_t^* = -0.1243x_t^*$? Although ORV is statistically different from $ORV_{\omega=0}$, the difference has no economic meaning if the two targeting rules yield similar welfare levels.

To answer these questions, I compare the welfare losses associated with each targeting rule in the first two rows in Table 3.[25] The welfare losses are moderate (0.25 and 0.35 percent of steady state consumption respectively). Two factors contribute to the moderate welfare losses. First, the sample period considered is characterized by small fluctuations of aggregate variables (aka "The Great Moderation"), perhaps due to a combination of good stabilization policies and good luck. As a consequence, the estimated values for the parameters that capture persistence and volatility of the inefficient shocks (i.e. cost-push shocks) were relatively small. Second, as mentioned earlier, the assumption that the central bank chooses the inflation rate and output gap that satisfy the targeting rules at every point in time without errors, is unrealistic. Policy errors, often called monetary policy shocks, unambiguously lower welfare, yet these are unaccounted for in the targeting rules.

The welfare gain associated with moving from the suboptimal to optimal targeting rule is about 0.1 $(= 0.35 - 0.25)$ percent of steady state consumption, which is again moderate. However I believe this magnitude of gain should not be dismissed. Reis (2009) reported that this gain in the U.S. in 2006, would be worth \$9,240million, which is roughly three times as large as the total expenses of the Federal Reserve Banks (which was \$3,264million in 2006). I am sympathetic to Reis (2009)'s view that monetary policies that yield 0.1 percent of welfare gain deserve serious consideration by policy makers, especially when the gains far exceed the costs associated with those policies.

What are the implications for interest rate rules? I consider this in the next three rows of Table 3. The estimated interest rate rule, in the third row, performs poorly relative to the targeting rules for two reasons. First, the interest rate rule was not derived as an optimality condition directly from the central bank's objective function. Since I let the data choose the response parameters in the interest rate rule which best captured actual dynamics, the estimates are not the optimal rule coefficients. Second, I did not exclude monetary policy disturbances associated with the interest rate rule in order to come up with a more realistic number for welfare losses. The welfare cost resulting from the estimated interest rate rule is 0.46 percent of steady state consumption relative to the Pareto optimal equilibrium and 0.21

[25] The model parameters are set to their posterior means.

percent relative to the optimal targeting rule case. I find that the interest rate rule achieves significant welfare gains when interest rates respond more aggressively to inflation and less to output gap, which is consistent with my previous argument that the central bank should put more weight on stabilizing inflation. Indeed, in any reasonable parameter spaces considered, I find it optimal to assign the largest value on ϕ_π and the smallest value on ϕ_x. For example, given the estimated interest smoothing coefficient ρ_r, the optimal pair of (ϕ_π, ϕ_x) is given by $(5, 0)$ in the parameter space of $[1, 5] \times [0, 5]$. In that case, the interest rate rule becomes quite close to the optimal targeting rule in terms of welfare costs.

Table 3: Welfare Costs of Alternative Policy Rule

	Alternative Policies	Welfare Cost[1]	Welfare Cost[2]
targeting rules	optimal targeting rule	0.25%	-
	suboptimal targeting rule	0.35%	0.10%
interest rate rules	estimated rule: $\phi_\pi = 1.45$, $\phi_x = 0.65$	0.46%	0.21%
	$\phi_\pi = 5$, $\phi_x = 0.65$	0.35%	0.10%
	$\phi_\pi = 5$, $\phi_x = 0$	0.31%	0.06%

1: Welfare loss relative to Pareto optimal equilibrium (as a fraction of steady state consumption)

2: Welfare loss relative to equilibrium under optimal targeting rule (as a fraction of steady state consumption)

The welfare gains from taking financial frictions into account in conducting monetary policies were moderate partly because I focused on the time period of "The Great Moderation" and consequently the estimated inefficient cost-push shocks were moderate. But there is no reason to believe that these conditions will persist. If shocks become more volatile and persistent, the central bank's failure to recognize the welfare consequences of financial frictions could lead to more substantial welfare losses. What would happen in the future if the model economy remains the same as today but cost-push shocks behave like those experienced in the 70s and early 80s? To address this question, I conduct a counter-factual analysis. I take the same model with the same parameter values (including those in the policy rules) but I use an AR(1) process for cost-push shocks that are estimated using the time series data from 1970:Q1 to 1986:Q2. The estimated AR(1) process for the cost-push shocks is:

$$\mu_t = \underset{(0.50,\ 0.69)}{0.61}\mu_{t-1} + \underset{(0.15,\ 0.26)}{0.2}\varepsilon_{\mu,t}.$$

Hence the cost-push shocks were more volatile and persistent in that time period. The numbers in the parenthesis indicate the lower and upper bound of 90% confidence interval.[26]

[26] In addition to cost-push shocks, the disturbaces in the IS curve and interest rate rule were more volatile in this sample period. But I chose to parameterize these disturbances at the previous values to focus on only one deviation. There were no substantial changes for the estimates of the other parameters.

Table 4 reports the welfare losses for each monetary policy rule. The welfare gain associated with moving from the suboptimal to optimal targeting rule is about 0.27 percent of steady state consumption, which is much larger than the previous case. In addition, moving from the estimated interest rule (in the third row) to the "optimal" interest rule (in the bottom row) yields a substantial welfare gains of 0.52 percent of steady state consumption.

Table 4: Welfare Costs of Alternative Policy Rule

	Alternative Policies	Welfare Cost[1]	Welfare Cost[2]
targeting rules	optimal targeting rule	0.65%	-
	suboptimal targeting rule	0.92%	0.27%
interest rate rules	estimated rule: $\phi_\pi = 1.45$, $\phi_x = 0.65$	1.24%	0.59%
	$\phi_\pi = 5$, $\phi_x = 0.65$	0.92%	0.27%
	$\phi_\pi = 5$, $\phi_x = 0$	0.72%	0.07%

1: Welfare loss relative to Pareto optimal equilibrium (as a fraction of steady state consumption)
2: Welfare loss relative to equilibrium under optimal targeting rule (as a fraction of steady state consumption)
cost-push shock follows: $\mu_t = 0.61\mu_{t-1} + 0.2\varepsilon_{\mu,t}$

Table 5 compares monetary policy rules in a period characterized by more severe fluctuations. This scenario is less-likely but possible: I assign the upper bound numbers of the 90th percent confidence intervals to ρ_μ and σ_μ. Again the welfare gains from taking into account financial friction and its policy consequences are quite large. The welfare gains amount to 1 percent of steady state consumption if the central bank adopts the optimal targeting rule instead of the suboptimal targeting rule. Moreover, adopting the "optimal" interest rule leads to welfare gain of more than two percent of steady state consumption.

Table 5: Welfare Costs of Alternative Policy Rule

	Alternative Policies	Welfare Cost[1]	Welfare Cost[2]
targeting rules	optimal targeting rule	1.75%	-
	suboptimal targeting rule	2.75%	1.00%
interest rate rules	estimated rule: $\phi_\pi = 1.45$, $\phi_x = 0.65$	3.95%	2.20%
	$\phi_\pi = 5$, $\phi_x = 0.65$	2.65%	0.90%
	$\phi_\pi = 5$, $\phi_x = 0$	1.80%	0.05%

1: Welfare loss relative to Pareto optimal equilibrium (as a fraction of steady state consumption)
2: Welfare loss relative to equilibrium under optimal targeting rule (as a fraction of steady state consumption)
cost-push shock follows: $\mu_t = 0.69\mu_{t-1} + 0.26\varepsilon_{\mu,t}$

Based on the quantitative exercise conducted in this section, I conclude that (i) the introduced financial frictions change the optimal targeting rule in the direction of favoring inflation stabilization and (ii) failing to recognize the inefficiency resulting from financial frictions can be very costly, especially when the economy experiences large and persistent economic shocks.

6 Summary

This paper has documented some implications of financial frictions for optimal monetary policy within a simple NK framework. I showed that the introduced financial frictions lead to imperfect risk-sharing among heterogeneous households, which increases both the benefit and cost of stabilizing inflation. Whether the benefit exceeds the cost depends on specific values of the model parameters. I estimated the parameters employing a full-information Bayesian technique. The estimation result suggested that the benefit of stabilizing inflation associated with financial frictions is likely to dominate the cost. Hence, I conclude that the central bank has mistakenly been under-stabilizing inflation and should place an even greater emphasis on stabilizing inflation going forward (unless the central bank believes in a representative household or asset market completeness).

I deliberately use a highly stylized model to make my main points in the simplest way possible. Consequently, while the exercise in this paper shows some suggestive evidence on the quantitative importance of financial frictions on monetary policies and welfare costs, I do not intend to make a final quantitative statement. Instead, the results in this paper encourage further quantitative investigation with more realistic and elaborate models such as the ones in Christiano et al. (2005) or Smets and Wouters (2003, 2007).

Appendix

A Proof

A.1 Proof of Proposition 1

From (6) one can derive the following equation:

$$c_t^R(i) - \frac{\phi}{\sigma+\phi} x_t^R(i) = c_{t+1}^R(i) - \frac{\phi}{\sigma+\phi} x_{t+1}^R(i), \tag{23}$$

which must hold for every time period t and for every state of the economy. (23) implies that $c_t^R(i) - \frac{\phi}{\sigma+\phi} x_t^R(i)$ should be some constant. Let

$$c_t^R(i) - \frac{\phi}{\sigma+\phi} x_t^R(i) = z,$$

for some constant z. Then it is necessary that $z = 0$ because $\int_0^1 c_t^R(i)di = \int_0^1 x_t^R(i)di = 0$.

A.2 Proof of Proposition 2

Log-linearizing a firm's first order condition (14) gives

$$\begin{aligned} E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[p_t^*(i) - p_{t+k}\right] &= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[w_{t+k}(i) - a_{t+k} + s_{t+k} - p_{t+k}\right] \\ &= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\varphi h_{t+k}(i) + \sigma c_{t+k}(i) - a_{t+k} + s_{t+k}\right] \\ &= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\varphi y_{t+k}(i) + \sigma c_{t+k}(i) - (1+\varphi) a_{t+k} + s_{t+k}\right] \\ &= E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\varphi y_{t+k}^R(i) + \sigma c_{t+k}^R(i) + (\sigma+\varphi) y_{t+k} - (1+\varphi) a_{t+k} + s_{t+k}\right] \end{aligned} \tag{24}$$

The household's intra-temporal first order condition,

$$w_t(i) - p_t = \varphi h_t(i) + \sigma c_t(i),$$

can be written as

$$w_t^R(i) = \varphi h_t^R(i) + \sigma c_t^R(i).$$

Add $h_t^R(i)\left(=y_t^R(i)\right)$ to both sides,

$$w_t^R(i)+h_t^R(i)=(1+\varphi)\,y_t^R(i)+\sigma c_t^R(i).$$

Note that from the definition of total income, $X_t(i)\equiv\frac{W_t(i)H_t(i)+\Pi_t-P_tT_t}{P_t}$, I can obtain the log-linearized relationship between relative total income and relative labor income, $x_t^R(i)=\left(\frac{\theta}{\theta-1}\right)\left[w_t^R(i)+h_t^R(i)\right]$, and that from proposition 1, we have $c_t^R(i)=\frac{\omega}{1+\omega}x_t^R(i)$. Combining these two and then using the household's intra-temporal first order condition gives:

$$\begin{aligned}c_t^R(i)&=\frac{\omega}{1+\omega}\frac{\theta}{\theta-1}\left[w_t^R(i)+h_t^R(i)\right]\\&=\frac{\omega}{1+\omega}\frac{\theta}{\theta-1}\left[(1+\varphi)\,y_t^R(i)+\sigma c_t^R(i)\right]\end{aligned}$$

Solving for $c_t^R(i)$ gives

$$c_t^R(i)=\gamma y_t^R(i), \tag{25}$$

where

$$\gamma\equiv\frac{1+\varphi}{\frac{\theta-1}{\theta}\frac{1+\omega}{\omega}-\sigma}$$

By substituting (25) into (24), I can rewrite the linearized first order condition as

$$\begin{aligned}&E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[p_t^*(i)-p_{t+k}\right] \qquad (26)\\&=E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[(\varphi+\sigma\gamma)\,y_{t+k}^R(i)+(\sigma+\varphi)y_{t+k}-(1+\varphi)a_{t+k}+s_{t+k}\right],\end{aligned}$$

Note log-linearizing the demand function gives

$$y_{t+k}^R(i)=-\theta\left[p_t^*(i)-p_{t+k}\right] \tag{27}$$

Then substituting (27) into (26) gives

$$\begin{aligned}&(1+\theta\,(\varphi+\sigma\gamma))\,E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[p_t^*(i)-p_{t+k}\right] \qquad (28)\\&=(\sigma+\varphi)E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[\underbrace{y_{t+k}-\left(\frac{1+\varphi}{\sigma+\varphi}\right)a_{t+k}}_{=y_{t+k}-y_{t+k}^E}+\left(\frac{1}{\sigma+\varphi}\right)s_{t+k}\right].\end{aligned}$$

Following the standard steps, I obtain the Phillips curve from (28):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(y_t - y_t^E \right) + \mu_t,$$

where

$$\kappa = \left\{ \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \right\} \left\{ \frac{\sigma + \varphi}{1 + \theta(\varphi + \sigma\gamma)} \right\}$$

$$\mu_t = \kappa (\sigma + \varphi)^{-1} s_t.$$

A.3 Proof of Proposition 3 & 4

I follow Woodford (2003) in deriving the utility-based loss function and calculate a Taylor expansion of each term of the utility function. Taking a second order expansion around the steady state, I obtain

$$U(C_t(i)) = U(C) + U_C(C_t(i) - C) + \frac{1}{2} U_{CC}(C_t(i) - C)^2 + O\left(\|\xi\|^3\right) \tag{29}$$

where $O\left(\|\xi\|^3\right)$ denotes all relevant terms that are of third or higher order, and $U(C_t(i)) = \frac{C_t(i)^{1-\sigma} - 1}{1-\sigma}$. I also take a second order Taylor expansion of C_t. Then I have

$$C_t(i) = C\left(1 + c_t(i) + \frac{1}{2} c_t(i)^2\right) + O\left(\|\xi\|^3\right) \tag{30}$$

where $c_t(i) \equiv \log \frac{C_t(i)}{C}$ as in the text. This implies

$$C_t(i) - C = C c_t(i) + \frac{1}{2} C c_t(i)^2 + O\left(\|\xi\|^3\right) \tag{31}$$

Substituting (31) into (29) gives

$$U(C_t(i)) = U(C) + U_C C c_t(i) + \frac{1}{2} U_C C c_t(i)^2 + \frac{1}{2} U_{CC} C^2 c_t(i)^2 + O\left(\|\xi\|^3\right) \tag{32}$$

Note that $U(C)$ is independent of monetary policy. We rewrite (32) as

$$U(C_t(i)) = U_C C \left\{ c_t(i) + \frac{1}{2} c_t(i)^2 + \frac{1}{2} \frac{U_{CC} C}{U_C} c_t(i)^2 \right\} + t.i.p + O\left(\|\xi\|^3\right)$$

where $t.i.p$ denotes all the terms independent of monetary policy. From the utility function I assume in the text, I have $\frac{U_{CC} C}{U_C} = -\sigma$. Also, in the steady state, $C(i) = Y$. Thus I finally obtain

$$U(C_t(i)) = U_C Y \left\{ c_t(i) + \frac{1}{2}(1-\sigma) c_t(i)^2 \right\} + t.i.p + O\left(\|\xi\|^3\right) \tag{33}$$

Integrating (33) gives

$$\int_0^1 U(C_t(i))di = U_C Y \left\{ E_i \left[c_t(i) \right] + \frac{1}{2}(1-\sigma) E_i \left[c_t(i)^2 \right] \right\} + t.i.p + O\left(\|\xi\|^3 \right)$$

Recall that the economy's resource constraint is given by

$$\int C_t(i)di = Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}. \tag{34}$$

Taking the second order expansions gives

$$\begin{aligned} E_i \left[c_t(i) \right] &= y_t + \frac{1}{2} y_t^2 - \frac{1}{2} E_i \left[c_t(i)^2 \right] + O\left(\|\xi\|^3 \right) \\ &= y_t + \frac{1}{2} y_t^2 - \frac{1}{2} Var_i \left[c_t(i) \right] - \frac{1}{2} E_i \left[c_t(i) \right]^2 + O\left(\|\xi\|^3 \right) \qquad (35) \\ E_i \left[c_t(i) \right]^2 &= y_t^2 + O\left(\|\xi\|^3 \right) \qquad (36) \end{aligned}$$

Substituting (35) and (36) into the above gives

$$\begin{aligned} \int_0^1 U(C_t(i))di &= U_C Y \left\{ y_t + \frac{1}{2} y_t^2 - \frac{1}{2} \sigma E_i \left[c_t(i)^2 \right] \right\} + t.i.p + O\left(\|\xi\|^3 \right) \\ &= U_C Y \left\{ y_t + \frac{1}{2} y_t^2 - \frac{1}{2} \sigma \left(Var_i \left[c_t(i) \right] + E_i \left[c_t(i) \right]^2 \right) \right\} + t.i.p + O\left(\|\xi\|^3 \right) \\ &= U_C Y \left\{ y_t + \frac{1-\sigma}{2} y_t^2 - \frac{\sigma}{2} Var_i \left[c_t(i) \right] \right\} + t.i.p + O\left(\|\xi\|^3 \right). \qquad (37) \end{aligned}$$

I also take a second order Taylor expansion of $V\left(H_t(i)\right) = \frac{H_t(i)^{1+\varphi}}{1+\varphi}$, which gives

$$V\left(H_t(i)\right) = V(H) + V_H (H_t(i) - H) + V_{HH} (H(i) - H)^2 + O\left(\|\xi\|^3 \right) \tag{38}$$

The second order approximation of $H(i)$ is:

$$H_t(i) = H\left(1 + h_t(i) + \frac{1}{2} h_t(i)^2 \right) + O\left(\|\xi\|^3 \right) \tag{39}$$

Substituting (39) into (38) gives

$$V\left(H_t(i)\right) = V_H H \left\{ h_t(i) + \frac{1}{2} h_t(i)^2 + \frac{1}{2} \frac{V_{HH} H}{V_H} h_t(i)^2 \right\} + t.i.p. + O\left(\|\xi\|^3 \right) \tag{40}$$

Since $\frac{V_{HH}H}{V_H} = \varphi$, I rewrite (40)as

$$V\left(H_t(i)\right) = V_H H\left\{h_t(i) + \frac{1}{2}(1+\varphi)h_t(i)^2\right\} + t.i.p. + O\left(\|\xi\|^3\right) \tag{41}$$

From the production function, I have

$$\begin{aligned} y_t(i) &= a_t + h_t(i) \Longrightarrow \\ h_t(i) &= y_t(i) - a_t \end{aligned} \tag{42}$$

Substituting (42) into (41), I obtain

$$\begin{aligned} V\left(H_t(i)\right) &= V_H H\left\{y_t(i) - a_t + \frac{1}{2}(1+\varphi)\left[y_t(i)^2 + a_t^2 - 2a_t y_t(i)\right]\right\} + t.i.p. + O\left(\|\xi\|^3\right) \\ &= V_H H\left\{y_t(i) + \frac{1}{2}(1+\varphi)y_t(i)^2 - (1+\varphi)a_t y_t(i)\right\} + t.i.p. + O\left(\|\xi\|^3\right) \end{aligned} \tag{43}$$

because $\frac{1}{2}(1+\varphi)a_t^2 - a_t$ is $t.i.p.$ By integrating (43) over $i \in [0,1]$, I obtain

$$\int_0^1 V\left(H_t(i)\right)di = V_H H\left\{\begin{array}{c} E_i\left[y_t(i)\right] + \frac{1}{2}(1+\varphi)Var_i\left[y_t(i)\right] + \\ \frac{1}{2}(1+\varphi)E_i\left[y_t(i)\right]^2 - (1+\varphi)a_t E_i\left[y_t(i)\right] \end{array}\right\} + t.i.p. + O\left(\|\xi\|^3\right) \tag{44}$$

Taking a second order approximation of the aggregators gives

$$y_t = E_i\left[y_t(i)\right] + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)Var_i\left[y_t(i)\right] + O\left(\|\xi\|^3\right)$$

This gives the two equations

$$E_i\left[y_t(i)\right] = y_t - \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)Var_i\left[y_t(i)\right] + O\left(\|\xi\|^3\right) \tag{45}$$

$$E_i\left[y_t(i)\right]^2 = y_t^2 + O\left(\|\xi\|^3\right) \tag{46}$$

Then substitute (45) and (46) into (44), obtaining

$$\begin{aligned} \int_0^1 V\left(H_t(i)\right)di = V_H H\Big\{&y_t + \frac{1}{2}(1+\varphi)y_t^2 - (1+\varphi)a_t y_t + \frac{1}{2}\left(\varphi + \theta^{-1}\right)Var_i\left[y_t(i)\right]\Big\} \\ &+ t.i.p. + O\left(\|\xi\|^3\right) \end{aligned}$$

Now recall that $H = Y/A$, and from the household's labor supply relation, we have

$$-\frac{V_H}{U_C} = \frac{W}{P} = A = \frac{Y}{H}$$

Thus, it follows that

$$-V_H H = U_C Y$$

This implies

$$\begin{aligned}
&\int_0^1 \{U(C_t(i)) - V(H_t(i))\} di \\
&= U_C Y \left\{ \begin{array}{c} \frac{1-\sigma}{2} y_t^2 - \frac{\sigma}{2} Var_i[c_t(i)] - \frac{1}{2}(1+\varphi) y_t^2 \\ +(1+\varphi) a_t y_t - \frac{1}{2}(\varphi + \theta^{-1}) Var_i[y_t(i)] \end{array} \right\} + t.i.p. + O\left(\|\xi\|^3\right) \\
&= U_C Y \left\{ \begin{array}{c} \frac{1-\sigma}{2} y_t^2 - \frac{\sigma}{2} Var_i[c_t(i)] - \frac{1}{2}(1+\varphi) y_t^2 \\ +(1+\varphi) a_t y_t - \frac{1}{2}(\varphi + \theta^{-1}) Var_i[y_t(i)] \end{array} \right\} + t.i.p. + O\left(\|\xi\|^3\right) \\
&= -\frac{U_C Y}{2} \left\{ \begin{array}{c} (\sigma + \varphi)\left(y_t - y_t^N\right)^2 \\ +(\varphi + \theta^{-1}) Var_i[y_t(i)] + \sigma Var_i[c_t(i)] \end{array} \right\} + t.i.p. + O\left(\|\xi\|^3\right)
\end{aligned}$$

The demand for $Y_t(i)$ is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$$

Then

$$\log Y_t(i) - \log Y_t = -\theta\left(\log P_t(i) - \log P_t\right)$$

This implies that

$$Var_i[\log Y_t(i) - \log Y_t] = \theta^2 Var_i[\log P_t(i) - \log P_t]$$

$$\Longrightarrow Var_i[\log Y_t(i) - \log Y] = \theta^2 Var_i[\log P_t(i) - \log P]$$

$$\Longrightarrow Var_i[y_t(i)] = \theta^2 Var_i[p_t(i)]$$

On the other hand, we have

$$c_t^R(i) = \gamma y_t^R(i) + O\left(\|\xi\|^2\right)$$

Thus

$$\begin{aligned}
Var_i[c_t(i)] &= \gamma^2 Var_i[y_t(i)] + O\left(\|\xi\|^3\right) \\
&= (\theta\gamma)^2 Var_i[p_t(i)] + O\left(\|\xi\|^3\right)
\end{aligned} \tag{47}$$

Thus

$$\begin{aligned}
&\int_0^1 \{U(C_t(i)) - V(H_t(i))\}\, di \\
&= -\frac{U_C Y}{2}\left\{(\sigma+\varphi)\left(y_t - y_t^N\right)^2 + \theta^2\left\{\left(\varphi+\theta^{-1}\right)+\sigma\gamma^2\right\} Var_i\left[p_t(i)\right]\right\} \\
&+ t.i.p. + O\left(\|\xi\|^3\right),
\end{aligned} \tag{48}$$

When prices are staggered as in the discrete time Calvo structure, Woodford (2003) has shown that

$$\begin{aligned}
\Delta_t &= \alpha\Delta_{t-1} + \frac{\alpha}{1-\alpha}\pi_t^2 + O\left(\|\xi\|^3\right) \Longrightarrow \\
&= \alpha^{t+1}\Delta_{-1} + \sum_{s=0}^{t}\alpha^{t-s}\left(\frac{\alpha}{1-\alpha}\right)\pi_s^2 + O\left(\|\xi\|^3\right),
\end{aligned} \tag{49}$$

where $\Delta_t \equiv Var_i\left[p_t(i)\right]$ is a measure of price dispersion. Therefore proposition 3 is proved by (47) and (49). If a new policy is conducted from $t \geqslant 0$, the first term, $\alpha^{t+1}\Delta_{-1}$, is independent of policy. If we take the discounted sum over time, we obtain

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p. + O\left(\|\xi\|^3\right) \tag{50}$$

Now plugging (50) into (48), the sum of households' utilities is given by

$$\begin{aligned}
&\sum_{t=0}^{\infty}\beta^t\int_0^1 \{U(C_t(i)) - V(H_t(i))\}\, di \\
&= -\frac{U_C Y}{2}\left[\sum_{t=0}^{\infty}\beta^t(\sigma+\varphi)\left(y_t - y_t^N\right)^2 + \frac{\alpha\theta\left\{(1+\varphi\theta)+\theta\gamma^2\sigma\right\}}{(1-\alpha)(1-\alpha\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2\right] \\
&+ t.i.p. + O\left(\|\xi\|^3\right)
\end{aligned}$$

And finally, I obtain

$$\sum_{t=0}^{\infty}\beta^t\int_0^1 \{U(C_t(i)) - V(H_t(i))\}\, di = -\frac{\Omega}{2}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2 + \lambda x_t^2\right) + t.i.p + O\left(\|\xi\|^3\right)$$

where

$$\lambda = \frac{(1-\alpha)(1-\alpha\beta)(\sigma+\varphi)}{\alpha\left\{1+\varphi\theta+\theta\sigma\gamma^2\right\}}\frac{1}{\theta},$$

$$\Omega \equiv YU_C(Y)\frac{(1-\alpha)(1-\alpha\beta)}{\alpha\left\{1+\varphi\theta+\theta\sigma\gamma^2\right\}}\frac{1}{\theta}.$$

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