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Working Paper

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Discussion Paper, No. 323

Provided in Cooperation with:

Department of Economics - Center for Economic Research, University of Minnesota

Suggested Citation: Nieva, Ricardo (2005) : An ANC payoff function for networks with sequentially Nash coherent plans, Discussion Paper, No. 323, University of Minnesota, Center for Economic Research, Minneapolis, MN

This Version is available at:

<https://hdl.handle.net/10419/58353>

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An ANC Payoff Function for Networks with Sequentially Nash Coherent Plans

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October 2005

Abstract

I add endogenous bargaining possibilities to develop criteria to determine which statements are credible in a three-player model with complete information where pairs, in a sequential order, can formulate simultaneous negotiation statements. Joint plans are credible if they are the outcome of a plan Nash bargaining problem—the pair bargains cooperatively over the equilibrium payoffs induced by tenable and reliable plans—unless one or both bargainers are indifferent to bargaining. Then, a credible plan is up to the future-request by the oldest pair ("of friends") among the past pairs that successfully cooperated and included one of the indifferent players. I interpret this model as an almost non cooperative (ANC) modification of the three-player Aumann-Myerson (1988) sequential network formation game. Whenever discussing a link two players can bargain non cooperatively out of the sum of their Myerson values (1977) in the prospective network and enunciate simultaneous negotiation statements. The disagreement plan suggests link rejection. Sequentially Nash (1950) coherent plans can be defined and exist. Analytical payoffs are unique. In strictly superadditive cooperative games the complete graph never forms.

Keywords: Credible Simultaneous Negotiation; Nash Bargaining; Sequential network formation

*Thanks for comments to Leonid Hurwicz, Maria Montero, Roger Myerson, Jun Wako. Ricardo Nieva is a visiting assistant professor at University of New Brunswick Saint John.

1 Introduction

Cooperation and bargaining should be studied using non cooperative models because rational decision making can be analyzed clearly. However, non-cooperative bargaining models result in multiple Nash equilibria. Myerson (1989) addressed this problem by adding negotiation in models with sole sequential negotiators. When the sole negotiator uses statements that can be credibly used—based on Myerson’s credibility criteria—according to their literal meanings, a reduced set of Nash equilibria is obtained.

It is useful to ask if a similar result extends to models with simultaneous negotiators, in particular, network formation models with simultaneous non cooperative bargaining. One problem with standard models is that players evaluate prospective networks according to analytical payoff allocation rules that are *fixed* or *static* in the sense that they depend only on the fixed network structure. Potential network payoffs should depend on the possibilities of players forming other networks. To achieve this, fixed payoff allocation rules are disregarded while allowing non cooperative bargaining over the total payoffs a network can achieve. However, it may be empirically relevant to consider such fixed rules if there is the possibility of pairs of players "bargaining non cooperatively over the sum of their implied fixed payoffs".

In its first part, this paper introduces two features in order to develop criteria to determine which statements can be *credibly* used according to their literal meanings in a three-player model with complete information where pairs, in a sequential order, can formulate simultaneous negotiation statements. First, simultaneous joint "similar" negotiation statements, a *joint plan* (plan¹), are credible "in most cases" if they are the solution outcome of an "endogenous" *plan bargaining problem* solved with the Nash (1950) bargaining rule (NBR). Each pair bargains cooperatively over the equilibrium payoffs induced by "tenable and reliable" plans. In reality, players often cooperate to improve outcomes. Second, one or both players in the pair of negotiators may be indifferent to suggesting any plan with *individually rational feasible (IRF) payoffs* as individual payoffs for one or both of them may be the same as the ones obtained if agreement is not reached. A credible plan is then one with *IRF* payoffs and "future-requested" in the negotiation statement by the "oldest pair" among the past pairs that included one of the indifferent players (Assumption one: *Oldest Friend (O-F) Focal Effect*) and that "successfully" cooperated (Assumption two). This mirrors reality as one often observes loyalty to oldest friends.

In its second part, this paper interprets this negotiation model as a modification of the three-player Aumann-Myerson (1988) (A-M) sequential network formation game where prospective networks are evaluated based on a fixed payoff allocation rule, that implied by the players’ *Myerson values* (1977). Whenever pairs are deciding to form a bilateral communication *link* and thus a new *communication network*, I allow them instead to bargain non cooperatively out of the sum of their Myerson values, in the

¹My plans should be distinguished from Myerson’s (1989) plans.

prospective network. They can also enunciate simultaneous negotiation statements about the non cooperative bargaining process even if they don't have a communication link. I assume they have a *partial communication technology*, an initial phone call, for example.

I then prove by construction that all possible plan bargaining problems are well defined in the modified A-M game and credible plans exist at the beginning of the game; these are defined as *sequentially Nash coherent plans*. Payoff predictions are unique and analytical. Link structures are also predicted for strictly superadditive cooperative games.

Negotiations, modelled as a communication game, can influence selection among different Nash equilibria provided one assumes that players understand the negotiator's statements and negotiators are committed to follow through. For example, in the battle of the sexes game with complete information and communication, there is a Nash equilibrium in which players ignore the male's suggestion to both go to the football game, but both players choose to go to the ballet concert. There are also equilibria where players don't ignore suggestions. Schelling (1960) would argue that players would *focus* on the equilibrium that has both players following the male's suggestion to attend the football game if the male is committed to his literal words.

When commitments are not guaranteed, Farrel (1993) and Myerson (1989) develop criteria to evaluate the credibility of different literal meanings in order to narrow down the number of Nash equilibria in games with sole negotiators. Players will play a Nash equilibrium strategy profile suggested, provided the suggestion passes a credibility test. Credible literal meanings will not be understood and then ignored, but understood and believed.

Whenever the male negotiates and suggests both going to the football game, his suggestion is *tenable*, because it is optimal for the female to go there if she believes he will go there. His suggestion is *reliable* because it is best for him to go if he expects her to follow his suggestion. His suggestion is credible or *coherent*, informally, "he means what he says", because it is the best for him out of all tenable and reliable suggestions. In particular, it is better for him than suggesting both going to the ballet concert.

If both players are allowed to formulate negotiation statements simultaneously, then in the associated communication game, there is a Nash equilibrium, where the most preferred suggestion by the male is followed and the female's most preferred one is ignored and vice versa. Even if they mean what they say, when statements conflict, neither Nash equilibrium can be focal because both players would not know what to focus on. Statements with similar suggestions that coincide may be the exception. I argue that if "bargaining over tenable and reliable similar suggestions" is possible, then players would focus on the equilibrium associated with the bargained suggestion. Such suggestion will be credible.

I adopt the latter informal argument to develop criteria for the credibility of statements in a more general environment that is susceptible to conflicting statements. I

consider three sequential negotiators that negotiate in pairs, where these pairs formulate negotiation statements "represented" by suggestions about actions to be played in an *immediate payoff relevant* game, a "promise-request in a correlated strategy". A *correlated strategy* is a randomization over action profiles. Representations of negotiation statements also consist of suggestions about actions in *future payoff relevant contingencies* in a three-player game after the immediate game, a "future-request in a vector of correlated strategies".

Assume that past negotiation statements by other players have no influence and that pairs of negotiators face a well defined *tenability correspondence* given a negotiation statement. This correspondence represents the set of all correlated strategies that could be rationally implemented by the players in "future contingencies of the communication game associated to the payoff relevant game" if they believe the negotiation statement by a player in the pair who is the sole negotiator. The associated negotiation statement is defined as tenable.

An individual's negotiation statement is reliable if given that the sole negotiator believes that players will obey her future-request, the promise-request in a correlated strategy is tenable and reliable. In this more general set up, reliability entails both players finding it rational to play according to the correlated strategy.

As part of assuming the availability of *endogenous* cooperative negotiation possibilities, players are able to formulate *similar* joint statements, ones with identical promise-requests and future-requests (a plan). Plans are tenable and reliable if any of the associated individual similar statements is tenable and reliable. The current pair of negotiators bargain over payoffs that would result if players play according to tenable and reliable plans. The payoffs obtained in case of disagreement, the *outside options*, are the ones induced by a given tenable and reliable *disagreement* plan. Cooperation is endogenous as not all pairs may end up cooperating. Plans are *Nash coherent* if the bargained payoffs are those predicted by the *non transferable utility* (NTU) NBR.

Assume instead that past negotiation statements by other players are influential. If assumptions one and two hold, I show that *O-F (Nash coherent) plans* can be defined and exist provided that the current plan bargaining problem is well defined. This the case if the plan bargaining problem in each contingency that follows the pair's negotiation statements and, thus, the tenability correspondence are well defined. O-F plans at the beginning of the game, sequentially Nash coherent plans, would then exist.

For interpreting this sequential negotiation model as a modification of the A-M model, I first elaborate on "tedious" notation in the original A-M model.

The Myerson values induce an ad hoc payoff allocation rule for N -player games where cooperation is incomplete as only players that have bilateral communication links or "friendship relationships" can coordinate actions and cooperate. These values have intuitive appeal as players with more friends get more payoffs in a given *graph*, defined as a set of links. However, Myerson values ignore the effects on payoffs

of a situation where players may prefer to cooperate in different *coalition structures* and act strategically when thinking of forming them (See Aumann and Dreze (1974)). Note that a coalition structure is induced by the *communication structure* represented in a given graph. The A-M game is an attempt to predict more reasonable payoffs in such games by endogenizing communication links and thus coalition structures.

In A-M, pairs of players propose indestructible bilateral communication links following a bridge-like rule order and evaluate induced communication structures using the Myerson values. Links are formed if the pair agrees. As in bridge, after the last link has been formed, each of the pairs must have a last chance to form an additional link. If then every pair rejects, the game *ends*. This game is of perfect information. Hence, it has subgame perfect equilibria in pure strategies. Each equilibrium has a unique graph formed at the end of play.

Next, I propose a second model, a multistage game with observed payoff relevant actions. This is interpreted as a non cooperative variation (a *cooperative transformation* in Myerson (1991 pp. 371)) of the original A-M model. It is useful to associate to *histories in A-M*, a sequence of links acceptances and rejections, an *immediate prospective graph*, the graph that will form if the corresponding pair proposing accepts its link.

The initial history in the first stage of the modified game has each player in the same pair as in A-M select a non-negative payoff proposal for each pair member. The third player proposes nothing. Any action profile selected has a link and a *next* prospective graph outcome. If payoff proposals *match* the link is formed and the immediate prospective graph results, otherwise, the link does not form. In any case, a next prospective graph follows according to the A-M rule of order. In order to match, proposals have to "coincide" and be *feasible*. To be feasible, payoffs proposed for each player have to add up the sum of the pairs' Myerson values in the immediate prospective graph.

Histories in stage two are the payoff proposal pairs chosen in stage one and have action sets that depend on histories because action sets are different depending on which next prospective graph resulted that in turns depends on whether a proposal match occurred or not. Each player in the new pair proposing according to the resulting next prospective graph, the new immediate prospective graph, selects, as in the initial history, a non negative payoff proposal pair. The third player proposes nothing. Link, next prospective graph outcomes and feasibility are derived analogously as in the initial history and so on in future histories except in the case where the A-M game would end after link decisions occur. In particular, if the immediate prospective graph is the one that has everyone linked, the *complete graph*, the only feasible proposal pair is the pair's Myerson values in the complete graph, the pair's Shapley (1953) values.

With respect to payoff outcomes, if the immediate prospective graph does not form and the game ends, payoffs in the *last proposal match*—the one that led to the formation of the last graph—are realized. The third player gets her Myerson value in

such last graph. Otherwise stage payoffs are zero unless the complete graph forms, in which case, the Shapley values are realized.

To formulate plan bargaining problems, I assume that each pair can formulate negotiation statements in the associated communication game represented by a correlated strategy in the immediate and future payoff relevant proposal games. The disagreement plan suggests "given unilateral link rejections"—for example suggesting both to propose given not feasible payoff proposals—in the immediate payoff proposal game.

This paper can be seen as an extension of *Myerson's (1989) coherent plans* for sole sequential negotiators to the case of pairs of sequential simultaneous negotiators whenever cooperative negotiation possibilities are endogenous (in Myerson (1984-85), bilateral cooperation is not endogenous) and, however, there is complete information. In a broader perspective, in contrast to Aumann and Hart (2003) and the literature reviewed in their paper that studies strategic information transmission as expanding the set of outcomes, my work emphasizes its study as *restricting* the set of outcomes. In particular, I focus on *long bounded* cheap talk whereas the authors focus on *long cheap talk*.

As my modification of the A-M model addresses the problem of the fixed nature of payoff allocation rules by allowing bargaining over sums of Myerson values, it may be situated and contrasted with the network formation bargaining literature (See Jackson (2004) for a review). Bargaining over what the network can achieve—disregarding fixed payoff allocation rules—in the form of proposals, occur multilaterally and simultaneously in Slikker and Van de Noweland (2001). Currarini and Morelli (2000) have instead a sequential model. Navarro and Perea (2001) use a bilateral sequential model, however, the latter authors' goal objective is to *implement* the Myerson value.²

This model can be seen also as predicting payoffs in an almost non cooperative way (ANC) whenever players think strategically in forming coalition structures (See Aumann and Dreze (1974)).

In section two, I solve a three-player simple majority game with A-M and then I illustrate how my assumptions induce unique payoff predictions by computing sequentially Nash coherent plans using the same game. In section three, I define plan bargaining problems, Nash coherent plans and O-F Nash coherent plans. In section four, notation for graphs is given and Myerson values are described. In section five, histories in the A-M model are defined and the multistage payoff relevant game is set up. Next, cooperative negotiation is added. In section six, existence of sequentially Nash coherent plans is proved by constructing recursively well defined plan bargaining problems. My predictions are partially characterized for strictly superadditive games. Conclusions follow.

²The reader may be interested in a related paper by Jackson (2005) which addresses the problem instead axiomatically by proposing payoff allocation rules that account for simultaneous possibilities of extra link formation.

2 An Example

Consider the three-player simple majority game given by:

$$\begin{aligned} v(1) &= 0, & v(2) &= 0, & v(3) &= 0, \\ v(13) &= 1 & v(23) &= 1 & v(12) &= 1, \\ v(123) &= 1. \end{aligned}$$

where, for example, $v(13)$ is the total wealth players 1 and 3 can assure if they collude and cooperate.

Graph g^{ij} is the one that only has a link between player i and j , ij . Graph g^{ij+jl} is the one that would result if links jl is added to graph g^{ij} for $i \neq j \neq l$, where $i, j, l \in \{1, 2, 3\}$. Graph g^N denotes the complete graph where all players are linked. Also, if I write that some values for player i and j are (x, y) , the first (second) value component refers to player i (j). Myerson values for different graphs are given in the following table (the first, second third component in the triplet corresponds to player 1, 2, and 3 respectively):

One-link	Values	Two-Link	Values	Complete	Values
g^{13}	$(\frac{3}{6}, 0, \frac{3}{6})$	g^{13+32}	$(\frac{1}{6}, \frac{1}{6}, \frac{4}{6})$	g^N	$(\frac{2}{6}, \frac{2}{6}, \frac{2}{6})$
g^{23}	$(0, \frac{3}{6}, \frac{3}{6})$	g^{12+23}	$(\frac{1}{6}, \frac{4}{6}, \frac{1}{6})$		
g^{12}	$(\frac{3}{6}, \frac{3}{6}, 0)$	g^{21+13}	$(\frac{4}{6}, \frac{1}{6}, \frac{1}{6})$		

Note how the player who has relatively more links or friends gets more.

In the rest of the paper, I assume that links 12, 23 and 13 are proposed in that order.

Claim 1: The A-M solution has three subgame perfect equilibrium outcomes in which either of the one link graph is the last to form.

Proof:

From any two link graph the complete graph follows as the players not linked get more if they link, $\frac{2}{6}$ instead of $\frac{1}{6}$. A one link graph is last to form as any player in that link would reject a second link as the complete graph would follow next in which case her payoff would go down from $\frac{3}{6}$ to $\frac{2}{6}$.

Suppose links 12 and 23 have been rejected. Link 13 would form as players 1 and 3 would expect to get half instead of zero payoffs in case the game would end after rejection. One stage backwards, player 3 is indifferent between linking or not with player 2. One more stage backwards, player 2 is indifferent between linking or not with player 1 if players expect link 23 to form. On the other hand, player 1 is indifferent between linking or not with 2 if players expect link 23 not to form and instead link 13 to form. Thus, depending on the decision of the indifferent player, there are several subgame perfect equilibria outcomes in which either of the one link graph forms.

Claim 2: In the three-player simple majority game, the unique sequentially Nash coherent plan has the first pair suggesting "half-each" payoffs and future-requesting plans that suggest consecutive rejection of the next two links in the order.

Proof:

Let the first two links in the rule of order 12 and 23 be rejected in stage 1 and 2 of the game respectively. Next to propose in stage 3 is pair (1, 3).³

Part 1

I. Suppose that players 1 and 3 have a candidate for a tenable plan that suggests⁴ a half-each payoff proposal match, that is, it recommends each one to propose $(\frac{3}{6}, \frac{3}{6})$, a payoff for player 1 and another one for player 3. Say such a match occurs and link 13 is accepted, link 12 is rejected in stage 4 and link 32 is being discussed in stage 5. I want to find out, to begin with, what are all the tenable future-request for players 1 and 3 on players 3 and 2 in this contingency.

First, note that players 3 and 2 could enunciate a tenable plan that suggests a proposal match such that player 2 is offered (out of the sum of their Myerson values in the immediate prospective graph g^{13+32} , $\frac{4}{6} + \frac{1}{6}$) less than what she would get in the complete graph, $\frac{2}{6}$. If link 32 forms with this match, this tenable plan would have to future-request players 1 and 2 to enunciate their unique O-F plan that suggests a proposal match (both propose their Shapley values) and thus form the third link 12. This is the case as the latter players' plan bargaining problem would be "essential", both gain by linking (Note that any plan for players 1 and 2 that suggests either unilateral rejections—say suggest both proposing not feasible payoff pairs proposals—or a proposal match are tenable and reliable as any of the corresponding payoff pair proposals profile is a Nash equilibrium of the last simultaneous proposal game). The expected payoffs for player 3 and 2 associated to their tenable plan (Plan a) would be $(\frac{2}{6}, \frac{2}{6})$, their Myerson values in the complete graph. Note also that whenever describing link or payoff outcomes, it is implicit that I am assuming that players are obedient to tenable plans (one is on "the equilibrium path").

Second, if instead players 3 and 2 can enunciate a tenable plan that suggests a proposal match such that player 2 is offered strictly more than $\frac{2}{6}$, this plan has to future-request players 2 and 1 to enunciate the unique O-F plan that suggests both unilaterally rejecting the third link (any pair of unilateral rejections could be chosen and held fixed for these purposes). Link 32 would be the last to form (Plan of type b). The associated expected payoffs pair (x_3^5, x_2^5) for players 3 and 2 would lie on the diagonal in figure 1 (See section 6) to the northwest of $\bar{\beta}_{32}^5 = (\frac{3}{6}, \frac{2}{6})$.

Third, if instead players 3 and 2's tenable plan suggests a proposal match that offers exactly $\frac{2}{6}$ to player 2, proposal match β^5 such that $\beta_{32}^5 = (\frac{3}{6}, \frac{2}{6}) = \bar{\beta}_{32}^5$ in figure 1, player 2 would be indifferent between forming or not the third link. As player 3 and 2 are the only relevant oldest pair of friends, there are three types of tenable plans if $\beta_{32}^5 = \bar{\beta}_{32}^5$. One type of plan would future-request an O-F plan that suggests link 12 to be formed (Plan d1). The other one would future-request an O-F plan that

³Note that if pair (1, 3) rejects the game ends with zero payoffs. If it accepts, pair (1, 2) follows; in turn, if (1, 2) rejects, pair (2, 3) is next; because every not linked pair must have a last opportunity to propose (as in bridge). If link 23 does not form the game ends, and so on.

⁴In the language of section 3, this plan has a promise-request in a degenerated correlated strategy that puts probability 1 on both proposing $(\frac{3}{6}, \frac{3}{6})$.

suggests link 12 to be rejected (Plan d2). The third one consist of mixes (Plans d3). The associated expected payoffs (x_3^5, x_2^5) for players 3 and 2 would be respectively $(\frac{2}{6}, \frac{2}{6})$, $\bar{\beta}_{32}^5 = (\frac{3}{6}, \frac{2}{6})$ and convex combinations of the latter pairs of payoffs..

As outside options for players 3 and 2 are $(\frac{3}{6}, 0)$, plan d2 with payoffs $(\frac{3}{6}, \frac{2}{6})$ is the only tenable and reliable one that has "strong Pareto efficient" payoffs (Note that the plan that suggests link 32 rejection is also tenable and reliable. On the other hand, plans d1, d3, a and b are tenable but not reliable!, as player 3 would gain by unilaterally rejecting). Thus, d2 is the unique Nash coherent plan for players 3 and 2. Player 1 would get in the latter case her Myerson value in graph $g^{13+32}, \frac{1}{6}$. See figure 1, however, set the outside options for players 3 and 2 $(\psi_3^5, \psi_2^5) = (\frac{3}{6}, 0)$.

Back to players' 1 and 3's discussion, as player 3 gets the same independently of link 32 forming or not, the O-F focal effect assumption implies that O-F plans whenever link 32 is being discussed are up to the oldest friends 1 and 3. Tenable plans for players 1 and 3 vary if the O-F plan they future-request either suggest link 32 rejection (type 1 plans), a proposal match $(\frac{3}{6}, \frac{2}{6})$ —and thereafter link 12 rejection—(plan 2) or mixes (type 3 plans). Associated expected payoffs for players 1, 2 and 3 would be respectively $(\frac{3}{6}, 0, \frac{3}{6})$, $(\frac{1}{6}, \frac{2}{6}, \frac{3}{6})$ and convex combinations between $(\frac{3}{6}, 0, \frac{3}{6})$ and $(\frac{1}{6}, \frac{2}{6}, \frac{3}{6})$.

One stage backwards, as of link 12 discussions in stage 4, one can now characterize all possible type 1 plans for players 1 and 3. As the outside option pair for players 1 and 2 is $(\frac{3}{6}, 0)$, using analogous reasons as in bargaining among players 3 and 2 above, such a tenable plan for players 1 and 3 would have to future-request an O-F plan that suggests either unilaterally rejecting link 12 (type 1.1 plan) or a proposal match with proposals $(\frac{3}{6}, \frac{2}{6})$ (type 1.2 plan) or a mix.(type 1.3 plans) Expected payoffs pair for players 1 and 3 would be respectively $(\frac{3}{6}, \frac{3}{6})$, $(\frac{3}{6}, \frac{1}{6})$ and convex combinations between $(\frac{3}{6}, \frac{3}{6})$ and $(\frac{3}{6}, \frac{1}{6})$. On the other hand, one can characterize the unique type 2 plan for players 1 and 3. As the outside options pair for players 1 and 2 is $(\frac{1}{6}, \frac{2}{6})$, their bargaining game is essential and such a tenable plan for players 1 and 3 would have to future-request an O-F plan for players 1 and 2 that suggests a proposal match. Also, analogously as before, an O-F plan that suggests link 23 rejection after link 12 forms would be future-requested. The NTU NBR yields payoffs of $(\frac{1}{6} + \frac{1}{6}, \frac{2}{6} + \frac{1}{6})$ for players 1 and 2. Player 3 would get her Myerson value in $g^{13+12}, \frac{1}{6}$. Under any plan of type 3, the bargaining game for players 1 and 2 is also essential, thus player 3 would get also $\frac{1}{6}$ and player 1 could not get more than $\frac{3}{6}$!

II. Suppose that players 1 and 3 have a candidate for a tenable plan that suggests proposal matches where player 3 is offered less than half.

If link 12 is rejected then in any O-F plan for players 3 and 2, they would suggest a proposal match as the plan bargaining game is essential (See figure 1 where player 3 is offered $\beta^3(3) = \frac{2}{6}$ and hence outside options are $(\psi_3^5, \psi_2^5) = (\frac{2}{6}, 0)$). Based on the analysis in I, link 23 would be the last link to form. In particular, if player 3's outside option is zero (Note that player 2's outside option is, as in I, again zero) the NTU NBR would give player 2 half of the sum of their Myerson values, that is, $\frac{2.5}{6}$. That is the most she would get. The least she may get is, following I, $\frac{2}{6}$ (See figure 1

where she gets exactly that).

One stage backwards, as player 1's outside option is $\frac{1}{6}$ and that of 2's is at most $\frac{2.5}{6}$, the plan bargaining game as of link 12 discussions is essential (as $\frac{1}{6} + \frac{2.5}{6} < \frac{5}{6}$, the sum of players 1 and 2's Myerson values) whenever player 3 is offered less than half. Analogously as in the case of type 2 plan in I, it can be shown that under any tenable plan by players 1 and 3 with future-requests consistent with the previous analysis, link 12 would form right after link 13 forms and then the third link 23 would be rejected.

III. Now suppose player 3 is offered more than half.

If link 12 is rejected then in any O-F plan for players 3 and 2, they suggest unilateral rejections. Note that as link 23 does not form, player 2 gets zero in g^{13} , and player 3 would get more than $\frac{3}{6}$.

One stage backwards as of link 12 discussions, as the outside option pair for players 1 and 2 is $(\psi_1^4, 0)$, where $\psi_1^4 < \frac{3}{6}$, as in II, a tenable plan for players 1 and 3 consistent with the previous analysis would have to future-request on players 1 and 2 an O-F plan that suggests a proposal match. Again, link 12 would be the last link to form.

Tenable plans in cases II, III and I, where in the latter case one does not include the tenable plan for players 1 and 3 that future-requests unilateral rejections of links 12 and 32—in that order—after link 13 forms (type 1.1 plan), have expected payoffs for players 1 and 3 that would give at least one player (either 1 or 3) less than a half and the other one at most $\frac{3}{6}$.

Part 2. Because the outside options are zero as of link 13 discussions, any plan that suggests a proposal match and a tenable future-request is not only tenable but reliable. Moreover, from Part 1, out of any tenable and reliable plan, type plan 1.1 is the only one that yields strong Pareto efficient payoffs, $(\frac{3}{6}, \frac{3}{6})$, if obeyed. Thus, it is the unique O-F plan as of link 13 discussions. Note that as of link 13 discussions no link has formed—as cooperation has been unsuccessful—so any statement by not linked pairs is ignored.

Part 3. One stage backwards, tenable plans for players 2 and 3 are analogous to the one in the bargaining problem for players 1 and 3. In contrast, outside options are zero for player 2 and a half for player 3. As players 2 and 3 have no preceding oldest friendship pair, the unique O-F plan suggests a half-half proposal match and future-requests consecutive rejection of the next two links in the order (it is a plan analogous to type 1.1 plan). At the beginning of the game, a similar argument can be applied as of link 12 discussions and the claim follows■

3 Simultaneous Negotiation Problems

3.1 A Two-Player Negotiation Problem

I consider the problem of two players i and j , the *negotiators*, when they have the opportunity to make simultaneous negotiation statements to players i , j and l in a

payoff relevant game to follow. I assume for now that there are no past statements that players i and j may know about at the time they negotiate.

The *immediate payoff relevant* action sets that negotiators i and j have available are denoted by the infinite sets B_i and B_j . Such a set B_l for player l has as trivial unique payoff irrelevant action a null vector. Denote by $B = B_i \times B_j \times B_l$ the associated *action profile set*. Also, a two-player action set of profiles for i and j is denoted by B_{ij} .

The *set of future joint payoff relevant strategies* is given by $\times Z$, an infinite Cartesian product of \check{Z}_p sets, $p \in \{1, \dots\}$. Each \check{Z}_p stands for the infinite action profile set in each *future payoff relevant contingency* p that may follow any of the negotiators' immediate payoff relevant action profile.

For any (β, z) , where $z \in \times Z$ and $\beta \in B$, $U_m(\beta, z)$ denotes the expected utility payoff outcome for player $m = i, j, l$ if β and subsequently z are played.

Wlg. (See section 5.2.4), it suffices to define a *correlated strategy* on an infinite strategy profile set R as a function ι from R to the Real interval $[0, 1]$ such that $(\iota(r))_{r \in R^c} \in \Delta R^c$ is a probability distribution over some finite strategy profile subset R^c of R , and $\iota(r) = 0$ if $r \notin R^c$. A given correlated strategy ι may be implemented with a mediator that randomly chooses a profile r of pure strategies in R^c with probability $\iota(r)$. Then the mediator would recommend each player, say i , j and l , *publicly* to implement the strategy r_i , r_j and r_l respectively.

A *negotiation statement* in the *communication game* associated to the payoff relevant game to follow for player i , μ_i , is *represented* on the one hand by a *promise-request* of play associated to a correlated strategy σ^i on B . For simplicity and tractability, her promise-request is the same for any negotiation statement μ_j player j may actually formulate. If the negotiator announces σ^i , regardless of any given statement μ_j for player j that may have occurred she is requesting player j to obey her mediator according to σ^i . She is also promising to obey her own mediator according to σ^i . The request to player l is trivial; abusing notation, I often refer informally to σ^i as a correlated strategy on B_{ij} ignoring player l . The set of all correlated strategies on B is denoted by \mathcal{U}^B .

On the other hand, a negotiation statement for player i , μ_i , consists also of *future-requests* on play in "future contingencies" of the communication game. For simplicity and tractability future-requests are assumed to be identical whenever a contingency or "history" in the communication game "shares" the same payoff relevant contingency or history of the payoff relevant game (See section 5.3 for a clarification and application of this assumption). Thus, formally and abusing notation, these future-requests can be represented by an infinite dimensional vector of correlated strategies ζ_i derived from $\times Z$ as follows

$$\zeta_i = \prod_p \zeta_{i,p},$$

where $\zeta_{i,p}$ is a correlated strategy on \check{Z}_p . If the negotiator announces ζ_i , she has future-requests on future players in any future contingency of the communication

game that corresponds—in the sense above—to the future payoff relevant future contingency p to obey her mediator according to the correlated strategy $\zeta_{i,p}$, the p -th component of ζ_i . The set of all vectors of correlated strategies on $\times Z$ is denoted by $\mathcal{U}^{\times Z}$.

A negotiation statement for player i is thus an element of $\mathcal{U} = \mathcal{U}^B \times \mathcal{U}^{\times Z}$ and it is denoted by $\mu_i = (\sigma_i, \zeta_i) \in \mathcal{U}$.

A promise-request in σ_j and future-request in ζ_j for player j are defined analogously and it is clear that her negotiation statement $\mu_j \in \mathcal{U}$.

To formalize the credibility, reliability and tenability of a negotiation statement whenever there are two simultaneous negotiators, one needs to deal first with the problem of conflicting simultaneous negotiation statements. To set up this problem precisely, I will define first a tenable and reliable statement for a player when she is the sole negotiator.

Let player i be the *sole* negotiator with negotiation statement $\mu_i = (\sigma_i, \zeta_i)$ given player j 's statement $\mu_j = (\sigma_j, \zeta_j)$, where the latter is to be regarded as noise. I assume in this section that there exists a well defined non empty *tenability correspondence* $Q : \mathcal{U} \rightarrow \mathcal{U}^{\times Z}$, where $Q(\sigma_i)$ represents the set of all vectors of correlated strategies that could be rationally implemented by the players in future contingencies⁵ following the negotiator's statement in the communication game if they would believe negotiation statement μ_i . A negotiation statement μ_i is *tenable* iff $\zeta_i \in Q(\mu_i)$. One writes then $\mu_i \in \underline{\mathcal{U}} \subset \mathcal{U}$.

Let $\mu_i = (\sigma_i, \zeta_i) \in \mathcal{U}$ and wlg. noise $\mu_j = (\sigma_j, \zeta_j)$ be given. Consider the following strategic form game $(B_i \times B_j, \pi_{ij}^{\mu_i})$, where payoffs are given by

$$\pi_{ij}^{\mu_i}(\beta_i, \beta_j) = \left[\sum_z [\zeta_i(z) | \beta] U_i(\beta, z), \sum_z [\zeta_i(z) | \beta] U_j(\beta, z) \right],$$

if (β_i, β_j) is played and $[\zeta_i(z) | \beta]$ is the probability that $U_i(\beta, z)$ is obtained if play—for simplicity of exposition—in the payoff relevant game is according to the given ζ_i and (β_i, β_j) is played.

Note that $\pi_l^{\mu_i}(\beta_i, \beta_j)$, the associated payoff to player l can be computed analogously and $\pi^{\mu_i}(\beta_i, \beta_j)$ would then refer to a payoff triplet for all players. Recall, β_l is trivial.

Suppose that players are expected to obey future request ζ_i . A request in σ_i by player i is tenable if it is optimal for player j to obey player i 's mediator given that player i is believed to fulfill his promise to obey the mediator. A promise in σ_i by player i is reliable if it is optimal for player i to obey the mediator given that player j is expected to obey the mediator. Equivalently, I will say that a promise-request in σ_i by player i is tenable and reliable given μ_i if σ_i is a *publicly correlated equilibrium* of $(B_i \times B_j, \pi_{ij}^{\mu_i})$. A statement $\mu_i = (\sigma_i, \zeta_i) \in \mathcal{U}$ is reliable if its promise-request in σ_i is tenable and reliable.

⁵Note that this correlated strategy prescribes rational behavior in future contingencies that follow a "mistake" that occurs whenever one player doesn't conform to such prescribed strategy.

Tenable and reliable statements μ_i will be said to belong to $\tilde{\mathcal{U}}$. As $(B_i \times B_j, \pi_{ij}^{\mu_i})$ will turn out to be a coordination game in the modification of the A-M model, $\mu_i \in \tilde{\mathcal{U}}$ will be also *self-signaling* (See Farrel and Rabin (1996) for a definition).

Analogously, one defines reliability of μ_j for player j whenever she is the sole negotiator and has her own mediator. Note that $\mu_i, \mu_j \in \tilde{\mathcal{U}}$, so μ_i is tenable and reliable whenever player i is the sole negotiator if and only if μ_j is tenable and reliable whenever player j is the sole negotiator.

In case neither of the negotiation statements by players i and j are noise, the tenability of one player's statement depends on the statement of the other one. If one has conflicting requests, who would players obey if they are willing to obey either of the negotiators, or equivalently, if both negotiators' statements are tenable whenever they are the sole negotiators? The subsections that follow address this problem.

A simultaneous negotiation problem for players i and j as just described is denoted by $\Phi_{ij} = (B, \times Z, U, Q)_{ij}$.

3.2 Nash Coherent Plans

3.2.1 Preliminary Definitions

We define for any two vectors x and y in \mathbb{R}^2

$x \geq y$ (x is *as least as good as* y) iff $x_i \geq y_i$ and $x_j \geq y_j$, and

$x > y$ (x is *strictly better* than y) iff $x_i > y_i$ and $x_j > y_j$, $i \neq j$.

A bargaining problem for agents i and j consists of a pair (F, ψ) , where F is a closed convex subset of \mathbb{R}^2 , $\psi = (\psi_i, \psi_j)$ is a vector in \mathbb{R}^2 and the set of *individually rational feasible allocations* (*IRF set*)

$$F \cap \{(x_i, x_j) \mid x_i \geq \psi_i \text{ and } x_j \geq \psi_j \text{ or } x_{ij} \geq \psi_{ij}\}$$

is non-empty and bounded. Here F represents the set of feasible payoff allocations or the *feasible set*, and ψ represents the disagreement payoff allocation or the *outside options*.

A bargaining game (F, ψ) is *essential* iff there exists at least one allocation x in F that is strictly better for agents than the disagreement allocation ψ , i.e., $x > \psi$.

A point x in F is *strongly (Pareto) efficient* iff there is no other point y in F such that $y \geq x$ and $x_w > y_w$ for at least one player $w \in \{i, j\}$. A point x in F is *weakly (Pareto) efficient* iff there is no other point y in F such that $y > x$. The *feasible frontier* is the set of feasible payoff allocations that are strongly Pareto efficient in F . The *IRF frontier* is the set of points in F that are strongly Pareto efficient in the *IRF set*.

3.2.2 A Plan Bargaining Problem

Before I develop a notion of credibility whenever negotiation statements are simultaneous by adding cooperative negotiation, I will define tenability and reliability in this context and define a plan bargaining problem.

Negotiation statements for both players are *similar* if $\mu_i = \mu_j$. A *joint plan* (plan) is a negotiation statement $\mu \in \tilde{\mathcal{U}}$ such that there exists similar statements for players 1 and 2 and $\mu_1 = \mu_2 = \mu$. A plan is tenable and reliable iff μ is tenable and reliable for player i or j whenever any of them is the sole negotiator.

A *plan bargaining problem* for players i and j derived from a simultaneous negotiation problem $\Phi_{ij} = (T, \times Z, U, Q)$ is a bargaining problem (F, ψ) with two characteristics:

1. For each $(x_i, x_j) \in F$, there exists an associated tenable and reliable plan $\mu = (\sigma, \zeta) \in \tilde{\mathcal{U}}$ such that

$$(x_i, x_j) = \sum_{\beta} \sigma(\beta) \pi_{ij}^{\mu}(\beta).$$

2. If disagreement occurs on $\psi = (x_i, x_j)$, then both agents commit to enunciate the associated *disagreement* plan $\hat{\mu} \in \tilde{\mathcal{U}}$.

Any such plan bargaining game will be denoted by (F, ψ, Φ_{ij}) .

3.2.3 Nash Effective Cooperative Negotiation and Credibility

Players i and j can carry out negotiations *Nash effectively* and *cooperatively* if given the simultaneous negotiation problem Φ_{ij} , they can construct and solve cooperatively the associated plan bargaining game (F, ψ, Φ_{ij}) , where any $(B_i \times B_j, \pi_{ij}^{\mu_i})$ is a coordination game, with the *non transferable utility* (NTU) *Nash Bargaining Rule* (NBR). The NTU NBR solution in any bargaining game (F, ψ) solves the following maximization problem:

$$\arg \max_{x \in F(h), x \geq \psi} (x_i - \psi_i) (x_j - \psi_j),$$

Let the payoff relevant action sets for players i and j and l be given by B together with a payoff relevant game to follow with joint strategy set $\times Z$ for players i, j and l . Players i and j can *cooperate* Nash effectively *with communication* if they can construct Φ_{ij} and carry out negotiations Nash effectively and cooperatively.

A plan μ is *credible* or Nash *coherent* if the payoff solution to a plan bargaining problem (F, Φ_{ij}, ψ) where players can negotiate Nash effectively and cooperatively has as associated plan μ .

Whenever I want to refer to players i and j 's set of Nash coherent plans in Φ_{ij} given ψ , I write $\eta(\Phi_{ij}, \psi) \subset \tilde{\mathcal{U}}$.

3.3 Oldest-Friends Nash Coherent Plans

I will be interested in developing credibility criteria for simultaneous statements in a situation where pairs of players, out of a total of three, take turns according to a rule of order to formulate cooperative negotiation plans in stages k of a multistage game, where $k = 1, \dots, K + 1$, and whenever cooperation possibilities are *endogenous*. Cooperation is endogenous in the sense that disagreement or unsuccessful cooperation is possible and "meaningful".

To formulate these criteria, I make two assumptions:

Assumption 1: Let one or both players in the pair of negotiators be indifferent between plans with payoffs in the IRF set. A credible plan is one that has a payoff in the IRF and that is future-requested in the negotiation statement by the *oldest pair* of negotiators—according to a rule of order—among the past pairs that included one of the indifferent players.

Assumption 2: Plans enunciated by older pairs of negotiators that did not co-operated successfully are ignored or are not credible.

As for assumption 1, the sequence of previous plans enunciated by different pairs, to be denoted by μ^{k-} , may influence the current negotiation problem, at stage k , and the ones that follow. To indicate this influence, the tenability correspondence will be denoted by $Q_{\mu^{k-}}$ and the negotiation problem is instead given by $\Phi_{ij, \mu^{k-}}$.

The plans that are predicted in this solution concept will be defined as *oldest-friends Nash coherent plans (O-F plans)* and they will be denoted by $\eta^f(\Phi_{ij, \mu^{k-}}, \psi^k)$.

Formally, the assumptions are made effective as follows:

Let $i, j, l \in \{1, 2, 3\}$, and $i \neq j \neq l$. Suppose players i and j successfully cooperated and have enunciated, as part of their future-request, the tenable and reliable plan $\gamma \in \tilde{U}_{il}$ and only then j and l successfully cooperated and future-requested $\delta \in \tilde{U}_{il}$ where it maybe that $\gamma \neq \delta$. Schematically, as the bargaining problem for i and l follows, one has the following physical order:

$(i, j) \quad (j, l) \quad (i, l).$

For all essential bargaining problems for i and l , I set

$$\eta^f(\Phi_{il, \mu^{k-}}, \psi^k) = \eta(\Phi_{il, \mu^{k-}}, \psi^k).$$

Otherwise:

Case 1. If $\nexists (x_i, x_l) \in IRF^k$ s.t. $x_i^k > \psi_i^k$, however $\exists (x_i, x_l) \in IRF^k$ s.t. $x_l^k > \psi_l$,

I set $\eta^f(\Phi_{il, \mu^{k-}}, \psi^k) = \gamma$;

Case 2. If $\nexists (x_i, x_l) \in IRF^k$ s.t. $x_l^k > \psi_l^k$, however $\exists (x_i, x_l) \in IRF^k$ s.t. $x_i^k > \psi_i^k$

I set $\eta^f(\Phi_{il, \mu^{k-}}, \psi^k) = \delta$;

Graphically, in the plane (x_i, x_l) , the IRF^k set for $(F^k, \psi^k, \Phi_{ij, \mu^{k-}})$ is a straight closed vertical and horizontal closed segment respectively.

Case 3. If $\nexists (x_i, x_l) \in IRF^k$ s.t. $x^k > \psi^k$

I set $\eta^f(\Phi_{il, \mu^{k-}}, \psi^k) = \gamma$

In words, there are 3 cases in which the assumptions turn out to imply a not essential $(F^k, \psi^k, \Phi_{ij, \mu^{k-}})$ to be "effectively" a singleton. As oldest friends' are the only statements that are credibly understood by their literal meanings, the only possible payoff $(x_i, x_l) \in IRF^k$ and associated plan to be bargained about by players i and l is the one that confirms the plan by the oldest pair of friends that has one of its member, i or l , indifferent between any payoff in IRF^k .

In addition, if one only has pair (i, j) enunciating as part of its future-request $\gamma \in \tilde{\mathcal{U}}_{il}$ and thus one has schematically,

$$(i, j) \quad (i, l),$$

then

Case 1. $\nexists (x_i^k, x_l^k) \in IRF^k$ s.t. $x_i^k > \psi_i^k$, however $\exists (x_i, x_l) \in IRF^k$ s.t. $x_l^k > \psi_l^k$

I set $\eta^f(\Phi_{il, \mu^{k-}}, \psi^k) = \gamma$

Case 2. $\nexists (x_i, x_l) \in IRF^k$ s.t. $x^k > \psi^k$

I set $\eta^f(\Phi_{il, \mu^{k-}}, \psi^k) = \gamma$

Note that in non essential bargaining games in this bilateral sequential negotiation environment an O-F plan may be the disagreement plan in which case unsuccessful bilateral cooperation occurs. Moreover as for assumptions 1 and 2 cooperating and thus not cooperating are both *meaningful*.⁶

In this context, players i and j can *endogenously cooperate Nash effectively with communication* if they can construct $\Phi_{ij, \mu^{k-}}$, carry out negotiations Nash effectively and cooperatively unless they have to use $\eta^f(\Phi_{ij, \mu^{k-}}, \psi^k)$.

4 Graphs and the Myerson Value

4.1 Notation for Graphs

Denote by $N = \{1, 2, 3\}$ the set of players. A graph g is a set of unordered pairs of distinct agents belonging to N . Each pair is represented by a link (non-directed segment) between the two players (nodes). Thus, g stands also for the set of links for graph g .

We denote by ij , or equivalently ji , the link that joins agents i and j , where $i \neq j \neq l$, $i, j, l \in N$. If $ij \in g$, we say that i and j are *directly linked* in graph g . Iff $ij, jl \in g$, we say that i and l are *indirectly linked* by j .

We use often ij as a superscript for referring to the graph g that contains only link ij , say g^{ij} . In turn, the superscript ijl would refer to the graph where only player j is directly linked to two agents. Later on, we will distinguish among different orderings of ijl representing the order in which links have been formed.

The graph where every pair is directly linked, or linked from now on, is called the *complete* graph, and is denoted by g^N . The empty graph where no pair is linked is represented by g^\emptyset . The set G of all possible graphs on N is $\{g : g \subseteq g^N\}$. We use, $g^{\theta+ij}$ when referring to the graph that results to adding link ij to graph g^θ , where $\theta \in \{\emptyset, il, ilj\}$ $i \neq j \neq l$, $i, j, l \in \{1, 2, 3\}$.

⁶In standard 2 player bargaining problems disagreement is not meaningful in the sense that in general it does not occur and if it "would occur" only the disagreement payoffs pair is obtained. In this paper disagreement is in contrast meaningful as different link structures and payoffs for the third player may occur after disagreement.

Let $B \subseteq N$, $g \subseteq G$, $i \in B$, $j \in B$ be given. Agents i and j are *connected* in B by g iff there is a *path* in g from i to j and stays within B . That is, iff i and j are directly or indirectly linked under some g' , where g' is such that $g' \subseteq g$ and $g' \subseteq G'$, and G' is the set of all graphs of B .

4.2 Payoffs in Communication Structures as Graphs

Let a *cooperative game* v be given with N as the player set. Given N , let CL be the set of all coalitions (non-empty subsets) of N , $CL = \{B \subseteq N, B \neq \emptyset\}$. A *characteristic function* $v : CL \rightarrow \mathbb{R}$ associates the maximum wealth or transferable utility (TU) *payoff* achievable if the coalition $B \in CL$ forms and coordinates effectively (and thus cooperates).

There are intermediate cases between N -player games that are played cooperatively and non-cooperatively. For predicting payoff outcomes in these cases, Myerson (1977) assumes that effective coordination can occur if pairs of players by establishing bilateral agreements or friendship relationships, represented by links of communication, are at least indirectly linked. In this context a set of links is denoted equivalently as a cooperation, communication or *cooperation structure*. Myerson (1977) derives axiomatically a cooperative solution for given cooperation structures, i.e., a graph g for $g \subseteq g^N$.

Formally, define $B|g$ as the unique *partition* of B in which groups of players are together iff they are connected in B by g . Loosely speaking, it is the collection of smaller coalitions, or connected *components* of $B|g$, into which B would break up, if players could only coordinate along the links in g .

Let a coalitional game v be given with N as player set and g as the cooperation structure. For each player i and given the graph g and the characteristic function v , the Myerson value for player i is denoted by $\phi_i^g = \phi_i^g(v)$.

I founded this practical method by Myerson (1977) to be useful to give intuition and to derive the Myerson values: Given v and g , define a coalitional game v^g by

$$v^g(S) := \sum v^g(S_j),$$

where the sum ranges over the connected components S_j^g of $S|g$. Then

$$\phi_i^g(v) = \phi_i(v^g)$$

where ϕ_i denotes the ordinary Shapley (1953) value for player i .

In words the Myerson value is the Shapley value of an auxiliary cooperative game where any given coalition gets all its worth provided all players in that coalition are at least indirectly linked. Otherwise the payoffs in that coalition are the sum of the worth of its subcoalitions that in contrast get all their worth (including possible trivial singleton coalitions).

I normalize three-player cooperative games by focusing in characteristic functions $v : CL \rightarrow [0, d]$ with

$$\begin{aligned} v(1) &= 0, & v(2) &= 0, & v(3) &= 0, & a \\ v(13) &= a, & v(23) &= b, & v(12) &= c, & v(123) &= d, \end{aligned}$$

where $d = 1$ and, for example, $v1$ is used instead of $v(\{1\})$.

5 A Multistage Game with Simultaneous Negotiation Statements

As negotiation statements are about a payoff relevant multistage game which in turn is derived from the A-M model, I first define useful concepts in these two base models.

5.1 The A-M Model

Consider g^N , where $N = 3$, i.e., $g = \{(1, 2), (2, 3), (1, 3)\}$. The rule of order according to which pairs of players propose links in A-M can be represented by the function $\rho_\emptyset : g^3 \rightarrow \{1, 2, 3\}$. Wlg., I will assume a fixed ρ_\emptyset , where

$$\rho_\emptyset(12) = 1 \quad \rho_\emptyset(23) = 2 \quad \rho_\emptyset(13) = 3.$$

The interpretation is that pair $(1, 2)$ in the initial history as of stage 1 discusses the *first* link 12 in the game. If 12 is rejected, 23 follows, and if 23 is in turn rejected, 13 follows. If 13 is rejected the game ends

If a first link ij has just been accepted I will write that a *first round of play* has been completed. Suppose that is the case. The rule of order for the *left out* pairs to propose a *second* link in the game,

$$\rho_{ij} : g^3 \setminus g^{ij} \rightarrow \{1, 2\}, \text{ for } i, j \in \{1, 2, 3\}, i \neq j,$$

is derived from ρ_\emptyset and one has:

$$\rho_{12}(23) = 1, \rho_{12}(13) = 2,$$

$$\rho_{23}(13) = 1, \rho_{23}(12) = 2 \text{ or}$$

$$\rho_{13}(12) = 1, \rho_{13}(23) = 2$$

depending on either link 12, 23 or 13 being the first to form respectively. The interpretation is analogous as before. In particular, if all left out pairs reject the game ends.

If two links have just been accepted, and thus a *second* round of play has been completed, the pair not linked yet is next. If the left out pair rejects, the game ends. If the *third* round of play has been completed (and thus, three links have formed) the game ends.

Given ρ_\emptyset , an *A-M-history* is a sequence of links acceptances and rejections. If the game ends, then an A-M *final* history is reached. Except for the latter, each history has an *immediate prospective graph*—the one that would result if the associated link being proposed forms. The immediate prospective graph that *may* result after link decisions have been made is defined as the *next* prospective graph. Unless otherwise stated let, *for now on*, $\theta \in \{\emptyset, il, ilj\}$ $i \neq j \neq l$, $i, j, l \in \{1, 2, 3\}$. Also, let k be the stage of the game one is at and ρ_\emptyset be given. An immediate prospective graph will be denoted by $g^{\theta+ij}$.

I assume that the order of ijl matters. Non final histories are then denoted uniquely by $h_{AM}^k(g^{\theta+ij})$. For example, only $h_{AM}^1(g^{\varnothing+12})$ stands for the initial history. If link 12 is rejected, the next history is denoted uniquely by $h_{AM}^2(g^{\varnothing+23})$ and so on. History $h_{AM}^5(g^{13+32})$, or equivalently $h_{AM}^5(g^{132})$, corresponds to link 13 being the first link to form, following ρ_{13} , link 12 being rejected so that link 32 is next to be discussed in stage 5. Analogously, $h_{AM}^3(g^{123+13})$ has third link 13 next to be proposed in stage 3 after link 12 formed in stage 1 and link 23 was accepted in stage 2.

With respect to payoff outcomes, let g^θ be the last graph to form at the end of the game. Then each player gets her Myerson value in graph g^θ . In particular, if in history $h_{AM}^k(g^{ilj+ij})$ link ij is accepted then players get their Myerson value in the complete graph. Otherwise payoffs are zero.

5.2 The Payoff Relevant Multistage Game

5.2.1 The Abstract model

Actions Sets and Histories I adopt a $K+1$ -multistage game with payoff relevant observed actions M based in Fudenberg and Tirole (1992).

In the first stage 1, all players $m = 1, 2, 3$ choose simultaneously from choice sets B_{m,h^1} , $m = 1, 2, 3$. I let the initial history be $h^1 = \varnothing$ at the start of play. At the end of each stage, all players observe the stage's action profile. Let $\beta^1 = (\beta_1^1, \beta_2^1, \beta_3^1)$ be the stage 1 action profile. At the beginning of stage 2 players know history h^1 that can be identified with β^1 given that h^1 is trivial. In general, actions for player m will depend on previous actions, so I let B_{m,h^2} denote the action set for player m at history h^2 . By iteration, histories in general are

$$h^k = (\beta^1, \beta^2, \dots, \beta^{k-1}),$$

and B_{m,h^k} is the action set for player m at stage k when the history is h^k . I let $K+1$ be the total number of stages in the game. By definition each h^{K+1} describes an entire sequence of actions from the start of the game on. I denote H^{K+1} as the set of all terminal histories that can be identified with the set of possible *outcomes* when the game is played.

Pure Strategies and Payoff Outcomes A pure strategy for player i is a contingent plan on how to play in each stage k for possible history h^k . If one lets H^k denote the set of all stage- k histories, and

$$B_{i,H^k} = \cup_{h^k \in H^k} B_{i,h^k},$$

a pure strategy for player i is a sequence of maps $\{s_i^k\}_{k=1}^K$, where each s_i^k maps H^k to the set of player i 's feasible actions B_{i,H^k} (i.e., satisfies $s_i^k(h^k) \in B_{i,h^k}$ for all $h^k \in H^k$). The set of all pure strategies for player i in the payoff relevant multistage game is denoted by S_i

A sequence of actions for a profile for such strategies $s \in S$ is called the *path* of the strategy profile, where S is the set of all strategy profiles: the stage 1 actions are $\beta^1 = s^1(h^1)$. Stage 2 actions are $\beta^2 = s^2(\beta^1)$. The stage 3 actions are $\beta^3 =$

$s^3(\beta^1, \beta^2)$ and so on. Since the terminal histories represent an entire sequence of play or path associated with a given strategy profile, one can represent each players' corresponding *overall's* payoff as a function $u_i : H^{K+1} \rightarrow \mathbb{R}$. Abusing notation, I denote the payoff vector to profile $s \in S$ as $u(s) = u(h^{K+1})$, as one can assign an outcome in H^{K+1} to each strategy profile $s \in S$.

Nash Equilibrium A pure-strategy *Nash equilibrium* in this context is a strategy profile s such that no player i can do better with a different strategy or, using standard Fudenberg and Tirole's (1992) notation, $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$ for all $s'_i \in S_i$.

Subgame Perfect equilibrium Since all players know the history h^k , of moves before stage k , one can view the game from stage k on with history h^k as an extensive form game in its own and denote it by $M(h^k)$. To define the payoff functions in this game, note that if the sequence of actions or path in stages k through K are β^k through β^K , the final history will be $h^{K+1} = (h^k, \beta^k, \dots, \beta^K)$. The payoffs for player i will be $u_i(h^{K+1})$.

Strategies in $M(h^k)$ are defined in a way where the only histories one needs consider are those consistent with h^k . Precisely, any strategy profile s of the whole game induces a strategy profile $s|h^k$ on any $M(h^k)$. For each i , $s_i|h^k$ is the *restriction* of s_i to the histories consistent with h^k . One denotes the restriction profile set by $S|h^k$.

Let histories h^{K+1} be such that $h^{K+1} = (h^k, \beta^k, \dots, \beta^K)$ and the associated subset of H^{K+1} be denoted by $H^{K+1}(h^k)$. As one can assign an outcome in $H^{K+1}(h^k)$ to each restriction profile $s|h^k$ where $s \in S$, the overall payoff vector to the restriction $s|h^k$, will be denoted abusing notation by $u(s|h^k)$. Thus, one can speak of Nash equilibria of $M(h^k)$.

A strategy profile s of a multi-stage game with observed actions M is a *subgame-perfect equilibrium* if, for every h^k , the restriction $s|h^k$ to $M(h^k)$ is a Nash equilibrium of $M(h^k)$.

5.2.2 Interpretation

I interpret the abstract model as a modification of the A-M model. In history h^1 , I define a next prospective graph stage outcome function that depends on an element of the stage action profile set B_{h^1} and the initial immediate prospective graph $g^{\emptyset+12}$ as follows:

First, let the stage 1 payoff pair set for agent $m = 1, 2$ be

$$B_{m,h^1} = \{\beta_m = (\beta(1), \beta(2)) \mid \beta(1) \geq 0, \beta(2) \geq 0\},$$

i.e., payoff pairs for agent m , are restricted to be two non-negative payoff proposals: one for player 1, $\beta(1)$, and one for player 2, $\beta(2)$.

For player 3, the choice set B_{3,h^1} is the singleton "do nothing", which I denote as choosing payoff proposal $(0, 0)$.

Player m 's payoff pair proposal is *feasible*, iff

$$\beta_m(1) + \beta_m(2) = \phi_1^{g^\varnothing+12} + \phi_2^{g^\varnothing+12}, m = 1, 2.$$

In words, proposal pairs by player m are feasible, iff they add up to the sum of both agents' (1 and 2) Myerson values in the immediate prospective graph $g^{\varnothing+12}$.

Proposals *coincide* iff $\beta_1 = \beta_2$. Proposals *match* for player's 1 and 2, iff their proposals are feasible and coincide. A payoff proposal by player m is called an *unilateral rejection* if β_m is not feasible. I define β to be a *proposal match* iff proposals for player's i and j match. Otherwise β is not a proposal match.

With these definitions, in history h^1 , the stage game link outcome and the next prospective graph—as defined in the A-M model—outcome depend on proposals chosen as follows. Link 12 forms if proposals match and so does graph $g^{\varnothing+12}$. The next prospective graph is g^{12+23} following the A-M rule of order. If payoff proposals don't match, link 12 is rejected and the next pair in the rule of order ρ_\emptyset follows, that is link 23 is proposed. The next prospective graph in this case is $g^{\varnothing+23}$.

It will be useful to index a history in the payoff relevant multistage game by its immediate prospective graph and the complete or partial sequence of payoff proposals that led to it as link, next prospective graph and payoff outcomes may depend on them.

The initial history is then arbitrarily indexed as $h_{g^{\varnothing+12}(\varnothing)}^1$. A generic history in any stage k that had the sequence $(\beta^1, \dots, \beta^{(k-1)})$ and led to immediate prospective graph $g^{\theta+ij}$ is denoted by $h_{g^{\theta+ij}(\beta^1, \dots, \beta^{(k-1)})}^k$. Whenever much specificity is not necessary, one writes $h_{g^{\theta+ij}(\beta^{(k-1)})}^k$, $h_{g^{\theta+ij}(\cdot)}^k$ or even $h_{(\cdot)}^k$.

In general, the stage k payoff proposal pair set in history $h_{g^{\theta+ij}(\cdot)}^k$ for player $m = i, j$ is equal to:

$$B_{m, h_{g^{\theta+ij}(\cdot)}^k} = \{\beta_m = (\beta(i), \beta(j)) \mid \beta(i) \geq 0, \beta(j) \geq 0\}.$$

For the third player l , the choice set is the singleton "do nothing", which we denote as proposing $(0, 0)$.

The payoff action profile set for players i, j and l is denoted by

$$B_{h_{g^{\theta+ij}(\cdot)}^k} = \left\{ B_{m, h_{g^{\theta+ij}(\cdot)}^k} \right\}_{m=i,j,l}$$

where $B_{l, h_{g^{\theta+ij}(\cdot)}^k}$ is trivial.

But for $g^{\theta+ij} = g^N$, player i 's payoff pair proposal is feasible, iff

$$\beta_i(i) + \beta_i(j) = \phi_i^{g^{\theta+ij}} + \phi_j^{g^{\theta+ij}}.$$

If $g^{\theta+ij} = g^N$, I define there to be only one feasible proposal pair, that associated to the Myerson values in the complete graph. This is given by

$$\beta_m = (\beta(i), \beta(j)) = (\phi_i^{g^N}, \phi_j^{g^N}), m = i, j.$$

As before, β is a proposal match iff proposals for player's i and j match. Sometimes, I refer to a proposal match and its components by simply β and $(\beta(i), \beta(j))$ instead of β and

$$[(\beta_i(i), \beta_i(j)), (\beta_j(i), \beta_j(j)), \beta_l] \text{ respectively.}$$

In history $h_{g^{\theta+ij}(\cdot)}^k$, link ij forms if proposals match and so does graph $g^{\theta+ij}$. If payoff proposals don't match, link ij is rejected. If that is the case according to the A-M game, only the next pair in the rule of order ρ_\emptyset may follow. Analogously, a next prospective graph may follow or not.

Following the A-M game, if the game ends, all agents move "nothing", a vector of zeros $\mathbf{0}$, thereafter in each stage until stage K . Outcomes are non existent or trivial as links cannot be formed anymore. If the game ends at stage K then the final history $K + 1$ follows.

Payoffs outcomes are realized if the game ends. In that case, the last pair that formed a link receives its payoff proposal match and the third player receives her Myerson value in the resulting last graph.

Formally, the overall's payoff function $u = (u_1, u_2, u_3)$ is constructed from payoff functions ν in all possible non final histories as follows:

If the immediate prospective graph is the complete graph, that is, the associated history $h_{g^{\theta+ij}(\cdot)}^k$ is such that $g^{\theta+ij} = g^N$, and link ij forms, then the three players get their Myerson value in the complete graph, i.e., player m gets $\nu_{m, h_{g^{\theta+ij}(\cdot)}^k}(\beta^k) = \phi_m^{g^N}$ for $m = i, j, l$,

where β^k is the proposal match that leads to g^N .

Suppose $i \neq j \neq l$, $i, j, l \in \{1, 2, 3\}$.

Let $\theta = ilj$. Suppose at history $h_{g^{\theta+ij}(\cdot)}^k$, β^k is not a proposal match and thus link ij does not form and hence, the game ends, then the stage payoffs for players i, j and l are given by

$$\begin{aligned}\nu_{i, h_{g^{\theta+ij}(\beta^{k-1})}^k}(\beta^k) &= \phi_i^{g^\theta}, \\ \nu_{j, h_{g^{\theta+ij}(\beta^{k-1})}^k}(\beta^k) &= \beta^{k-1}(j) \text{ and} \\ \nu_{l, h_{g^{\theta+ij}(\beta^{k-1})}^k}(\beta^k) &= \beta^{k-1}(l),\end{aligned}$$

where β^{k-1} is the *last proposal match* that occurred in stage $k - 1$ where the last link lj was accepted and thus g^θ formed.

Analogously, let $\theta = il$. Suppose that β^k is not a proposal match and that the game would end in such a case, then payoffs are

$$\begin{aligned}\nu_{i, h_{g^{\theta+ij}(\beta^{k-2})}^k}(\beta^k) &= \beta^{k-2}(i), \\ \nu_{j, h_{g^{\theta+ij}(\beta^{k-2})}^k}(\beta^k) &= \phi_j^{g^\theta} \text{ and} \\ \nu_{l, h_{g^{\theta+ij}(\beta^{k-2})}^k}(\beta^k) &= \beta^{k-2}(l),\end{aligned}$$

where $k - 2$ is the stage where the last proposal match, β^{k-2} , occurred.

Let $\theta = \emptyset$. Suppose that β^k is not a proposal match and that the game would end in such a case, then payoffs are

$$\nu_{h_{g^{\emptyset+13}(\cdot)}^3}(\beta^3) = (\phi_1^{g^\emptyset}, \phi_2^{g^\emptyset}, \phi_3^{g^\emptyset}) = (0, 0, 0),$$

where it is clear that $k = 3$ and that the first (second) component in the vector corresponds to player 1's (2's) payoff. In any other $k = 1, \dots, K$ stage history payoffs

are zero.

There is no discounting. Thus, player m 's, for $m = 1, 2, 3$, overall payoff at the h^{K+1} terminal history that has as past history the outcome where the game "ends" at stage $k \leq K$ with a graph g^θ , where $\theta = ilj$, is given by

$$u_m(h^{K+1}) = \nu_{m, h_{g^{\theta+ij}(\beta^{k-1})}^k}(\beta^k).$$

For example, $u_j(h^{K+1}) = \beta^{k-1}(j)$ (See above).

Analogously, if the game ends with a graph g^θ , where $\theta = il$, one has

$$u_m(h^{K+1}) = \nu_{m, h_{g^{\theta+ij}(\beta^{k-2})}^k}(\beta^k).$$

If the game ends with a graph g^θ , where $\theta = \emptyset$, one has

$$u(h^{K+1}) = \nu_{h_{g^{\emptyset+13(\cdot)}}^3}(\beta^3) = (\phi_1^{g^\emptyset}, \phi_2^{g^\emptyset}, \phi_3^{g^\emptyset}) = (0, 0, 0).$$

If the game ends with a graph $g^{\theta+ij} = g^N$ then

$$u_m(h^{K+1}) = \nu_{m, h_{g^{\theta+ij}(\cdot)}^k}(\beta^k) = \phi_m^{g^N}$$

where, as pointed out earlier on, β^k is a proposal match.

5.2.3 Existence of Multiple Equilibria

Whenever individual action sets are continuous in an extensive form game, there is no assurance there will be subgameperfect equilibria. In my case, one can show existence by construction. Wlg., one can restrict the search to pure strategies as mixed action stages, mixed payoff proposals, would have zero probability of inducing any payoff proposal match. Once this is done, its not hard to see that this "divide-the-dollar-like" multistage game has infinite equilibria.

5.2.4 Vectors of Correlated Strategies

I will be interested in defining negotiation statements as a vector of correlated strategies. As mixed payoff proposals would have zero probability of inducing any payoff proposal match, the following formalization that uses my definition of correlated strategies is wlg.

A vector of correlated strategies is a sequence of maps $\{\omega^k\}_{k=1}^K$, where each ω^k maps H^k to the set of correlated strategies on elements of B_{H^k} (i.e., $\omega^k(h^k)$ is a correlated strategy on B_{h^k} for all $h^k \in H^k$). I denote by $W|h^1$ the set of all vectors of correlated strategies in history h^1 .

Given $\omega|h^1 \in W|h^1$, I am interested in the probability $[\omega|h^1](s)$ of the path $(\beta^1, \beta^2, \dots, \beta^K)$ corresponding to strategy profile $s \in S$. This will be given by the expression

$$[\omega|h^1](s) = \omega_{h^1}^1(\beta^1) * \omega_{(\beta^1)}^2(\beta^2) * \omega_{(\beta^1, \beta^2)}^3(\beta^3) *, \dots, * \omega_{(\beta^1, \beta^2, \dots, \beta^{K-1})}^K(\beta^K).$$

Let $\omega|h^k \in W|h^k$ be the set of all vectors of correlated strategies in the subgame that begins in history h^k . It will be also of interest to know the probability $[\omega|h^k](s|h^k)$ of the path $(h^k, \beta^k, \dots, \beta^K)$ corresponding to the restriction $s|h^k$ of $s \in S$ on $M(h^k)$ for any $h^k \in H^k$ for all k . This will be given by the expression

$$[\omega|h^k](s|h^k) = \omega_{h^k}^k(\beta^k) * \omega_{(h^k, \beta^k)}^{k+1}(\beta^{k+1}) *, \dots, * \omega_{(h^k, \beta^k, \dots, \beta^{K-1})}^K(\beta^K).$$

5.3 Sequentially Nash Coherent Plans

I want to add endogenous effective Nash cooperation with communication—as defined at the end 3.3 based on 3.2.3—each time pairs of players discuss a link in the payoff relevant multistage game.

In order to do so, I assume that at every relevant "history" of the associated communication game a player $m = i, j, l$ that moves non trivially can formulate a negotiation statement using a partial communication technology. These statements are represented for simplicity by correlated strategies about immediate payoff proposals and future joint payoff relevant strategies in "corresponding histories" in the payoff relevant game. Different vectors of correlated strategies that are enunciated at different stages of the communication game by the same player may be implemented by having respectively different mediators that, at each stage, make (wlg.) a public announcement or recommendation observed by all players. For simplicity in the notation, the mediating technology will be "implicit".

Next, I denote the set of future joint payoff relevant strategies at stage $k = K$ in the histories where the immediate prospective graph is the complete one, that is, in $h_{g^N(\cdot)}^K$ with $g^N = g^{\theta+ij}$, as

$$\times Z_{h_{g^N(\cdot)}^K} = \prod_{\beta^K} \mathbf{0}^{[h_{g^N(\cdot)}^K, \beta^K]},$$

where $\beta^K \in B_{h_{g^N(\cdot)}^K}$. This is the Cartesian product of trivial action sets, three dimensional $\mathbf{0}$ -vectors, corresponding to all final histories $h_{(\cdot)}^{K+1} = [h_{g^N(\cdot)}^K, \beta^K]$ that follow if $h_{g^N(\cdot)}^K$ would be reached and some β^K is played.

If $h_{(\cdot)}^k$ is a trivial history where players move nothing, and indexing by an immediate prospective graph is not appropriate, and $k = K$, one has $\times Z_{h_{(\cdot)}^K} = \mathbf{0}$. If instead $k < K$ then $\times Z_{h_{(\cdot)}^k} = S| [h_{g^N(\cdot)}^k, \mathbf{0}]$, the restriction of S to history $[h_{g^N(\cdot)}^k, \mathbf{0}]$.

For all other histories, one has

$$\times Z_{h_{g^{\theta+ij}(\cdot)}^k} = \prod_{\beta^k} S| [h_{g^{\theta+ij}(\cdot)}^k, \beta^k].$$

Let $h_{(\cdot)}^{k'}/[h_{g^{\theta+ij}(\cdot)}^k, \beta^k]$, with $k' \geq k$, be a history *induced* by $[h_{g^{\theta+ij}(\cdot)}^k, \beta^k]$. Such a induced history is one that can be reached following $h_{g^{\theta+ij}(\cdot)}^k$ at substage k' and after β^k was played, including history $[h_{g^{\theta+ij}(\cdot)}^k, \beta^k]$.

It will be useful to group the components of $z^k \in \times Z_{h_{g^{\theta+ij}(\cdot)}^k}$ according to all histories induced by $[h_{g^{\theta+ij}(\cdot)}^k, \beta^k]$, where $\beta^k \in B_{h_{g^{\theta+ij}(\cdot)}^k}$ is given. For these purposes, I define the *group component*

$$z^k_{h^k_{(.)}/[h^k_{g^{\theta+ij}(.),\beta^k}]}$$

to be an element of

$$\times Z_{h^k_{(.)}/[h^k_{g^{\theta+ij}(.),\beta^k}]} = S| [h^k_{g^{\theta+ij}(.),\beta^k}].$$

Such a component is an element of the Cartesian product of the action sets of all such induced histories.

Note that

$$\times Z_{h^k_{(.)}/[h^k_{g^{\theta+ij}(.),\beta^k}]} = B[h^k_{g^{\theta+ij}(.),\beta^k}] \times \left(\times Z_{h^k_{g^{\theta+ij}(.),\beta^k}} \right).$$

This implies that a group component $z^k_{h^k_{(.)}/[h^k_{g^{\theta+ij}(.),\beta^k}]}$ is an element of the Carte-

sian product of the immediate payoff pair proposals set in history $[h^k_{g^{\theta+ij}(.),\beta^k}]$ and this history's set of future joint payoff relevant strategies.

Also, the component of $z^k \in \times Z_{h^k_{g^{\theta+ij}(.),\beta^k}}$ that stands for action sets in $h^k_{(.)}$ will be denoted as $z^k_{h^k_{(.)}}$.

Now one can define utility functions at earlier histories $h^k_{(.)}$ where the arguments are immediate payoff relevant actions and future joint payoff relevant strategies by using:

$$U_{h^k_{(.)}}(\beta^k, z^k) = u(s|h^k),$$

where $s|h^k = (\beta^k, z^k)$. This expression refers to the expected utilities for the three players if β^k and subsequently $z^k \in \times Z_{h^k_{(.)}}$ are played following $h^k_{(.)}$.

To formulate a negotiation problem, I assume that a *history of the communication game* (the multistage payoff relevant game with negotiation statements) $\hat{h}^k_{(.)}$ includes in the subscript $(.)$, in addition to a sequence of past payoff relevant actions, a sequence of past statements $(\mu^1, \dots, \mu^{k-1}) = \mu^{k-}$ and a sequence of past recommendations by different mediators. It is clear that any such history has a *corresponding history* in the payoff relevant game. For the most (See below), the notation on recommendations is left implicit. One should set $B = B_{h^k_{(.)}}$ and $\times Z = \times Z_{h^k_{(.)}}$ and $U = U_{h^k_{(.)}}$.

The negotiation problem is trivial in histories where players move nothing.

Wlg., and as a way of illustration, assume link ij is accepted, and the rule of order has next links il , and jl being proposed in that order, link il is rejected and link jl is accepted. One defines O-F plans in histories $\hat{h}^k_{g^{ijl+il}(.)}$.

The set of future joint strategies $\times Z_{\hat{h}^k_{g^{ijl+il}(.)}} = \times Z_{h^k_{g^{ijl+il}(.)}}$ is the Cartesian product of infinitely many one point sets (Note that $B_{\hat{h}^k_{g^{ijl+il}(.)}} = B_{h^k_{g^{ijl+il}(.)}}$). So the tenability correspondence in a history $\hat{h}^k_{g^{ijl+il}(.)}$ of the communication game is trivially defined as

$$Q_{\hat{h}^k_{g^{ijl+il}(.)}}(\mu^k) = \mathcal{U}^{\times Z_{\hat{h}^k_{g^{ijl+il}(.)}}}.$$

Mostly, I will write only $\mathcal{U}^{\times Z^k}$, in this case a singleton, an infinite Cartesian product of functions that put probability one on the unique element of the trivial action set profiles at each future history of the game without communication (This latter game is used for simplicity in the notation. See below). One should set the tenability correspondence $Q_{\mu^k-}(\mu)$, in section 3, equal to $Q_{\hat{h}_{g^{ijl+il}(\cdot)}^k}(\mu^k)$.

For any associated given strategic form game $(B_i^k \times B_j^k, \pi^{\mu^k})$, where $\mu^k = (\sigma^k, \zeta^k) \in \mathcal{U}^k$, to be well defined, one sets for any given $\beta^k \in B_{h_{g^{\theta+ij}(\cdot)}^k}$ (or $B_{\hat{h}_{g^{ijl+il}(\cdot)}^k}$) and the unique trivial $z^k \in \times Z^k$

$$[\zeta^k(z^k) | \beta^k] = 1.$$

Recall that $[\zeta^k(z^k) | \beta^k]$ is the probability that $U^k(\beta^k, z^k)$ is obtained if play is according to a given ζ and (β_i, β_j) is played.

The outside options in the associated plan bargaining problem are $\psi^k = (x_i^k, x_l^k)$ that has as associated *degenerate* disagreement plan $\hat{\mu}^k = (\hat{\sigma}_{\hat{\beta}^k}^k, \hat{\zeta}^k)$ where $\hat{\sigma}_{\hat{\beta}^k}^k$ is a *degenerate correlated strategy* that puts probability 1 on $\hat{\beta}^k$ which is composed by given (wlg.) unilateral rejections. As deviating yields the same expected payoffs, $\hat{\beta}_{il}^k$ is a Nash equilibrium of $(B_i^k \times B_l^k, \pi^{\hat{\mu}^k})$, so $\hat{\mu}^k$ is tenable and reliable and hence the outside options belong to the feasible set.

One completes the formulation of the negotiation problem in the notation of section 3 in history $\hat{h}_{g^{ijl+il}(\cdot)}^k$, if the sequence of past statements is given by μ^{k-} , by setting $\Phi_{il, \mu^{k-}} = \Phi_{\hat{h}_{g^{ijl+il}(\cdot)}^k}$ or simply Φ^k .

Recall that to each history in the communication game $\hat{h}_{g^{ijl+il}(\cdot)}^k$, there are associated future-requests by pairs ij and jl that formed in that order. By assumption 2, the ones of pair (i, l) that rejected its link are ignored. Suppose that $(F, \Phi, \psi)_{\hat{h}_{g^{ijl+il}(\cdot)}^k}$ is well defined. Assumption 1 and 2 ensure that $\eta_{\hat{h}_{g^{ijl+il}(\cdot)}^k}^f(\Phi^k, \psi^k)$ can be defined and exists for any such possible history.

In general, suppose that one has inductively defined a non empty O-F plan set in any $\hat{h}_{g^{\theta+ij}(\cdot)}^k$, denoted by $\eta_{\hat{h}_{g^{\theta+ij}(\cdot)}^k}^f(\Phi^k, \psi^k) \neq \emptyset$, for all $\theta \in \{\emptyset, il, ilj\}$ $i \neq j \neq l$, $i, j, l \in \{1, 2, 3\}$.

Let $\mu^k = (\sigma^k, \zeta^k)$. As it was done with $z^k \in \times Z_{h_{g^{\theta+ij}(\cdot)}^k}$, one groups the components of $\zeta^k \in \mathcal{U}^{\times Z^k}$ according to all histories *induced* by $[h_{g^{\theta+ij}(\cdot)}^k, \beta^k]$, where $\beta^k \in B_{h_{g^{\theta+ij}(\cdot)}^k}$ is given. Formally, one lets ζ^k be an element of

$$\prod_{\beta^k \in B_{h_{g^{\theta+ij}(\cdot)}^k}} W | [h^k, \beta^k]$$

and denotes the β^k -component of ζ^k by

$$\zeta_{h_{(\cdot)}^{k'}/[h_{g^{\theta+ij}(\cdot),\beta^k}^k]}^k \in W| [h^k, \beta^k],$$

where $W| [h^k, \beta^k]$ is the set of vectors of correlated strategies in the subgame in history $[h^k, \beta^k]$. Also the component of ζ^k associated to a correlated in the action profile set in $h_{(\cdot)}^{k'}$ will be denoted as $\zeta_{h_{(\cdot)}^{k'}}$.

Let $\mu^k = (\sigma^k, \zeta^k)$ have $\zeta^k \in Q_{h_{g^{\theta+ij}(\cdot)}^k}(\mu^k)$, that is, μ^k is tenable. The future-request ζ^k should be such that for any $\beta^k \in B_{h_{g^{\theta+ij}(\cdot)}^k}$

$$\zeta_{h_{(\cdot)}^{k'}/[h_{g^{\theta+ij}(\cdot),\beta^k}^k]}^k \in \eta_{h_{(\cdot)}^{k+1}}^f(\Phi^{k+1}, \psi^{k+1}),$$

where $\hat{h}_{(\cdot)}^{k+1} = [\hat{h}_{g^{\theta+ij}(\cdot)}^k, \beta^k, \mu^k, r^k]$, for all recommendations $r^k \in B_{h_{g^{\theta+ij}(\cdot)}^k}$. That is, any group component of ζ^k , should equal the identical O-F plans in the histories that follow $\hat{h}_{g^{\theta+ij}(\cdot)}^k$, or better yet $\hat{h}_{g^{\theta+ij}(\mu^{k-})}^k$, after players i and j formulated plan μ^k , and actions played were β^k and any recommendation occurred. Such O-F plans are required to be identical because the assumption is that correlated strategies in future contingencies as of history $\hat{h}_{g^{\theta+ij}(\mu^{k-})}^k$ in the communication game may only differ depending on past sequences of payoff relevant actions, that is, they are the same irrespective of past recommendations by different mediators and these recommendations' corresponding past negotiation statements by pairs different and younger than the current pair in $\hat{h}_{g^{\theta+ij}(\mu^{k-})}^k$.⁷ From now on, to save on notation, I will ignore indexing histories in the communication game explicitly by past recommendations and write instead $[\hat{h}_{g^{\theta+ij}(\cdot)}^k, \beta^k, \mu^k]$, whenever indexing by μ^{k-} is not relevant.

It is implicit that if one of such histories $\hat{h}_{(\cdot)}^{k+1}$ may be such that the game ends if link ij is rejected, $\eta_{h_{(\cdot)}^{k+1}}^f(\Phi^{k+1}, \psi^{k+1}) = \mathcal{U}^k$, a trivial plan, as the set of stage actions thereafter is a vector of zeros. The same is done for other trivial histories where players move nothing.

By the inductive assumption $Q_{h_{g^{\theta+ij}(\cdot)}^k}(\mu^k) \neq \emptyset$.

For any associated given strategic form game $(B_i^k \times B_j^k, \pi_{ij}^{\mu^k})$, where $\mu^k = (\sigma^k, \zeta^k) \in \mathcal{U}^k$, to be well defined, one sets for any given $\beta^k \in B_{h_{g^{\theta+ij}(\cdot)}^k}$ and $z^k \in \times Z^k$

$$[\zeta^k(z^k) | \beta^k] = [\omega | h^k](s | h^k),$$

where the vector of correlated strategies $\omega | h^k = \ddot{\mu}^k = (\ddot{\sigma}_{\beta^k}^k, \zeta^k)$ puts probability 1 on β^k in the immediate proposal game and is consistent with ζ^k thereafter. Also, $s | h^k = (\beta^k, z^k)$.

As before, the outside options in the associated plan bargaining problem are $\psi^k =$

⁷It is clear that correlated strategies may be different depending on the μ^{k-} associated to $\hat{h}_{g^{\theta+ij}(\mu^{k-})}^k$.

(x_i^k, x_j^k) that has as an associated tenable and reliable $\hat{\mu}^k = (\hat{\sigma}_{\hat{\beta}^k}, \hat{\zeta}^k)$, i.e., $\hat{\mu}^k \in \tilde{\mathcal{U}}^k$, where $\hat{\beta}^k$ is composed, wlg., by given unilateral rejections.

In general, history $\hat{h}_{g^{\theta+ij}(\cdot)}^k$ has future-requests by pairs corresponding to links in θ that formed in a given order. The ones of pairs that rejected a link are ignored. Assume that $(F, \Phi, \psi)_{\hat{h}_{g^{\theta+ij}(\cdot)}^k}$ is well defined. Assumption 1 and 2 ensure that $\eta_{\hat{h}_{g^{\theta+ij}(\cdot)}^k}^f(\Phi, \psi)$ exists for any possible history.

Suppose O-F plans exist for all histories, only then our inductive assumption is justified. Then, one says that sequentially Nash coherent plans exist at the beginning of play.

Basically, to check for existence of sequentially Nash coherent plans, it suffices to show that the feasible set in plan bargaining games in all histories is closed. The proof in section 6 is by construction.

6 Existence Theorem

In each history $\hat{h}_{g^{\theta+ij}(\cdot)}^k$, it will be useful to distinguish tenable expected payoffs that are feasible by matching payoff proposals and hence forming a link. Formally, $(x_i^k, x_j^k) \in PMF^k$, the *proposal match payoff feasible set* in $\hat{h}_{g^{\theta+ij}(\cdot)}^k$, if there exists a payoff proposal match $\beta^k \in B_{h_{g^{\theta+ij}}^k}$ and $\mu^k = (\sigma_{\beta^k}^k, \zeta^k) \in \underline{\mathcal{U}}^k$, i.e., μ^k is tenable, such that $\pi_{ij}^{\mu^k}(\beta_{ij}^k) = (x_i^k, x_j^k)$. The set of strong Pareto efficient points of PMF^k is the frontier of PMF^k .

Plan bargaining problems $(F, \Phi, \psi)_{\hat{h}_{g^{\theta+ij}(\cdot)}^k}$ are classified in two types. In *type 1*, there exists a better payoff proposal match.

There exists a *better payoff proposal match* if some element of PMF^k is as least as good as the outside options, that is, $(x_i^k, x_j^k) \geq \pi_{ij}^{\hat{\mu}^k}(\hat{\beta}_{ij}^k)$, for some $(x_i^k, x_j^k) \in PMF^k$.

Note that the β_{ij}^k associated to a better payoff proposal match is a Nash equilibrium of $(B_i^k \times B_j^k, \pi^{\mu^k})$ and so the plan $\mu^k = (\sigma_{\beta^k}^k, \zeta^k)$ is tenable and reliable. By definition, such (x_i^k, x_j^k) belongs to the feasible set. Any other action profile $\ddot{\beta}^k$ type not analyzed so far has $\ddot{\beta}_{ij}^k$ that is not a Nash equilibrium of $(B_{ij}^k \times B_j^k, \pi^{\ddot{\mu}^k})$. Thus, feasible sets are convex combinations of outside options and payoffs associated to better payoff proposal matches. Such convex combinations have corresponding non degenerate tenable and reliable plans.

In *type 2*, there are not better payoff proposal matches and the only feasible payoff pair is the one associated to the disagreement plan, hence the associated link will not form. Payoff proposal matches associated to elements in PMF^k don't induce Nash equilibria in their associated $(B_i^k \times B_j^k, \pi^{\mu^k})$ as it is always better to unilateral reject.

As it will become clearer in the construction proof, the existence of these two types imply that feasible and *IRF* sets in each history coincide.

Theorem 1: Sequentially Nash coherent plans exist for three-player normalized cooperative games with the Myerson value as a payoff allocation rule.

Proof:

As feasible and *IRF* sets coincide, for the O-F bargaining game to be well defined at the initial history it suffices to show that the *IRF* sets are closed in any possible future history.

1. The Plan Bargaining Problem in $\hat{h}_{g^{132+12}(\cdot)}^6$

Outside options in histories with the same last proposal match $\beta^5, \hat{h}_{g^{132+12}(\beta^5)}^6$, are $(\psi_1^6, \psi_2^6) = (\phi_1^{132}, \beta^5(2))$. The PMF^6 consists only of payoffs in the complete graph $(\phi_1^{g^N}, \phi_2^{g^N})$.

Note that player 1 can in general do better in the complete graph because $\phi_1^{132} \leq \phi_1^{g^N} \Leftrightarrow 0 \leq c$, (See diagram in Appendix). Recall that $\beta^5(3) + \beta^5(2) = \phi_3^{g^{132}} + \phi_2^{g^{132}}$. Denote $\phi_3^{g^{132}} + \phi_2^{g^{132}} - \phi_2^{g^N} = \frac{2d+a+b-c}{6} > 0$ by $\bar{\beta}^5(3)$ and $\phi_2^{g^N}$ by $\bar{\beta}^5(2)$.

Thus, the bargaining game is of type 2 iff $\beta^5(2) > \bar{\beta}^5(2) = \phi_2^{g^N}$. The IRF^6 consists just of ψ_{12}^6 . Otherwise, the IRF^6 contains the unique element of the PMF^6 , $(\phi_1^{g^N}, \phi_2^{g^N})$, that now is associated to a better proposal match, the one that leads to link 12 forming and hence the complete graph. The IRF^6 consists of convex combinations of the outside options (ψ_1^6, ψ_2^6) and $(\phi_1^{g^N}, \phi_2^{g^N})$.

In any case, the IRF^6 is closed, thus, assumption 1 and 2 ensure that for any $\hat{h}_{g^{132+12}(\cdot)}^6$ one can compute $\eta_{\hat{h}_{g^{132+12}(\cdot)}^6}^f(\Phi^6, \psi^6)$.

2. The Plan Bargaining Problem in $\hat{h}_{g^{13+32}(\cdot)}^5$

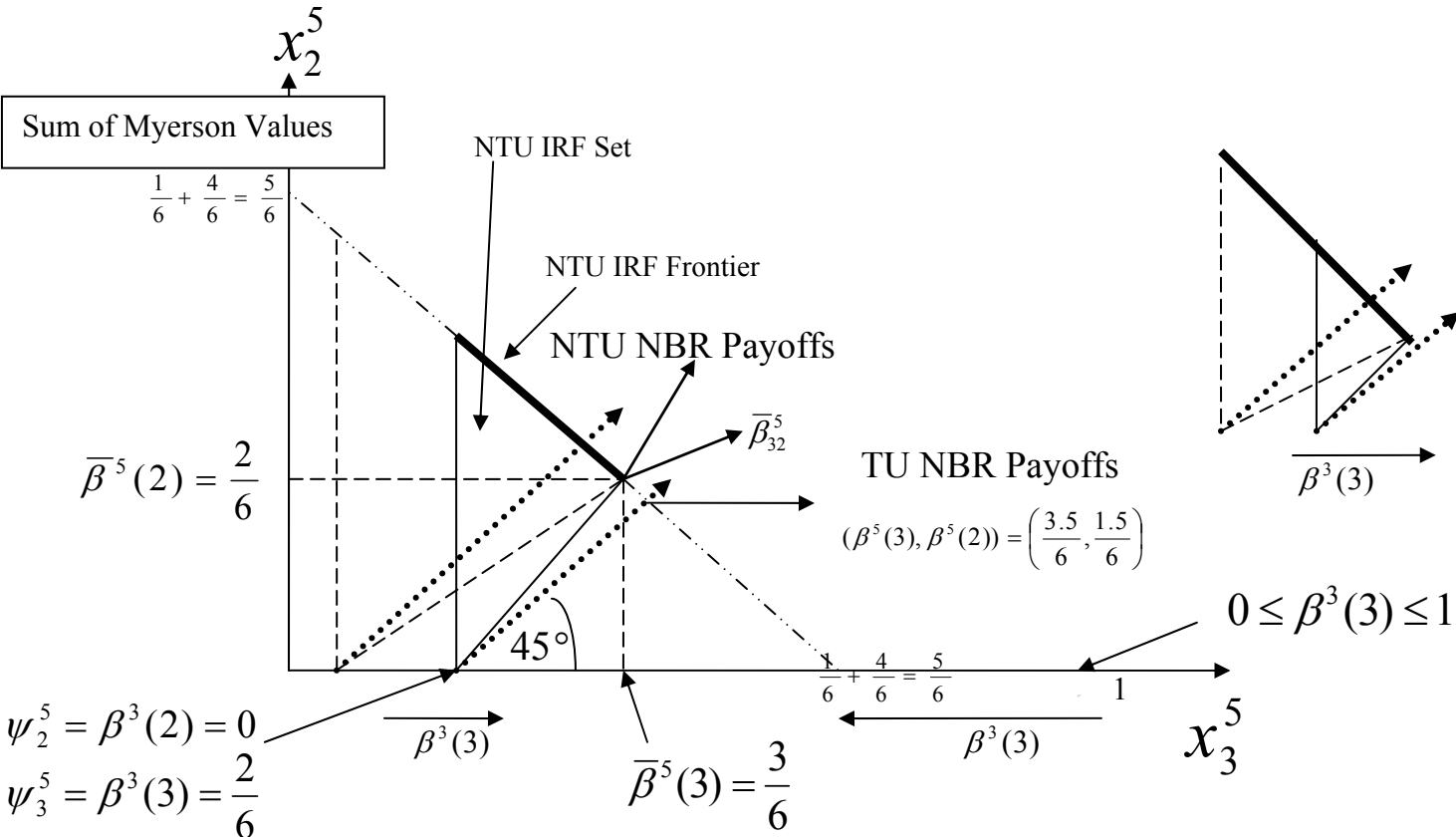
Outside options in histories $\hat{h}_{g^{13+32}(\beta^3)}^5$ are $(\psi_3^5, \psi_2^5) = (\beta^3(3), \phi_2^{g^{13}} = 0)$.

It suffices to check that the IRF^5 is closed in bargaining games of type 1. In what follows of (2) I assume that $\beta^3(3)$ induces such type. Let $\check{\mu}^5 = (\check{\sigma}_{\check{\beta}^5}^5, \check{\zeta}^5) \in \tilde{U}^5$, where $\check{\mu}^k$ stands for a tenable and reliable plan in stage k . Assume history $\hat{h}_{g^{132+12}(\check{\mu}^5, \check{\beta}^5, \beta^3)}^6$ is reached.

From 1, whenever $\check{\beta}^5(2) > \bar{\beta}^5(2)$, $\check{\mu}^6 \in \eta_{\hat{h}_{g^{132+12}(\check{\mu}^5, \check{\beta}^5, \beta^3)}^6}^f(\Phi^6, \psi^6)$ is a disagreement plan, i.e., $\check{\mu}^6 = \hat{\mu}^6$. By assumption, player 3 and 2's $\check{\mu}^5 = (\check{\sigma}_{\check{\beta}^5}^5, \check{\zeta}^5)$ has to future-request $\hat{\mu}^6$, formally, $\check{\zeta}_{h(\cdot)/[h_{g^{13+32}(\beta^3)}^5, \check{\beta}^5]}^5 = \hat{\mu}^6$. Associated payoffs $\pi_{32, \hat{h}_{g^{13+32}(\beta^3)}^5}^{\check{\mu}^5}(\check{\beta}_{32}^5) = \check{\beta}_{32}^5$ are illustrated in figure 1 by the segment in bold not including $\bar{\beta}_{32}^5$ (Note that in the simple majority game $c > 0$ and $\bar{\beta}^5(2) = \phi_2^{g^N} = \frac{2}{6}$).

Assume that $c = 0$ (See theorem 2 for the case $c > 0$) and hence player 1 in

Players 3 and 2's Bargaining Game-Figure 1



Link 12 and 23 were rejected in stage 1 and 2 respectively. In stage 3, link 13 formed with payoff proposal match β^3 with $(\beta^3(1), \beta^3(3)) = (\frac{4}{6}, \frac{2}{6})$. Link 12 was rejected. In stage 5, link 32 is proposed with induced outside options (ψ_3^5, ψ_2^5) . Link 32 is last to form with payoffs (x_3^5, x_2^5) equal to an associated proposal match with $(\beta^5(3), \beta^5(2)) = (\frac{3}{6}, \frac{2}{6}) = \bar{\beta}_{32}^5$.

such $\overset{\circ}{h}^6$ is indifferent to further linking. If $\check{\mu}^5$ has $\check{\beta}^5(2) \leq \bar{\beta}^5(2)$, the bargaining game in $\overset{\circ}{h}_{g^{132+12}}^6(\check{\mu}^5, \check{\beta}^5, \beta^3)$ is of type 1, however, it is not essential. By assumption 1, future-requests in $\check{\mu}^5$ of O-F plans in $\overset{\circ}{h}_{g^{132+12}}^6(\check{\mu}^5, \check{\beta}^5, \beta^3)$ depend on μ^3 , so it is useful to write $\overset{\circ}{h}_{g^{132+12}}^6(\mu^3, \check{\mu}^5, \check{\beta}^5, \beta^3)$. If $\check{\mu}^6$ is an O-F plan, then its promise-request $\check{\sigma}^6$ may entail a payoff proposal match, the given unilateral rejection or a "mix".

At first glance, there may be a jump in payoffs whenever $\check{\beta}^5(2) = \bar{\beta}^5(2)$ depending on μ^3 . However, in any case, payoffs are always $\pi_{32, \overset{\circ}{h}_{g^{13+32}}^5(\mu^3, \beta^3)}^{\check{\mu}^5}(\check{\beta}_{32}^5) = (\bar{\beta}^5(3), \bar{\beta}^5(2))$.

If $\check{\beta}^5(2) < \bar{\beta}^5(2)$ payoffs $\pi_{32, \overset{\circ}{h}_{g^{13+32}}^5(\mu^3, \beta^3)}^{\check{\mu}^5}(\check{\beta}_{32}^5)$ are equal to convex combinations between $(\bar{\beta}^5(3), \bar{\beta}^5(2))$ and $(\check{\beta}^5(3), \check{\beta}^5(2))$ depending on μ^3 .

It follows that in any $\overset{\circ}{h}_{g^{13+32}}^5(\mu^3, \beta^3)$, the IRF^5 is closed. Also, the IRF^5 frontier has right side endpoint $(\bar{x}_3^5, \bar{x}_2^5)$ in the plane (x_3^5, x_2^5) equal to or to the southeast of $\bar{\beta}_{32}^5$ (depending on μ^3). Thus, $\eta_{\overset{\circ}{h}_{g^{13+32}}^5(\mu^3, \beta^3)}^f(\Phi^5, \psi^5)$ can be computed.

3. Plan Bargaining Problem in $\overset{\circ}{h}_{g^{13+12}}^4(\cdot)$

The outside options in any $\overset{\circ}{h}_{g^{13+12}}^4(\cdot)$ depend on β^3 and μ^3 (See 2) as follows:

$$\psi_{12}^4 = \pi_{12, \overset{\circ}{h}_{g^{13+12}}^4(\beta^3, \mu^3)}^{\hat{\mu}^4}(\hat{\beta}_{12}^4) = \sum_{\beta^5} \check{\sigma}^5(\beta^5) \pi_{12, \overset{\circ}{h}_{g^{13+32}}^5(\beta^3, \mu^3, \hat{\mu}^4, \hat{\beta}^4)}^{\check{\mu}^5}(\beta_{32}^5) \quad (*)$$

where $\check{\mu}^5 = \hat{\zeta}_{h_{(\cdot)}^4 / [h_{g^{13+12}}^4(\beta^3), \hat{\beta}^4]}$, the group component of $\hat{\zeta}^4$ that contains correlated strategies in histories induced by $[h_{g^{13+12}}^4(\beta^3), \hat{\beta}^4]$. Also, as $\hat{\mu}^4$ is tenable, $\check{\mu}^5$ is O-F in $\overset{\circ}{h}_{g^{13+32}}^5(\beta^3, \mu^3, \hat{\mu}^4, \hat{\beta}^4)$, i.e.,

$$\check{\mu}^5 \in \eta_{\overset{\circ}{h}_{g^{13+32}}^5(\beta^3, \mu^3, \hat{\mu}^4, \hat{\beta}^4)}^f(\Phi^5, \psi^5).$$

Analogously as in $\overset{\circ}{h}_{g^{13+32}}^5(\cdot)$, one can prove (not straightforward) that the IRF^4 set is always closed in any $\overset{\circ}{h}_{g^{13+12}}^4(\cdot)$, assuming now $b = 0$. Hence, one can compute $\eta_{\overset{\circ}{h}_{g^{13+12}}^4(\cdot)}^f(\Phi^4, \psi^4)$.

4. The Plan Bargaining Problem in $\overset{\circ}{h}_{g^{\varnothing+13}}^3(\cdot)$

Players 1 and 2s' outside options are $\psi_{13}^3 = (\phi_1^{g^{\varnothing}}, \phi_3^{g^{\varnothing}}) = (0, 0)$. As for (1), (2) and (3), \tilde{U}^3 can be derived. I argue that the IRF^3 set and frontier are closed if payoffs in the IRF^3 set are continuous on "appropriate subsets" of the tenable and reliable set \tilde{U}^3 composed of degenerate plans $\check{\mu}^3 = (\check{\sigma}_{\check{\beta}^3}^3, \check{\zeta}^3)$. It can be shown that all

associated payoffs in such subsets correspond to all what is achievable by degenerate elements of \tilde{U}^3 . Assume \tilde{U}^3 is known.

Back to $\overset{\circ}{h}^5_{g^{13+32}(\check{\mu}^3, \check{\beta}^3, \hat{\mu}^4, \hat{\beta}^4)}$

As $\overline{x}_3^5 \geq \overline{\beta}^5(3) > 0$ and $\check{\beta}^3(1) + \check{\beta}^3(3) = \phi_1^{g^{13}} + \phi_3^{g^{13}} = a \geq 0$, different $\check{\mu}^3 \in \tilde{U}^3$ that differ in $\check{\beta}^3$ induce a total of three *classes* of bargaining games (See figure 1, albeit $c > 0$):

Class 1: If $\check{\beta}^3$ is such that $\psi_3^5 = \check{\beta}^3(3) = \overline{x}_3^5$, player 3 will be indifferent between forming or not link 32. The bargaining game in $\overset{\circ}{h}^5_{g^{13+32}(\check{\mu}^3, \check{\beta}^3, \hat{\mu}^4, \hat{\beta}^4)}$ will be not essential but it is of type 1.

Class 2: If $\check{\beta}^3$ is such that $\check{\beta}^3(3) < \overline{x}_3^5$, then the bargaining game is of type 1 and agent 3 is better off if link 32 forms.

Class 3: If $\check{\beta}^3$ is such that $\check{\beta}^3(3) > \overline{x}_3^5$, then the bargaining game is of type 2.

The case $a = \overline{x}_3^5$ exhibits the first two classes of bargaining games. If $a < \overline{x}_3^5$ then only the second class results. The case $a > \overline{x}_3^5$ exhibits the three classes.

Depending on these three ranges of a , one needs to consider at most two "types of families" of subsets of \tilde{U}^3 . Wlg., I focus on the case $a > \overline{x}_3^5$ where one can distinguish two types of families.

Consider the expected payoff function associated to the O-F plan

$$\check{\mu}^5 = (\check{\sigma}^5, \check{\zeta}^5) \in \eta_{\overset{\circ}{h}^5_{g^{13+32}(\check{\mu}^3, \check{\beta}^3, \hat{\mu}^4, \hat{\beta}^4)}}^f(\Phi^5, \psi^5).$$

This is given by

$$\sum_{\beta^5} \check{\sigma}^5(\beta^5) \pi_{12, \overset{\circ}{h}^5_{g^{13+32}(\check{\mu}^3, \check{\beta}^3, \hat{\mu}^4, \hat{\beta}^4)}}^{\check{\mu}^5}(\beta_{32}^5) \quad (**).$$

As \tilde{U}^3 is known, one can redefine $\check{\mu}^5$ (including $\check{\sigma}^5$) and $\check{\beta}^3$, wlg., as some given auxiliary function of degenerate plans $\check{\mu}^3 = (\check{\sigma}_{\check{\beta}^3}^3, \check{\zeta}^3) \in \tilde{U}^3$, where $\check{\zeta}^3$ future-requests $\check{\mu}^5$, and $\hat{\mu}^4$ and $\hat{\beta}^4$ are redefined as constant functions of $\check{\mu}^3$. Reinterpret from now on this payoff function as a function of only $\check{\mu}^3$.

Denote a generic subset of a *first type of family of subsets* of \tilde{U}^3 by $\overrightarrow{\tilde{U}}^3(Q^5)$. For any $\overrightarrow{\tilde{U}}^3(Q^5)$, if $\check{\mu}^3 \neq \check{\mu}^3$ and $\check{\mu}^3, \check{\mu}^3 \in \overrightarrow{\tilde{U}}^3(Q^5)$, the respective induced tenability correspondences are such that $\check{Q}^5 = \check{Q}^5 = Q^5$. Respective future-requests have O-F plans $\check{\mu}^5$ and $\check{\mu}^5$ with $\check{\zeta}^5 = \check{\zeta}^5 = \zeta^5(Q^5)$, a function of Q^5 , and with promise-requests that may be mixes over proposal matches only, that is, any $\beta \in B_{\overset{\circ}{h}^5_{g^{13+32}(\cdot)}}$ that has $\check{\sigma}^5(\beta) > 0$ ($\check{\sigma}^5(\beta) > 0$) is a proposal match. Also, $\hat{\beta}^3(3) \neq \check{\beta}^3(3)$ belong to the closed interval $[0, \overline{x}_3^5]$. Recall from 2 that Q^5 depends on $\check{\mu}^3$, hence there is a family of subsets of the first type. Each subset in the family is indexed by Q^5 . Assume for now that appropriate subsets of $\overrightarrow{\tilde{U}}^3(Q^5)$ exist with an appropriate metric. One can

show (See below) that the payoff function above in equation (**) is continuous on any appropriate subset of $\overrightarrow{\mathcal{U}}^3(Q^5)$.

As player 1's payoff is always constant and equal to $\varphi^{g^{132}}$, one has also that

$$\sum_{\beta^5} \check{\sigma}^5(\beta^5) \pi_{\check{h}^5}^{\check{\mu}^5}_{g^{13+32}(\check{\mu}^3, \check{\beta}^3, \hat{\mu}^4, \hat{\beta}^4)}(\beta_{32}^5),$$

with range on \mathbb{R}^3 , is continuous on any appropriate subset of $\overrightarrow{\mathcal{U}}^3(Q^5)$.

By analogous arguments, this latter payoff function is continuous on any appropriate subset of $\overleftarrow{\mathcal{U}}^3(\check{Q}^5)$ in a second type of family that is derived almost identically as before: Any such $\overleftarrow{\mathcal{U}}^3(\check{Q}^5)$ has instead $\check{\sigma}_{\check{\beta}^5}^5 = \check{\sigma}_{\check{\beta}^5}^5 = \hat{\sigma}_{\check{\beta}^5}^5$, that is, future-requests in $\check{\mu}^3$ and $\check{\mu}^3$ have O-F plans $\check{\mu}^5$ and $\check{\mu}^5$ that promise-request the disagreement plan. Also, $\check{\beta}^3(3) \neq \hat{\beta}^3(3) \in [\bar{x}_3, a]$.

Graphically, as long as $\check{\beta}^3(3) \neq \hat{\beta}^3(3)$ different plans, $\check{\mu}^3$ and $\check{\mu}^3$, in any given appropriate subset of $\overrightarrow{\mathcal{U}}^3(Q^5)$ ($\overleftarrow{\mathcal{U}}^3(\check{Q}^5)$) induce bargaining games with the same PMF^5 frontier but with different outside options that move along the horizontal axis in the plane (x_3^5, x_2^5) . (See figure 1, albeit $c > 0$)

Back to $\check{h}^4_{g^{13+12}(\check{\mu}^3, \check{\beta}^3)}$

Using equation (*), one obtains

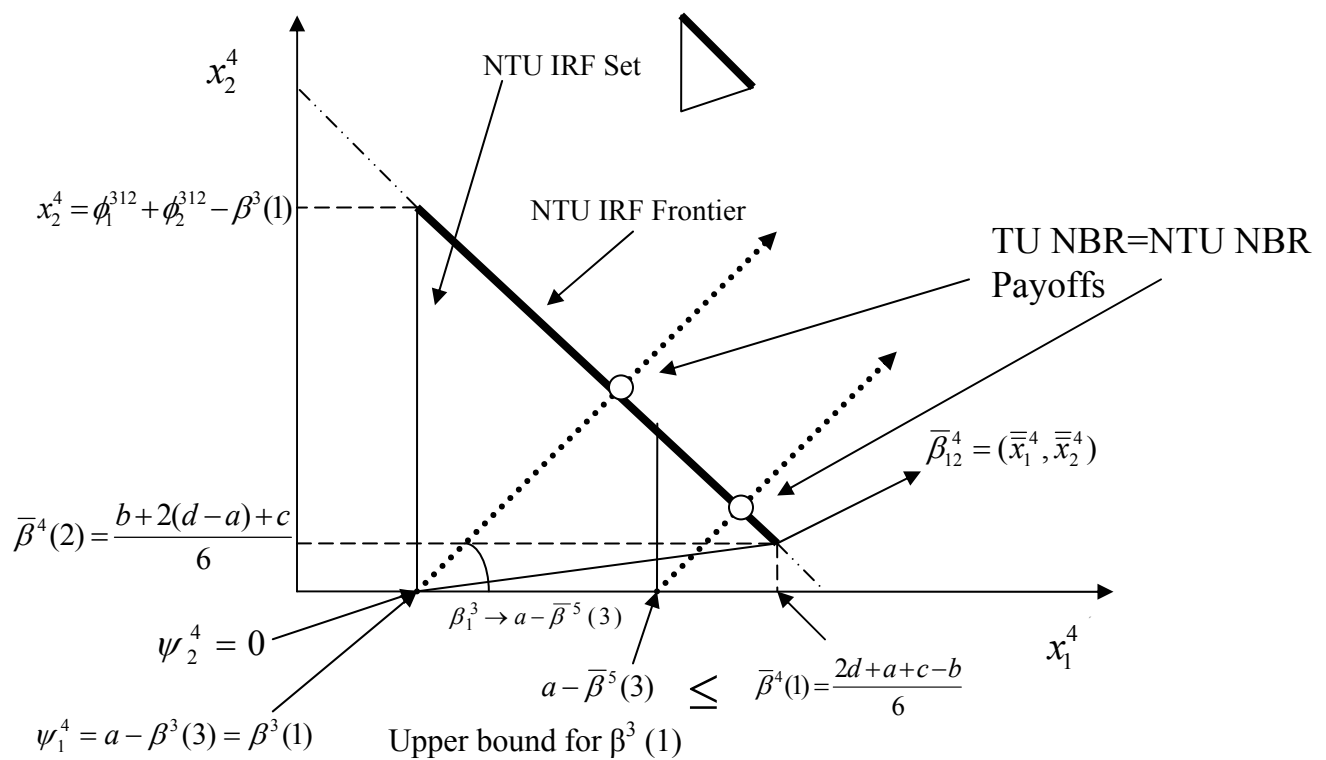
$$\psi_{12}^4 = \pi_{12, \check{h}^4}^{\hat{\mu}^4}_{g^{13+12}(\check{\mu}^3, \check{\beta}^3)}(\hat{\beta}_{12}^4) = \sum_{\beta^5} \check{\sigma}^5(\beta^5) \pi_{12, \check{h}^5}^{\check{\mu}^5}_{g^{13+32}(\check{\mu}^3, \check{\beta}^3, \hat{\mu}^4, \hat{\beta}^4)}(\beta_{32}^5)$$

As the last expression in this equation is now a function of $\check{\mu}^3$, outside options ψ_{12}^4 are continuous on any appropriate subset of $\overrightarrow{\mathcal{U}}^3(Q^5)$ ($\overleftarrow{\mathcal{U}}^3(\check{Q}^5)$).

I proceed by constructing appropriate subsets of $\overrightarrow{\mathcal{U}}^3(Q^5)$. Player 2's outside option, ψ_2^4 weakly increases (while $\psi_1^4 \leq \bar{\beta}^4(1)$ is constant, where $\bar{\beta}^4$ is defined analogously as $\bar{\beta}^5$ is) whenever $\check{\mu}^3 \in \overrightarrow{\mathcal{U}}^3(Q^5)$ has a lower $\check{\beta}^3(3) \in [0, \bar{\beta}^5(3)]$. Thus, there may exist some $\check{\beta}^3(3)$ where player 1 is indifferent between linking or not with agent 2. As before one may distinguish 3 classes of bargaining games depending on parameter cases. Also, one may have to distinguish two different *sub-types of conditional* (on Q^5) *families of subsets of* $\overrightarrow{\mathcal{U}}^3(Q^5)$, where subsets in these sub-types of conditional families are denoted by either $\overrightarrow{\mathcal{U}}^{3.1}(Q^5, Q^4)$ or $\overrightarrow{\mathcal{U}}^{3.2}(Q^5, Q^4)$. These subsets will have elements $\check{\mu}^3 \in \overrightarrow{\mathcal{U}}^3(Q^5)$ that future-request on players 1 and 2 a mix over proposal matches and a disagreement plan respectively and are defined as *appropriate* subsets of $\overrightarrow{\mathcal{U}}^3(Q^5)$.

Consider payoffs associated to O-F plan $\check{\mu}^4 \in \eta_{\check{h}^4}^f_{g^{13+12}(\check{\mu}^3, \check{\beta}^3)}(\Phi^4, \psi^4)$

Players 1 and 2's Bargaining Game-Figure 2



$$\sum_{\beta^5} \check{\sigma}^4(\beta^4) \pi_{12, \check{h}^4}^{\check{\mu}^4}_{g^{13+12}(\check{\mu}^3, \check{\beta}^3)} \left(\check{\beta}_{12}^4 \right).$$

As before, after reinterpreting the payoff expression, one can show that this is continuous on any appropriate $\overrightarrow{\mathcal{U}}^{3.1}(Q^5, Q^4)$ or $\overrightarrow{\mathcal{U}}^{3.2}(Q^5, Q^4)$. Informally, the outside options ψ_{12}^4 are continuous on the latter appropriate sets and the NBR payoffs in the associated bargaining games are continuous on the outside options for a fix PMF^4 frontier (Recall, composition of continuous functions are continuous). So is

$$\sum_{\beta^5} \check{\sigma}^4(\beta^4) \pi_{\check{h}^4}^{\check{\mu}^4}_{g^{13+12}(\check{\mu}^3, \check{\beta}^3)} \left(\check{\beta}_{12}^4 \right)$$

with range in \mathbb{R}^3 .

Analogously, this payoff function is continuous, if necessary, on any element of two sub-types of conditional families of subsets of $\overleftarrow{\mathcal{U}}^3(\check{Q}^5)$, denoted either by $\overleftarrow{\mathcal{U}}^{3.1}(\check{Q}^5, \check{Q}^4)$ or $\overleftarrow{\mathcal{U}}^{3.2}(\check{Q}^5, \check{Q}^4)$. (Figure 2, where $(\overline{x}_1, \overline{x}_2) = \overline{\beta}^4$, is useful to illustrate this claim's proof).

It follows that $\pi_{13, \check{h}^3}^{\check{\mu}^3}_{g^{\varnothing+13}(\cdot)}(\beta^3)$ is continuous if necessary on any element on these four conditional families of subsets, for any possible $Q^5(\check{Q}^5)$.

Note that as $\check{\beta}^3$ varies along a closed interval associated with any given such appropriate subset, the only components of $\check{\mu}^3 = (\check{\sigma}_{\check{\beta}^3}^3, \check{\zeta}^3)$ that vary are $\check{\sigma}_{\check{\beta}^3}^3, \check{\zeta}_{h^4}^3_{g^{13+12}(\check{\beta}^3)} = \check{\mu}^4$ and $\check{\zeta}_{h^5}^3_{g^{13+32}(\check{\beta}^3, \check{\beta}^4)} = \check{\mu}^5$. It can be shown that any such appropriate subset, now completely characterized, is a metric space (See a metric in the Appendix) and my earlier claims on continuity can be justified.

In any $\check{h}^3_{g^{\varnothing+13}(\cdot)}$, convex combinations over the payoffs associated to plans in any given appropriate subset and the outside options ψ_{13}^3 yield a closed IRF^3 set and frontier. So, O-F plans exist.

In turn, the outside options in any $\check{h}^2_{g^{\varnothing+23}(\cdot)}$ can be derived and the IRF^2 set and frontier are closed by similar arguments—now assuming, whenever appropriate and in that order $a = 0$ and $c = 0$. O-F plans exist. The same is the case for $\check{h}^1_{g^{\varnothing+12}(\cdot)}$, assuming whenever appropriate and in that order $b = 0$ or $a = 0$. After using the results in theorem 2, the theorem follows for all parameter values. ■

Theorem 2: If $a, b, c > 0$, the complete graph never forms.

Proof:

In contrast to part 2 in theorem 1, if $\check{\beta}^5$ has $\check{\beta}^5(2) \leq \overline{\beta}^5(2)$, player 1 gains by forming link 12 and O-F plans don't depend on μ^3 anymore. Bargaining games in $\check{h}^6_{g^{132+12}(\check{\mu}^5, \check{\beta}^5, \beta^3)}$ are of type 1.

In particular, if $\check{\beta}^5(2) = \overline{\beta}^5(2)$ the bargaining game in such \check{h}^6 is not essential and

given that player 2 is indifferent, the bargaining outcome depends on $\check{\mu}^5$. A $\check{\mu}^5$ may have players 2 and 3 future-request an O-F plan that promise-requests a disagreement plan in $\check{h}_{g^{132+12}(\check{\mu}^5, \check{\beta}^5, \beta^3)}^6$, a payoff proposal match or a mix. In the first case, payoffs $\pi_{32, \check{h}_{g^{132+12}(\beta^3)}^5}^{\check{\mu}^5}(\check{\beta}_{32}^5)$ would be "assured" to be $(\bar{\beta}^5(3), \bar{\beta}^5(2))$. In the second case payoffs are $(\phi_3^N, \phi_2^N) \neq (\bar{\beta}^5(3), \bar{\beta}^5(2))$. Note that as $\phi_3^N < \bar{\beta}^5(3)$ and $\phi_2^N = \bar{\beta}^5(2)$, this payoff pair is not in the PMF^5 frontier. These payoffs are not strongly Pareto efficient. The same is the case if a mix would be future-requested.

If $\check{\beta}^5(2) < \bar{\beta}^5(2)$, the bargaining game is essential and $\check{\mu}^5$ future-requests in $\check{h}_{g^{132+12}(\check{\mu}^5, \check{\beta}^5, \beta^3)}^6$ a proposal match, in which case payoffs are again (ϕ_3^N, ϕ_2^N) .

Thus, the IRF^5 set and frontier are closed in the bargaining game in any $\check{h}_{g^{132+12}(\beta^3)}^5$. Moreover, it is never credible to have $\check{\mu}^5$ that future-requests a plan that promise-requests a proposal match in $\check{h}_{g^{132+12}(\check{\mu}^5, \check{\beta}^5, \beta^3)}^6$ with positive probability as the Nash Bargaining solution predicts strong Pareto efficient payoffs. Theorem 2 follows as the analysis for the cases $a, b > 0$ are similar. ■

As a corollary of theorem 2, the complete graph never forms in strictly superadditive games. Also, note that any sequentially Nash coherent plan is a subgameperfect publicly correlated equilibrium (Myerson 1991, pp. 334).

7 Conclusions

This paper adds effective endogenous cooperation possibilities (See end of section 3) to a modification of the A-M model, where pairs of players bargain non cooperatively over the sum of their Myerson values in the prospective network. Negotiation statements at each history of the communication game are credible, in most cases, if they are the outcome of a plan bargaining problem where feasible payoffs are those induced by tenable and reliable plans. The disagreement tenable and reliable plan promise-requests link rejection. If one or both bargainers are indifferent to any IRF plan, a credible plan is the one future-requested by the oldest pair among the past pairs that successfully cooperated and included one of the indifferent players. Credible statements or sequentially Nash coherent plans, exist and analytical payoffs predicted are unique.

In a slightly different communication environment, in a preliminary version of this paper, among other results, it is shown that all payoff predictions in that model are efficient. I conjecture that the same results hold in the model of this paper.

A first natural future work is that of a non cooperative implementation of my plan bargaining problem as then the credibility of a plan would not rely on some binding or commitment element.

It should be relevant to check if sequentially Nash coherent plans can be defined

and exists whenever payoff allocation rules different than the Myerson value are used or whenever N players are allowed.

8 Appendix

An Appropriate Metric

Let $\check{\mu}^3 = (\check{\sigma}_{\check{\beta}^3}^3, \check{\zeta}^3)$ be an element of any given appropriate subset of $\vec{\mathcal{U}}^3(Q^5)$ ($\overleftarrow{\mathcal{U}}^3(\vec{Q}^5)$). Given $\check{\sigma}_{\check{\beta}^3}^3$ define $\gamma \in [0, 1]$ so as to satisfy

$$\gamma (\phi_1^{13} + \phi_3^{13}, 0) + (1 - \gamma) (0, \phi_1^{13} + \phi_3^{13}) = \check{\beta}_{13}^3.$$

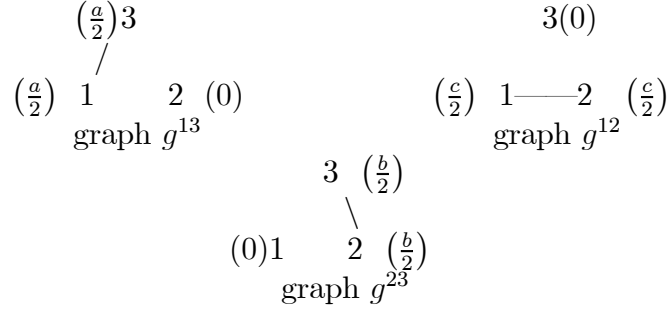
Let $\check{\zeta}_p^3 = (\check{\sigma}^k, \check{\zeta}^k)$ be the component of $\check{\zeta}^3$ that corresponds to a history $h_{g^{\theta+ij}(\cdot)}^k$ that follows $h_{g^{13}(\cdot)}^3$. For each correlated strategy $\check{\zeta}_p^3$ define $\gamma \in [0, 1]$ so as to satisfy

$$\gamma (\phi_i^{g^{\theta+ij}} + \phi_j^{g^{\theta+ij}}, 0) + (1 - \gamma) (0, \phi_i^{g^{\theta+ij}} + \phi_j^{g^{\theta+ij}}) = \sum_{\beta^k \in B_{h_{g^{\theta+ij}(\cdot)}^k}} \check{\sigma}^k (\beta^k) \pi_{ij, h_{g^{\theta+ij}(\cdot)}^k}^{\check{\zeta}_p^3} (\check{\mu}^3, \check{\beta}^3, \dots) (\check{\beta}_{ij}^k).$$

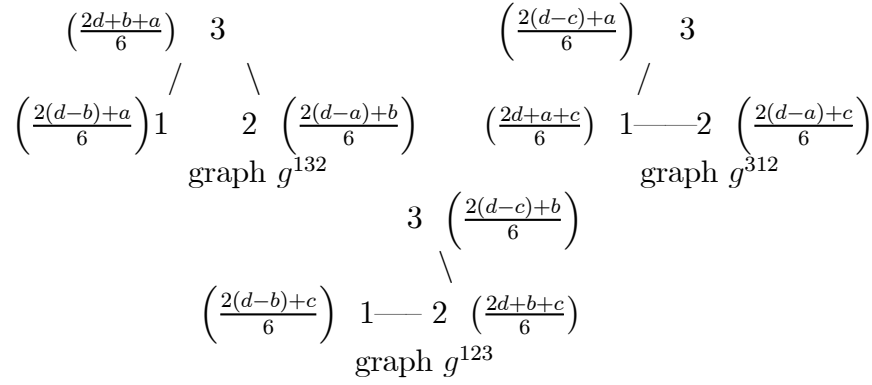
Define the vector of gammas associated to $\check{\mu}^3$ as $\left(\gamma(\check{\sigma}^3), \left\{ \gamma(\check{\zeta}_p^3) \right\}_p \right)$. The distance between two different $\check{\mu}^3$ could be given by any standard infinite dimensional distance between their associated vector of gammas. Such a weird metric is necessary specially as for the complex *IRF* frontiers of histories $\hat{h}_{g^{13+32}(\cdot)}^5$ and $\hat{h}_{g^{13+12}(\cdot)}^4$ whenever $c = 0$ and $b = 0$ respectively.

Diagram: Myerson Values for Normalized Games

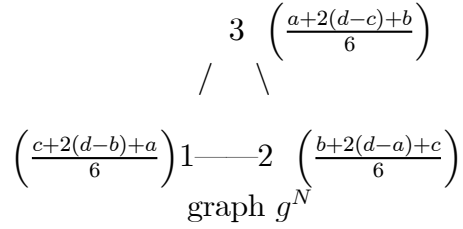
One-link Graphs



Two-Link Graphs



Complete graph



References

- [1] Aumann R. Dreze J.(1974) "Cooperative Games with Coalition Structures" *International Journal of Game Theory* Vol. 3, Issue 4 page 217-237 Physica-Verlag,Vienna
- [2] Aumann R. Hart O.(2003) "Long Cheap Talk" *Econometrica*, vol. 71, No. 6:1619-660.
- [3] Aumann R., Myerson R.(1988) " An endogenous formation of links between players and coalitions: an application of the Shapley Value" in *The Shapley value: Essays in honour of Lloyd Shapley* (A. Roth, Ed.), pp, 175-191, Cambridge, UK.: Cambridge University Press
- [4] Belleflamme P. (2000) "Stable Coalition Structures with Open Membership and Asymmetric Firms" *Games and Economic Behavior* Vol. 30, No. 1,pp. 1-21 January 1,
- [5] Binmore K. (1988) "Game Theory and the Social Contract II The MIT Press. Cambridge, Massachusetts, London, England.
- [6] Binmore K. and Dasgupta P. (1987) "The Economics of Bargaining" Basil Blackwell LTd.
- [7] Bloch F. "Sequential Formation of Coalitions in Games with Externalities and Fixed Payoff Division" *Games and Economic Behavior* Vol. 14, No. 1, May 1, 1996 pp. 90-123
- [8] Chatterjee, K., B. Dutta, D. Ray, and K. Sengupta(1993) " A Noncooperative theory of Coalitional Bargaining" *Rev. Econ. Stud.* 60, 463-477.
- [9] Currarini S.and Morelli M. 2000 "Network Formation with sequential Demands" *Review of Economic Design* 229-249 Vol 5(3) pp 229 249
- [10] Dutta, B. , A. van den Nouweland and S. Tijs, "Link Formation in Cooperative Situations" *International Journal of Game Theory* 27 (1998) 245-256
- [11] Farrel, J. (1993) "Meaning and Credibility in Cheap-Talk Games" *Games and Economic Behavior*,5,514-531.
- [12] Farrel, J., and M. Rabin (1996): "Cheap Talk," *Journal of Economic Perspectives*, 10,103-118.
- [13] Fudenberg, D. and J. Tirole (1991) *Game Theory*. The MIT Press.
- [14] Jackson, M (April 2005) "Allocation Rules for Network Games".*Games and Economic Behavior*,51,128-154

- [15] Jackson, M (2004) A Survey of Models of Network Formation: Stability and Efficiency. In *Group Formation in Economics, Networks, Clubs and Coalitions*, edited by Gabrielle Demange and Myrna Wooders, Cambridge University Press, Cambridge U.K.
- [16] Jackson, M and A. Wolinsky (1996) "A strategic Model of social and economic Networks" *Journal of economic Theory* 71,44-71
- [17] Jackson M and Bloch, F. "The formation of Networks with Transfers among Players", forthcoming *Journal of Economic Theory* .
- [18] Mutuswami S. Winter E. (2002) "Subscription Mechanisms for Network Formation" *Journal of Economic Theory* 106, 242-264
- [19] Myerson R. (1977) "Graphs and Cooperation in Games" *Mathematics of Operation Research* 2, 225-9.
- [20] Myerson R. "(1980) "Conference structures and fair allocation rules" *Int. Journal of Game Theory*, Vol 9 Issue 3 page 169-82 Physica-Verlag, Vienna
- [21] Myerson R. (1984) "Two-Person Bargaining Problems with Incomplete Information" *Econometrica* 52:461-487.
- [22] Myerson R. (1985) "Analysis of Two Bargaining Problems with Incomplete Information." In A. E. Roth, ed., *Game Theoretic Models of Bargaining*, pp. 115-147. Cambridge: Cambridge University Press.
- [23] Myerson R. (1986) "Multistage Games with Communications" *Econometrica* 54:323-358.
- [24] Myerson R. (1989) "Credible Negotiation Statements and Coherent Plans" *Journal of economic Theory* 48, 264-303
- [25] Myerson R. (1991) "Game Theory: Analysis of Conflict" Harvard University Press., Cambridge, Massachusetts.
- [26] Navarro and Perea (2001) "Bargaining in Networks and the Myerson Value, " mimeo: Universidad Carlos III de Madrid.
- [27] Nash, J, (1950) "The Bargaining Problem" *Econometrica* 18:155-162.
- [28] Nash, J, (1953) "Two Person Cooperative Games" *Econometrica* 21:128-140.
- [29] Ray, D. and R. Vohra,(1999) "A Theory of Endogenous Coalition Structures" *Games and Economic Behavior* 26, 286,336.
- [30] Schelling, T. C. (1960) "The strategy of Conflict". Harvard University Press., Cambridge, Massachusetts.

- [31] Slikker and van den Nouweland (2001) *Social and Economic Networks in Cooperative Game Theory* Kluwer Academic Publishers
- [32] Watts, E.(2001) "A dynamic model .of network formation" *Games and Economic Behavior*,34,331-341.