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**The Concept of Multidimensional Poverty:
Accounting for Dimensional Poverty**

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The Concept of Multidimensional Poverty: Accounting for Dimensional Poverty

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Abstract

Increasing efforts have recently been directed towards the question of how to incorporate the idea of a multidimensional poverty concept into traditional poverty measurement. In response, several suggestions have been made to derive different classes of multidimensional poverty measures. In this paper we focus on five axiomatically derived classes of multidimensional poverty measures. Each of these classes follow the unidimensional approach to progressively weight the respective distances to the threshold levels in order to account for poverty intensity.

In this paper we claim that this approach, though reasonable in a unidimensional setting, does not suffice in a multidimensional setting. An additional aspect of poverty intensity should be considered which we denote as dimensional poverty: the number of dimensions in which individuals are deprived. There exists no luminous explanation why a weighting scheme should account for one aspect of poverty intensity while at the same time ignoring the other one.

In this paper we introduce a multiple cutoff method to identify the poor which allows us to extend the five classes of poverty measures to include an additional weighting scheme in order to account for dimensional poverty. We find that the additional weight has no effect on the axiomatic basis of the classes of poverty measures other than a partial violation of the well-known subgroup decomposability axiom.

Keywords: Multidimensional Poverty, Multidimensional Poverty measures, Axiomatic Approach, Poverty Intensity

JEL-Codes: I32

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1 Poverty Analysis: An Introduction

Poverty reduction has become one of the main goals of development efforts. Examples are the adoption of the PRSP (Poverty Reduction Strategy Papers) approach and the Millennium Development Goals (MDGs) by the majority of international development organisations. With the target date 2015 of the MDGs approaching, interest in this topic is likely to be additionally stirred. However, a sound poverty reduction strategy obviously requires a sound analysis of poverty. Poverty analysis itself has always been a core issue of development economics. A lot of research has been dedicated towards this subject and with the persistent public interest in poverty reduction this trend is presumed to continue.

Obviously, the first issue every poverty analysis has to address is the question of how poverty should be defined and measured. This paper contributes to this core issue and proceeds as follows. After introducing the main approaches to the definition and measurement of poverty, chapter 1 shows that current measurement approaches that try to account for poverty intensity miss an important aspect. While utilising a weighting scheme to put greater emphasis on the shortfalls *within* a poverty dimension, current approaches ignore the relationship *between* poverty dimensions. We introduce a weighting scheme to make up for this failure. Having done so, the chapter concludes with an explanation why the rest of the paper focuses on one particular approach to poverty measurement – the axiomatic approach – which is discussed in detail in chapter 2. In particular, the chapter introduces the main axioms developed so far and discusses their advantages and disadvantages. Chapter 3 introduces five main classes of multidimensional poverty measures which have been derived by different combinations of the axioms presented in chapter 2. We extend each of these classes by our weighting scheme and discuss the effects the extension has on the axiomatic basis of the respective measures. Chapter 4 concludes.

1.1 Defining Poverty

The first issue every poverty analysis has to address is the question of how poverty should be defined. Over time, experts and academics suggested a variety of concurrent definitions for poverty and the discussion continues to be controversial. The difficulty is that the definition should be as inclusive as possible, i.e. comprising preferably every aspect of deprivation, but also lean enough to be applicable in poverty measurement. One main controversy is whether poverty should be defined as a unidimensional or as a multidimensional concept¹.

¹ There are of course many controversies, like for instance the discussion whether poverty should be defined in absolute or relative terms. However, since this paper contributes to the dimensional issue, we will in the following concentrate on providing a brief overview of the arguments concerning this specific issue.

It has long been acknowledged that poverty is indeed a multidimensional phenomenon. However, for a long time period insufficient income (or expenditure) was commonly accepted as an appropriate indicator to reflect the multidimensional character of poverty.

Recently, the unidimensional definition of poverty came increasingly under criticism. A higher budget surely improves an individual's ability to fulfil his/her basic needs. However, the underlying a priori restriction assigning a weight of one to a single dimension (i.e. income/expenditure) and zero weights to every other potential dimension of poverty has increasingly been questioned as being too severe a constraint. All the more since taking a unidimensional approach to poverty measurement inevitably leads to a loss of information on dimension-specific shortfalls which are completely neglected.

Additionally, an income-based definition of poverty presupposes that a market exists for all dimensions of poverty and that the respective prices reflect the utility weights all households assign to these dimensions. However markets, especially in developing countries, may be imperfect (e.g. many small-scale farmers lack access to formal credit markets due to insufficient collaterals) or even not exist at all – which surely is the case for public goods like, for example, immunization programmes, and also poverty dimensions like literacy, life expectancy, security, etc.

Empirical studies cast further doubts on a very close correlation between income (or expenditure) and basic needs. As Thorbecke (2008, p. 4) puts it: *“There is not guarantee that individuals with incomes at – or even above – the poverty line would actually allocate their incomes so as to purchase the minimum basic needs”*. Lipton and Ravallion (1995) for instance studied the relationship between nutrition and income. Their findings support the view that it is not the income level per se which is important for poverty measurement but rather how this income is spent. For instance, there exist examples of household heads who spend their resources on tobacco, alcohol, gambling or narcotics instead of satisfying the minimum caloric requirements of their children (Thorbecke 2008, p. 5).

Based on these and related arguments many development economists criticised the income method. Two of the earliest ones, whose approaches are by now very well known, are the basic needs approach (1970s)² and Sen's capability approach (1980s)³. May poverty be

² In the basic needs approach, poverty measurement is based on a chosen set of goods and services which are considered to meet the basic needs of individuals and allow for a minimum level of quality of life. Thereby, quality of life does not only comprise those basic needs needed for survival (e.g. nutrition, water, health, garments etc.) but also security (e.g. housing, safety, employment etc.) and empowerment (e.g. education, voice etc.). However, one of the main problems of this approach is the determination of what basic needs are, additionally hampered by the fact that basic needs differ between persons as they depend on individual characteristics like age, sex, activity etc.

³ In the capability approach, poverty measurement is based on so called 'functionings' and 'capabilities' (Sen 1985, 1992). Functionings denote specific achievements, something a person is able to do or to be, like being healthy, well-nourished but also having self-esteem and the possibility to participate in social life. Capabilities indicate the freedom an individual enjoys in terms of choosing between different sets of functionings. Based on this approach, an individual is considered to be poor if he/she lacks the possibility to achieve a minimum level of capabilities to function. This approach underlines the weakness of the income (expenditure) method by pointing

defined in terms of basic needs or functionings⁴, ever increasing research efforts are directed towards the issue to directly account for the multidimensional character of poverty instead of relying solely on income as an imperfect proxy.

The discussion proves that there exists reasonable doubt that a unidimensional definition of poverty, though easily applicable, is a good proxy for the multidimensional phenomenon of poverty. But does it really lead to different results when it comes to empirical application? The answer seems to be yes. Analysing poverty in South Africa, Klasen (2000) indeed finds evidence that uni- and multidimensional approaches to poverty measurement lead to quite different results. They diverge greatly in identifying the poorest sections of the population as well as in identifying the impact of race, headship, residence, and household size. Those differences sure enough have considerable consequences for targeting. Analysing individual well-being in Catalonia, Ramos (2005) defines six dimensions of well-being which are all rather weakly correlated with equivalent income⁵, indicating that economic resources do not necessarily lead to higher levels of well-being⁶. In fact, he comes to find that only one third of the income poor are also poor in a multidimensional setting. He concludes that poverty analysis focussing on income related indicators alone clearly misses important aspects of well-being.

Though we of course acknowledge the obvious advantages of a unidimensional definition of poverty – especially with respect to its applicability – we believe there is enough evidence by now to conclude that it is nevertheless an inadequate definition. Therefore, our paper seeks to contribute to research efforts to operationalise a multidimensional poverty definition for poverty measurement.

The following sub-chapter provides an overview of the current debate of how to apply a multidimensional poverty definition to poverty measurement. In his famous 1976 article, Sen identified two main steps which poverty measurement comprises; i) the identification of the poor and ii) the aggregation of the identified characteristics into an overall indicator. There exists a variety of competitive approaches for both steps and we will briefly introduce the most influential ones in the following sub-chapters.

out that income should not be analysed independently from its actual possibility to be transformed in functionings. The adequacy of income to escape poverty varies with personal characteristics and circumstances. Consider, for instance, a person suffering from a kidney disease who needs dialyse treatment. This person may have more income than another person but because of the costly treatment his/her possibility to achieve a certain set of functionings can be very limited. However, again the question remains of how to determine an appropriate set of functionings.

⁴ All multidimensional poverty indices developed on the basis of the capability approach are based on the measurement of functionings rather than capabilities. The reason is that “*while one may agree with Sen that capability is the crux of the matter in poverty measurement, the practical implementation of the capability approach is inherently difficult because one has to have information on the opportunity sets of functionings.*” (Tsui 2002, p. 72).

⁵ Ramos (2005) utilises the modified OECD equivalence scales to equalize income, assigning a weight of 1 to the first adult, of 0.5 to every other adult in the household and of 0.3 to children.

⁶ Ramos (2005) defines well-being poverty as the same percentage of the population which is defined poor according to equivalent income.

1.2 Measuring Poverty

Poverty measurement comprises an identification and an aggregation step. For each step, we briefly introduce the main approaches which are utilised in current research activities. Having done so, we introduce an alternative identification method which we denote as ‘multiple cutoff’ method. Precisely, we suggest to distinguish between different degrees of poverty in order to account for dimensional poverty. We will show that this adjustment in the identification step allows us to expand the way in which poverty measures account for poverty intensity in the aggregation step.

However, before turning to these issues we will first provide a brief overview of the notation we will utilise in the following.

Let $i = 1, \dots, n$ denote the number of individuals in a population and $j = 1, \dots, k$ the number of attributes (basic needs) identified for the measurement of multidimensional poverty. Individual i thus possesses a k -row vector of attributes, $x_i \in R_+^k$ ⁷, which is the i th row of an $n \times k$ matrix $X \in K^n$. K^n denotes the set of all $n \times k$ matrices whose entries are non-negative reals. Let $K = \prod_{n \in N} K^n$, where N is the set of positive integers. The j th column of X accordingly denotes the distribution of attribute j among the n individuals of the population. Therefore, the (i, j) th entry of X gives the quantity individual i possesses of attribute j . It should be noted that this quantitative specification of attributes precludes the possibility that a variable can take a qualitative form. Finally, let $z \in Z$; $Z \subset R_+^k$ be the vector of the respective threshold levels chosen for the different attributes.

1.2.1 The Identification Step

Having introduced our basic notation, we will now turn to the question of how to identify the poor within our multidimensional framework. The identification step comprises two steps. The first step identifies the deprived, whereas, based on the results of the first step, the second step finally identifies the poor.

1.2.1.1 First Step: Identification of the Deprived

When it comes to the identification of the deprived, we follow the approach taken by Bourguignon and Chakravarty (2003) in utilising functioning failures in terms of shortfalls from certain pre-specified minimum (threshold) levels of attributes as indicators for deprivation. In addition, we draw on the work of Donaldson and Weymark (1986) to differentiate between a weak and a strong definition of deprivation. Under the *weak*

⁷ This restriction of endowments on positive values only is common in poverty measurement literature since some axioms (e.g. scale invariance, which will be introduced later in the text) make no sense if endowments could take values less or equal to zero.

definition, the group of individuals who are deprived with respect to a certain attribute comprise all those who fail to achieve the threshold level of this attribute. Or, expressed mathematically, individual i is deprived with respect to attribute j if $x_{ij} < z_j$. Accordingly, the *strong definition* of deprivation additionally includes all individuals who meet exactly the respective threshold level. Thus, under this definition, individual i is deprived with respect to attribute j if $x_{ij} \leq z_j$.

The differentiation between the two definitions does not make any substantial difference empirically, not in view of somewhat arbitrary threshold levels (Zheng 1997). It does, however, make a difference from a theoretic point of view. Donaldson and Weymark (1986) show that the choice of the strong definition may affect the axioms a poverty measure satisfies, that is it may result in diverse impossibilities. For this reason, we will utilise the weak definition and follow the argument of Zheng (1997) by claiming that if it is the objective of a poverty reducing strategy to rise the endowments of the deprived up to a certain threshold level then individuals at the threshold level should indeed not be considered as deprived since no effort has to be made to make them non-deprived.

Thus, individual i is deprived with respect to attribute j if $x_{ij} < z_j$. For any $X \in K$, let $S_j(X)$ or simply S_j denote the set of individuals who are deprived with respect to attribute j .

1.2.1.2 Second Step: Identification of the Poor

Once the deprived have been identified, the next issue to be answered is how deprived an individual has to be in order to be poor. Three approaches for the identification of the poor can be differentiated, the ‘union’, the ‘intersection’ and the ‘dual cutoff’ method. We will briefly discuss the different approaches before we introduce our own approach which we denote as ‘*multiple cutoff*’ method.

The ‘union’ method

One identification method is known as the ‘union’ method, identifying an individual as poor if his/her achievements fall short of the respective threshold level(s) of at least one dimension. In this case, any deprived person will also be classified as poor. Or, expressed mathematically, individual i is poor if $\exists j \in \{1, 2, \dots, k\} : x_{ij} < z_j$. This approach surely accounts for the unique importance of every single dimension since it excludes the possibility of substitution between poor and non-poor attributes, i.e. the ability to compensate the shortfall in one dimension by the overachievement in (an)other dimension(s). If an individual fails to achieve the threshold level in one dimension, he/she is considered to be poor irrespectively of how much he/she has of other attributes. The disadvantage of this approach is that it does not differentiate between individuals who are poor in one dimension and individuals who are poor in many or even all dimensions. Thus, it is overly inclusive, leading to exaggerated poverty rates.

The ‘intersection’ method

A second identification method is the ‘intersection’ method, denoting an individual as poor whenever his/her achievements fall below the respective threshold level in every single dimension. Or, expressed mathematically, individual i is poor if $x_{ij} < z_j \forall j$. The approach certainly identifies the most deprived but is overly constrictive, leading to minimised poverty rates. For instance, Bourguignon and Chakravarty (1997) doubt the appropriateness of this approach by pointing out that in case longevity and income are two poverty dimensions, an old beggar would be considered as non-poor according to the intersection method.

The ‘dual cutoff’ method

In a recent paper, Alkire and Foster (2008) proposed a way to combine both methods by introducing two forms of cut-offs. Whereas the first cut-off identifies the deprived, the second cut-off defines how deprived an individual has to be to be considered poor. This second cut-off is defined as a minimum number of dimensions, say d . Or, expressed mathematically, individual i is poor if $x_{ij} < z_j$ for $j \in \{1, 2, \dots, k\}$ and $\# j \geq d$. This approach includes both the ‘union’ as well as the ‘intersection’ method as special cases where $d = 1$ and $d = k$, respectively. However, one drawback of this approach is that the choice of the second cut-off d is rather arbitrary.

The ‘multiple cutoff’ method

In our paper, we refrain from the choice of a single or dual cutoff since we believe it wastes too much information which could be very valuable in the aggregation step. Instead, we suggest to differentiate between different categories or degrees of poverty. The greater the number of dimensions in which an individual fails to achieve the threshold level, the more dimensional poor is he or she. Thus, with k dimensions of multidimensional poverty, there exist up to k groups of poor individuals. This procedure captures the warrantable requirement of the union method to account for the unique importance of every single poverty dimension, since a person will be denoted as poor whenever he/she fails to achieve the respective threshold in at least one of the identified poverty dimensions. However, by differentiating individuals according to the number of dimensions in which they fail to reach the threshold levels, it also captures the requirement of the intersection method to account for poverty severity. As we will show in the following subchapter, this procedure will enable us to enrich the development of poverty indicators in the aggregation step.

1.2.2 The Aggregation Step

As we have seen in the proceeding subchapter, the identification of the poor is solely based on dimensional aspects of poverty. However, this is a severely restrictive focus on poverty. Consider a situation with two poor persons and two attributes whose respective threshold

levels are (4,4). Now consider the following two distributions of attributes: $X = \begin{pmatrix} 3 & 3 \\ 3 & 4 \end{pmatrix}$ and

$\hat{X} = \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix}$. Irrespectively which identification method we utilise, both distributions would yield the same number of poor⁸. Yet, the situation of the poor under distribution \hat{X} is obviously much more severe than that of the poor under distribution X .

This is the very idea behind distribution-sensitive poverty measures developed in the unidimensional setting: Instead of merely counting the number of the poor, these measures rather sum up the distances to the respective thresholds according to which individuals are deprived.

This approach, though perfectly adequate in the unidimensional setting, is not simply transferable to the multidimensional setting. First of all, it inevitably leads to double-counting, whenever individuals are deprived in more than one dimension. This problem is taken care of by Bourguignon and Chakravarty's (2003) identification function⁹.

However, a second problem arises when transferring the principle of distribution-sensitive poverty measures to the multidimensional case which has not been addressed so far. This problem is connected with a second feature of those measures which goes back to Sen (1976). He claimed that the increase in poverty due to a reduction in the income of the poor should be higher the lower the initial income of the poor. Thus, the resulting distribution-sensitive poverty measures do not merely sum up the distances to the respective thresholds according to which individuals are deprived, but rather attribute greater weight to larger distances.

In the unidimensional case this procedure is perfectly adequate to account for poverty intensity. Yet, in the multidimensional case, another aspect of poverty intensity occurs. This other aspect is the dimensional poverty.

There exists no luminous explanation why a weighting scheme should be utilised to account for differences in poverty intensity *within* a poverty dimension but not *between* dimensions. It is well conceivable that an increase in one poverty dimension in which an individual is deprived should cause a larger decline in the poverty measure if the recipient is also deprived in other poverty dimensions. Thus, we suggest to introduce an additional weighting scheme. A scheme that guarantees that the poverty reduction due to an increase in one poverty

⁸ The union method would identify both persons as poor, the intersection method one person, the dual cutoff method either one or both (depending on the definition of d), while our method would classify one person as poor to degree one and one person as poor to degree two.

⁹ By defining the poverty indicator variable as $\rho(x_i; z) = 1$ if $\exists j \in \{1, 2, \dots, k\} : x_{ij} < z_j$ and 0 otherwise, the number of poor in a multidimensional setting is simply given by $n_p(X) = \sum_{i=1}^n \rho(x_i; z)$.

dimension will be higher the higher the number of dimensions in which the threshold levels are not achieved¹⁰.

In particular, this paper introduces a weight $\mu_i \in [0,1]$, depending on the number of dimensions $s_i = 0, \dots, k$ in which an individual i ; $i = 1, \dots, n$; fails to achieve the minimum threshold level. For instance, a simple way to define μ_i could be $\mu_i = s_i/k$.¹¹ With μ_i being increasing in s , this weighting scheme provides a way to account for an individual's degree of dimensional poverty¹².

These considerations in mind, we will now turn to the actual aggregation step which finally leads us to the multidimensional poverty index. As Maasoumi and Lugo (2008) point out, a poverty index can be disentangled into the individual poverty function $p(x_i; z)$ and the aggregator function which together form the poverty index $P(X; z) = 1/n \sum_{i=1}^n p(x_i; z)$.

Thus, a multidimensional poverty index is a non-constant real-valued function $P: K \times Z \rightarrow R$. For any $X \in K$, $z \in Z$, $P(X; z)$ gives the extent of poverty associated with matrix X and threshold vector z . In other words, though we have chosen a multidimensional perspective for poverty measurement, the magnitude of overall poverty is indicated by a real number. This numerical representation of overall multidimensional poverty meets the concern of Sen (1981) who considered the monetary approach as having an edge over the direct approach since it specifies the poverty degree by a numerical value.

Current research seems to concentrate on four main approaches when it comes to the actual derivation of multidimensional poverty indices (e.g. Deutsch and Silber 2005, Kakwani and Silber 2008, Maasoumi and Lugo 2008): i) the *fuzzy set approach*, ii) the *distance function approach*, iii) the *information theory approach*, and iv) the *axiomatic approach*. In our paper we will utilise the axiomatic approach as being the most appropriate for the purpose of our research. Thus, we will briefly introduce the four approaches before turning to a more detailed discussion of the axiomatic approach in chapter 2.

1.2.2.1 The "Fuzzy Set" approach

The theoretical foundation of the fuzzy set approach was developed by Zadeh (1965) and is based on the consideration that it may sometimes be impossible to precisely define certain sets of objects. Whereas it is always possible to decide for some objects whether they belong to a certain set or not there may very well remain objects for which such a precise statement

¹⁰ Note that the only reason for this additional weighting scheme is to account for dimensional poverty. It does not say anything about possible interactions between the attributes.

¹¹ However, what would be the best way to define μ_i should be tested in practice, that is by applying real data. We will conduct a practice test in a second step following this paper.

¹² Note that when it comes to counting the number of the poor, we will obviously have to draw on the usually utilised poverty indicator variable $\rho(x_i; z)$ introduced by Bourguignon and Chakravarty (2003).

cannot be made. As a consequence, these objects are considered to form a kind of fuzzy subset.

Sure enough, this approach is relevant in the case of poverty measurement. We already pointed out how challenging it is to precisely define notions like poverty and well-being. Considering the case of poverty, some individuals lack so many resources that they certainly should be classified as poor whereas the welfare level of others is as high that they should certainly not be classified as poor at all. However, there will be individuals for whom such a classification is not that clear. This is especially the case if a multidimensional approach to poverty measurement is chosen since there will be many individuals who will be poor for some criteria and non-poor for others. The fuzzy set approach would attribute those individuals different degrees of membership of the set of the poor¹³.

Let $x \in X$ be any element of a set X and $A \in X$ being a fuzzy subset of X . $A = \{x, \lambda_A(x)\} \forall x \in X$ is characterised by a membership function $\lambda_A(x) \rightarrow R \in [0,1]$ denoting the degree of membership of element x to the fuzzy set A . $\lambda_A(x)$ takes the value zero if x does not belong to A and unity if x completely belongs to A . However, if $0 < \lambda_A(x) < 1$, x belongs partially to A , with its degree of membership given by $\lambda_A(x)$.

The use of fuzzy measures in the poverty context allows for an imprecise borderline between the poor and the nonpoor which seems to be a promising way to capture the vagueness inherent in poverty measurement. However, even this approach does not completely solve the problem of arbitrariness: Now it is not the choice of poverty line or threshold levels which is arbitrary but instead the choice of the precise boundaries of the imprecise borderline (Qizilbash and Clark 2005). Yet, the very advantage of this approach is at the same time the root of a disadvantage when it comes to its practical application. It is the very impreciseness of the approach which renders poverty comparisons extremely difficult. This is the main reason why this paper does not follow the fuzzy set approach.

1.2.2.2 The Distance Function approach

The concept of distance functions is widely used in efficiency analysis, especially in production economics, as a mean to summarize the range of information resulting from the multi-output nature of production into merely one dimension.

Distance functions can be differentiated according to their input or output orientation. Intuitively, the output distance function $D_{out}(x,y)$ measures the extent to which an output vector $y \in R_{++}^M$ can be proportionally increased given a fixed input vector $x \in R_{++}^N$. Let $P(x)$ be defined as the output set of all output vectors y which can be produced by the input vector x . The production possibility frontier $PPF(x)$ depicts the maximum among the output

¹³ Note that fuzzy poverty measures are not measures of the intensity of poverty, but instead address the vagueness of poverty (Qizilbash and Clark 2005).

combinations which can be produced given input vector x . The distance function for a specific output combination measures the distance between this combination and the $PPF(x)$ as the inverse of the factor by which the production could be increased for a given input vector x , that is

$$D_{out}(x, y) = \min\{\theta : (y / \theta) \in P(x)\}$$

where θ is a scalar measuring the distance to $PPF(x)$ ¹⁴.

Similarly, the input distance function $D_{in}(x, y)$ measures the extent to which an input vector $x \in R_{++}^N$ can be proportionally contracted given a fixed output vector $y \in R_{++}^M$. Let $L(y)$ be defined as the input set of all input vectors x which can produce the output vector y . The isoquant $IQ(y)$ depicts the minimum among the input combinations which can produce a certain output vector y . The distance function for a specific input combination measures the distance between this combination and the $IQ(y)$ as the inverse of the factor by which the input quantities could be reduced for a given output vector y , that is

$$D_{in}(x, y) = \max\{\rho : (x / \rho) \in L(y)\}$$

where ρ is a scalar measuring the distance to $IQ(y)$ ¹⁵.

This approach obviously shows methodological similarities to the analysis of well-being. Both approaches try to summarize the information resulting from a multidimensional phenomenon into just one single dimension. Thereby, the distance function approach does not require any assumptions concerning prices or behaviour. This certainly is a valuable property which would also be desirable in the context of poverty measurement. Based on these considerations, Lovell et al. (1994) were the first to apply the distance function approach to the analysis of well-being.

The input vector x is interpreted as a vector of resources and the output vector y as a vector of functionings. A multidimensional index of well-being is derived by first utilising the input distance functions to build several measures of resources which in a second step are transformed into a scalar measure of functionings via an output distance function.

As a matter of fact, the distance function approach indirectly aggregates the various attributes of an individual into a single cardinal index with individual i being poor whenever his/her aggregate index falls below some chosen poverty line. However, this approach is severely restrictive. In fact, the multidimensionality of poverty is reduced to a mere generalisation of

¹⁴ It may be proven (see Coelli et al. 1998) that $D_{out}(x, y)$ is non-decreasing, positively linearly homogenous and concave in y and decreasing in x . Additionally, $D_{out}(x, y) \leq 1$ if y belongs to $P(x)$ and $D_{out}(x, y) = 1$ if y lies on $PPF(x)$.

¹⁵ It may be proven (see Coelli et al. 1998) that $D_{in}(x, y)$ is non-decreasing, positively linearly homogenous and concave in x and decreasing in y . Additionally, $D_{in}(x, y) \leq 1$ if x belongs to $L(y)$ and $D_{in}(x, y) = 1$ if x lies on $IQ(y)$.

the income-based approach by utilising a broader concept of income. It is therefore inapplicable to the idea underlying this paper.

1.2.2.3 The Information Theory approach

Cover and Thomas (2006, p. 1) provide the following description of the information theory approach: “*Information Theory answers two fundamental questions in communication theory: What is the ultimate data compression (answer: the entropy H), and what is the ultimate transmission rate of communication (answer: the channel capacity C).*” There exists a specific result concerning the entropy which is extremely valuable in the context of inequality and poverty measurement: two entropies are equal if, and only if, underlying distributions are identical. Thus, “*the basic measure of divergence between two distributions is the difference between their entropies*” (Maasoumi and Lugo 2008, p. 8).

Thus, it is of no surprise at all that the concept of entropy has been applied several times to the analysis of multidimensional inequality, first by Maasoumi in 1986. In 1997 Miceli happened to be the first to apply the approach to the analysis of multidimensional poverty. The idea is that since all poverty indices are functions of the distribution of identified attributes and the distribution of attributes contains all the information about the attributes, a poverty index should be required to have a distribution as ‘close’ as possible to the multivariate distribution of attributes. In other words, a poverty index should diverge least from the distributional information provided by the distribution of attributes (Maasoumi 1986). This is the same as requiring to minimise the respective entropies.

Let $x_{ij} \in X$ be the endowment of individual i ($i = 1, \dots, n$) with attribute j ($j = 1, \dots, k$). Following Maasoumi (1986), each vector of attributes (x_{i1}, \dots, x_{ik}) is aggregated into a single scalar x_{ic} . Individual i will be classified as poor whenever his/her scalar x_{ic} falls below a pre-specified poverty line. An aggregation function is utilised to derive the composite index $x_c = (x_{1c}, \dots, x_{nc})$. The special feature of this function is that it is chosen to minimise the distance between the n vectors (x_{i1}, \dots, x_{ik}) and the composite index x_c . Drawing on the findings of Information Theory, the class of generalized divergence measures, which had been extended by Maasoumi (1986) to fit to the multidimensional case, is utilised for this minimisation problem:

$$D_\gamma(x_c, X; \alpha) = \begin{cases} 1/(\gamma(\gamma+1)) \sum_{j=1}^m \alpha_j \{ \sum_{i=1}^n x_{ic} [(x_{ic}/x_{ij})^\gamma - 1] \} & \text{when } \gamma \neq 0, -1 \\ \sum_{j=1}^m \alpha_j [\sum_{i=1}^n x_{ic} \log(x_{ic}/x_{ij})] & \text{when } \gamma \rightarrow 0 \\ \sum_{j=1}^m \alpha_j [\sum_{i=1}^n x_{ij} \log(x_{ij}/x_{ic})] & \text{when } \gamma \rightarrow 1 \end{cases}$$

The minimisation of this class of proximity measures then leads to the desired composite indices x_{ic}

$$x_{ic} \propto \begin{cases} [\sum_{j=1}^m \delta_j (x_{ij})^{-\gamma}]^{-(1/\gamma)} & \text{when } \gamma \neq 0, -1 & (\text{harmonic mean}) \\ [\prod_{j=1}^m (x_{ij})^{\delta_j}] & \text{when } \gamma \rightarrow 0 & (\text{geometric mean}) \\ [\sum_{j=1}^m \delta_j (x_{ij})] & \text{when } \gamma \rightarrow 1 & (\text{arithmetic mean}) \end{cases}$$

where $\delta_j = \alpha_j / \sum_{j=1}^m \alpha_j$.

What remains to be chosen are the weights δ_j , γ and the respective poverty line.

The idea behind this approach is very charming indeed. However, similar to the distance function approach, the identification of the poor takes place *after* the respective endowments have been aggregated. Again, this approach reduces the multidimensionality of poverty to a mere generalisation of the income approach. Yet, in contrast to the distance function approach, the information theory approach has the valuable advantage that it can easily be utilised as an add-on in almost every poverty measurement approach¹⁶. Once a certain class of multidimensional poverty indices has been derived, it can be tested for their information efficiency according to the information theory approach.

1.2.2.4 The Axiomatic Approach

In his famous 1976 article, Sen came up with a first list of axioms which reasonable poverty indices should satisfy and which should be utilised to evaluate and construct poverty indices. Until now, very few studies have attempted to axiomatically derive multidimensional poverty measures. The first attempt was made by Chakravarty, Mukherjee and Ranade in 1998. To the best of our knowledge there exist currently four main papers dealing with the axiomatic approach to multidimensional poverty measurement: Chakravarty, Mukherjee and Ranade (1998); Tsui (2002); Bourguignon and Chakravarty (2003); and Chakravarty and Silber (2008). Each of these approaches defined various axioms and combined them in different ways in order to derive different classes of multidimensional poverty measures. In the next chapter we introduce and discuss the axioms defined in the four papers.

¹⁶ Note, however, that the information theory approach reaches far beyond being a simple add-on for other methods. Indeed, it opens the way to more generalised poverty measures which are more sophisticated than, for instance, the average functions of the axiomatic approach (Maasoumi and Lugo 2008).

2 The Axiomatic Approach

In the following we provide an overview of the axioms defined in the four main papers dealing with an axiomatic approach to multidimensional poverty measurement: Chakravarty, Mukherjee and Ranade (1998); Tsui (2002); Bourguignon and Chakravarty (2003); and Chakravarty and Silber (2008). Thereby, we differentiate between three groups of axioms: i) the group of core axioms which consists of independent and mainly uncontroversial axioms, ii) the group of implied / non-restrictive axioms, and iii) the group of controversial axioms.

2.1 The group of core axioms

Core axioms are easily acceptable, independent axioms which are essentially generalizations of the axioms proposed in the unidimensional case. So far, the following nine main core axioms have been introduced for the multidimensional context.

Anonymity (AN)¹⁷: For any $(X; z) \in K \times Z : P(X; z) = P(\Pi X; z)$, where Π is any permutation matrix¹⁸ of appropriate order.

AN states that any characteristic of persons other than the attributes j are irrelevant for measuring poverty.

Continuity (CN): For any $z \in Z, P$ is continuous on K .

CN requires P to vary continuously with x_{ij} and is essentially a technical requirement. It precludes oversensitivity of the poverty index, i.e. abrupt changes in P for small changes in X .

Principle of Population (PP)¹⁹: For any $(X; z) \in K \times Z; m \in N : ^{20}P(X^m; z) = P(X; z)$ where X^m is the m -fold replication of X .

PP ensures that the poverty index depends on the distributions of the attributes j and their shortfalls below z rather than on population size. Thus, by facilitating the transformation of different-sized matrices into one size, PP makes cross population and cross time comparisons of poverty possible.

Focus (SF)²¹: For any $(X; Z); (Y; Z) \in K \times Z$: if

- i) for any i such that $x_{ij} \geq z_j, y_{ij} = x_{ij} + \delta$, where $\delta > 0$,

¹⁷ Also known as “Symmetry” (e.g. Tsui 2002).

¹⁸ A permutation matrix is a square (0,1)-matrix of any order that has exactly one entry 1 in each row and each column and 0's elsewhere.

¹⁹ Also known as “Replication Invariance” (e.g. Tsui 2002, Zheng 1997).

²⁰ N being the set of positive integers.

²¹ Bourguignon and Chakravarty (2003) additionally differentiate between a strong and a weak version of the focus axiom. The axiom we introduced as Focus would be the strong version. The definition of the weak version is as follows: For any $(X; Z); (Y; Z) \in K \times Z$ if for some $i x_{ij} \geq z_j \forall j$ and i) for any $j, y_{ij} = x_{ij} + \delta$, where $\delta > 0$, ii) $y_{it} = x_{it} \forall t \neq j$, and iii) $y_{rs} = x_{rs} \forall r \neq i$ and $\forall s$, then $P(Y; z) = P(X; z)$. In contrast to SF, WF says that the poverty index is independent of the attribute levels of the non-poor persons only. WF follows directly from SF.

- ii) $y_{ij} = x_{ij} \forall t \neq i$, and
- iii) $y_{is} = x_{is} \forall s \neq j$ and $\forall i$,

then $P(X; z) = P(Y; z)$.

SF demands that giving a person more of an attribute with respect to which this person is not poor then the poverty index will not change even if this person is poor with respect to some other attribute(s). In other words, SF demands independence from non-poor attributes.

Subgroup Decomposability (SD): For any $X^1, X^2, \dots, X^m \in K$ and $z \in Z$:

$$P(X^1, X^2, \dots, X^m; z) = \sum_{i=1}^m \frac{n_i}{n} P(X^i; z)$$

with n_i being the population size of subgroup X^i , $i = 1, \dots, m$ and $\sum_{i=1}^m n_i = n$.

SD allows the decomposition of a population into several subgroups according to ethnic, spatial or other criteria so that overall poverty is the population share weighted average of the respective subgroup poverty levels. SD is a valuable property for policy makers since it enables the calculation of percentage contributions of different subgroups to overall poverty and thus to identify those subgroups most afflicted by poverty. Note that any decomposable poverty index also satisfies subgroup consistency.

Another decomposability postulate which deals with the decomposability of attributes is Factor Decomposability.

Factor Decomposability (FD): For any $(X, z) \in K \times Z$:

$$P(X; z) = \sum_{j=1}^k a_j P(x_j; z_j)$$

with $a_j > 0$ being the weight attached to attribute j , $j = 1, \dots, k$ and $\sum_{j=1}^k a_j = 1$.

FD allows calculating the contribution of attribute combinations to overall poverty. In connection with SD it allows for a two-way poverty breakdown, calculating the contribution of different subgroup-attribute combinations to overall poverty. Those axioms allow for better targeting of antipoverty policies. However, FD requires the poverty measure to be additive, a rather severe restriction preventing the fulfilment of some desirable axioms as will be seen later on.

In his well-known article from 1976 Sen requested poverty indices to be sensitive to inequality among the poor, so that, whenever poverty among the poor decreases, poverty indices should fall. Distribution-sensitive poverty measures provide a clearer picture of poverty intensity and changes. In particular, they i) distinguish between poverty eliminating, alleviating and redistributing policies, and ii) channel assistance to the poorest first whereas not distribution-sensitive measures tend to help the least poor first.

One well-known partial order ranking distributions of attributes by their degrees of inequality is the so called Pigou-Dalton transfer.

Pigou-Dalton transfers: Matrix X is said to be obtained from matrix Y by a *Pigou-Dalton progressive transfer* of attribute j from one poor person to another if for some persons i, m :

- i) $y_{mj} < y_{ij} < z_j$,
- ii) $x_{mj} - y_{mj} = y_{ij} - x_{ij} > 0$, $x_{ij} \geq x_{mj}$,
- iii) $x_{rj} = y_{rj} \forall r \neq i, m$;
- iv) $x_{rs} = y_{rs} \forall s \neq j$ and $\forall r$.

$$\begin{pmatrix} y_{i1} & \dots & y_{ij} & \dots & y_{ik} \\ M & & M & & M \\ y_{i1} & \dots & z_j > y_{ij} > y_{mj} & \dots & y_{ik} \\ M & & M & & M \\ y_{m1} & \dots & y_{mj} < y_{ij} < z_j & \dots & y_{mk} \\ M & & M & & M \\ y_{n1} & \dots & y_{nj} & \dots & y_{nk} \end{pmatrix} \xrightarrow{\text{transfer } \theta \text{ units}} \begin{pmatrix} x_{i1} = y_{i1} & \dots & x_{ij} = y_{ij} & \dots & x_{ik} = y_{ik} \\ M & & M & & M \\ x_{i1} = y_{i1} & \dots & x_{ij} = y_{ij} - \theta \geq x_{mj} & \dots & x_{ik} = y_{ik} \\ M & & M & & M \\ x_{m1} = y_{m1} & \dots & x_{mj} = y_{mj} + \theta \leq x_{ij} & \dots & x_{mk} = y_{mk} \\ M & & M & & M \\ x_{n1} = y_{n1} & \dots & x_{nj} = y_{nj} & \dots & x_{nk} = y_{nk} \end{pmatrix}$$

One says that matrix X is obtained from matrix Y by a Pigou-Dalton progressive transfer of attribute j from one poor person to another if X and Y are exactly the same except that the – with respect to attribute j – richer poor i has θ units less of attribute j in X than in Y whereas the poorer poor m has θ units more. It is quite reasonable to argue that under such a progressive transfer poverty should not increase.

A generalization of this principle leads to the following axiom.

Transfers Principle (TP): For any $z \in Z$, and X, Y of the same dimension, if $X^P = BY^P$ and B is not a permutation matrix, then $P(X; z) \leq P(Y; z)$, where $X^P(Y^P)$ is the attribute matrix of the poor corresponding to $X(Y)$ and $B = (b_{ij})$ is some bistochastic matrix ($b_{ij} \geq 0$; $\sum_i b_{ij} = \sum_j b_{ij} = 1$) of appropriate order.

TP requires that a transformation of the attribute matrix Y^P of the poor in Y into the corresponding matrix X^P by an equalising operation does not increase poverty.

However, as Tsui (2002) points out, this axiom says nothing about the implications of a correlation increasing switch²².

Correlation increasing switch: Matrix X is said to be obtained from matrix Y by a *correlation increasing switch* of attribute j from one poor person to another if for some persons i, m :

- i) $y_{mh} < y_{ih} < z_h$; $y_{mj} > y_{ij} < z_j$,
- ii) $x_{mh} = y_{mh}$, $x_{ih} = y_{ih}$; $x_{mj} = y_{ij}$, $x_{ij} = y_{mj}$
- iii) $x_{rj} = y_{rj} \forall r \neq i, m$;
- iv) $x_{rs} = y_{rs} \forall s \neq h, j$ and $\forall r$.

²² Please note that this is the expression which Bourguignon and Chakravarty (2003) utilise. Tsui (2002) himself calls this kind of specific transfer a ‘basic arrangement-increasing transfer’ (Tsui 2002).

$$\begin{pmatrix} y_{11} & \Lambda & y_{1h} & \Lambda & y_{1j} & \Lambda & y_{1k} \\ M & & M & & M & & M \\ y_{i1} & \Lambda & z_h > y_{ih} > y_{mh} & \Lambda & y_{ij} < y_{mj} < z_j & \Lambda & y_{ik} \\ M & & M & & M & & M \\ y_{m1} & \Lambda & y_{mh} < y_{ih} < z_h & \Lambda & z_j > y_{mj} > y_{ij} & \Lambda & y_{mk} \\ M & & M & & M & & M \\ y_{n1} & \Lambda & y_{nh} & \Lambda & y_{nj} & \Lambda & y_{nk} \end{pmatrix} \xrightarrow{\text{Switch}} \begin{pmatrix} x_{11} = y_{11} & \Lambda & x_{1h} = y_{1h} & \Lambda & x_{1j} = y_{1j} & \Lambda & x_{1k} = y_{1k} \\ M & & M & & M & & M \\ x_{i1} = y_{i1} & \Lambda & x_{ih} = y_{ih} > x_{mh} & \Lambda & x_{ij} = y_{mj} > x_{nj} & \Lambda & x_{ik} = y_{ik} \\ M & & M & & M & & M \\ x_{m1} = y_{m1} & \Lambda & x_{mh} = y_{mh} < x_{ih} & \Lambda & x_{mj} = y_{ij} < x_{ij} & \Lambda & x_{mk} = y_{mk} \\ M & & M & & M & & M \\ x_{n1} = y_{n1} & \Lambda & x_{nh} = y_{nh} & \Lambda & x_{nj} = y_{nj} & \Lambda & x_{nk} = y_{nk} \end{pmatrix}$$

That is, under a correlation increasing switch individuals having a higher amount of one attribute get a higher amount of (an)other attribute(s) through a rank reversing transfer. Tsui (2002) argues that such a transfer should not decrease poverty.

However, as for instance Bourguignon and Chakravarty (2003) criticise, this presupposes that attributes are substitutes²³. But some attributes might as well be complements. For instance, as Ravallion (1996) points out, low income may be both, cause and result of poor health and schooling. Also, there seems to be a certain degree of complementarity between the dimensions education and nutrition. Poor nutrition, especially in the years of early childhood, may lead to persistent health effects which lower educational performance. Thus, one could think of a situation where it might be possible to actually reduce poverty if better access to education is granted to those children who do not suffer from undernutrition. In this case a correlation increasing switch would lead to a decrease in aggregate poverty.

Therefore, Bourguignon and Chakravarty (2003) amend Tsui's axiom called Poverty-Nondecreasing Rearrangement (Tsui 2002) by limiting it to substitute attributes only, denoting it as Nondecreasingness under Correlation Increasing Switch.

Nondecreasingness under Correlation Increasing Switch (NDC): For any $(X; z) \in K \times Z$, if $Y \in K$ is obtained from X by a correlation increasing switch of two *substitute* attributes between two poor individuals, then $P(X; z) \leq P(Y; z)$.

Additionally, they introduce the following corresponding axiom for complement attributes.

Nonincreasingness under Correlation Increasing Switch (NIC): For any $(X; z) \in K \times Z$, if $Y \in K$ is obtained from X by a correlation increasing switch of two *complement* attributes between two poor individuals, then $P(X; z) \geq P(Y; z)$.

Though poverty measures should account for joint interaction among attributes, this is a challenging requirement. Especially since the relationship between attributes may change with the respective time horizon, being substitutes in the short-run and complements in the long-run (Thorbecke 2008). These changing and highly complex interrelations between attributes limit the practical applicability of these axioms.

²³ Substitutability means that a person can trade-off attributes for one another (e.g. less clothing for better nutrition) while still remaining on the same iso-utility curve. Complementarity means this trade-off does not exist: an increase in one attribute raises the marginal utility of another attribute.

2.2 The group of implied / non-restrictive axioms

The following group of axioms comprises all the reasonable axioms that are either direct implications of the core axioms introduced in the proceeding subchapter or are not restrictive at all. Altogether, we differentiate five axioms in this group of implied and/or non-restrictive axioms.

Monotonicity (MN): For any $(X; Z); (Y; Z) \in K \times Z$ if:

- i) for any $i, y_{ij} = x_{ij} + \delta$, where $x_{ij} < z_j, \delta > 0$,
- ii) $y_{ij} = x_{ij} \forall t \neq i$, and
- iii) $y_{is} = x_{is} \forall s \neq j$ and $\forall i$,

then $P(Y; z) \leq P(X; z)$.

MN requires that the poverty index does not increase if – ceteris paribus – the condition of person i who is poor with respect to attribute j improves. It's most important implication is the fact that it rules out every measure which is based on a simple counting of the poor – like the headcount ratio. MN follows directly from TP and CN.

Nondecreasingness in Subsistence Levels (NS)²⁴: For any $X \in M$, $P(X; z)$ is non-decreasing in $z_j \forall j$.

NS requires that, everything else equal, the community with higher threshold levels should not have a lower poverty. NS follows directly from TP and MN.

Non-Poverty Growth (NG): For any $(X; z) \in K \times Z$, if Y is obtained from X by adding a rich person to the population, then $P(Y; z) \leq P(X; z)$.

NG requires the poverty index to be nonincreasing in the population size of the non-poor. It follows directly from MN, FC and PP.

Normalization (NM): For any $(X; z) \in K \times Z$: i) $P(X; z) = 1$ if $x_{ij} = 0 \forall i, j$ and ii) $P(X; z) = 0$ if $x_{ij} \geq z_j \forall i, j$. Thus, $P(X; z) \in [0, 1]$

NM is a cardinality property of a poverty index which simply requires the measure to be equal to zero in case all individuals are non-poor (i.e. where there is zero poverty). Thus, the resulting index can assume any non-negative value. Quite often the index is additionally required to assume a value of one in case all individuals are poor, that is an index range between zero and one. Obviously, this property does not impose very much restriction on the poverty index.

²⁴ This is a multidimensional extension of the Increasing Poverty Line axiom (Zheng 1997) – though in a softened form.

Subgroup Consistency (SC): For any n and k such that X_1 and Y_1 are $n \times k$ matrices and X_2 and Y_2 are $m \times k$ matrices, with²⁵ $X^T := [X_1^T, X_2^T]$ and $Y^T := [Y_1^T, Y_2^T]$, $P(X; z) > P(Y; z)$ whenever $P(X_1; z) > P(Y_1; z)$ and $P(X_2; z) = P(Y_2; z)$.

SC demands the poverty index not to increase if the degree of poverty of a population subgroup decreases. SC follows directly from SD.

2.3 The group of controversial axioms

This last group of axioms comprises three axioms which are well-known but cannot be easily justified and are thus discussed controversially.

Poverty Criteria Invariance (PI): Let z, \tilde{z} be such that $z_j \neq \tilde{z}_j$; then $P(X, z) \leq P(Y, z) \Leftrightarrow P(X, \tilde{z}) \leq P(Y, \tilde{z})$ whenever $X(z) = X(\tilde{z})$ and $Y(z) = Y(\tilde{z})$.

Suppose the vector of poverty thresholds is adjusted from z to \tilde{z} . If the new poverty thresholds identify the same group of people as poor, then the *ordinal ranking* of X and Y should remain unchanged. In other words, a change in the thresholds which does not alter the number of the poor should not lead to a significant change in the evaluation of poverty. However, as Tsui (2002) points out, PI precludes possible changes in shortfalls, i.e. from $z - x_{ij}$ to $\tilde{z} - x_{ij}$, which may very well reverse ordinal rankings if differential ethical weights are assigned to shortfalls of attributes.

Translation-Scale Invariance (TI): For any $(X; z) \in K \times Z$: $P(X; z) = P(X + \Gamma; z + t)$, where Γ is any matrix with identical rows $t = (t_1, \dots, t_k)$.

TI requests that adding a constant to the income of each person as well as to the respective thresholds does not change the degree of poverty. Like in the unidimensional case TI is the characterisation of an absolute poverty measure.

Scale Invariance (SI)²⁶: For any $(X; z) \in K \times Z$: $P(X; z) = P(X'; z')$ where $X' = \Lambda X$; $z' = \Lambda z$ with Λ being the diagonal matrix $\text{diag}(\lambda_1, \dots, \lambda_m)$, $\lambda_i > 0 \forall i$.

Different attributes may have different scales of measurement. SI ensures that the poverty index is invariant under scale transformation of attributes and thresholds, that is the poverty index does not change when the matrix X and the vector z are multiplied by the same diagonal matrix Λ . In other words, only the relative distance of all attributes from their respective poverty thresholds matters for poverty measurement. Which is why poverty measures satisfying SI form the group of the relative poverty measures. Note that for SI to make sense the values of attributes are required to be positive, a fact we accounted for in defining $x_i \in R_+^k$.

²⁵ X^T is the transpose of matrix X .

²⁶ Tsui (2002) calls this axiom Ratio-Scale Invariance (RS).

Neither Scale nor Translation-Scale Invariance can be denoted as core axioms, since both requirements only function as characterisations of certain classes of poverty indices. Furthermore, Zheng (1994) shows that there exists no distributions-sensitive poverty measure that can be both relative and absolute.

Having introduced the main axioms usually discussed in multidimensional poverty measurement, we now turn to the five most well-known classes of multidimensional poverty indices which have been derived so far.

3 Some Classes of Multidimensional Poverty Measures

Based on the work of Chakravarty, Mukherjee and Ranade (1998); Tsui (2002); Bourguignon and Chakravarty (2003); and Chakravarty and Silber (2008), we differentiate five main classes of multidimensional poverty measures. They have all been derived from different selections of the axioms introduced in the second chapter. Of course, there is no such thing as a ‘best’ class of poverty measures. Rather, the choice of any measure will always depend on the respective research question. Yet, we will also see that one out of the five classes of poverty measures considered in this paper seems to be inferior when compared to the other classes.

In the following we will introduce the different classes and discuss their main advantages and disadvantages. Also, we will extend each class of indices according to our weighting scheme. At the end of the chapter we will provide an overview of the axioms the different classes satisfy.

In 1998 Chakravarty, Mukherjee and Ranade showed that the only poverty measure satisfying CN, FC, SD, FD, SI, MN, TR and NM is of the form $P(X; z) = 1/n \sum_{i=1}^n \sum_{j=1}^k a_j f\left(\frac{x_{ij}}{z_j}\right)$ where $f: [0, \infty] \rightarrow R^1$ is continuous, non-increasing, convex, $f(0) = 1$ and $f(t) = c \forall t \geq 1$, where $c < 1$ is a constant, $a_j > 0$ and $\sum_{j=1}^k a_j = 1$ are constants.

Three out of the five classes of poverty measures we focus on in this paper are of this special form, i.e. i) the multidimensional Foster-Greer-Thorbecke class of indices, ii) the multidimensional Watts class of indices, and iii) the first multidimensional Chakravarty class of indices.

i) The Multidimensional Foster-Greer-Thorbecke (FGT) class of indices:

This class of indices is a multidimensional extension of the Foster-Greer-Thorbecke index from 1984.

$$P_{FGT}(X; z) = \frac{1}{n} \sum_{j=1}^k \sum_{i \in S_j} a_j \left(1 - \frac{x_{ij}}{z_j} \right)^{\theta_j}$$

with $a_j > 0$; $\sum_{j=1}^k a_j = 1$; $\theta_j > 1$

The introduction of our weighting scheme leads to the following extension:

$$P_{FGT}^*(X; z) = \frac{1}{n} \sum_{j=1}^k \sum_{i \in S_j} a_j \mu_i \left(1 - \frac{x_{ij}}{z_j} \right)^{\theta_j}$$

with $a_j > 0$; $\sum_{j=1}^k a_j = 1$; $\mu_i \in [0,1]$; $\theta_j > 1$

The parameter θ_j reflects different perceptions of poverty, it can be interpreted as an indicator for poverty aversion. For a given X , $P_{FGT}^{(*)}$ decreases as θ_j increases, that is a smaller θ_j gives greater emphasis to the poorest among the poor within one dimension.

This class of poverty indices does not satisfy the axioms NDC and NIC; a direct result of the strong restriction of additivity implied by FD.

ii) The first multidimensional Chakravarty class of indices (C_1):

This class of indices is a direct multidimensional extension of the Chakravarty index from 1983.

$$P_{C_1}(X; z) = \frac{1}{n} \sum_{j=1}^k a_j \sum_{i \in S_j} \left[1 - \left(\frac{x_{ij}}{z_j} \right)^{c_j} \right]$$

with $a_j > 0$; $\sum_{j=1}^k a_j = 1$; $c_j \in (0,1)$

The introduction of our weighting scheme leads to the following extension:

$$P_{C_1}^*(X; z) = \frac{1}{n} \sum_{j=1}^k a_j \sum_{i \in S_j} \mu_i \left[1 - \left(\frac{x_{ij}}{z_j} \right)^{c_j} \right]$$

with $a_j > 0$; $\sum_{j=1}^k a_j = 1$; $c_j \in (0,1)$; $\mu_i \in [0,1]$

As in the case of the FGT class of indices, the parameter c_j can be interpreted as an indicator for poverty aversion. Note that in this case for a given X , $P_{C_1}^{(*)}$ increases as c_j increases, meaning that a larger c_j gives greater emphasis to the poorest among the poor in dimension j .

This class of indices satisfies exactly the same axioms as the FGT class of indices.

iii) Multidimensional Watts (W) class of indices:

This class of indices is a direct multidimensional extension of the Watts index from 1968.

$$P_w(X; z) = \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^k a_j \log \left(\frac{z_j}{\hat{x}_{ij}} \right)$$

$$\text{with } \hat{x}_{ij} = \min\{z_j, x_{ij}\}; a_j > 0; \sum_{j=1}^k a_j = 1$$

With the introduction of our weighting scheme leading to the following extension:

$$P_w^*(X; z) = \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^k \mu_i a_j \log \left(\frac{z_j}{\hat{x}_{ij}} \right)$$

$$\text{with } \hat{x}_{ij} = \min\{z_j, x_{ij}\}; a_j > 0; \sum_{j=1}^k a_j = 1; \mu_i \in [0,1]$$

This class of indices satisfies the same axioms as the former classes of indices. However, it additionally satisfies Poverty Criteria Invariance (PI). Chakravarty, Deutsch and Silber (2008) show that a great advantage of this class of indices is that it can be decomposed in five²⁷ elements which enable the identification of the causal factors of poverty: i) the Watts poverty gap ratio; ii) the Bourguignon-Theil index of inequality among the poor; iii) the overall headcount ratio; iv) the weights of the various dimensions; v) a measure of correlation between the various dimensions; and (vi) a measure for the intensity of dimensional poverty). The decomposability is a valuable property for the implementation of poverty reduction strategies. A disadvantage of this class of poverty measures is that it has no dimension-specific indicator of poverty perception like the multidimensional FGT class (θ_j) and the first multidimensional Chakravarty class of indices (c_j).

The last two classes of multidimensional poverty measures we consider diverge from the basic form we introduced in the beginning of this chapter. In particular, both classes reject the additivity needed for the indices to satisfy FD as being too severe a restriction. In utilising more sophisticated aggregation functions, they are able to achieve sensitivity towards correlation increasing switches. The two classes we will focus on are i) the second multidimensional Chakravarty class of indices, and ii) the multidimensional Bourguignon-Chakravarty class of indices.

i) Second multidimensional Chakravarty class of indices (C_2):

This class of indices is another direct multidimensional extension of the Chakravarty index from 1983, introduced by Tsui (2002).

$$P_{C_2}(X; z) = \frac{1}{n} \sum_{i=1}^n \left[\prod_{j=1}^k \left(\frac{z_j}{\hat{x}_{ij}} \right)^{r_j} - 1 \right]$$

$$\text{with } \hat{x}_{ij} = \min\{z_j, x_{ij}\}; r_j \in [0,1]$$

²⁷ Six if our additional weight μ_i is included.

And again the introduction of our weighting scheme leading to the following extension:

$$P_{C_2}^*(X; z) = \frac{1}{n} \sum_{i=1}^n \mu_i \left[\prod_{j=1}^k \left(\frac{z_j}{\hat{x}_{ij}} \right)^{r_j} - 1 \right]$$

with $\hat{x}_{ij} = \min\{z_j, x_{ij}\}$; $r_j \in [0,1]$; $\mu_i \in [0,1]$

This class of indices is rather restrictive since it satisfies NDC but not NIC, therefore implicitly assuming attributes to be substitutes. This is a quite severe limitation indeed, since if the effort is made to generate a more sophisticated class of poverty indices that accounts for possible interactions between attributes, it should not limit itself by assuming merely one interaction. In this sense, this class of indices seems to be indeed inferior to the following class of indices proposed by Bourguignon and Chakravarty (2003).

ii) Multidimensional Bourguignon-Chakravarty (BC) class of indices:

$$P_{BC}(X; z) = \frac{1}{n} \sum_{i=1}^n \left[\sum_{j=1}^k a_j \left(1 - \frac{\hat{x}_{ij}}{z_j} \right)^\alpha \right]^{\frac{\delta}{\alpha}}$$

with $\hat{x}_{ij} = \min\{z_j, x_{ij}\}$; $a_j > 0$; $\sum_{j=1}^k a_j = 1$; $\alpha > 1$; $\delta \begin{matrix} \geq \\ \leq \end{matrix} \alpha$

The introduction of our weighting scheme leads to the following extension of this class of poverty indices:

$$P_{BC}^*(X; z) = \frac{1}{n} \sum_{i=1}^n \mu_i \left[\sum_{j=1}^k a_j \left(1 - \frac{\hat{x}_{ij}}{z_j} \right)^\alpha \right]^{\frac{\delta}{\alpha}}$$

with $\hat{x}_{ij} = \min\{z_j, x_{ij}\}$; $a_j > 0$; $\sum_{j=1}^k a_j = 1$; $\mu_i \in [0,1]$; $\alpha > 1$; $\delta \begin{matrix} \geq \\ \leq \end{matrix} \alpha$

This class of indices is the only one out of the five classes satisfying NDC as well as NIC – depending on the choice of parameters. Attributes may be substitutes ($\delta > \alpha$) or complements ($\delta < \alpha$). As Chakravarty and Silber (2008) point out this class of indices is less simple than Tsui's multidimensional extension because the constant elasticity is defined between shortfalls instead of attributes and does not necessarily equal one. Obviously this family of indices does not satisfy FD.

The following table provides an overview of the main axioms and classes of indices identified by Chakravarty, Mukherjee and Ranade (1998); Tsui (2002); Bourguignon and Chakravarty (2003); and Chakravarty and Silber (2008) in the multidimensional setting.

Axioms	FGT	FGT*	C ₁	C ₁ *	W	W*	C ₂	C ₂ *	BC	BC*
<i>Anonymity (AN)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
<i>Continuity (CN)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
<i>Principle of Population (PP)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
<i>Focus (FC)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
<i>Subgroup Decomposability (SD)</i>	✓	✗	✓	✗	✓	✗	✓	✗	✓	✗
<i>Factor Decomposability (FD)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗
<i>Transfer Principle (TP)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✓ ²⁸	✓
<i>Nondecreasingness under Correlation increasing switch (NDC)</i>	✗	✗	✗	✗	✗	✗	✓	✓	✓	✓
<i>Nonincreasingness under Correlation increasing switch (NIC)</i>	✗	✗	✗	✗	✗	✗	✗	✗	✓	✓
<i>Monotonicity (MN)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
<i>Non-Decreasingness in Subsistence Levels (NS)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
<i>Non-Poverty Growth (NG)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
<i>Normalization (NM)</i>	✓	✓	✓	✓	✓ ²⁹	✓	✓ ²⁸	✓	✓	✓
<i>Subgroup Consistency (SC)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
<i>Scale Invariance (SI)</i>	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
<i>Poverty Criteria Invariance (PI)</i>	✗	✗	✗	✗	✓	✓	✓	✓	✗	✗
<i>Translation Scale Invariance (TI)</i>	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

As has been pointed out before there does not exist such thing as a best class of poverty indices. Rather, the choice of a specific class of measures has to always depend on the respective research question. When surveying the table above it can be seen that the classes of indices differ in merely four axioms: Factor Decomposability (FD), Nondecreasingness under Correlation increasing switch (NDC), Nonincreasingness under Correlation increasing switch (NIC), and Poverty Criteria Invariance (PI). Thereby it is of special importance to keep in mind that a trade-off exists between FD and NDC/NIC rendering it impossible for classes of poverty indices to fulfil both axioms at the same time.

Thus, if the research question is concerned with the different possible interactions of attributes, it should be ensured that the class of poverty indices utilised satisfies NDC/NIC. In this case, the multidimensional Bourguignon-Chakravarty class of indices would be suitable. However, if the research objective is a practical application, and especially if it is based on a broader definition of poverty, the utilisation of this class of poverty indices is not advisable. To ascertain the degree of substitutability or complementarity between attributes on a pair-wise basis is difficult enough, to do it for combinations of n attributes taken up to n at a time seems to be at least for current state of the art more or less utopian. This is the reason why multidimensional poverty measurement usually deals with merely two (and at most four) attributes in empirical applications while showing that in theory their methods can be extended to deal with n dimensions (Thorbecke 2008). Hence, for this kind of research one of the first three classes of poverty indices would seem to be more appropriate.

As far as the five classes of poverty indices considered in this paper are concerned, our extension should have no influence on the actual choice of poverty measures since it's effect

²⁸ TP is satisfied whenever δ is greater than unity.

²⁹ In case that all individuals are non-poor the index is zero. However, the index is not defined if all individuals are poor.

on all of them is the same: it leads to a partial violation of the Subgroup Decomposability axiom. More precisely, SD is only satisfied for the group of extreme dimensional poor, i.e. those individuals who fail to reach the threshold level in every single dimension. Though the violation of SD is of course a disadvantage of our extension, its consequence is softened a bit by the fact that it is still fulfilled for the group of the extreme dimensional poor. SD can still be utilised to identify the contribution of different subgroups to extreme dimensional poverty and hence to construct a poverty profile accordingly.

4 Conclusion

In our paper we deal with the issue of how to account for poverty intensity. In the unidimensional setting, distribution sensitive poverty measures are utilised to account for the differences in the respective distances to the threshold level.

In our paper we claim that in a multidimensional setting there exists another aspect of poverty intensity which we call dimensional poverty – the number of dimensions in which individuals fail to reach the threshold levels. This second aspect has been ignored so far, yet without any luminous explanation.

It is well conceivable that an increase in one poverty dimension in which an individual is deprived should cause a larger increase in the poverty measure if the recipient is also deprived in other poverty dimensions. Thus, in our paper we differentiate between different degrees of dimensional poverty by introducing an additional, individual weight.

In particular, we extend five classes of well-known multidimensional poverty measures by our additional weight. We find that our extension has no other effect on the axiomatic basis of each class of poverty indices considered than a partial violation of the Subgroup Decomposability axiom. Of course, our extension has to be applied to real data in order to ensure that it leads to meaningful results in practice and the author will do so in a second step following this paper.

With regard to future research it is an important fact to note that our extension does not contribute to the discussion about possible interactions between attributes. The issue of how to account for the fact that attributes might be substitutes or complements and that their interaction might change with the time frame chosen is far from being solved. This surely is an important issue for future research.

References

- Alkire, S. / J. Foster (2008): Counting and multidimensional poverty measurement, *OPHI Working Paper No. 7*
- Bourguignon, F. / S. R. Chakravarty (2003): The measurement of multidimensional poverty, *Journal of Economic Inequality* 1: 25-49
- Chakravarty, S. R. (1983): A new index of poverty, *Mathematical Social Sciences* 6(3): 307-313
- Chakravarty, S. R. / C. D'Ambrosio (2006): The measurement of social exclusion, *Review of Income and Wealth* 52(3): 377-398
- Chakravarty, S. R. / J. Deutsch / J. Silber (2008): On the Watts multidimensional poverty index and its decomposition, *World Development* 36(6): 1067-1077
- Chakravarty, S. R. / D. Mukherjee / R. R. Ranade (1998): On the family of subgroup and factor decomposable measures of multidimensional poverty, *Research on Economic Inequality* 8: 175-194
- Coelli, T. / D. S. Prasada Rao / G. E. Battese (1998): An introduction to efficiency and productivity analysis, Boston: Kluwer Academic Publishers
- Cover, T. M. / J. A. Thomas (2006): Elements of Information Theory, second edition, New Jersey: John Wiley & Sons Inc.
- Deutsch, J. / J. Silber (2005): Measuring multidimensional poverty: an empirical comparison of various approaches, *Review of Income and Wealth* 51(1): 145-174
- Donaldson, D. / J. A. Weymark (1986): Properties of fixed population poverty indices, *International Economic Review* 27: 667-688
- Foster, J. (1984): On economic poverty: a survey of aggregate measures, in: R. L. Basmann and Rhodes, G. F. (eds.), *Advances in Econometrics*, 3, Connecticut: JAI
- Foster, J. / J. Greer / E. Thorbecke (1984): A class of decomposable poverty measures, *Econometrica* 52(3): 761-766
- Foster, J. / A. Shorrocks (1991): Subgroup consistent poverty indices, *Econometrica* 59: 687-709
- Kakwani, N. / J. Silber (2008): Introduction: on quantitative approaches to multidimensional poverty measurement, in: *Quantitative Approaches to Multidimensional Poverty Measurement*, Kakwani, N. / J. Silber (eds.), New York: Palgrave Macmillan: xviii-xxiv
- Klasen, S. (2000): Measuring poverty and deprivation in South Africa, *Review of Income and Wealth*, 46(1): 33-58
- Kolm, S-Ch. (1977): Multidimensional egalitarianisms, *The Quarterly Journal of Economics*, 91(1): 1-13
- Lipton, M. / M. Ravallion (1995): Poverty and policy, in: *Handbook of Development Economics*, Vol. 3, editors: Behrmann, J. / T. N. Srinivasan, Amsterdam: North-Holland
- Lovell, C. A. K. / S. Richardson / P. Travers / L. Wood (1994): Resources and functionings: a new view of inequality in Australia, in: *Models and Measurement of Welfare and Inequality*, Eichhorn, W. (ed.), Heidelberg: Springer-Verlag
- Maasoumi, E. / M. A. Lugo (2008): The information basis of multivariate poverty assessments, in: *Quantitative Approaches to Multidimensional Poverty Measurement*, Kakwani, N. / J. Silber (eds.), New York: Palgrave Macmillan

- Maasoumi, E. (1986): The measurement and decomposition of multidimensional inequality, *Econometrica* 54(4): 991-997
- Qizilbash, M. / D. A. Clark (2005): The capability approach and fuzzy poverty measures: an application to the South African context, *Social Indicators Research* 74: 103-139
- Ramos, X. (2005): Using efficiency analysis to measure individual well-being with an illustration for Catalonia, Working paper No. 1748, IZA
- Sen, A. K. (1992): Inequality reexamined, Cambridge: Harvard University Press
- Sen, A. K. (1985): Commodities and capabilities, Amsterdam: North-Holland
- Sen, A. K. (1976): Poverty: an ordinal approach to measurement, *Econometrica* 44(2): 219-231
- Thorbecke, E. (2008): Multidimensional poverty: conceptual and measurement issues, in: *The Many Dimensions of Poverty*, editors: Kakwani, N. / J. Silber, New York: Palgrave MacMillan
- Tsui, K.Y. (2002): Multidimensional poverty indices, *Social Choice and Welfare* 19: 69-93
- Vélez, C.E. / M. Robles (2008): Determining the parameters of axiomatically derived multidimensional poverty indices: an application based on reported well-being in Colombia, in: *Quantitative Approaches to Multidimensional Poverty Measurement*, editors: Kakwani, N. / J. Silber, New York: Palgrave Macmillan
- Watts, H. (1968): An economic definition of poverty, in: *On understanding poverty*, editor: Moynihan, D. P., New York: Basic Books
- Zadeh, L. A. (1965): Fuzzy sets, *Information and Control* 8: 338-353
- Zheng, B. (1994): Can a poverty index be both relative and absolute?, *Econometrica* 62(6): 1453-1458
- Zheng, B. (1997): Aggregate poverty measures, *Journal of Economic Surveys* 11(2): 123-162