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by

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The Dark Side of Reciprocity*

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Abstract

Whether friendship or competitive relationships deserve to be encouraged in the workplace is not obvious a priori. In this paper we derive the conditions under which a profit-maximizing employer finds it convenient to induce a rat race among workers exhibiting horizontal reciprocity in order to obtain underpaid or unpaid extra effort. We characterize the optimal compensation scheme under both symmetric and asymmetric information about workers’ actions, and we also derive conditions for our result to hold in the presence of vertical reciprocity.

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1 Introduction

Relying on the relevance of other-regarding preferences in workplaces, we provide a behavioral explanation for the extra effort provision in organizations (i.e., overtime, additional task, etc). In many situations employees exert extra effort, work overtime or accept to carry out tasks which are not included in their job contract. In particular, unpaid overtime seems quite common in modern industrialized societies.¹

The most intuitive reason why workers agree to exert extra effort is the attempt to gain a better future position, typically through promotion or career advancement. However, empirical evidences show that quite often exertion of extra effort does not lead to better job conditions (Booth et al., 2003, and Meyer and Wallette, 2005) and that even workers at the end of their career or at the top of their organization’s hierarchy work unpaid overtime (Pannenberg, 2005).

In what follows, we offer a complementary explanation for the exertion of extra effort, which is focused on the role of workers’ other-regarding preferences in competitive work environments. We develop a principal multi-agents model in which agents exhibit reciprocity concerns toward colleagues.

Reciprocity between colleagues is likely to occur. As a matter of fact, the workplace is

¹For instance, in 2001 the average European wage earner was compensated for only about 5 out of 9 hours overtime per week (Eurostat, 2004). In Canada, the percentage of employees working overtime increased from 18.6% in 1997 to 22.6% in 2007, and 11.4% of overtime in 2007 was unpaid (Statistics Canada, 2008). Similar occurrences are characteristic of Australia, Japan, and the U.S. (Mizunoya, 2001).
characterized by a high density of social relationships which concur in determining the effectiveness of incentive schemes (Rotemberg, 2006). However, the nature of social interactions is ambivalent. For instance, in the workplace, an employee may develop new friendships which have a positive impact on job satisfaction (Clark, 2005), while in other circumstances s/he may feel stressed by the extreme competition with colleagues or by peer pressure (Heywood et al., 2005), and, in extreme cases, may even experience sabotage by workmates (Lazear 1989). Whether friendships and positive social interactions, rather than competitive relationships, deserve to be encouraged to achieve organizational goals is not obvious a priori. As discussed by Heywood et al. (2005), this should rather depend on job attributes and organizational characteristics.

In this paper, we characterize the conditions under which a profit-maximizing employer finds it convenient to offer highly competitive compensation schemes to workers motivated by reciprocity even when such schemes may be perceived as unfair by workers and/or may result in negative relationships between colleagues.

Our main result is that employees’ horizontal reciprocity concerns are exploited in order to elicit extra effort without (full) compensation. The optimal mechanism is a relative compensation scheme which in equilibrium induces extra effort provision and, by relying on workers’ negative reciprocity, does not provide for the payment of any monetary compensation. In particular, the optimal compensation scheme assigns a high monetary payment to the worker who exerts extra effort when his colleague refuses to do so, and no compensation otherwise. Even when only a jointly exerted extra effort but not an individual action is observable, the compensation scheme offered by the employer traps workers in a situation which resembles a prisoner’s dilemma. Therefore, the worker who chooses to exert extra ef-
fort precludes his/her colleague from gaining his/her highest monetary compensation and, in this way, induces her colleague’s negative attitude toward her/him. It follows that a worker motivated by negative reciprocity is willing to exert underpaid (or unpaid) extra effort in order to preclude that his colleague gains from being the only one exerting extra effort. Each of the two will then exert extra effort, whereas it would be better for both not to exert any extra effort at all. This result seems to match job habits in financial and professional services, where strong work pressure induces an extreme time competition among employees which resembles a "rat race" (Landers et al., 1996).²

First, we consider the case in which agents exhibit only horizontal reciprocity (i.e., concerns about the fairness of coworker’s actions). Second, we consider the case in which workers exhibit both horizontal and vertical fairness (i.e., concerns about the fairness of the employer’s actions). In fact, while vertical reciprocity may be plausibly ignored in contexts where social distance between employer and workers is high, as in large firms (Henning-Schmidt et al., 2010), it may affect both the employer’s and employees’ actions in those contexts which are characterized by low social distance. In our case, in particular, workers may consider the relative compensation scheme as an unfair offer and react to it by not accepting to exert extra effort. Consequently, the employer, anticipating the workers’ negative reaction, may prefer to offer a different compensation scheme. We consider two extreme scenarios. In the first, we assume additivity of vertical and horizontal reciprocity concerns. In this case, the relative compensation scheme defined in the presence of horizontal reciprocity concerns remains the optimal one. In the second, we assume that horizontal and vertical reciprocity concerns...

²Moreover, our results are consistent with the evidence reported by Van Echtelt et al. (2007) after analyzing the Time Competition Survey on a sample of Dutch firms. The authors report that work pressure (defined as workers’ negative motivation) is predictive of doing additional unpaid work.
reciprocity are mutually exclusive, as evidenced in Eisenkopf and Teyssier (2009). In this case, we identify the conditions under which a profit-maximizing employer still prefers a relative compensation scheme (inducing negative reciprocity) to an individual compensation scheme (inducing positive reciprocity).

Therefore, in the presence of workers motivated by vertical reciprocity concerns, our main result holds, evidencing that even in this case, the employer exploits uses employees’ horizontal reciprocity concerns to obtain extra effort provision at no monetary cost.

Our results crucially depend on the presence of workers’ reciprocity concerns. Extra effort may be elicited from standard agents through adequate compensation schemes. However, while compensation schemes designed for standard agents are required to pay positive monetary compensation in equilibrium in order to induce extra effort, in our model, the presence of workers’ reciprocity concerns suggests that an optimal compensation scheme is one that does not pay any monetary prize in equilibrium.

The paper is organized as follows. Section 2 briefly discusses the related literature. Section 3 illustrates the model and discusses the definition of horizontal reciprocity. Section 4 characterizes the optimal contract under both symmetric and asymmetric information. Section 5 presents some extensions to the base model. Section 6 introduces vertical reciprocity concerns. Section 7 concludes. All proofs are contained in the Appendix.

2 Related Literature

Our results add to the recent literature which investigates how organizations can motivate workers by substituting social for monetary incentives (Bandiera et al., 2009; Dur and Sol,
Designing effective incentive schemes has a crucial relevance in determining the success of an organization. Recent empirical evidence, both from laboratories and the field, have assessed the existence and relevance of other-regarding preferences as motivators of human behavior. In workplaces, as well as in other contexts, individuals are not motivated solely by self-interest but also care - positively or negatively - about material payoffs from relevant others whom they choose as referents. Therefore, when designing incentive schemes, other-regarding preferences deserve to be adequately taken into account. Workers’ effort may be affected not only by monetary compensation but also by the way in which other-regarding preferences respond to own and other workers’ payoffs.

Our analysis focuses on reciprocity which identifies the willingness to respond fairly to kind action and unfairly to unkind actions (Rabin, 1993). Reciprocity seems to be one of the most relevant factors in motivating workplace behaviors (Akerlof, 1928, Englmaier and Leider (forthcoming)). We concentrate on horizontal reciprocity in order to capture what, according to Social Comparison Theory, is a natural tendency: people make comparisons, especially to others similar to themselves (Festinger, 1954). Differently from vertical fairness, reciprocity among peers has not been extensively analyzed in the workplace. Studies on other-regarding preferences among peers have, theoretically and empirically, focused on peer pressure (Kandel and Lezaer, 1992; Mas and Moretti, 2009), conformism (Gächter and Töni, 2002) and Rotemberg (2006), respectively, review experimental and theoretical results on other-regarding preferences in the workplace.

4Intrinsic motivation, crowding-out (Gneezy and Rustichini, 2000), and over-justification effects (Bénabou and Tirole, 2006) are some of the best known examples of unexpected negative effects resulting from errors in the incentive systems’ design.

5Vertical reciprocity has been extensively analyzed since the seminal paper by Akerlof (1982). For a survey of experimental results, see Fehr and Gächter, (2002).
2009), inequity aversion (Rey Biel, 2008; Englmaier and Wambach, 2010), social interactions (Dur and Sol, 2010), and altruism (Rotemberg, 1994), but not specifically on reciprocity among colleagues. Moreover, mutual help among employees (Corneo and Rob, 2003), the social sanctioning of free riders (Carpenter and Matthews, 2009), and social support among coworkers (Mossholder et al., 2005) may be interpreted as manifestations of reciprocity. In the workplace repetitive interactions and team work create an environment in which each worker may affect the team’s activity and the compensation of other team members. In such contexts, horizontal reciprocity matters because each worker compares what he (and other team mates) earn with what he would have obtained as a consequence of alternative choices made by his colleagues.\(^6\)

We model reciprocity as in Cox et al. (2007), where distribution of the material outcomes and the kindness (unkindness) of others’ choices affect a person’s emotional state. This emotional state, then, determines the marginal rate of substitution between own and others’ payoffs and affects the person’s subsequent choices. Differently from the approaches where reciprocity is modeled in terms of beliefs regarding intentions (Rabin 1993), Cox et al.’s (2007) formulation defines reciprocity in sequential games where the fairness judgment is essentially based on actual behaviors rather than beliefs and expectations. Therefore, this formulation gains in tractability and still captures the relevance of intentions since the distribution of material outcomes is intended to reveal the others’ intentions\(^7\). Consistently with Cox et al. (2007), in our sequential model, the fairness of the colleague’s strategy is

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\(^6\)In Kahneman et al. (1986), this definition refers to a comparison between what the worker (and other team mates) earn and what s/he thinks s/he (and other team mates) are entitled to.

\(^7\)Empirical evidences have shown that simple models of other-regarding preferences, e.g., inequality aversion, are not able to capture the intention component of agents’ action. See Falk et al. (2003).
evaluated by looking at the material consequences for the worker’s utility function, as we are convinced that, at least in the workplaces, the reciprocal response between workers is driven by actual behaviors more than by beliefs and expectations.

Results similar to ours have been obtained in a different framework by the theoretical studies of Rey-Biel (2008) and Dur and Sol (2009). In Rey-Biel (2008), a profit-maximizing employer exploits the inequity aversion of his/her workers to induce effort without fully compensating its cost, while offering a relative performance contract. While in Rey-Biel’s paper workers exhibit inequality aversion deriving disutility from differences between themselves and others, in our model each worker evaluates the colleague’s fairness by comparing the material consequences of the chosen strategy against those of the unchosen one. Moreover, while Rey-Biel (2008) does not consider workers’ other-regarding preferences toward the employer, we include them in the analysis since it is reasonable to assume that, in some work contexts, employees may form a judgment about the fairness or equity of the employer’s offer.

Our paper proposes a complementary perspective to that provided by Dur and Sol (2010), who have evidenced the circumstances where it may be in the interest of managers to encourage friendship formation between employees in order to attract and retain workers. In their model, workers can devote part of their effort to social interaction with their colleagues. In equilibrium, positive reciprocity arises because a worker who is treated kindly will care more about the well-being of his/her colleagues. This implies an increase in job satisfaction which offsets lower wages. Similarly, in our model, even if the characterization of reciprocity is different, monetary incentives and other-regarding preferences are substitute means that

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8 Appendix A1 discusses Cox et al.’s (2007) formulations and derives the ones discussed here. For a discussion on the role of beliefs and expectations as well as real behavior in conceptualizing reciprocity, see Perugini et. al. (2003).
an employer may use in order to obtain a desired output. In our model, reciprocity is related to what happens in the workplace (and hence is deeply affected by the incentive system), while in Dur and Sol’s (2009) model, "being kind" is equivalent to showing "interest in the colleague’s personal life, offering a drink after working hours...", (Dur and Sol (2009), p. 2). Consequently, while Dur and Sol study the case where workers’ positive social relations at work are also beneficial for the manager, we characterize a situation where the manager finds it convenient to have workers competing even at the cost of deteriorating the social relationships in the workplace.

Finally, we show that horizontal reciprocity may provide a rationale for the composition of teams of workers, even if production technology induces negative externalities among the workers’ efforts. Gould and Winter (2009) show that the presence of strategic interdependencies among workers’ actions affects worker’s action choice. In their model, depending on the value of the project, a employer may find it optimal to employ only one or two workers because of the strategic substitutability of production technology. We show that workers’ reciprocity is a reason for composing teams of two workers in situations where one standard worker would otherwise be employed. Our result is based on the endogenous complementarity (Potter and Suetens, 2009) of workers’ actions induced by reciprocity, which mitigates the impact of the negative externalities imposed on the workers by production technology.

3 The Model

Extra effort provision is modeled in a frame where a risk neutral employer \((P)\) engages a team of two risk neutral workers: \(A_i\), with \(i \in \{1, 2\}\), where the index refers to the timing
of the worker’s action\(^9\). Employer and workers contract some activities additional to those included in the job contract, typically an *extra task* or *overtime*. For this reason we assume that the participation constraints are satisfied\(^10\). The employer asks each worker to exert *extra effort*. Let \( a_i \in \{0, e\} \) with \( i = 1, 2 \) be the worker’s decision, where \( a_i = e > 0 \) and \( a_i = 0 \) indicate whether the worker accepts to exert extra effort or not. The cost of exerting extra effort is \( c(e) = c > c(0) = 0 \). We assume that workers are identical with respect to productivity and disutility of effort and that they can observe their colleagues’ choice. Finally, let \( X(\gamma, a_i, a_j) = \gamma(a_i + a_j) \) be the production function with \( \gamma > 0 \)^{11}. The timing of the extra effort game is as follows: at \( t = 0 \), the employer offers a compensation scheme for extra effort provision: \( w_i(a_i, a_j) \), for \( i, j = \{1, 2\} \) and \( i \neq j \). At \( t = 1 \), \( A_1 \), having observed the compensation scheme, decides whether or not to exert extra effort. At \( t = 2 \), \( A_2 \), having observed both the compensation scheme and the action chosen by the team mate, chooses \( a_2 \). Then production is realized and compensations are paid. We solve the game by backward induction. The employer maximizes the following profit function with respect to the workers’ effort:

\[
\Pi = \gamma(a_i + a_j) - (w_i + w_j)
\]

\(^9\)Henceforth, we will assume that the employer is female and that the employees are male.

\(^{10}\)In the rest of the paper, we will use the term "game" to denote the "extra effort provision game." We assume that there is no interdependence between this game and the "normal effort" or "normal working hours" game.

\(^{11}\)We only need to assume that the production function is increasing in agents’ effort, and we focus on the case in which employer’s profit is maximized when agents exert extra effort. Our results are not affected by the functional form of the production function. Therefore, we assume a linear function in order to keep the frame as simple as possible.
If $\gamma > \frac{\sigma_i}{c_i}$, with $i = 1, 2$, the employer earns her highest profit when both workers exert extra effort\textsuperscript{12}.

Let $M_i$ denote the worker’s material payoff:

$$M_i(w_i, c_i) = w_i(a_i, a_j) - c_i(a_i)$$

that is, the compensation received minus the cost\textsuperscript{13} of exerting extra effort.

Workers maximize the following utility function:

$$U_i(M_i, M_j, \rho_i^h, r_i) = M_i + \rho_i^h r_i \sigma_j M_j$$

where the exogenous parameter $\rho_i^h \in [0, 1]$ measures the impact of horizontal reciprocity concern on worker $i$’s utility function. We define as standard those workers with $\rho_i^h = 0$ and who care only about their own material payoff. Reciprocal workers are those workers who have $\rho_i^h > 0$ and also care about the colleague’s material payoff. The reciprocity term $r_i, \sigma_j$ determines the sign (positive or negative) of worker $i$’s reciprocity. Denote by $H_i$ and $L_i$, respectively, the highest and the lowest material payoff for $A_i$. Let $\sigma_j$ and $\sigma'_j$ be two strategies of $A_j$, with $\sigma_j \neq \sigma'_j$. Let $b_i^*(\sigma_j) = \sigma_i^*$ be $A_i$’s best response to strategy $\sigma_j$ chosen by $A_j$, such that $M_i(\sigma_i^*, \sigma_j) \geq M_i(\sigma_j, \sigma_j) \forall \sigma_i \neq \sigma_i^*$.

The reciprocity term of $A_i$, given that $A_j$ chooses the strategy $\sigma_j$, is defined as follows:

$$r_{i, \sigma_j} = \frac{M_i(b_i^*(\sigma_j), \sigma_j) - M_i(b_i^*(\sigma'_j), \sigma'_j)}{H_i - L_i} \in [-1, +1]$$

The reciprocity term in (4) is determined by the difference between the maximum material payoff that $A_i$ can obtain - given the strategy $\sigma_j$ chosen by $A_j$ - and the maximum material payoff that $A_i$ can obtain - given the strategy $\sigma'_j$ chosen by $A_j$.

\textsuperscript{12}In section (6.4) we modify the employer’s profit function, allowing for the employer to care about the fairness of the compensation scheme offered to the workers.

\textsuperscript{13}We assume that $c$ is the material equivalent of the disutility from exerting extra effort.
payoff that \( A_i \) could have obtained under the alternative strategy choice \( \sigma_j \). This difference is then normalized by \( H_i - L_i \).\(^{14}\) We assume that \( r_{i,\sigma_j} = 0 \) if the \( H_i = L_i \) is equal to 0. When \( r_{i,\sigma_j} > (\leq) 0 \), \( A_i \) positively (negatively) evaluates \( A_j \)'s material payoff. Hence, if \( M_j > 0 \) \((< 0)\), \( M_j \) enters \( A_i \)'s utility function as a positive (negative) externality.

The reciprocity term accounts for the intentionality of \( A_j \)'s choices. \( A_i \) evaluates \( A_j \)'s kindness by comparing how the \( A_j \)'s chosen and not chosen strategies affect his own material payoff.\(^{15}\)

In what follows, we design the optimal compensation scheme that the employer should offer to induce workers to exert extra effort. We assume that workers are already in the firm and that the participation constraints are satisfied. To avoid trivial solutions, we also assume that the employer cannot trigger her workers with negative compensations nor promise unlimited compensations even if they are not paid in equilibrium. Hence, we fix a budget \( B > 0 \) and assume \( w_i \geq 0 \), for both \( i \in \{1, 2\} \) such that \( w_1 + w_2 \leq B \).

### 4 The Optimal Compensation Scheme

In this section, we derive the optimal compensation schemes both for the case where the employer observes the workers' actions (section 4.1) and the case where she does not observe any individual actions but only the final output produced (section 4.2). In both sections, 4.1 and 4.2, we assume that the employer observes the employees' type \( \rho_i^h \) and that employees observe each other's action. Finally, in section 4.3 we characterize the optimal compensation scheme requiring the least payment to be offered out of equilibrium.

\(^{14}\)The magnitude of \( r_{i,\sigma_j} \) is determined by the numerator of eq.\((4)\).

\(^{15}\)The relevance of unchosen alternatives constitutes the main difference with respect to distributional models à la Fehr and Schmidt, (1999), where only the final relative distribution matters, Falk et al. (2003).
4.1 The symmetric information case

When the employer observes employees’ actions, the compensation scheme may be conditional on the latter. Let us use $w^S_i(a_i, a_j)$ for $i, j = 1, 2$ with $j \neq i$ to denote the optimal compensation scheme for standard workers ($\rho^h_i = 0$). This scheme will be used as a benchmark. The optimal compensation scheme $w^S_i(a_i, a_j)$ is such that, irrespectively of the action chosen by the team mate, each worker receives a compensation $w^S_i(e_i, a_j) = c$ if he exerts extra effort and $w^S_i(0, a_j) = 0$ otherwise,\(^{16}\) for both $i, j = 1, 2$, with $j \neq i$. The employer pays a compensation equal to $2c$ and obtains $\Pi^S = 2(\gamma e - c)$ as profit.

The following proposition describes the optimal compensation scheme when workers are reciprocal.

**Proposition 1** Under symmetric information and $\rho^h_i > 0$ for $i = 1, 2$, the optimal compensation scheme is a tournament that induces negative horizontal reciprocity. Each worker receives a monetary compensation equal to $B$ if and only if he is the only one exerting extra effort, and no compensation otherwise. When $B \geq (\frac{1}{\min\{\rho^e_i, \rho^e_j\}} + 1)c$, $\Pi = 2\gamma e$ and no monetary compensations are paid in equilibrium. When $B \in \left(0, (\frac{1}{\min\{\rho^e_i, \rho^e_j\}} + 1)c\right)$, then the employer still obtains $\Pi = 2\gamma e - (w_i + w_j)$ by paying each employee a monetary compensation positive but lower than $2w^S_i$.

**Proof.** See Appendix A.2. □

The optimal compensation scheme in proposition 1 induces a unique equilibrium in dominant strategies, in which $A_2$ exerts extra effort, irrespectively of the action of the $A_1$, and $A_1$ exerts extra effort as well.

\(^{16}\)This is only one of several possible optimal compensation schemes. Note that $w_1(e_1, 0)$ and $w_1(0, 0)$ refer to output levels that, given the incentives provided to $A_2$, are never produced. This implies $w_1(e_1, 0)$ and $w_1(0, 0)$ can take any value in the interval $[0, B]$. Depending on the values specified for each of them, we have different optimal compensation schemes implementing $2\gamma e$ at the cost of $2c$. 
Figure 1 illustrates the optimal compensation scheme. The intuition of the result is as follows. Consider $A_2$ first. Suppose $A_1$ has chosen his action. If $A_2$’s action does not affect $A_1$’s material payoff, then $A_2$ chooses the action that maximizes his own material payoff.

If $A_2$’s action modifies $A_1$’s material payoff, then $A_2$ chooses the action that maximizes his own utility, which is not necessarily the action that provides him with the maximum $M_2$. In this case horizontal reciprocity plays a role since $A_1$’s material payoff enters as an externality into $A_2$’s utility function. If $A_1$ chooses $a_1 = e$, this prevents $A_2$ from gaining his highest material payoff $w_2(0,e_2) = B$ and therefore motivates $A_2$ to adopt a negative attitude toward $A_1$. For this reason $A_2$ prefers to exert extra effort even if this reduces his material payoff.

Since $A_1$’s choice of extra effort enters into $A_2$’s utility function as a negative externality and this externality is increasing with the value of $w_1(e_1,0)$, $P$ will find it convenient to fix out of equilibrium the highest possible compensation for $w_1(e_1,0) = B$. It follows that $A_1$ will prefer to exert unpaid extra effort to avoid such a large negative externality. In fact, if the negative externality is higher than the cost of exerting extra effort, $A_2$ will work for free. In this way he will avoid the situation of not exerting extra effort, while $A_1$ does so, receiving $w_1(e_1,0) = B$. By a similar argument, $A_1$ anticipates $A_2$’s behavior and chooses to exert extra effort.

The minimum level of payment that must be offered out of equilibrium to induce unpaid extra effort is $B = \left( \frac{1}{\min \{ h_i, h_j \}} + 1 \right) c$. Note that $B$ is increasing with the disutility of effort $c$.
and decreasing with $\rho_i^h$, with $i = 1, 2$. Intuitively, for any given compensation offered out of equilibrium, the higher the impact of workers’ horizontal reciprocity concerns, the easier it becomes for the employer to induce unpaid extra effort. Note that when $\rho_i^h \to 1$, $A_i$ weights $A_j$’s material payoff almost as his own. In this case, the $B$ that must be offered out of equilibrium approximates $2w_i^S(e_1, e_2) = 2c$, which is the budget required to induce extra effort by standard workers. Similarly, the greater the disutility of workers’ effort, the larger is the $B$ that must be offered to exploit reciprocity concerns.\footnote{By offering this compensation scheme, the employer puts her workers in a situation similar to a sequential prisoner’s dilemma, where each worker is unable credibly to commit to not providing overtime once the colleague has declined to do so. Of course, one could reasonably object that a repetition of this game could provide agents with an incentive for colluding. However, we believe that the one-shot nature of our game better captures the non-regularity of overtime demand.}

In our model, if the employer demands extra effort from both employees, a compensation scheme inducing positive reciprocity is always more costly than a compensation scheme offered to standard employees as long as we do not remove the assumption that the employer maximizes a profit function as in eq(1).\footnote{See Appendix A.3 for a formal proof. In Appendix A.4, we also show that, when the optimal compensation scheme designed for standard workers is offered to reciprocal workers, the ORPs are neutralized.}

In addition, note that when the employer is able to observe $\rho_i^h$, she always prefers to demand extra effort from reciprocal types because she obtains the highest output at no monetary cost.

**Proposition 2** The employer prefers to employ reciprocal workers rather than standard workers.

**Proof.** See Appendix A.5. ■

In the Appendix, we rank the employer’s preferences regarding the composition of teams.
We show that a team composed of two reciprocal workers is always preferred to a team composed of a standard worker and a reciprocal worker. Hence, a team composed of a standard and a reciprocal worker is always preferred to a team composed exclusively of standard workers.

4.2 The asymmetric information case

In this section and the rest of the paper, we assume that $P$ only observes the employees’ type $\rho_i^h$ and the output level produced by the team. Under asymmetric information, a complete compensation scheme specifies the rewards offered to each worker conditional on the total output and its being profit maximizing. In this regard, three different output levels can be defined: $2\gamma e > \gamma e > 0$, depending on whether, respectively, two workers, one worker, or any worker exert extra effort.

As under symmetric information, we use the case with standard workers as a benchmark. In this case, the scheme assigns to each worker a compensation equal to $w_i(2\gamma e) = c$, if $2\gamma e$ is produced, and no compensation otherwise. The employer obtains $\Pi^S = 2(\gamma e - c)$ by paying a compensation equal to $2w_i(2\gamma e) = 2c$.

**Proposition 3** Under asymmetric information, if $\rho_i^h > 0$ for both $i = 1, 2$, then the optimal compensation is an asymmetric payment scheme that induces negative horizontal reciprocity. $A_1$ receives a positive monetary compensation equal to $B^A$ if and only if $\gamma e$ is produced and $A_2$ receives a positive monetary compensation equal to $B^A$ if and only if none exerts extra effort.

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19 There are indeed many situations in which managers cannot monitor workers, while the workers can observe each other: e.g., in professional jobs and research activities.

20 As in the symmetric information case, this is only one of several possible compensation schemes that maximize the employer’s profit. Given the incentives provided to $A_2$, $\gamma e$ is never produced. Hence, depending on the value specified for $w_1(\gamma e) \in [0, B]$, we have different optimal compensation schemes implementing $2\gamma e$ at a cost equal to $2c$.

21 Note that, since both the employer (principal) and the workers (agents) are risk neutral, under asymmetric information we do not observe loss of efficiency due to the distortion in the risk allocation among the parties.
extra effort. When \( B^A \geq \max \left\{ \frac{c}{\rho_1^2}, \frac{c^{1+1/(4\rho_2^1)}}{2\rho_2^2} \right\} \), \( P \) obtains \( \Pi = 2\gamma e \) without paying any compensation in equilibrium, while when \( B^A \in \left( 0, \max \left\{ \frac{c}{\rho_1^2}, \frac{c^{1+1/(4\rho_2^1)}}{2\rho_2^2} \right\} \right) \), the employer obtains \( \Pi = 2\gamma e \) by paying as compensations a sum lower than \( 2w_1(2\gamma e) \).

**Proof.** See the Appendix A.6. ■

The optimal compensation scheme in proposition 3 induces a unique equilibrium, which survives to the iterated elimination of dominated strategies. In equilibrium, \( A_2 \) exerts extra effort in the first subgame but not in the second, and \( A_1 \) exerts extra effort. The intuition of this result is similar to the one valid for proposition 1. Inspection of figure 2 shows that the main difference with respect to the symmetric information case is that \( P \) cannot condition the compensation scheme on the individual actions but only on the output level.

[Figure 2 about here]

Consider \( A_2 \). If \( A_1 \) does not exert extra effort, \( A_2 \) will not do so because this action maximizes his material payoff: \( w_2(0) = B^A \). If \( A_1 \) exerts extra effort, \( A_2 \) has an incentive to exert extra effort as well. When \( A_2 \) is motivated by negative reciprocity, then not exerting extra effort (allowing \( A_1 \) to gain \( w_1(\gamma e) = B^A \)) may be even worse than working unpaid. The negative attitude of \( A_2 \) follows from the fact that \( A_1 \), by choosing to exert extra effort rather than decline it, prevents him from obtaining his highest material payoff.

The key assumption behind this result is that, while we assume that \( P \) cannot monitor workers’ actions, we also assume that she is still able to distinguish reciprocal workers from standard ones. This enables her to offer an information revelation scheme, inducing unpaid extra effort under asymmetric information. Given that each worker observes his colleague’s
action, the employer exploits the second mover’s reaction to offer an information revelation incentive scheme.

The minimum level of payment that $P$ must offer out of equilibrium to induce unpaid extra effort is different for each worker, and we denote it by $B_A^A = \max\left\{ \frac{c}{\rho_i^h}, c \frac{1+(1+4\rho_i^h)^{\frac{1}{2}}}{2\rho_i^h}\right\}$. Note that $B_A^{22}$ is increasing in the disutility of effort and decreasing in $\rho_i^h$, with $i = 1, 2$. This result implies that when workers have the same $\rho^h$, worker 2 requires the highest payment out of equilibrium to exert unpaid extra effort, that is, $w_2 = B_A = c \frac{1+(1+4\rho_i^h)^{\frac{1}{2}}}{2\rho_i^h}$. Finally, note that, for both $\rho^h \to 1$ the $B_A^A$ that must be offered out of equilibrium is slightly higher than $w_i^S(e_1, e_2)$. As the $\rho^h \to 0$, $B_A^A$ offered out of equilibrium goes to $+\infty$.

### 4.2.1 The least budget-demanding optimal compensation scheme

In the previous sections, we assumed that $P$ has an unlimited amount of money $B$ to be offered out of equilibrium. As highlighted above, depending on $B$, several optimal compensation schemes may be defined. However, it is likely that in some situations the budget is limited, i.e., due to binding financial constraints. Since the credibility of the payments fixed out of equilibrium plays a crucial role in our framework, it makes sense to identify the optimal scheme requiring the lowest possible level of $B$. Let us provide the following definition to such a scheme.

**Definition 1** The least budget-demanding (LBD) optimal compensation scheme is the optimal compensation scheme requiring the smallest payment $B$ to be offered out of equilibrium such that both $A_1$ and $A_2$ exert unpaid extra effort.

22 Where the index "A" allows for its distinction from the $B$ offered under symmetric information.
In this respect, we can show that:

**Proposition 4** For any $\rho_i^h$ and $\rho_j^h$, with $\rho_i^h > \rho_j^h$, an LBD optimal compensation scheme always exists, assigning the first move to the $A_j$ (leader) and the second to the $A_i$ (follower). The optimal compensation scheme is an asymmetric compensation scheme such as the one described in proposition 2.

**Proof.** See the Appendix A.7. ■

This result contains an implication particularly useful for job design if only a limited budget is available to $P$. Since she knows the reciprocity concern of each worker, she will always find it convenient to assign the second move to the worker with the higher $\rho_i^h$, thereby obtaining the desired outcome at no cost.

## 5 Extensions

In this section, we present some extensions to the base model. In section 5.1, we consider the case where the employer has a limited budget, i.e., lower than the monetary compensation required to induce workers to exert unpaid extra effort in equilibrium, as indicated in propositions 1 and 3. In section 5.2, we consider a production technology that exhibits negative externalities between employees’ actions such that for the employer it would be optimal to employ only one worker if the workers exhibit $\rho_i^h = 0$.

### 5.1 The optimal compensation scheme with budget constraint

In the previous sections, we assumed that $P$ has a budget sufficient to induce unpaid extra effort. Let $B^F$ denote such a feasible budget. Here, we analyze the case where $B^F$ is lower than the level required in propositions 1 and 3, respectively.
Proposition 5 When $0 < B^F < B$ ($B^A$), the employer obtains $\Pi = 2\gamma e - 2w^S$ ($\Pi = 2\gamma e - 2w(2\gamma e)$) by paying the employees a sum of compensations lower than $2w^S (2w(2\gamma e))$. Savings are increasing in the amount of the feasible budget.

Proof. See the Appendix A.8. ■

The result can be explained by the substitutability between reciprocity concerns and monetary incentives, i.e., material payments. When $B^F \in [B; +\infty)$, reciprocity concerns and incentives are perfect substitutes. When $0 < B^F < B$, reciprocity concerns and monetary incentives are imperfect substitutes. Therefore, in the second case, in order to obtain the highest output, $P$ must pay in equilibrium a positive amount of compensation, which is still lower than $2w^S(2w(2\gamma e)) = 2c$. This implies that, even if extra effort needs to be compensated with a monetary payment, some savings may still be achieved with respect to the benchmark case. This result highlights that, in our model, reciprocal workers are always preferred to standard workers.

5.2 Production technology with negative externalities

In the previous sections, we assumed a functional form of the production function which did not impose any technological interdependencies among the workers. Now let us consider a production technology with negative externalities: $X'(\gamma, \beta, a_i, a_j) = \gamma(a_i + a_j) - \beta(a_i a_j)$ with $\gamma > \beta > 0$, where $\beta$ measures the level of negative externality from exertion of joint extra effort. It is also assumed that two workers exerting extra effort are more productive than one: $X(2\gamma e - \beta e^2) > X(\gamma e) > 0$, and furthermore assumed that the employer maximizes

---

23According to Potter and Suetens (2009), a game is characterized by strategic complements (substitutes) if $\forall i, j$ and $i \neq j$: $\frac{\partial^2 u_i}{\partial a_i \partial a_j} > 0$ ($< 0$). Games characterized by strategic substitutability or strategic complementarity have externalities (at least locally): this follows from the fact that $\frac{\partial^2 u_i}{\partial a_i \partial a_j} > 0$ ($< 0$) implies $\frac{\partial u_i}{\partial a_i} > 0$ ($< 0$).
her profits if only one standard worker exerts extra effort: \( \Pi(\gamma, 0, e_j) > \Pi(\gamma, e_i, e_j) > 0 \) for \( i = 1, 2 \) and \( i \neq j \).

If both these assumptions hold, we obtain the following result:

**Proposition 6** Under a production technology characterized by negative externalities: \( X'(\gamma, \beta, a_i, a_j) \) and for \( \beta \in \left( \frac{\gamma - \varepsilon}{\varepsilon}; \frac{\gamma}{2} \right) \), the employer will employ one worker, if he exhibits standard preferences, while she will form a team of two workers if they are reciprocal.

Our model complements the findings by Gould and Winter (2009), who analyze how the effort choices of selfish workers interact according to the production technology. In their model, a principal can employ one or two workers to sequentially carry out an individual task, which contributes to the success of a project. When production technology exhibits strategic complementarity (substitutability), the task completion by one worker contributes more (less) to the success of the entire project if the other worker also completes his/her task.

We show that workers’ reciprocity is a reason for composing teams of two workers in situations where otherwise one standard worker would be employed. The intuition of this result is that, since reciprocity induces endogenous complementarity among workers (Potter and Suetens, 2009), it mitigates the negative externalities imposed by production technology. Therefore, by hiring reciprocal workers and offering them a compensation scheme such as that defined in propositions 1 and 3, the employer obtains the desired output at no monetary cost.
6 Vertical Reciprocity

The results presented in the previous sections are based on the assumption that vertical reciprocity does not affect workers’ motivation. However, in many situations this assumption may not hold\(^{24}\). Thus, in this section we allow for the employer to choose between a relative and an individual compensation scheme, and we assume that besides horizontal fairness concerns, workers also exhibit vertical reciprocity.

In section 6.1, we define vertical reciprocity. Subsequently, in section 6.2 we show that our main result, defined in proposition 1, is robust to this extension as long as vertical and horizontal reciprocity concerns are additive. In section 6.3, we derive the conditions under which the employer still prefers to offer a relative compensation scheme when workers’ vertical and horizontal reciprocity concerns are mutually exclusive. Finally, in section 6.4 we assume that the employer exhibits concerns about how workers perceive the compensation scheme offered to them. We then derive the conditions under which the employer prefers to offer an individual compensation scheme inducing positive vertical reciprocity rather than a relative compensation scheme inducing negative vertical reciprocity.

6.1 The vertical reciprocity formulation

In order to introduce vertical reciprocity, let us define both an actions set for the employer and a vertical reciprocity component for employees’ utility function.

We assume that \( P \) will offer either a relative or an individual compensation scheme, as shown in figure 3. Individual compensation coincides with that defined in section 4.1 for standard workers, where compensations are as follows: \( w^i_s(e_i, a_j) = c \) if worker works extra

\(^{24}\)See, e.g., Fehr and Gächter (2002) for a survey.
and \( w_i^S(0, a_j) = 0 \) otherwise, for \( i, j = 1, 2 \), with \( j \neq i \). The relative compensation scheme coincides with the tournament defined in proposition 1, section 4.1, where \( A_i \) receives a compensation equal to \( w_i(e_i, 0) = B \) if he is the only one exerting extra effort, and no compensation otherwise: \( w_i(0, 0) = w_i(e_i, 0) = w_i(e_i, e_j) = 0 \).

As shown previously, if \( P \) offers the relative compensation scheme, both workers, being motivated by negative horizontal reciprocity, exert extra effort to prevent the other from obtaining a reward. It follows that the employer will acquire the workers’ extra effort at no cost. On the contrary, when individual compensation is offered, each worker decides to exert extra effort, and hence his compensation is independent from his colleague’s choice, \( r^h = 0 \).

The relevance of vertical reciprocity in \( A_i \)'s utility function is captured by the term \( \rho_i^v \in [0, 1] \). The vertical reciprocity term \( r_{i,\sigma_P} \) determines the magnitude of \( A_i \)'s reciprocity toward \( P \). Denote by \( H_i \) and \( L_i \), respectively, the highest and lowest material payoff for \( A_i \). Let \( \sigma_P \) and \( \sigma_P' \) be two strategies of \( P \), with \( \sigma_P \neq \sigma_P' \). Define \( b_i^*(\sigma_P) = \sigma_i^* \) as \( A_i \)'s best response to strategy \( \sigma_P \) chosen by \( P \) such that \( U_i(b_i^*(\sigma_P), b_P(\sigma_i)) \geq U_i(\sigma_i, \sigma_P) \forall \sigma_i \neq \sigma_i^* \), for any \( i = 1, 2 \). Denote by \( M_i^* \) \( A_i \)'s material payoff associated to the best response such that \( M_{i,\sigma_P}^* \in U_i(b_i^*(\sigma_P), b_P(\sigma_i)) \), for any \( i = 1, 2 \). Given that \( P \) chooses the strategy \( \sigma_P \), the vertical reciprocity of \( A_i \) is defined as follows:

\[
r_{i,\sigma_P} = \frac{M_{i,\sigma_P}^* - M_{i,\sigma_P'}^*}{H_i - L_i} \in [-1, +1] \tag{4.1}
\]

The vertical reciprocity term in (4.1) is determined by comparing the material payoff that \( A_i \) obtains in the SPNE of each of the two subgames determined by the choice of \( P \). As
for horizontal reciprocity we normalize by \( H_i - L_i \). Note that when \( r_{i,s_p} > 0 \) \((< 0)\), \( A_i \) will consider as fair (unfair) \( P \)'s offer and, consequently, he will care positively (negatively) about \( P \)'s profit.

In the next two sections, we separately analyze the case in which horizontal and vertical reciprocity concerns are simultaneously present in the workers’ utility function and the case in which they are mutually exclusive.

6.2 Case I: additivity

In this section, we assume that vertical and horizontal reciprocity concerns are additive such that workers maximize the following utility function:

\[
U_i(M_i, M_j, \Pi, \rho_i^h, \rho_i^v, r_i) = M_i + \rho_i^h r_{i,s_j} M_j + \rho_i^v r_{i,s_p} \Pi
\]  

(3.1)

The following proposition describes the optimal compensation scheme in this case:

**Proposition 7** Under symmetric information and \( \rho_i^h, \rho_i^v > 0 \) for \( i = 1, 2 \), the optimal compensation scheme is a tournament that induces workers’ negative reciprocity both toward colleagues and employer. Each worker receives a monetary compensation equal to \( B^{HV} \) if and only if he is the only one exerting extra effort, and no compensation otherwise, where \( B^{HV} \geq \max \{ \varphi_1, \varphi_2 \} \) with \( \varphi_i = \frac{c(1+\rho_i^h+\rho_i^v)+\sqrt{c(1+\rho_i^h+\rho_i^v)^2+4\rho_i^h\rho_i^v c \gamma e}}{2\rho_i^h} \) for \( i = 1, 2 \). The employer obtains \( \Pi = 2\gamma e \).

**Proof.** See Appendix A.9. ■

The optimal compensation scheme in proposition 7 induces a unique SPNE in dominant strategies. In equilibrium, \( P \) offers a tournament scheme, and if she can promise out of equilibrium a budget \( B^{HV} = \max \{ \varphi_1, \varphi_2 \} \), she earns the highest profits \( \Pi = 2\gamma e \) by exploiting unpaid extra effort. Under this compensation scheme, workers motivated by negative reciprocity both toward colleagues and employer exert extra effort without being
compensated for it. Even in the presence of negative vertical reciprocity, they prefer, rather than refuse, to exert extra effort. This result recalls the compensation scheme defined in proposition 2, with the difference that $B^{HV} > B$ for any $\rho^v > 0$. In fact, when workers exhibit vertical reciprocity in addition to horizontal reciprocity, the minimum payment promised off equilibrium in order to induce unpaid extra effort should also compensate the disutility related to the unkind offer made by the employer.

Notice that, if $P$ had offered the individual compensation scheme, this would have induced positive vertical reciprocity by the workers. In particular, any individual compensation scheme assigning to the workers a positive compensation for exertion of extra effort induces positive vertical reciprocity. This follows from the fact that such a compensation scheme would be compared to the alternative relative compensation scheme which, in equilibrium, does not pay any monetary compensation. However, even if $P$ could induce extra effort at a lower cost than the one demanded by standard workers, her preferred choice would remain the relative compensation scheme since it induces extra effort by both workers at no cost.

6.3 Case II: mutual exclusivity

In this section, we assume that vertical and horizontal reciprocity concerns are mutually exclusive. Experimental findings evidence, on the one hand, that both envy between the workers and reciprocity toward the employer are relevant in determining tournaments’ effectiveness and, on the other hand, that these two fairness concerns seem to be mutually exclusive (see Eisenkopf and Teyssier, 2009). A formulation in line with such findings is:

$$U_i(M_i, M_j, \Pi, \rho^h_i, \rho^v_i, r_i) = \begin{cases} M_i + \rho^h_i r_{i,\sigma_j} M_j & \text{if } \rho^h_i > \rho^v_i, \\ M_i + \rho^v_i r_{i,\sigma_p} \Pi & \text{if } \rho^h_i \leq \rho^v_i. \end{cases}$$

(3.2)
Equation (3.2) indicates that the reciprocity concerns which have the highest weight will prevail in the worker’s utility function. Therefore, we have two cases. If horizontal reciprocity concerns are stronger, \( \rho_i^h > \rho_i^v \), we return to the situation described in section 4.1, and the optimal compensation scheme is defined in proposition 1.

If, instead, the reciprocity toward \( P \) is stronger, i.e., \( \rho_i^v > \rho_i^h \), the optimal compensation scheme is described in proposition 8 below.

**Proposition 8** Under symmetric information and \( \rho_i^v > \rho_i^h \geq 0 \) for \( i = 1, 2 \),
a) if \( \gamma e \geq 2c(1 + \rho_i^v) \), the optimal compensation scheme chosen by the employer is a tournament that induces negative vertical reciprocity. Each worker receives a monetary compensation equal to \( B^V = \frac{c(1-\rho_i^v)+\sqrt{[-c(1-\rho_i^v)]^2+4c\gamma e}}{2} \) if he exerts extra effort, and no compensation otherwise. In equilibrium only the first mover exerts extra effort, and the employer earns profits \( \gamma e - B^V \);

b) if \( \gamma e < 2c(1 + \rho_i^v) \) the optimal compensation scheme chosen by the employer is the individual payment scheme that induces positive vertical reciprocity. Each worker receives a monetary compensation equal to \( c \) if he exerts extra effort, and no compensation otherwise. In equilibrium both workers exert extra effort, and the employer obtains \( \Pi = 2(\gamma e - c) \).

**Proof.** See Appendix A.10. ■

In the case where \( \rho_i^v > \rho_i^h \), the optimal compensation scheme is illustrated in figure 3. When the individual compensation scheme is offered, both workers exert extra effort and receive a monetary compensation equal to \( w^S(e_i) = c \). Therefore, in this subgame’s SPNE the employer obtains \( \Pi = 2(\gamma e - c) \) as profits\(^{25}\).

\(^{25}\)When the individual compensation scheme is offered, it induces positive vertical reciprocity by the workers. This implies that under this compensation scheme, workers are willing to exert extra effort for a monetary compensation equal to \( k_i c \), where \( k_i = \frac{(B^V+\rho_i^v\gamma e)-(B^V+\rho_i^v\gamma e)^2-4B^V\gamma e}{2\rho_i} < 1 \). Therefore, by employing reciprocal workers, the employer obtains extra effort by paying a compensation lower than the one designed for standard agents. In this case, the results do not change qualitatively, although the complexity of the analysis increases (the proofs are available upon request). Hence, for simplicity, we consider the individual compensation scheme defined in proposition 8.
When the relative compensation scheme is offered, $A_2$ judges as unfair $P$’s offer and, therefore, to prevent her from earning her highest profits, he prefers not to exert extra effort. The difference in $A_2$’s behavior is mainly explained by the irrelevance of the horizontal reciprocity concerns. In fact, $A_2$ does not suffer if $A_1$ receives high material payoff $B^V$, and he prefers that $A_1$, rather than $P$, gains something.

$A_1$, anticipating the shirking behavior of $A_2$, experiences two contrasting feelings. On the one hand, exerting extra effort provides him with the highest monetary compensation $B^V$; on the other hand, choosing this action, he suffers a loss of utility due to negative reciprocity. Since he is the only one exerting extra effort, he still lets $P$ earn positive profits. $P$’s profit in the SPNE of this subgame is equal to $(\gamma e - B^V)$.

Whether the tournament or the individual compensation scheme are going to be offered, depends on $P$’s profits. When $\gamma e \geq 2c(1 + \rho_1^u)$, the tournament maximizes $P$’s profits even if, after/by offering a tournament, only one worker exerts extra effort. In the other case, the most profitable compensation scheme is the individual compensation scheme which offers to the reciprocal agents $w^S$.

6.4 Employer with fairness concern about workers

In this section, we extend the model presented in section 6.1 by assuming that $P$ exhibits concern about the workers’ perception of the compensation scheme offered. In designing the compensation schemes for their workers, managers seem to care about how those compensation schemes are perceived (Agell and Lundborg, 1999). The utility function of the employer becomes:
\[ U_p = \Pi(\gamma, a_i, a_j, r_i, r_j, \sigma_p) + \delta [r_i, r_j, \sigma_p] \]  

(1.1)

where \( \delta \in [0, 1] \) indicates the impact of fairness concern on the employer’s utility function.

The optimal compensation is described in proposition 9 below.

**Proposition 9** Under symmetric information, if \( \rho^v_i, \rho^h_i \geq 0 \) two are the possible cases. When vertical and horizontal reciprocity concerns are additive, if \( \delta > \frac{B}{2} \), the optimal compensation scheme is an individual payment scheme that induces positive vertical reciprocity.

If vertical and horizontal reciprocity concerns are mutually exclusive, the individual compensation scheme is always optimal.

In equilibrium both workers exert extra effort, and the employer obtains \( \Pi = 2(\gamma - c) \).

**Proof.** See Appendix A.11

Introducing concerns about the workers’ fairness perception in the employer utility function may therefore determine whether the individual compensation scheme is preferred to the relative compensation scheme. This happens both in the case where horizontal and vertical reciprocity are simultaneously present in the workers’ utility functions and the case where they are mutually exclusive.

### 7 Discussion

In this paper, we have presented a stylized model using horizontal reciprocity to provide a rationale for unpaid extra effort. We have shown that when the employer has a budget sufficient to offer credible compensations out of equilibrium, she can always induce reciprocal workers to exert productive extra effort without fully compensating its cost. This result holds both under symmetric and asymmetric information. We have also identified the minimal budget required to support a scheme inducing unpaid extra effort. In addition, we have
shown that when the employer has a budget below that amount, even when positive monetary compensation is paid, some savings can still be made by exploiting the workers’ reciprocity concerns. These results may have important implications for the ideal team composition. The employer always prefers teams of reciprocal workers rather than teams with one standard and one reciprocal worker. Consequently, a "one standard/one reciprocal" team is always preferred to a team composed only of standard workers. We have also developed an extension of the basic model which highlights the importance of horizontal reciprocity in the design of incentive systems characterized by a production technology imposing negative externalities among the workers. Under this extension, the employer demands extra effort from one worker if he is standard, while she prefers to employ teams of two workers if they are reciprocal. These results are derived on the assumption that vertical reciprocity concerns do not play any role in large firms, where the social distance between employer and workers is high (Henning-Schmidt et al., 2010).

Given the relevance of vertical reciprocity in itself and the potential interactions between vertical and horizontal fairness, we have extended the base model by incorporating fairness toward the employer. If the workers consider the relative compensation scheme as an unfair offer, it may be that the employer, anticipating the workers’ negative reaction, may prefer to offer a different compensation scheme. Therefore, we have considered the case where the employer can choose between a relative and an individual compensation scheme, and we have shown that our main results are robust to this extension as long as vertical and horizontal reciprocity concerns are additive. When horizontal and vertical reciprocity are mutually exclusive we derive the conditions under which a profit-maximizing employer would still prefer a tournament to an individual compensation scheme. Finally, we have analyzed the
case where the employer exhibits fairness concerns about workers and derived the conditions under which an individual compensation scheme inducing positive vertical reciprocity is preferred to a relative compensation scheme.

A final point deserves to be addressed. Unlike in our model, where the employer can determine how to assign the order of moves to the employees, there is also the case where extra effort is demanded from the workers simultaneously. In this scenario, employees face a simultaneous prisoner’s dilemma, where the dominant strategy for each worker is to exert extra effort so that the NE in pure strategies supports the outcome in which both workers exert unpaid (underpaid) extra effort. Even in a simultaneous move game, our main result holds: the employer prefers to employ reciprocal workers, thus obtaining unpaid extra effort. However, we chose a sequential game, being convinced that, at least in workplaces, it is actual behavior rather than beliefs and expectations that drive the reciprocal response between workers. Consistently, Cox et al.’s (2007) model has defined reciprocity in sequential games, where the fairness judgment is essentially based on actual behaviors rather than, as in the psychological game theory literature, on beliefs and expectations.

Our simple model emphasizes that the optimal contract for reciprocal workers differs considerably from the optimal contract for standard workers. In particular, an employer dealing with workers motivated by reciprocity will always benefit from a relative performance contract which uses competition between employees to achieve the desired outcome. The higher the payment that the employer can promise out of equilibrium, the easier it will be to induce reciprocal workers to exert underpaid or unpaid extra effort. Our results may have significant implications for real firms: if managers can credibly promise certain benefits to reciprocal workers out of equilibrium, they can exploit the employees’ other-regarding preferences as
sources of nonmonetary incentives to enhance productivity. Professional services, research institutions, and the knowledge industry are organizational settings in which the workers’ willingness to work hard to obtain career advancement or bonuses can be exploited by employers to induce competition. In this regard, we argue that, for real managers, our less striking result - underpaid rather than unpaid extra effort - may be the most important one because it provides a reliable source of economic advantage for the organization, minimizing the possible drawback associated with unpaid extra effort, that is, a negative attitude toward the employer.

A Appendix

A.1 Utility for reciprocal workers

We define the utility function of reciprocal workers using a simplified formulation of reciprocity presented in Cox et al. (2007, p. 22). Let consider the formulation presented in their paper (eq.1):²⁶

\[
U_i(M_i, M_j, \theta_i(s, r)) = \begin{cases} 
\frac{M_i^{\alpha} + \theta_i(s, r)M_j^{\alpha}}{\alpha} & \text{if } \alpha \in (-\infty, 0) \cup (0, 1], \\
(M_iM_j)^{\theta_i(s, r)} & \text{if } \alpha = 0,
\end{cases} \quad ((A.1.1))
\]

where player \(j\) is the first mover and player \(i\) the second mover, \(U_j\) and \(U_i\) represent the utility function of each player and \(M_i\) and \(M_j\) are the material payoffs each player receives, \(\alpha\) is the parameter of elasticity of substitution among the players’ utility functions and \(\theta(r, s)\) stands for the emotional state. Depending on the value of \(\alpha\) preferences may be linear (if \(\alpha = 1\)) or strictly

²⁶The functional form is tested through experiments on a dictator game, a Stackelberg duopoly game, a mini-ultimatum game and an ultimatum game with both random and contest role assignment.
convex (if $\alpha < 1$). Cox et al. (2007) uses the concept of emotional state, $\theta$, to characterize the attitude of player $i$ toward player $j$. It represents the willingness to pay own payoff for other’s payoff. The emotional state is assumed to be increasing both in the status, $s$, and in the level of reciprocity, $r$. The status is defined as the "generally recognized asymmetries in players’ claims or obligations" (p. 23) while the reciprocity corresponds to the difference between the maximum payoff that player $i$ can afford given the choice made by $j$ and a reference payoff "neutral in some appropriate sense" (p. 23).

Our definition of reciprocity is a simplified version of the functional form proposed by Cox et al (2007). In particular, we impose $\alpha = 1$ and by assuming identical workers, we abstract from the status concern. Finally, for the sake of simplicity, we assume that the emotional state is a linear function of reciprocity, i.e., $\theta(r_i) = \rho_i r_i$ where $\rho_i \in [0, 1)$ represents the impact of reciprocity concern on worker $i$’s utility function, and $r_i, r_j$ is the reciprocity term accounting for worker $j$’s fairness.

### A.2 Proof of Proposition 1

According to proposition 1 the optimal compensation scheme is:

$$w_i(e_i, e_j) = w_i(0, e_j) = w_i(0, 0) = 0; \quad w_i(e_i, 0) = B, \quad \text{for } i, j \in \{1, 2\} \text{ with } i \neq j \quad (A.2.1)$$

Note that for $A_1$, strategies and actions coincide. On the contrary, for $A_2$ strategies are defined as follows: $\sigma_2^a = \{e, e\}; \quad \sigma_2^b = \{e, 0\}; \quad \sigma_2^c = \{0, e\} \text{ and } \sigma_2^d = \{0, 0\}$.

In equilibrium, reciprocity for $A_1$ and $A_2$ are respectively defined as:

$$r_{1, \sigma_2^2} = \frac{-w_1(e_1, 0) + c}{w_1(e_1, 0)} < 0$$

$$r_{2, \sigma_2^2} = \frac{-w_2(0, e_2) + c}{w_2(0, e_2)} < 0$$
To induce both workers to exert extra effort in equilibrium, the following incentive compatibility constraints (hereafter, ICCs) must hold:

\[
\begin{align*}
    w_1(e_1, e_2) - c + \rho_1^h r_{1,\sigma_2} w_2(e_1, e_2) &\geq w_1(0, e_2) + \rho_1^h r_{1,\sigma_2} w_2(0, e_2), \\
    w_2(e_1, e_2) - c + \rho_2^h r_{2,\sigma_2} w_1(e_1, e_2) &\geq w_2(e_1, 0) + \rho_2^h r_{2,\sigma_2} w_1(e_1, 0).
\end{align*}
\] (A.2.2)

By substituting (A.2.1) respectively into (A.2.2) and (A.2.3) we obtain:

\[
\begin{align*}
    0 &\geq c + \rho_1^h (-B + c), \\
    0 &\geq c + \rho_2^h (-B + c).
\end{align*}
\] (A.2.4, A.2.5)

Rearranging (A.2.4) and (A.2.5) yields

\[
B \geq c \left( \frac{1}{\rho_i^h} + 1 \right) \quad \text{for } i = 1, 2 ,
\] (A.2.6)

where \( B \) is the monetary compensation to be offered out of equilibrium to induce both workers to exert unpaid extra effort \((w_i(e_i, e_j) = 0)\).

We proceed now proving that the compensation scheme in (A.2.1) induces a unique equilibrium in dominant strategies in which \(A_2\) exerts extra effort both in the first and in the second subgame and \(A_1\) exerts extra effort.

First we show that \(\sigma_2^* = (e, e)\) is the dominant strategy for \(A_2\). If \(A_1\) chooses to exert extra effort, \(a_1 = e_1\), the reciprocity for \(A_2\) is given by:

\[
r_{2,e_1} = \frac{\max\{0 - c, 0\} - \max\{(B - c), 0\}}{(B - c) - (0 - c)} = -\frac{B - c}{B} < 0.
\] (A.2.7)

The utility \(A_2\) gets if he exerts extra effort is: \(-c(1 + \rho_2^h r_{2,e_1})\), while the utility from not exerting it is: \(\rho_2^h r_{2,e_1} (B - c)\).
exertion of extra effort is the optimal action for $A_2$ in first subgame if

$$-c(1 + \rho_2^h r_{2,e_1}) > \rho_2^h r_{2,e_1} (B - c) \quad (A.2.8)$$

and rearranged it yields $\rho_2^h r_{2,e_1} B + c < 0$. Using (A.2.7) we obtain: $\rho_2^h B + c(1 + \rho_2^h) < 0$, yielding $B \geq c \left(\frac{1}{\rho_2^h} + 1\right)$ which is more restrictive than (A.2.6). Therefore, we have proven that in the first subgame the optimal action for $A_2$ is $a_2 = e_2$.

Suppose $A_1$ chooses $a_1 = 0$, the reciprocity for $A_2$ is given by:

$$r_{2,0} = \frac{\max \{(B - c), (0)\} - \max \{(0 - c), 0\}}{(B - c) - (0 - c)} = \frac{B - c}{B} > 0. \quad (A.2.9)$$

The utility $A_2$ gets if he exerts extra effort is: $B - c$, while the utility from not exerting it is 0. So, when $B > c$ the optimal action for $A_2$ in the second subgame is $a_2 = e_2$. This is always satisfied since by (A.2.6) $B > c$.

Consider $A_1$. We want to prove that $a_1 = e_1$ is the $A_1$'dominant strategy.

If $A_2$ plays $\sigma = \sigma_2^a$, the reciprocity for $A_1$ is given by:

$$r_{1,\sigma_2^a} = \frac{\max \{(0 - c), (0)\} - \max \{(B - c), 0\}}{(B - c) - (0 - c)} = -\frac{B - c}{B} < 0. \quad (A.2.10)$$

The utility $A_1$ gets if he exerts extra effort is $-c(1 + \rho_1^h r_{1,\sigma_2^a})$, while the utility from not exerting it is: $\rho_1^h r_{1,\sigma_2^a} (B - c)$. Of small extra effort is the optimal action for $A_1$ if:

$$-c(1 + \rho_1^h r_{1,\sigma_2^a}) > \rho_1^h r_{1,\sigma_2^a} (B - c) \quad (A.2.11)$$

By substituting (A.2.10) in to (A.2.11) and simplifying it, (A.2.11) yields $-c > -\rho_1^h B$, which always holds when (A.2.6) holds. Therefore, we have proved that, when $B \geq c \left(\frac{1}{\rho_1^h} + 1\right)$ and given that $A_2$ plays $\sigma_2^a$, the optimal action for $A_1$ is $a_1 = e_1$. 
Suppose now that $A_2$ chooses $\sigma^b_2 = \{e, 0\}$, in this case $A_1$’s reciprocity is:

$$r_{1, \sigma^b_2} = \frac{\max \{(0 - c), (0)\} - \max \{(B - c), 0\}}{(B - c) - (0 - c)} = -\frac{B - c}{B} < 0. \tag{A.2.12}$$

The utility $A_1$ gets if he exerts extra effort is $-c(1 + \rho^b_1 r_{1, \sigma^b_2})$, while the utility from not exerting it is: $\rho^b_1 r_{1, \sigma^b_2} (B - c)$.

Suppose $A_2$ chooses $\sigma^c_2 = \{0, e\}$, in this case $A_1$’s reciprocity is:

$$r_{1, \sigma^c_2} = \frac{\max \{(B - c), (0)\} - \max \{(0 - c), 0\}}{(B - c) - (0 - c)} = \frac{B - c}{B} > 0. \tag{A.2.13}$$

The utility $A_1$ gets if he exerts extra effort is $B - c$, while the utility from not exerting it is $\rho^b_1 r_{1, \sigma^c_2} (B - c)$ where $r_{1, \sigma^c_2} = \frac{B - c}{B}$.

Last, suppose $A_2$ chooses $\sigma^d_2 = \{0, 0\}$, in case $A_1$’s reciprocity is:

$$r_{1, \sigma^d_2} = \frac{\max \{(B - c), (0)\} - \max \{(0 - c), 0\}}{(B - c) - (0 - c)} = \frac{B - c}{B} > 0. \tag{A.2.14}$$

The utility $A_1$ gets if he exerts extra effort is $B - c$, while the utility from not exerting it is 0. To exert extra effort is always better than not exerting it if $B > c$, which is the case if (A.2.6) holds.

Therefore, $a_1 = e$ is $A_1$’s dominant strategy.

### A.3 A Compensation Scheme Inducing Positive Reciprocity

#### A.3.1 Symmetric information case

In this section we prove that a compensation inducing positive reciprocity for the exertion of extra effort by both workers is more costly than the compensation scheme for standard workers. The
total compensation paid to standard workers is $w_1^s(e_1, e_2) + w_2^s(e_1, e_2) = 2c$.

Now, consider $A_1$. When $A_2$ chooses strategy $\sigma_2^a$ then the reciprocity of $A_1$ is:

$$r_{1,\sigma_2^a} = \frac{\max \{w_1(e_1, e_2) - c, w_1(0, e_2)\} - \max \{w_1(e_1, 0) - c, w_1(0, 0)\}}{H_1 - L_1}. \quad (A.3.1.1)$$

Since $H_1 - L_1 > 0$ then $r_{1,\sigma_2^a} > 0$ if the numerator is positive. As $w_1(e_1, 0) = w_1(0, 0) = 0$ then $\max \{w_1(e_1, 0) - c, w_1(0, 0)\} = w_1(0, 0) = 0$, and it suffices to show that

$$\max \{w_1(e_1, e_2) - c, w_1(0, e_2)\} > 0.$$ This inequality holds in two cases:

(1a) if $\max \{w_1(e_1, e_2) - c, w_1(0, e_2)\} = w_1(e_1, e_2) - c > 0$. This implies $w_1(e_1, e_2) > c$;

(2a) if $\max \{w_1(e_1, e_2) - c, w_1(0, e_2)\} = w_1(0, e_2) > 0$.

In these cases, $r_{1,\sigma_2^a} = \frac{w_1(0, e_2)}{w_1(0, e_2) + c} > 0$.

Similarly, reciprocity for $A_2$,

$$r_{2,e} = \frac{\max \{w_2(e_1, e_2) - c, w_2(e_1, 0)\} - \max \{w_2(0, e_2) - c, w_2(0, 0)\}}{H_2 - L_2}; \quad (A.3.1.2)$$

is positive if the numerator is positive.

As $w_2(0, e_2) = w_2(0, 0) = 0$, then $\max \{w_2(0, e_2) - c, w_2(0, 0)\} = w_2(0, 0) = 0$.

Therefore, $r_{2,e_1} > 0$ if

(1b) if $\max \{w_2(e_1, e_2) - c, w_2(e_1, 0)\} = w_2(e_1, e_2) - c > 0$. This implies $w_2(e_1, e_2) > c$;
(2b) if \( \max\{w_2(e_1, e_2) - c, w_2(e_1, 0)\} = w_2(e_1, 0) > 0. \)

In these cases, \( r_{2,e} = \frac{w_2(e_1, 0)}{w_2(e_1, 0) + c} > 0. \)

By substituting these results respectively into \( A_1 \) and \( A_2 \) ICCs (A.2.2 and A.2.3) we obtain:

\[
\begin{align*}
& w_1(e_1, e_2) - c + \rho_1^h \frac{w_1(0, e_2)}{w_1(0, e_2) + c} w_2(e_1, e_2) \geq w_1(0, e_2) + \rho_1^h \frac{w_1(0, e_2)}{w_1(0, e_2) + c} w_2(0, e_2), \\
& w_2(e_1, e_2) - c + \rho_2^h \frac{w_2(e_1, 0)}{w_2(e_1, 0) + c} w_1(e_1, e_2) \geq w_2(e_1, 0) + \rho_2^h \frac{w_2(e_1, 0)}{w_2(e_1, 0) + c} w_1(e_1, 0).
\end{align*}
\]

By combining \( 1a \) and \( 1b \) with \( 2a \) and \( 2b \), we analyze the four possible cases where reciprocity is positive for both workers.

- **Case 1a and 1b.** A compensation scheme where \( w_1(e_1, e_2) > c \) and \( w_2(e_1, e_2) > c \) are paid is necessarily more costly than the scheme proposed to standard workers which costs \( 2c \).

- **Case 2a and 2b.** Rearranging the ICCs:

\[
\begin{align*}
& w_1(e_1, e_2) - c - w_1(0, e_2) + \rho_1^h \frac{w_1(0, e_2)}{w_1(0, e_2) + c} w_2(e_1, e_2) \geq 0, \\
& w_2(e_1, e_2) - c - w_2(e_1, 0) + \rho_2^h \frac{w_2(e_1, 0)}{w_2(e_1, 0) + c} w_1(e_1, e_2) \geq 0.
\end{align*}
\]

Note that both constraints are never satisfied for \( w_1(e_1, e_2) < c \) and \( w_2(e_1, e_2) < c \).

- **Case 1a and 2b (case 2a and 1b is symmetric).** We need to prove \( w_1(e_1, e_2) + w_2(e_1, e_2) < 2c \).

Rearranging the ICC for \( A_2 \) we obtain \( w_2(e_1, e_2) \geq w_2(e_1, 0) + c - \rho_2^h \frac{w_2(e_1, 0)}{w_2(e_1, 0) + c} w_1(e_1, e_2). \)

By subtracting this inequality from \( w_1(e_1, e_2) + w_2(e_1, e_2) < 2c \) yields

\[
w_1(e_1, e_2)(1 - \rho_2^h \frac{w_2(e_1, 0)}{w_2(e_1, 0) + c}) + w_2(e_1, 0) - c < 0. \]

Since by \( 1a \), \( w_1(e_1, e_2) > c \), this inequality is never satisfied and consequently any saving can be made under positive reciprocity.
A.3.2 Asymmetric information

The same arguments used in section A.3.1 can be used to prove the result under asymmetric information. Note that in this case the reciprocity for worker 1 and 2 are respectively:

\[ r_{1,e} = \max\{w_1(2\gamma e) - c, w_1(0)\} - \max\{w_1(\gamma e) - c, w_1(\gamma e)\} \]
\[ H_1 - L_1 \] (A.3.2.1)

\[ r_{2,e} = \max\{w_2(2\gamma e) - c, w_2(\gamma e)\} - \max\{w_2(\gamma e) - c, w_2(0)\} \]
\[ H_2 - L_2 \] (A.3.2.2)

A.4 Standard Compensation Scheme for Reciprocal Workers

A.4.1 Symmetric information case

Consider the set of optimal compensation scheme for standard workers. Applying it to reciprocal workers yields:

\[ w_1(e_1, e_2) = c; \quad w_1(e_1, 0) \in [0, B]; \quad w_1(0, e_2) = 0; \quad w_1(0, 0) \in [0, B]; \] (A.4.1.1)

\[ w_2(e_1, e_2) = c; \quad w_2(e_1, 0) = 0; \quad w_2(0, e_2) = c; \quad w_2(0, 0) = 0; \]

By substituting (A.4.1.1) in the ICC for \( A_1 \) (A.2.2) we can easily see that since \( A_1 \)'s choices do not affect the material payoff of \( A_2 \) and that the reciprocity component in the utility function cancels since \( w_2(e_1, e_2) = w_2(0, e_2) \). The ICC of \( A_1 \) coincides with the ICC of standard workers.

Now consider \( A_2 \) and substitute (A.4.1.1) in (A.2.3). It easy to see that when \( w_1(e_1, 0) = c \) the reciprocity component of the utility function is neutralized. Note that, when \( w_1(e_1, 0) \neq c \), substituting A.4.1.1 in the definition of reciprocity in (A.3.1.2) \( r_{2,e} = 0 \), (see section 2).

A.4.2 Asymmetric information case

Applying the set of optimal compensation schemes for standard worker to reciprocal worker:
\[ w_1(2\gamma e) = c; \quad w_1(\gamma e) \in [0, B]; \quad w_1(0) = 0; \quad w_2(2\gamma e) = w_2(\gamma e) = 0, \quad w_2(0) = B^A. \]

By substituting this compensation scheme in the ICCs of each worker it can be shown that each action chosen by one worker does not affect the material payoff of the other, so for this reason, the reciprocity component in the utility function cancels out. Under asymmetric information the multiplicity of optimal compensation schemes does not play any role, since, by calculating reciprocity of \( A_2 \) from (A.1.3.2) when (A.4.2.1) is offered, we obtain: \( r_{2,e} - \frac{c}{c} = 0 \).

### A.5 Proof of Proposition 2

In this section we prove that the employer has the following rank over team composition: team composed by two reciprocal workers are always preferred to teams composed by a standard worker and a reciprocal worker. Consistently, the latter team composition will be always preferred over team composed by two standard workers.

A team of standard workers produces \( 2\gamma e \) at a cost equal to \( 2c \). In subsection A.2 we show that a team of reciprocal workers produces the same output at zero cost for the employer. Let us consider the case of a team composed by a standard worker and a reciprocal worker.

Suppose \( \rho_1^h = 0, \rho_2^h > 0 \). To induce \( A_1 \) to exert extra effort a compensation scheme as the one described in subsection 3.1 \( w_1^S(e, a_2) = c \) and \( w_1^S(0, a_2) = 0 \) must be offered. On the contrary, \( A_2 \) chooses \( e_2 \) if paid according to (A.2.1). By substituting (A.2.1) in (A.2.3) we obtain:

\[
w_2(e_1, e_2) \geq c - \rho_2^h \frac{B}{B + c} [w_1(e_1, 0) - c].
\]
Since the employer wants to maximize her profit, she will offer a \( w_2(e_1,e_2) \) such that the ICC holds with equality. At this point:

- if \( w_1(e_1,0) - c > 0 \) then \( w_2(e_1,e_2) < c \) and \( A_2 \) will exert under-paid extra effort. Hence by offering \( w_1(e_1,0) = B > c \), the employer gets the output \( 2\gamma e \) by paying a sum of compensation lower than \( 2c \);

- if \( \frac{B}{B+c}(B-c) \geq \frac{c}{\rho_2} \), \( A_2 \) will exert unpaid extra effort. In this case, the employer obtains \( 2\gamma e \) by paying a sum of compensations equal to \( c \).

A.6 Proof Proposition 3

According to proposition 3 the optimal compensation scheme is:

\[
\begin{align*}
  w_1(2\gamma e) &= w_1(0) = 0; \quad w_1(\gamma e) = B^A; \\
  w_2(2\gamma e) &= w_1(\gamma e) = 0; \quad w_1(0) = B^A.
\end{align*}
\]  

(A.6.1)

The definitions of reciprocity for \( A_1 \) and \( A_2 \) in equilibrium are:

\[
\begin{align*}
  r_{1,\sigma_b} &= \frac{-w_1(\gamma e)}{w_1(\gamma e) + c}, \quad r_{2,\sigma} = \frac{-w_2(0)}{w_2(0) + c}.
\end{align*}
\]

In equilibrium, to induce both workers to exert extra effort, the following ICCs must hold:

\[
\begin{align*}
  w_1(2\gamma e) - c + \rho_1^h r_1 w_2(2\gamma e) - c \geq w_1(0) + \rho_1^h r_1 w_2(0), \quad (A.6.2) \\
  w_2(2\gamma e) - c + \rho_2^h r_2 w_1(2\gamma e) \geq w_2(\gamma e) + \rho_2^h r_2 w_1(\gamma e). \quad (A.6.3)
\end{align*}
\]

By substituting (A.6.1) respectively into (A.6.2) and (A.6.3) we obtain:

\[
\begin{align*}
  0 \geq c + \rho_1^h \frac{-w_1(\gamma e)}{w_1(\gamma e) + c} [w_2(0) - c] \quad (A.6.4) \\
  0 \geq c + \rho_2^h \frac{-w_2(0)}{w_2(0) + c} w_1(\gamma e) \quad (A.6.5)
\end{align*}
\]
Assume $w_1(\gamma e) = w_2(0) = B^A$. Rearranging (A.6.4) and (A.6.5) yields

$$B^A \geq \frac{c}{\rho^h_1}, \quad (A.6.6)$$

$$\frac{\rho^h_2}{c} (B^A)^2 - B^A - c \geq 0, \quad (A.6.7)$$

where $B^A \geq \frac{c}{\rho^h_1}$ is the monetary payment the employer must offer out of equilibrium in order to induce $A_1$ to exert unpaid extra effort ($w_1(2\gamma e) = 0$).

Solving $\frac{\rho^h_2}{c} (B^A)^2 - B^A - c = 0$ yields

$$B^A_1, B^A_2 = c \frac{1 \pm (1 + 4\rho^h_2)^{\frac{1}{2}}}{2\rho^h_2}. \quad (A.6.8)$$

Due to limited liability constraint the negative root makes no sense. Finally, the employer will offer out of equilibrium a level of $B$ such that:

$$B^A \geq \max \left\{ \frac{c}{\rho^h_1}, c \frac{1 + (1 + 4\rho^h_2)^{\frac{1}{2}}}{2\rho^h_2} \right\}. \quad (A.6.9)$$

Now we have to show that the compensation scheme in (A.6.1) induces a unique equilibrium which survives the iterated elimination of dominated strategies. In this equilibrium $A_2$’s dominant strategy is $\sigma^b_2 = \{e, 0\}$ and $A_1$’s best reply is $a_1 = e$.

Consider $A_2$. Suppose $a_1 = e$, then the reciprocity of $A_2$ is:

$$r_{2,e} = \frac{\max \{(0 - c), (0)\} - \max \{(0 - c), B^A\}}{(B^A) - (0 - c)} = -\frac{B^A}{B^A + c} < 0. \quad (A.6.10)$$

The utility $A_2$ gets if he exerts extra effort is $-c + \rho^h_2 r_{2,e}(-c)$, while the utility from not exerting it is: $\rho^h_2 r_{2,e}(B^A - c)$. of small extra effort is the optimal action for $A_2$ if

$$-c + \rho^h_2 r_{2,e}(-c) > \rho^h_2 r_{2,e}(B^A - c). \quad (A.6.11)$$
Substituting (A.6.10) in to (A.6.11) and simplifying it, (A.6.11) yields 

\[-c(B^A + c) > -\rho_2^b (B^A)^2\]

which holds when \(B^A \geq c \frac{1 + (1 + 4\rho_2^h)^{\frac{1}{2}}}{2\rho_2^h} \).

Suppose now \(a_1 = 0\), then the reciprocity of \(A_2\) is:

\[r_{2,0} = \max \{ (0 - c), (B^A) \} - \max \{ (0 - c), 0 \} = \frac{B^A}{B^A + c} > 0. \tag{A.6.12}\]

The utility \(A_2\) gets if he exerts extra effort is \(-c + \rho_2^b r_{2,0} B^A\), while the utility from not exerting it is \(B^A\). Not exerting extra effort in the second subgame is the optimal action for \(A_2\) if 
\(B^A > -c + \rho_2^b r_{2,0} (B^A)\) holds, which is always the case, since \(B^A > \rho_2^b r_{2,0} B^A\), given that \(\rho_2^b < 1\) and \(r_{2,0} < 1\). Therefore, we have proved that \(\sigma_2^b\) is the \(A_2\)’s dominant strategy.

Now we want to prove that \(a_1 = c\) is \(A_1\)’s best reply to \(\sigma_2^b\). When \(A_2\) chooses \(\sigma = \sigma_2^b\) this is the reciprocity for \(A_1\):

\[r_{1,\sigma_2^b} = \max \{ (0 - c), 0 \} - \max \{ (B^A - c), B^A \} = -\frac{B^A}{B^A + c} < 0. \tag{A.6.13}\]

The utility \(A_1\) gets if he exerts extra effort is \(-c + \rho_1^h r_{1,\sigma_2^b} (-c)\), while the utility from not exerting it is: \(\rho_1^h r_{1,\sigma_2^b} B^A\). Exertion of extra effort is the optimal action for \(A_1\) if 
\(-c + \rho_1^h r_{1,\sigma_2^b} (-c) > \rho_1^h r_{1,\sigma_2^b} B^A\) holds. This is always the case when \(B^A \geq c \frac{\rho_1^h}{\rho_1^h}\).

A.7 Proof of Proposition 4

Let prove that the LBD optimal compensation scheme assigns the second move to the worker with the highest \(\rho\). Let start from (A.6.9) . It contains two conditions that refer to the payment that must be offered in equilibrium to the first and second mover: \(B_1 \geq c \frac{1 + (1 + 4\rho_2^h)^{\frac{1}{2}}}{2\rho_2^h}\) and \(B_2 \geq c \frac{1 + (1 + 4\rho_2^h)^{\frac{1}{2}}}{2\rho_2^h}\).

If,

\[\forall \rho_1^h \neq \rho_2^h, \text{ if } \rho_1^h (1 + \rho_1^h) \geq \rho_2^h \Rightarrow c \frac{1 + (1 + 4\rho_2^h)^{\frac{1}{2}}}{2\rho_2^h} > c \frac{\rho_1^h}{\rho_1^h} \tag{A.7.1}\]
Suppose, without loss of generality, that \( \rho_i^h > \rho_j^h \).

If the first move is assigned to \( i \), \( A_1 = i \), then \( \rho_{1=i}^i (1 + \rho_{1=i}^i) > \rho_{2=i}^h \) and the binding condition is \( B_2 \geq \frac{c^{1+(1+4\rho_i^h)^\frac{1}{2}}}{2\rho_i^h} \).

Suppose, on the contrary, that the second move is assigned to \( i \), \( A_2 = i \). We have two possibilities:

1) if \( \rho_{1=j}^i (1 + \rho_{1=j}^i) < \rho_{2=i}^h \), then \( B_1 \geq \frac{c}{\rho_j^h} \) is the binding condition;

2) if \( \rho_{1=j}^i (1 + \rho_{1=j}^i) \geq \rho_{2=i}^h \), then \( B_2 \geq \frac{c^{1+(1+4\rho_i^h)^\frac{1}{2}}}{2\rho_i^h} \) is the binding condition.

However, we know that \( \forall \rho, B_2 > B_1 \):

\[
\frac{1 + (1 + 4\rho_i^h)^\frac{1}{2}}{2\rho_i^h} > \frac{c}{\rho_i^h}
\]  
(A.7.2)

which becomes \( \rho_i^h > 0 \).

Consider case 1). By assigning the second move to worker \( i \) the binding condition would be \( B_1 = \frac{c}{\rho_j^h} \), while by assigning to him the first move \( B_2 = \frac{c^{1+(1+4\rho_i^h)^\frac{1}{2}}}{2\rho_i^h} \). From (A.7.2) it follows that \( B_1 < B_2 \).

Consider now case 2). By assigning the second move to worker \( i \), the binding condition would be \( B_2 = \frac{c^{1+(1+4\rho_i^h)^\frac{1}{2}}}{2\rho_i^h} \), while assigning to him the first move \( B_2 = \frac{c^{1+(1+4\rho_i^h)^\frac{1}{2}}}{2\rho_i^h} \). Again, from (A.7.2) we see that \( B_2 < B_2 \). Therefore, we have proved that by assigning the second move to the worker with the highest \( \rho^h \), the LBD optimal compensation scheme is offered.

A.8 Proof of Proposition 5

In this section we want to prove that, when \( B^F > 0 \) is lower than the level inducing workers to exert unpaid extra effort, the employer could always obtain extra effort by paying in equilibrium a total compensation lower than to \( 2c \).
A.8.1 Symmetric information

Denote by $B^F$ the feasible budget and assume $B^F < c \left( \frac{1}{\min\{\rho^h, \rho^h\}} + 1 \right)$. In this case the ICCs for $A_1$ and $A_2$ are given by:

$$w_1(e_1, e_2) \geq c - \rho^h \frac{B^F + c}{B^F - w_1(e_1, e_2)} [B^F - w_2(e_1, e_2)], \quad (A.8.1.1)$$
$$w_2(e_1, e_2) \geq c - \rho^h \frac{B^F + c}{B^F - w_2(e_1, e_2)} [B^F - w_1(e_1 e_2)]. \quad (A.8.1.2)$$

In order to maximizes her profit, the employer will set $w_1(e_1, e_2)$ and $w_2(e_1, e_2)$ such that the previous ICCs hold with equality. Let check if $w_1(e_1, e_2) + w_2(e_1, e_2) < 2c$. Rearranging it suffices to show

$$c - \rho^h \frac{B^F + c}{B^F - w_1(e_1, e_2)} [B^F - w_2(e_1, e_2)] + c - \rho^h \frac{B^F + c}{B^F - w_2(e_1, e_2)} [B^F - w_1(e_1 e_2)] < 2c,$$

$$-\rho^h \frac{B^F + c}{B^F - w_1(e_1, e_2)} [B^F - w_2(e_1, e_2)] - \rho^h \frac{B^F + c}{B^F - w_2(e_1, e_2)} [B^F - w_1(e_1 e_2)] < 0,$$

which are always verified since by assumption $w_1(e_1, e_2) + w_2(e_1, e_2) \leq B^F$.

A.8.2 Asymmetric information

When $B^F < \frac{c}{\rho^h}$ and $B^F < \frac{c(1+(1+4\rho^h)^2)}{2\rho^h}$ the ICCs for $A_1$ and $A_2$ becomes respectively:

$$w_1(2\gamma e) \geq c - \rho^h \frac{B^F}{B^F + c - w_1(X^2\gamma e)} [B^F + c - w_2(2\gamma e)], \quad (A.8.2.1)$$
$$w_2(2\gamma e) \geq c - \rho^h \frac{B^F}{B^F + c} [B^F - w_1(2\gamma e)]. \quad (A.8.2.2)$$
The employer obtains $2\gamma e$ paying a sum of compensations lower than $2c$ if:

$$c - \rho_1 \frac{B^F}{B^F + c - w_1(2\gamma e)} [B^F + c - w_2(2\gamma e)] + c - \rho_2 \frac{B^F}{B^F + c} [B^F - w_1(2\gamma e)] < 2c \quad (A.8.2.3)$$

Since $w_1(2\gamma e) + w_2(2\gamma e) \leq B^F$ then the inequality are always verified.

**A.9 Proof of Proposition 7**

For the employer actions and strategies coincide. Denote with $RC_P$ and $IC_P$ the employer’s choice of the relative and the individual compensation scheme respectively. According to proposition 7 the optimal compensation scheme is:

$$w_i(e_i, e_j) = w_i(0, e_j) = w_i(0, 0) = 0; \quad w_i(e_i, 0) = B^{HV}, \quad \text{if } \sigma_P = RC_P; \quad (A.9.1)$$

$$w_i(e_i, a_j) = c; \quad w_i(0, a_j) = 0, \quad \text{if } \sigma_P = IC_P, \text{ for } i, j \in \{1, 2\} \text{ with } i \neq j.$$

In the subgame identified by $\sigma_P = RC_P$, the SPNE is $(e_1, (e_2, e_2))$ and both workers obtain a material payoff equal to $-c$. Therefore, $M^*_{i, RC_P} = -c$ for $i, j = 1, 2$. As shown in Appendix A.2, in this subgame the horizontal reciprocity for $A_1$ and $A_2$ are: $r_{1, \sigma_2} = \frac{-B^{HV} + c}{B^{HV}}$ and $r_{2, \epsilon} = \frac{-B^{HV} + c}{B^{HV}}$ respectively. In the subgame identified by $\sigma_P = IC_P$, the SPNE is $(e_1, (e_2, e_2))$ and therefore, $M^*_{i, IC_P} = 0$, while $\rho^h$.

First, consider the workers’ choices if $\sigma_P = RC_P$.

From eq. (4.1) we can calculate vertical reciprocity for both workers:

$$r_{1, RC_P} = r_{2, RC_P} = -\frac{c}{B^{HV}}.$$

To induce both workers to exert extra effort the following ICCs must hold:

$$w_1(e_1, e_2) - c + \rho_1^{h} r_{1, \sigma_2} (w_2(e_1, e_2) - c) + \rho_1^{w} r_{1, RC_P}(2\gamma e) \geq 0 \quad (A.9.2)$$

$$w_1(0, e_2) + \rho_1^{h} r_{1, \sigma_2} (w_2(0, e_2) - c) + \rho_1^{w} r_{1, RC_P}(\gamma e - w_2(0, e_2));$$

$$w_2(e_1, e_2) - c + \rho_2^{h} r_{2, \epsilon} w_1(e_1, e_2) + \rho_2^{w} r_{2, RC_P}(2\gamma e) \geq 0 \quad (A.9.3)$$

$$w_2(e_1, 0) + \rho_2^{h} r_{2, \epsilon} w_1(e_1, 0) + \rho_2^{w} r_{2, RC_P}(\gamma e - w_1(e_1, 0)).$$
By substituting (A.9.1) respectively into (A.9.2) and (A.9.3) we obtain:

\[-c + \rho_1^h B^{HV} + c(-c) + \rho_1^v B^{HV} (2\gamma e) \geq \rho_1^h B^{HV} + c(B^{HV} - c) + \rho_1^v B^{HV} (\gamma e - B^{HV}); \tag{A.9.4}\]

\[-c + \rho_2^h B^{HV} + c(-c) + \rho_2^v B^{HV} (2\gamma e) \geq \rho_2^h B^{HV} + c(B^{HV} - c) + \rho_2^v B^{HV} (\gamma e - B^{HV}); \tag{A.9.5}\]

rearranging:

\[\rho_1^h (B^{HV})^2 - c(1 + \rho_1^v + \rho_1^h)B^{HV} - \rho_1^v c\gamma e \geq 0, \tag{A.9.6}\]

\[\rho_2^h (B^{HV})^2 - c(1 + \rho_2^v + \rho_2^h)B^{HV} - \rho_2^v c\gamma e \geq 0. \tag{A.9.7}\]

(A.9.6) and (A.9.7) yields

\[B^{HV} = \frac{c(1 + \rho_1^v + \rho_1^h) \pm \sqrt{c(1 + \rho_1^v + \rho_1^h)^2 + 4\rho_1^v \rho_1^h c\gamma e}}{2\rho_1^h} \quad \text{for } i = 1, 2, \tag{A.9.8}\]

from which, by excluding the negative solution due to limited liability, we obtain:

\[B^{HV} \geq \frac{c(1 + \rho_1^v + \rho_1^h) + \sqrt{c(1 + \rho_1^v + \rho_1^h)^2 + 4\rho_1^v \rho_1^h c\gamma e}}{2\rho_1^h} \quad \text{for } i = 1, 2, \tag{A.9.8}\]

where \(B^{HV}\) is the monetary compensation to be offered out of equilibrium to induce both workers to exert unpaid extra effort. The employer obtains \(\Pi = 2\gamma e\). A2 proves that exerting extra effort is optimal for the workers once a relative compensation, such as the one described in (A.9.1), is offered.

Consider the workers’ choices when \(\sigma_P = IC_P\).

From eq. (4.1) we can calculate vertical reciprocity:

\[r_i,RC_P = \frac{c}{B^{HV}} \quad \text{for } i = 1, 2.\]

To induce both workers to exert extra effort the following ICCs must hold:

\[w_1(e_1, e_2) - c + \rho_1^h r_{1,IC_P} [2\gamma e - (w_1(e_1, e_2) + w_2(e_1, e_2))] \geq 0; \tag{A.9.9}\]

\[w_1(0, e_2) + \rho_1^v r_{1,IC_P} (\gamma e - w_2(0, e_2));\]

\[w_2(e_1, e_2) - c + \rho_2^h r_{2,IC_P} [2\gamma e - (w_1(e_1, e_2) + w_2(e_1, e_2))] \geq 0; \tag{A.9.10}\]

\[w_2(e_1, 0) + \rho_2^v r_{2,IC_P} (\gamma e - w_1(e_1, 0)).\]

By substituting (A.9.1) in (A.9.9) and in (A.9.10) we obtain:

\[\rho_i^v (\gamma e - c) \leq 0; \quad \text{for } i = 1, 2. \tag{A.9.11}\]

which are always satisfied since \(\gamma e \geq c\).
The next step is to prove that exerting extra effort is the optimal strategy for each worker. Considering $A_2$, first, (A.8.11) shows that when $A_1$ does exert effort then it is optimal for him to exert extra effort. When $A_1$ does not exert extra effort then for $A_2$ it is optimal to work extra hours if
\[
w_2(0, e_2) - c + \rho^v_{2, r_{2, r_2}} \left[ \gamma e - w_2(e_1, e_2) \right] \geq w_2(0, 0);
\]
from which $\rho^v_{2, r_{2, r_2}} \geq 0$, that holds. This proves that exerting extra effort is a dominant strategy for $A_2$.

Consider now $A_1$. (A.9.9) shows that when $A_2$ exert effort it is dominant for $A_2$ to exert extra effort as well. In case $A_2$ does not exert effort, then $A_1$ prefers to work extra if:
\[
w_1(e_1, 0) - c + \rho^v_{1, r_{1, r_1}} \left[ \gamma e - w_1(e_1, 0) \right] \geq w_1(0, 0);
\]
from which, by substituting (A.9.1) we obtain: $\rho^v_{1, r_{1, r_1}} \geq 0$, that holds. For $A_1$ exerting effort is a dominant strategy.

Consider now the choice of the employer. If she offers the relative compensation scheme then she earns $\Pi = 2\gamma e$, while if she offers the individual compensation scheme she earns $\Pi = 2(\gamma e - c)$. Therefore, the relative compensation scheme will be preferred to the individual one as long as this latter requires a positive sum of compensations to be paid to the workers who exert extra effort.

A.10 Proof of Proposition 8

We only consider the case where $\rho^h_i \geq \rho^v_i$ and therefore, $U_i = M_i + \rho^v_i r_{i, \sigma_i} \Pi$. For the proof of the case $\rho^h_i \geq \rho^v_i$ see A.2.

According to proposition 8 the optimal compensation scheme is:
\[
\begin{align*}
  w_i(e_i, e_j) &= w_i(0, e_j) = w_i(0, 0) = 0; \quad w_i(e_i, 0) = B^V, \quad \text{if } \gamma e \geq 2c(1 - \rho_i); \\
  w_i(e_i, a_j) &= c; \quad w_i(0, a_j) = 0, \quad \text{if } \gamma e < 2c(1 - \rho_i), \quad \text{for } i, j \in \{1, 2\} \text{ with } i \neq j.
\end{align*}
\]

a) The tournament is offered

Consider first $A_2$. When the tournament is offered and $A_1$ exerts extra effort then $A_2$ will exert extra effort if the following ICC holds:
\[
w_2(e_1, e_2) - c + \rho^v_{2, r_{2, r_2}} \left[ 2\gamma e - \left( w_1(e_1, e_2) + w_2(e_1, e_2) \right) \right] \geq w_2(e_1, 0) + \rho^v_{2, r_{2, r_2}} \left[ \gamma e - w_1(e_1, 0) \right].
\]
By substituting (A.10.1) in (A.10.2) we obtain:
\[
(B^V + \gamma e)\rho^v_{2, r_{2, r_2}} \geq c.
\]
From eq. (4.1), we can calculate vertical reciprocity which equals to $\rho^v = -\frac{c}{B^V}$. By substituting it, (A.10.3) becomes:
\[
-c - \rho^v \left( \frac{c}{B^V} \right) 2\gamma e \geq \rho^v \left( \frac{c}{B^V} \right) \left( \gamma e - B^V \right);
\]
which, rearranged, yields:
\[-c - \rho_2^v v \left( \frac{c}{B^V} \right) (\gamma e + B^V) \geq 0.\]  \hfill (A.10.5)

Note that (A.10.5) never holds \( \forall B^V \geq 0 \). Therefore, when the tournament is offered, \( A_2 \) will not exert extra effort if \( A_1 \) does.

Consider now \( A_1 \). Given that \( A_2 \) will not exert extra effort, \( A_1 \) will exert extra effort if the following ICC holds:
\[ w_1(e_1, 0) - c + \rho_1^v r_{1,x_p} (\gamma e - w_1(e_1, 0)) \geq w_1(0, 0) + \rho^v_1 r_{1,x_p} 0. \]  \hfill (A.10.6)

By substituting (A.10.1) and the reciprocity in (A.10.6) we obtain
\[ B^V - c + \rho_1^v \frac{c}{B^V} (\gamma e - B^V) \geq 0. \]  \hfill (A.10.7)

Rearranging (A.10.7), it becomes
\[ \varphi(B^V) = B^V - B^V c(1 - \rho_1^v) - \rho_1^v c \gamma e \geq 0. \]  \hfill (A.10.8)

Solving \( \varphi(B^V) = 0 \) yields
\[ B_1^V, B_2^V = \frac{c(1 - \rho_1^v) \pm \sqrt{[-c(1 - \rho_1^v)]^2 - 4(-\rho_1^v c \gamma e)}}{2}. \]  \hfill (A.10.9)

Due to limited liability constraint the negative root makes no sense. The employer will offer a \( B^V \) such that:
\[ B^V = \frac{c(1 - \rho_1^v) + \sqrt{[-c(1 - \rho_1^v)]^2 + 4\rho_1^v c \gamma e}}{2}. \]  \hfill (A.10.10)

Therefore, if the tournament is offered, \( A_1 \) will exert extra effort when \( A_2 \) refuses to do so.

Consider now the case in which \( A_1 \) does not exert effort. \( A_2 \) will exert effort if the following ICC holds:
\[ w_2(0, e_2) - c + \rho_2^v r_{2,x_p} [\gamma e - w_2(0, e_2)] \geq 0. \]  \hfill (A.10.11)

By substituting (A.10.1) and the reciprocity in (A.10.11) we obtain
\[ B^V - c + \rho_2^v \frac{c}{B^V} (\gamma e - B^V) \geq 0; \]  \hfill (A.10.12)

Note that (A.10.12) is equivalent to (A.10.7). It follows that, when \( A_1 \) does not exert effort \( A_2 \) will exert effort if \( B^V \geq \frac{c(1 - \rho_1^v) + \sqrt{[-c(1 - \rho_1^v)]^2 + 4\rho_1^v c \gamma e}}{2} \).

When the tournament is offered, the optimal strategy for the \( A_2 \) is \( \{0, e\} \) and \( A_1 \)'s best reply is \( a_1 = e \). The employer earns profits equal to \( \Pi = \gamma e - B^V \).

\[ \text{b) The individual compensation scheme is offered} \]
The individual compensation scheme in (A.10.1) coincides with the optimal compensation scheme defined for standard agents in section 4.1. Notice that when this compensation scheme is offered, from eq (4.1), vertical reciprocity is equal to \( \frac{c}{B} \).

Both \( A_1 \) and \( A_2 \) will exert effort if the following ICC hold:

\[
\begin{align*}
\nu_i(e_i, a_j) - c + \rho_i^e r_{i,T} \left[ 2\gamma e - (w_i(e_i, a_j) + w_j(e_i, a_j)) \right] &\geq w_i(0, a_j) + \rho_i^e r_{i,T} \left[ \gamma e - w_j(0, a_j) \right] \\
& \text{for } i, j \in \{1, 2\} \text{ with } i \neq j.
\end{align*}
\]

(A.10.13)

which by substituting (A.10.1) becomes:

\[
\rho_i^e \left( \frac{c}{B} \gamma e - c \right) \geq 0 \quad \text{for } i, j \in \{1, 2\} \text{ with } i \neq j;
\]

(A.10.14)

which always holds.

Therefore, when the individual compensation scheme is offered, each agent exerts extra effort and the employer obtains \( \Pi = 2(\gamma e - c) \).

**Choosing between tournament and individual compensation scheme.**

Now, whether the employer will offer a tournament rather than an individual compensation scheme depends on which is the compensation scheme that maximizes her profits, that is:

\[ \gamma e - B^V \geq 2(\gamma e - c). \]

(A.10.15)

Therefore, when (A.10.15) holds, the employer will offer the tournament. By substituting in to (A.10.15) \( B^V \) from (A.10.12) we obtain:

\[
\gamma e - \frac{c(1 - \rho_1^e)}{2} - \sqrt{\frac{[-c(1 - \rho_1^e)]^2 + 4 \rho_1^e c \gamma e}{2}} \geq 2(\gamma e - c);
\]

(A.10.16)

which can be rewritten as:

\[
3c - 2\gamma e + \rho_1^e c \geq \sqrt{[-c(1 - \rho_1^e)]^2 + 4 \rho_1^e c \gamma e};
\]

(A.10.17)

which can be reorganized as:

\[
(3c - 2\gamma e + \rho_1^e c)^2 \geq [-c(1 - \rho_1^e)]^2 + 4 \rho_1^e c \gamma e.
\]

(A.10.18)

After some calculations, (A.10.13) becomes:

\[
2c(1 + \rho_1^e)(c - \gamma e) + \gamma e(\gamma e - c) \geq 0;
\]

(A.10.19)

which holds if

\[ \gamma e \geq 2c(1 + \rho_1^e). \]

(A.10.20)

Therefore, when \( \gamma e \geq 2c(1 + \rho_1^e) \) the employer will offer a tournament rather than an individual compensation scheme since it ensures the highest profits.
A.11 Proof of Proposition 9

Case 1: additivity of vertical and horizontal reciprocity.

The employer will prefer to offer the individual compensation scheme if

\[ 2(\gamma e - c) + \delta(r_{1,IRP} + r_{2,IRP}) \geq 2\gamma e + \delta(r_{1,Tp} + r_{2,Tp}); \]  

which, considering that \( r_{1,IRP} = r_{2,IRP} = \frac{c}{\delta} \) and \( r_{1,Tp} = r_{2,Tp} = \frac{c}{\delta} \) could be rearranged as

\[ -2c + 4\delta \frac{c}{\delta} \geq 0; \]  

from which we obtain

\[ 2\delta \geq B; \]  

which, holds if

\[ \delta \geq \frac{B}{2}. \]  

Case 2: mutual exclusivity of vertical and horizontal reciprocity.

The employer will prefer to offer the individual compensation scheme if

\[ 2(\gamma e - c) + \delta(r_{1,IRP} + r_{2,IRP}) \geq \gamma e - B + \delta(r_{1,Tp} + r_{2,Tp}); \]  

which, considering that \( r_{1,IRP} = r_{2,IRP} = \frac{c}{\delta} \) and \( r_{1,Tp} = r_{2,Tp} = \frac{c}{\delta} \) could be rearranged as

\[ B + \gamma e - 2c + 2\delta \frac{c}{\delta} \geq 2\delta \left( -\frac{c}{\delta} \right); \]  

Which could be rearranged as

\[ B + \gamma e + 4\delta \frac{c}{\delta} \geq 2c; \]  

which is always satisfied, since, by assumption \( B > c \) and \( \gamma e > c \).

References


Figure 1. The optimal compensation scheme under symmetric information

Figure 2. The optimal compensation scheme under asymmetric information
Figure 3. Presence of vertical and horizontal reciprocity