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Increasing workload in a stochastic environment

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**Increasing Workload in a
Stochastik Environment**

Philipp Weinschenk





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INCREASING WORKLOAD IN A STOCHASTIC ENVIRONMENT

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Abstract

We show that a steeply increasing workload before a deadline is compatible with time-consistent preferences. The key departure from the literature is that we consider a stochastic environment where success of effort is not guaranteed.

JEL Classification: D91, J22, D81, D11.

Keywords: Increasing Workload, Deadline, Stochasticity.

1. INTRODUCTION

Life is full of tasks which have to be completed by a prespecified date. For example, a husband needs to repair something before the mother-in-law will visit; a student may have to complete a proof for the next math class; a researcher must complete a presentation for a conference. People sometimes delay or procrastinate with the completion of such tasks.¹ Their workload is steeply increasing before the deadline.

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¹For empirical evidence, see, for example, Ellis and Knaus (1977), Solomon and Rothblum (1984), and McCown et al. (1987).

The literature explains this phenomenon by use of time-inconsistent, present-biased preferences (O’Donoghue and Rabin, 1999, 2001, 2007; Akerlof, 1991; Brocas and Carrillo, 2001). In contrast, typical models with time-consistency predict that a person distributes her effort equally across time (O’Donoghue and Rabin, 2007). Because of discounting, a time-consistent person may optimally choose a slightly increasing workload (Fischer, 2001). However, “quantitatively, the fully rational model appears to require an extremely high rate of time preference or elasticity of intertemporal substitution to generate serious procrastination” (Fischer 2001, p. 249). That is, a steeply increasing workload is seen as incompatible with the time consistency assumption. We show that in a stochastic environment this is no longer true. That is, a time-consistent person may optimally choose a steeply increasing workload. Therefore, we provide—to the best of our knowledge—the first explanation why a steeply increasing workload can be fully rational.

In our model, we suppose that a person has to solve a task (e.g., prove a mathematical result). The rough intuition why a steeply increasing workload is fully rational is as follows. When there is a lot of time left, the person does not have to worry much about the deadline. Then she optimally exerts some risky approaches with low effort costs (e.g., looking in related papers). These are unlikely to be successful, but have low risk-adjusted effort costs. When the task is not solved shortly before the deadline, the solution gets urgent. Then the person optimally chooses relatively safe approaches which require a lot of effort (e.g., create a self-reliant proof). Although these have high risk-adjusted effort costs, they are very likely successful.

We validate and strengthen the results when we change from a discrete to a continuous approach space. For a specific family of effort cost functions, we illustrate our findings and yield a further result: not only the expected per-period effort costs, conditional that the task is not yet solved, increase in time; the unconditional expected effort costs also increase.

Our findings are not a critique of the concept of present-biased pref-

erences. We think it is extremely useful to explain phenomena like undesired delay. Our point is that a steeply increasing workload is perfectly compatible with the assumption of time consistency and can therefore not be seen as a proof for time inconsistency.

Our paper is related to Weitzman (1979) who studies the problem of searching for the best outcome when there are alternative sources. To explore a source, the person has to invest. Unlike in our model, there is no deadline. The problem is one of selection and optimal stopping.² In our problem, the stopping rule is degenerated: stop if and only if an approach was successful or the deadline is reached. Nonetheless, our selection problem is nontrivial because of the deadline. Roberts and Weitzman (1981) determine the optimal funding criteria in a multiple stage investment problem, where information about the final benefit is revealed bit by bit. In their setting, the only decision is to stop or continue. That is, there is no selection problem.

In Bisin and Hyndman (2009), a person's effort costs evolve stochastically in continuous time and she has to invest once to complete the task. In contrast to our model, the success of effort is guaranteed. They show that it may be optimal to "wait for better times". Formally, the optimal decision rule is to wait to exert effort until costs are less than a certain threshold.

The paper proceeds as follows. In the next section, we describe the model with a discrete approach space. We analyze it in Section 3. In Section 4, we consider a continuous approach space. In Section 5, we discuss and extend the model. Finally, we offer some concluding remarks in Section 6.

²When stopping, the person receives the maximum reward she has thus far uncovered. Sources differ in their potential rewards as well as in their costs. The reward space is much richer than in our model, where an approach can either be successful or not.

2. THE MODEL

The person has to solve some task within $T \geq 2$ periods. There is a finite number of different classes. Each class contains at least T identical approaches; we discuss this assumption in Section 5.1. The character of an approach of class i is determined by the class-specific effort cost $c_i \in \mathbb{R}_{\geq 0}$ and the class-specific probability that the task is solved $p_i \in [0, 1]$. In each period, the person exerts one approach and each approach can be conducted at most once. The success probabilities are uncorrelated.

The person is risk-neutral and minimizes the expected effort costs of the solution. When two or more classes have the same success probabilities, they must have, by definition, different costs. All classes except the one with the lowest cost are inferior and not important for the person's problem. Call the set of all classes which are not inferior in this sense \mathcal{I} . We number the classes belonging to \mathcal{I} with $0, 1, \dots, n$ and arrange them according to their success probabilities: $p_i < p_j$ for $i < j$.

We assume that there is a default class 0 with $c_0 = p_0 = 0$. When the person exerts an approach of class 0, she essentially does nothing. We also suppose that there is a safe class which is surely successful: $p_n = 1$.³ In Section 5.3, we assume that completion of the task is not mandatory and show that this allows us to drop the assumption that there is a safe class. We normalize all costs so that $c_n = 1$. We want to answer the following related questions. When does the person optimally exert which approach? How does the person's workload evolve over time?

3. ANALYSIS

The optimal stopping rule is simple: when the person has solved the task, she optimally exerts approaches of class 0. But which approach should she exert when the task is not yet solved? The person's strategy for this case is denoted by $\mathbf{s} = (s_1, \dots, s_t, \dots, s_T)$, where $s_t \in \mathcal{I}$. This

³Otherwise we would have the problem that the person cannot guarantee to solve the task. Then the person's problem is no longer well defined.

vector specifies which approach the person exerts in t , conditional that no previous approach was successful. By permuting the elements of the vector \mathbf{s} such that the elements are arranged in an ascending order of their classes, we obtain a new vector which we call \mathbf{S} . This vector can be interpreted as the arranged vector of all approaches which are conducted in some period, given that the task is not solved.⁴

The person can, in principle, apply a mixed strategy. Mixed strategies can only be weakly optimal when the person is indifferent between several approaches. This is generically never the case and the person is equally well-off when she plays only pure strategies. Therefore, we will concentrate on pure strategy equilibria. But the concept of mixed strategies is useful to yield some insights.

3.1. THE RELEVANT SET

We call a class dominated if there is some other class or mixtures of classes which yield the same success probability with lower costs. We denote the set of classes which are not dominated the relevant set \mathcal{I}^R , where $\mathcal{I}^R \subseteq \mathcal{I}$. Note that a person's strategy \mathbf{s} need not contain all elements of \mathcal{I}^R . But all components of an optimal strategy \mathbf{s} have to be elements of \mathcal{I}^R ; otherwise the person could improve.

LEMMA 1: *The set of relevant classes \mathcal{I}^R forms a weakly convex and weakly increasing line in the $p - c$ space. No class is below this line. The classes 0 and n are always part of \mathcal{I}^R .*

PROOF: See appendix.

Geometrically, the set of relevant classes can be found as follows. Form the convex hull around all classes. The relevant set is then given by the classes on the lower part of the convex hull. This can be seen in Figure 1, where the relevant set are the classes on the bold lines.

⁴For example, if $\mathbf{s} = (2, 1, 1, 3)$ then $\mathbf{S} = (1, 1, 2, 3)$. The same vector \mathbf{S} is yielded for other strategies, like $\mathbf{s} = (3, 1, 1, 2)$ or $\mathbf{s} = (3, 1, 2, 1)$.

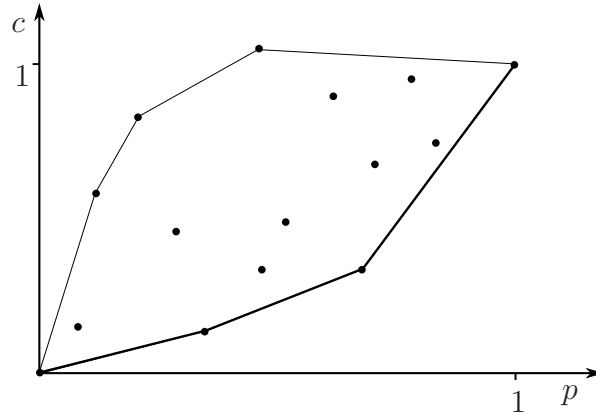


Figure 1: The set \mathcal{I} , the convex hull, and the set \mathcal{I}^R .

When the relevant set consists only of classes 0 and n , then the person's problem is trivial: exert once an approach of class n in some period and otherwise conduct approaches of class 0. The same is true when there is no $i \in \{1, \dots, n-1\}$ with $c_i/p_i < 1$. Because approaches of the classes 0 (n) fail (succeed) for sure, there is no role for stochasticity. To avoid this and to make the problem interesting, we impose the following assumption.

ASSUMPTION 1: *There is at least one class of approaches for which $p_i > 0$ and $c_i/p_i < 1$.*

3.2. THE CLASSES 0 AND n

The person has to solve the task no later than the end of T . Her strategy \mathbf{s} must therefore contain an approach of the safe class n . But is it worthwhile for the person to exert an approach of another class before eventually exerting a safe approach? Investing in approach $i \in \{1, n-1\}$ in $t-1$ and in approach n in t (when there was no success in $t-1$) is strictly better than the other way round (actually, i is never exerted) if and only if

$$p_i > c_i. \quad (1)$$

That is, with probability p_i approach i is successful and the effort costs in t of $c_n = 1$ are avoided. This benefit must exceed the costs of c_i .

Because of Assumption 1, there exists at least one class for which (1) holds. Hence, the person optimally exerts an approach of class n only in the last period T (given that the task is not yet solved).

We already know that the person optimally exerts approaches of class 0 once the task is solved. But can it be optimal to exert approaches of class 0 before? The answer is no. The intuition is that by doing so the person would forego the opportunity to solve the task with some other, relatively cheap approach.

LEMMA 2: *The person exerts an approach of class n only in period T and when the task is not yet solved. Approaches of class 0 are only exerted once the task is solved.*

PROOF: See appendix.

3.3. THE INCREASING WORKLOAD RESULT

Consider the subproblem where the ordered vector of approaches which is possibly exerted \mathcal{S} is given. In which sequence should the approaches which \mathcal{S} contains be exerted?

Denote the expected costs of solving the task under the optimal strategy, given that the task is not solved, measured at the beginning of period t , by $E_t[C]$. Suppose an approach of class i is exerted in t and, possibly, when this approach is not successful, an approach of class j in $t+1$. Then the expected costs of the solution at the beginning of t are

$$(1 - p_i)(1 - p_j)E_{t+2}[C] + c_i + (1 - p_i)c_j, \quad (2)$$

whereas they are

$$(1 - p_i)(1 - p_j)E_{t+2}[C] + c_j + (1 - p_j)c_i \quad (3)$$

when the sequence of the approaches i and j is permuted. Hence, it is optimal to conduct i before j if

$$\frac{c_i}{p_i} < \frac{c_j}{p_j}. \quad (4)$$

When (4) holds with $>$ it is optimal to exert j before i and the person is indifferent when (4) holds with equality. This observation yields the following lemma.

LEMMA 3: *Given the ordered vector of approaches which is possibly exerted \mathbf{S} and that the task is not yet solved, the person conducts the approaches in an ascending order of risk-adjusted costs c_i/p_i .*

Because Lemma 3 holds for all vectors \mathbf{S} , it implies that as long as the task is not yet solved, the person conducts approaches with weakly increasing risk-adjusted costs. The intuition for this result is that the expected costs of a solution are minimized when the person starts with approaches which have low risk-adjusted costs.

One may presume that as long as the task is not solved, it is always optimal to exert approaches of the class with the lowest risk-adjusted costs, except in the last period T . This conclusion is wrong. To see this, consider the following example. There are three periods, $T = 3$, and the following four classes:

$$\begin{aligned} c_0 &= 0, & p_0 &= 0, \\ c_1 &= 0.1, & p_1 &= 0.4, & c_1/p_1 &= 1/4; \\ c_2 &= 0.4, & p_2 &= 0.8, & c_2/p_2 &= 1/2; \\ c_3 &= 1, & p_3 &= 1, & c_3/p_3 &= 1. \end{aligned}$$

In $t = 3$, the person optimally exerts an approach of class 3. Therefore, $E_3[C] = 1$. In $t = 2$, the expected costs of completion are $0.1 + 0.6 \times 1 = 0.7$ when the person exerts an approach of class 1, whereas they are $0.4 + 0.2 \times 1 = 0.6$ when she chooses an approach of class 2. Hence, in $t = 2$, she optimally chooses an approach of class 2. In $t = 1$, exerting an approach of class 1 causes expected costs of $0.1 + 0.6 \times 0.6 = 0.46$, which is lower than the costs when choosing an approach of class 2: $0.4 + 0.2 \times 0.6 = 0.52$. Therefore, the optimal strategy is $\mathbf{s} = (1, 2, 3)$.

From Lemma 1 we know that the relevant set forms a weakly convex and weakly increasing line in the $p - c$ space and the class 0 is in the relevant set. Hence, the property of increasing risk-adjusted costs c_t/p_t

implies that also the non-adjusted costs c_t and the success probability p_t are weakly increasing in the time t . When it comes to the periods $T - 1$ and T we can be sure that there is a strict increase because of Lemma 2.

PROPOSITION 1: *Given that the task is not yet solved, with proceeding time the person conducts approaches which become weakly more costly and more promising in terms of success. There is a strict increase from $T - 1$ to T .*

That is, the workload (measured by the effort cost c_t) is weakly increasing in t until the person has success. Intuitively, when there is a lot of time left, the person does not care much about the deadline and therefore tries some cheap and risky approaches. These are unlikely to be successful, but have low risk-adjusted costs. When the task is not solved shortly before the deadline, solving gets urgent. Then the person chooses expensive but safe approaches. Although these have high risk-adjusted costs, they are very likely successful.

We finally show that the expected costs of a solution are increasing in time t .

PROPOSITION 2: *$E_t[C] \in (c_t/p_t, c_T/p_T)$ for all $t < T$ and $E_t[C]$ is strictly increasing in time t .*

PROOF: See appendix.

The intuition is that the expected cost of a solution, $E_t[C]$, is increasing because the person exerts approaches which become incrementally inefficient in terms of risk-adjusted costs.

4. CONTINUOUS APPROACH SPACE

In this section, we assume that there is a continuum of approaches. We validate and strengthen the results we have shown in the previous section with a discrete approach space. For a specific family of effort cost functions, we illustrate our findings and yield a further interesting result:

not only do the expected per-period effort costs increase in time, conditional that the task is not yet solved; the unconditional expected costs also increase.

Suppose that the person can choose in every period the success probability $p \in [0, 1]$. This causes effort costs of $c = f(p)$. In the previous section, where we have considered a discrete approach space, we have seen that the relevant set contains the costless class 0 and that the relevant set is convexly and increasingly ordered in a $p - c$ diagram. Considering a continuous approach space, we therefore now assume that $f(0) = 0$, $f'(\cdot) > 0$, and $f''(\cdot) > 0$. Additionally we assume that $f(\cdot)$ is twice continuously differentiable.

The person's problem can be solved recursively. When period T arrives, and the task is not yet solved, she has to choose $p_T^* = 1$, which results in costs of $f(1)$. When we go back to period $t < T$, she minimizes the expected costs of a solution

$$\min_{p_t} f(p_t) + (1 - p_t)E_{t+1}[C]. \quad (5)$$

This yields the first-order condition⁵

$$f'(p_t^*) - E_{t+1}[C] \stackrel{!}{=} 0. \quad (6)$$

One can write

$$E_t[C] = f(p_t^*) + (1 - p_t^*)E_{t+1}[C]. \quad (7)$$

Note that $p_T^* = 1$, $E_T[C] = f(1)$, (6), and (7) determine the optimal sequence of success probabilities $p^* = (p_1^*, \dots, p_t^*, \dots, p_T^*)$ which the person chooses when the task is not yet solved. When the task is solved, it is optimal to choose $p_t = 0$.

From (6) we see that

$$\frac{dp_t^*}{dE_{t+1}[C]} = \frac{1}{f''(p_t^*)} > 0. \quad (8)$$

⁵One can directly show that with a continuous approach space (i) it is never optimal to choose $p = 0$ when the task is not yet solved and (ii) $p = 1$ is only chosen in T and when the task is not solved. Therefore, corner solutions are not an issue.

As with a discrete approach space, we have that $E_t[C] \in (f(p_t^*)/p_t^*, f(1)/1)$ for all $t < T$ and that $E_t[C]$ is increasing in t . The reasons are almost the same as in the proof of Proposition 2. Together with (8), this implies that p_t^* is increasing in time t . Because $f'(\cdot) > 0$, the per-period effort costs $f(p_t^*)$ are increasing, too. Because of the assumptions made on $f(\cdot)$, we yield that $f(p_t^*)/p_t^*$ is also increasing. The next proposition summarizes.

PROPOSITION 3: *Given that the task is not yet solved, the workload, $f(p_t^*)$, is strictly increasing in time t . Also, p_t^* , $f(p_t^*)/p_t^*$, and $E_t[C]$ are strictly increasing in t .*

Because of the strict increases, these findings are stronger than the ones of Proposition 1.

4.1. A SPECIFIC FAMILY OF EFFORT COST FUNCTIONS

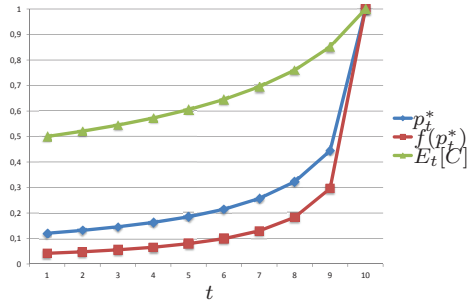
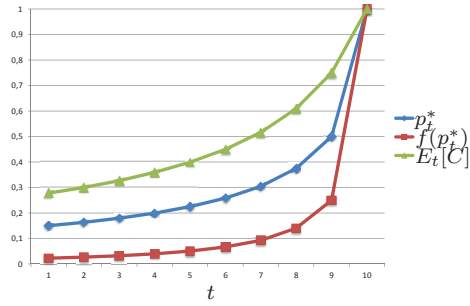
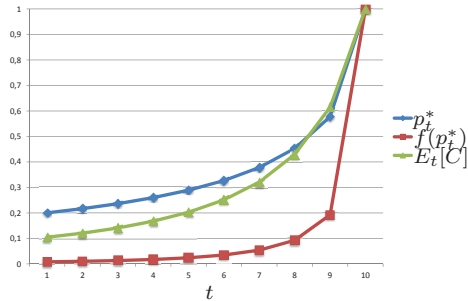
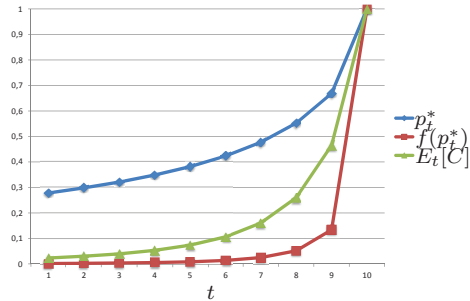
We now consider the following family of effort cost functions:

$$f(p) = \alpha p^\gamma, \tag{9}$$

with $\alpha > 0$, $\gamma > 1$, and $p \in [0, 1]$. Because the expected costs of a solution $E_t[C]$ are also linear in α , one can normalize $\alpha = 1$.

Figures 2-5 show how the optimal success probability of a period p_t^* (blue line), the per-period effort costs $f(p_t^*)$ (red line), and the expected costs of a solution $E_t[C]$ (green line) evolve over time for different values of the parameter γ . The numerical data can be found in the appendix. The diagrams show the case where the person has ten periods to solve the task. When a shorter horizon, $T < 10$, is considered, one has to cut off the first $10 - T$ periods of the figures.

The figures illustrate that the optimally chosen success probabilities p_t^* are increasing in γ . The reason is that a higher γ implies lower effort costs for all interior levels of p . Also the per-period effort costs for the optimal success probability are lower, which results in lower expected costs of a solution. Obviously, in the last period, $t = 10$, all variables are the same for all values of the parameter γ .

Figure 2: $\gamma = 1.5$.Figure 3: $\gamma = 2$.Figure 4: $\gamma = 3$.Figure 5: $\gamma = 5$.

The figures also clarify that in our stochastic environment a steeply increasing workload can be optimal. It is hard to qualify what “steeply” means. But we think that, for example, the quadrupling of the effort costs $f(p_t^*)$ from the penultimate to the last period in the quadratic cost specification ($\gamma = 2$) can be seen as a steep increase.

Propositions 1 and 3 have shown that the per-period effort costs are increasing in time, conditional that the task is not yet solved. Put differently, we showed that the workload of the person is increasing, given that she works. Next, we explore the unconditional expected effort costs. That is, we measure the expected workload, taking into account that the person may quit working because she already solved the task.⁶

Time has two counteracting effects on the unconditional expected effort costs. First, in early periods the task is less likely solved than in

⁶These unconditional expected effort costs can also be interpreted as the average per-period effort costs when there is a continuum of persons and each person plays her own game.

late periods. Remember, once the task is solved, the person chooses not to work in any of the remaining periods. Only when the task is not solved does the person try to solve the task by choosing a success probability $p_t^* > 0$, which results in positive effort costs of $f(p_t^*) > 0$. Therefore, this effect predicts that the unconditional expected effort costs are higher in the early periods than in the late periods. Second, from before we know that when the task is not yet solved, the person invests more in the late than in the early periods. Formally, $f(p_t^*)$ is increasing in t . This effect predicts that the unconditional expected effort costs are increasing in time. We conducted a numerical analysis; see the appendix. It suggests that the latter effect always dominates. That is, the person's effort costs are on average higher in late than in early periods.

PROPOSITION 4: *Given that the task is not yet solved in period $\tau \leq t$, for all $\gamma > 1$, the unconditional expected effort costs in period t are increasing in t .*

So Proposition 4 says that the person's expected workload is increasing until the deadline T .

Returning to the illustration with $T = 10$, the ratio between the unconditional expected effort costs of period 10 and 1 is 2.33 for $\gamma = 1.5$, 2.58 for $\gamma = 2$, 3.01 for $\gamma = 3$, and 3.70 for $\gamma = 5$.⁷ These numbers indicate that (i) the ratio is increasing in γ and (ii) the expected workload, measured by the unconditional expected effort costs, is quite substantially increasing between periods 1 and 10. As one can see from the tables in the appendix, around 80 percent of this increase occurs in the last step from period 9 to 10. That is, the expected workload increases relatively moderately until period 9; then there is a sharp increase in period 10.

⁷The numerical data as well as a richer series of the unconditional expected effort costs are given in the appendix.

5. DISCUSSION AND EXTENSIONS

In this section, we explore whether our results are robust when we consider (i) a scarce approach space, where the approach space is not only discrete, but where each class contains not necessarily at least T approaches, (ii) discounting of future effort costs, and (iii) a setting where completion of the task is no longer mandatory, but instead rewarded.

5.1. SCARCE APPROACHES

Up to now, we have assumed that approaches are abundant. What happens when the approach space is discrete and approaches are scarce in the sense that each class contains not at least T approaches?⁸

Consider the extreme example where each class contains only one approach. Suppose that the person has three periods to solve the task, and there are four classes:

$$\begin{aligned} c_0 &= 0, & p_0 &= 0, \\ c_1 &= 0.1, & p_1 &= 0.2, & c_1/p_1 &= 1/2; \\ c_2 &= 0.2, & p_2 &= 0.6, & c_2/p_2 &= 1/3; \\ c_3 &= 1, & p_3 &= 1, & c_3/p_3 &= 1. \end{aligned}$$

From Lemma 3 we know that the person optimally conducts the approaches in an ascending order of risk-adjusted costs c_i/p_i . Hence, the person first exerts approach 2, then 1, and then 3 (given that no success is reached). Therefore, when approaches are scarce, the person's workload (measured in terms of the effort costs c_i) is not necessarily monotonically increasing. The relationship can be non-monotonic. However, Assumption 1 guarantees that the workload is always increasing from $T - 1$ to T (when the task is not yet solved) because we know from before that (i) approach n is exerted last⁹ and (ii) no approaches with risk-adjusted

⁸Our analysis showed that our results stay completely valid when each class contains at least $T - 1$ approaches.

⁹With a scarce approach space, there may exist multiple optima. However, when there is some, even slight, discounting of effort costs and the person is able to do

costs above one are conducted. That is, even if approaches are scarce, the model predicts that the workload is at least increasing shortly before the deadline.

Additionally, with many classes it may be no problem that each class contains only few approaches. In Section 4, we considered the extreme case where there is a continuum of classes and showed that the person will optimally exert approaches of different classes in every period.

5.2. DISCOUNTING

We implicitly assumed that there is no discounting. What happens when future costs are discounted with rate $\delta \in (0, 1)$? We consider the simplest case where there is a continuous approach space, effort costs are $f(p) = p^\gamma$, with $\gamma > 1$, and two periods to complete the task, $T = 2$. In $t = 2$, the person chooses $p_2^* = 1$. In $t = 1$, she minimizes the present value of the expected costs of completion

$$\min_{p_1} p_1^\gamma + \delta(1 - p_1). \quad (10)$$

This yields

$$p_1^* = \left(\frac{\delta}{\gamma} \right)^{\frac{1}{\gamma-1}}. \quad (11)$$

Because $\gamma > 1 > \delta$, with discounting we also get that $p_1^* < p_2^* = 1$. That is, with discounting the workload is also increasing. Observe that discounting has a negative impact on p_1^* , which makes the workload increase even more sharply over time. Because p_1^* is continuous in δ , discount rates close to 1 lead to results which are quantitatively very similar to the ones without discounting.

5.3. REWARD FOR COMPLETION

We assumed that the person has to complete the task until the deadline T . Some real-world situations may be better described by assuming that a task does not have to be completed for sure, but rather that completion

nothing as often as she pleases, it is uniquely optimal to exert approach n last.

is rewarded (or equivalently, that failure is punished). Let the reward the person receives for completing the task within the deadline be R . Consider again the case where there are two periods, $T = 2$, and a continuous approach space. In $t = 2$, the person is no longer forced to choose $p_2 = 1$. She maximizes the expected reward minus her effort costs:

$$\max_{p_2} p_2 R - f(p_2). \quad (12)$$

When the reward is sufficiently high, the person chooses to solve the task for sure: $p_2^* = 1$ if $R \geq f'(1)$. Otherwise, p_2^* solves

$$f'(p_2^*) = R. \quad (13)$$

In $t = 1$, she maximizes the expected reward minus her effort costs and also takes into account that by completing the task now she foregoes the continuation payoff of reaching period $t = 2$,

$$\max_{p_1} p_1 R - f(p_1) - p_1(p_2^* R - f(p_2^*)). \quad (14)$$

The continuation payoff is positive whenever $\lim_{p \rightarrow 0} f'(p) < R$. We assume that this condition is satisfied, otherwise the problem is trivial and the person always chooses zero effort. Comparing (12) and (14), we see that the person is less eager to complete the task in $t = 1$ than in $t = 2$. The reason is that a failure in the first period still leaves the possibility open that the task is completed in the second period. This finding indicates that we should still get the result that the workload is increasing.

We now prove this by showing that $p_1^* < p_2^*$, which implies $f(p_1^*) < f(p_2^*)$. Suppose, contrary to our claim, that $p_1^* > p_2^*$. Then p_2^* is smaller than one and solves (13). The first-order condition of (14) is

$$f'(p_1) = R - (p_2^* R - f(p_2^*)). \quad (15)$$

Comparing (13) and (15) clarifies that p_1^* cannot exceed p_2^* because the continuation payoff is positive. The same line of arguments implies that whenever $p_2^* < 1$, we must have $p_1^* < p_2^*$. It remains to show that whenever $p_2^* = 1$, we have $p_1^* < 1$. When $p_2^* = 1$, the continuation payoff of reaching

period two is $R - f(1)$. Using this result, we see from (15) that $p_1^* = 1$ requires

$$f'(1) \geq f(1). \quad (16)$$

This cannot hold because we assumed that $f(0) = 0$ and $f', f'' > 0$.

Observe that when completion is rewarded, but not mandatory, we can relax the assumption that there is a class of approaches which are successful for sure. The problem is then still well-defined. Alternatively, when completion of the task is not mandatory, we can assume that high levels of effort are prohibitively costly so that the person will never choose them. From above we know that we still get the result that the workload is increasing.

Is the workload also increasing when the approach space is discrete? It is straightforward to show that when completion is rewarded, but not mandatory, Lemma 3 also remains valid. This lemma is the key to show that the workload is increasing.

6. CONCLUDING REMARKS

We show that in a stochastic environment a person optimally chooses an increasing workload. Time-inconsistent preferences are not needed to explain a steeply increasing workload.

We model stochasticity very simply. There exist alternative, possibly more complicated ways to capture stochasticity. Because the intuition for our results seems to be quite general (and not specific to the way we modeled stochasticity), we expect that our results still hold in alternative formulations of stochasticity.

O'Donoghue and Rabin (2008) consider a multiple step setting where the completing of a project requires several steps of effort. A possible direction for future research would be to study such a multiple step setting with stochasticity.

APPENDIX A: PROOFS

PROOF OF LEMMA 1

We prove the weak convexity property by contradiction. Suppose, in contradiction, that the person exerts an approach of a class i and there are two classes j and k so that $p_i = \psi p_j + (1-\psi)p_k$ and $c_i > \psi c_j + (1-\psi)c_k$, with $\psi \in (0, 1)$. Then the person can improve by playing a mixed strategy with weight ψ on the approach of class j and weight $1-\psi$ on the approach of class k . When the relevant set of classes forms a weakly convex line, then these classes are not dominated in the aforementioned sense by some other classes or mixtures of them. This proves the weak convexity property.

Next, we prove that no class is below the line in the $p - c$ space formed by \mathcal{I}^R . Suppose, in contradiction, that there is a class i which does not belong to \mathcal{I}^R and is below the line formed by \mathcal{I}^R in a $p - c$ diagram. Because \mathcal{I}^R forms a weakly convex and weakly increasing line in the $p - c$ space, see above, class i cannot be dominated. This is shown in Figure 6. The bold line shows some hypothetical \mathcal{I}^R . The grey area is the space where classes are dominated. Because the class i is below this area, it cannot be dominated by this hypothetical \mathcal{I}^R . This also holds for all other possible \mathcal{I}^R to which class i does not belong. Therefore, class i is not dominated and has, by the definition of \mathcal{I}^R , to belong to \mathcal{I}^R ; a contradiction.

Finally, because the class 0 has $p_0 = 0$ it cannot be replicated by a mixture of other classes. The same is true for class n because $p_n = 1$. Hence, the classes 0 and n cannot be dominated and are part of \mathcal{I}^R . Because of this and $c_i \in \mathbb{R}_{\geq 0}$, it is straightforward that the line formed by the relevant set is weakly increasing. \square

PROOF OF LEMMA 2

We have already shown that an approach of class n is only conducted in period T and when the task is not yet solved. Next, we show that approaches of class 0 are only exerted once the task is solved. We prove

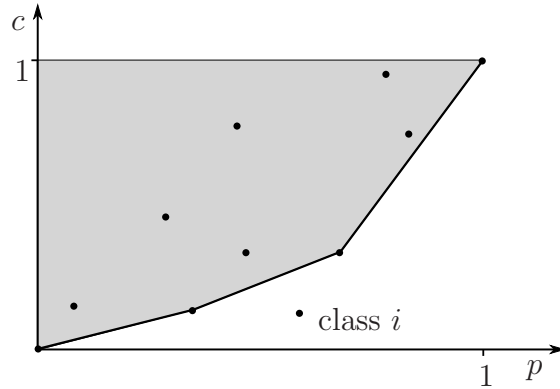


Figure 6: Graphical proof that class i cannot be dominated.

by contradiction.

Suppose that the person's strategy is $\mathbf{s}' = (s'_1, \dots, s'_t = 0, \dots, s'_T = n)$. That is, even when there is no success until t the person exerts an approach of class 0 in t . Then the person yields the same expected payoff when she chooses the strategy $\mathbf{s}'' = (s''_1 = s'_1, \dots, s''_t = s'_{t+1}, \dots, s''_{T-1} = s'_T = n, s''_T = s'_t = 0)$. That is, with \mathbf{s}'' the person exerts the approaches in the same order as with \mathbf{s}' until $t-1$. From t on, she exerts all approaches one period earlier until $T-1$. In T , she exerts an approach of class 0. But there is a third strategy \mathbf{s}''' , which in terms of payoffs dominates \mathbf{s}'' and therefore also \mathbf{s}' : $\mathbf{s}''' = (s'''_1 = s''_1, \dots, s'''_{T-2} = s''_{T-2}, \dots, s'''_{T-1} = i, s'''_T = n)$, where i is an approach of a class with $c_i/p_i < 1$. The reason why \mathbf{s}''' dominates \mathbf{s}'' is simple: From before, see especially (1), we know that the person optimally exerts an approach of class n only in T . This is not the case for strategy \mathbf{s}'' . The person can improve by using \mathbf{s}''' . \square

PROOF OF PROPOSITION 2

From before we know that in T the person will exert an approach of class n . Hence,

$$E_T[C] = c_n/p_n = c_T/p_T = 1. \quad (17)$$

One directly gets that the expected costs of the solution are a weighted average of the expected costs of the solution in the next period and the

risk-adjusted costs of the current period:

$$E_t[C] = (1 - p_t)E_{t+1}[C] + p_t \frac{c_t}{p_t}. \quad (18)$$

We know that before T the person will conduct approaches with risk-adjusted costs below one and in an ascending order of risk-adjusted costs (see Lemma 3 for the last point). Hence, $\frac{c_{T-1}}{p_{T-1}} < 1$. Therefore,

$$E_{T-1}[C] = (1 - p_{T-1})E_T[C] + p_{T-1} \frac{c_{T-1}}{p_{T-1}} \in \left(\frac{c_{T-1}}{p_{T-1}}, \frac{c_T}{p_T} \right). \quad (19)$$

For period $T - 2$ we get

$$E_{T-2}[C] = (1 - p_{T-2})E_{T-1}[C] + p_{T-2} \frac{c_{T-2}}{p_{T-2}}. \quad (20)$$

Because $E_{T-1}[C] \in \left(\frac{c_{T-1}}{p_{T-1}}, \frac{c_T}{p_T} \right)$ and $\frac{c_{T-2}}{p_{T-2}} \leq \frac{c_{T-1}}{p_{T-1}}$ we get that

$$E_{T-2}[C] \in \left(\frac{c_{T-2}}{p_{T-2}}, \frac{c_T}{p_T} \right). \quad (21)$$

The arguments can be repeated which yields that

$$E_t[C] \in \left(\frac{c_t}{p_t}, \frac{c_T}{p_T} \right) \quad (22)$$

for all $t < T$.

From (19) we see that $E_{T-1}[C] < E_T[C]$. Is $E_t[C]$ also increasing in t in earlier periods? Applying (22) for $t + 1$, we see from (18) that $E_t[C]$ is strictly increasing in t because of $\frac{c_t}{p_t} \leq \frac{c_{t+1}}{p_{t+1}}$. \square

PROOF OF PROPOSITION 4

We first consider the periods $t \leq T - 2$. Applying (6) for the specific functional form $f(p) = \alpha p^\gamma$ yields

$$\gamma p_t^{\gamma-1} \stackrel{!}{=} E_{t+1}[C] \quad (23)$$

and for period $t + 1$

$$\gamma p_{t+1}^{\gamma-1} \stackrel{!}{=} E_{t+2}[C]. \quad (24)$$

Similarly, (7) for period $t + 1$ is

$$E_{t+1}[C] = p_{t+1}^\gamma + (1 - p_{t+1})E_{t+2}[C]. \quad (25)$$

Plugging (25) into (23) yields

$$\gamma p_t^{\gamma-1} = p_{t+1}^\gamma + (1 - p_{t+1})E_{t+2}[C]. \quad (26)$$

Plugging (24) into this equation yields

$$p_t = \left(p_{t+1}^\gamma \left(\frac{1-\gamma}{\gamma} + p_{t+1}^{-1} \right) \right)^{\frac{1}{\gamma-1}}. \quad (27)$$

We first want to prove that at the beginning of every period $t \leq T - 2$ the effort costs in t are lower than the expected effort costs of $t + 1$. That is, we want to show that

$$p_t^\gamma < (1 - p_t)p_{t+1}^\gamma. \quad (28)$$

Plugging (27) into (28) yields, after substituting p for p_{t+1} ,

$$p^\gamma \left(1 - \left(p^\gamma \left(\frac{1-\gamma}{\gamma} + p^{-1} \right) \right)^{\frac{1}{\gamma-1}} \right) - \left(p^\gamma \left(\frac{1-\gamma}{\gamma} + p^{-1} \right) \right)^{\frac{\gamma}{\gamma-1}} > 0. \quad (29)$$

We conducted a numerical analysis with the software Mathematica. The Figures 7-9 illustrate the results. The numerical results show that for $p \in (0, 1]$ and $\gamma > 1$, (29) indeed holds. That is, the expected effort costs are increasing until period $T - 1$.

Finally, we want to show that at the beginning of period $T - 1$ the effort costs of $T - 1$ are also lower than the expected effort costs of T .

When T is reached $p_T = 1$. Using (23) implies that

$$p_{T-1} = \left(\frac{1}{\gamma} \right)^{\frac{1}{\gamma-1}}. \quad (30)$$

But this is just (27) with $p_{t+1} = 1$. So the former steps also apply for $t = T - 1$. \square

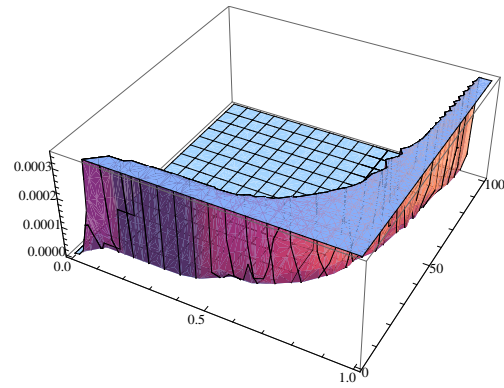


Figure 7: The inequality (29) for $p \in (0, 1]$ and $\gamma \in [1, 100]$.

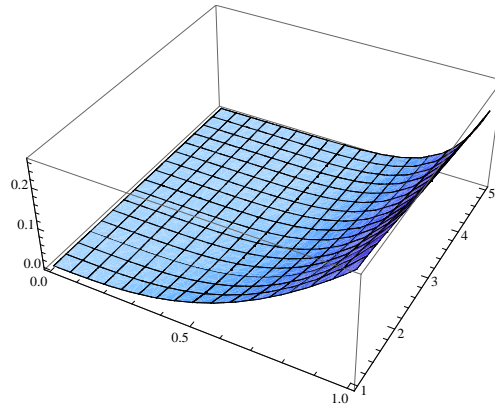


Figure 8: The inequality (29) for $p \in (0, 1]$ and $\gamma \in [1, 5]$.

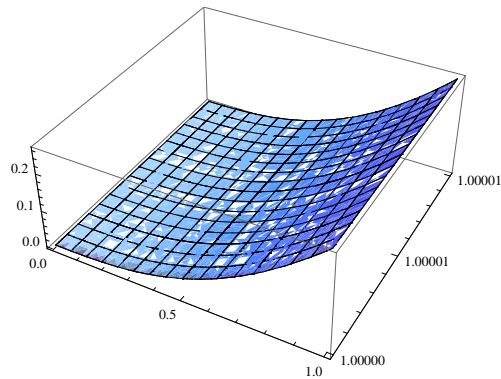


Figure 9: The inequality (29) for $p \in (0, 1]$ and $\gamma \in [1, 1.00001]$.

APPENDIX B: NUMERICAL RESULTS

NUMERICAL RESULTS OF THE EXAMPLE

Here are the numerical data of the $T = 10$ example for different values of γ . The calculations were made with the software Microsoft Excel. The unconditional expected per-period effort costs are denoted by $E[f(p_t^*)]$.

t	p_t	$f(p_t)$	$E[f(p_t^*)]$	$E_t[z]$
1	0.12058806	0.041875158	0.041875158	0.499949295
2	0.13193795	0.04792417	0.042145087	0.520886874
3	0.145757348	0.055647504	0.042480421	0.544848959
4	0.162985877	0.065799858	0.042909093	0.572672711
5	0.185129507	0.079655078	0.043478108	0.60557264
6	0.214783103	0.09954058	0.044273711	0.645400179
7	0.256896533	0.130207909	0.045475005	0.695170469
8	0.322511812	0.183154856	0.047533864	0.760274424
9	0.444444444	0.296296296	0.05209699	0.851851852
10	1	1	0.097681856	1

Table 1: Numerical results for $\gamma = 1.5$.

t	p_t	$f(p_t)$	$E[f(p_t^*)]$	$E_t[z]$
1	0.150178593	0.02255361	0.02255361	0.277803576
2	0.16355346	0.026749734	0.022732497	0.300357185
3	0.179699396	0.032291873	0.022954038	0.327106919
4	0.199624333	0.039849875	0.02323624	0.359398792
5	0.224918499	0.050588331	0.0236093	0.399248667
6	0.258270264	0.066703529	0.024128424	0.449836998
7	0.3046875	0.092834473	0.024907785	0.516540527
8	0.375	0.140625	0.026234235	0.609375
9	0.5	0.25	0.02914915	0.75
10	1	1	0.058298299	1

Table 2: Numerical results for $\gamma = 2$.

t	p_t	$f(p_t)$	$E[f(p_t^*)]$	$E_t[z]$
1	0.200609649	0.008073381	0.008073381	0.104585932
2	0.21690417	0.010204781	0.008157604	0.120732694
3	0.236316697	0.013197243	0.008261464	0.141142257
4	0.259903612	0.01755646	0.008393132	0.167536743
5	0.289293373	0.024211152	0.008566252	0.202649663
6	0.327163966	0.035018408	0.008805665	0.251071967
7	0.378338133	0.054155223	0.009162529	0.321108782
8	0.452806	0.092840296	0.009764854	0.429419228
9	0.577350269	0.19245009	0.011076146	0.615099821
10	1	1	0.024324905	1

Table 3: Numerical results for $\gamma = 3$.

t	p_t	$f(p_t)$	$E[f(p_t^*)]$	$E_t[z]$
1	0.278513901	0.001675847	0.001675847	0.023382131
2	0.298148091	0.002355918	0.001699762	0.030085519
3	0.321121843	0.003414674	0.00172911	0.039509191
4	0.348458374	0.005137532	0.001766118	0.053167886
5	0.38168894	0.008101172	0.001814493	0.073718014
6	0.423246156	0.013582043	0.00188096	0.106122704
7	0.477334268	0.024780671	0.001979329	0.160450877
8	0.552233245	0.051358548	0.002144084	0.259573562
9	0.668740305	0.133748061	0.002500163	0.465007756
10	1	1	0.006192264	1

Table 4: Numerical results for $\gamma = 5$.

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