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Fiscal Competition and a Potential Growth Effect of Centralization

Kersten Kellermann*

December 2007

Abstract

I discuss a dynamic version of the standard tax competition model, where world capital supply is not fixed. The time horizon of the welfare maximizing government is assumed to be infinite, whereas the household sector is designed according to the two-period overlapping-generations model. Thus, social evaluation is involved which is not directly based on individual preference ordering. In the case of decentralized public decision making, the model produces a type of inefficiency caused by a head tax on immobile workers. From the viewpoint of the global economy, head taxes reduce global saving and create a negative externality on global capital supply. The lower-level government thus finances the public goods supply with too little reliance on source taxes. Centralization of investment decisions has the potential for welfare improvement. It can lead to an increase in private and public investment and enhances transitory growth.

Keywords: Public investment, Tax competition, Fiscal competition, Social time preference, Federalism, Centralization

JEL-Klassifikation: H70; H10

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Fiscal Competition and a Potential Growth Effect of Centralization

1. Introduction

In standard tax competition models, such as Zodrow and Mieszkowski (1986) or Wilson (1986), a tax on each jurisdiction’s fixed factor achieves a first-best allocation. Each jurisdiction views such a tax as a nondistortionary source of revenue and therefore follows first-best rules for public good provision. In the basic single-period models of tax competition, only a source tax on interregional mobile capital income creates a positive fiscal externality and consequently, the government sets its tax rates and public good supply at inefficiently low levels. In this paper I discuss a dynamic version of the standard tax competition model, where the basic assumption that mobile capital is fixed in supply for the world economy is relaxed. Households are assumed to be immobile across lower-level jurisdictions. In the model discussed, the planning horizon of the government is infinite in time, whereas the household sector is designed according to the two-period overlapping-generations model. Thus, social evaluation is involved which is not directly based on individual preference ordering.

The described modification of the standard tax competition model produces a type of inefficiency caused by a head tax on workers, although labor supply is price inelastic. This head tax is equivalent to a wage tax. From the viewpoint of the central government, head taxes reduce global saving and thus create an externality to the rest of the world by reducing public and private investment. These externalities are not taken into account by the lower-level governments that have no incentives to stimulate private saving because it has no effect on the level of in-state investment under the open economy assumption. As a result, the lower-level government finances the public goods supply with too much reliance on head taxes.

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1 Earlier versions of this paper were presented at the Annual Meeting of the Austrian Economic Association (May 2007, Klagenfurt), at the “II. Workshop on Fiscal Federalism: Competition and Cooperation among Governments” (June 2007, Sevilla), at the PET 2007 conference (July 2007, Nashville), and at the 63rd Congress of the IIPF (August 2007, Warwick). I am grateful to all participants for helpful comments.

2 Bucovetsky and Wilson (1991) describe a positive externality that is induced by a wage tax; therefore, wage taxes are set suboptimally from the viewpoint of the entire or global economy. The crucial presumption made by Bucovetsky and Wilson (1991) is that global labor supply increases in response to a wage tax induced change of the world return of capital.

3 Wildasin (2003) analyzes fiscal competition among jurisdictions in an explicitly dynamic framework, where it is costly and time-consuming to adjust the private capital stock, so the degree of capital mobility between jurisdictions is imperfect. In the model discussed in this paper, I allow for perfect capital mobility.

4 Jones (2005) surveys some arguments as to why there should be a difference between the way we view saving versus consumption decisions collectively and the way we view these decisions individually.
The model contributes to the discussion about the various effects of decentralization on economic growth inspired by an initial paper of Oates (1993). Oates takes the view that better targeting of growth-enhancing infrastructure investment in lower-level jurisdictions could raise an economy’s growth rate. Lejour and Verbon (1997) discuss the impact of tax competition in a two-country endogenous growth model where a source tax on imperfect mobile capital is used to finance transfers to immobile workers. In their model, the well-known positive fiscal externality of a source tax on the global tax base is counteracted by a negative growth externality of a source tax with respect to the creation of global wealth. If the growth externality overstates the standard positive fiscal externality of a source tax, tax competition leads to too much redistribution and decelerates growth. Brueckner (1999) uses an overlapping-generation model where the public good demand of young and old people differs. Decentralization allows for local segmentation of young and old into homogenous communities, so that the amount of public goods provided can be tailored to suit their preferences. Since public good provision affects private consumption, decentralization alters the incentive to save. In response, the transitory growth rates can either enhance or reduce. In a more recent paper, Brueckner (2006) uses an endogenous growth model to explore the connection between decentralization and economic growth. However, in this analysis, federalism unambiguously contributes to faster economic growth in the long run by increasing the incentive to invest in human capital. Edwards (2005) discusses a time consistency problem of the revenue maximizing central government that stems from a lack of tax competition. In contrast, tax competition commits the regional government not to expropriate tax payers in the long run, which again stimulates human capital investment and growth. The wasteful leviathan state is also considered by Rauscher (2005). Although the selfish policy makers have the same namely infinite planning horizon as the private households, the rent-seeking government’s preferences deviate from the private ones. In his model, the effects of increasing mobility of the tax base on economic growth are ambiguous and depend on the rate of substitution between the government’s own consumption and its political support. In a recent paper, Rauscher (2007) focuses on the impact of tax competition on innovation in the public sector. The model shows the counter-intuitive result that increased mobility of the tax base induces the leviathan government to spend a larger share of its budget on unproductive activities.

The model discussed in this paper emphasizes a potential impact of centralization on transitory growth as well as long-term labor productivity that has so far been neglected in the literature. The intuition of what is shown in the model is the following: as is known from the early work of
Phelps (1961, 1965) and Diamond (1965), private savings can be too low or too high from the perspective of a welfare-maximizing government. Too low a level of private saving leads to a global under-accumulation of capital, whereas too high a level of private saving leads to over-accumulation. In the latter case, the economy can even end up in a situation of dynamic inefficiency, where lower savings would not reduce long-term consumption. From the viewpoint of the central government, capital supply is endogenously determined by savings, so that the way in which taxes affect global saving is taken into account by the public decision maker. Centralization of competences has the potential of welfare improvement. It can lead to an increase in private and public investment and thus enhances transitory growth. The results of the model are in line with empirical studies done by Davoodi and Zou (1998), Zhang and Zou (1998) and Xie, Zou and Davoodi (1999). However, Brueckner (2006) cites numerous more recent empirical studies that find a positive relation between decentralization and growth.

The rest of the paper is structured as follows. Section 2 presents a dynamic model of a small open economy where private capital formation and saving decisions are influenced by fiscal instruments as a source-based tax, and a head tax on workers and the supply of productive infrastructure. In the first step of the analysis, private households and firms take all policy instruments parametrically. In a second step, governments decide about policies, taking the first order conditions of the private sector as given. Section 3 is devoted to the question of how governments design an optimal investment and financing policy. According to the so-called golden rule of public sector borrowing, public investment expenditure is financed by public borrowing (see Kopits and Symansky, 1998). However, in each period, the tax revenue must cover the debt service plus public capital depreciation. To make the model more applied, alternative specifications of public inputs are distinguished according to the degree of homogeneity of the aggregate production function. Note that this typification is not crucial for the results presented in the paper. In section 3.1, we look at a small open jurisdiction in which the markets for goods and inputs are perfectly competitive and both goods and private capital are perfectly mobile across borders. According to the neoclassical approach, the public decision maker seeks to promote social welfare, while acting as a price-taker in the capital market. Decentralized public decision making leads to an allocation where the private marginal rate of time preference equals the marginal productivity of private and public capital. However, as discussed in section 3.2, this situation does not represent a welfare maximum if social and private preferences differ. Section 4 presents some conclusions.
2. Public Infrastructure Investment

There are two types of governmental units in the model, namely a central government and a system of lower-level governments. The lower-level jurisdictions might be states. Each of them are indexed by superscripts $j$ for $j = 1...R$. In each jurisdiction, different factors of production are applied: labor $L_i^j$, private capital $K_i^j$ and three kinds of public capital $G_i^j$ (for $i = P, K, L$) which are used by private firms located in the jurisdiction to produce one homogeneous good $Y_i^j$ at time $t$ (see Feehan, 1989). The price of $Y_i^j$ is normalized to unity. The labor supply grows at an exogenous rate $n^j$, thus $L_{t+1}^j = (1+n^j)L_t^j$. Capital is simply non-consumed output. The aggregate production function

$$A(G_{p,t}^j)F(K_{p,t}^j, G_{k,t}^j, L_{t}^j, G_{L,t}^j) = Y_t^j$$  \hspace{1cm} (1)$$

exhibits positive diminishing marginal products with respect to each input, for all factors $L_i^j, K_i^j, G_i^j > 0$, and the Inada conditions hold. All factors are complements in the sense that the second-order cross derivations of $A(G_{p,t}^j)F(K_{p,t}^j, G_{k,t}^j, L_{t}^j, G_{L,t}^j)$ are positive. Total factor productivity $A(G_{p,t}^j)$ is a convex function of the local public capital stock $G_{p,t}^j$.

The production function, which is assumed to be the same in all lower-level jurisdictions, has increasing returns of scale in all inputs, but $F(K_{p,t}^j, G_{k,t}^j, L_{t}^j, G_{L,t}^j)$ is linearly homogeneous in private capital, labor, and the two kinds of publicly provided private inputs $G_{L,t}^j$ and $G_{k,t}^j$ together. The empirical literature on local public expenditure shows that several publicly provided inputs have roughly the same amount of rivalry as private goods (see Büttner et al., 2004). The government provides such private inputs because exclusion is not possible and a market solution is thus unsatisfactory. Since these public inputs are free of charge, they can be

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4 From an empirical point of view, it is difficult to divide public capital expenditure into productive and consumptive investment programs. In addition, some current expenditure on health or education could also be considered as growth-enhancing. However, most relevant economic studies focus on public investment in infrastructure such as highways, other transportation facilities or communication systems owned by the public sector. In recent empirical analysis, different methodologies have been followed to investigate the impact of public investment on economic activities. In most of this work, measures of public investment are found to increase aggregate output, yet there are exceptions (see Romp and de Haan, 2005).
interpreted as unpaid factors. A further property of the production technology is that it faces diminishing returns to the accumulation of private and public capital together. The public capital yields only production benefits, so that households are not immediate beneficiaries of public capital.

Since the private production function exhibits constant returns in \( L_t, K_t, G_{L,t} \) and \( G_{K,t} \), the output \( Y_t \) can be decomposed into the imputed shares of private capital, labor, and government capital \( G_{L,t} \) and \( G_{K,t} \) according to Euler’s theorem. We assume that the publicly provided input \( G_{L,t} \) generates a rent to private labor and \( G_{K,t} \) does the same to private capital. Therefore, the income share of the publicly provided private capital is distributed to private capital and labor. An example for investments that increase \( G_{L,t} \) and improve labor income is public expenditure in education. With respect to \( G_{K,t} \), one may think of public investment in modern transportation, communication systems and energy infrastructure that, for example, improve the exploitation of new technologies. Note that public ownership of capital does not generate rents that directly enter the public budget.

In the private sector, private capital has two costs to the firm: the rental price \( r_t \) and a source-based tax on capital revenue, where \( \tau_t \) denotes the capital tax rate. Firms invest capital up to the point where the marginal revenue of private investment equals the costs. In jurisdiction \( j \) the marginal revenue of private capital

\[
\frac{r_t}{1-\tau_t} = \frac{\partial Y_t}{\partial K_t} + \frac{\partial Y_t}{\partial G_{K,t}} G_{K,t} J
\]

is the sum of the partial product of private capital \( \partial Y_t / \partial K_t \) plus the output share of public capital appropriated by each unit of private capital \( (\partial Y_t / \partial G_{K,t})(G_{K,t} / J) \). The government
effectively provides a firm with \((G_{K,t}^j / K_t^j)\) units of public capital for each unit of private capital that it employs\(^5\).

Thus, in the lower-level jurisdiction \(j\) the aggregate domestic output

\[
Y_t^j = r_t^jK_t^j + W_t^j + \left( \frac{\partial Y_t^j}{\partial K_t^j} K_t^j + \frac{\partial Y_t^j}{\partial G_{K,t}^j} G_{K,t}^j \right) \tau_t^j
\]

(3)

can be decomposed in the gross wage income \(W_t^j\), the source tax revenue of the local government, and in an income share that goes to the mobile factor \(r_t^j K_t^j\). The interest rate \(r_t^j\) can thus be interpreted as the marginal opportunity costs of private capital to the jurisdiction. The wage income \(W_t^j\) includes a rent generated by \(G_{L,t}^j\).

The household sector of jurisdiction \(j\) is designed according to the two-period overlapping-generations model. An individual born at time \(t\) supplies a fixed amount of labor and receives a wage income \(w_t^j = W_t^j / L_t^j\). Further, \(t_t^j\) denotes a head tax that everybody has to pay during the time of youth. From the perspective of the private agents, the head tax \(t_t^j\), the source tax rate \(\tau_t^j\), and the public capital supply are taken parametrically. Each young person consumes \(c_t^{y,j}\) of net wage income and saves the remainder \(s_t^j = w_t^j - t_t^j - c_t^{y,j}\). In the second period of life, the individual consumes all wealth, both interest and principal \(c_{t+1}^{o,j} = s_t^j (1 + r_{t+1})\). The decision problem for young people is to maximize a log utility function \(u(c_t^{y,j}, c_{t+1}^{o,j}) = \ln c_t^{y,j} + 9 \ln c_{t+1}^{o,j}\) subject to the private budget constraint \(c_t^{y,j} = w_t^j - t_t^j - c_{t+1}^{o,j} / (1 + r_{t+1})\). The parameter \(9\), with \(0 < 9 < 1\), denotes the subjective discount factor. The first-order condition for a private utility maximum is that the marginal time preference of private households

\[
\left( \frac{\partial u}{\partial c_t^{y,j}} / \frac{\partial u}{\partial c_{t+1}^{o,j}} \right) - 1 = \frac{c_{t+1}^{o,j}}{9 c_t^{y,j}} - 1 = r_{t+1}^j \quad \text{equals the marked rate of interest. The consumption of the old is} \quad c_{t+1}^{o,j} = 9 c_t^{y,j} (1 + r_{t+1}^j) \quad \text{and the private budget constraint in the household’s optimum equals}
\]

\(^5\) For a given level \(G_{K,t}\), the inflow of private capital causes congestion costs for the private capital already located in the jurisdiction, since the provision with public capital per unit of private capital shrinks.
\[
\begin{align*}
  w_i^j &= t_i^j + c_i^{y,j}(1 + \vartheta). \\
  \end{align*}
\] (4)

The utility of a resident \( u(c_i^{y,j}, \vartheta, r_{i+1}^j) = \vartheta \ln(1 + r_{i+1}^j) + (1 + \vartheta) \ln c_i^{y,j} \) can thus be expressed as a function of consumption of the young \( c_i^{y,j} \), the subjective discount factor \( \vartheta \), and the interest rate \( r_{i+1}^j \). Private savings are a fixed fraction of available wage income \( \left( w_i^j - t_i^j \right) \left( \frac{(\vartheta)}{(1 + \vartheta)} \right) = s_i^{y,j} \). As is known from the work of Phelps (1961, 1965) and Diamond (1965), from the perspective of a welfare-maximizing government, these savings can be too low or too high, which leads to an under- or over-accumulation of private capital, respectively. In the next section, it can be shown that the central government takes this problem into account, whereas the lower-level government neglects it.

### 3. Public Decision Making

The public sector of jurisdiction \( j \) is designed as follows: the government’s total tax revenue is the sum of the capital tax revenue \( \Omega_i^j = \frac{\tau_i^j t_i^j}{1 - \tau_i^j} C_i^j \) and the head tax revenue \( T_i^j = t_i^j L_i^j \). Both tax rates \( \tau_i^j \) and \( t_i^j \) are allowed to be time-variant. A third financing instrument for government expenditure is public borrowing \( (B_{i,t+1}^j - B_{i,t}^j) \). Public consumption is neglected in the model, thus public expenditure consists of public net investment \( \sum_{i=P,K,L} (G_{i,t+1}^j - G_{i,t}^j) \), public depreciation \( D_i^j \), and the debt service. Empirically, there are unresolved questions about how to measure depreciation of the public capital stock (see European Commission, 2003, p. 104). Therefore, I treat public depreciation as exogenous. The public budget constraint in jurisdiction \( j \) and period \( t \) is thus given by

\[
\begin{align*}
  D_i^j + r_i^j B_i^j + \sum_{i=P,K,L} (G_{i,t+1}^j - G_{i,t}^j) = \Omega_i^j + T_i^j + (B_{i,t+1}^j - B_{i,t}^j).
\end{align*}
\]
In each period, the tax revenue plus public borrowing must cover the debt service $r_i B_t^j$, public capital depreciation $D_t^j$ plus public net investment. To rule out any Ponzi-game, the budget constraint of the public sector is divided in a current budget

$$D_t^j + r_i B_t^j = K_t^j \frac{r_i^j}{(1 - \tau_i^j)} \tau_t^j + T_t^j$$

and a public investment budget

$$\left( B_{t+1}^j - B_t^j \right) = \sum_{i=P,K,L} \left( G_{t+1}^j - G_{t+1}^j \right).$$

The government uses public sector borrowing just in order to finance public investment and accumulate a public capital stock. Transfers can only be financed by taxes and would occur as negative taxes.

Social welfare is a function not only of the utility of individuals who are members of society at present, but of the utility of all future members of society as well. Compared to private households, the government is less myopic and thus takes responsibility for unborn generations and their descendants. The government has the objective to maximize a social welfare function

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6 The special treatment of capital expenditure points to the “golden rule of public sector borrowing”. According to this fiscal rule, government deficit is accepted if accompanied by an increase in assets so that the government’s net asset position does not deteriorate. Thus, current expenditure must be covered by current receipts, while for investment expenditure, recourse to debt is allowed. In the United Kingdom and Germany, the government’s deficit-limiting budget rules basically follow the golden rule. Kopits and Symansky (1998) cite other countries that introduced balanced-budget rules of the golden rule type.

7 Since (5) and (6) are not formulated as inequality constraints, the government is restricted to debt financing of public investment and can not choose the financing instrument freely. Kellermann (2007) shows that a benevolent government that cares about future generations and is less myopic than private households uses taxes to finance public investment, because debt financing increases the marginal opportunity cost of public investment. This result holds at least in the medium and long term.

8 The transversality condition $\lim_{i \to \infty} (1/(1 + r^*)) G_{t+i} = 0$ implies that $r^* > n$ holds. From the public budget constraint of period $t$, the intertemporal budget constraint

$$B_t^j = \sum_{i=1}^{\tau_t} \left[ Y_{t+i}^j - w_i^j L_{t+i}^j - \left( D_t^j + \sum_{i=P,K,L} G_t^j - G_t^j \right) \right] + \frac{1}{(1 + r^*)} B_{t+i}^j$$

with $r_t = r^*$ can be derived.

9 This can be justified by the assumption that private households delegate intertemporal altruism to the government.
of the form \( \Psi = u(c^{0,j}_t) + \sum_{i=1}^{T} (1+\lambda)^{t-i} u(c^{x,j}_{i,j}, r^j_{i+1}, 9) \), where the utility of future generations is discounted at a rate \( \lambda > 0 \). The instruments at the government’s disposal in period \( t \) are investment \( G^j_{i,t+1} \) respectively borrowing \( B^j_{i,t+1} \), the source tax revenue \( \Omega^j_i \), and the head tax revenue \( T^j_i \).

In the following, the discussion focuses on the impact of tax competition on the marginal social rate of time preference of lower-level jurisdictions compared to that of a central jurisdiction. According to the social discount rate literature surveyed by Jones (2005), a public investment project should be undertaken if the marginal social opportunity costs of public capital are lower than or equal to the rate of return of the government’s project. Since the central requirement of intertemporal allocative efficiency holds if the marginal social costs of public capital are evaluated by the marginal social rate of time preference, the latter represents the crucial measure. However, the described requirement says little about desirable investment policy. The reason is that the marginal social costs of public capital are quite difficult to quantify. They do not only depend on the way public investments are financed but also on the link between domestic saving and domestic investment and thus on the degree of capital mobility. Lind (1990) emphasizes that the existence of a world with integrated capital markets fundamentally changes the analyses of the opportunity costs of public programs and policies. In the model discussed in this paper, the degree of capital mobility differs for the central jurisdiction and the lower-level jurisdiction.

### 3.1. Public investment policy of the lower-level government

First, we discuss the case of a single lower-level jurisdiction \( j \) that can be interpreted as a small open economy which competes for mobile capital. The private households and firms, and the government are price takers in the world capital market, thus \( r^j_t = r^* \) holds. In such a jurisdiction, domestic saving does not determine investment. The supply of capital is rather taken as totally elastic, so an increase in public borrowing does not directly crowd out private investment. The optimization problem of the regional government can be described as the Lagrangian expression
\[
\Gamma (K^j_i, B^j_i, G^j_{i,t}, c^j_{i,t}) = \Psi + \sum_{t=1}^{\infty} \mu^i_t \left[ \Omega^j_i + T^j_i - D^i_t - r^* B^i_t \right] \\
+ \sum_{t=1}^{\infty} \theta^j_i \left[ \left( B^j_{i,t+1} - B^j_i \right) - \sum_{i=P,K,L} \left( G^j_{i,t+1} - G^j_{i,t} \right) \right] \\
+ \sum_{t=1}^{\infty} \delta^j_i \left[ W^j_i - T^j_i - (1 + \delta) L^j_i c^j_{i,t} \right] + \sum_{t=1}^{\infty} \rho_i \left[ Y^j_i - W^j_i - K^j_i r^* - \Omega^j_i \right]
\]

where \( \mu^j_t, \rho^j_t, \delta^j_t, \theta^j_t \) denote the Lagrange parameters. The constraints of the public optimization problem are the two public budget constraints (5) and (6), the private sector budget constraint (4), and the optimality condition of the private firms (2). Furthermore, an additional constraint holds that \( G^j_{i,t} \) are given.

According to the first-order conditions of the optimization problem presented in Appendix A, the optimal source tax rate\(^{10}\) is

\[
\tau^{\text{opt}}_{i,t+1} = \frac{\partial Y^j_{i,t+1}}{\partial G^j_{K,t+1}} \left( \frac{Y^j_{i,t+1} - W^j_{i,t+1}}{G^j_{K,t+1}} \right)
\]

and the optimality condition for public investment is

\[
\left[ \frac{\partial Y^j_{i,t+1}}{\partial G^j_{i,t+1}} \right] = \left( r^* - \frac{\partial W^j_{i,t+1}}{\partial K^j_{i,t+1}} \right) \left[ \frac{\partial Y^j_{i,t+1}}{\partial K^j_{i,t+1}} + \frac{\partial W^j_{i,t+1}}{\partial K^j_{i,t+1}} + r^* \right] + \frac{\partial W^j_{i,t+1}}{\partial K^j_{i,t+1}} = r^* .
\]

Condition (8) shows that in the small open jurisdiction, the opportunity costs of debt-financed public inputs equal the market price of investment \( r^* \) if the source tax is chosen optimally. The costs of the

\(^{10}\) The optimal public policy is to tax away the rents appropriated by the private capital, so that the corresponding tax revenue equals exactly the share of public capital income \( \left( \frac{\partial Y^j_{i,t+1}}{\partial G^j_{K,t+1}} \right) G^j_{K,t+1} \). The rent of the other publicly provided private input \( G^j_{i,t+1} \) goes to labor. \( G^j_{i,t+1} \) does not generate any factor rent. If the production elasticity of \( G^j_{k,t+1} \) is zero, so that public capital generates no rent to private capital, then \( \tau^{\text{opt}}_{i,t+1} = 0 \) (see Kellermann, 2006). It is certainly not a surprising result that rents earned by mobile factors provide an efficient and thus highly desirable means of raising tax revenue. Zodrow (2006) quotes several arguments supporting source-based capital income taxes on account of rents.

13
publicly provided capital consist only of the resource costs. Congestion costs caused by the inflow of private capital, with respect to the private capital already located in the jurisdiction, do not occur in this optimality condition. The same holds for tax-distortion costs. If the tax instruments are chosen optimally, no excess burden is caused by shifting money from the private to the public sector.

To analyze the intertemporal equilibrium of the small open jurisdiction, the behavior of households and the behavior of the private and public investors must be considered simultaneously. The relevant optimality condition of the private firms located in the jurisdiction is given by equation (2). Together with the optimality condition of taxation, it has to be combined with the household optimum and the optimal investment strategy of the lower-level government represented by equation (8). The equilibrium is characterized by an intertemporal allocation where the household’s marginal rate of substitution between consumption at time t+1 and consumption at time t is equal to the marginal rate of transformation from public and private production

\[
\left( \frac{\partial u}{\partial c_{t+1}}, \frac{\partial u}{\partial c_t} \right) = \frac{1}{1 + \frac{\partial Y_t}{\partial K_t}} = \frac{1}{1 + \frac{\partial Y_{t+1}}{\partial G_{t+1}}}.
\]

Thus, the local jurisdiction ends up in a situation where the intertemporal efficiency condition is satisfied. The household’s marginal rate of substitution between consumption of the old and consumption of the young equals the marginal rate of transforming future consumption in current consumption. Since the interest rate \( r^* \) is the same in all jurisdictions, this allocation is also spatially efficient.

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11 Wilson (1997) discusses a model, where each jurisdiction faces an infinitely elastic supply of mobile residences. The inflow of new residences induces congestion cost. In his model existing residents have to be compensated new residents, so that the congestion costs enter rule of the lower-level optimal public good supply.
3.2. Public investment policy of the central government

A question that remains is whether the lower-level jurisdiction equilibrium described in equation (8) is a desirable outcome from the viewpoint of a benevolent central government. The primary problem of public borrowing in a small open jurisdiction is the burden induced by public debt that has to be covered by taxes. If the government imposes a source tax on capital income for which the revenue exceeds a potential factor rent absorbed by private capital, this source tax will lead to an overly capital outflow and thus decrease welfare. On the other hand, a head tax reduces private consumption and welfare directly. Since in an open economy with high capital mobility the link between domestic saving and domestic investment is rather weak, the impact of taxation on private capital supply is neglected by the lower-level jurisdiction.

Keeping this in mind, we now suppose that the small open economy considered above is part of a big region consisting of numerous jurisdictions. In the extreme case where the big region is a closed economy, the sum of domestic saving equals exactly the sum of private and public investment. The capital market equilibrium is thus determined by the fundamental identity

$$\sum_{j=1}^{R} \theta L_t^j c_t^j = \sum_{j=1}^{R} (K_{t+1}^j + B_{t+1}^j),$$

where $$\sum_{j=1}^{R} \theta L_t^j c_t^j = \sum_{j=1}^{R} L_t^j s_t^j$$ is the sum of gross savings over all jurisdictions $$j$$ (for $$j = 1…R$$), that represents global capital supply. $$\sum_{j=1}^{R} (K_{t+1}^j + B_{t+1}^j)$$ is the global gross capital demand of private firms and the government. Considering the identity of gross saving and gross investment, the global interest rate is no longer exogenous but determined by the global capital market equilibrium and thus is potentially reactive to public policy decisions$^{12}$. In the following, we show that the public financing and investment strategy of the central government that takes the global constraint in capital supply into account can differ from the lower-level jurisdiction’s strategy if the social time preferences differ from the private household time preferences. If the welfare-maximizing government has a rate of substitution of future for present consumption that differs from the private household’s intertemporal rate of marginal substitution, then the efficiency condition of the lower-level jurisdiction does not represent a

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$^{12}$ Empirical investigations by Bayoumi and Rose (1993), Bayoumi and Sterne (1993) and Sinn (1992) show that the correlation of regional saving and investment quotas are rather low, whereas on the national level, saving and investment co-movement is remarkably stable. The cited results are derived from intra-national cross-region analyses on the state level in the UK, Canada and the USA, respectively.
welfare maximum. Under these circumstances, there is scope for welfare improvements by centralizing the responsibility of investment policy.

The fiscal instruments at the central government’s disposal are the source tax \( \Omega_j^i = \frac{\tau_i r_i^j - K_i^j}{1 - \tau_i} \) and the head tax \( T_i^j = t_i L_i^j \). The according tax rates are allowed to be time-variant but are the same for all jurisdictions. Thus, \( \tau_{i+1}^j = \tau_{i+1}^j \) and \( t_{i+1}^j = t_{i+1}^j \) hold. With respect to public investment engagement, the central government treats all jurisdictions equally, so that public capital supply per worker is the same in each location. This assumption is in line with Oates (1972, p. 11) who states that if all public goods are supplied by a central government, one should expect a tendency toward uniformity in public programs across all communities. The local public good supply \( G_{p,i+1}^j \) is an equal fraction (1/R) of the total central government’s total supply \( G_{p,i+1}^j = \frac{G_{p,i+1}^j}{R} \). The publicly provided private capital stocks \( G_{K,i+1}^j \) and \( G_{L,i+1}^j \) are regionally allocated according to

\[
\frac{G_{L,i+1}^j}{L_{i+1}^j} = g_{L,i+1}^j = g_{L,i+1}^j \quad \text{and} \quad \frac{G_{K,i+1}^j}{L_{i+1}^j} = g_{K,i+1}^j = g_{K,i+1}^j.
\]

The production technology, the natural growth rate \( n_j^i = n \) as well as private preferences are assumed to be the same in all jurisdictions. Because fiscal parameters are the same in all jurisdictions, interregional capital mobility leads to the equalization of the per-capita capital stock \( k_{i+1}^j = k_{i+1}^j \), per-capita output\(^{13} \) \( y_{i+1}^j = y_{i+1}^j \), wage income \( w_{i+1}^j = w_{i+1}^j \), and private consumption \( c_{i+1}^j = c_{i+1}^j \). The social welfare function of the central government

\[
\Psi^C = \sum_{j=1}^{R} \sum_{i=1}^{\infty} \frac{L_i^j}{1 + \lambda} \ln \left[ \varrho \ln \varrho + (1 + \varrho) \ln c_{i+1}^j + \varrho \ln(1 + r_{i+1}) \right],
\]

which is the weighted sum of the lower-level jurisdictions’ welfare functions, can thus be expressed as

\[
\Psi^C = \sum_{i=1}^{\infty} (1 + \lambda) \ln \left[ \varrho \ln \varrho + (1 + \varrho) \ln c_{i+1}^j + \varrho \ln(1 + r_{i+1}) \right].
\]

\(^{13}\) Local per-capita output can be expressed as \( A(\frac{G_{c_i}}{R})R(\frac{K_{i+1}^j}{L_{i+1}^j}, \frac{G_{K,i+1}^j}{L_{i+1}^j}, \frac{G_{L,i+1}^j}{L_{i+1}^j}, 1) = y_{i+1}^j \).
The constraints of the public optimization problem are the central government budget constraints given by \[ D_t + r_t B_t = \Omega_t + T_t \] and \[ \sum_{i=P,K,L} \left( G_{t+i} - G_t \right) = (B_{t+1} - B_t), \] the private sector budget constraint (4) and the optimality condition of the private firms (2), which are the same for each jurisdiction. Further, the capital market must be in its equilibrium. The optimization problem of the central government can be described as the Lagrangian expression

\[ \Pi(K_t, B_t, G_t, c_t^Y, r_t) = \Psi^C + \sum_{j=1}^{\infty} \mu_t \left[ \Omega_t + T_t - D_t - r_t B_t \right] + \sum_{i=1}^{\infty} \theta_t \left[ (B_{t+1} - B_t) - \sum_{i=k,L} \left( G_{t+i} - G_t \right) \right] + \sum_{j=1}^{\infty} \rho_{t+j} \left[ y_{t+j} - w_{t+j} - k_{t+j} r_{t+j} - \frac{\Omega_{t+1}}{L_{t+1}} \right] + \sum_{i=1}^{\infty} \delta_{t+i} \left[ w_{t+i} - t_{t+i} - (1 + \Theta) c_{t+i}^Y \right] + \sigma_t \left[ 9 L_t c_t^Y - (B_{t+1} + K_{t+1}) \right]. \]

Applying the first-order conditions of the optimization problem – presented in Appendix B – the optimality condition of public investment faced by the central government is

\[ \left[ \frac{\partial Y_{t+1}^j}{\partial G_{t+1}^j} \right] = \left[ \frac{\partial Y_{t+1}^j}{\partial K_{t+1}^j} \right] = \left[ \left( 1 + \lambda (1 + n) \frac{c_{t+1}^0}{c_t^0} - 1 \right) - \frac{r_{t+1}}{\Theta} \right] \frac{(1 + \Theta)}{\Theta} + r_{t+1}. \] (10)

Equation (10) shows that the central government realizes an allocation so that the partial marginal productivity of private capital equals the marginal productivity of public capital. In this respect, the central government behaves as the lower-level government. The right-hand side of equation (10) shows the marginal opportunity costs of public investments in the central jurisdiction. They differ from the market rate of interest \( r_{t+1} \), if \( \left( 1 + \lambda (1 + n) \frac{c_{t+1}^0}{c_t^0} - 1 \right) \neq r_{t+1} \). In this case, the marginal time preference of a social planner differs from the marginal time preference of the private household \( r_{t+1} \). The opportunity costs of public capital can be interpreted as a weighted average of the market rate of interest and the social planner’s marginal time preference given by

\[ \left( 1 + n \left( 1 + \lambda \right) \frac{c_{t+1}^0}{c_t^0} - 1 \right)^{14}. \]

\[^{14}\text{The social planner’s marginal time preference is deduced in Appendix C.}\]
3.3. Policy adjustment after centralization

To compare explicitly the investment and taxation policy of the central and the lower-level government, we describe the policy adjustment after centralization. In the initial situation, an autonomous lower-level government chooses a taxation and investment policy that satisfies the conditions (7) and (8). This situation can be characterized as a steady state in the sense that capital endowment per worker and consumption per head are constant over time. We discuss three different cases. First, we make the assumption that the world interest rate \( r^*_w \), taken by the lower-level jurisdiction as exogenous, equals the long-term social planner’s time preference \( \{(1 + \lambda)(1 + n) - 1\} = r^*_c \). In this case, the optimality condition of the central government (10) reduces to the optimality condition of the lower-level government. The opportunity costs of private and public capital in the central jurisdiction equal the interest rate. The central government has no incentive to adjust its tax or investment policy. Thus, the centralization of investment policy from the lower-level to the central jurisdiction will neither enhance nor reduce the social opportunity costs of public investment. There is no scope for welfare improvement. Since public investment and tax parameters are kept constant, private investment will stay constant too. As a result, centralization has neither welfare nor growth or labor productivity-enhancing effects.

However, the situation changes if initially \( \{(1 + \lambda)(1 + n) - 1\} < r^*_c \). This high global interest rate \( r^*_w \) is caused by a relatively low private household’s propensity to save. According to Pigou (1932), the benevolent government is the guardian of the interests of future generations as well as of the interests of the present generation. Therefore, public decision makers ought to have a lower marginal rate of substitution of future for present consumption than private households. The welfare-maximizing social rate of time preference can thus be lower than the individual time preference. Pigou’s position is that the goal of public policy is presumably to synthesize a time preference cleansed of the myopic individual preferences (see Marglin, 1963). For \( \{(1 + \lambda)(1 + n) - 1\} < r^*_c \), centralization will lead to a policy adjustment because condition (10) is violated by the lower-level jurisdictions’ investment decisions.

The comparison of (8) and (10) shows that the marginal opportunity costs of public investment reduce after centralization. What causes this decline? The answer is straightforward:
Centralization leads to the internalization of a negative fiscal externality on the supply of global saving induced by a local head tax. From the perspective of a lower-level jurisdiction, the supply of capital is sufficiently elastic to largely eliminate the crowding out of private savings associated with a head tax. In each single jurisdiction, the increase of a head tax may reduce private saving. Yet, this crowding out effect is ignored by the lower-level government because it has no effect on the level of in-state investment under the open economy assumption. However, the described crowding out of private saving represents a reduction in global capital supply that harms the welfare of all other regions. The head tax on workers causes a negative externality although the households are immobile. The central jurisdiction takes the displacement of private saving by a head tax into account.

Centralization thus has not only the potential for welfare improvement but also a positive effect on private and public investment, transitory output growth and long-term labor productivity. For the central government, the described decline of the marginal opportunity costs of public investments creates an incentive to search for a policy that increases private and public investment. For an exogenous labor supply, this policy enhances transitory output growth as well as long-term labor productivity. A prerequisite for an expansion of investments is the expansion of private savings. Since aggregate private savings $S_t = \sum_{j=1}^{R} g_L c_i^j = \left(\frac{\theta}{1+\theta}\right) \sum_{j=1}^{R} \left(W^j_{t+1} - T^j_{t+1}\right)$ are a fixed fraction of net wage income, the central government has only one way to stimulate the supply of capital by increasing available income $\sum_{j=1}^{R} \left(W^j_{t+1} - T^j_{t+1}\right)$\textsuperscript{15}. The adequate instrument to do so is the reduction of the head tax, as can easily be shown\textsuperscript{16}. Thus, to meet condition (10), the central government simultaneously increases its investment expenditure and reduces the head tax.

\textsuperscript{15} What can be said about the long-term equilibrium interest rate of the central jurisdiction? According to equation (10), in the steady state we end up in a situation where

\[
\left[\frac{\partial Y_i}{\partial K_i}\right] = \left[\frac{\partial Y}{\partial K}\right] = \left\{\left[1+\lambda_i(1+n)-1\right] - \frac{1}{1+\theta}\right\} \frac{1+\theta}{\theta} + \tau = \frac{r}{1-\tau} > 0. \]

For this condition to hold, $\tau > 0$ and $r < \left[\left(1+\lambda_i(1+n)-1\right)\right]$ must be satisfied, too. Since $\tau > 0$, we end up with an equilibrium interest rate that is even lower than the social planner’s time preference. However, this interest rate does not represent the marginal user costs of capital to the private firm as is the case in the lower-level jurisdiction.

\textsuperscript{16} An increase of public investment, where the debt burden is financed by a head tax, always leads to a decline of available income, if $\left(\partial Y_i / \partial G_i\right) = \left(\partial Y / \partial K_i\right)$. Using equations (3) and (5) the marginal increase of the available income by an increase of public capital supply can be expressed as

\[
\frac{\partial \left[W^j - T^j\right]}{\partial G^j_i} = \left\{\left[\frac{\partial Y_i}{\partial G_i} / \partial G_i\right][K^j_i + G^j] + \frac{\partial Y_i}{\partial K_i} \right\} < 0.
\]
The first measure leads to an expansion of public debt and thus to a rise of the debt burden, the second leads to a reduction in current tax revenue that has to be compensated by an increase in the source tax revenue. Compared to the policy of the lower-level government, the central government will thus substitute head taxes by source taxes. Note that in the central jurisdiction a source tax has no distortionary effect since investment is determined by the supply of saving and the problem of capital flight can not occur.

Finally, we look at a third case where initially \( \{(1+\lambda)(1+n)-1\} > r^* \) holds. Starting from this position, the centralized government will follow a policy that reduces private saving by increasing head taxes. As a result, private and public investment will shrink. For the current public budget constraint (5) to hold, the source tax revenue has to decline. Source taxes could even become negative and thus a subsidy to private firms. Note that in the extreme case, where \( n > r^* = \frac{\partial Y_{t,1}/\partial K_{t,1}}{\partial K_{t,1}} \) holds, the lower-level jurisdiction is trapped in a situation of dynamic inefficiency and over-accumulation. Saving could thus be reduced to increase private consumption of the present generation without lowering the consumption of future generations. However, the benevolent government of the lower-level jurisdiction has now a fiscal instrument that could be applied to escape from the state of dynamic inefficiency.

4. Conclusions

According to Wilson and Wildasin (2004, p. 1067), tax competition is defined as “…noncooperative tax setting by independent governments, under which each government’s policy choices influence the allocation of a mobile tax base among regions represented by these governments”. In his basic article, Wilson (1986, p. 297) uses a further and concurrently a narrower definition of tax competition by writing that tax competition is a situation “…where public service outputs and tax rates are too low in the sense that a federal government could raise the nation’s welfare by requiring each region to increase its public service output”. Both definitions hold with respect to the model discussed in this paper. However, my concern is not merely about the spatial allocation of capital, but how tax competition influences intertemporal allocation and explicitly the marginal time preferences of public decision makers.
The model presented in this paper shows that in a situation where the social planner’s time preference is lower than the private household’s time preference, centralization of investment responsibility from the lower-level to a central government leads to a policy adjustment. The central government has an incentive to search for a policy that increases private and public investment by reducing head tax paid by immobile workers. The first measure leads to an expansion of public debt and thus to a rise of the debt burden, the second leads to a reduction in current tax revenue that has to be compensated by an increase in the source tax revenue. Compared to the policy of the lower-level government, the central government will thus substitute head taxes by source taxes. The model distinguishes two further cases: if the social planner’s time preference equals the private household’s time preference, centralization has neither welfare nor growth or labor productivity-enhancing effects. If it is assumed that the social planner’s time preference exceeds the private household’s time preference, centralization induces a welfare-improving reduction of private and public investment.

What explains these differences in the tax and investment policy of the lower-level and the central jurisdiction? From the perspective of a lower-level jurisdiction, the supply of capital is sufficiently elastic to eliminate largely the crowding out of private savings associated with a head tax. In each single jurisdiction a head tax effects private saving. Yet, this effect is ignored by the lower-level government. In other words, there is a fiscal externality caused by a head tax on workers. The central jurisdiction takes the displacement of private saving by a head tax into account. Centralization thus has the potential for welfare improvement. Under certain empirically not irrelevant circumstances, centralization creates an effect that can positively influence private and public investment, transitory output growth, and long-term labor productivity.
References


Appendix A: Lower-level Jurisdiction

\[ \frac{\partial \Gamma}{\partial (\Omega^j_{i+1})} = \mu^j_{i+1} - \rho^j_{i+1} = 0 \]  
(A1)

\[ \frac{\partial \Gamma}{\partial (T^j_{i+1})} = \mu^j_{i+1} - \delta^j_{i+1} = 0 \]  
(A2)

\[ \frac{\partial \Gamma}{\partial G^j_{i+1}} = \rho^j_{i+1} \left[ \frac{\partial Y^j_{i+1}}{\partial G^j_{i+1}} - \frac{\partial W^j_{i+1}}{\partial G^j_{i+1}} \right] + \delta^j_{i+1} \left[ \frac{\partial W^j_{i+1}}{\partial G^j_{i+1}} \right] - \theta^j_{i+1} + \theta^j_{i+1} = 0 \]  
(A3)

\[ \frac{\partial \Gamma}{\partial B^j_{i+1}} = -\mu^j_{i+1} r^* - \theta^j_{i+1} + \theta^j_{i} = 0 \]  
(A4)

\[ \frac{\partial \Gamma}{\partial K^j_{i+1}} = \rho^j_{i+1} \left[ \frac{\partial Y^j_{i+1}}{\partial K^j_{i+1}} - \frac{\partial W^j_{i+1}}{\partial K^j_{i+1}} - r^* \right] + \delta^j_{i+1} \left[ \frac{\partial W^j_{i+1}}{\partial K^j_{i+1}} \right] = 0 \]  
(A5)

\[ \frac{\partial \Gamma}{\partial c^j_{i+1}} = \frac{\partial \Psi}{\partial c^j_{i+1}} - \delta^j_{i+1} \left[ c^j_{i+1} \right] = 0 \]  
(A6)
Appendix B: Central Jurisdiction

\[ \frac{\partial \Pi}{\partial \Omega_{t+1}} = \mu_{t+1} - \frac{\rho_{t+1}}{L_{t+1}} = 0 \]  
\[ (B1) \]

\[ \frac{\partial \Pi}{\delta (T_{t+1})} = \mu_{t+1} - \frac{\delta_{t+1}}{L_{t+1}} = 0 \]  
\[ (B2) \]

\[ \frac{\partial \Pi}{\partial g_{K_{t+1}}} = \rho_{t+1} \left[ \frac{\partial y_{t+1}}{\partial g_{K_{t+1}}} - \frac{\partial w_{t+1}}{\partial g_{K_{t+1}}} \right] + \delta_{t+1} \left[ \frac{\partial w_{t+1}}{\partial g_{K_{t+1}}} \right] - \theta_{t} L_{t+1} + \theta_{t} + L_{t+1} = 0 \]  
\[ (B3) \]

\[ \frac{\partial \Pi}{\partial g_{L_{t+1}}} = \rho_{t+1} \left[ \frac{\partial y_{t+1}}{\partial g_{L_{t+1}}} - \frac{\partial w_{t+1}}{\partial g_{L_{t+1}}} \right] + \delta_{t+1} \left[ \frac{\partial w_{t+1}}{\partial g_{L_{t+1}}} \right] - \theta_{t} L_{t+1} + \theta_{t} + L_{t+1} = 0 \]  
\[ (B3') \]

\[ \frac{\partial \Pi}{\partial G_{P_{t+1}}} = \rho_{t+1} \left[ \frac{\partial y_{t+1}}{\partial G_{P_{t+1}}} - \frac{\partial w_{t+1}}{\partial G_{P_{t+1}}} \right] + \delta_{t+1} \left[ \frac{\partial w_{t+1}}{\partial G_{P_{t+1}}} \right] - \theta_{t} L_{t+1} + \theta_{t} + L_{t+1} = 0 \]  
\[ (B3'') \]

\[ \frac{\partial \Pi}{\partial B_{t+1}} = -\mu_{t+1} r_{t+1} + \theta_{t} - \theta_{t} - \sigma_t = 0 \]  
\[ (B4) \]

\[ \frac{\partial \Pi}{\partial K_{t+1}} = \rho_{t+1} \left[ \frac{\partial y_{t+1}}{\partial K_{t+1}} - \frac{\partial W_{t+1}}{\partial K_{t+1}} - r_{t+1} \right] + \delta_{t+1} \left[ \frac{\partial W_{t+1}}{\partial K_{t+1}} \right] - \sigma_t = 0 \]  
\[ (B5) \]

\[ \frac{\partial \Pi}{\partial c_{t+1}} = \frac{\partial \psi^C}{\partial c_{t+1}} - \delta_{t+1} [1 + g] + \sigma_{t+1} g L_{t+1} = 0 \]  
\[ (B6) \]

\[ \frac{\partial \Pi}{\partial t_{t+1}} = \frac{\partial \psi^C}{\partial t_{t+1}} - \mu_{t+1} B_{t+1} - \rho_{t+1} \frac{K_{t+1}}{L_{t+1}} = 0 \]  
\[ (B7) \]
Appendix C: Social Planner

The social planner has the power to dictate the choice of private consumption over time and seeks to maximize the social welfare function subject to the closed economy’s budget constraints.

According to the latter, the output of all regions together \( \sum_{j=1}^{R} Y_j^j \) in a given period of time is the sum of private consumption of young and old people \( \sum_{j=1}^{R} (L_j^j c_t^y + L_{j+1}^j c_{t+1}^y) \), private and public investment and depreciation \( \sum_{j=1}^{R} \left( \sum_{i=k+1}^{L_j} \left( G_{j,i+1}^j - G_{j,i}^j \right) + \left( K_{j,i+1}^j - K_{j,i}^j \right) + D_{i+1}^j \right) \). The social planner’s intertemporal rate of substitution of future and present consumption can be derived from the optimization problem given by

\[
\Gamma (c_t^y, c_{t+1}^y, K_{t+1}, G_{t+1}) = \sum_{i=1}^{T} \sum_{j=1}^{R} \left( 1 + \lambda \right)^{i-1} \left( \ln c_t^y + \theta \ln c_{t+1}^y \right) + \sum_{i=1}^{T} \eta_t \left[ \sum_{j=1}^{R} \left( L_j^j c_t^y + L_{j+1}^j c_{t+1}^y \right) + \sum_{i=k+1}^{L_j} \left( G_{j,i+1}^j - G_{j,i}^j \right) + \left( K_{j,i+1}^j - K_{j,i}^j \right) + D_i^j \right] - \sum_{j=1}^{R} Y_j^j
\]

where \( \eta_t \) denotes the Lagrange parameter.

\[
\frac{\partial \Gamma}{\partial c_t^y} = \frac{L_t^j}{\sum_{j=1}^{R} L_t^j} \left( 1 + \lambda \right)^{t-1} + \eta_t L_t^j = 0 \quad (C1)
\]

\[
\frac{\partial \Gamma}{\partial c_{t+1}^y} = \frac{L_t^j}{\sum_{j=1}^{R} L_t^j} \left( 1 + \lambda \right)^{t} + \eta_t L_t^j = 0 \quad (C2)
\]

\[
\frac{\partial \Gamma}{\partial K_{t+1}} = \eta_t - \eta_{t+1} \left( 1 + \frac{\partial Y_{t+1}^j}{\partial K_{t+1}^j} \right) = 0 \quad (C3)
\]

\[
\frac{\partial \Gamma}{\partial G_{t+1}^j} = \eta_t - \eta_{t+1} \left( 1 + \frac{\partial Y_{t+1}^j}{\partial G_{t+1}^j} \right) = 0 \quad (C4)
\]

The optimality condition of public investment faced by the central planner is given by:

\[
\frac{\partial Y_{t+1}^j}{\partial K_{t+1}^j} = \frac{\partial Y_{t+1}^j}{\partial G_{t+1}^j} = (1 + n) \left( 1 + \lambda \right) c_t^y - 1 \quad (C5)
\]