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## Saving rates and portfolio choice with subsistence consumption

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## **Saving Rates and Portfolio Choice with Subsistence Consumption\***

Carolina Achury<sup>1</sup>, Sylwia Hubar<sup>2</sup>,  
and Christos Koulovatianos<sup>3</sup>

January 26, 2011

### **Abstract:**

We analytically show that a common across rich/poor individuals Stone-Geary utility function with subsistence consumption in the context of a simple two-asset portfolio-choice model is capable of qualitatively and quantitatively explaining: (i) the higher saving rates of the rich, (ii) the higher fraction of personal wealth held in risky assets by the rich, and (iii) the higher volatility of consumption of the wealthier. On the contrary, time-variant “keeping-up-with-the-Joneses” weighted average consumption which plays the role of moving benchmark subsistence consumption gives the same portfolio composition and saving rates across the rich and the poor, failing to reconcile the model with what micro data say.

**JEL Classification:** G11, D91, E21, D81, D14, D11

**Keywords:** Elasticity of Intertemporal Substitution, Stone-Geary Preferences, Two-asset Portfolio, Household Portfolios, Wealth Inequality, Controlled Diffusion

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## 1. Introduction

A vast literature studying the connection between consumption and portfolio choice has been trying to explain a number of empirical regularities that are considered as stylized facts. First, households with higher lifetime income exhibit higher saving rates.<sup>1</sup> Second, poorer stockholding households hold a lower fraction of their financial wealth in risky assets compared to wealthier ones.<sup>2</sup> Third, consumption growth of wealthier stockholders is more volatile than that of poorer stockholders.<sup>3</sup>

A fourth empirically motivated perception is that high-income households (who are also stockholders) exhibit higher elasticity of intertemporal substitution (EIS) compared to poorer non-stockholding households.<sup>4</sup> This distinction has led some researchers to assume exogenously different EIS across households as a building block for their models (typically, constant-EIS utility functions where the EIS parameter differs).<sup>5</sup> A typical criticism to assuming exogenously heterogeneous EIS is that one can generate any desired result through “(trivially) assuming convenient preferences”. For instance, assuming that some individuals are born with a higher EIS will tend to directly imply a higher saving rate for them, that these individuals will turn out to be richer, that they will tend to hold more stocks, and that their consumption will be more volatile. Apart from the accusation that assumptions and conclusions are too close, another criticism is that the potentially quantifiable heterogeneity

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<sup>1</sup> See, for example, Dynan, Skinner, and Zeldes (2004). They report a strongly positive correlation between saving rates and lifetime income and a less strong correlation between lifetime income and marginal propensity to save.

<sup>2</sup> A recent study that examines and documents this fact comprehensively is Wachter and Yogo (2010, Table 8).

<sup>3</sup> For example, Malloy et al. (2009, Table 1, p. 2439) compare consumption growth of the top third (and richest) stockholders with this of all other stockholders and report a substantial difference.

<sup>4</sup> See, for example, Guvenen (2009, Section 3), for evidence on this statement.

<sup>5</sup> For example, Guvenen (2009) and De Graeve, Dossche, Emiris, Sneessens, and Wouters (2009) use exogenously heterogeneous EIS as a key assumption in their analysis. Notably, Carroll (2002) provides arguments that the pattern of facts is against exogenous variation in risk tolerance, while in Carroll (2000) he proposes an alternative explanation of the high saving rates of the rich based on bequest motives.

of utility functions is difficult to establish empirically. Although we are not against behavioral approaches to finance and consumer theory, in this paper we explore the possibility of reconciling models with the three empirical regularities about saving, stockholding, and consumption volatility through a single utility function.

The goal of our analysis is precisely set. Using the simple Merton (1969, 1971) model as a vehicle for our analysis, we confront two alternative concepts of subsistence consumption with each other, in order to examine their promise for simultaneously resolving consumption/savings and portfolio-choice puzzles. The first concept is the standard constant subsistence level of consumption that we examine using a typical Stone-Geary (time-separable) momentary utility function of the form  $u(c(t)) = \{[c(t) - \chi]^{1-1/\eta} - 1\} / (1 - 1/\eta)$ , where  $\chi, \eta > 0$ . The second concept is the “keeping-up-with-the-Joneses” time-variant subsistence level of consumption given by,  $u(c(t), \bar{C}(t)) = \{[c(t) - \gamma\bar{C}(t)]^{1-1/\eta} - 1\} / (1 - 1/\eta)$ , where  $\gamma, \eta > 0$  and  $\bar{C}(t)$  stands for average consumption in a certain community. This second utility function implies a fully external habit with time separability. We focus on the fundamental Merton (1969, 1971) framework in order to obtain simple analytical solutions that allow for comparative static analysis which is robust in a way that numerical solutions cannot offer.

What we find is sharp. The Stone-Geary formulation with *time-invariant* subsistence consumption meets all four empirical regularities at the micro level: it generates (i) saving rates, (ii) risky-asset portfolio shares, (iii) consumption-growth volatility, (iv) endogenous consumption-choice dependent EIS, all four positively dependent on initial or current financial wealth. *Time-invariant* subsistence consumption fails in producing a stationary relative wealth distribution with different EIS across the rich and the poor. On the contrary, the Stone-Geary formulation with *time-variant* subsistence consumption produces a stationary

relative wealth distribution with different EIS across the rich and the poor throughout the whole equilibrium path, but fails to reconcile the three first empirical regularities appearing above. It implies that, (i) saving rates, (ii) risky-asset portfolio shares, (iii) consumption-growth volatility, are all the same across the rich and the poor.

We use our analytical formulas to perform a calibration exercise finding that our time-invariant subsistence model simultaneously matches the range of risky asset holdings in SCF data (6% to 22% for poor vs. rich stockholders), gives saving rates ranging from 6% (poor) to 17% (rich), and coefficients of relative risk aversion ranging from 14 (poor) to 4.5 (rich). These quantitative findings are robust to varying levels of time-invariant subsistence needs within a range of estimates in the literature.

We show that poor households with time-invariant subsistence needs choose to exit their poverty slowly, rather in a fast and temporarily painful way, in order to smooth the high disutility of consuming close to their subsistence needs. This strategy explains the low saving rates of the poor. Another part of this investment strategy of the poor is not taking high investment risks, thus holding lower shares of risky assets. On the contrary, external habits make subsistence needs grow over time together with aggregate consumption. So, poor households cannot afford to have a lower saving rate and to take fewer risks, because the pressure they will be feeling in the future will increase, as subsistence needs increase. For this reason the poor are catching up with others and have the same saving rates and portfolio shares, despite that they always have lower EIS than others.

Our work opens new and specific questions for future research. An environment with liquidity constraints and imperfect correlation between income shocks and risky asset returns is substantially different than the simple model we study in this paper. Nevertheless, it is an open question whether the two subsistence-consumption setups retain an influence by the

qualitative features we uncover here. It is interesting to examine whether there is scope for quantitative explanations of stockholding shares or even stock-market participation through the use of a single utility function for all household with time-invariant consumption.<sup>6</sup>

Our focus on subsistence consumption and on the use of a common across agents utility function has similarities with the approach of Wachter and Yogo (2010) who propose non-homothetic utility but distinguish between basic goods and luxuries, assuming that people are less risk averse about luxuries than about necessities. Wachter and Yogo (2010) also obtain the result that the rich invest more in risky assets, simply because what they are risking is mostly luxury consumption.<sup>7</sup> The role of subsistence consumption is similar: the rich are willing to take more risk, because the chances that any given wealth shortfall will jeopardize their ability to consume the subsistence consumption are much smaller than for the poor, whose consumption is hovering around subsistence. We think that subsistence consumption offers more parsimony as our approach allows to work with the consumer basket rather than with micro-level consumer data, facing the additionally tedious task of having to distinguish between luxuries and necessities. Perhaps this aspect is more appealing to macroeconomists who work on idiosyncratic-risk heterogeneous agent models in order to study asset markets among other questions.

Concerning the technical contribution of our paper, we are not the first who have technically investigated the two-asset Merton (1969, 1971) model using the Stone-Geary utility function with time-invariant subsistence consumption. Karatzas et al. (1986) and Sethi et al. (1992) mention explicit solutions to this problem among their other results regarding the possibility of investor bankruptcy. Weinbaum (2005) is another study making use of the

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<sup>6</sup> In our simple model all households participate in the stock market as a result of our requirement for interior and analytically tractable solutions.

<sup>7</sup> Yogo (2006) has used a similar approach to Wachter and Yogo (2010) distinguishing between durable and nondurable consumption in order to analyze the cyclical behavior of stock returns.



ability of the model to land an explicit solution in order to apply it to the bond market. Yet, we offer a simpler solution approach based on undetermined coefficients to the case of time-invariant subsistence (our Proposition 1 in Section 2). On the contrary, we are not aware of other studies that analytically work out the time-variant subsistence formulation.<sup>8</sup> For solving the model using “keeping-up-with-the-Joneses” preferences, it is crucial to use an aggregation result that greatly simplifies the problem. Such an aggregation result appears in Koulovatianos (2005, Theorem 3), and we demonstrate how to use this aggregation result in the context of solving a portfolio-choice problem. Most importantly, we are not aware of any study that uses the explicit solution of the two-asset Merton (1969, 1971) model with Stone-Geary preferences in order to address empirical consumption/savings and portfolio choice regularities.

We examine the time-invariant subsistence consumption model in Section 2, while in Section 3 we analyze the time-variant subsistence model. In Section 4 we present our calibration exercise, and in Section 5 we make concluding remarks.

## 2. Time-invariant Subsistence Consumption

Time is continuous, with  $t \in [0, \infty)$ . Consider the Merton (1969, 1971) two-asset model, where an investor having initial wealth holdings  $k_0 > 0$  has the opportunity to invest in a risky asset (investing a fraction  $\phi$  of her wealth in the risky asset) and a risk-free asset.<sup>9</sup> So, the investor chooses the consumption path  $(c(t))_{t \geq 0}$  and the path of portfolio composition

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<sup>8</sup> External habits in the utility function have been introduced, for example, in the analyses of Abel (1990), and Campbell and Cochrane (1999), while portfolio choice with external habits in utility has been studied numerically by Chan and Kogan (2002) and Wachter (2006). Examples of portfolio-choice analyses with internal habits are Gomes and Michaelides (2003), Polkovnichenko (2007), and Brunnermeier and Nagel (2008).

<sup>9</sup> If we assume  $n \geq 2$  risky assets, the mutual fund theorem developed in Merton (1971) and also outlined in Karatzas et al. (1986, Section 5), justifies why the model can be equivalently reduced to a single risky investment.

over time  $(\phi(t))_{t \geq 0}$ , that maximizes her expected utility,

$$E_0 \left\{ \int_0^\infty e^{-\rho t} \frac{[c(t) - \chi]^{1 - \frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} dt \right\}, \quad (1)$$

with  $\rho, \chi, \eta > 0$ . The household's budget constraint is,

$$dk(t) = \{ [\phi(t)R + (1 - \phi(t))r^f] k(t) - c(t) \} dt + \sigma \phi(t) k(t) dz(t), \quad (2)$$

where  $R$  is the mean rate of return of the risky asset,  $r^f$  is the risk-free rate ( $R > r^f$ ), and  $dz(t) = \varepsilon(t) \sqrt{dt}$ , where  $\varepsilon(t) \sim N(0, 1)$ , i.e.  $dz(t)$  is a Brownian motion.

## 2.1 Decision Rules and Dynamics of Financial Wealth

The Hamilton-Jacobi-Bellman equation (HJB) is given by,

$$\rho J(k) = \max_{c \geq 0, \phi} \left\{ \frac{(c - \chi)^{1 - \frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} + J'(k) \{ [\phi R + (1 - \phi)r^f] k - c \} + \frac{(\sigma \phi k)^2}{2} J''(k) \right\}. \quad (3)$$

The first-order conditions are,

$$(c - \chi)^{-\frac{1}{\eta}} = J'(k), \quad (4)$$

$$\phi = \frac{J'(k)}{-J''(k)} \frac{R - r^f}{k \sigma^2}. \quad (5)$$

We make two assumptions, technical in nature, that enable us to secure that solutions exist and that they are interior. The rationale behind these assumptions becomes obvious in the process of proving Proposition 1 which appears below.<sup>10</sup>

**Assumption 1** *Initial conditions are restricted so that,*

$$k_0 > \frac{\chi}{r^f}.$$

<sup>10</sup>The need for some parametric restrictions arises even without the presence of subsistence consumption. Yet, subsistence consumption places additional parametric constraints which become obvious in our model here due to its ability to land analytical results.

**Assumption 2** *Parameter  $\eta$  is restricted to be strictly below the strictly positive value,  $\bar{\eta}$ , characterized below:*

$$0 < \eta < \bar{\eta} \equiv \frac{1 - \frac{r^f - \rho}{\frac{1}{2} \left( \frac{R - r^f}{\sigma} \right)^2} + \left\{ \left[ \frac{r^f - \rho}{\frac{1}{2} \left( \frac{R - r^f}{\sigma} \right)^2} \right]^2 + 2 \frac{r^f + \rho}{\frac{1}{2} \left( \frac{R - r^f}{\sigma} \right)^2} + 1 \right\}^{\frac{1}{2}}}{2} .$$

It is easy to verify that  $\bar{\eta} > 0$  for any values of  $\sigma, \rho, R, r^f > 0$ .<sup>11</sup> Moreover, placing such a parametric restriction on parameter  $\eta$  may not prevent the quantitative matching of observed/estimated elasticities of intertemporal substitution (EIS), since in this model  $EIS = \eta(1 - \chi/c)$ .<sup>12</sup> Proposition 1 provides the analytical solution to the model.

### Proposition 1

*Under Assumptions 1 and 2, the solution to the problem expressed by the HJB equation given by (3) is a decision rule for consumption,*

$$c^* = C(k) = \xi k + \psi ,$$

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<sup>11</sup>Notice that

$$\left[ 1 - \frac{r^f - \rho}{\frac{1}{2} \left( \frac{R - r^f}{\sigma} \right)^2} \right]^2 < \left[ \frac{r^f - \rho}{\frac{1}{2} \left( \frac{R - r^f}{\sigma} \right)^2} \right]^2 + 2 \frac{r^f + \rho}{\frac{1}{2} \left( \frac{R - r^f}{\sigma} \right)^2} + 1 .$$

<sup>12</sup>Barsky et al. (1997) offer plausible numbers based on a combination of a survey approach and objective observations. Blundell, Browning, and Meghir (1994) have estimated that the *EIS* of households in the top income quintile is about three times that of households in the bottom quintile of the distribution. In his calibration exercise for explaining asset prices, Guvenen (2009) uses values 0.1 for (poorer) non-stockholders and 0.3 for (richer) stockholders. With the aid of parameter  $\chi$ , such values can be matched. It can easily be verified that using a parametrization according to US estimates of stock and bond returns (see, for example, Guvenen (2009, Table II, p. 1725)),  $R = 8\%$ ,  $r^f = 2\%$ ,  $\sigma = 20\%$ , and with  $\rho = 1.5\%$ , the implied level of the upper bound  $\bar{\eta}$  is  $\bar{\eta} \cong 1.25$ . If we use  $\rho = 4.5\%$  and the stock/bond returns above (for example, aggregate-economy models with idiosyncratic risk imply that  $r^f < \rho$ ), then the implied value for  $\bar{\eta}$  is  $\bar{\eta} \cong 1.8$ . In any case, the implied value for  $\bar{\eta}$  gives ample space for plausible calibration exercises such as this we perform in Section 4 below.

where

$$\xi = \rho\eta + (1 - \eta)r^f - \frac{\eta(\eta - 1)}{2} \left( \frac{R - r^f}{\sigma} \right)^2,$$

and

$$\psi = \eta\chi \frac{r^f - \rho + \frac{\eta-1}{2} \left( \frac{R-r^f}{\sigma} \right)^2}{r^f},$$

a decision rule for portfolio choice,

$$\phi^* = \Phi(k) = \eta \frac{R - r^f}{\sigma^2} \left( 1 - \frac{\chi}{k} \right),$$

while the value function is given by,

$$J(k) = -\frac{1}{\rho \left( 1 - \frac{1}{\eta} \right)} + \xi^{-\frac{1}{\eta}} \frac{\left( k - \frac{\chi}{r^f} \right)^{1 - \frac{1}{\eta}}}{1 - \frac{1}{\eta}}.$$

**Proof** See the Appendix.  $\square$

On a technical note, the role of Assumption 2 is to secure that  $\xi$  in Proposition 1 is strictly positive. The role of Assumption 1 is obvious after looking at the functional form of the value function,  $J(k)$ , in Proposition 1, a crucial condition for guaranteeing that the problem is well-defined and that the solution is interior. It is also easy to verify that  $c = \xi \left( k - \chi/r^f \right) + \chi$ , which reveals another role of both Assumptions 1 and 2, which is to meet the requirement that  $c \geq \chi > 0$ .<sup>13</sup>

Proposition 2 reveals the dynamics of household wealth, which secure that, in equilibrium, *discretionary lifetime resources* (defined as  $k - \chi/r^f$ ) are always strictly positive, i.e.,  $k^*(t) > \chi/r^f$  for all  $t \geq 0$ .

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<sup>13</sup>In principle, the constraint  $c \geq \chi$  must be imposed a-priori. Assumption 2 contributes to  $c \geq \chi$  being always slack by placing an upper bound to a parameter that directly affects the EIS of the investor (the upper bound of EIS is  $\lim_{c \rightarrow \infty} \eta(1 - \chi/c) = \eta$ ). An upper bound for EIS implies a bound in the investor's tolerance for fluctuations in consumption, which prevents  $c$  from hitting the  $\chi$  bound in equilibrium. If Assumption 2 is violated, then the way to solve the problem is by imposing  $c \geq \chi$  explicitly. On the contrary, under Assumption 2 (and Assumption 1), the constraint  $c \geq \chi$  can be safely ignored. Moreover, the requirement of Assumption 2 holds also in the case where  $\chi = 0$ , the original parametric example appearing in Merton (1969). In this particular case the constraint  $c \geq \chi$  coincides with the standard requirement of non-negative consumption ( $c \geq 0$ ).

## Proposition 2

Under Assumptions 1 and 2, the dynamics of financial wealth,  $k$ , are fully characterized by,

$$k^*(t) - \frac{\chi}{r^f} = e^{\eta \left[ r^f - \rho + \frac{1}{2} \left( \frac{R-r^f}{\sigma} \right)^2 \right] t + \eta \left( \frac{R-r^f}{\sigma} \right) z(t)} \left( k_0 - \frac{\chi}{r^f} \right) \quad (6)$$

where  $z(t) = \int_0^t dz(s)$  with  $\int$  being the stochastic (Itô) integral.

**Proof** See the Appendix.  $\square$

## 2.2 Characterization of Saving and Portfolio Choices

With Propositions 1 and 2 at hand we proceed to characterizing the savings, consumption, and stock-holding behavior of a price-taking household. Proposition 3 provides necessary and sufficient conditions for having a saving rate which is increasing in wealth.

### Proposition 3

Under Assumptions 1 and 2, the saving rate of a household is strictly increasing in wealth if and only if,

$$\eta > \underline{\eta} \equiv -1 - \frac{r^f - \rho}{\frac{1}{2} \left( \frac{R-r^f}{\sigma} \right)^2} . \quad (7)$$

**Proof** Fix any time instant  $t \geq 0$ . Based on Proposition 1, direct substitution of the decision rule for consumption,  $C(k)$ , and also for portfolio choice,  $\Phi(k)$ , into the household's budget constraint given by (2), after some algebra, gives the equilibrium savings level at time  $t$ ,

$$S^*(t) = \zeta \left[ k^*(t) - \frac{\chi}{r^f} \right] ,$$

with

$$\zeta \equiv \eta \left[ \frac{\eta + 1}{2} \left( \frac{R - r^f}{\sigma} \right)^2 + r^f - \rho \right] .$$

The saving rate,  $s^*(t)$ , is

$$s^*(t) = \frac{S^*(t)}{\phi^*(t)(R - r^f)k^*(t) + r^f k^*(t)} \quad (8)$$

and after substituting  $\Phi(k)$  from Proposition 1 into (8) it is

$$s^*(t) = \frac{\zeta}{\eta \left( \frac{R - r^f}{\sigma} \right)^2 + \frac{r^f}{1 - \frac{r^f}{k^*(t)}}}$$

which implies that  $ds^*(t)/dk^*(t) > 0 \Leftrightarrow \zeta > 0 \Leftrightarrow (7)$ . $\square$

Since  $\eta > 0$  the mechanics of Proposition 3 apply if  $\max\{0, \underline{\eta}\} < \eta < \bar{\eta}$ , while it is easy to verify that  $\underline{\eta} < \bar{\eta}$  for all acceptable values of parameters  $\rho$ ,  $\sigma$ ,  $R$ , and  $r^f$ . It is also easy to verify from equation (6) that the expected value of  $k$  of any household will grow at a strictly positive rate if and only if,

$$r^f - \rho + \frac{1 + \eta}{2} \left( \frac{R - r^f}{\sigma} \right)^2 > 0 ,$$

which is equivalent to the condition given by equation (7), the necessary and sufficient condition for having saving rates that increase in wealth. Corollary 1 expresses this key observation that allows to see how the parametric constraint given by (7) is linked with the monotonicity of savings in Proposition 3.

### Corollary 1

*Under Assumptions 1 and 2, the household's expected wealth grows at a strictly positive rate if and only if the household's saving rate is strictly increasing in wealth.*

Corollary 1 reveals the model's mechanics. When the household sees the possibility of following a trajectory of expected wealth that grows over time, the household essentially sees a potential for escaping from the pressure of subsistence needs in the future. This foreseen potential to be alleviated from consuming too close to subsistence needs allows to afford a strategy characterized by a lower saving rate while poor. Poorer households may choose a lower saving rate because consuming less while poor is too painful due to the presence of subsistence needs. So, the potential to escape subsistence in the future can smooth out the disutility of currently consuming close to subsistence. Corollary 1 confirms that this is exactly the logic behind the trajectory of intertemporal decisions the household has in mind: the poor choose to escape poverty slower, rather than faster and in the most painful way. Yet, whether it is optimal to select such a path depends on the model's parameters, i.e., whether inequality (7) is met.

Regarding the role of inequality (7), the two preference parameters  $\eta$  and  $\rho$  are of key importance. Parameter  $\eta$  is tightly connected to EIS which makes vivid monotonic dynamics of consumption and wealth more tolerable or desirable (e.g., an increasing wealth/consumption trajectory is more likely to be chosen as optimal). Moreover, higher EIS is linked with higher saving rates throughout the infinite horizon. Inequality (7) places a lower bound on parameter  $\eta$  ( $\underline{\eta}$ ), encouraging preference for higher savings and for vividly monotonic trajectories of wealth and consumption. Yet, this lower bound for  $\eta$  is related to  $\rho$  since a high value for  $\rho$  discourages overall savings. So, through inequality (7) a high level of  $\rho$  necessitates a further increase in  $\eta$ .<sup>14</sup> Nevertheless, it is easy to verify that the parametric constraint given by (7) gives sufficient freedom to combine a wide range of plausible calibration parameter values. In addition, when (7) is not met, ( $\eta \leq \underline{\eta}$ ) implies that  $\zeta \leq 0$ , which means that savings are

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<sup>14</sup>Unlike the case of using Epstein-Zin preferences,  $\eta$  and  $\rho$  cannot be calibrated independently in our setup.

always negative.<sup>15</sup>

Proposition 4 is our portfolio-share result.

**Proposition 4**

*Under Assumptions 1 and 2, the portfolio share of risky assets is strictly positive and increasing in wealth.*

**Proof** Immediate from  $\Phi(k)$  of Proposition 1.  $\square$

Proposition 5 examines the relationship between the coefficient of variation of consumption and initial financial wealth and also the relationship between consumption growth and current wealth.

**Proposition 5**

*Under Assumptions 1 and 2, the coefficient of variation of consumption is strictly increasing in initial financial wealth and the variance of the growth rate of consumption is increasing in current financial wealth.*

**Proof** See the Appendix.  $\square$

Proposition 4 shows that the same utility function with subsistence consumption for all households leads to a theoretical prediction which is closer to the empirical observation made by Wachter and Yogo (2010) that household portfolio shares of risky assets rise in wealth. Proposition 5 is consistent with the empirical observations made by Malloy et al. (2009, Table 1, p. 2439) that consumption growth is more volatile for top (and richer) stockholders compared to all other stockholders.

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<sup>15</sup>We thank an anonymous referee for noticing this subtle connection revealed by Corollary 1 and for motivating us to focus on this connection in order to provide an explanation for the parametric constraint given by (7).



## 2.3 Dynamics of inequality

A key feature of this simple partial-equilibrium analysis with price-taking households is that inequality in financial wealth increases over time if everybody's financial wealth grows over time, as long as (7) holds (see Corollary 1 above). Considering two individuals, a “rich” and a “poor” according to their initial total asset holdings,  $k_{r,0} > k_{p,0}$  (subscripts correspond to “r: rich” and “p: poor”), equation (6) implies that

$$\frac{k_r^*(t) - \frac{\chi}{r^f}}{k_p^*(t) - \frac{\chi}{r^f}} = \frac{k_{r,0} - \frac{\chi}{r^f}}{k_{p,0} - \frac{\chi}{r^f}}$$

which leads to,

$$\frac{k_r^*(t)}{k_p^*(t)} = \frac{k_{r,0} - \frac{\chi}{r^f}}{k_{p,0} - \frac{\chi}{r^f}} - \frac{\chi}{r^f} \cdot \frac{k_{r,0} - k_{p,0}}{k_{p,0} - \frac{\chi}{r^f}} \cdot \frac{1}{k_p^*(t)}. \quad (9)$$

Equation (9) implies that as  $k_p^*(t)$  increases over time, inequality in relative wealth increases over time.

The feature that the model does not lead to a steady-state stationary distribution of relative wealth may be considered as unattractive. Chan and Kogan (2002) using an external-habit model resolve the issue of having a long-run growth model with a stationary relative wealth distribution. Scholars who study simulated models propose the preference formulation of next model's section (for example, see Guvenen (2009, footnote 7, p. 1723) and his discussion in the paper's conclusions regarding the extension of his model to long-run growth analysis). We show that in the context of our model's aggregate stock market shocks this proposed preference formulation of “keeping-up with the Joneses” benchmark subsistence consumption leads to less promising results: despite that it is capable of producing a stationary relative wealth distribution and EIS that is increasing in wealth, it nevertheless fails to meet all other micro data empirical regularities.

### 3. Time-variant Benchmark Subsistence Consumption

The structure of the model is the same as above, with the sole difference that utility is of the form

$$E_0 \left\{ \int_0^\infty e^{-\rho t} \frac{[c(t) - \gamma \bar{C}(t)]^{1-\frac{1}{\eta}}}{1 - \frac{1}{\eta}} dt \right\}, \quad (10)$$

where  $\gamma \in (0, 1)$  and with  $\bar{C}(t)$  being average consumption in the economy in period  $t$ .<sup>16</sup> Moreover, we would like to restrict  $\gamma$  so that all individuals in the economy have consumption  $c(t) > \gamma \bar{C}(t)$ . This amounts to a restriction on  $\gamma$  which is driven by the distribution of initial asset holdings. Yet, in order to identify such a parametric restriction involving initial conditions and parameters, the model must be first solved under the working assumption of having interior solutions.<sup>17</sup> Preferences given by equation (10) assume "benchmark consumption levels".<sup>18</sup> Such a utility function captures the "keeping-up with the Joneses" idea of having consumption standards influenced by average consumption standards, with the

<sup>16</sup>For this formulation, see, for example, Guvenen (2009, footnote 7, p. 1723). These preferences fall into the category studied in Koulovatianos (2005, Theorem 3) that gives perfect linear aggregation under certainty. This means that, provided that all solutions are interior, also under aggregate uncertainty the aggregate level of consumption,  $\bar{C}(t)$ , corresponds to the choice of a (perhaps fictitious) consumer with wealth holdings equal to the aggregate wealth level of the economy,  $\bar{K}(t)$ . This property is proved to be true in equilibrium.

<sup>17</sup>Notice in the model with subsistence consumption above that the initial capital stock of the poorest household,  $\underline{k}_0$  is restricted to  $\underline{k}_0 > \chi/r^f$ , which guarantees that all households in the economy have well-defined problems and interior solutions. Notice also that we were able to identify that restriction only after having solved the problem under the working assumption of interior solutions.

<sup>18</sup>This is also a variant of the preference formulation in Chan and Kogan (2002), which focuses on the "external habit", in contrast to the formulation of Constantinides (1990), which focuses on the "internal habit." The difference in our formulation is that it is not the stock of external habit but the flow of external habit that influences behavior. As it will be shown below, in equilibrium, consumption paths of all agents grow parallelly. Given that in our model prices are exogenous, it would be difficult to empirically distinguish between the external and internal habit from simulated data from our model. A recent empirical study investigating the relative importance of external vs. internal habits is Grishchenko (2009). Notice that the elasticity of intertemporal substitution (EIS) is

$$EIS = \eta \left( 1 - \gamma \frac{\bar{C}}{c} \right),$$

so, for  $c = \bar{C}$ ,  $EIS = (1 - \gamma)$ , and the formula  $\bar{C}/c = (1/\gamma)(1 - EIS/\eta)$  reveals that empirically plausible levels of the  $EIS$  can match observed consumption ratios  $\bar{C}/c$ .

quantitative impact of this influence moderated through parameter  $\gamma$ . Clearly, we assume a community of investors such that each investor is a price taker (for example, members of the community invest in a globalized international market portfolio of stocks) and keeps track of average consumption,  $\bar{C}$ , of his/her (local) community. The path of average consumption,  $\bar{C}$ , over time is generated through the decision-making process of a fictitious household that possesses average initial wealth  $\bar{K}_0$ .<sup>19</sup> This property is due to that perfect linear aggregation holds in our model, and this we confirm once we derive our solution below.

The budget constraint of the individual is,

$$dk(t) = \{[\phi(t)R + (1 - \phi(t))r^f]k(t) - c(t)\}dt + \sigma\phi(t)k(t)dz(t) , \quad (11)$$

however, in this case, keeping track of the dynamics of average consumption,  $\bar{C}(t)$ , is also necessary. Because the (perhaps fictitious) average household possessing  $\bar{K}(t)$  units of wealth also solves an optimal control problem, the household possessing  $k(t)$  units of wealth will form a value function which depends on both the current level of its financial wealth,  $k(t)$ , and the current level of the poorest household's financial wealth,  $\bar{K}(t)$ , i.e. the value function will be of the form  $J(k, \bar{K})$ . In doing so, the household with  $k$  needs to keep track of the dynamics of the average household's budget constraint (with wealth holdings  $\bar{K}$ ), although the household with  $k$  does not control  $\bar{C}(t)$  or  $\bar{K}(t)$ . Below we reconfirm that the decision rules of all individuals imply perfect linear aggregation. This budget constraint is,

$$d\bar{K}(t) = \{[\bar{\Phi}(t)R + (1 - \bar{\Phi}(t))r^f]\bar{K}(t) - \bar{C}(t)\}dt + \sigma\bar{\Phi}(t)\bar{K}(t)dz(t) , \quad (12)$$

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<sup>19</sup>This household is called a “representative consumer (RC)”, a fictitious household who possesses average wealth and who has a utility function composed by the utility functions of all other household types (this holds in the case of preference heterogeneity – in the present paper all agents have the same preferences, so, in our case, RC's utility function is the same as everyone else's), and whose choices coincide with all aggregated choices of the community under any price regime. For further details on the concept of RC see Caselli and Ventura (2000) and Koulovatianos (2005).

and we incorporate it in the HJB equation of household with holdings  $k$  as,

$$\begin{aligned} \rho J(k, \bar{K}) = \max_{c \geq 0, \phi} & \left\{ \frac{(c - \gamma \bar{C})^{1 - \frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} + J_k(k, \bar{K}) \{ [\phi R + (1 - \phi) r^f] k - c \} + \right. \\ & + J_{\bar{K}}(k, \bar{K}) \{ [\bar{\Phi} R + (1 - \bar{\Phi}) r^f] \bar{K} - \bar{C} \} + \\ & \left. + \frac{(\sigma \phi k)^2}{2} J_{kk}(k, \bar{K}) + \frac{(\sigma \bar{\Phi} \bar{K})^2}{2} J_{\bar{K}\bar{K}}(k, \bar{K}) + \sigma^2 \phi \bar{\Phi} k \bar{K} J_{k\bar{K}}(k, \bar{K}) \right\} \end{aligned} \quad (13)$$

where  $J_x$  denotes the first partial derivative with respect to variable  $x \in \{k, \bar{K}\}$ ,  $J_{xx}$  is the second partial derivative with respect to  $x$ , and the notation for the cross-derivative is obvious. For an individual other than the (perhaps fictitious) individual with wealth holdings equal to  $\bar{K}$ , the paths  $\bar{C}(t)$  and  $\bar{\Phi}(t)$  are generated through two decision rules,  $\bar{C}(t) = C(\bar{K}(t))$  and  $\bar{\Phi}(t) = \Phi(\bar{K}(t))$  which are consistent with consumer optimization of the individual with wealth holdings  $\bar{K}(t)$  for all  $t \geq 0$ , and also the budget constraint given by (12). First-order conditions are,

$$(c - \gamma \bar{C})^{-\frac{1}{\eta}} = J_k(k, \bar{K}) \quad (14)$$

$$\phi = \frac{R - r^f}{\sigma^2} \frac{J_k(k, \bar{K})}{-J_{kk}(k, \bar{K}) k} + \bar{\Phi} \cdot \frac{J_{k\bar{K}}(k, \bar{K}) \bar{K}}{-J_{kk}(k, \bar{K}) k} \quad (15)$$

Proposition 6 states the solution to the above problem. Yet, interiority of solutions involves making a parametric constraint which involves parameter  $\gamma$ , given by Assumption 3.

**Assumption 3** *Parameter  $\gamma$  and initial conditions are restricted so that,*

$$\gamma < \frac{k_0}{\bar{K}_0},$$

where  $k_0$  is the initial wealth of the poorest household and  $\bar{K}_0$  is the average initial wealth.

Using Assumption 3 we proceed to characterizing the interior solution of the model, which is given by Proposition 6.

**Proposition 6**

*Under Assumptions 2 and 3, the solution to the problem expressed by the HJB equation given by (13) is a decision rule for consumption,*

$$c^* = C(k) = \xi k ,$$

where

$$\xi = \rho\eta + (1 - \eta)r^f - \frac{\eta(\eta - 1)}{2} \left( \frac{R - r^f}{\sigma} \right)^2 ,$$

a decision rule for portfolio choice,

$$\phi^* = \Phi(k) = \eta \frac{R - r^f}{\sigma^2} ,$$

while the value function is given by,

$$J(k, \bar{K}) = -\frac{1}{\rho \left(1 - \frac{1}{\eta}\right)} + \xi^{-\frac{1}{\eta}} \frac{(k - \gamma \bar{K})^{1 - \frac{1}{\eta}}}{1 - \frac{1}{\eta}} .$$

**Proof** See the Appendix.□

Corollary 2 characterizes the role of the decision rules implied by Proposition 6.

**Corollary 2**

*Under Assumption 2, the solution to the problem expressed by the HJB equation given by (13) implies that the saving rate, the portfolio composition, the coefficient of variation of personal consumption, and the variance of the growth rate of consumption is the same across richer and poorer individuals.*

**Proof** Immediate after noticing that the dynamics of financial wealth of any household follows a geometric Brownian motion over time with the same coefficients for all households, irrespective of initial conditions.  $\square$

In the absence of idiosyncratic labor-income shocks all households are subject to the same aggregate shocks to stock-market-index returns, and inequality in the distribution of relative wealth will be increasing over time. Moreover, in our analysis these aggregate shocks follow a random walk, and decision makers take into account that each shock realization has a permanent effect on wealth accumulation.

The results of Propositions 3 through 5 point out that all empirical regularities (personal saving rates, risky-asset portfolio shares, and consumption volatility, all being increasing in wealth, in addition to having EIS increasing in wealth) are qualitatively consistent with the model's mechanics. Through the aid of Corollary 1 above we have explained that, since  $\chi$ -type subsistence needs stay constant over time, a poor household can escape the pressure of survival in the future, through growing wealth. For this reason, a poor with  $\chi$ -type subsistence needs can afford to have lower saving rates and portfolios with lower expected returns, choosing to escape poverty and subsistence consumption at a slower pace. On the contrary, in the case of  $\gamma\bar{C}(t)$ -type agents, subsistence needs grow over time. This means that the household cannot foresee escape from pressing subsistence needs, so it cannot afford to save at lower saving rates or to choose less risky portfolios with lower expected returns than others. Instead, a poor household with  $\gamma\bar{C}(t)$ -type subsistence needs must catch up with the others.

The exceptionally sharp result of Corollary 2, that the saving rates and portfolio shares of rich and poor are exactly the same, shows that the whole community follows parallel wealth trajectories. So, despite that EIS is always different across households, instilling the average

consumption trajectory into an investor’s expectations in a way that this can be parallel to her own consumption trajectory, creates a rebalancing effect: the equilibrium trajectory is the same as if preferences were homothetic, coinciding with the case where the external habit is eliminated, i.e. setting  $\gamma = 0$ .<sup>20</sup>

Another source contributing to the sharp result of Corollary 2 is the aggregation property which is inherent in the Stone-Geary formulation. Previous papers on aggregation such as Chatterjee (1994) and Caselli and Ventura (2000) emphasize that time-invariant and time-variant subsistence consumption can lead to wealth-dependent saving rates. What we find here is that the “keeping-up-with-the-Joneses” formulation tends to rebalance saving rates and, as it has become clear by this study, it rebalances portfolio choice among the rich and the poor as well. This happens because the benchmark subsistence consumption is subject to aggregation as well (it is proportional to the consumption level of the “representative consumer” – see Caselli and Ventura (2000) or Koulovatianos (2005) for a definition of the concept). Yet, so far it has not been obvious that this feature can survive under portfolio choice, which is a key contribution of this paper.<sup>21</sup>

#### 4. Calibration Exercise

Our targets in this section are three. First, we check whether our model is capable of matching observed stockholding shares across income classes based on data from the Survey of Consumer Finances (SCF year 2007, focusing on households that hold stocks only). Second, we examine whether the model’s implied saving rates are consistent with measured values

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<sup>20</sup>This coincidence of results can be verified by the fact that parameter  $\gamma$  does not appear in any of the decision rules of Proposition 6, but only in the value function.

<sup>21</sup>Nevertheless, whenever a time-variant level of benchmark consumption is exogenous, the knife-edge result of Corollary 2 generally fails (on this see Caselli and Ventura (2000) and Koulovatianos (2005, Theorem 3) – the latter also provides necessary conditions for the aggregation result with time-variant subsistence consumption).

reported by Dynan, Skinner, and Zeldes (2004). Third, after inserting income/wealth data in the model, we check whether the implied levels of the elasticity of intertemporal substitution are broadly consistent with those estimated across income classes and used in the finance literature. The most crucial aspect of our quantitative exercise is to check whether the above three targets can be met simultaneously using the same utility function for all (rich and poor) stockholding households.

The spirit of our calibration exercise is to challenge the analytical formulas we have provided for the case of time-invariant subsistence needs. Most of our benchmark calibrating parameters are standard in a broad portfolio-choice literature and appear in Table 1.<sup>22</sup> Yet, there are two issues requiring special attention. First, unlike typical full-fledged simulated models in the literature, our definition of resources available to a household is inflexible, as it does not explicitly distinguish between asset holdings and labor income. In order to make the definition of our model’s resource variable “ $k$ ” consistent with observed wealth/labor-income data and also with notions of stockholding shares, we define  $k$  as the sum of wealth and life-time income in the data. Second, generally agreed estimates of the time-invariant subsistence consumption parameter  $\chi$  may be difficult to defend, and it may be argued that results are quantitatively sensitive to the choice of  $\chi$ .<sup>23</sup> We treat these two issues of  $k$  and  $\chi$  separately before we move to explaining our benchmark results and our sensitivity analysis.

#### 4.1 Definition and calibration of variable “ $k$ ”

We construct  $k$  based on available wealth/labor-income SCF data for different income classes.

To be precise, a richer version of the budget constraint given by (2), would distinguish

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<sup>22</sup>Below we provide some more details about the choice of each parameter appearing in Table 1.

<sup>23</sup>We thank Dirk Krueger for raising this point to us.



between assets,  $a$ , and labor income,  $y$ , having, for example, the form,

$$da(t) = \{[\phi(t)R + (1 - \phi(t))r^f]a(t) + y(t) - c(t)\}dt + \sigma\phi(t)a(t)dz_a(t) , \quad (16)$$

and

$$dy(t) = \nu(y, t)dt + \theta(y, t)dz_y(t) , \quad (17)$$

where  $z_y(t) = \rho_{a,y}z_a(t) + \sqrt{1 - \rho_{a,y}^2}z(t)$ , with  $z(t)$  being a standard Brownian motion independent of  $z_a(t)$  ( $z_a(t)$  is also a standard Brownian motion) and with  $\rho_{a,y} \in (-1, 1)$  denoting the correlation coefficient between asset returns and the income process.<sup>24</sup> Multiplying (16) by an appropriate integrating factor, applying the conditional expectations operator at time 0 on both sides, solving the resulting equation forward, while applying the transversality condition, gives

$$EPVC = a_0 + EPVY \quad (18)$$

with

$$EPVC \equiv \int_0^\infty E_0 \left[ e^{-\int_0^t r(\tau)d\tau} c(t) \right] dt, \quad EPVY \equiv \int_0^\infty E_0 \left[ e^{-\int_0^t r(\tau)d\tau} y(t) \right] dt ,$$

where  $y(t)$  solves equation (17),  $r(t) \equiv \phi(t)R + (1 - \phi(t))r^f$  is the return to investment subject to portfolio choice, and “ $EPVC$ ” and “ $EPVY$ ” stand for expected present value of consumption and income. So, our definition of  $k$  should be perceived as a broad measure of resources, namely  $k_0 = a_0 + EPVY$ .<sup>25</sup> In order to calculate  $EPVY$  in our calibration exercise

<sup>24</sup>Such a general formulation has been analyzed by Henderson (2005, p. 1241), while more specific versions of equation (17) have been studied by Duffie et al. (1997, p. 755) and Koo (1998).

<sup>25</sup>To cross-check the agreement between equation (18) and the interpretation  $k_0 = a_0 + EPVY$  for addressing our model, solving equation (2), and taking conditional expectations gives

$$k_0 = \int_0^\infty E_0 \left[ e^{-\int_0^t r(\tau)d\tau} c(t) \right] dt \equiv EPVC \quad (19)$$

So, equation (19) and the constraint given by Assumption 1 which is necessary for meeting the transversality condition now becomes,

$$EPVC > \frac{\chi}{r^f} , \quad (20)$$

we discount life-time earnings according to interest rates implied by observed portfolio choices of each income class.

In Table 2, which focuses on households holding stocks only, column (1) presents data for family net worth “ $a$ ” (SCF data from year 2007, with amounts expressed in 2007 US dollars), while column (4) gives average after-tax/transfer annual household incomes for each different income category of stockholders.<sup>26</sup> In order to derive a proxy for  $EPVY$ , the entries of column (4) in Table 2 are discounted using  $r_y \equiv \phi_y R + (1 - \phi_y) r^f$ , where  $\phi_y$  denotes the entries of column (5) in Table 2 (subscripts “ $y$ ” denote income-class dependent variables), while  $R$  and  $r^f$  are set to the calibrated values we show in Table 1. Following Wachter and Yogo (2010),  $\phi_y$  is defined as the fraction of net worth ( $a_y$ ) held in the form of stocks and other equity (so,  $\phi_y \equiv (\text{value of stocks \& other equity held}) / a_y$ ). The subcategory “other equity” includes three SCF variables, “other residential property”, “equity in nonresidential property”, and “business equity”, and in our calibration exercise we assume that “other equity” is as risky as stocks.<sup>27</sup> Entries in column (7) of Table 2 are the data values for  $\Phi(k)$

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which means that the present value of expected lifetime consumption should exceed the present value of lifetime subsistence consumption discounted by the risk-free rate. Equation (18) implies that the requirement given by (20) becomes

$$EPVC = a_0 + EPVY > \frac{\chi}{r^f},$$

and from

$$a_0 > - \left( EPVY - \frac{\chi}{r^f} \right). \quad (21)$$

The expression  $EPVY - \chi/r^f$  can be seen as the present value of lifetime discretionary income. As long as the present value of lifetime discretionary income is strictly positive, financial wealth (net worth of assets),  $a$ , in (21) can be negative, as it is the case for some households in SCF data. In more descriptive models using explicit exogenous income process and liquidity constraints, technical concerns for retaining a well-defined objective function of households may arise in addition to keeping inequality (21). We thank Chris Carroll for pointing out this technical issue to us.

<sup>26</sup>Estimates of income taxes and transfers that lead to the effective marginal tax rates appearing in column (3) of Table 2, are based on work by Grant et al. (2010, Table 2 therein).

<sup>27</sup>Values about fractions of “other equity” held by different income and age groups that are reported in published SCF tables suffer from aggregation biases that may mask the true stockholding pattern across the population. Subcategories of “other equity” such as business equity or residential equity are vulnerable to age effects, due to age-related constraints such as credit-history requirements for undertaking such investments, or fixed initial investment costs. On the contrary, such age-related participation constraints to stock markets are weaker. To overcome such aggregation problems we have projected components of “other equity” using

that our calibration exercise targets to match.

## 4.2 Calibrating subsistence consumption

One key empirical question is how to estimate the time-invariant component of subsistence consumption levels. Some recent work gives strong empirical support for the existence of subsistence consumption in utility functions.<sup>28</sup> So, in our exercise we calibrate  $\chi$  using subsistence estimates from the survey data appearing in Koulovatianos et al. (2007, Table 4) regarding Germany, France, Cyprus, India, China, and Botswana.<sup>29</sup> Specifically, these estimates suggest monthly subsistence costs per person between 111 and 302 US dollars of year 2004 across different countries and family types. Since we match savings and stockholding data evaluated in 2007 US dollars, we pick the value of USD 230 per month as benchmark.<sup>30</sup>

This amount implies that the annual subsistence cost per person is USD 2,760, while we perform sensitivity analysis by letting per-person subsistence be as low as USD 150 per month (USD 1,800 per person and per year) and as high as USD 300 per month (USD 3,600 per

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the working hypothesis that, approximately, behavior regarding stockholding by each income group may be similar to this for holding other risky assets *during a household's whole lifetime*. For all details about the derivation of the entries appearing in column (5) of Table 2, see Table A2 in the Online Data Appendix. The pattern of values appearing in column (5) of Table 2 matches the pattern of risky-asset shares for the age group of 46-55 years during 1989-2004 (see Wachter and Yogo (2010, Table 8)), even after taking into account that our own group categorization is in income percentiles while in Wachter and Yogo (2010, Table 8)) group categorization is by net worth. This matching of portfolio shares between our column (5) of Table 2 and the age group of 46-55 years in Wachter and Yogo (2010, Table 8) indicates that our pattern-projection strategy of aggregating these shares across the US population is effective.

<sup>28</sup>In particular, Donaldson and Pendakur (2006) identify family-type subsistence consumption using a restriction on equivalence scales, called “Generalized Absolute Equivalence Scale Exactness (GAESE)” through demand-system analysis. Koulovatianos et al. (2007, 2008) propose a survey approach for identifying subsistence consumption and also for testing GAESE. Their work gives strong support to GAESE, finding also that in six countries and 49 cases they examine subsistence consumption is always present playing an important role in comparisons of material comfort among individuals living in different family types.

<sup>29</sup>The estimated subsistence levels for Germany based on Koulovatianos et al. (2007, Table 4) obtained in year 1999 are very close to the German estimates for year 2006 (see this by comparing family-type regression estimates by Koulovatianos et al. (2008, Table 1) with Koulovatianos et al. (2007, Table 4)). Moreover, Canadian subsistence consumption estimates by Donaldson and Pendakur (2006, p. 262, Table 4) are not far from these appearing in Koulovatianos et al. (2007, Table 4).

<sup>30</sup>In what follows, we use “USD” in order to indicate 2007 US Dollars, unless we indicate otherwise.

person and per year).<sup>31</sup>

In Table 1 we present our benchmark value for  $\chi$ . Since our SCF data refer to resources measured at the household level, we take into account the average household size in the US during the period under examination. Based on Luxembourg Income Study data, during the period 1986-2004 the average US household size was 2.54 persons.<sup>32</sup> So, the value for  $\chi$  we use in our calibration exercise is  $2.5 \times$  (annual subsistence needs per person).

### 4.3 Calibration results

In Table 1 we present our benchmark calibration parameters. We set  $R$ ,  $r^f$ , and  $\sigma$  to 7%, 3%, and 20% which are close to values implied by data (see, for example, Guvenen (2009, Table II, p. 1725) where  $R \simeq 8\%$ ,  $r^f \simeq 2\%$ , and  $\sigma \simeq 20\%$  in the US), and close to values used in portfolio choice literature (e.g., Gomes and Michaelides (2003) choose values  $R = 6\%$ ,  $r^f = 2\%$ , and  $\sigma = 18\%$  as their benchmark parameter values). In brief, we stick to the 4% equity premium (instead of the historical 6%) that is standardized in the portfolio-choice literature, and we raise the risk-free rate to 3% in order to obtain more empirically plausible saving rates. Nevertheless,  $r^f$  can be set to 2% combined with  $R = 6\%$  and values for  $\rho$  and  $\eta$  can be set within an admissible calibration range in order to match portfolio choice targets or saving rate targets in isolation. We have chosen these calibration parameter values in order to meet three targets simultaneously: (i) portfolio choice shares, (ii) saving rates, and (iii) plausible elasticities of intertemporal substitution. In order to jointly meet these three objectives we also choose  $\eta = 0.23$  and  $\rho = 2.5\%$ , that best serve these multiple goals of our exercise. The spirit of our calibration exercise is not to resolve

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<sup>31</sup>Using  $r^f = 3\%$  in what follows, the assumption that subsistence costs equal USD 230 per month implies lifetime subsistence costs USD 92,000 per-person (infinitely-lived person), while monthly USD 150 implies lifetime subsistence costs of USD 60,000. On the other extreme, monthly needs of USD 300 per person mean (infinite-) lifetime subsistence needs of USD 120,000.

<sup>32</sup>See Table A1 in the Online Data Appendix for details.

all quantitative questions in the literature, but to make an essential demonstration: that our model does not tend to set restrictive quantitative bounds against meeting multiple data targets due to parametric constraints that jeopardize existence of equilibrium or due to other vital technical issues. Our main purpose is to show that adding time-invariant subsistence consumption to a utility function can serve as a promising modeling ingredient for quantitative explorations through more descriptive simulated models. So, in what follows we do not perform a sensitivity analysis on our benchmark calibration values for  $R$ ,  $r^f$ ,  $\sigma$ ,  $\rho$ , and  $\eta$ . Instead, our sensitivity analysis focuses on varying values for the parameter that is most crucial in this paper: parameter  $\chi$ .

On the horizontal axis of both Figures 1 and 2 appear different income categories of stockholding households as defined in Table 2 (moving from the left to the right of the horizontal axis household income and lifetime resources increase). The solid line and the dashed line of Figure 1 plot the data and benchmark calibration values for  $\Phi(k_y)$ , where the values are taken from columns (7) and (8) of Table 2. So, Figure 1 shows that the model, similarly to the data, implies that portfolio shares increase from about 6% to 22% as lifetime resources increase. At the wealth levels of the richest,  $\Phi(k_y)$  converges to the model's upper bound of risky-asset holdings (which equals  $\eta(R - r^f) / \sigma^2$  as  $k \rightarrow \infty$ ) for all values of  $\chi$  we examine. On the contrary, varying subsistence parameter  $\chi$ , influences the choices  $\Phi(k)$  for the poorest. Yet, given the wide range of values for  $\chi$  we have used, the sensitivity of  $\Phi(k)$  to varying subsistence needs is not high.

While Figure 1 does not reveal an accurate goodness of fit of our simple model to the data, it nevertheless shows that our model is successful at capturing the observed range of difference between holdings of risky assets across the rich and the poor. So, since this simple framework does not exhibit limitations in generating sufficient quantitative differences in

portfolio shares, we think that the concave shape of function  $\Phi(k)$  can be smoother by adding other real-world characteristics to a richer model, such as separating stocks from other risky assets, housing-value risks, etc.

At the same time, Figure 2 plots the model's implications for the saving rates across the rich and the poor (see equation (8)), with values robustly well within the range of estimates provided by Dynan et al. (2004, Figure 1, p. 419). Finally, Table 3 shows the model's implied coefficients of relative risk aversion which fall well within the range of values used in finance literature that considers consumption risk as its building block, such as Malloy et al. (2009), and also matches empirical estimates such as those reported by Barsky et al. (1997).

We believe this simple exercise shows that introducing time-invariant subsistence to a single utility function does not imply prohibitive quantitative bounds for addressing multiple quantitative targets. In this sense we think that time-invariant consumption shows promise for extensions to more descriptive simulated models.

## 5. Concluding remarks

Many researchers who work on explaining stockholding patterns and who want to simultaneously meet empirical regularities of saving and consumption tend to dislike the idea of assuming different preferences between rich and poor households. We have investigated whether it is possible to use a common utility function across the rich and the poor in order to jointly explain three stylized facts, namely that the rich have: (i) higher saving rates, (ii) a larger fraction of their wealth held in stocks, and (iii) higher consumption-growth volatility. Since it is broadly accepted as a stylized fact that the rich exhibit higher EIS, it seems promising to achieve the above goal through assuming a utility function with subsistence

consumption.

Here, we have distinguished and studied two types of subsistence consumption introduced into Stone-Geary preferences: (a) a time-invariant level of subsistence consumption, and (b) a time-variant level of subsistence consumption,  $\gamma\bar{C}(t)$  ( $\bar{C}(t)$  is average consumption in a community at time  $t$ , and  $\gamma > 0$ ), with the latter formulation capturing the external-habit idea of “keeping-up-with-the-Joneses” preferences. We have analytically studied savings and portfolio choice in the simple two-asset Merton (1969, 1971) model.

In the case with time-invariant subsistence consumption, even this simple framework appears capable of quantitatively addressing all three stylized facts above. Through a simple calibration exercise we found that plausible parameter values jointly imply portfolio shares ranging from 6% to 22% for poor vs. rich stockholders (matching SCF data between 1989-2007), saving rates ranging from 6% to 17% (poor vs. rich), consistently with values estimated by Dynan et al. (2004), and coefficients of relative risk aversion ranging from 14 to 4.5 (poor vs. rich, i.e., not far from estimates reported by Malloy et al. (2009)). These results are robust to varying time-invariant subsistence needs within an empirically plausible range.

On the contrary, Stone-Geary preferences with time-variant subsistence needs of the form  $\gamma\bar{C}(t)$  imply that saving rates and portfolio shares are exactly the same across the rich and the poor. This extreme result occurs despite that EIS is higher for the rich in equilibrium for this formulation, too.

The difference between the two formulations is explained through a key feature of the setup with time-invariant subsistence: a loose parametric restriction implies that the saving rate is strictly increasing in wealth if, and only if, expected wealth exhibits a strictly positive growth rate (see Corollary 1). This means that a poor investor with wealth close to lifetime subsistence needs, foresees a wealth path that can take her far from subsistence consumption

in the future. This foreseen potential for alleviation from the subsistence low bound makes the investor afford to have a lower saving rate when poor in the beginning, and a temporarily slower, less painful wealth accumulation profile. As part of the same plan, a poorer investor also chooses a less risky portfolio with lower expected returns. As we have demonstrated, these lifetime resource-allocation plans are reflected on current decisions in a quantitatively important way.

Unlike the investor with  $\chi$ -type subsistence needs, the needs of the  $\gamma\bar{C}(t)$ -subsistence-type investor grow over time. This means that the external-habit investor cannot foresee any potential for alleviation from the subsistence low bound, which means that she cannot afford to save at a lower rate or to invest in a portfolio with lower expected return. The investor who has this external habit cannot afford to escape poverty in a slow and least painful way. Instead, she feels she must accumulate wealth at the fast pace of the richer (catching up). This knife-edge result of everyone saving at the same rate originates from exact aggregation properties instilled by time-variant Stone-Geary preferences (see Koulovatianos (2005, Theorem 3)).

It is interesting to introduce Stone-Geary preferences with subsistence consumption in a portfolio-choice model with liquidity constraints and (imperfectly correlated) income- and risky-asset shocks. Through an analysis similar to this of Haliassos and Michaelides (2003), it is an open question whether such preferences are capable of quantitatively explaining the empirically observed saving and portfolio dependence on wealth despite the liquidity constraints. It is also interesting to examine whether such preferences can explain stockholding puzzles (in our present analysis all investors have been stockholders). Another open question is about the role of the key mechanism that we have identified, i.e., that it is the growth potential of wealth that makes  $\chi$ -type subsistence investors accumulate wealth so slowly



compared to the  $\gamma\bar{C}(t)$ -type investors. It is interesting to examine whether this mechanism is robust to introducing idiosyncratic income risk in a stationary-equilibrium setup via the potential for wealth mobility, or to introducing life-cycle characteristics to households. For pursuing such extensions, we believe our analysis allows us to point at a direction: using time-invariant preferences seems more promising than using subsistence consumption of the form  $\gamma\bar{C}(t)$ . If anything, our analysis suggests a formulation where subsistence has at least a time-invariant component, such as the form  $\chi + \gamma\bar{C}(t)$ .

## 6. Appendix - Proofs

### Proof of Proposition 1

We make a guess on the functional form of the value function, namely,

$$J(k) = a + b \frac{(k - \omega)^{1 - \frac{1}{\eta}}}{1 - \frac{1}{\eta}}, \quad (22)$$

which implies,

$$J'(k) = b(k - \omega)^{-\frac{1}{\eta}}, \quad (23)$$

and

$$J''(k) = -\frac{1}{\eta} b(k - \omega)^{-\frac{1}{\eta} - 1}. \quad (24)$$

From (23) and (4) it is,

$$c = b^{-\eta} k + \chi - b^{-\eta} \omega. \quad (25)$$

Moreover, substituting (23) and (24) into (5) gives,

$$\phi = \eta \frac{R - r^f}{\sigma^2} \left(1 - \frac{\omega}{k}\right). \quad (26)$$

Substituting (22), (25), (23), (26), and (24) into the HJB given by (3) becomes,

$$\begin{aligned} \rho a + \rho b \frac{(k - \omega)^{1 - \frac{1}{\eta}}}{1 - \frac{1}{\eta}} &= \frac{b^{1 - \eta} (k - \omega)^{1 - \frac{1}{\eta}}}{1 - \frac{1}{\eta}} - \frac{1}{1 - \frac{1}{\eta}} + \\ &+ b(k - \omega)^{-\frac{1}{\eta}} \left[ \eta \frac{(R - r^f)^2}{\sigma^2} \left(1 - \frac{\omega}{k}\right) k + r^f k - b^{-\eta} k - \chi + b^{-\eta} \omega \right] - \\ &- \frac{1}{2\eta} \left( \eta \frac{R - r^f}{\sigma} \right)^2 b(k - \omega)^{1 - \frac{1}{\eta}} \end{aligned} \quad (27)$$

Setting

$$a = -\frac{1}{\rho \left(1 - \frac{1}{\eta}\right)}, \quad (28)$$

dividing both sides of (27) by  $b(k - \omega)^{1 - \frac{1}{\eta}}$  and re-arranging terms, it is,

$$\begin{aligned} \left[ \frac{\rho - b^{-\eta}}{1 - \frac{1}{\eta}} - \frac{\eta}{2} \left( \frac{R - r^f}{\sigma} \right)^2 \right] k - \left[ \frac{\rho - b^{-\eta}}{1 - \frac{1}{\eta}} - \frac{\eta}{2} \left( \frac{R - r^f}{\sigma} \right)^2 \right] \omega = \\ = (r^f - b^{-\eta}) k + b^{-\eta} \omega - \chi. \end{aligned} \quad (29)$$

In order that the guess we made for  $J(k)$  be operative, it must be that we can find  $b$  and  $\omega$  such that both the coefficient of  $k$  in equation (29) and the constant part must both be equal to zero. So, (29) implies

$$\frac{\rho - b^{-\eta}}{1 - \frac{1}{\eta}} - \frac{\eta}{2} \left( \frac{R - r^f}{\sigma} \right)^2 = r^f - b^{-\eta}, \quad (30)$$

which leads to,

$$b^{-\eta} = \rho\eta + (1 - \eta)r^f - \frac{\eta(\eta - 1)}{2} \left( \frac{R - r^f}{\sigma} \right)^2. \quad (31)$$

Moreover, the constant terms of (29) should sum up to zero, so

$$\left[ \frac{\rho - b^{-\eta}}{1 - \frac{1}{\eta}} - \frac{\eta}{2} \left( \frac{R - r^f}{\sigma} \right)^2 \right] \omega = \chi - b^{-\eta} \omega$$

and combining it with (30) becomes,

$$(r^f - b^{-\eta}) \omega = \chi - b^{-\eta} \omega,$$

or

$$\omega = \frac{\chi}{r^f}. \quad (32)$$

So, after substituting (32) into (25), the decision rule for consumption becomes,

$$c = b^{-\eta} k + \chi \left( 1 - \frac{b^{-\eta}}{r^f} \right)$$

and after substituting (31) it is,

$$c = \left[ \rho\eta + (1 - \eta)r^f - \frac{\eta(\eta - 1)}{2} \left( \frac{R - r^f}{\sigma} \right)^2 \right] k + \eta\chi \frac{r^f - \rho + \frac{\eta - 1}{2} \left( \frac{R - r^f}{\sigma} \right)^2}{r^f}, \quad (33)$$

confirming the statement of the Proposition. It remains to verify that Assumption 2 guarantees that  $\xi > 0$ . From (33) the requirement that  $\xi > 0$  implies,

$$\eta^2 - \left[ 1 - \frac{r^f - \rho}{\frac{1}{2} \left( \frac{R-r^f}{\sigma} \right)^2} \right] \eta - \frac{r^f}{\frac{1}{2} \left( \frac{R-r^f}{\sigma} \right)^2} < 0. \quad (34)$$

The discriminant of the quadratic polynomial with respect to  $\eta$  given by (34) is

$$\left[ \frac{r^f - \rho}{\frac{1}{2} \left( \frac{R-r^f}{\sigma} \right)^2} \right]^2 + 2 \frac{r^f + \rho}{\frac{1}{2} \left( \frac{R-r^f}{\sigma} \right)^2} + 1 > 0,$$

which means that real roots exist, while the constant term of (34) reveals that both roots are different from zero and that they have opposite signs. Since there is a parametric restriction that  $\eta > 0$ , the formula for  $\bar{\eta}$  appearing in the statement of the proposition is the positive root of the quadratic polynomial with respect to  $\eta$  given by (34).  $\square$

### Proof of Proposition 2

Direct substitution of the decision rule for consumption,  $C(k)$ , and also for portfolio choice,  $\Phi(k)$ , into the household's budget constraint given by (2), after some algebra, gives,

$$dk = \eta \left[ \frac{\eta + 1}{2} \left( \frac{R - r^f}{\sigma} \right)^2 + r^f - \rho \right] \left( k - \frac{\chi}{r^f} \right) dt + \eta \left( \frac{R - r^f}{\sigma} \right) \left( k - \frac{\chi}{r^f} \right) dz \quad (35)$$

Applying Itô's lemma on (35) yields,

$$d \ln \left( k - \frac{\chi}{r^f} \right) = \eta \left[ \frac{1}{2} \left( \frac{R - r^f}{\sigma} \right)^2 + r^f - \rho \right] dt + \eta \left( \frac{R - r^f}{\sigma} \right) dz \quad (36)$$

and after integrating (36) using Itô's stochastic integral (while setting  $z(0) = 0$  by convention), proves the proposition.  $\square$

## Proof of Proposition 5

Fix any  $t > 0$ . From Proposition 1 it is  $C(k^*(t)) = \xi k^*(t) + \psi$ . From Proposition 2 we can see that

$$E[C(k^*(t))] = \xi E[\nu(t)] \left(k_0 - \frac{\chi}{r^f}\right) + \xi \frac{\chi}{r^f} + \psi \quad (37)$$

where

$$\nu(t) \equiv e^{\eta \left[ r^f - \rho + \frac{1}{2} \left( \frac{R-r^f}{\sigma} \right)^2 \right] t + \eta \left( \frac{R-r^f}{\sigma} \right) z(t)} .$$

Since  $\psi = \chi(1 - \xi/r^f)$ , (37) implies,

$$E[C(k^*(t))] = \xi E[\nu(t)] \left(k_0 - \frac{\chi}{r^f}\right) + \chi . \quad (38)$$

So, apparently, the coefficient of variation is,

$$CoeffVar(C(k^*(t))) = \frac{\xi \{Var[\nu(t)]\}^{\frac{1}{2}}}{\xi E[\nu(t)] + \frac{\chi}{k_0 - \frac{\chi}{r^f}}}$$

and since both  $E[\nu(t)]$  and  $\{Var[\nu(t)]\}^{1/2}$  are strictly positive,<sup>33</sup> this last equation implies that  $dCoeffVar(C(k^*(t)))/dk_0 > 0$ . Regarding the volatility of consumption growth, direct substitution of the decision rule for consumption,  $C(k)$ , and also for portfolio choice,  $\Phi(k)$ , into the household's budget constraint given by (2), after some algebra, gives,

$$dk = \theta \left(k - \frac{\chi}{r^f}\right) dt + \eta \left(\frac{R-r^f}{\sigma}\right) \left(k - \frac{\chi}{r^f}\right) dz \quad (39)$$

where

$$\theta = \eta \left[ \frac{\eta + 1}{2} \left(\frac{R-r^f}{\sigma}\right)^2 + r^f - \rho \right] .$$

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<sup>33</sup>Notice that

$$E[\nu(t)] = e^{\eta \left[ r^f - \rho + \frac{1}{2} \left( \frac{R-r^f}{\sigma} \right)^2 \right] t} ,$$

and

$$Var[\nu(t)] = e^{2\eta \left[ r^f - \rho + \frac{1}{2} \left( \frac{R-r^f}{\sigma} \right)^2 \right] t} \left\{ e^{\eta^2 \left( \frac{R-r^f}{\sigma} \right)^2 t} - 1 \right\} .$$

Using  $C(k^*(t)) = \xi k^*(t) + \psi = \xi [k^*(t) - \chi/r^f] + \chi$ , and applying Itô's lemma on (39) gives,

$$d \ln [C(k^*)] = \left\{ \xi \theta \frac{\hat{k}^*}{\xi \hat{k}^* + \chi} - \frac{1}{2} \left[ \xi \eta \left( \frac{R - r^f}{\sigma} \right) \right]^2 \left( \frac{\hat{k}^*}{\xi \hat{k}^* + \chi} \right)^2 \right\} dt + \xi \eta \left( \frac{R - r^f}{\sigma} \right) \frac{\hat{k}^*}{\xi \hat{k}^* + \chi} dz$$

where  $\hat{k}^* = k^* - \chi/r^f$ , and which implies that

$$\text{Var} (d \ln [C(k^*)]) = \left[ \xi \eta \left( \frac{R - r^f}{\sigma} \right) \right]^2 \left( \frac{\hat{k}^*}{\xi \hat{k}^* + \chi} \right)^2 dt ,$$

an increasing function of  $k^*$ .  $\square$

### Proof of Proposition 6

Let's assume that, indeed, such interior solutions are guaranteed. Our guess for the value function is,

$$J(k, \bar{K}) = a + b \frac{(k - \gamma \bar{K})^{1 - \frac{1}{\eta}}}{1 - \frac{1}{\eta}} . \quad (40)$$

So, (40) implies,

$$J_k(k, \bar{K}) = b (k - \gamma \bar{K})^{-\frac{1}{\eta}} , \quad (41)$$

$$J_{kk}(k, \bar{K}) = -\frac{1}{\eta} b (k - \gamma \bar{K})^{-\frac{1}{\eta} - 1} , \quad (42)$$

$$J_{\bar{K}}(k, \bar{K}) = -\gamma b (k - \gamma \bar{K})^{-\frac{1}{\eta}} , \quad (43)$$

$$J_{\bar{K}\bar{K}}(k, \bar{K}) = -\frac{\gamma^2}{\eta} b (k - \gamma \bar{K})^{-\frac{1}{\eta} - 1} , \quad (44)$$

$$J_{k\bar{K}}(k, \bar{K}) = \frac{\gamma}{\eta} b (k - \gamma \bar{K})^{-\frac{1}{\eta} - 1} . \quad (45)$$

Combining (14) with (41) gives,

$$c = b^{-\eta} (k - \gamma \bar{K}) + \gamma \bar{C} , \quad (46)$$

while combining (15) with (41), (42) and (45) implies,

$$\phi = \eta \frac{R - r^f}{\sigma^2} \left( 1 - \gamma \frac{\bar{K}}{k} \right) + \bar{\Phi} \gamma \frac{\bar{K}}{k} . \quad (47)$$

When equations (46) and (47) are substituted into the individual's budget constraint, equation (11), the result is,

$$\begin{aligned} dk = \left\{ \left[ \eta \left( \frac{R - r^f}{\sigma} \right)^2 - b^{-\eta} + r^f \right] k + \gamma \left[ b^{-\eta} + (R - r^f) \bar{\Phi} - \eta \left( \frac{R - r^f}{\sigma} \right)^2 \right] \bar{K} - \gamma \bar{C} \right\} dt \\ + \left[ \eta \frac{R - r^f}{\sigma} k + \gamma \left( \sigma \bar{\Phi} - \eta \frac{R - r^f}{\sigma} \right) \bar{K} \right] dz . \end{aligned}$$

This last equation implies exact linear aggregation of the equilibrium law of motion of financial wealth among rich and poor. This means that the guess we have made is consistent with the way we have set up the problem, and it remains to see whether there exists a constant term  $b$  that validates the solution. Linear aggregation allows us to substitute for  $\bar{K}$  and  $\bar{\Phi}$  in equation (47) in order to characterize the portfolio choice of the household with average wealth. Doing so leads to,

$$\bar{\Phi} = \eta \frac{R - r^f}{\sigma^2} , \quad (48)$$

and substituting (48) into (47) implies

$$\phi = \bar{\Phi} = \eta \frac{R - r^f}{\sigma^2} , \quad \text{for all } k > 0 . \quad (49)$$

Moreover, by linearly aggregating equation (46) we obtain

$$c = b^{-\eta} k , \quad \text{for all } k > 0 , \quad (50)$$

which includes  $\bar{K}$ . Substituting equations (40) through (47) into the HJB equation given by (13), and imposing  $a = -1/[\rho(1 - 1/\eta)]$ , we arrive, after some algebra to,

$$\frac{\rho - \frac{1}{\eta} b^{-\eta}}{1 - \frac{1}{\eta}} - r^f = (R - r^f) \frac{\phi k - \gamma \bar{\Phi} \bar{K}}{k - \gamma \bar{K}} - \frac{\sigma^2}{2\eta} \left( \frac{\phi k - \gamma \bar{\Phi} \bar{K}}{k - \gamma \bar{K}} \right)^2 . \quad (51)$$

Substituting that  $\phi = \bar{\Phi}$  from (49) into (51) leads to the expression stated in the proposition. Given this interior solution, the role of Assumption 3 is reconfirmed in a straightforward manner.  $\square$



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$\eta$	$R$	$r^f$	$\sigma$	$\rho$	$\chi$
0.23	7%	3%	20%	2.50%	6900 <sup>a</sup>

**Table 1** Calibrating parameters.

<sup>a</sup> Annual subsistence cost per person in 2007 US Dollars. For our calculations we use the convention that the average US household has size 2.5 persons, an average calculated by broad coverage Luxembourg Income Study (LIS) data referring to years 1986-2004, which are closest to the data of the calibration exercise (see also Table A1 in the Online Data Appendix, and Koulovatianos et al. (2010, Figure 2)).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<i>Percentile of income</i>	Family Net Worth (Data) <sup>a</sup>	Before Tax family income (Data) <sup>a</sup>	Effective marginal tax rate (Data - %) <sup>b</sup>	After Tax family income (Data) <sup>c</sup>	Fraction (%) of net worth held in stocks/equity (Data) <sup>d</sup>	Total lifetime resources (Data) <sup>e</sup>	Fraction (%) of total resources held in stocks/equity (Data) <sup>f</sup>	Fraction (%) of total resources held in stocks (Model) <sup>g</sup>	Saving Rate (Model-%) <sup>g</sup>	EIS (Model) <sup>g</sup>
Less than 20	65.45	10.57	-1.83%	10.77	28.28	314.97	5.88	6.20	5.65	0.07
20–39.9	120.62	25.90	2.78%	25.18	30.68	692.67	5.34	15.36	12.58	0.16
40–59.9	175.12	43.88	6.47%	41.04	39.26	1045.62	6.58	17.94	14.28	0.18
60–79.9	284.03	69.86	14.28%	59.88	48.87	1464.84	9.48	19.39	15.20	0.20
80–89.9	449.24	104.97	22.63%	81.22	53.37	1999.15	11.99	20.35	15.80	0.21
90–100	2173.55	301.00	29.27%	212.90	62.56	5983.38	22.73	22.12	16.85	0.22

**Table 2** Key income-wealth distribution facts of stockholding households and benchmark calibration exercise (model with time-invariant subsistence costs).

<sup>a</sup> Amounts (in thousands of 2007 US Dollars) are averages of years 1989-2007 and are taken from the Survey of Consumer Finances (SCF, 2007).

<sup>b</sup> Calculations based on data taken from the Federation of Tax Administrators at 444 N. Capital Street, Washington DC, referring to year 2003. See also Grant et al. (2010, Table 2).

<sup>c</sup> Derived from columns (2) and (3) through direct application of the effective marginal tax rate on before-tax family income (in thousands of 2007 US Dollars).

<sup>d</sup> Data taken/projected from SCF in year 2007 (averages of years 1989-2007 – see Table A2 and its notes in the Online Data Appendix for details).

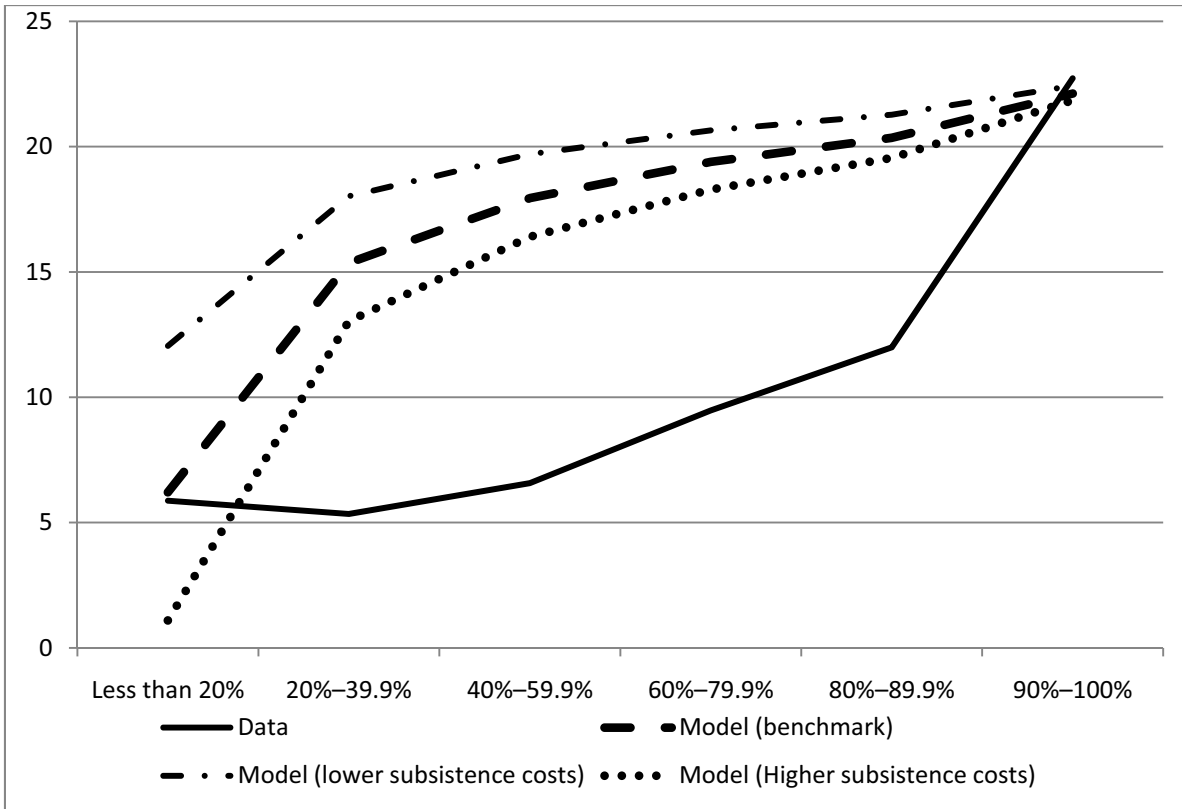
<sup>e</sup> Numbers appearing in column (6) correspond to variable “ $k$ ” in the model, and are measured (in thousands of 2007 US Dollars). Column (6) is constructed through adding column (1) to the present value of after tax income (see  $EPVY$  in the text) derived from annual incomes appearing in column (4). The discount factor to calculate  $EPVY$  uses a calibrated proxy of each income class’ effective interest rate, using  $\varphi R + (1-\varphi)r^f$ , where  $\varphi$  corresponds to the entries of column (5), while  $R$  and  $r^f$  are the calibrated values from Table 1. Family  $EPVY$  is calculated based on life duration of 78 years of an average family’s income (overlapping) earners.

<sup>f</sup> Entries in column (8) are the product of columns (1) and (5) divided by column (6) and correspond to  $\Phi(k)$  in the model (see the text for details).

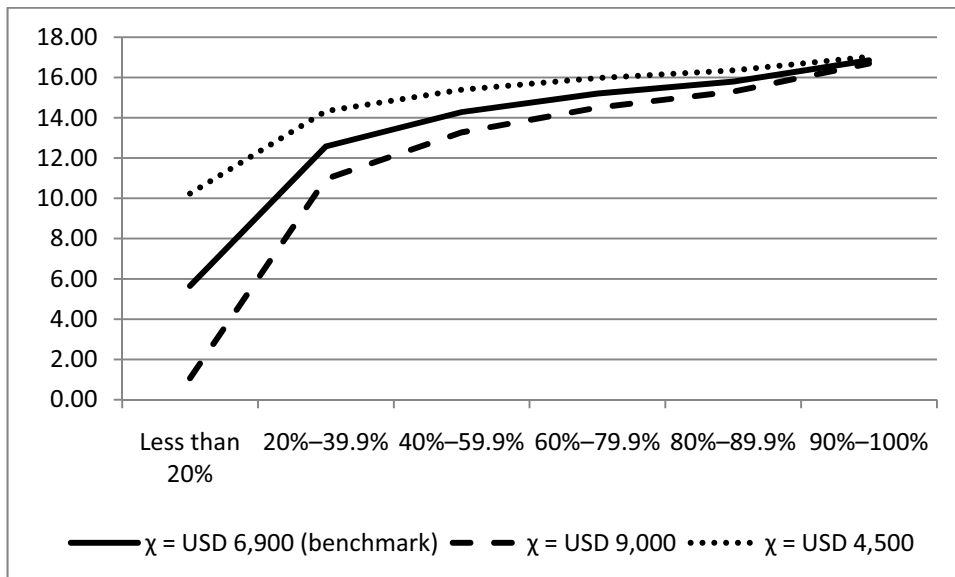
<sup>g</sup> Calculations are based on inserting values from column (6), which are proxies of variable “ $k$ ”, into formulas derived from the model of Section 2.

<i>Percentile of income</i>	$\chi = \text{USD } 6,000$ (benchmark)	$\chi = \text{USD } 9,000$	$\chi = \text{USD } 4,500$
<b>Less than 20%</b>	14.35	57.85	7.85
<b>20%–39.9%</b>	6.26	7.27	5.42
<b>40%–59.9%</b>	5.43	5.88	4.99
<b>60%–79.9%</b>	5.05	5.32	4.78
<b>80%–89.9%</b>	4.84	5.01	4.65
<b>90%–100%</b>	4.50	4.55	4.45

**Table 3** Model’s implied coefficients of relative risk aversion for different levels of subsistence consumption.



**Figure 1** Fraction (%) of total resources held in stocks (it corresponds to  $\Phi(k)$  in the model) plotted against different income categories (see income percentiles on the horizontal axis). Data Source is SCF in year 2007 (column (7) of Table 2). The model’s reported  $\Phi(k)$  for our benchmark calibration uses  $\chi = \text{USD } 6,900$  (column (8) of Table 2 – monthly subsistence cost per-person USD 230, base year 2007), the model’s  $\Phi(k)$  for “lower subsistence costs” uses  $\chi = \text{USD } 4,500$  (monthly subsistence cost per-person USD 150), while reported  $\Phi(k)$  for “higher subsistence costs” uses  $\chi = \text{USD } 9,000$  (monthly subsistence cost per-person USD 300).



**Figure 2** Model’s saving rates (%) for different levels of subsistence consumption.

# ONLINE DATA APPENDIX

for

## Saving Rates and Portfolio Choice with Subsistence Consumption

by

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### General Notes

All data appearing in Table A1 have been retrieved from the Luxembourg Income Study (LIS) database. In order to retrieve summary statistics from the LIS database, a Stata code can be run through email, after obtaining authorization and an access password from LIS.

Data appearing in Tables A3 through A6 are summary statistics obtained from the Survey of Consumer Finances (SCF) 2007 Chartbook. All manipulations of these summary statistics are explained in detail in the notes of Table A2 below.



	1986	1991	1994	1997	2000	2004	Average 1986-2004
<b>Number of family members</b>	2.55	2.60	2.45	2.58	2.55	2.53	2.54

**Table A1:** Average number of persons living together in the same household in the US in different years. Source: Luxembourg Income Study.

	(1)	(2)	(3)	(4)
<i>Percentile of income</i>	<b>Financial Assets as % of Net Worth<sup>a</sup></b>	<b>Stocks as % of Net Worth<sup>b</sup></b>	<b>Other equity as % of Net Worth<sup>c</sup></b>	<b>Stocks &amp; other equity as % of Net Worth<sup>d</sup></b>
<b>Less than 20</b>	35.54	8.78	19.49	28.28
<b>20–39.9</b>	37.30	9.85	20.83	30.68
<b>40–59.9</b>	39.08	13.01	26.26	39.26
<b>60–79.9</b>	43.94	17.49	31.39	48.87
<b>80–89.9</b>	44.56	19.27	34.10	53.37
<b>90–100</b>	43.70	22.31	40.26	62.56

**Table A2** Explanation of how the fraction of total resources held in stocks/equity (column (5) of Table 2 in the paper) is derived from SCF data.

<sup>a</sup> These percentages are derived by dividing the amounts of the last column of Table A3 by the amounts of the last column of Table A4.

<sup>b</sup> These percentages are derived by multiplying the percentages of column (1) of this table by the percentages of the last column of Table A5.

<sup>c</sup> Other equity includes three SCF variables, “other residential property”, “equity in nonresidential property”, and “business equity”. Data about “other equity components” that are immune from age aggregation biases are not readily available for each percentile of income from SCF tables. So, numbers appearing in column (3) of this table have been projected using the working hypothesis that, approximately, behavior regarding stockholding by each income group may be similar to this for holding other risky assets. So, percentages of the last column of Table A5 have been normalized so as to sum up to 100%, and have been multiplied by 28.72%, which is the percentage of “other equity” in total assets, as we have defined it above. In particular, we multiply the percentages of the last column of Table A6 that refer to the three equity categories “other residential property”, “equity in nonresidential property”, and “business equity”, by 63.8%. The value 63.8% is the average amount of nonfinancial assets as a share of total assets for years 1989-2007 (see the bottom number of the last column of Table A6). The outcome of this multiplication is 5.75% for “other residential property”, 5.29% for “equity in nonresidential property”, and 17.68% for “business equity”, which sum up to 28.72%.

<sup>d</sup> These are the numbers appearing in column (5) of Table 2 in the paper. They are derived by adding columns (2) and (3) of this table.

	1989	1992	1995	1998	2001	2004	2007	Average 1989-2007
<i>Percentile of income</i>								
<b>Less than 20</b>	19.30	14.90	21.50	23.90	28.20	25.40	29.60	<b>23.26</b>
<b>20–39.9</b>	37.90	32.10	43.90	52.80	52.10	46.80	49.30	<b>44.99</b>
<b>40–59.9</b>	50.90	50.70	57.70	66.80	94.40	79.70	78.80	<b>68.43</b>
<b>60–79.9</b>	78.80	74.60	93.20	124.20	175.70	161.60	165.50	<b>124.80</b>
<b>80–89.9</b>	114.60	127.20	164.40	215.50	275.00	266.90	237.60	<b>200.17</b>
<b>90–100</b>	584.00	554.50	664.90	984.30	1283.30	1209.40	1368.80	<b>949.89</b>

**Table A3:** Mean value of financial assets for families with holdings (in thousands of 2007 US dollars). Source: Survey of Consumer Finances (2007).

	1989	1992	1995	1998	2001	2004	2007	Average 1989-2007
<i>Percentile of income</i>								
<b>Less than 20</b>	39.8	48.2	60.3	63.1	62.3	78.5	105.9	<b>65.5</b>
<b>20–39.9</b>	109.6	93.8	108.5	124.4	138.2	135.3	134.5	<b>120.6</b>
<b>40–59.9</b>	165.2	146.8	137.2	160.1	191.0	215.0	210.6	<b>175.1</b>
<b>60–79.9</b>	219.9	202.2	216.0	259.9	345.2	372.5	372.6	<b>284.0</b>
<b>80–89.9</b>	360.0	329.9	349.6	419.7	529.9	541.5	614.2	<b>449.2</b>
<b>90–100</b>	1,619.3	1,394.5	1,487.0	1,976.3	2,644.3	2,788.0	3,305.6	<b>2,173.6</b>

**Table A4:** Family net worth, by selected characteristics of families, 1989–2007 surveys (in thousands of 2007 dollars). Source: Survey of Consumer Finances (2007).

	1989	1992	1995	1998	2001	2004	2007	Average 1989-2007
<i>Percentile of income</i>								
<b>Less than 20</b>	13.08	13.92	13.51	22.55	39.01	32.15	38.78	<b>24.71</b>
<b>20–39.9</b>	10.19	16.17	27.93	28.88	36.21	31.25	34.22	<b>26.41</b>
<b>40–59.9</b>	16.67	20.99	27.75	38.29	47.34	43.38	38.61	<b>33.29</b>
<b>60–79.9</b>	21.60	28.19	35.61	47.04	51.62	41.83	52.67	<b>39.80</b>
<b>80–89.9</b>	26.04	32.90	40.90	49.26	57.62	48.04	47.92	<b>43.24</b>
<b>90–100</b>	35.13	40.18	45.83	62.23	59.89	57.19	56.82	<b>51.04</b>

**Table A5:** Stock holdings as share of financial assets of different income groups (%). Source: Survey of Consumer Finances (2007).

	1989	1992	1995	1998	2001	2004	2007	Average 1989-2007
<i>Type of nonfinancial asset</i>								
<b>Vehicles</b>	5.6	5.7	7.1	6.5	6.0	5.1	4.5	<b>5.8</b>
<b>Primary residence</b>	45.7	47.0	47.3	47.0	47.3	50.3	48.3	<b>47.6</b>
<b>Other residential property</b>	8.1	8.5	8.2	8.7	8.2	10.2	11.2	<b>9.0</b>
<b>Equity in nonresidential property</b>	11.3	10.9	7.6	7.6	8.2	7.1	5.4	<b>8.3</b>
<b>Business equity</b>	27.1	26.4	27.7	28.6	28.8	25.8	29.6	<b>27.7</b>
<b>Other</b>	2.2	1.5	2.2	1.6	1.5	1.5	1.0	<b>1.6</b>
<b>Total</b>	100	100	100	100	100	100	100	100
<b>Nonfinancial assets as a share of total assets</b>	68.9	68.3	63.1	59.1	57.4	64.2	65.7	<b>63.8</b>

**Table A6:** Value of nonfinancial assets of all families, distributed by type of asset, 1989–2007 surveys (in %). Source: Survey of Consumer Finances (2007).

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