

A Service of

ZBW

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Lin, Carl

Working Paper Decomposing excess returns in stochastic linear models

IZA Discussion Papers, No. 6237

**Provided in Cooperation with:** IZA – Institute of Labor Economics

*Suggested Citation:* Lin, Carl (2011) : Decomposing excess returns in stochastic linear models, IZA Discussion Papers, No. 6237, Institute for the Study of Labor (IZA), Bonn, https://nbn-resolving.de/urn:nbn:de:101:1-201202285068

This Version is available at: https://hdl.handle.net/10419/58700

#### Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

#### Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



# WWW.ECONSTOR.EU

IZA DP No. 6237

# Decomposing Excess Returns in Stochastic Linear Models

Carl Lin

December 2011

Forschungsinstitut zur Zukunft der Arbeit Institute for the Study of Labor

# Decomposing Excess Returns in Stochastic Linear Models

### **Carl Lin**

Beijing Normal University and IZA

Discussion Paper No. 6237 December 2011

IZA

P.O. Box 7240 53072 Bonn Germany

Phone: +49-228-3894-0 Fax: +49-228-3894-180 E-mail: iza@iza.org

Any opinions expressed here are those of the author(s) and not those of IZA. Research published in this series may include views on policy, but the institute itself takes no institutional policy positions.

The Institute for the Study of Labor (IZA) in Bonn is a local and virtual international research center and a place of communication between science, politics and business. IZA is an independent nonprofit organization supported by Deutsche Post Foundation. The center is associated with the University of Bonn and offers a stimulating research environment through its international network, workshops and conferences, data service, project support, research visits and doctoral program. IZA engages in (i) original and internationally competitive research in all fields of labor economics, (ii) development of policy concepts, and (iii) dissemination of research results and concepts to the interested public.

IZA Discussion Papers often represent preliminary work and are circulated to encourage discussion. Citation of such a paper should account for its provisional character. A revised version may be available directly from the author.

IZA Discussion Paper No. 6237 December 2011

# ABSTRACT

## **Decomposing Excess Returns in Stochastic Linear Models**

We present a theorem helpful in estimating the mean and variance of a linear function with arbitrary multivariate randomness in its coefficients and variables. We derive a generalized decomposition result from two random linear functions in which the result can be applied to most models using event study analysis. Taking the 1989 minimum wage hike as an example, we found that the apparent lack of an effect is a consequence of two off-setting forces: 1) a negative effect arising from firm-specific traits and 2) a positive effect arising from market performance. In sum, we bring to the analysis a method that helps provide additional insights and can be applied to much of the work using event study.

JEL Classification: G14, J31, J38

Keywords: excess returns, minimum wage, decomposition, event study

Corresponding author:

Carl Lin School of Economics and Business Administration Beijing Normal University No.19 Xin Jie Kou Outer St. Haidian Dist. Beijing 100875 China E-mail: csmlin@bnu.edu.cn

#### 1. Introduction

Social scientists are often interested in the economic effects of events and government regulations. Does a regulation confer net benefits on consumers at the expense of regulated firms? Do regulated firms receive net benefits at the expense of consumers? Our interest is in making predictions about the effects of event or regulation on the value of the regulated firms.

The effects of these events/regulations on firms are typically obtained by estimating excess returns arising from the events/regulations. Often these models contain linear functions in which some components (coefficients or variables) are random. To accommodate randomness in the components, several simplifying assumptions (e.g. normality, independence of observed data) are made. Brown and Rutemiller (1977) state that when the coefficients or variables are known to be random in a general and multivariate fashion, concise specification of the randomness exhibited by the linear function is, at best, extremely complicated, usually requiring severe and unrealistic restrictions on the density functions of the random components.

This paper presents a method of estimating the mean and variance of a linear function with arbitrary multivariate randomness in its coefficients and variables. A generalized decomposition result derived from the differential between two random linear functions can be applied to models (single or multi-factor) in an event study. As an example, we take the market model in Card and Krueger (1995) showing this new approach helps provide additional insights. Section 2 presents the theoretical framework. In section 3 we provide an example to show how to apply the generalized method to decompose two random linear functions. Section 4 is the concluding remarks.

#### 2. Theoretical Framework

#### 2.1. The Means and Variances of Two Random Vectors

**Lemma:** Let V be an inner product space over  $F = \mathbb{R}$  where an inner product on V is a function  $\langle , \rangle \colon V \times V \to F$ . Also let X,  $Y \subseteq V$  be two random n-element vectors. Denote the expectation of these vectors as  $E\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \overline{X} \\ \overline{Y} \end{pmatrix}$  and  $e = \begin{pmatrix} e_X \\ e_Y \end{pmatrix}$  is the deviation of each

random vector from its expectation with the variance-covariance matrix

$$E\left\langle e, e^{T} \right\rangle = \begin{pmatrix} e_{X} \\ e_{Y} \end{pmatrix} \begin{pmatrix} e_{X} \\ e_{Y} \end{pmatrix}^{T} = \begin{pmatrix} \sum_{XX} & \sum_{XY} \\ \sum_{YX} & \sum_{YY} \end{pmatrix}.$$
  
If  $Z = \left\langle X^{T}, Y \right\rangle$ , then  
 $Z = \left\langle X^{T}, Y \right\rangle = \left\langle (\overline{X} + e_{X})^{T}, (\overline{Y} + e_{Y}) \right\rangle = \overline{X}^{T} \overline{Y} + \overline{X}^{T} e_{Y} + e_{X}^{T} \overline{Y} + e_{X}^{T} e_{Y}.$ 

The next result plays a key role in the later sections.

**Theorem:** If V is an inner product space and  $Z = \langle X^T, Y \rangle \subseteq V$ , then

1)

$$E(Z) = \overline{X}^T \overline{Y} + tr(\Sigma_{XY}),$$

2)

$$V(Z) = E(e_{X}^{T}e_{Y}e_{Y}^{T}e_{X}) + \mu^{T} \sum \mu + 2\bar{X}_{Y}^{T}E(e_{X}e_{X}^{T}e_{Y}) + 2\bar{X}_{X}^{T}E(e_{Y}e_{Y}^{T}e_{X}) - (tr(\sum_{XY}))^{2},$$

where  $\mu^T \sum \mu = \overline{X}^T \sum_{YY} \overline{X} + \overline{Y}^T \sum_{XX} \overline{Y} + 2\overline{X}^T \sum_{YX} \overline{Y}$ .

 $\begin{aligned} & \textbf{Proof. Since } E(e_{X}) = 0 \ \text{and } E(e_{Y}) = 0, \ \text{then} \\ & E(Z) = E(\overline{X}^{T}\overline{Y} + \overline{X}^{T}e_{Y} + e_{X}^{T}\overline{Y} + e_{X}^{T}e_{Y}) = \overline{X}^{T}\overline{Y} + tr(\Sigma_{XY}). \end{aligned}$   $\begin{aligned} & \text{Note that } V(Z) = E(Z^{2}) - (E(Z))^{2}, \\ & E(Z^{2}) = E[\left\langle X^{T}, Y \right\rangle]^{2} \\ & = E[\left\langle (\overline{X} + e_{X})^{T}, (\overline{Y} + e_{Y}) \right\rangle]^{2} \\ & = E[e_{X}^{T}e_{Y}e_{Y}^{T}e_{X} + e_{X}^{T}e_{Y}\overline{Y}^{T}e_{X} + e_{X}^{T}e_{Y}e_{Y}^{T}\overline{X} + e_{X}^{T}\overline{Y}e_{Y}^{T}\overline{X} + e_{X}^{T}\overline{Y}\overline{Y}^{T}e_{X} \\ & + e_{X}^{T}\overline{Y}e_{Y}^{T}\overline{X} + e_{X}^{T}\overline{Y}\overline{Y}^{T}\overline{X} + \overline{X}^{T}e_{Y}e_{Y}^{T}\overline{X} + \overline{X}^{T}e_{Y}\overline{Y}^{T}e_{X} + \overline{X}^{T}e_{Y}e_{Y}^{T}\overline{X}] \\ & = E[e_{X}^{T}e_{Y}e_{Y}^{T}e_{X} + \overline{X}^{T}\overline{Y}\overline{Y}^{T}e_{X} + \overline{X}^{T}\overline{Y}e_{Y}^{T}\overline{X} + \overline{X}^{T}e_{Y}e_{Y}^{T}\overline{X}] \\ & = E[e_{X}^{T}\overline{Y}e_{Y}e_{X} + \overline{X}^{T}\overline{Y}\overline{Y}\overline{Y}^{T}e_{X} + \overline{X}^{T}\overline{Y}e_{X}^{T}\overline{Y} + (\overline{X}^{T}\overline{Y})^{2} + 2\overline{X}^{T}e_{Y}e_{Y}^{T}e_{X} + 2\overline{Y}^{T}e_{X}e_{X}^{T}e_{Y} \\ & \quad + 2\overline{X}^{T}\overline{Y}e_{X}^{T}e_{X} + \overline{X}^{T}e_{Y}e_{X}^{T}\overline{Y} + 2\overline{Y}^{T}e_{X}\overline{X}^{T}\overline{Y} + 2\overline{X}^{T}e_{Y}\overline{X}^{T}\overline{Y}], \end{aligned}$ and  $E\left\langle e_{Y}, e_{Y}^{T}\right\rangle = \sum_{YY}, E\left\langle e_{X}, e_{X}^{T}\right\rangle = \sum_{XX}, E\left\langle e_{X}^{T}, e_{Y}\right\rangle = tr(\Sigma_{XY}). \end{aligned}$ 

Therefore,

$$\begin{split} E(Z^2) &= E[\left\langle X^T, Y \right\rangle]^2 \\ &= E(e_x^T e_y e_y^T e_x) + \bar{X}^T \sum_{YY} \bar{X} + \bar{Y}^T \sum_{XX} \bar{Y} + (\bar{X}^T \bar{Y})^2 + 2\bar{X}^T E(e_y e_y^T e_x) + 2\bar{Y}^T E(e_x e_x^T e_y) \\ &+ 2\bar{X}^T \bar{Y} tr(\sum_{XY}) + 2\bar{X}^T \sum_{YX} \bar{Y}. \end{split}$$

Let  $\mu^T \sum \mu = \overline{X}^T \sum_{YY} \overline{X} + \overline{Y}^T \sum_{XX} \overline{Y} + 2\overline{X}^T \sum_{YX} \overline{Y}$ , we have

$$\begin{split} V(Z) &= E(Z^{2}) - (E(Z))^{2} \\ &= [E(e_{X}^{T}e_{Y}e_{Y}^{T}e_{X}) + \overline{X}^{T}\sum_{YY}\overline{X} + \overline{Y}^{T}\sum_{XX}\overline{Y} + (\overline{X}^{T}\overline{Y})^{2} + 2\overline{X}^{T}E(e_{Y}e_{Y}^{T}e_{X}) + 2\overline{Y}^{T}E(e_{X}e_{X}^{T}e_{Y}) \\ &+ 2\overline{X}^{T}\overline{Y}tr(\sum_{XY}) + 2\overline{X}^{T}\sum_{YX}\overline{Y}] - [(\overline{X}^{T}\overline{Y})^{2} + 2\overline{X}^{T}\overline{Y}tr(\sum_{XY}) + (tr(\sum_{XY}))^{2}] \\ &= E(e_{X}^{T}e_{Y}e_{Y}^{T}e_{X}) + 2\overline{X}^{T}E(e_{Y}e_{Y}^{T}e_{X}) + 2\overline{Y}^{T}E(e_{X}e_{X}^{T}e_{Y}) + \mu^{T}\sum\mu - (tr(\sum_{XY}))^{2}. \end{split}$$

The corollary next are special cases of the theorem. Three common cases that researchers often encounter are introduced.

**Corollary:** 1) If  $e_x$  and  $e_y$  are stochastically independent, the elements of  $e_x$  are correlated with one another, the elements of  $e_y$  are correlated with one another, then we have

$$E(Z) = \overline{X}^T \overline{Y},$$

and

$$V(Z) = \overline{X}^T \sum_{YY} \overline{X} + \overline{Y}^T \sum_{XX} \overline{Y} + tr(\sum_{XX} \sum_{YY}).$$

2) If  $e_x$ ,  $e_y$  and all elements of  $e_x$  and  $e_y$  are stochastically independent, then

$$V(Z) = \overline{X}^T \sum_{YY} \overline{X} + \overline{Y}^T \sum_{XX} \overline{Y} + tr(\sum_{XX} \sum_{YY}).$$

where  $\sum_{XX}$  and  $\sum_{YY}$  are diagonal matrices.

3) If 
$$e_x$$
 and  $e_y$  are correlated,  $e_x \sim N(0, \Sigma_x)$  and  $e_y \sim N(0, \Sigma_y)$ , then  

$$V(Z) = \mu^T \Sigma \mu + tr(\Sigma_{XX} \Sigma_{YY}) + (tr(\Sigma_{XY}))^2.$$

Next section we apply the theorem to decompose the differential between two random linear functions.

#### 2.2. Decomposing Excess Returns

Decomposition techniques for linear regression models have been used for many decades. As stated in Powers, Yoshida and Yun (2011), decomposition is widely used in social research to quantify the contributions to group differences in average predictions from regression models. The technique utilizes the output from regression models to parcel out components of a group difference in a statistic (such as a mean or proportion) which can be attributed to differences between groups (i.e., differences in characteristics, endowments, or attributes) and to differences in the effects of characteristics (i.e., differences in the returns, coefficients, or behavioral responses). Applying the theorem from the previous section, we introduce a method allowing decomposition of a firm's excess return into differences in economy-wide and individualistic factors.

To illustrate the idea, Figure 1 shows the time line of estimation and event windows which are used to decompose excess returns of a firm in an event study.  $T_1$  to  $T_2$  is the "estimation window" which is used to estimate the normal performance stock return of a firm,  $\hat{R}_i^N$ .  $T_2 + 1$  to  $T_3$  is the "event window" and  $T_0$  is the event day.  $\hat{R}_i^P$  is the predicted return of a firm's stock from  $T_0 + 1$  to  $T_3$  after the event day.

Figure 1 Time Line of Decomposing Excess Returns in an Event Study



Suppose the return-generating process is  $R = X^T \beta + e$ , the equation identifies the return R is a linear combination of X and  $\beta$ . X is a vector containing the predictors and  $\beta$  contains the slope parameters and the intercept, e is the error term. In the estimation and event windows, the return-generating processes can be written as:

$$R_N = X_N^T \beta_N + e_N$$

$$R_P = X_P^T \beta_P + e_P,$$
(1)

where N denotes normal performance return and P denotes predicted return.

Under linear regression model, the estimated and predicted returns can be obtained as:

$$\hat{R}_{N} = \overline{X}_{N}^{T} \hat{\beta}_{N}$$

$$\hat{R}_{P} = \overline{X}_{P}^{T} \hat{\beta}_{P}.$$
(2)

The mean difference between predicted return and normal return is called excess return AR can be decomposed as

$$AR = \hat{R}_{p} - \hat{R}_{N}$$

$$= \overline{X}_{p}^{T} \hat{\beta}_{p} - \overline{X}_{N}^{T} \hat{\beta}_{N}$$

$$= (\overline{X}_{p}^{T} \hat{\beta}_{p} - \overline{X}_{N}^{T} \hat{\beta}_{p}) + (\overline{X}_{N}^{T} \hat{\beta}_{p} - \overline{X}_{N}^{T} \hat{\beta}_{N})$$

$$= (\overline{X}_{p} - \overline{X}_{N})^{T} \hat{\beta}_{p} + \overline{X}_{N}^{T} (\hat{\beta}_{p} - \hat{\beta}_{N}).$$
(3)

Since  $\overline{X}$  and  $\hat{\beta}$  are uncorrelated by assumption and assume that the two samples  $\overline{R}_N$  and

 $\overline{R}_P$  are independent, the mean and the variance for excess return AR are

$$E(AR) = (\overline{X}_P - \overline{X}_N)^T \hat{\beta}_P + \overline{X}_N^T (\hat{\beta}_P - \hat{\beta}_N),$$

and

$$V(\hat{R}_{N} - \hat{R}_{P}) = V(\hat{R}_{N}) + V(\hat{R}_{P})$$

$$= V((\overline{X}_{P} - \overline{X}_{N})^{T} \hat{\beta}_{P}) + V(\overline{X}_{N}^{T} (\hat{\beta}_{P} - \hat{\beta}_{N})).$$
(4)

Applying the corollary, the first and second term in equation (4) become

$$\hat{V}((\overline{X}_{P} - \overline{X}_{N})^{T} \hat{\beta}_{P}) = (\overline{X}_{P} - \overline{X}_{N})^{T} \hat{V}(\hat{\beta}_{P})(\overline{X}_{P} - \overline{X}_{N}) + \hat{\beta}_{P}^{T} (\hat{V}(\overline{X}_{P}) + \hat{V}(\overline{X}_{N}) \hat{\beta}_{P} + tr(\hat{V}(\overline{X}_{P} - \overline{X}_{N}) \hat{V}(\hat{\beta}_{P})),$$

and

$$\hat{V}(\overline{X}_N^T(\hat{\beta}_P - \hat{\beta}_N)) = \overline{X}_N^T(\hat{V}(\hat{\beta}_P) + \hat{V}(\hat{\beta}_N))\overline{X}_N + (\hat{\beta}_P - \hat{\beta}_N)^T\hat{V}(\overline{X}^N)(\hat{\beta}_P - \hat{\beta}_N) + tr(\hat{V}(\overline{X}_N)\hat{V}(\hat{\beta}_P - \hat{\beta}_N)).$$

At the firm level, let

$$\hat{\beta}_{P} = \begin{bmatrix} \hat{\alpha}_{P} \\ \hat{\beta}_{P1} \\ \hat{\beta}_{P2} \\ \vdots \\ \hat{\beta}_{Pn} \end{bmatrix}, \hat{\beta}_{N} = \begin{bmatrix} \hat{\alpha}_{N} \\ \hat{\beta}_{N1} \\ \hat{\beta}_{N2} \\ \vdots \\ \hat{\beta}_{Nn} \end{bmatrix}, \overline{X}_{P} = \begin{bmatrix} 1 \\ \overline{X}_{P1} \\ \overline{X}_{P2} \\ \vdots \\ \overline{X}_{PN} \end{bmatrix}, \overline{X}_{N} = \begin{bmatrix} 1 \\ \overline{X}_{N1} \\ \overline{X}_{N2} \\ \vdots \\ \overline{X}_{Nn} \end{bmatrix},$$

equation (3) can be expressed as

$$AR = \overline{R}_{P} - \overline{R}_{N}$$

$$= (\overline{X}_{P} - \overline{X}_{N})^{T} \hat{\beta}_{P} + \overline{X}_{N}^{T} (\hat{\beta}_{P} - \hat{\beta}_{N})$$

$$= \begin{bmatrix} 1 - 1 \\ \overline{X}_{P1} - \overline{X}_{N1} \\ \overline{X}_{P2} - \overline{X}_{N2} \\ \vdots \\ \overline{X}_{Pn} - \overline{X}_{Nn} \end{bmatrix}^{T} \begin{bmatrix} \hat{\alpha}_{P} \\ \hat{\beta}_{P1} \\ \hat{\beta}_{P2} \\ \vdots \\ \hat{\beta}_{Pn} \end{bmatrix} + \begin{bmatrix} 1 \\ \overline{X}_{N1} \\ \overline{X}_{N2} \\ \vdots \\ \overline{X}_{Nn} \end{bmatrix}^{T} \begin{bmatrix} \hat{\alpha}_{P} - \hat{\alpha}_{N} \\ \hat{\beta}_{P1} - \hat{\beta}_{N1} \\ \hat{\beta}_{P2} - \hat{\beta}_{N2} \\ \vdots \\ \hat{\beta}_{Pn} - \hat{\beta}_{Nn} \end{bmatrix}$$

$$= \hat{\alpha}_{P} - \hat{\alpha}_{N} + (\overline{X}_{Pi} - \overline{X}_{Ni})^{T} \hat{\beta}_{Pi} + \overline{X}_{Ni}^{T} (\hat{\beta}_{Pi} - \hat{\beta}_{Ni}), \ i = 1, 2, ..., n.$$
(5)

Consequently, excess return (AR) of firm i on any day during the prediction period can be calculated by,

$$AR_{i} = \hat{R}_{Pi} - \hat{R}_{Ni}$$

$$= \hat{R}_{Pi} - (\hat{\alpha}_{Ni} + \hat{\beta}_{Ni} \overline{R}_{Nm})$$

$$= \hat{\alpha}_{Pi} - \hat{\alpha}_{Ni} + \hat{\beta}_{Pi} (\overline{R}_{Pm} - \overline{R}_{Nm}) + \overline{R}_{Nm} (\hat{\beta}_{Pi} - \hat{\beta}_{Ni}),$$
(6)

where

 $\hat{R}_{Pi} = \text{the predicted return of firm } i;$   $\hat{R}_{Ni} = \text{the estimated normal performance return of firm } i;$   $\hat{\alpha}_{Ni} = \text{the estimated intercept from the estimation period of firm } i;$   $\hat{\alpha}_{Pi} = \text{the estimated intercept from the post-event day period of firm } i;$   $\hat{\beta}_{Ni} = \text{the estimated slope from the estimation period of firm } i;$   $\hat{\beta}_{Pi} = \text{the estimated slope from the post-event day period of firm } i;$   $\hat{R}_{Nm} = \text{the mean market performance return from the estimation period; }$   $\overline{R}_{Pm} = \text{the mean market performance return from the post-event day period. }$ 

At industry level, the mean excess return of an industry containing N firms is,

$$\overline{AR_{i}} = \frac{1}{N} \sum_{i=1}^{N} (\hat{\alpha}_{P_{i}} - \hat{\alpha}_{N_{i}}) + \frac{1}{N} \sum_{i=1}^{N} \hat{\beta}_{P_{i}} (\overline{R}_{P_{m}} - \overline{R}_{N_{m}}) + \frac{1}{N} \sum_{i=1}^{N} \overline{R}_{N_{m}} (\hat{\beta}_{P_{i}} - \hat{\beta}_{N_{i}}).$$
(7)

After the estimation period, we can get the ex post estimated systematic risk  $\hat{\beta}_p$  and ex post individualistic component  $\hat{\alpha}_p$ .  $R_{Pm}$  is the ex post mean market return. Therefore, equation (7) can be expressed as,

$$\overline{AR_{i}} = \underbrace{\frac{1}{N} \sum_{i=1}^{N} (\hat{\alpha}_{Pi} - \hat{\alpha}_{Ni})}_{\text{Due to differences in firm-specific traits}} + \underbrace{\frac{1}{N} \sum_{i=1}^{N} \hat{\beta}_{Pi} (\overline{R}_{Pm} - \overline{R}_{Nm})}_{\text{Due to differences in systematic risks}} + \underbrace{\frac{1}{N} \sum_{i=1}^{N} \overline{R}_{Nm} (\hat{\beta}_{Pi} - \hat{\beta}_{Ni})}_{\text{Explained by the market}}}_{\text{Explained by the market}}$$

The mean excess returns  $\overline{AR_i}$  of industry *i* can then be decomposed into three terms. The first and second terms represent the parts that are not explained by the market. More precisely, the first term represents how much of the excess returns can be attributed to differences in firm-specific traits. The second term represents the mean excess returns which can be attributed to differences in systematic risks,  $\beta$ . The third term represents the part that is explained by the market which is equivalent to differences in market performance.

#### 3. Two Examples

To illustrate the approach, we take Chapter 10 in the book *Myth and Measurement* by Card and Krueger (1995) as examples. Card and Krueger (1995) quantify the impact of minimum-wage legislation on firm profits. Their results show mixed evidence that news about a minimum wage hike induces investors to adjust their valuation of firms downward. Excess returns associated with news about the 1989 minimum-wage legislation are generally unsystematic. They conclude that in the sample of events they have examined, news about a minimum wage hike rarely seems to have effect on shareholder wealth. In this section we replicate their results and employ the approach to re-examine the effect of 1989 minimum wage hike.

From Card and Krueger (1995) Table A.10.1, we collect daily stock return data on the same sample of 110 publicly-traded firms that are particularly likely to have been affected by the 1989 minimum wage increase. Daily stock returns are obtained from the Center for Research in Security Prices (CRSP). Then we estimate the "normal performance" of firm i in the past one year before the minimum-wage legislation using equation (9):

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}.$$
 (9)

To be consistent with Card and Krueger (1995), equation (9) is estimated using data on returns in 1987 to get the normal performance of each firm. Next, the mean predicted return of each company after an event from day 1 to day 10 is obtained by estimating equation (9). Mean excess returns (AR) are then calculated and decomposed for each company on each day. Lastly, using equation (8) the result attributes the excess returns immediately to differences in firm-specific traits, systematic risks and market performances.

Two legislation events in Card and Kruger (1995) are re-examined here. The descriptions are based on the title of the *Wall Street Journal's* article on the event.

#### a. March 4, 1988 - Headline: Panel Votes to Sharply Boost Minimum Wage

Card and Krueger (1995) predict a negative effect on the wealth of sample companies. They show that the cumulative excess return is decreasing after March 4 as shown in Figure 2(A), but neither cumulative excess return nor mean excess return is statistically significant from zero. Table 1, however, offers a different perspective than Card and Krueger (1995). By decomposing excess return, we find that even though the post-event mean excess return is only 0.077% and not significant, the strong pull and push between market and non-market forces play very active roles. The market performs exceptionally well from day 1 to day 10 (compared to its 1987 performance) which should drive the profits of the sample firms up by a large magnitude. Nevertheless, the news of March 4 generates another strong but negative effect on the sample companies which offsets most of the increase. The three-fold results in Table 1 confirm the findings.

### b. September 27, 1988 - Headline: *Democrats' Bid to Boost Minimum Wage Thwarted by GOP Filibuster*

According to Card and Krueger (1995), this event contains the strongest evidence that investors view a minimum-wage hike as having negative consequences for corporate profits. Figure 2(B) shows the cumulative excess returns around the time of the final cloture vote on the Republican-led filibuster of the Kennedy-Hawkins minimum-wage bill. The cumulative excess return in the 10-day interval around the successful filibuster was nearly 4%. Table 1 reports mean excess returns in the 10-day interval as 0.42% and significant. In the 10 days, 81.6% of the mean excess return cannot be explained by the market which means the event has a significant and large effect on the sample companies; on the other hand, only 18.4% can be explained by the market. This point is supported by looking at the three-fold decomposition. The difference in systematic risks is small and not significant. Therefore, our results confirm Card and Krueger (1995), showing that firm-specific traits account for more than 80% of the good news to the firms to the event on September 27, 1988. Market performance contributes only 18%.

#### Figure 2 Replication of Mean Excess Return and Cumulative Excess Return in Card and Krueger (1995)



Table 1 Results of Decomposing Excess Retur	ns
---------------------------------------------	----

	Cumulative Excess Return				
Event	in		Decomposition of Mean Excess Return (%)		
	Card and Krueger (1995)		_		
	T = -10 to 10	T = 1 to 10		T = 1 to 10	Share
March 4, 1988 Panel votes bill to sharply boost minimum wage.	013	0276	Mean Excess Return	.077749	100%
			Explained by the Market	.44996***	578.72%
			Not Explained by the Market	37221***	-478.72%
			Mean Excess Return	.077749	100%
			Due to Differences in Market Performances	.44996***	578.72%
			Due to Differences in Systematic Risks	9.2706e-04	1.20%
			Due to Differences in Firm-specific Traits	37313***	-479.92%
September 27, 1988 Democrats' bid to boost minimum wage this year is thwarted by GOP filibuster.	.039**	.0320***	Mean Excess Return	.42005***	100%
			Explained by the Market	.07721***	18.38%
			Not Explained by the Market	.34283***	81.62%
			Mean Excess Return	.42005***	100%
			Due to Differences in Market Performances	.07721***	18.38%
			Due to Differences in Systematic Risks	-4.2676e-04	10%
			Due to Differences in Firm-specific Traits	.34326***	81.72%

Note: The sample size ranges between 102 and 108. \* Significant at the 10% level. \*\* Significant at the 5% level. \*\*\* Significant at the 1% level.

#### 4. Concluding Remarks

We present a theorem which helps in estimating the mean and variance of a linear function with arbitrary multivariate randomness in its coefficients and variables. Using the theorem, we derive a generalized decomposition result from two random linear functions in which the result can be applied to most models using event study analysis. Taking the 1989 minimum wage hike in Card and Krueger (1995) as an example, we find the apparent lack of an effect is a consequence of two off-setting forces: 1) a negative effect arising from firm-specific traits and 2) a positive effect arising from market performance. In sum, we bring to the analysis a method that helps provide additional insights and can be applied to much of the work using event study analysis.

#### References

- Blinder, Alan S. 1973. "Wage Discrimination: Reduced Form and Structural Estimates." *The Journal of Human Resources*, 8(4), 436-55.
- Brown, G. G. and H. C. Rutemiller. 1977. "Means and Variances of Stochastic Vector Products with Applications to Random Linear-Models." *Management Science*, 24(2), 210-16.
- Campbell, John Y.; Andrew W. Lo and A. Craig MacKinlay. 1997. *The Econometrics* of Financial Markets. Princeton, New Jersey: Princeton University Press.
- Card, David and Alan B. Krueger. 1995. *Myth and Measurement: The New Economics of the Minimum Wage*. Princeton: Princeton University Press.
- Lin, Carl. 2011. "Give Me Your Wired and Your Highly Skilled: Measuring the Impact of Immigration Policy on Employers and Shareholders," IZA Discussion Paper No. 5754., IEB Working Paper Series [2011/17],
- Oaxaca, Ronald L. 1973. "Male-Female Wage Differentials in Urban Labor Markets." *International Economic Review*, 14(3), 693-709.
- **Powers, Daniel A.; Hirotoshi Yoshioka and Myeong-Su Yun.** 2011(forthcoming). "Mvdcmp: Multivariate Decomposition for Nonlinear Response Models." *The Stata Journal.*
- Roman, Steven. 2008. Advanced Linear Algebra. New York: Springer.
- Schwert, G. William. 1981. "Using Financial Data to Measure Effects of Regulation." *Journal of Law and Economics*, 24(1), 121-58.
- Yun, Myeong-Su. 2004. "Decomposition Differences in the First Moment." *Economics Letters*, 82(2), 275-80.