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# intra-daily volatility spillovers between the US and german stock markets

by Vasyl Golosnoy, Bastian Gribisch, Roman Liesenfeld



# Intra-Daily Volatility Spillovers between the US and German Stock Markets

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## Abstract

Using a novel three-phase model based upon a conditional autoregressive Wishart (CAW) framework for the realized (co)variances of the US Dow Jones and the German stock index DAX, we analyze intra-daily volatility spillovers between the US and German stock markets. The proposed model explicitly accounts for three distinct intraday periods resulting from the non-synchronous and partially overlapping opening hours of the two markets. We find evidence of significant short-term volatility spillovers from one intraday period to the next within both markets ('heat-wave effects') as well as across the two markets ('meteor-shower effects'). Furthermore, we find that during the subprime crisis the general persistence of short-term volatility shocks is considerably higher and the spillovers effects between the US and the German stock markets are significantly larger than before the crisis, indicating substantial volatility contagion effects.

*JEL classification:* C32, C58, G17

*Keywords:* Conditional autoregressive Wishart model, Impulse response analysis, Observation-driven models, Realized covariance matrix, Subprime crisis.

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# 1 Introduction

A common finding of empirical studies devoted to asset-return variances and covariances across international financial markets is their high degree of contemporaneous and temporal interdependence. This interdependence, which plays an important role for international portfolio allocation and financial risk management, is often attributed to information transmissions across financial markets. This view is based upon the hypothesis that the arrival process of economic news and the trading dynamics in response to news are key determinants of the short-run dynamics of asset-return volatility (see, e.g., Kyle, 1985). Against the background of an apparently increasing integration of international financial markets it is interesting to see to what extent a volatility shock generated by news in one market spills over onto the volatility observed in the next market to trade. As noted by Hamao et al. (1990) and Wongswan (2006), such spillovers could represent a causal phenomenon across markets that trade sequentially; alternatively they could reflect shocks which are generated by news relevant to the global economy and impinging concurrently on the volatility across international markets. Similarly, it is of interest whether those spillover effects are more pronounced during periods of very high volatility associated with severe financial crises like that of 2007-2009. While this crisis had its origin in the US sub-prime mortgage market, it spread out increasing the volatility across international financial markets above and beyond a level which can be explained by a ‘regular’ fluctuation. A potential channel of such a volatility contagion is that an initially local crisis in one country generates news that prompt investors to fundamentally reassess the general vulnerability of other national markets (see, e.g., Bekaert, et al., 2011).

The strand of empirical literature concerned with volatility spillovers on international financial markets goes back to the early papers of Engle, et al. (1990) and Hamao, et al. (1990), in which GARCH models fitted to intra-day returns are used to measure the volatility transmissions from one period to the next within markets (‘heat waves’) and across markets

(‘meteor showers’). The former study uses four intra-day returns per day of the yen-US dollar exchange rate associated with four distinct geographic market segments with non-synchronous trading hours (Tokyo, Europe, New York, Pacific), and reports significant spillovers between the different market segments, indicating that volatility in international markets behaves like a meteor shower. Hamao, et al. (1990) rely on close-to-open and open-to-close returns and find spillovers from the US to the Japanese stock market but not conversely.

More recent studies examining volatility transmissions between international markets use high-frequency return data in order to construct realized variances or ranges between the largest and smallest log prices as precise estimates for the volatility of low-frequency returns and model those estimates directly. This offers the advantage that those volatility measures are typically more informative about the true volatility than the corresponding conditional variances obtained from GARCH models, which leads to an improved inference about volatility transmissions across markets. Such approaches are found, e.g., in Engle, et al. (2012), Bubák, et al. (2011) for markets with synchronous trading hours and in Melvin and Melvin (2003), Dimpfl and Jung (2012), Chiang and Wang (2011), for markets with nonsynchronous business hours. The study of Engle, et al. (2012) uses a multivariate multiplicative error model (MEM) for the vector of daily volatilities approximated by the daily ranges and applies this approach to measure the volatility transmissions across eight East Asian stock markets and to examine changes in the transmission mechanism during the 1997-1998 East Asian crisis. In order to analyze the short-term interdependence of the realized variances for the exchange rates of four European currencies against the US dollar, Bubák, et al. (2011) propose a multivariate version of the heterogeneous autoregressive (HAR) model of Corsi (2009). Melvin and Melvin (2003) investigate volatility spillovers of the Deutsche mark-US dollar and yen-US dollar exchange rate across geographical market segments, while Dimpfl and Jung (2012) examine spillovers across the stock markets in Europe, the US and Japan. Both studies rely on structural vector autoregressive (VAR) models for the realized volatilities accounting for

the time differences in trading hours of the markets under consideration. Using a range-based conditional autoregressive volatility model for the stock markets of the G7 countries, Chiang and Wang (2011) examine changes in the volatility transmission mechanism due to the subprime mortgage crises.

In the present paper, we investigate the short-term interdependence of the realized variances and covariance of the US Dow Jones and the German stock index DAX. For this purpose, we propose a novel sequential phase model accounting for the three distinct geographical intra-day trading periods of the US and German stock market: (1) the Germany-US trading overlap period, (2) the US-only trading period, and (3) the Germany-only trading period. Our approach consists of three separate reduced-form time-series specifications, one for each intra-day period. For the covariance matrix of the Germany-US trading overlap period we use the Conditional Autoregressive Wishart (CAW) model of Golosnoy, et al. (2012) extended to include the lagged variances of the other two intra-day periods as additional covariates. The two variances of the US-only and the Germany-only trading periods are assumed to follow a corresponding conditional autoregressive Gamma distribution, which obtains from the CAW model for the covariance matrix as a natural marginal specification for the variances. The resulting sequential three-phase model facilitates a detailed analysis of the short-term causal effects of news generating intra-day volatility in one market onto subsequent trading on this and the other market, i.e. both meteor-shower and heat-wave effects. We supplement this analysis of the direct causal effects by an impulse-response analysis, which provides information not only about the direct but also the indirect effects of volatility shocks. As such, the impulse-response analysis also accounts, e.g., for the indirect effect of a volatility shock during the afternoon trading on the German market on its volatility at the next morning via the US trading which has taken place in the meantime. In addition, we use our framework in order to investigate whether the short-term volatility transmission mechanism is significantly different during the recent subprime crisis than before and after the crisis which would indicate

volatility contagion effects.

By accounting for the interdependence between the variances as well as the covariance our approach differs from existing empirical studies on volatility transmissions across stock markets with overlapping trading hours like those of Engle, et al. (2012), Dimpfl and Jung (2012) and Chiang and Wang (2011), which solely investigate the dynamic interdependence of variance measures. As such, our approach allows to account for two potential channels of volatility spillovers, namely, via a direct volatility transmission from one market to the other through its variance and via an indirect transmission through its covariance. Furthermore, our approach explicitly accounts for the contemporaneous interdependence between the variances during the overlapping trading periods which is ignored in the studies of Engle, et al. (2012) and Dimpfl and Jung (2012). In order to properly identify the direct causal effects of news on subsequent volatility on the domestic and foreign markets, it is critical to explicitly account for the indirect effects transmitted via the covariance and for the contemporaneous dependence among the variances.

The rest of the paper is organized as follows. In Section 2, we describe the schedule of trading hours of the German and US stock market and the adjustments made to remove long-run trend effects from the realized variance and covariance series. In Section 3, we introduce the sequential three-phase model and discuss its properties. In Section 4, we present the design of the impulse response analysis for our three phase model. The empirical results are presented in Section 5, while Section 6 concludes.

## 2 Trading Times, Data and Adjustments

In our analysis of the short-term volatility spillovers for the German and US stock market on the intra-daily basis we account for their non-synchronous opening hours, leading to three distinct intra-day trading periods. The different trading intervals in Central European Time

(CET) associated with different trading regimes are illustrated in Figure 1. Assume that a global business day  $t$  starts with the opening of the New York Stock Exchange at 3:30 pm CET. From 3:30 pm to 5:30 pm the US and German stock market are simultaneously open. This joint trading period of two hours length is referred to as period 1 of a trading day. In period 2 which lasts from 5:30 pm to 10:00 pm, only the US market is open. The last interval of a trading day referred to as period 3, starts at 9:30 am when the German market opens and ends with the re-opening of the US market at 3:30 pm.

To model the dynamic process of intra-daily volatilities accounting for this chronological ordering of overlapping and non-overlapping trading periods we propose a sequential three-phase model, which treats the volatility for the three intra-day periods separately by specifying three dynamic reduced-form models, one for each intra-day period. In order to obtain volatility measures for the three intra-day periods, we use high-frequency data to construct realized variances and covariances as direct estimates of the corresponding variances and covariances of returns. Our data consists of synchronized 1-minute prices sampled with previous-tick interpolation for the German stock index DAX and the US Dow Jones industrial index (DJ). The sample period begins at January 2, 1996 and ends on December 29, 2010 covering  $T = 3645$  trading days. For intra-day period 1 the realized covariance matrix for the DAX and DJ can be computed as  $V_{t,1}^{(us,g)} = \sum_{i=1}^{n_1} y_{t,i} y'_{t,i}$ , where  $y'_{t,i} = (y_{t,i}^{(us)}, y_{t,i}^{(g)})$  is the vector of the DJ and DAX log returns computed for the 5-minute interval  $i$  in period 1 of trading day  $t$ . The number of 5-minute intervals in this period is  $n_1 = 24$ . In the sequel, the diagonal (variance) elements of the period-1 realized covariance matrix  $V_{t,1}^{(us,g)}$  are denoted by  $v_{t,1}^{(us)}$  and  $v_{t,1}^{(g)}$  and the off-diagonal (covariance) element by  $v_{t,1}^{(us,g)}$ . The realized variance of the DJ in period 2 and that of the DAX in period 3 can be computed analogously as  $v_{t,2}^{(us)} = \sum_{i=n_1+1}^{n_2} [y_{t,i}^{(us)}]^2$  and  $v_{t,3}^{(g)} = \sum_{i=n_2+1}^{n_3} [y_{t,i}^{(g)}]^2$ , respectively, where the number of 5-minute intervals for the second period is  $n_2 - n_1 = 54$  and that for the third period  $n_3 - n_2 = 78$ . These realized variance and covariance measures are further refined by averaging over subsampling subgrids per intra-day



period in order to cope with market microstructure noise (see, Zhang, et al., 2005). Finally, the realized variances and covariances of the three intra-day periods are normalized by the length of the respective intra-day period.

Figure 2 shows the time series plots of the resulting realized (co)variances for the three intra-day periods. These plots reveal a common cyclical long-term behavior across the five variance and covariance time series with its largest peak during the subprime crisis starting in 2008. A number of authors attribute those long-term shifts in the volatility to changes in the global macroeconomic and financial environment and interpret them as evidence against global stationarity – see, e.g., Engle, et al. (2009) and the literature cited therein<sup>1</sup>. In order to capture those long-run movements Engle and Rangel (2008), Engle, et al. (2009) and Hafner and Linton (2010) use component volatility models with a long-run and a short-run component, where the former is associated with the state of the economy while the latter is related to day-to-day liquidity concerns and the arrival of news process triggering trading activities in response to news. Here we are not interested to explain or model the long-term volatility rather than the short-term volatility transmission between the two stock markets from one intra-day period to the next. Therefore, we remove the common long-run shifts from the realized (co)variances prior to the analysis of the short-term patterns. For this purpose we follow Hafner and Linton (2010) and use a nonparametric kernel procedure to estimate the long-run components of the covariance matrix  $V_{t,1}^{(us,g)}$  and the variances  $v_{t,2}^{(us)}$  and  $v_{t,3}^{(g)}$ . The corresponding estimates for the long-run components of the three intra-day periods are obtained as

$$M_{t,1} = \frac{\sum_{s=1}^T \mathcal{K}(\frac{t-s}{hT}) \cdot V_{t,1}^{(us,g)}}{\sum_{s=1}^T \mathcal{K}(\frac{t-s}{hT})}, \quad m_{t,2} = \frac{\sum_{s=1}^T \mathcal{K}(\frac{t-s}{hT}) \cdot v_{t,2}^{(us)}}{\sum_{s=1}^T \mathcal{K}(\frac{t-s}{hT})}, \quad m_{t,3} = \frac{\sum_{s=1}^T \mathcal{K}(\frac{t-s}{hT}) \cdot v_{t,3}^{(g)}}{\sum_{s=1}^T \mathcal{K}(\frac{t-s}{hT})}, \quad (1)$$

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<sup>1</sup>For an initial investigation, we fitted our model proposed in Section 3 below to the raw realized (co)variances  $V_{t,1}^{(us,g)}$ ,  $v_{t,2}^{(us)}$  and  $v_{t,3}^{(g)}$  and found that they can not be represented by a covariance stationary specification due to an explosive behavior in their conditional means.

respectively, where  $h$  denotes the bandwidth and  $\mathcal{K}(\cdot)$  is a scalar-valued kernel function. Here we use a two-sided quartic kernel function and set the bandwidth to  $h = 0.05$ , such that about 10% of the data are used for local averaging<sup>2</sup>. Note that we use the same weighting scheme and bandwidth for all five variance and covariance time series, which imposes implicitly a common pattern in the long-run dynamics for all of them. This restriction could be justified by the finding that the long-run movements of the five time series appear to be very similar (see Figure 2).

To detrend the realized (co)variances, we normalize them by their estimated long-run components given by Equation (1) and plotted in Figure 2. In particular, the detrended realized covariance matrix for intra-day period 1, denoted by  $R_{t,1}^{(\text{us,g})}$ , obtains as

$$R_{t,1}^{(\text{us,g})} = C_{t,1}^{-1} V_{t,1}^{(\text{us,g})} (C_{t,1}^{-1})', \quad \text{with} \quad M_{t,1} = C_{t,1} C_{t,1}', \quad (2)$$

where  $C_{t,1}$  is the lower-triangular Cholesky factor of the period-1 long-run component  $M_{t,1}$ . The detrended realized variances of the intra-day periods 2 and 3 are constructed analogously by

$$r_{t,2}^{(\text{us})} = v_{t,2}^{(\text{us})} / m_{t,2}, \quad r_{t,3}^{(\text{g})} = v_{t,3}^{(\text{g})} / m_{t,3}, \quad (3)$$

respectively.

Figure 3 shows the plots of the detrended realized variance and covariance time series and reveals that the normalization make them more homogeneous. This allows us to focus on the short-term dynamic structure under the assumption of global stability. Descriptive statistics for the detrended realized (co)variance series are provided in Table 1. The mean of the variances is close to unity and that of the covariance close to zero, which is to be expected given the normalization rule given by Equations (2) and (3). The empirical distribution of the

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<sup>2</sup>We also experimented with other values for the bandwidth ranging from 0.025 to 0.075. However, the qualitative results of our analysis of the short-term volatility patterns reported below remain essentially unchanged for those alternative values of the bandwidth.

variances is leptokurtic and slightly skewed to the right while that of the covariance is skewed to the left. The Ljung-Box statistics including 50 lags indicate strong serial correlation.

### 3 Econometric Specification

To model the short-term dynamic structure of the volatility of the US and German stock market related to the news arrival process and to analyze the information transmission effects between and within markets, we use a sequential three-phase approach for the detrended volatility of the three intra-day periods given by  $\{R_{t,1}^{(\text{us,g})}, r_{t,2}^{(\text{us})}, r_{t,3}^{(\text{g})}\}_{t=1}^T$ . Since the sequentially ordered intra-day periods are non-overlapping, the volatility originating from previous intra-day periods is a pre-determined variable for the current period. This suggests the following sequential factorization of the conditional joint density of  $(R_{t,1}^{(\text{us,g})}, r_{t,2}^{(\text{us})}, r_{t,3}^{(\text{g})})$  given the information set  $\mathcal{F}_{t-1}$  available at the end of day  $t - 1$ :

$$f(R_{t,1}^{(\text{us,g})}, r_{t,2}^{(\text{us})}, r_{t,3}^{(\text{g})} | \mathcal{F}_{t-1}) = f(R_{t,1}^{(\text{us,g})} | \mathcal{F}_{t-1}) \cdot f(r_{t,2}^{(\text{us})} | R_{t,1}^{(\text{us,g})}, \mathcal{F}_{t-1}) \cdot f(r_{t,3}^{(\text{g})} | r_{t,2}^{(\text{us})}, R_{t,1}^{(\text{us,g})}, \mathcal{F}_{t-1}). \quad (4)$$

Based upon this natural decomposition of the daily joint distribution, we specify three separate reduced form models, one for each of the three intra-day periods designed to measure the volatility transmission effects for the two stock markets from one intra-day period to the coming ones.

#### 3.1 Period 1: Germany-US trading overlap

We start to detail our specification for the covariance matrix of period 1,  $R_{t,1}^{(\text{us,g})}$ . A particular convenient and flexible dynamic specification for this symmetric positive definite matrix of dimension  $2 \times 2$  is provided by the CAW model proposed by Golosnoy, et al. (2012), which assumes a central Wishart distribution for  $R_{t,1}^{(\text{us,g})} | \mathcal{F}_{t-1}$ . The specific CAW model for  $R_{t,1}^{(\text{us,g})}$

adopted here includes preceding period-2 and period-3 variances as additional explanatory variables and takes the form

$$R_{t,1}^{(\text{us,g})} | \mathcal{F}_{t-1} \sim \mathcal{W}_2(\nu_1, S_{t,1}/\nu_1), \quad (5)$$

$$S_{t,1} = G_1 G_1' + \sum_{i=1}^{q_1} \sum_{\ell=1}^{\bar{q}_1} A_{i\ell,1} R_{t-i,1}^{(\text{us,g})} A_{i\ell,1}' + \sum_{i=1}^{p_1} \sum_{\ell=1}^{\bar{p}_1} B_{i\ell,1} S_{t-i,1} B_{i\ell,1}' + \sum_{i=1}^{z_1} \sum_{\ell=1}^{\bar{z}_1} D_{i\ell,1} \bar{R}_{t-i}^{(\text{us,g})} D_{i\ell,1}', \quad (6)$$

where  $\mathcal{W}_2$  denotes the law of a central Wishart distribution for a  $2 \times 2$  matrix,  $\nu_1 > 2$  is the scalar degree of freedom, and  $S_{t,1}/\nu_1$  represents the  $2 \times 2$  positive definite scale matrix, such that the conditional mean is  $E(R_{t,1}^{(\text{us,g})} | \mathcal{F}_{t-1}) = S_{t,1}$ . In the linear autoregressive recursion for the conditional mean (6), which resembles the BEKK-GARCH specification of Engle and Kroner (1995),  $\bar{R}_t^{(\text{us,g})} = \text{diag}(r_{t,2}^{(\text{us})}, r_{t,3}^{(\text{g})})$  is a diagonal matrix containing the variances of period 2 and 3, and  $G_1$ ,  $A_{i\ell,1}$ ,  $B_{i\ell,1}$  and  $D_{i\ell,1}$  are  $2 \times 2$  parameter matrices, where  $G_1$  has a lower-triangular form. While the summation limits  $(q_1, p_1, z_1)$  determine the number of lagged terms, the limits  $(\bar{q}_1, \bar{p}_1, \bar{z}_1)$  control the generality of the process. The most general process ensures that the number of parameters in the matrices  $A_{i\ell,1}$ ,  $B_{i\ell,1}$  and  $D_{i\ell,1}$  is equal to the number of marginal effects of the different elements in the lagged  $R_{t,1}^{(\text{us,g})}$ ,  $S_{t,1}$ , and  $\bar{R}_t^{(\text{us,g})}$  matrices on the distinct elements in  $S_{t,1}$ . However, the model as specified is unidentified. Sufficient conditions for identification are given by Engle and Kroner (1995, Proposition 2.3). For a model with  $(\bar{q}_1, \bar{p}_1, \bar{z}_1) = (1, 1, 1)$ , for example, these conditions are that the main diagonal elements of  $G_1$  and the first diagonal element for each of the matrices  $A_{i1,1}$ ,  $B_{i1,1}$  and  $D_{i1,1}$  are restricted to be positive.

The contemporaneous dependence of the volatility for the two markets implied by the CAW model becomes manifest in the behavior of the conditional covariance matrix of the realized (co)variance, denoted by  $\text{Var}[\text{vec}(R_{t,1}^{(\text{us,g})}) | \mathcal{F}_{t-1}]$ . Under the conditional Wishart distribution

in Equation (5) this conditional covariance matrix is (see Muirhead, 1982)

$$\text{Var}[\text{vec}(R_{t,1}^{(\text{us},g)})|\mathcal{F}_{t-1}] = \frac{1}{\nu_1}(I + K_{44})(S_{t,1} \otimes S_{t,1}), \quad (7)$$

where  $I$  is the identity matrix,  $\text{vec}(\cdot)$  denotes the operator that stacks all columns of a matrix into a vector, and  $K_{44}$  is the commutation matrix defined so that  $K_{44}\text{vec}(W) = \text{vec}(W')$  for any  $4 \times 4$  matrix  $W$ .

As discussed in Golosnoy, et al. (2012), the CAW model (5)-(6) can be interpreted as a state-space model with  $S_{t,1}$  as a state variable measured by the observable matrix  $R_{t,1}^{(\text{us},g)}$  so that  $S_{t,1}$  can be regarded as the ‘true’ integrated covariance matrix of period 1 for a broad class of continuous-time stochastic volatility processes approximated by  $R_{t,1}^{(\text{us},g)}$  (see Barndorff-Nielsen and Shephard, 2004). The dynamic specification assumed for  $S_{t,1}$  is designed to capture complex dynamic interactions across the covariance and variances for the returns of the US and German stock market of period 1 as well as their dependencies from the preceding variances for the corresponding returns of periods 2 and 3. It is easy to see that the direct volatility spillover effects from one market to future trading periods of the other market are directed by the non-diagonal elements in  $A_{i\ell,1}$ ,  $B_{i\ell,1}$  and  $D_{i\ell,1}$  parameter matrices. For a specification with  $q_1 = \bar{q}_1 = z_1 = \bar{z}_1 = 1$  and  $p_1 = 0$  with parameter matrices  $G_1 = (g_{jk})$ ,  $A_{11,1} = (a_{jk})$  and  $D_{11,1} = (d_{jk})$ , for example, the conditional mean of the period-1 DJ variance  $s_{t,1}^{(\text{us})}$  obtains from Equation (6) as

$$s_{t,1}^{(\text{us})} = g_{.11}^2 + a_{.11}^2 r_{t-1,1}^{(\text{us})} + 2a_{.11}a_{.12}r_{t-1,1}^{(\text{us},g)} + a_{.12}^2 r_{t-1,1}^{(g)} + d_{.11}^2 r_{t-1,2}^{(\text{us})} + d_{.12}^2 r_{t-1,3}^{(g)}. \quad (8)$$

Hence, the effect of a shock in the period-3 DAX variance  $r_{t-1,3}^{(g)}$  to the next period-1 DJ variance is directed by  $d_{.12}$  and that of a shock in the period-1 DAX variance  $r_{t-1,1}^{(g)}$  by  $a_{.12}$ , respectively. However, note that the contemporaneous correlation among the period-1 (co)variances  $r_{t-1,1}^{(g)}$ ,  $r_{t-1,1}^{(\text{us})}$  and  $r_{t-1,1}^{(\text{us},g)}$  (see Equation 7) implies that the shock in the period-1 DAX variance

$r_{t-1,1}^{(g)}$  can spill over onto the next day period-1 DJ variance also indirectly via the variance  $r_{t-1,1}^{(us)}$  and the covariance  $r_{t-1,1}^{(us,g)}$ .

### 3.2 Periods 2 and 3: US-only and Germany-only trading

Since the conditional Wishart distribution assumed for the period-1 realized covariance matrix  $R_{t,1}^{(us,g)}$  implies that its diagonal variance elements follow a conditional Gamma distribution it is natural to assume such a conditional Gamma distribution also for the realized variances of period 2 and 3. The particular reduced form model used for the period-2 DJ variance  $r_{t,2}^{(us)}$  including preceding period-1 covariance matrices and period-3 variances as additional explanatory variables takes the form

$$r_{t,2}^{(us)} | R_{t,1}^{(us,g)}, \mathcal{F}_{t-1} \sim \mathcal{G}(\nu_2/2, 2s_{t,2}/\nu_2), \quad (9)$$

$$s_{t,2} = g_2 + d'_{0,2} R_{t,1}^{(us,g)} d_{0,2} + \sum_{i=1}^{q_2} a_{i,2} r_{t-i,2}^{(us)} + \sum_{i=1}^{p_2} b_{i,2} s_{t-i,2} + \sum_{i=1}^{z_2} d'_{i,2} R_{t-i,1}^{(us,g)} d_{i,2} + \sum_{i=1}^{w_2} c_{i,2} r_{t-i,3}^{(g)}, \quad (10)$$

where  $\mathcal{G}$  denotes the law of a Gamma distribution,  $\nu_2/2$  is the shape parameter of the Gamma distribution, and  $2s_{t,2}/\nu_2$  represents its scale parameter such that  $s_{t,2}$  is the conditional mean, i.e.,  $s_{t,2} = E(r_{t,2}^{(us)} | R_{t,1}^{(us,g)}, \mathcal{F}_{t-1})$ . The linear autoregressive recursion for  $s_{t,2}$  given by Equation (10) is characterized by the scalar parameters  $g_2, a_{i,2}, b_{i,2}, c_{i,2}$ , which are restricted to be positive, and the two-dimensional parameter vectors  $d_{i,2}$ . As mentioned in the context of the CAW model above,  $s_{t,2}$  can be interpreted as the true integrated variance of period 2 measured by  $r_{t,2}^{(us)}$ . The direct volatility transmission effects from the German stock market to the period-2 DJ volatility are driven by the second elements of the vectors  $d_{i,2}$  and the parameters  $c_{i,2}$ . Additionally, we have an indirect transmission of an impulse in the period-1 DAX variance  $r_{t-i,1}^{(g)}$  through its contemporaneous correlation with period-1 DJ variance  $r_{t-i,1}^{(us)}$  and the corresponding covariance  $r_{t-i,1}^{(us,g)}$ .

The final component of our sequential three-phase model for the volatility of the US and German stock market consists of a reduced form specification for the period-3 variance of the DAX  $r_{t,3}^{(g)}$ , which takes a similar form as that for the period-2 DJ variance, namely

$$r_{t,3}^{(g)} | r_{t,2}^{(us)}, R_{t,1}^{(us,g)}, \mathcal{F}_{t-1} \sim \mathcal{G}(\nu_3/2, 2s_{t,3}/\nu_3), \quad (11)$$

$$s_{t,3} = g_3 + d'_{0,3} R_{t,1}^{(us,g)} d_{0,3} + c_{0,3} r_{t,2}^{(us)} + \sum_{i=1}^{q_3} a_{i,3} r_{t-i,3}^{(g)} + \sum_{i=1}^{p_3} b_{i,3} s_{t-i,3} + \sum_{i=1}^{z_3} d'_{i,3} R_{t-i,1}^{(us,g)} d_{i,3} + \sum_{i=1}^{w_3} c_{i,3} r_{t-i,2}^{(us)}. \quad (12)$$

Here the spillover parameters are given by the first elements of the two-dimensional vectors  $d_{i,3}$  and the coefficients  $c_{i,3}$  driving the direct transmission effects.

### 3.3 Model properties, estimation and diagnostics

The three-phase model introduced in Equations (5)-(12) is expected to accommodate a large variety of dynamic patterns in the process of intra-day realized variances and covariances. In order to obtain the stability conditions of this process ensuring the existence of the stationary mean, we use its VARMA representation. Let  $r_t = (\text{vech}(R_{t,1}^{(us,g)})', r_{t,2}^{(us)}, r_{t,3}^{(g)})'$  and  $s_t = (\text{vech}(S_{t,1})', s_{t,2}, s_{t,3})'$ , where  $\text{vech}(\cdot)$  denotes the operator that stacks the lower triangular portion, including the diagonal of a matrix, into a vector. Then the system of interdependent recursions (6), (10), and (12) from one day to the next can be written as (see, Golosnoy, et al., 2012)

$$s_t = g + \Delta_0 r_t + \sum_{i=1}^q \mathcal{A}_i r_{t-i} + \sum_{i=1}^p \mathcal{B}_i s_{t-i}, \quad (13)$$

where  $g$  is a 5-dimensional vector and  $\Delta_0, \mathcal{A}_i, \mathcal{B}_i$  are  $5 \times 5$  matrices which are straightforward functions of the parameters characterizing the three recursions. The lag orders are  $q = \max\{q_1, z_1, q_2, z_2, w_2, q_3, z_3, w_3\}$  and  $p = \max\{p_1, p_2, p_3\}$ . Next we exploit that  $r_t$  can be written as  $r_t = s_t + v_t$ , where  $v_t$  is a martingale difference with  $E(v_t) = 0$  and  $E(v_t v_s') = 0$  for

all  $s \neq t$  so that the VARMA representation of  $r_t$  obtains as

$$r_t = g^* + \sum_{i=1}^{\max\{p,q\}} (\mathcal{A}_i^* + \mathcal{B}_i^*) r_{t-i} - \sum_{i=1}^p \mathcal{C}_i^* v_{t-i}^* + v_t^*, \quad (14)$$

where  $g^* = (I - \Delta_0)^{-1}g$ ,  $\mathcal{A}_i^* = (I - \Delta_0)^{-1}\mathcal{A}_i$ ,  $\mathcal{B}_i^* = (I - \Delta_0)^{-1}\mathcal{B}_i$ ,  $\mathcal{C}_i^* = \mathcal{B}_i^*(I - \Delta_0)$  and  $v_t^* = (I - \Delta_0)^{-1}v_t$ . It follows that the three-phase model is stable with a stationary mean  $E(r_t) = [I - \sum_{i=1}^{\max\{p,q\}} (\mathcal{A}_i^* + \mathcal{B}_i^*)]^{-1}g^*$  if and only if all eigenvalues of the matrix  $\sum_{i=1}^{\max\{p,q\}} (\mathcal{A}_i^* + \mathcal{B}_i^*)$  are less than 1 in modulus (see, e.g. Lütkepohl, 2005).

The parameter vector  $\theta$  of the three-phase model (5)-(12) is made up of the coefficients in the autoregressive specifications for the conditional means  $S_{t,1}$ ,  $s_{t,2}$ , and  $s_{t,3}$  plus the degree of freedom  $\nu_1$  and shape parameters  $\nu_2$  and  $\nu_3$ . They can be estimated by maximizing the log-likelihood function

$$\mathcal{L}(\theta) = \sum_{t=1}^T \ln f(R_{t,1}^{(\text{us},g)} | \mathcal{F}_{t-1}) + \sum_{t=1}^T \ln f(r_{t,2}^{(\text{us})} | R_{t,1}^{(\text{us},g)}, \mathcal{F}_{t-1}) + \sum_{t=1}^T \ln f(r_{t,3}^{(g)} | r_{t,2}^{(\text{us})}, R_{t,1}^{(\text{us},g)}, \mathcal{F}_{t-1}). \quad (15)$$

Since there are no parametric restrictions across the log-likelihood components for the three intra-day periods, we can maximize the complete log-likelihood by separately maximizing the three components provided in Equation (15).

For identification of the orders for each of the three model components, that is  $(q_1, p_1, z_1, \bar{q}_1, \bar{p}_1, \bar{z}_1)$  for the period-1 CAW specification,  $(q_2, p_2, z_2, w_2)$  for the period-2 Gamma model, and  $(q_3, p_3, z_3, w_3)$  for the period-3 Gamma model, we use Schwarz's (1978) information criterion. The order identification is supplemented by diagnostic checks based upon the standardized Pearson residuals. For the period-1 CAW model they obtain as

$$(u_{t,1}^{(\text{us})}, u_{t,1}^{(\text{us},g)}, u_{t,1}^{(g)})' = \text{Var}[\text{vech}(R_{t,1}^{(\text{us},g)}) | \mathcal{F}_{t-1}]^{-1/2} \left\{ \text{vech}(R_{t,1}^{(\text{us},g)}) - E[\text{vech}(R_{t,1}^{(\text{us},g)}) | \mathcal{F}_{t-1}] \right\}, \quad (16)$$

where  $\text{Var}[\text{vech}(R_{t,1}^{(\text{us},g)}) | \mathcal{F}_{t-1}]^{-1/2}$  is the inverse Cholesky factor of the conditional covariance



matrix of  $\text{vech}(R_{t,1}^{(\text{us,g})})$ , given in Equation (7). The Pearson residuals for the period-2 and period-3 Gamma specifications are constructed analogously by

$$u_{t,2}^{(\text{us})} = \frac{r_{t,2}^{(\text{us})} - E(r_{t,2}^{(\text{us})} | R_{t,1}^{(\text{us,g})}, \mathcal{F}_{t-1})}{\text{Var}(r_{t,2}^{(\text{us})} | R_{t,1}^{(\text{us,g})}, \mathcal{F}_{t-1})^{1/2}}, \quad u_{t,3}^{(\text{g})} = \frac{r_{t,3}^{(\text{g})} - E(r_{t,3}^{(\text{g})} | r_{t,2}^{(\text{us})}, R_{t,1}^{(\text{us,g})}, \mathcal{F}_{t-1})}{\text{Var}(r_{t,3}^{(\text{g})} | r_{t,2}^{(\text{us})}, R_{t,1}^{(\text{us,g})}, \mathcal{F}_{t-1})^{1/2}}, \quad (17)$$

respectively. The corresponding conditional variances are given as

$$\text{Var}(r_{t,2}^{(\text{us})} | R_{t,1}^{(\text{us,g})}, \mathcal{F}_{t-1}) = \frac{2s_{t,2}^2}{\nu_2}, \quad \text{Var}(r_{t,3}^{(\text{g})} | r_{t,2}^{(\text{us})}, R_{t,1}^{(\text{us,g})}, \mathcal{F}_{t-1}) = \frac{2s_{t,3}^2}{\nu_3}. \quad (18)$$

For a correctly specified model, these residuals are serially uncorrelated and not predictable by past realized (co)variances. In order to test this implication, we regress each of the residual series  $u_{t,i}^{(\cdot)}$  on a constant and past realized (co)variances of all three intra-day periods and test the joint hypothesis that all coefficients other than the constant are equal to zero by using the  $F$ -statistic.

## 4 Impulse-Response Analysis

The marginal effects given by the entries of  $\Delta_0$ ,  $\mathcal{A}_i$ , and  $\mathcal{B}_i$  in Equation (13) measure the direct causal impact of volatility shocks on future variances. However, as noted earlier, there are also indirect volatility transmission channels. In order to examine the compound effect of volatility shocks in one market on subsequent volatility in both markets we use an impulse-response (IR) analysis. Since our three-phase volatility model is nonlinear the standard IR technique à la Sims (1980) developed for linear time series models is not applicable. Hence, we rely on the nonlinear IR strategy of Gallant, et al. (1993) which involves a comparison of forecasts obtained when perturbing the vector of conditioning arguments in the conditional density (conditional mean profile) to baseline forecasts produced without such a perturbation (baseline profile). Using this approach we analyze the effects of shocks to the DJ and DAX

variances appearing in the three different intra-day periods, i.e., shocks to  $r_{t,1}^{(\text{us})}$ ,  $r_{t,1}^{(\text{g})}$ ,  $r_{t,2}^{(\text{us})}$ , and  $r_{t,3}^{(\text{g})}$ , by tracing them period by period through the system.

To simplify the notation for the following presentation we replace the two time indices used for the (co)variances, i.e. the index for the trading day  $t \in \{1, 2, \dots, T\}$  and that for the intra-day period for a given trading day  $m \in \{1, 2, 3\}$ , by a single time index for the sequence of consecutive intra-day periods, say,  $\tau = \tau(t, m)$  such that  $\tau \in \{1, 2, \dots, 3T\}$ ; next we set

$$\mathbf{r}_\tau = \begin{cases} (r_\tau^{(\text{us})}, r_\tau^{(\text{us,g})}, r_\tau^{(\text{g})})', & \text{if } \tau \text{ is a Germany-US overlap period } \tau(t, 1) \\ r_\tau^{(\text{us})}, & \text{if } \tau \text{ is a US-only trading period } \tau(t, 2) \\ r_\tau^{(\text{g})}, & \text{if } \tau \text{ is a Germany-only trading period } \tau(t, 3) \end{cases}, \quad (19)$$

and denote the lags of  $\mathbf{r}_{\tau+1}$  by  $x_\tau = (\mathbf{r}'_\tau, \mathbf{r}'_{\tau-1}, \dots)'$ . Then the  $j$ -step-ahead forecast,  $j = 1, 2, \dots$ , at time period  $\tau$  of the (co)variance  $\mathbf{r}_{\tau+j}$  for a given value of the conditioning arguments  $x$  is

$$\hat{\mathbf{t}}_j(x) = \text{E}(\mathbf{r}_{\tau+j} | x_\tau = x). \quad (20)$$

Under our three-phase model, those forecasts are easily obtained by recursion based upon the conditional expectations (6), (10) and (12). Let  $\delta$  denote a perturbation to the contemporaneous value of  $\mathbf{r}_\tau$  and define  $x^0 = (\hat{\mu}', \hat{\mu}', \hat{\mu}', \dots)'$  and  $x^+ = (\hat{\mu}' + \delta', \hat{\mu}', \hat{\mu}', \dots)'$ , where  $\hat{\mu}$  denotes the sample means of  $\mathbf{r}_\tau$  associated with the three periods. Note that  $\delta$  and  $\hat{\mu}$  represent 3-dimensional vectors for period 1 and scalars for periods 2 and 3. Then the impulse response is defined in terms of the relative net effect of a perturbation  $\delta$ , i.e.,

$$\left\{ [\hat{\mathbf{t}}_j(x^+) - \hat{\mathbf{t}}_j(x^0)] ./ \hat{\mathbf{t}}_j(x^0) \right\}_{j=1}^\infty, \quad (21)$$

where  $./$  denotes the element-wise division.

In the application below, we set  $\delta$  for a period-2 DJ shock and a period-3 DAX shock to

unity, which is roughly one sample standard deviation of  $r_{t,2}^{(us)}$ , and  $r_{t,3}^{(g)}$ . In order to specify the vector  $\delta$  for a typical shock of unity to one of the period-1 variances accounting for the contemporaneous correlation structure among the period-1 variables, we follow Gallant et al. (1993) and rely on conditional expectations. In particular, when considering a perturbation of unity to the period-1 DJ variance  $r_{\tau}^{(us)}$ , then the remaining entries of the vector  $\delta$  are specified such that the corresponding elements of  $\hat{\mu} + \delta$  are equal to the conditional expectations of  $r_{\tau}^{(us,g)}$  and  $r_{\tau}^{(g)}$  given the value for  $r_{\tau}^{(us)}$ . These conditional expectations are approximated by the non-parametric Nadaraya-Watson kernel smoother with a rule-of-thumb bandwidth selection (see, e.g., Li and Racine 2007, p. 66ff). We construct  $(1 - \alpha)$  percent confidence bands around the IR function by drawing a sample of 10,000 simulated values for the parameter vector  $\theta$  from the asymptotic distribution of the ML estimator for  $\theta$ . For each simulated  $\theta$  value we compute the IR function and put an interval around the IR function obtained for the ML estimates, just wide enough to include  $(1 - \alpha)$  percent of the simulated IR functions (see, Gallant, et al. 1993).

## 5 Empirical Results

### 5.1 Parameter estimates and marginal effects

Table 2 reports the ML parameter estimates for the three-phase model given by Equations (5), (6) and (9)-(12) fitted to the full sample data described in Section 2 together with the results of diagnostic checks on the standardized residuals defined in Equations (16) and (17). The orders of the model components have been selected using the Schwarz-information criterion and are given by  $(q_1, p_1, z_1, \bar{q}_1, \bar{p}_1, \bar{z}_1) = (2, 3, 1, 1, 1, 2)$  for the period-1 component,  $(q_2, p_2, z_2, w_2) = (2, 3, 0, 1)$  for the period-2 component and  $(q_3, p_3, z_3, w_3) = (2, 3, 0, 0)$  for the period-3 component.

The largest eigenvalue of the estimated characteristic matrix  $\sum_{i=1}^3(\mathcal{A}_i^* + \mathcal{B}_i^*)$  (see Equation 14) is given by 0.92 indicating that the short-term volatility process of the German and US stock market across the three intra-day periods is stable in mean, though with a fairly high persistence in the conditional mean. The results of the  $F$ -test for residual predictability using 50 lags reveals that the model successfully accounts for the joint dynamics of the DAX and DJ volatility. The standardized residuals for all three intra-day periods pass the  $F$ -test at the 1% significance level. Further increasing the model order beyond the Schwarz-preferred specification did not significantly improve the results of the diagnostic checks.

Table 3 reports the implied ML estimates of the marginal effects, which are given by the elements of the matrices  $\Delta_0$ ,  $\mathcal{A}_i$  and  $\mathcal{B}_i$  in the vector representation of the three-phase model (see Equation 13). They reveal evidence for both heat-wave and meteor-shower effects. Across all intra-day periods the variances of both markets depend significantly on their own lags (heat waves) as well as on the lagged variances of the other market (meteor showers). Next, we find that the covariance of the DAX and DJ returns in period 1 ( $r_{t,1}^{(\text{us,g})}$ ) has a significant effect on the subsequent period-3 and period-1 DAX variances. Since this covariance itself depends significantly on lagged DJ and DAX variances, it appears to be a further volatility transmission channel in addition to the transmission directly through the variances.

In order to analyze the immediate causal impact of variance shocks, we provide in Figure 4 a diagram of the marginal effects of each intra-day variance on the next-period variance of the respective domestic and foreign market together with the associated estimates taken from Table 3. The diagram reveals that variance shocks in each intra-day period have a significant effect on the next-period variance of the home market. Furthermore, it appears that the importance of those heat-wave effects depends on the currency of the transmitted information: more recent domestic news from the immediately preceding intra-day period is more important than older domestic news from a period separated by a non-trading period.

Next, we also find for both stock markets significant causal effects of news which has gener-

ated volatility abroad. In general, those meteor-shower effects are somewhat smaller than the heat-wave effects. Similar to the heat waves, the importance of the meteor showers critically depends on the currency of the information and, additionally, on whether the news is from a period with or without a trading overlap. Specifically, news causing volatility on the German market during the trading-overlap period 1 does not have a significant direct causal effect on the DJ variance in the next trading period 2. This implies that the volatility impulse of global news hitting the markets when they are simultaneously trading is transmitted immediately during period 1 via the US variance to the subsequent US trading, which reflects the US economy's leading role for international stock markets. In sharp contrast to the DAX volatility of the joint-trading period 1, the DAX volatility in period 3 when the US market is closed has a relatively strong and significant causal effect on the next day period-1 DJ variance. Note that this effect of the DAX shock on the period-1 US volatility is even larger than the period-1 US response to a domestic shock from the previous trading period. This relatively strong impact of the period-3 DAX volatility is consistent with the result reported by Dimpfl and Jung (2012), that the European markets morning trading has significant impact on the US volatility. It can be explained by the fact that during period 3 the German market processes and aggregates global news generated after the closing of the US market (including news from the Asian markets), which hit the US market when re-opening at the next day in period 1 as new information.

Turning to the causal importance of the US market for the German volatility, we find a significant marginal effect of the US volatility in period 2 when the German market is closed on the DAX variance in the next trading period. Interestingly, this effect from the US to the German market appears to be significantly smaller than the corresponding effect of the German to the US market, namely the impact of a period-3 DAX shock on the next period-1 DJ volatility. This seems to contradict the general assessment that it is the US stock market which is the leading market. However, this apparent contradiction can be resolved by

accounting for the fact that news generating US volatility in period 2 hits the German market with a time delay, which is caused by the time gap between the closing of the US market and the opening of the German market, and after the news is processed by the trading on the Asian markets; in contrast, the news generating DAX volatility in period 3 arrives the US market immediately without such a time delay.

Taken all together, we can conclude that the short-run volatility dynamics of the German and US stock market are driven by both heat-wave and meteor-shower effects and that the importance of their immediate impacts on the next period to trade critically depends on how current the corresponding news is.

## 5.2 Impulse-response analysis

Although the marginal effects discussed above are suggestive about the impact of volatility shocks on the future volatility on the home and foreign market, they do not provide the complete picture. In particular, consider, e.g., a shock on the DAX volatility in period 1. Its direct impact on the subsequent volatility of the US market in period 2 and on the German market in period 3 is measured by the respective marginal effects. However, the period-1 DAX shock may also influence the period-2 US volatility indirectly, namely through a simultaneous change of the period-1 US variance and the covariance. Similarly, there is an indirect effect of the period-1 on the period-3 DAX volatility via an increase of the period-2 US volatility. Information about the compound impact consisting of the direct and indirect effects of shocks on subsequent volatility is provided by the IR function defined in Section 4. The values of the perturbation  $\delta$  and of the conditioning argument for the base case  $x_0 = (\hat{\rho}', \hat{\mu}', \hat{\rho}', \dots)$  which we used to compute the IR functions according to Equation (21) are summarized in Table 4.

Figure 5 displays the multi-period IR functions tracing trading period by trading period the response to volatility shocks occurring in the three different intra-day phases. Shocks in

all periods lead to significant responses on the respective home and foreign market which die out approximately after two months. In general, the responses of both markets to domestic shocks are larger than to foreign shocks. Next we note that the immediate responses after one trading period essentially confirm the results gleaned from the immediate marginal effects discussed above. A notable exception is the significant spillover of a period-1 DAX shock onto the DJ volatility, while the corresponding direct marginal effect was found to be negligible (see Figure 4). This implies that the period-1 DAX volatility influences the subsequent US trading only indirectly via its contemporaneous correlation with the period-1 DJ variance. Hence, it appears that spillovers of shocks occurring on the German market during trading-overlap periods onto the subsequent US trading mainly reflect global news impinging concurrently on the volatility of both markets, rather than a causal phenomenon.

### 5.3 Impact of the subprime crisis

Recent studies, including those of Bubak, et al. (2011), Chiang and Wang (2011) and Engle, et al. (2012) report evidence for significant changes of the dynamic volatility transmission mechanism on international financial markets during financial crises. This finding and the enormous impact of the recent subprime crisis on the international financial markets suggest to treat the period during the subprime crisis covered by our sample differently from the periods before and after the crisis. For this purpose we allow for structural changes in the volatility spillover mechanism modeled by the three-phase specification. A potential source of such changes could be the boost of intensity at which news hits the international financial markets when entering a turbulent crisis period – especially for news from the country where the crisis originated. Additionally, those changes could also reflect an increase of the time of trading it takes to settle the differences in the traders’ interpretation of news within and across regional markets, which can be expected due to a higher level of general uncertainty and the increased amount of information to be processed during crisis episodes.

In order to analyze the effects of the subprime crisis on the short-term volatility transmission mechanism, we extend our model specification and allow the parameter values to be different during the crises than before and after the crises, while using the same orders for the three model components as selected in Section 5.1 above. This is implemented by means of a subprime crisis dummy, say  $CR_t$ , i.e. by defining the  $i$ th parameter of the model as  $\theta_i = (1 - CR_t) \cdot \theta_{i0} + CR_t \cdot \theta_{i1}$  for all  $i$ , where  $\theta_{i0}$  denotes the value of the parameter before and after the crisis and  $\theta_{i1}$  is the value during the crisis. Following Bekaert, et al. (2011), we define the subprime crisis to start at August 2007 and to end March 2009, shown as the dark-gray shaded area in Figures 2 and 3.

The ML estimation results for this extended three-phase model reveal that the inclusion of the subprime-crisis dummy substantially improves the fit of the model. The  $p$ -value of the likelihood-ratio test of the null hypothesis that the parameter values during the crisis period are equal to those before and after the crisis equals  $8.4 \cdot 10^{-14}$ , indicating a strong rejection of the null. Next we find that the parameter estimates (not presented here) imply a significant increase of the largest eigenvalue of the characteristic matrix (see Equation 14) from 0.90 during the non-crisis periods to 0.96 during the crisis. Hence, the crisis leads to a substantial rise of the general persistence of volatility shocks in the German and US stock market.

Figure 6 provides the estimates of the immediate marginal effects of domestic and foreign volatility shocks on the next-period variances for both the non-crisis and the crisis period. The results indicate that before and after the crisis those effects remain typically very close to those obtained under the model specification without a crisis-dummy (see Figure 4). Next, the comparison between the marginal effects obtained for the non-crisis periods and those during the crisis reveals that the crisis had a major impact on the short-term volatility transmission mechanism. In particular, the crisis is associated with a substantial strengthening of the meteor-shower effects, e.g. the size of the marginal effect of the period-3 DAX variance on the subsequent period-1 DJ variance increased during the crisis by 10% and that of the period-2



DJ on the period-3 DAX variance by even 58%. Obviously, this particularly large increase of the immediate causal effect of the DJ on the DAX variance, dominating the increase of the causal effects of the DAX on the DJ variance, reflects the fact that the subprime crisis had its origin in the US subprime mortgage market and spread out across international stock markets via various economic and financial links, including mortgage-backed securities widely held by financial firms all over the world. Due to those links the investors' need to closely monitor the US market increased during the crisis in order to gather new critical information about investments in the German market. This in turn might have significantly intensified the causal effects in particular of US business news on the volatility in the German market. We also note that our results concerning the meteor-shower effects during the crisis and the non-crisis periods are in line with the findings of Diebold and Yilmaz (2009) and Engle, et al. (2012) who also report a substantial strengthening of the inter-market volatility linkages of regional markets induced by financial crises.

In Figure 7 we provide the IR functions of shocks occurring in the three intra-day periods obtained for the crisis and non-crisis periods. Here we use the same values of the perturbation and of the conditioning arguments for the base case as selected in Section 5.2 (see Table 4). Note that Figure 7 does not display confidence bands for the crisis period. The reason for this is that the volatility model is very close to non-stability during the crisis episode, as indicated by the largest eigenvalue of the characteristic matrix given by 0.96. Hence, using the asymptotic normal distribution of the ML-estimator in order to simulate artificial values of the model parameters for the construction of the confidence bands (as described in Section 4) often leads to simulated values violating the stability condition. The comparison of the IR functions during the crisis with those for non-crisis periods shows that the crisis leads to a substantial increase of the time it takes for volatility shocks to die out. This is fully in line with our finding that during the crisis episode the largest eigenvalue of the characteristic matrix is well above its value before and after the crisis. As mentioned above, this general

rise of persistence in the volatility process could reflect an intensified information clustering and/or an increase of time needed by the investors to process and interpret new information. In particular, while the looming crisis initially appeared to be a local phenomenon bound to the US market of subprime mortgages, it became progressively evident that this crisis has a global dimension. International financial markets experienced a clustering of news concerning the potential devaluation of various investments, which finally culminated in the Lehman Brothers Inc. bust of 2008. This intensified news clustering was accompanied by a large and long lasting uncertainty about the crisis implications on the real and financial sector of the global economy, which hindered a fast interpretation and pricing of news across international financial markets.

## 6 Conclusion

A novel sequential three-phase model is proposed in order to assess the short-term interdependence of the realized variances and covariance of the non-synchronously traded US Dow Jones and German DAX stock market indices. Our model contributes to the literature by explicitly accounting for the chronological ordering of overlapping and non-overlapping trading periods. Moreover, our approach embodies the realized covariance as an additional indirect transmission channel for volatility shocks and accounts for the contemporaneous interdependence between the (co)variances during partially overlapping trading times.

Considering the 15-year period from 1996 to 2010, we remove the common long-run trend from the realized (co)variance series and focus on the analysis of short-run volatility transmission patterns. Besides the analysis of direct marginal effects, an impulse response analysis serves for the quantification of compound (direct and indirect) effects of volatility shocks onto volatilities in subsequent periods. Our empirical results show that both own market heat-weave and cross market meteor-shower effects are present across all intra-day trading

periods. In general, the impact of heat-wave effects is larger than of meteor-shower effects. Furthermore, we find that the importance of those effects critically depends on the information currency, that means more recent news appear to be more important than older news. Finally, we find considerable changes in the short-run volatility transmission mechanism during the recent subprime crisis period with a substantially stronger persistence of volatility shocks. The subprime crisis period shows up with much more pronounced meteor-shower effects of volatility shocks from the U.S. market, where the crisis originates, to the German market volatility.

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*Table 1. Descriptive statistics for detrended realized variances and covariance.*

	Period 1			Period 2	Period 3
	$r_{t,1}^{(\text{us})}$	$r_{t,1}^{(\text{g})}$	$r_{t,1}^{(\text{us,g})}$	$r_{t,2}^{(\text{us})}$	$r_{t,3}^{(\text{g})}$
Mean	0.97	0.95	0.00	0.95	0.96
Std. dev.	1.06	1.02	0.42	1.40	1.05
Skewness	8.54	8.38	-1.15	23.39	10.33
Kurtosis	132.03	142.46	117.88	912.44	191.98
Minimum	0.07	0.04	-8.29	0.06	0.02
Maximum	24.77	26.57	8.91	60.71	27.55
LB(50)	2003.2311	2092.11	138.80	849.12	1969.47

Table 2. ML-Parameter Estimates for the BIC selected sequential three-phase CAW model.

Period-1 component								
param.	estimate		param.	estimate		param.	estimate	
$A_{11,1}$	.346*	.004	$A_{21,1}$	.344*	−.032			
	.023	.355*		−.079*	.221*			
$B_{11,1}$	.001	−.001	$B_{21,1}$	.087	.066	$B_{31,1}$	.274*	−.011
	−.001	.001		.030	.277*		−.018	.390*
$D_{11,1}$	.504*	.598*	$D_{12,1}$	★★	★★			
	−.075*	.139*		.223*	.650*			
$\nu_1$	9.864*							
Period-2 component								
param.	estimate		param.	estimate		param.	estimate	
$a_{1,2}$	.136*		$a_{2,2}$	.094*				
$b_{1,2}$	<.001		$b_{2,2}$	.122*		$b_{3,2}$	.119*	
$c_{1,2}$	.141*							
$d'_{0,2}$	.618*	−.020						
$\nu_2$	6.370*							
Period-3 component								
param.	estimate		param.	estimate		param.	estimate	
$a_{1,3}$	.217*		$a_{2,3}$	.066*				
$b_{1,3}$	.102		$b_{2,3}$	.035		$b_{3,3}$	.129*	
$c_{0,3}$	.161*							
$d'_{0,3}$	.186*	.421*						
$\nu_3$	8.307*							
Log-lik.: -6217.12			BIC: 12861.22			Max. eigenvalue: 0.92		
$p$ -values for $F$ -test on residual predictability (50 lags)								
$u_{t,1}^{(us)}$	$u_{t,1}^{(g)}$	$u_{t,1}^{(us,g)}$	$u_{t,2}^{(us)}$	$u_{t,3}^{(g)}$				
0.037	0.015	0.105	1.000	0.825				

Note: selected model orders are  $(q_1, p_1, z_1, \bar{q}_1, \bar{p}_1, \bar{z}_1) = (2, 3, 1, 1, 1, 2)$  in period 1;  $(q_2, p_2, z_2, w_2) = (2, 3, 0, 1)$  in period 2;  $(q_3, p_3, z_3, w_3) = (2, 3, 0, 0)$  in period 3; ★★ indicates identifying restrictions setting parameter values to zero. The max. eigenvalue refers to the estimated matrix  $\sum_{i=1}^{\max(p,q)} (\mathcal{A}_i^* + \mathcal{B}_i^*)$  in Equation (14); \* denotes significance at the 1% level.

Table 3. *ML-Estimates of the marginal effects for the BIC selected sequential three-phase model.*

Period-1 component								
Depend. Variable	$s_{t-1,1}^{(us)}$	$s_{t-1,1}^{(g)}$	$s_{t-1,1}^{(us,g)}$	$r_{t-1,1}^{(us)}$	$r_{t-1,1}^{(g)}$	$r_{t-1,1}^{(us,g)}$	$r_{t-1,2}^{(us)}$	$r_{t-1,3}^{(g)}$
$s_{t,1}^{(us)}$	<.001	<.001	<.001	.120*	<.001	.003	.254*	.357*
$s_{t,1}^{(g)}$	<.001	<.001	<.001	<.001	.126*	.016	.055*	.442*
$s_{t,1}^{(us,g)}$	<.001	<.001	<.001	.008	.001	.123*	-.038*	.083*
	$s_{t-2,1}^{(us)}$	$s_{t-2,1}^{(g)}$	$s_{t-2,1}^{(us,g)}$	$r_{t-2,1}^{(us)}$	$r_{t-2,1}^{(g)}$	$r_{t-2,1}^{(us,g)}$		
$s_{t,1}^{(us)}$	.008	.004	.012	.119*	.001	-.022		
$s_{t,1}^{(g)}$	<.001	.077*	.017	.006	.049*	-.035*		
$s_{t,1}^{(us,g)}$	.003	.018	.026	-.027*	-.007	.079*		
	$s_{t-3,1}^{(us)}$	$s_{t-3,1}^{(g)}$	$s_{t-3,1}^{(us,g)}$					
$s_{t,1}^{(us)}$	.075*	<.001	-.006					
$s_{t,1}^{(g)}$	<.001	.152*	-.014					
$s_{t,1}^{(us,g)}$	-.005	-.004	.107*					
Period-2 component								
Depend. Variable	$s_{t-1,2}^{(us)}$	$r_{t-1,2}^{(us)}$	$r_{t-1,3}^{(g)}$	$r_{t,1}^{(us)}$	$r_{t,1}^{(g)}$	$r_{t,1}^{(us,g)}$		
$s_{t,2}^{(us)}$	<.001	.136*	.141*	.382*	<.001	-.025		
	$s_{t-2,2}^{(us)}$	$r_{t-2,2}^{(us)}$	$r_{t-2,3}^{(g)}$	$s_{t-3,2}^{(us)}$				
$s_{t,2}^{(us)}$	.122*	.094*	<.001	.119*				
Period-3 component								
Depend. Variable	$s_{t-1,3}^{(g)}$	$r_{t,2}^{(us)}$	$r_{t-1,3}^{(g)}$	$r_{t,1}^{(us)}$	$r_{t,1}^{(g)}$	$r_{t,1}^{(us,g)}$		
$s_{t,3}^{(g)}$	.102	.162*	.217*	.035*	.177*	.157*		
	$s_{t-2,3}^{(g)}$	$r_{t-1,2}^{(us)}$	$r_{t-2,3}^{(g)}$	$s_{t-3,3}^{(g)}$				
$s_{t,3}^{(g)}$	.035	<.001	.066*	.129*				

Note: the marginal effects are given by the elements of the coefficient matrices  $\Delta_0$ ,  $\mathcal{A}_i$  and  $\mathcal{B}_i$  characterizing the vector representation of the three-phase model – see Equation (13); \* denotes significance at the 1% level.



*Table 4. Shock scenario of the impulse response analysis.*

Shock	$\hat{\mu}'$			$\delta'$		
	(us)	(us,g)	(g)	(us)	(us,g)	(g)
period-1 DAX	0.97	0.00	0.95	0.69	0.07	1
period-1 DJ	0.97	0.00	0.95	1	-0.02	0.63
period-2 DJ	0.95			1		
period-3 DAX			0.96			1

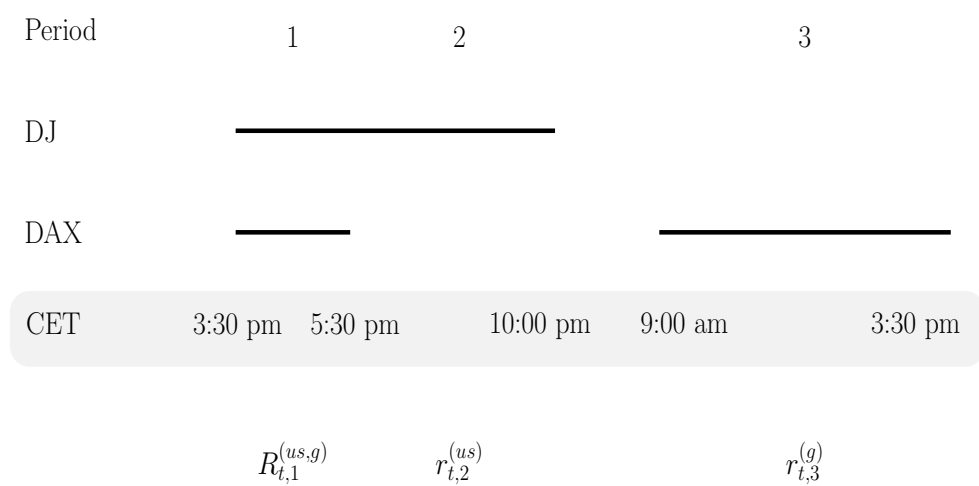


Figure 1. Daily trading hours for the US (DJ) and the German (DAX) stock markets.

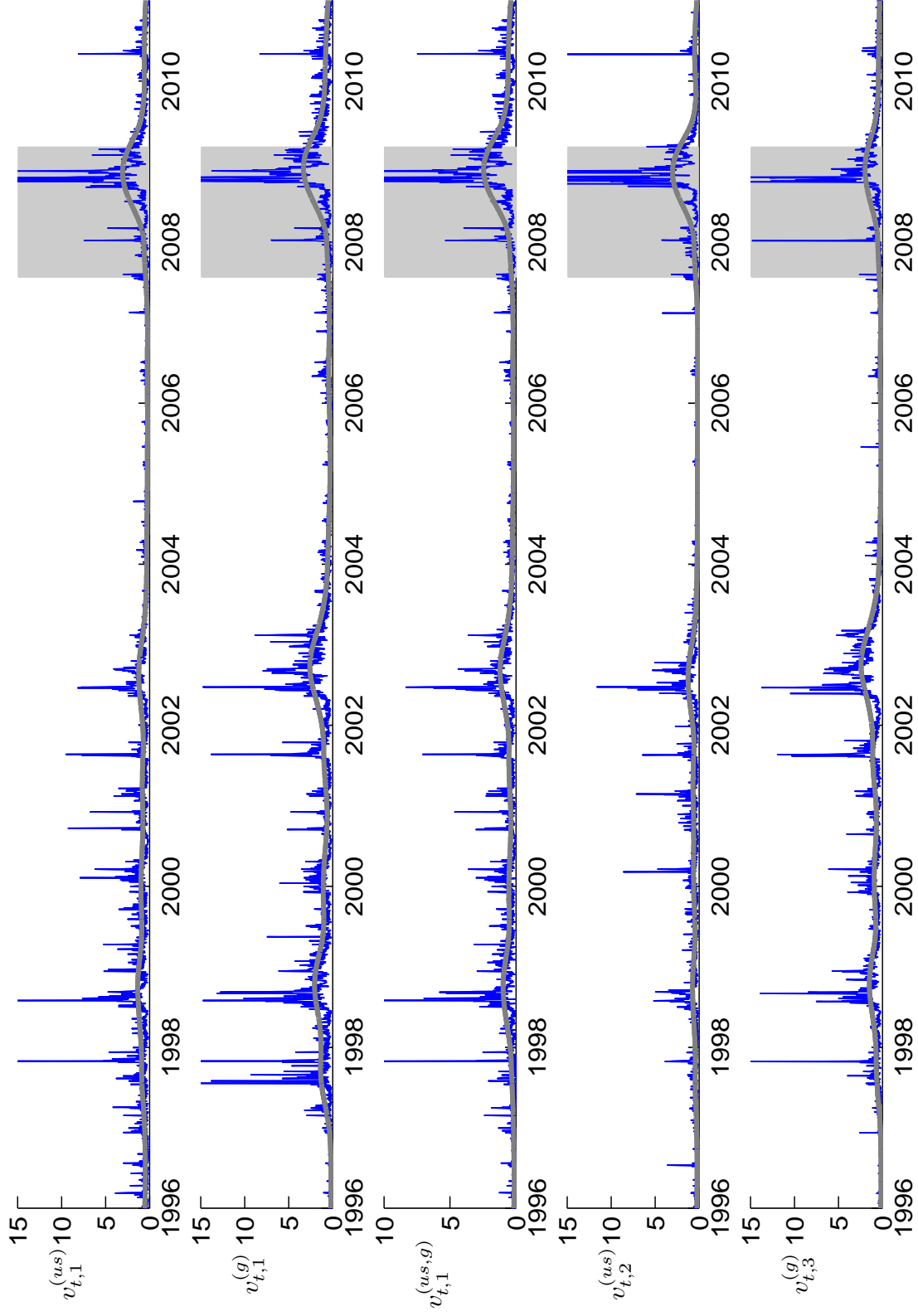


Figure 2. Time series of the realized (co)variances for the German DAX ( $g$ ) and US Dow Jones ( $us$ ) stock indices for the three intraday periods as in Figure 1 together with their estimated long-run component (thick gray line). The gray shaded area marks the subprime crisis period as defined in Section 5.

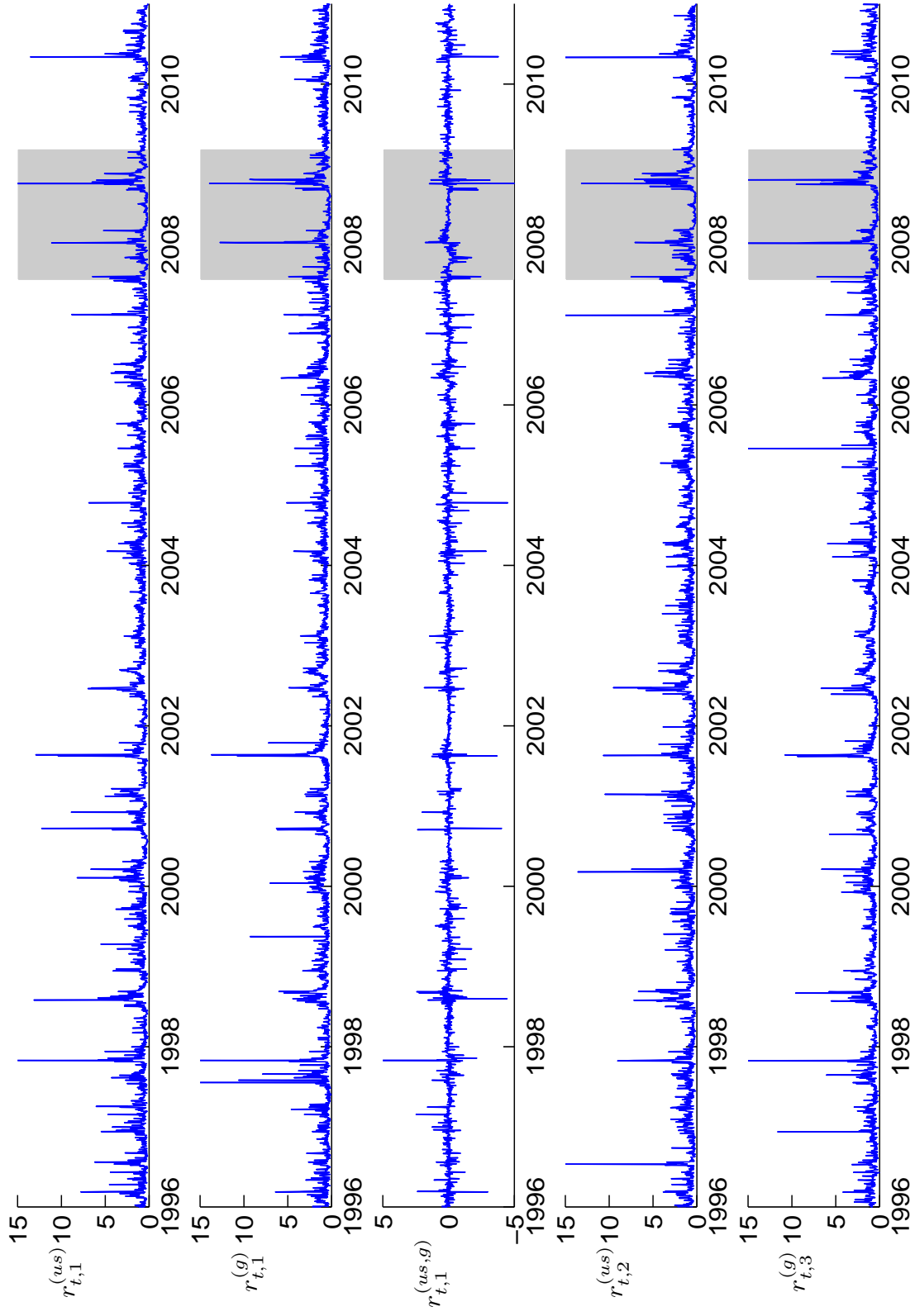


Figure 3. Time series of the detrended realized (co)variances for the German stock index DAX ( $g$ ) and US Dow Jones stock index ( $us$ ) for the three intra-day periods illustrated in Figure 1. The gray shaded area marks the subprime crisis period as defined in Section 5.

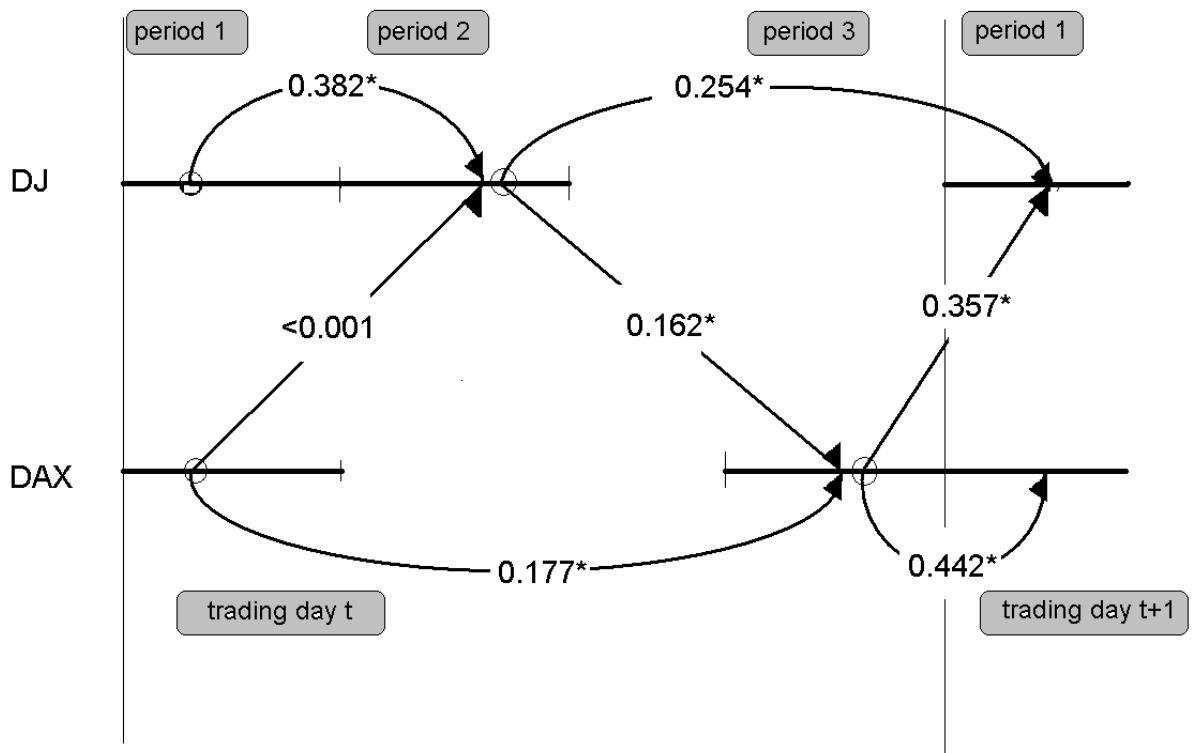


Figure 4. Estimates of the direct marginal effects of intra-day variances on the next-period variance at home and abroad; \* denotes significance at the 1% level.

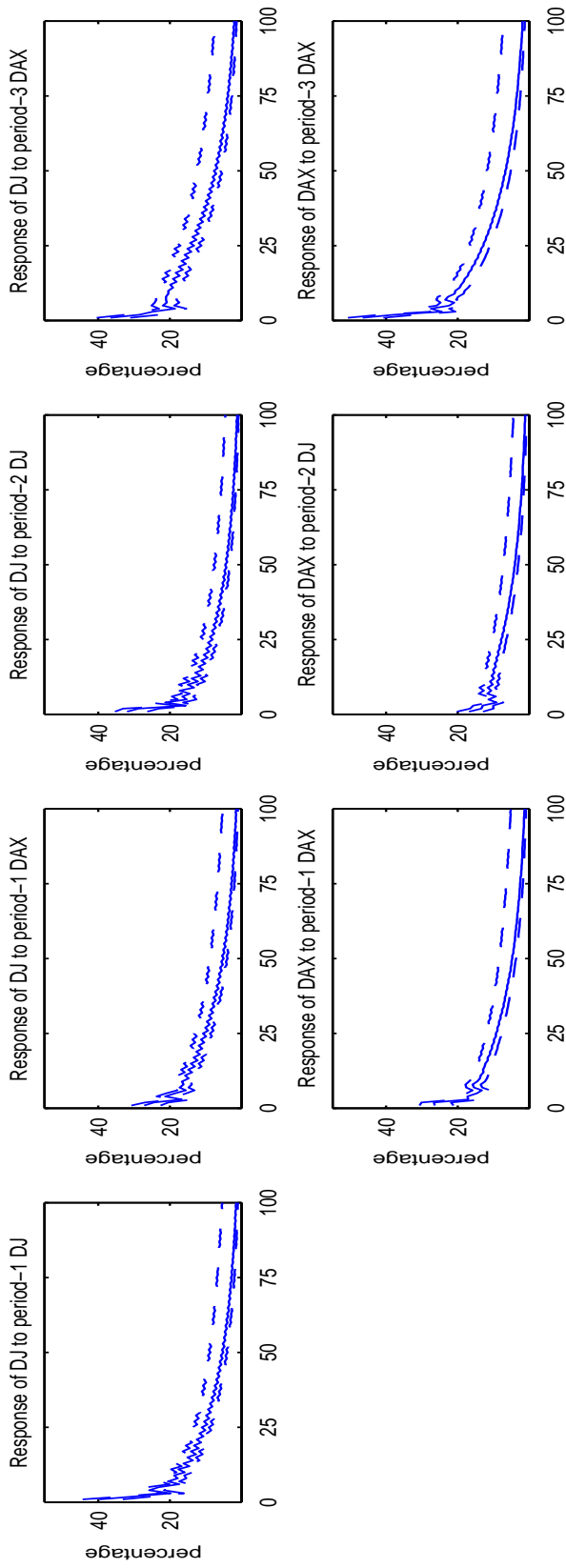


Figure 5. Estimated IR functions computed according to Equation (21). The dashed lines indicate 99% confidence bounds.

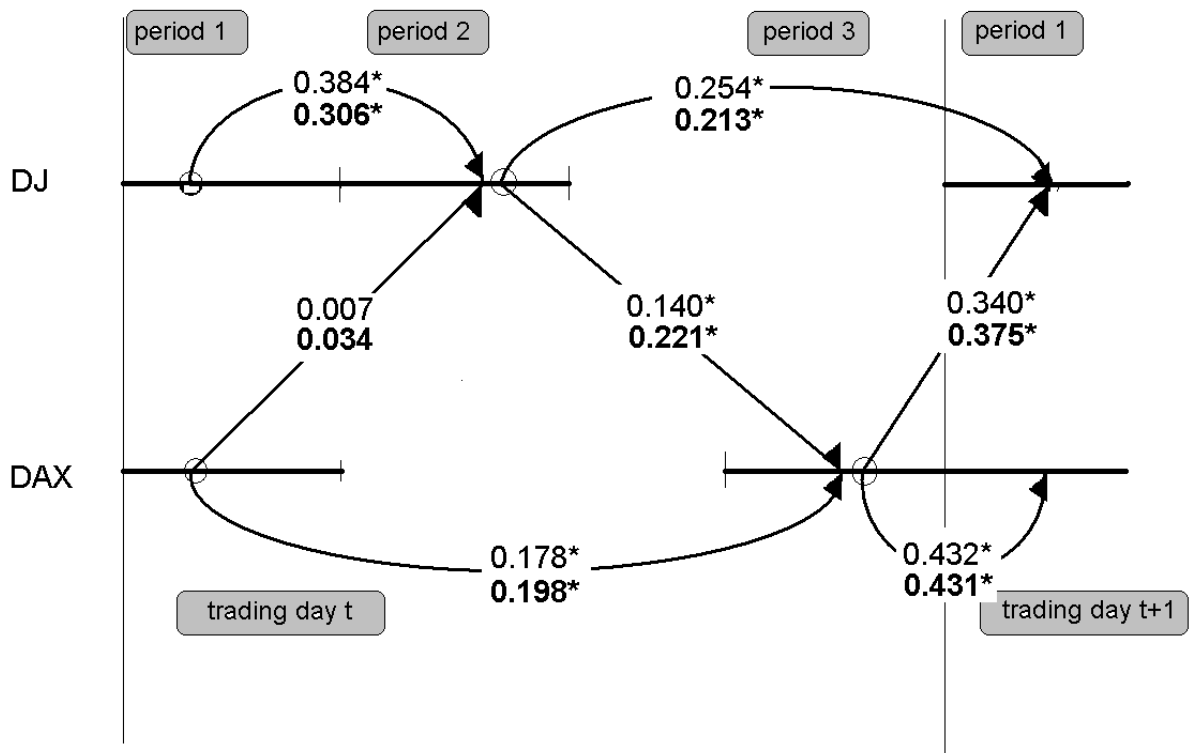


Figure 6. Estimates of the direct marginal effects of intra-day variance on the next-period variance at home and abroad during non-crisis periods and during the subprime crisis (bold numbers); \* significant at the 1% level.

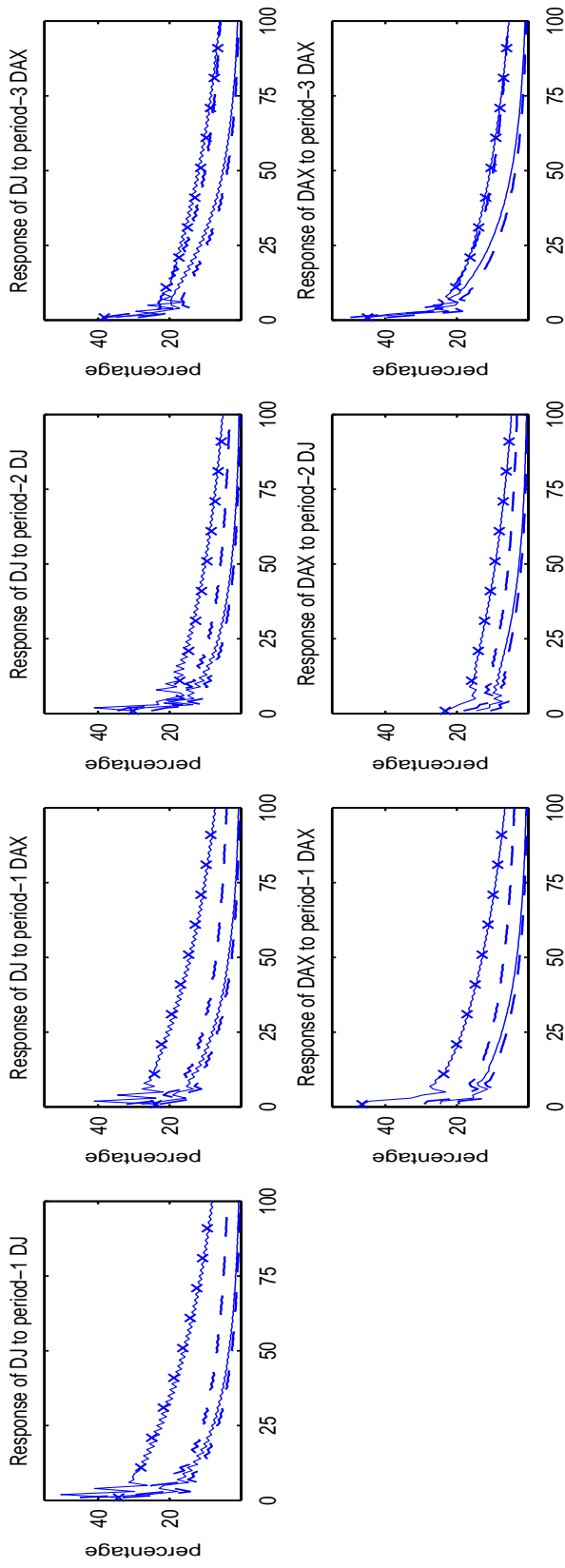


Figure 7. Estimated IR functions during non-crisis periods (solid line) and during the subprime crisis (crossed line) computed according to Equation (21). The dashed lines indicate 99% confidence bounds for non-crisis IR functions.