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Optimal Retirement Asset Decumulation Strategies: The Impact of Housing Wealth

Wei Sun, Robert K. Triest, and Anthony Webb

Abstract

A considerable literature examines the optimal decumulation of financial wealth in retirement. We extend this line of research to incorporate housing, which comprises the majority of most households' non-pension wealth.

We estimate the relationship between the returns on housing, stocks, and bonds, and simulate a variety of decumulation strategies incorporating reverse mortgages. We show that homeowner's reversionary interest, the amount that can be borrowed through a reverse mortgage, is a surprisingly risky asset. Under our baseline assumptions we find that the average household would be as much as 24 percent better off taking a reverse mortgage as a lifetime income relative to what appears to be the most common strategy: delaying tapping housing wealth until financial wealth is exhausted and then taking a line of credit. In addition, the results show that housing wealth displaces bonds in optimal portfolios, making the low rate of participation in the stock market even more of a puzzle.

JEL Codes: D14, D91, G11, J14

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This paper, which may be revised, is available on the web site of the Federal Reserve Bank of Boston at <http://www.bos.frb.org/economic/wp/index.htm>. A substantively similar version appears on the web site of the Center for Retirement Research at Boston College.

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Housing represents the majority of the non-pension wealth of most households entering retirement. Despite its importance in household balance sheets, little attention has been paid to strategies for managing and decumulating housing wealth during retirement. Declining Social Security replacement rates, declines in defined benefit pension coverage, increasing longevity, and increasing health care costs are making it ever harder for households to maintain their customary standard of living in retirement. Whether from choice or necessity, households may increasingly turn to their house to fund post-retirement consumption.

Housing differs fundamentally from other household assets in that it not only provides an important flow of services, but also has a significant residual value in excess of the service flow received during the lifetime of the owners.¹ For a household approaching retirement, the flow of housing services provided by home ownership is very attractive. It is equivalent to a lifetime annuity indexed to the cost of housing. Without home ownership, rent would comprise a significant proportion of household expenditure, so the insurance against fluctuations in the cost of housing services provided by home ownership is very desirable.²

The value of the house in excess of the flow of services over the remaining lives of the current owners is more difficult to characterize. Some authors do not attempt a detailed characterization but arbitrarily assume that none, half, or all of the value of the house is available for non-housing consumption. Munnell and Soto (2005) discuss alternative treatments and argue that what is available for non-housing consumption, at least in theory, is the present value of the eventual sale proceeds, the reversionary interest.

Reverse mortgages offer households a mechanism whereby they can access the majority of the reversionary interest. In contrast to a conventional forward mortgage

¹ Consumer durables also generate service flows, but they are less important in magnitude and typically have much lower resale values than houses.

² Home ownership does not protect against fluctuations in other components of housing costs, such as property taxes, utility expenses, and maintenance expenditures. The bitterness with which many of the elderly view property tax increases is suggestive of the value they place on insurance against increases in housing costs.

that requires regular payments of interest, the loan plus accumulated interest of a reverse mortgage is repayable only when neither borrower lives in the house. The amount repayable is capped at the sale proceeds. Home Equity Conversion Mortgages (HECMs), the product with over 90 percent of the U.S. market, allow borrowers to take their reverse mortgage in the form of a lump sum, a lifetime income, or a line of credit.³ As shown by Figure 1, take-up of HECMs has grown rapidly, albeit from a very low base, rising from 7,781 in 2001 to 43,131 in 2005.⁴ This paper investigates three related issues: (1) what is the optimal age to take a reverse mortgage, (2) in what form should it be taken, and (3) what effect does the availability of HECMs have on the optimal allocation of financial wealth.

We model this as both a portfolio allocation and a consumption decision. In addition to holding stocks and bonds, the household also owns a house. But it can't simply sell the house and consume the proceeds because it needs somewhere to live. It can, however, through the mechanism of a reverse mortgage, "sell" the present value of the eventual sale proceeds—the reversionary interest—either immediately upon retirement or, if it is not liquidity constrained, at a later date.⁵ The optimal strategy will depend on the expected returns to stocks, bonds, and the reversionary interest, the variances and covariances of those returns, and the household's attitude towards risk. It may also be affected by liquidity constraints.

Using a reduced-form vector autoregression (VAR), we quantify the relationship between the returns to stocks, bonds, and an investment in the reversionary interest over the period 1975 to 2005. We show that the capital return to housing, as measured by the Office of Federal Housing Enterprise and Oversight (OFHEO) house price index was quite modest and exhibited only small fluctuations—the mean and standard deviation were only 1.9 and 3.7 percent, respectively. In contrast, an

³ www.nrmlaonline.org "About the HECM."

⁴ www.nrmlaonline.org "Annual Origination Volume for Home Equity Conversion Mortgages;" volume is that accumulated to September 30th each fiscal year, as reported by the U.S. Department of Housing and Urban Development.

⁵ The household retains a small stake in the reversionary interest as it or its heirs is entitled to any excess of the sale proceeds over the loan plus accumulated interest.

investment in the reversionary interest provided a much higher return, but at very high risk. For households of age 65, the mean and standard deviation of the real return to an investment in the reversionary interest amounted to 16.0 and 40.6 percent, respectively. At high interest rates, the outstanding debt on a reverse mortgage will accrue more rapidly, and the amount that can be borrowed on a reverse mortgage is therefore inversely related to interest rates. The effect is substantial and the much greater standard deviation of the return to an investment in the reversionary interest than to housing overall is simply the result of movements in interest rates. The return is so much higher than the capital return to housing partly because the percentage of the house that can be borrowed on a reverse mortgage increases with age, but mainly because interest rates fell during the period, resulting in dramatic increases in the proportion of the value of the house that could be borrowed.

We then simulate asset return histories based on our VAR and use these histories to run Monte-Carlo simulations of the returns to alternative strategies for decumulating retirement wealth, inclusive of the reversionary interest, relative to a default of taking a reverse mortgage at age 65 and investing the proceeds in financial assets. We calculate the expected utility of each strategy, and then calculate reverse-mortgage equivalent wealth, the factor by which the wealth of a household adopting the default strategy must be multiplied so that its expected utility equals that provided by the alternative. When reverse-mortgage equivalent wealth exceeds 1.00, the household would, in expectation, be better off choosing the alternative. We test the sensitivity of our results to alternative assumptions about the means and variances of the returns on the various asset classes.

We find that over a wide variety of assumptions about asset returns, the optimal strategy for all but the most risk-tolerant households is to take a reverse mortgage in the form of a lifetime income. We are informed by the National Reverse Mortgage Lenders Association that only a small minority of borrowers choose this option, with most choosing a line of credit instead. Our findings appear to be yet another manifestation of the widely documented reluctance of households to annuitize

their wealth in retirement. There are substantial differences in reverse-mortgage equivalent wealth between strategies, and in our base case a household with average housing and financial wealth and a coefficient of risk aversion of 5 would be 24 percent better off taking a lifetime income at age 65 relative to taking a line of credit when financial wealth is exhausted.

We also find that including the reversionary interest in the household's portfolio results in an increase in the optimal allocation of financial assets to stocks, regardless of the strategy adopted for decumulating the reversionary interest. For example, at a coefficient of risk aversion of 5, the optimal allocation of financial assets when reverse mortgages are unavailable is 55 percent in stocks.⁶ But the same household will optimally invest 100 percent of its financial wealth in stocks if it plans to take a reverse mortgage in the form of a lifetime income when its financial wealth is exhausted.

The remainder of the paper is organized as follows. In section 1, we explain how reverse mortgages are structured and outline the literatures on reverse mortgages and on the decumulation of financial wealth in retirement. In section 2, we outline the literature on returns to investments in housing. In section 3, we present the VAR model that we use to determine the relationships between returns on housing and financial assets, and inflation and interest rates. In section 4, we describe the alternative strategies for accessing housing wealth that are evaluated later in the paper. We present our simulation results in section 5. Section 6 concludes.

1. Reverse Mortgages and Retirement Wealth Decumulation

1.1 Reverse mortgages

A reverse mortgage enables a household to consume part of the reversionary interest in its house while continuing to live in it. HECM loans, the product with 90 percent of the market, are available only to individuals and couples aged 62 or older. Housing equity can be withdrawn in the form of an income payable for as long as the

⁶ For this case, we assume that the household simply enjoys the imputed rent and that the house and any remaining financial assets pass on death as an unintended bequest.

borrowers continue to live in the house, a strategy that we refer to as the “lifetime-income plan,” a lump sum, a line of credit, or payments for a fixed number of years that may be shorter than the borrowers’ remaining time in the house. We do not comment further on the fixed-period option, as our calculations show that, for plausible strategies, it is dominated by the other options. Plans can also be combined. In contrast to a conventional home loan, the interest is capitalized. The loan, plus accumulated interest, must be repaid when neither borrower occupies the house as his principal residence.⁷ The amount owed is capped at the sale proceeds of the property. As the borrower is not required to make any payments on the loan, eligibility does not depend on the borrower’s income or credit rating.

The loan interest rate is set at 1.5 percentage points above the one-year constant maturity Treasury rate. The maximum percentage of the house that can be borrowed is calculated by reference to a formula based on the age of the younger borrower and the yield on the 10-year constant-maturity Treasury bond. The percentage is lower at younger ages because loans to younger borrowers will likely remain outstanding for longer periods and therefore accrue more interest per dollar borrowed. As previously mentioned, the percentage is also lower at higher interest rates, because at higher interest rates the interest will accrue more rapidly.

Although women generally live longer than men, Federal law prohibits the FHA from taking gender into account when setting loan-to-valuation ratios. Interestingly, the loan limit for married couples is identical to that for single individuals, even though the joint life expectancy of a couple exceeds those of single men and women. As of 2006, the FHA further restricts loans to percentages of maximum appraised values ranging from \$200,160 in rural areas to \$362,790 in high-cost metropolitan areas. At age 65, these values translate into loan limits of \$97,307 to \$182,934.⁸

⁷ There are also requirements relating to tax and insurance payments and maintenance of the property.

⁸ Appraised value limits can be found at <https://entp.hud.gov/idapp/html/hicostlook.cfm>. Interest rates prevailing on September 14, 2006 were assumed in these calculations.

The FHA requires the borrower to purchase a mortgage insurance policy that insures the lender against the losses that would arise in the event that the loan plus accumulated interest exceeds the sale proceeds of the property. The premium is 2 percent of the lesser of the appraised value and the county FHA loan limit, deducted from the loan advance, plus a 0.5-percent per year interest supplement. The premium does not vary with the percentage of the maximum allowable loan that is being borrowed. Other costs include an origination fee of a maximum of 2 percent of the lesser of the appraised value and the county loan limit, plus closing costs that the AARP informs us average \$2,000 to \$3,000. Although not strictly a closing cost, the present value of servicing costs to age 100 is also deducted from the loan. We calculate that this deduction amounts to \$5,127 at age 65.⁹

Households can return for a new loan. This is an expensive undertaking, requiring that they incur all their closing costs again, subject only to a credit for the mortgage insurance premium paid previously. Home mortgages are not portable, so reverse mortgages may restrict households' ability to move—for example, to an assisted living facility.

The initial amount that can be borrowed on a line of credit equals the amount that could be borrowed as a lump sum. The credit-line borrowing limit, inclusive of accumulated interest, increases at the prevailing one-year Constant Maturity Treasury Bill rate, plus the 1.5 and 0.5 percentage point supplements referred to above.

The amount of lifetime income is determined by reference to a closely related formula, again based on age and the 10-year Treasury bond rate at the time the plan commences. The monthly payments, plus accumulated interest, are charged to the borrower's account, so the amount outstanding increases over time.

An alternative to the lifetime-income plan is to take a lump-sum advance and use it to purchase an immediate annuity. In contrast to lifetime-income plan rates, which do not take account of gender or marital status, annuity income rates are highest for

⁹ We again assume September 14, 2006, interest rates and also assume a servicing cost of \$30 per month.

single men and lowest for joint lives. In further contrast, the entire cost of an immediate annuity is paid in advance. Immediate annuities redistribute wealth from those who die soon after purchase to those who live longer than expected, whereas reverse mortgages redistribute wealth only to the extent that the loan grows to exceed the value of the property. These “mortality credits” substantially increase annuity yields, particularly at older ages.

The overall impact of these differences in product design is that the annuity strategy generally provides a higher income, particularly at older ages and for single men. To illustrate, the payments under the lifetime-income plan commencing at age 65 equaled 7.45 percent of the amount that could be borrowed as a lump sum.¹⁰ The age 65 annuity rates for single males, single females, and joint lives with a 100-percent survivor benefit were 8.47, 7.88, and 7.14 percent, respectively.¹¹ At age 85, the lifetime-income plan yielded 10.58 percent, and the annuities yielded 17.06, 15.44, and 15.29 percent, respectively.

Increases in interest rates reduce the income payable per dollar of housing wealth under both the annuity and the lifetime-income strategies. Although annuity rates are more favorable at higher interest rates, this is insufficient to offset the reduction in the reverse mortgage loan-to-valuation ratio.¹²

Households that delay taking a reverse mortgage face a number of financial risks. The value of the house may decrease. The 10-year Constant Maturity Treasury rate may increase, reducing the percentages of the value of the house that can be borrowed either as a lump sum or under the lifetime-income plan. And the amount of income that

¹⁰ These calculations are correct for interest rates and other parameters, as of September 14, 2006.

¹¹ Source: www.immediateannuities.com and AARP reverse mortgage calculator.

¹² To calculate the relationship between interest rates and annuity income, we calculated the degree of actuarial unfairness of annuities, using prices quoted at www.immediateannuity.com, assuming the Treasury strip interest rate and population mortality for the appropriate birth cohort. We calculated annuity rates at other interest rates, assuming the same mortality rates and degree of actuarial unfairness. The annuity rates were then multiplied by the percentage of the value of the house that could be borrowed at the interest rate in question.

each dollar of housing wealth produces may also decline. These risks may be correlated. For example, an increase in interest rates may lead to a reduction in house prices.

At some cost, a household can partially protect itself from these risks by setting up a line of credit immediately on retirement for subsequent use. If, by the time the household is ready to commence withdrawals, interest rates and house prices have moved in favorable directions, the household can apply for a new line of credit. If, on the other hand, interest rates and house prices have moved in unfavorable directions, the household can commence withdrawals based on the original line of credit.

1.2 Calculations of potential consumption gains from reverse mortgages

Previous research shows that reverse mortgages can modestly increase post-retirement consumption. Venti and Wise (1991) estimated the increase at 10 percent, while Rasmussen, Megbolugbe, and Morgan (1995) estimated the increase at 25 percent among those with incomes of less than \$30,000, the higher estimate reflecting the fact that low-income homeowners often have substantial amounts of housing wealth relative to their income.

The above calculations of the amounts by which reverse mortgages could increase post-retirement consumption assume as a counterfactual that housing equity passes as an unintended bequest. But Venti and Wise (1991) and Walker (2004) show that this is not generally what happens; the house is often sold, albeit usually in advanced old age and after a precipitating shock such as the death of a spouse. Little is known about what happens to the sale proceeds. It is possible that they are spent on long-term care and medical expenses and that the house provides self-insurance against such expenses, or simply against living unusually long.

Even if the house does provide insurance against the perils of advanced old age, this is not an argument against taking a reverse mortgage, only against taking a

reverse mortgage and then failing to protect oneself against those perils.¹³ Such protection might take the form of the purchase of annuities and long-term care insurance, or simply restricting one's current consumption. However, the calculation is complicated by Medicaid rules, discussed in section 4.2, which generally treat housing more favorably than financial wealth and provide an incentive to decumulate financial wealth while preserving housing wealth.

1.3 Techniques for determining optimal asset decumulation strategies

One approach when, as is usually the case, the problem is not analytically tractable, is to use numerical optimization techniques to determine optimal strategies. This approach is computationally intensive. Including housing wealth as well as financial assets increases both the number of strategies to be considered and the number of asset classes with stochastic returns. Using numerical optimization to solve a model that included housing wealth would necessitate simplifying assumptions about asset returns, and available asset allocation and decumulation strategies, that would significantly detract from the realism of the model.¹⁴

The alternative approach that we adopt is to use Monte-Carlo simulations, as exemplified by Albrecht and Maurer (2002), Dus, Maurer, and Mitchell (2005), Blake, Cairns, and Dowd (2003), and Horneff, Maurer, Mitchell, and Dus (2006). The simulation approach does not consider every possible strategy, only those regarded as plausible alternatives. In particular, it assumes that households persevere with a predetermined decumulation strategy even when it may no longer be optimal to do so. Although this approach may fail to identify the optimal strategy, it might well be an advantage to consider only a subset of rule-of-thumb strategies that households might plausibly implement.

¹³ Stucki (2004) investigates the possibility of using reverse mortgages to manage the financial risk of long-term care.

¹⁴ The distribution of asset returns is typically discretized using Gaussian quadrature, a task that becomes extremely complex when there are multiple asset classes with correlated returns that also depend on past returns.

In the above papers, alternative strategies are compared to a benchmark of full annuitization at retirement; our benchmark, however, is taking a reverse mortgage immediately at retirement and adding the proceeds to financial wealth. A number of yardsticks can be used to compare alternatives. For example, Albrecht and Maurer (2002) ranked strategies based on the probability that the household would outlive its wealth. Although this “probability of ruin” metric is readily understandable, it can mislead. It ignores the additional return households get if they beat the target. This was not an issue for Albrecht and Maurer, as their alternative strategies all involved consuming a fixed amount until funds were exhausted. The use of their yardstick can also produce the paradoxical result that households with high withdrawal targets will minimize their probability of ruin by choosing extremely risky strategies when they might be better off reducing their withdrawal rate.

When the strategies allow for the possibility of increased consumption if returns are unusually favorable and, conversely, for decreased consumption if returns are unusually poor, an alternative approach is to consider the magnitudes of the shortfalls and surpluses, resulting in a partial ordering of decumulation plans. Dus, Maurer, and Mitchell (2005) consider three strategies in which consumption responds to asset returns: setting current consumption equal to a fixed percentage of current wealth, to $1/E(T)$, or to $1/T$, where $E(T)$ is remaining life expectancy and T is the maximum possible remaining life expectancy.

But unless one puts additional structure on the household’s preferences—see Sarin and Weber (1993)—it may not be possible to provide a complete ordering of all the strategies. An alternative to the above approaches is therefore to specify a utility function and to evaluate the strategies in expected utility terms, as in Blake, Cairns, and Dowd (2003) and Horneff, Maurer, Mitchell, and Dus (2006).¹⁵ This is the approach that

¹⁵ In the above simulations, consumption varied with asset returns. If one were to attempt a utility-based analysis of strategies that involved consuming a fixed amount until financial wealth was exhausted, one would end up with an ordering that was identical to that obtained under the “probability of ruin” approach.

we adopt.

2. Historical Data

2.1 Comparing housing returns to those on financial assets

The amount that the household can borrow on a reverse mortgage, what we term the reversionary interest, can be thought of as an asset. The household can “sell” this asset immediately on retirement or can retain ownership for a period of time. The household’s decision should depend on the expected returns on the various assets in its portfolio, including the reversionary interest, the riskiness of those returns, their covariances, and the household’s consumption needs.

The value of the reversionary interest depends on age, the Treasury bond interest rate, and the value of the house. Holding that interest rate constant, the expected return will exceed the expected return on the house because the percentage of the value of the house that can be borrowed increases with age. But fluctuations in interest rates mean that the amount the household can borrow is quite volatile. Figure 2 shows the percentage amounts that could have been borrowed from 1975 to 2005 on a \$200,000 house at ages 65, 75, and 85, net of closing costs, and assuming that the HECM program had been in existence throughout that period. The amount that could be borrowed at age 65 ranged from 4.8 percent of the value of the house in 1981 to 51.3 percent in 2002.

Table 1 compares the means and standard deviations of the real returns on housing, a diversified domestic equity portfolio, and one- and 10-year Treasury bond yields over the period 1975–2005. In addition to reporting the capital return on housing, we also show the return on the reversionary interest. This equals the percentage increase in its value, in constant prices, net of closing costs. The percentage effect of a given change in interest rates on the amount that can be borrowed on a reverse mortgage decreases with age, so the riskiness of the return to postponing a reverse mortgage likewise decreases with age. We therefore report the return on the reversionary interest at ages 65, 75, and 85.

The period 1975–2005 was one that produced unusually good real returns for bonds: 4.4 percent, compared with an average of 2.9 percent for the period 1926–2005.¹⁶ The real return on stocks greatly exceeds that on bonds, but at the cost of higher risk. Stock returns were 9.2 percent during 1975–2005, compared with their long-run average of 9.1 percent (1926–2005), yielding an equity premium of 4.8 percent compared with a long-run average of 6.2 percent.

The average increase in real house prices over the period 1975–2005 was only 1.9 percent, with a standard deviation of 3.7 percent. We characterize this return as modest, given that the period includes the recent housing boom. Even this may be something of a historical aberration. Shiller (2006) calculates that there has been little increase in real house prices over the period 1890–2000.

In contrast, both the real return on the reversionary interest and the standard deviation of that return were very substantially higher. The means and standard deviations were 16.0 and 40.6 at age 65, 10.2 and 23.4 at age 75, and 6.9 and 13.5 at age 85. The higher average return to an investment in the reversionary interest reflects not only the fact that the percentage of the value of the house that can be borrowed increases with age, but also the substantial declines in nominal interest rates during this period. An investment in the reversionary interest has quite different characteristics from one in the house itself, with a much higher mean and standard deviation.

The role of interest rate movements during the sample period can be highlighted through a few calculations. If interest rates had remained constant at 1975 levels, the mean returns on the reversionary interest would have been only 7.5 percent at age 65, 6.5 percent at age 75, and 5.3 percent at age 85. Fluctuations in interest rates also contributed substantially to the volatility of the return to the reversionary interest. Holding interest rates constant, the standard deviations fall to 4.3, 4.1, and 4.0 percent.

Households invest not in house price indices but in a particular house. Case and Shiller (1989) report substantial differences among cities in rates of appreciation over the period 1970–1986. More recently, Gyourko, Mayer, and Sinai (2006) also find

¹⁶ Ibbotson Associates (2006).

evidence of persistence in differences in the rate of house price appreciation. Since anticipated house price appreciation is an important determinant of the optimal timing of a reverse mortgage, it follows that optimal timing may depend on the city in which the house is located. Such differences may not necessarily imply differences in total rates of return, because current house prices in areas where there is an expectation of rapid future increases may be bid up to the level where the reduction in imputed rental return just compensates for the additional anticipated capital appreciation.

There is also evidence that the returns to investing in a particular house are considerably riskier than an analysis of either national house price indices or even indices for specific cities would suggest. Flavin and Yamashita (2002) analyze PSID data for 1968 to 1992. Every year, the PSID asks householders how much their house would sell for if it were put on the market at the date of the interview, enabling the authors to calculate annual rates of house price appreciation. They calculated that the mean and standard deviation of the real return to individual houses, inclusive of imputed rent, amounted to 6.6 and 14.2 percent, respectively. They assumed that the imputed rent equals a fixed 5 percent of the house value, plus a constant 33-percent tax rate multiplied by self-reported property tax. Thus, as a close approximation, the standard deviation of their capital return will also equal 14.2 percent. This is much higher than the 3.5-percent standard deviation of the real return on the Census Bureau house price index.

A possible concern about Flavin and Yamashita's approach is that the standard deviation of returns may be inflated by reporting error. This seems to be unfounded. Case and Shiller (1989) constructed house price indices for four cities using repeat sale data and estimated that the standard deviation of the returns on individual houses was about 15 percent, close to Flavin and Yamashita's estimate. To summarize, although home ownership insures households against changes in the cost of housing services, the evidence is clear that the reversionary interest is a very risky asset.

2.2 The covariance of housing returns with returns on financial assets

The attractiveness of the reversionary interest as an investment will depend not only on the mean and variance of its real return, but also on the covariance of that return with interest rates and with the returns on stocks and bonds. Unfortunately, there does not appear to be a stable and predictable relationship between interest rates, the primary determinant of bond returns, and house prices, which together with interest rates are the primary determinants of the return on the reversionary interest.

Economic theory indicates that the user cost of housing should be an important determinant of house prices. The user cost will reflect interest rates, depreciation, maintenance, and taxes. It will also include anticipated changes in the value of the house. An increase in inflation will lead to an increase in nominal interest rates, holding the real interest rate constant. The increase in nominal interest rates will increase the value of the mortgage interest tax deduction, reducing the user cost of housing, and, according to the user cost model, result in increases in house prices.

Poterba (1984) found evidence to support this hypothesis. He analyzed house price movements in the 1970s and calculated that the accelerating inflation of that decade could have accounted for a 30-percent increase in real house prices.¹⁷ In the 1980s, nominal interest rates and tax rates both declined, and changes to the tax code decreased the proportion of taxpayers who benefited from itemizing. These changes reduced the value of the mortgage interest tax deduction and increased user costs, as did increases in real interest rates. According to the user cost model, these trends should have led to a substantial reduction in house prices.

In fact, real house prices declined only very slightly during the 1980s. Mankiw and Weil (1989) argued that house buying by the baby boomers was the major cause of the increase in real housing prices and forecast a real price decline as smaller, subsequent birth cohorts entered the housing market. But Poterba, Weil, and Shiller (1991) found little evidence to support explanations based on demographics or changes in either user costs or construction costs. In the absence of convincing alternative

¹⁷ Poterba examined movements in the price of housing structures, exclusive of land.

explanations, they concluded that home owners may not have rational expectations and may incorporate extrapolations of past appreciation into their user cost calculations. They commented on, but did not investigate in detail, the possibility that relaxations in credit constraints may have led to an increase in house prices.

Starting in the late 1990s, there was yet another rapid increase in prices in some markets. This increase coincided with exceptionally low nominal interest rates and further financial liberalization.

We conclude that models that explain movements in house prices in one period may have very little predictive power in other periods, when monetary and tax policy and the structure of financial institutions may be quite different. The above papers provide little guidance on what rate of house price appreciation to expect and what the current relationship might be between returns on housing and financial assets.

An alternative approach is to use reduced-form vector autoregressions to identify the historical relationship between the returns in housing and financial markets. Our simulations require that we capture the auto-covariance structure of asset returns, but do not depend on a particular structural model of asset price determination. An important advantage of reduced-form VARs is that although forecasts made with such models assume stable relationships between the variables included in the VAR, they do not require us to make an explicit choice between competing theories of the determinants of house prices.

Sutton (2002) estimates VARs for the United States, the United Kingdom, Canada, Australia, the Netherlands, and Ireland. He finds that a 100-basis-point decrease in real interest rates increases real house prices by 0.5 to 1.5 percent. He also finds plausibly sized effects of shocks to GNP and stock prices. For example, over a three-year time horizon, a 1-percent increase in GNP is associated with a 1-to 4-percent increase in house prices, and a 10-percent increase in stock prices is associated with a 1-to-2-percent increase in house prices after three years; the increase is 5 percent in the United Kingdom, although these latter increases may reflect the tendency of stock prices to anticipate increases in GNP.

3. Modeling Asset Returns and Interest Rates

The Flavin and Yamashita data capture the relationship between the returns to housing, stocks, and bonds. Unfortunately, this relationship is not suitable for our purposes. The amount that can be borrowed on a reverse mortgage depends on the yields on the one-year Treasury bill and the 10-year Treasury bond, neither of which is included in their analysis. In addition, they make the analytically convenient assumption that returns are independent and identically distributed (i.i.d.), whereas Cho (1996) finds evidence of serial correlation in housing returns that might increase the riskiness of housing as an investment.

We therefore estimate a reduced-form VAR including both one- and 10-year bond yields. To avoid estimating separate equations for every housing market, we use national house price data and then test the sensitivity of our results to different assumptions about both the mean and the standard deviation of the return to housing. The covariance matrix that we obtain is then used in our Monte-Carlo simulations.

Our VAR consists of equations for the nominal quarterly yield on one-year U.S. Treasury Bills, the nominal quarterly yield on 10-year U.S. Treasury Notes, the real gross quarterly rate of capital gains on home ownership (using the OFHEO repeat sales price index), real quarterly GDP growth, and the quarterly rate of consumer price inflation (using the CPI-U, the consumer price index for all urban consumers, commonly referred to as "CPI"). The equation for each variable included eight quarterly lags of its own values as well of those of the other five variables. Following the typical treatment in the finance literature, we treated equity returns as exogenous to innovations in the processes driving the other variables and we included eight quarterly lags of real quarterly equity returns (using the total return on the S&P 500 index, with dividend reinvestment) in each equation.¹⁸ Our sample period extends from the first quarter of 1975 through the

¹⁸ Quarterly returns and growth rates for monthly variables were calculated based on their end-of-quarter or third-month-in-quarter values.

fourth quarter of 2005.¹⁹ Results from estimation of the VAR are shown in Appendix Table 1.

Table 2 compares the means and correlations of the returns for the historical period used in estimating the VAR with the mean of the corresponding simulated moments for the 35-year period commencing in 2006. In simulating the returns using our VAR estimates, we first take random draws for stock returns and the error terms for each equation for each quarter of the forecast period.²⁰ These random elements are then combined with the VAR coefficients to generate 10,000 simulations of yields and asset returns for 2006 to 2040.

4. Calculating Optimal Strategies with Housing Wealth

4.1 Our model

There are a variety of ways in which a household can liquefy the reversionary interest in its house by using a reverse mortgage. In this section, we compare several alternative strategies. We focus on three related questions, namely, what is the optimal age at which to take a reverse mortgage, in what form should the household take the proceeds, and what effect does the option to take a reverse mortgage have on the optimal allocation of financial assets between stocks and bonds. We consider married couples. We assume that the household has the mean amounts of financial and housing wealth for the median 20 percent of married couples turning 65 between 1994 and 2000—\$90,667 and \$101,333, respectively, as reported by Dushi and Webb (2004). We also assume that both the husband and the wife are 65 in 2006, which means that they face 1941 birth-cohort mortality, as forecast by the Social Security Administration. We do not consider single men or women, as the median individual in these categories has extremely small amounts of both housing and financial wealth. We ignore Social

¹⁹ The sample period was dictated by availability of the OFHEO house price index.

²⁰ We assumed that the quarterly stock returns were independent draws from a univariate normal distribution with mean and variance obtained from data for our estimation period. The distribution of the VAR innovations was assumed to be multivariate normal, with zero means and covariances estimated from the VAR residuals.

Security and defined benefit pension income, or equivalently assume that this income is used to meet fixed living costs.

Our benchmark is the household's consumption, assuming the household takes a reverse mortgage when the husband attains age 65. In our benchmark, the household allocates its financial wealth, including the proceeds of its reverse mortgage, between stocks and bonds, maintains this allocation with annual rebalancing, and consumes 7.2 percent of its current financial wealth each year. The household allocates its financial wealth between stocks and bonds so as to maximize expected lifetime utility, discounted by annual survival probabilities and a 3-percent rate of time preference. We follow the literature—see, for example, Brown and Poterba (2000)—by assuming a constant relative risk aversion utility function of the following form:

$$U_m(C_t^m, C_t^f) = \frac{(C_t^m + \lambda C_t^f)^{1-\gamma}}{1-\gamma}, U_f(C_t^f, C_t^m) = \frac{(C_t^f + \lambda C_t^m)^{1-\gamma}}{1-\gamma}, \quad (1)$$

where λ measures the extent to which each spouse benefits from his partner's consumption; C_t^m, C_t^f denote the consumption of the husband and wife at time t ; and γ is the coefficient of risk aversion. When λ equals 1, all consumption is joint. When λ equals 0, none of the household's consumption is joint. We assume λ equals 0.5. Household utility is assumed equal to the sum of the utilities of the spouses, and consumption is assumed to be equated across spouses.

Horneff, Maurer, Mitchell, and Dus (2006) included a bequest motive in their utility function. We chose not to include a bequest motive, as there is no consensus as to how it should enter into the utility function and all of our strategies result in at least some likelihood of a bequest.

We assume that management charges on stocks and bonds amount to 43 and 25 basis points, respectively.²¹ We assume that households invest in corporate bonds at a fixed 100-basis-point premium over the 10-year Treasury bond. We disregard income

²¹ These amounts are equal to the current expense ratios on Vanguard Diversified Equity and Long-Term Investment Grade Bond funds.

taxation, both for simplicity and also because the median 20 percent of married couple households are unlikely to face significant liabilities. Closing costs equal the \$2,073 that the AARP inform us was assumed in their reverse mortgage calculator.²²

The expected real returns on stocks and bonds in the simulation period are 9.3 and 3.7 percent, respectively, before management charges, with standard deviations of 16.9 and 10.3 percentage points. Given our assumed withdrawal rate of 7.2 percent of total wealth, it follows that there is a high probability that the value of the household's assets will decline, although the household will never exhaust them. The assumed withdrawal rate is somewhat higher than the 4 percent or so that is often suggested by financial advisers, but is chosen to facilitate comparison with Horneff, Maurer, Mitchell, and Dus (2006).²³ We also report benchmarks and alternatives, using both 5.0-percent and 10.0-percent withdrawal rates, but find that the choice of decumulation rate has little effect on the optimal strategy.

We then compare our base case with the following alternative strategies for using a reverse mortgage to make the household's reversionary interest available for consumption:

Simulations with a lump-sum advance

The household postpones taking its reverse mortgage until age 70, 75, 80, 85, or until it has exhausted its financial wealth.²⁴ In each period prior to taking its reverse mortgage, the household consumes 7.2 percent of the current total value of its financial wealth and reversionary interest. If the household has insufficient financial wealth to pay for planned consumption, it takes a reverse mortgage immediately. To implement this strategy, the household must be able to ascertain the value of its house and be able

²² The calculator is located at <http://www.rmaarp.com/>

²³ A summary of withdrawal rate recommendations is posted at <http://www.retireearlyhomepage.com/safewith.html>. Horneff et al. chose 7.2 percent to facilitate comparison with the purchase of an immediate annuity.

²⁴ Although the household will never exhaust its total wealth (including housing wealth), it can exhaust its financial wealth.

to determine the amount it could obtain on a reverse mortgage. This information is readily available on the Internet.²⁵

When the household takes a reverse mortgage it adds the proceeds to its stock of financial wealth and from then on consumes 7.2 percent a year of its current financial wealth. Upon taking a reverse mortgage, the household is allowed to select a revised allocation of financial wealth between stocks and bonds, which it maintains with annual rebalancing until death.

HECM rules permit a household to reapply for an additional loan if the current house value and 10-year Treasury bond interest rate permit. It is difficult to determine the optimal strategy in relation to further advances; the household faces the decision whether to take a small advance now or delay in the hope of being able to obtain a larger advance later. Our simulations indicate that transaction costs are such that it will only rarely be possible for households to obtain significant further advances, and we therefore assume that they take only a single loan for the maximum possible amount.

Simulations with a line of credit

We consider two alternatives. In the first, the household initially consumes 7.2 percent of the current total value of its financial wealth and reversionary interest. The household establishes a line of credit when it no longer has sufficient financial wealth to pay for planned consumption. It then takes a periodic withdrawal equal to 7.2 percent of the total of the current balance available for withdrawal on the line of credit plus any residual financial wealth. In the second, the household establishes its line of credit at age 65 and immediately commences drawing at a rate equal to 7.2 percent of the current undrawn balance.

Simulations with a lifetime income

We consider two alternatives. In the first, the household again initially consumes 7.2 percent of the current total value of its financial wealth and reversionary

²⁵ www.zillow.com enables homeowners to track the approximate value of their house.

interest. The household takes a lifetime-income reverse mortgage when it no longer has sufficient financial wealth to pay for planned consumption. This strategy can result in a significant change in income when the reverse mortgage is taken, as the income payment rate will typically be higher than the 7.2-percent withdrawal rate being taken until that time. But the monthly income for life is fixed in nominal terms so that the boost to income declines over time. In the second, the household takes a lifetime income immediately on retirement. In addition to consuming its lifetime income, it consumes 7.2 percent a year of its financial wealth.

4.2 Medicaid and health shocks

Medicaid eligibility rules treat housing more favorably than financial assets. In general, individuals will become eligible for Medicaid only after they have spent almost all of their financial assets. In contrast, housing wealth may be passed to a surviving spouse. Medicaid rules relating to expenditure on long-term care are somewhat less stringent, but still favor housing over financial wealth for many households. Depending on the state of residence, financial wealth of \$19,908 to \$99,540 is completely protected under so called “spousal protection rules,” and partial protection may be available up to \$199,080. Households wishing to protect assets for the benefit of a surviving spouse will often have an incentive to hold housing in preference to financial wealth. We do not model these incentives, as they depend on the household’s assessment of the probabilities of incurring expenditure on medical and long-term care, their state of residence, financial assets, and the amounts of other income received by the husband and wife.

5. Simulation Results

5.1 Optimal reverse mortgage strategies—base case

Table 3 reports our base-case results. We calculate the household’s expected utility if it takes a reverse mortgage at age 65 and invests the proceeds in a utility-

maximizing portfolio of stocks and bonds (we term this the default strategy) and compare the default strategy with the expected utilities of alternative strategies. These comprise taking a reverse mortgage at ages 70, 75, 80, or 85, or when its financial wealth is exhausted, or taking a line of credit or a lifetime income either at age 65 or when the household's financial wealth is exhausted. We calculate reverse-mortgage equivalent wealth. As mentioned previously, this is the factor by which the wealth of a household choosing the default strategy must be multiplied so that its expected utility equals that of the household choosing the alternative. When reverse-mortgage equivalent wealth of a particular strategy exceeds 1.00, that strategy offers a higher expected utility than the default. This measure is analogous to the calculation of "annuity equivalent wealth" in Brown and Poterba (2000), the amount the household would require by way of compensation for the loss of the right to annuitize its retirement wealth.

Regardless of the value of the coefficient of risk aversion, taking a lump sum at age 65 is always preferable to taking a lump sum at a later time. The strategy of taking a reverse mortgage in the form of a line of credit once financial wealth is exhausted, which the National Reverse Mortgage Lenders Association tells us is most frequently chosen, performs particularly badly. At a coefficient of risk aversion of 5, a household taking a line of credit when its financial wealth is exhausted would require a 24-percent increase in its wealth to compensate it for being denied the opportunity to take a lifetime income at age 65.

At higher levels of risk aversion, taking a reverse mortgage in the form of a lifetime income, either at retirement or when financial wealth is exhausted, is preferable to taking a lump sum at age 65. As mentioned in section 2, an alternative to taking a lifetime income from a reverse mortgage is to take a lump sum and use that to purchase an immediate annuity from an insurance company. At age 65, these two alternatives produce very similar incomes, but when a reverse mortgage is taken at older ages the strategy of applying the proceeds to the purchase of an immediate annuity yields a substantially higher income. Therefore, the dominant strategy for households is

probably to spend down their financial wealth, take a reverse mortgage, and use the proceeds to buy an annuity.

Regardless of the level of risk aversion, taking a lump sum, whether at age 65 or when financial wealth is exhausted, is preferable to taking a line of credit at the corresponding age. Regardless of the coefficient of risk aversion, taking a lifetime income, whether at age 65 or when financial wealth is exhausted, is also preferable to taking a line of credit at the corresponding age.

Investment allocations vary in predictable ways. In the default strategy, the optimal allocation to equities varies from 100 percent at a coefficient of risk aversion of 2, to 55 percent at a coefficient of risk aversion of 5. These investment allocations also apply when we close the reverse mortgage market and assume that the house passes as an unintended bequest. Households taking a lifetime income or a line of credit at age 65 allocate larger proportions of their financial wealth to stocks—77 and 71 percent, respectively, at a coefficient of risk aversion of 5—than households that take a lump sum at 65. Households that postpone taking a reverse mortgage until they have exhausted their financial wealth invest even larger percentages in stocks, 100 percent for those taking a lifetime income, 99 percent for those taking a lump sum, and 90 percent for those taking a line of credit, at a coefficient of risk aversion of 5. Although households that delay start out with the same amount of financial wealth as those that take a lifetime income or line of credit at age 65, they decumulate their financial wealth more rapidly. On average, over the course of their retirement, financial wealth constitutes a smaller proportion of their total wealth, and they respond by investing that wealth more aggressively. For the same reason, households with a coefficient of risk aversion of 5 that postpone taking a lump sum until age 70 invest 100 percent of their financial wealth in stocks prior to taking a reverse mortgage and decrease their financial wealth invested in stocks to 44 percent subsequently.

Figures 3 (a) and (b) show the means and standard deviations of the income flows resulting from the various strategies. We present results calculated at a coefficient of risk aversion of 5. The strategy of postponing taking a reverse mortgage until

financial wealth is exhausted and then taking the reverse mortgage in the form of a lifetime income provides the highest mean income at almost all ages, but with a standard deviation that sharply increases at very advanced ages. This increased variance is due to the household's investment in two risky assets: the reversionary interest and a portfolio of financial assets that is 100 percent invested in stocks. The strategy of taking a lifetime income at age 65 provides a lower mean income at older ages, but at substantially reduced risk at all ages, since the household is no longer exposed to the risks of investing in the reversionary interest. Taking a line of credit when financial wealth is exhausted—the strategy adopted by most households in the real world—performs particularly badly, being among the riskiest strategies at all but the oldest ages, while providing a modest and declining income.

The two lifetime-income strategies can each be compared with the corresponding lump-sum strategy. Taking a lifetime income at age 65 provides a higher average income at all ages than taking a lump sum, albeit at slightly higher risk at older ages. Taking a lifetime income when financial wealth is exhausted is clearly preferable to taking a lump sum when financial wealth is exhausted, providing a much higher average income at similar levels of risk at all but the most advanced ages.

The numbers at other degrees of risk aversion reflect the impact of risk aversion on portfolio allocations to equities. At a coefficient of risk aversion of 2, both the mean and the standard deviation of the age 65 lump-sum strategy are higher than when the coefficient of risk aversion equals 5. But the mean and standard deviation of the income from the strategy of taking a lifetime income when financial wealth is exhausted is unchanged, since households adopting this strategy invest 100 percent in equities regardless of the degree of risk aversion.

5.2 Optimal Strategies—Alternative assumptions regarding asset returns

We consider the implications of alternative assumptions about asset returns. Although our results vary in predictable ways, we find that our key conclusion—that

taking a lifetime income either at 65 or when financial wealth is exhausted dominates the alternatives—still holds.

Table 4 compares reverse-mortgage equivalent wealth under alternative assumptions regarding asset returns. When the housing return is increased by 2 percentage points, it becomes relatively more attractive to postpone taking a reverse mortgage. But at coefficients of risk aversion of 3, 4, and 5, the optimal strategy is still to take a lifetime income, although now it is clearly more advantageous to delay until financial wealth is exhausted. At a coefficient of risk aversion of 2, the dominant strategy remains to take a reverse mortgage as a lump sum at age 65 and invest everything in stocks.

When the housing return is decreased by 2 percentage points, the optimal strategy is to take a lifetime income at age 65, unless the coefficient of risk aversion equals 2, in which case the optimal strategy is again to take a lump sum at age 65 and invest everything in stocks.

When the stock return is decreased by 3 percentage points, the optimal strategy is to take a lifetime income when financial wealth is exhausted, regardless of the degree of risk aversion. The second-best choice is to take a lifetime income at age 65, again regardless of the degree of risk aversion. The lifetime-income option is always preferable to taking a lump sum at age 65 and investing the proceeds in stocks. The optimal portfolio allocation to stocks decreases substantially when we assume a lower return on stocks.

At a 5-percent withdrawal rate, the optimal strategy is to take a lifetime income at age 65. The lifetime-income approach now offers a substantially higher immediate income than the alternatives. At a 10-percent withdrawal rate, the optimal strategy is again to take a lifetime income at age 65, unless the coefficient of risk aversion equals 2, in which case the household should take a lump sum at age 65. In this case, however, the withdrawal rate is so high that households that do not take a lifetime income risk very low income in advanced old age due to relatively depleted wealth.

When we increase the standard deviation of housing returns to 15 percent, we find that it is optimal for all but the most risk tolerant to take a lifetime income at age 65. The most risk tolerant should take an immediate lump-sum reverse mortgage and again invest the proceeds entirely in stocks. Delaying taking a lifetime income now becomes highly unattractive.

In our simulations, the average nominal interest rate on the 10-year Treasury bond increases rapidly from a historically low rate of 4.7 percent to a long-run average of 7.5 percent. The lump sum, line of credit, and lifetime income that households can obtain on a reverse mortgage are all inversely related to nominal interest rates. The increases in nominal interest rates increase the attractiveness of taking a reverse mortgage immediately on retirement, relative to postponing. To check whether our results were robust to alternative assumptions regarding initial interest rates, we ran simulations with retirement dates chosen at random from the years 2026 to 2046. Table 5 reports our results. Under these alternative assumptions, it is clearly optimal to first consume one's financial wealth and then take a reverse mortgage in the form of a lifetime income. Except when the coefficient of risk aversion is 2, the second-best strategy is to take a reverse mortgage in the form of a lifetime income immediately on retirement.

6. Conclusion

Housing constitutes much of the non-pension wealth of the majority of households. As a result of inadequate savings rates, declines in Social Security replacement rates, increased life expectancy, and the demise of traditional defined benefit private-sector pensions, it seems likely that increasing numbers of households will need to tap their housing wealth in order to maintain their standard of living in retirement. Yet, there has been virtually no research to date on how households can best accomplish this. This paper helps to fill this gap by analyzing alternative strategies for using reverse mortgages to make house equity available to fund consumption while homeowners continue to enjoy the housing services provided by their house.

We show that the amount available to borrow through a reverse mortgage—the reversionary interest in the house—is a risky asset with a relatively high mean return. Households can take a reverse mortgage as a lump sum, a lifetime income, or a line of credit. Choosing among the alternative strategies for using a reverse mortgage to liquefy housing wealth is equivalent to a portfolio-choice decision, where at some point the reversionary interest is sold and put into a form where the funds are more readily available for consumption.

Currently, households that tap their housing wealth for consumption tend to choose a strategy that we show tends to perform very badly—waiting until financial wealth is exhausted and then taking a line of credit. This strategy involves holding onto the risky reversionary interest for a long time, but then exchanging it for an asset that has a relatively low yield.

Our simulations show that households would be substantially better off taking their reverse mortgage as a lifetime income, a result that is robust to alternative assumptions about rates of return and that mirrors findings in the annuitization literature. But households appear overwhelmingly to choose the line of credit option. Their reluctance to take the lifetime-income option mirrors the reluctance of households to annuitize their financial wealth. It is an open question as to why this is the case. The presence of bequest motives is one possibility, but even in this case there are better alternative to the behaviors typically exhibited. The failure of households to understand either annuities or the lifetime-income option in reverse mortgages may be playing an important role.

The consequences of this misunderstanding for household welfare are huge. Our simulations show that for plausible parameter values a household would require a 24-percent increase in total wealth in order to compensate it for having to convert its reversionary interest into a line of credit when it had run through its financial wealth rather than converting its reversionary income into a lifetime income at the start of retirement. Policies to educate households regarding the advantages of using a reverse

mortgage to generate a lifetime income near the start of retirement have the potential to greatly increase retirees' welfare.

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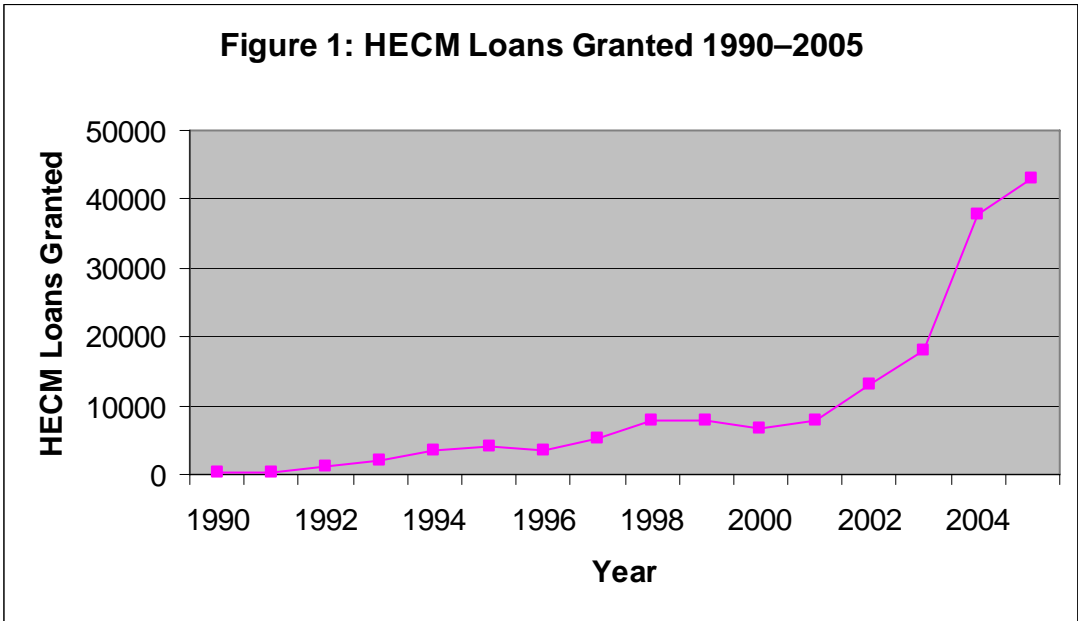
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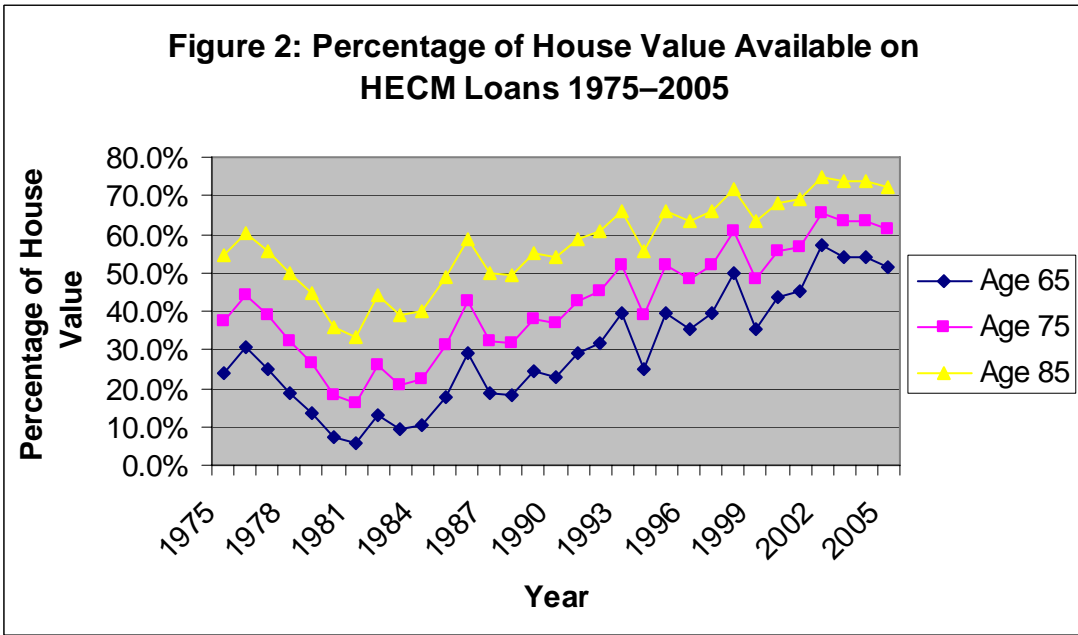
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Source: National Reverse Mortgage Lenders Association/U.S. Department of Housing and Urban Development, Fiscal Year October 1–September 30



Notes: 1) This figure assumes a \$200,000 house, a 1.5-percentage point lender’s margin, and the closing cost estimates used by the AARP in their online reverse mortgage loan calculator. These closing costs are an origination fee and a mortgage insurance premium each equal to 2 percent of the home’s appraised value, miscellaneous closing costs of \$2,074, and a servicing fee of \$30 per month. 2) HECM loans have been available only since 1990, so amounts for 1975 to 1989 represent the percentages that could have been borrowed had they been available.

Figure 3 (a): Mean Returns to Alternative Decumulation Strategies, Constant Relative Risk Aversion Coefficient=5

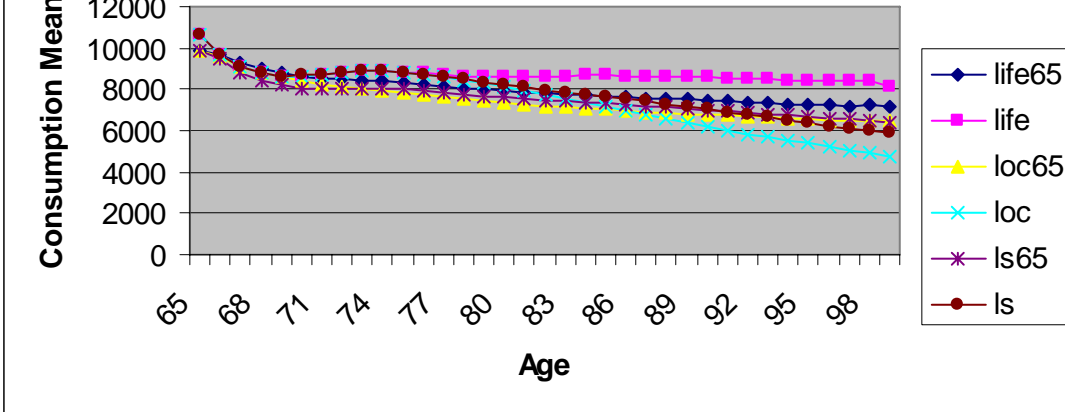
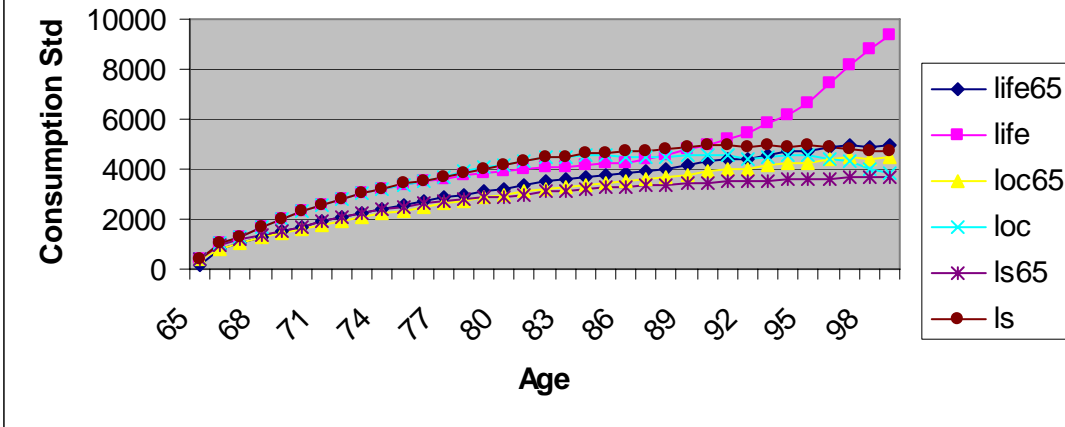


Figure 3 (b): Standard Deviation of Returns to Alternative Decumulation Strategies, Constant Relative Risk Aversion Coefficient=5



Notes: In the legends, “life” refers to taking a lifetime income, “loc” refers to taking a line of credit, and “ls” refers to taking a lump sum. A suffix of “65” means that the strategy is taken at age 65, while no suffix means that the strategy is taken when financial wealth is exhausted (that is, at a delay).

Table 1: Real Returns on Housing and Financial Assets 1975-2005

| | Mean | Standard Deviation |
|---------------------------------|-------|--------------------|
| One Year Treasury Bill | 2.2% | 2.0% |
| Ten Year Treasury Bond | 4.4% | 10.3% |
| S&P 500 incl dividends | 9.2% | 15.5% |
| Housing - Capital Return | 1.9% | 3.7% |
| Housing - Reversionary Interest | | |
| Postpone from 65 to 66 | 16.0% | 40.6% |
| Postpone from 75 to 76 | 10.2% | 23.4% |
| Postpone from 85 to 86 | 6.9% | 13.5% |

Table 2: Comparison of Historical and Simulated Data

| | | One year yield | Ten year yield | Stock return | House price | Inflation | real GDP growth |
|--------------------------|------------------|-------------------|-------------------|-----------------|----------------|-----------|--------------------|
| 1975-2005 | Mean | 6.593 | 7.610 | 9.178 | 1.880 | 4.359 | 3.195 |
| | SD | 3.292 | 2.656 | 15.483 | 3.715 | 2.980 | 2.027 |
| 2006-2040 | Mean | 6.677 | 7.471 | 9.349 | 2.067 | 4.334 | 2.842 |
| | SD | 2.938 | 2.287 | 16.884 | 3.559 | 2.599 | 2.184 |
| Historical correlation | | | | | | | |
| | One year yield | 1.000 | | | | | |
| | Ten year yield | 0.924 | 1.000 | | | | |
| | Stock return | -0.006 | -0.038 | 1.000 | | | |
| | House price appn | -0.574 | -0.632 | -0.055 | 1.000 | | |
| | Inflation | 0.790 | 0.663 | -0.223 | -0.452 | 1.000 | |
| | real GDP growth | -0.056 | -0.050 | 0.133 | 0.323 | -0.194 | 1.000 |
| Simulated correlation | | | | | | | |
| | One year yield | 1.000 | | | | | |
| | Ten year yield | 0.787 | 1.000 | | | | |
| | Stock return | -0.138 | -0.217 | 1.000 | | | |
| | House price appn | -0.845 | -0.979 | 0.226 | 1.000 | | |
| | Inflation | 0.065 | -0.545 | 0.092 | 0.415 | 1.000 | |
| | real GDP growth | 0.135 | 0.712 | -0.179 | -0.608 | -0.946 | 1.000 |
| * before management cost | | | | | | | |

Table 3: Results - Base Case

| Strategy | Coefficient of risk aversion | 2 | 3 | 4 | 5 |
|---|------------------------------|-------|-------|-------|-------|
| Lump sum at age | 65 Mean income | 11868 | 11037 | 8785 | 7570 |
| | Standard deviation | 9489 | 7915 | 4380 | 3013 |
| | Investment allocation | 100 | 93 | 70 | 55 |
| | 70 Equivalent wealth | 0.846 | 0.886 | 0.913 | 0.914 |
| | Mean income | 8440 | 7767 | 7420 | 7195 |
| | Standard deviation | 4764 | 3621 | 3211 | 3019 |
| | Investment allocation at 65 | 100 | 100 | 100 | 100 |
| | Investment allocation at 70 | 79 | 61 | 51 | 44 |
| | 75 Equivalent wealth | 0.865 | 0.892 | 0.888 | 0.846 |
| | Mean income | 8943 | 8295 | 7974 | 7780 |
| | Standard deviation | 5697 | 4348 | 3862 | 3632 |
| | Investment allocation at 65 | 100 | 100 | 100 | 100 |
| | Investment allocation at 75 | 93 | 73 | 62 | 55 |
| | 80 Equivalent wealth | 0.876 | 0.912 | 0.915 | 0.876 |
| | Mean income | 9090 | 8699 | 8496 | 8328 |
| | Standard deviation | 5009 | 4202 | 3896 | 3695 |
| | Investment allocation at 65 | 100 | 100 | 100 | 99 |
| | Investment allocation at 80 | 90 | 71 | 60 | 53 |
| | 85 Equivalent wealth | 0.882 | 0.922 | 0.927 | 0.888 |
| | Mean income | 9794 | 9495 | 9343 | 9220 |
| | Standard deviation | 5118 | 4414 | 4138 | 3955 |
| | Investment allocation at 65 | 100 | 100 | 100 | 99 |
| | Investment allocation at 85 | 91 | 72 | 62 | 56 |
| Lump sum when financial wealth exhasuted | Equivalent wealth | 0.883 | 0.922 | 0.928 | 0.889 |
| | Mean income | 8381 | 8193 | 8100 | 7995 |
| | Standard deviation | 4276 | 4160 | 4148 | 4072 |
| | Investment allocation at 65 | 100 | 100 | 100 | 99 |
| Line of credit at | 65 Equivalent wealth | 0.922 | 0.967 | 0.981 | 0.981 |
| | Mean income | 9267 | 9267 | 8801 | 7803 |
| | Standard deviation | 5017 | 5017 | 4221 | 2856 |
| | Investment allocation at 65 | 100 | 100 | 89 | 71 |
| Line of credit when financial wealth exhasuted | Equivalent wealth | 0.853 | 0.888 | 0.894 | 0.872 |
| | Mean income | 8169 | 8169 | 8127 | 8127 |
| | Standard deviation | 3805 | 3805 | 3745 | 3745 |
| | Investment allocation at 65 | 100 | 100 | 100 | 90 |
| Lifetime income at | 65 Equivalent wealth | 0.959 | 1.020 | 1.054 | 1.067 |
| | Mean income | 9576 | 9576 | 9340 | 8260 |
| | Standard deviation | 5045 | 5045 | 4628 | 3038 |
| | Investment allocation at 65 | 100 | 100 | 97 | 77 |
| Lifetime income when financial wealth exhausted | Equivalent wealth | 0.924 | 0.997 | 1.054 | 1.077 |
| | Mean income | 8855 | 8855 | 8855 | 8815 |
| | Standard deviation | 3783 | 3783 | 3783 | 3714 |
| | Investment allocation at 65 | 100 | 100 | 100 | 100 |

Table 4: Comparison of Reverse Mortgage Equivalent Wealth

| Strategy | Risk aversion | Lump sum | | Lifetime income | | Line of credit | |
|--------------------------------|---------------|-----------|--------------|-----------------|--------------|----------------|--------------|
| | | exhausted | 65 exhausted | 65 exhausted | 65 exhausted | 65 exhausted | 65 exhausted |
| Base case | 2 | 0.883 | 0.959 | 0.924 | 0.922 | 0.853 | |
| | 3 | 0.922 | 1.020 | 0.997 | 0.967 | 0.888 | |
| | 4 | 0.928 | 1.054 | 1.054 | 0.981 | 0.894 | |
| | 5 | 0.889 | 1.067 | 1.077 | 0.981 | 0.872 | |
| House return +2% | 2 | 0.952 | 0.959 | 0.997 | 0.922 | 0.915 | |
| | 3 | 1.006 | 1.020 | 1.083 | 0.967 | 0.967 | |
| | 4 | 1.027 | 1.054 | 1.156 | 0.981 | 0.987 | |
| | 5 | 0.998 | 1.067 | 1.200 | 0.981 | 0.964 | |
| House return -2% | 2 | 0.815 | 0.959 | 0.852 | 0.922 | 0.791 | |
| | 3 | 0.833 | 1.020 | 0.906 | 0.967 | 0.804 | |
| | 4 | 0.813 | 1.054 | 0.942 | 0.981 | 0.786 | |
| | 5 | 0.755 | 1.067 | 0.943 | 0.981 | 0.750 | |
| Stock return -3% | 2 | 0.991 | 1.047 | 1.090 | 0.985 | 0.978 | |
| | 3 | 0.978 | 1.067 | 1.142 | 0.989 | 0.979 | |
| | 4 | 0.916 | 1.079 | 1.164 | 0.986 | 0.951 | |
| | 5 | 0.843 | 1.090 | 1.177 | 0.979 | 0.927 | |
| 5% withdrawal rate | 2 | 0.919 | 1.021 | 0.941 | 0.931 | 0.911 | |
| | 3 | 0.968 | 1.075 | 0.994 | 0.976 | 0.959 | |
| | 4 | 1.002 | 1.108 | 1.038 | 1.006 | 0.992 | |
| | 5 | 1.006 | 1.113 | 1.058 | 1.017 | 0.997 | |
| 10% withdrawal rate | 2 | 0.825 | 0.958 | 0.843 | 0.909 | 0.754 | |
| | 3 | 0.830 | 1.071 | 0.932 | 0.946 | 0.750 | |
| | 4 | 0.791 | 1.168 | 1.026 | 0.940 | 0.723 | |
| | 5 | 0.724 | 1.243 | 1.108 | 0.930 | 0.688 | |
| 15% housing standard deviation | 2 | 0.849 | 0.959 | 0.885 | 0.922 | 0.823 | |
| | 3 | 0.848 | 1.020 | 0.918 | 0.967 | 0.820 | |
| | 4 | 0.782 | 1.054 | 0.914 | 0.981 | 0.764 | |
| | 5 | 0.659 | 1.067 | 0.861 | 0.981 | 0.672 | |

Note: Gray shading indicates that reverse-mortgage equivalent wealth is greater than 1.00.

Table 5: Households Attaining Age 65 2026-2046

| Strategy | Coefficient of risk aversion | 2 | 3 | 4 | 5 | |
|-----------------------------|---|-------------------|-------|-------|-------|-------|
| Lump sum at age 65 | Mean income | 9731 | 8228 | 7516 | 7030 | |
| | Standard deviation | 7047 | 4194 | 3178 | 2641 | |
| | Investment allocation | 93 | 71 | 59 | 50 | |
| | 70 Equivalent wealth | 1.023 | 1.021 | 1.005 | 0.983 | |
| | Mean income | 9364 | 8318 | 7718 | 7319 | |
| | Standard deviation | 6227 | 4217 | 3393 | 2965 | |
| | Investment allocation at 65 | 100 | 100 | 92 | 84 | |
| | Investment allocation at 70 | 92 | 70 | 58 | 50 | |
| | 75 Equivalent wealth | 1.020 | 1.010 | 0.978 | 0.930 | |
| | Mean income | 9096 | 8224 | 7583 | 7226 | |
| | Standard deviation | 5870 | 4147 | 3272 | 2912 | |
| | Investment allocation at 65 | 100 | 92 | 76 | 68 | |
| | Investment allocation at 75 | 94 | 72 | 60 | 51 | |
| | 80 Equivalent wealth | 1.033 | 1.031 | 1.008 | 0.966 | |
| | Mean income | 9430 | 8748 | 8163 | 8020 | |
| | Standard deviation | 5502 | 4141 | 3378 | 3196 | |
| | Investment allocation at 65 | 100 | 92 | 76 | 76 | |
| | Investment allocation at 80 | 95 | 74 | 63 | 56 | |
| | 85 Equivalent wealth | 1.041 | 1.041 | 1.019 | 0.975 | |
| | Mean income | 9898 | 9401 | 8970 | 8702 | |
| | Standard deviation | 5263 | 4219 | 3598 | 3291 | |
| | Investment allocation at 65 | 100 | 92 | 76 | 65 | |
| | Investment allocation at 85 | 93 | 73 | 63 | 56 | |
| | Lump sum when financial wealth exhasuted | Equivalent wealth | 1.041 | 1.042 | 1.021 | 0.976 |
| | Mean income | 8471 | 8038 | 7543 | 7234 | |
| Standard deviation | 4316 | 3722 | 3072 | 2805 | | |
| Investment allocation at 65 | 100 | 92 | 76 | 65 | | |
| Line of credit at 65 | Equivalent wealth | 0.993 | 1.017 | 1.046 | 1.080 | |
| | Mean income | 8746 | 7791 | 7350 | 7066 | |
| | Standard deviation | 4849 | 2987 | 2381 | 2098 | |
| | Investment allocation at 65 | 100 | 77 | 64 | 55 | |
| | Line of credit when financial wealth exhasuted | Equivalent wealth | 1.004 | 1.008 | 0.996 | 0.978 |
| Mean income | | 8230 | 7884 | 7693 | 7471 | |
| Standard deviation | | 3879 | 3253 | 2989 | 2760 | |
| Investment allocation at 65 | | 100 | 86 | 78 | 65 | |
| Lifetime income at 65 | | Equivalent wealth | 1.028 | 1.052 | 1.080 | 1.115 |
| | Mean income | 9027 | 8169 | 7681 | 7389 | |
| | Standard deviation | 5043 | 3303 | 2580 | 2262 | |
| | Investment allocation at 65 | 100 | 78 | 64 | 55 | |
| | Lifetime income when financial wealth exhausted | Equivalent wealth | 1.088 | 1.130 | 1.166 | 1.209 |
| Mean income | | 8907 | 8838 | 8433 | 8213 | |
| Standard deviation | | 3942 | 3748 | 3026 | 2670 | |
| Investment allocation at 65 | | 100 | 97 | 85 | 76 | |

Appendix Table 1: VAR Estimation Results

| | Dep Var = one year interest rate | | Dep Var = 10 year interest rate | | Dep Var = real housing return | | Dep Var = inflation | | Dep Var = real GDP growth | |
|---------------------------|-------------------------------------|-------|------------------------------------|-------|----------------------------------|-------|------------------------|-------|------------------------------|-------|
| | Coeff | SE | Coeff | SE | Coeff | SE | Coeff | SE | Coeff | SE |
| One Year rate | | | | | | | | | | |
| L1. | 0.336 | 0.159 | -0.217 | 0.096 | -1.073 | 0.487 | 0.572 | 0.288 | -0.170 | 0.421 |
| L2. | 0.068 | 0.188 | 0.060 | 0.114 | 1.165 | 0.576 | -0.387 | 0.341 | -1.241 | 0.498 |
| L3. | 0.412 | 0.195 | 0.090 | 0.118 | -0.749 | 0.597 | -0.013 | 0.353 | 0.344 | 0.516 |
| L4. | 0.261 | 0.197 | 0.382 | 0.119 | 0.053 | 0.603 | 0.013 | 0.357 | 1.539 | 0.521 |
| L5. | -0.078 | 0.204 | -0.086 | 0.123 | 0.171 | 0.623 | -0.371 | 0.369 | -1.280 | 0.539 |
| L6. | 0.124 | 0.199 | 0.142 | 0.120 | -0.966 | 0.608 | 0.889 | 0.360 | 0.443 | 0.526 |
| L7. | -0.321 | 0.197 | -0.337 | 0.119 | 0.506 | 0.603 | -0.344 | 0.357 | -0.448 | 0.521 |
| L8. | -0.090 | 0.174 | -0.022 | 0.105 | 0.337 | 0.532 | -0.127 | 0.315 | 0.653 | 0.460 |
| Ten Year rate | | | | | | | | | | |
| L1. | 0.041 | 0.258 | 0.861 | 0.156 | -1.128 | 0.791 | -0.003 | 0.468 | 0.036 | 0.683 |
| L2. | -0.022 | 0.334 | -0.041 | 0.202 | 1.263 | 1.023 | -0.469 | 0.605 | 1.206 | 0.884 |
| L3. | 0.163 | 0.328 | 0.177 | 0.199 | 0.455 | 1.005 | 0.205 | 0.595 | -1.192 | 0.869 |
| L4. | -0.321 | 0.328 | -0.498 | 0.198 | 2.075 | 1.004 | -0.919 | 0.594 | -1.679 | 0.868 |
| L5. | -0.181 | 0.334 | -0.049 | 0.202 | -3.398 | 1.022 | 1.689 | 0.604 | 1.275 | 0.883 |
| L6. | -0.135 | 0.347 | -0.057 | 0.210 | 2.035 | 1.062 | -1.738 | 0.629 | -0.412 | 0.918 |
| L7. | 0.111 | 0.352 | 0.125 | 0.213 | -1.148 | 1.078 | 0.865 | 0.638 | 0.467 | 0.932 |
| L8. | 0.405 | 0.274 | 0.340 | 0.166 | 0.104 | 0.839 | 0.096 | 0.496 | -0.474 | 0.725 |
| House capital gain | | | | | | | | | | |
| L1. | 0.013 | 0.039 | 0.020 | 0.024 | 0.381 | 0.120 | 0.001 | 0.071 | 0.218 | 0.104 |
| L2. | -0.006 | 0.041 | 0.006 | 0.025 | -0.057 | 0.124 | -0.010 | 0.074 | -0.010 | 0.107 |
| L3. | 0.014 | 0.039 | 0.006 | 0.023 | 0.506 | 0.118 | -0.074 | 0.070 | 0.026 | 0.102 |
| L4. | 0.043 | 0.040 | 0.011 | 0.024 | -0.286 | 0.124 | 0.288 | 0.073 | 0.197 | 0.107 |
| L5. | -0.078 | 0.040 | -0.035 | 0.024 | 0.206 | 0.124 | -0.111 | 0.073 | -0.035 | 0.107 |
| L6. | -0.017 | 0.044 | -0.057 | 0.026 | 0.109 | 0.133 | -0.018 | 0.079 | -0.206 | 0.115 |
| L7. | 0.034 | 0.037 | 0.006 | 0.023 | -0.300 | 0.114 | 0.118 | 0.068 | -0.185 | 0.099 |
| L8. | 0.062 | 0.038 | 0.063 | 0.023 | 0.267 | 0.117 | -0.060 | 0.069 | 0.035 | 0.101 |
| CPI | | | | | | | | | | |
| L1. | 0.052 | 0.071 | 0.039 | 0.043 | 0.124 | 0.218 | 0.152 | 0.129 | 0.303 | 0.188 |
| L2. | 0.078 | 0.071 | 0.078 | 0.043 | -0.207 | 0.216 | 0.050 | 0.128 | 0.070 | 0.187 |
| L3. | 0.022 | 0.068 | 0.024 | 0.041 | 0.211 | 0.209 | 0.316 | 0.124 | -0.016 | 0.180 |
| L4. | 0.069 | 0.071 | 0.019 | 0.043 | -0.501 | 0.218 | 0.396 | 0.129 | 0.258 | 0.188 |
| L5. | -0.067 | 0.070 | -0.025 | 0.042 | 0.119 | 0.213 | 0.082 | 0.126 | 0.022 | 0.184 |
| L6. | 0.055 | 0.067 | -0.003 | 0.041 | 0.109 | 0.205 | 0.022 | 0.122 | -0.124 | 0.177 |
| L7. | 0.086 | 0.067 | 0.033 | 0.041 | -0.166 | 0.206 | 0.075 | 0.122 | -0.377 | 0.178 |
| L8. | 0.098 | 0.069 | 0.051 | 0.041 | 0.532 | 0.210 | -0.131 | 0.124 | -0.063 | 0.181 |
| GDP growth | | | | | | | | | | |
| L1. | 0.111 | 0.041 | 0.046 | 0.025 | 0.134 | 0.126 | -0.001 | 0.075 | 0.134 | 0.109 |
| L2. | 0.115 | 0.041 | 0.059 | 0.025 | 0.058 | 0.124 | 0.078 | 0.073 | 0.187 | 0.107 |
| L3. | 0.000 | 0.041 | 0.022 | 0.025 | 0.208 | 0.127 | -0.004 | 0.075 | -0.039 | 0.109 |
| L4. | -0.006 | 0.041 | -0.023 | 0.025 | -0.372 | 0.126 | 0.195 | 0.075 | -0.005 | 0.109 |
| L5. | 0.009 | 0.039 | -0.039 | 0.023 | 0.203 | 0.119 | 0.038 | 0.070 | 0.047 | 0.103 |
| L6. | -0.037 | 0.039 | -0.031 | 0.023 | -0.010 | 0.119 | 0.024 | 0.070 | -0.040 | 0.103 |
| L7. | 0.050 | 0.036 | 0.024 | 0.022 | 0.102 | 0.109 | 0.099 | 0.065 | -0.003 | 0.094 |
| L8. | -0.114 | 0.031 | -0.072 | 0.019 | 0.132 | 0.094 | -0.171 | 0.056 | -0.197 | 0.081 |
| Stock return | | | | | | | | | | |
| L1. | 0.002 | 0.003 | 0.001 | 0.002 | -0.002 | 0.009 | -0.001 | 0.006 | 0.017 | 0.008 |
| L2. | 0.009 | 0.003 | 0.004 | 0.002 | 0.017 | 0.009 | -0.004 | 0.005 | 0.029 | 0.008 |
| L3. | -0.001 | 0.003 | -0.003 | 0.002 | 0.000 | 0.009 | 0.009 | 0.006 | 0.008 | 0.008 |
| L4. | 0.002 | 0.003 | 0.001 | 0.002 | -0.006 | 0.010 | 0.000 | 0.006 | -0.001 | 0.008 |
| L5. | -0.002 | 0.003 | -0.003 | 0.002 | -0.008 | 0.010 | 0.005 | 0.006 | -0.015 | 0.008 |
| L6. | 0.007 | 0.003 | 0.005 | 0.002 | 0.003 | 0.009 | -0.014 | 0.006 | 0.007 | 0.008 |
| L7. | 0.006 | 0.003 | 0.004 | 0.002 | 0.010 | 0.009 | -0.008 | 0.006 | -0.003 | 0.008 |
| L8. | -0.001 | 0.003 | 0.000 | 0.002 | 0.012 | 0.009 | -0.019 | 0.005 | 0.000 | 0.008 |
| Constant | -0.231 | 0.121 | -0.003 | 0.073 | -0.118 | 0.371 | -0.019 | 0.220 | 0.325 | 0.321 |