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Which Parameters Drive Approximation Inaccuracies?☆

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Abstract

This paper identifies parameters responsible for welfare reversals when the basic New Keynesian model is approximated. In our setting, a reversal occurs when the Ramsey policy under timeless perspective commitment ceases to be dominant against the Taylor rule after approximating the model. We find that the parameters involved are the degree of persistence in the autoregressive shock process and the labor elasticity of real output.

JEL classification: E30; E52; E61

Key words: Optimal Monetary policy; Approximations; Welfare Analysis; Timeless Perspective

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1 Introduction

Several ways have been proposed to compute a welfare optimizing monetary policy. Kahn et al. (2003) and King and Wolman (1999) derive first-order conditions by maximizing household utility subject to the model economy. Another method by Kahn et al. (2003), Kim and Kim (2006), and King and Wolman (1999) consists in computing a first-order approximation of the exact first-order conditions. While welfare in the nonlinear environment is measured by household utility, the approximated setup requires a quadratic welfare loss function along the lines of Benigno and Woodford (2004, 2006) and Woodford (1999, 2003). Alternatively, these authors propose the minimization of the loss function using a first-order approximation of the model economy as a constraint. In a first step we compute the Ramsey monetary policy under timeless perspective commitment and compare the resulting welfare loss to the interest rate rule by Taylor (1993). In a second step, the same is done in the approximated model following Kahn et al. (2003), Kim and Kim (2006), and King and Wolman (1999). We find contradictory policy recommendations when using the approximated model (see Sienknecht (2010)). This result may depend on parameters influencing the curvature of the target function and therefore the degree of nonlinearity in the model. An increasing curvature would make welfare reversals more likely. However, certain parameters are more influential than others. The aim of this paper is to identify these parameters and to check if their commonly designated values (calibration values) are high enough to induce the reversal effect. We proceed as follows: Section 2 presents the model and monetary policy alternatives. Section 3 approximates them. Section 4 presents graphical simulation results and explores parameter regions which cause welfare reversals. A few concluding remarks can be found in section 5.

2 The Model

The standard New Keynesian model for a cashless economy consists of final goods producers, intermediate goods firms, households, and the monetary authority. The non-neutrality of monetary policy is due to nonlinear adjustment costs in the spirit of Hairault and Portier (1993). The following sections present the model in detail.

2.1 Final Goods Producers

Final goods producers demand a continuum of monopolistically offered intermediate goods $Y_t(i) \in [0, 1]$ which are assembled towards the final product Y_t with a CES production technology. The final product is sold in a perfectly competitive market. A final good producer maximizes his profits

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad (1)$$

subject to

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_t}{\epsilon_t-1}} di \right)^{\frac{\epsilon_t-1}{\epsilon_t}} \quad (2)$$

The elasticity of substitution between input varieties ϵ_t is assumed to vary over time according to a first-order autoregressive process with a shock impulse variable e_t :

$$\left(\frac{\epsilon_t}{\epsilon} \right) = \left(\frac{\epsilon_{t-1}}{\epsilon} \right)^\rho \exp \{e_t\} \quad , \quad 0 \leq \rho < 1 \quad (3)$$

where ϵ is the steady state elasticity and ρ gives the degree of persistence in the shock process. The first-order condition for profit maximization yields the demand schedule for input variety i :

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_t} Y_t \quad (4)$$

Inserting this equation into the CES technology (2) yields the aggregate price index for the bundle Y_t :

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon_t} di \right)^{\frac{1}{1-\epsilon_t}} \quad (5)$$

2.2 Intermediate Goods Producers

Intermediate firm production $Y_t(i)$ requires labor hours $N_t(i)$:

$$Y_t(i) = N_t(i)^{1-\alpha} \quad , \quad 0 \leq \alpha < 1 \quad (6)$$

where $\alpha > 0$ represents decreasing marginal productivity. Total real costs are

$$TC_t(i) = \frac{W_t}{P_t} N_t(i) \quad (7)$$

Taking the derivative of (7) subject to the production function (6) yields real marginal costs

$$MC_t(i) = \left(\frac{1}{1-\alpha} \right) \frac{W_t}{P_t} Y_t(i)^{\frac{\alpha}{1-\alpha}} \quad (8)$$

An intermediate firm chooses $P_t(i)$ in order to maximize real profits subject to the demand schedule (4). By doing so, the firm faces real quadratic costs of price adjustment (Hairault and Portier (1993)):

$$Q_t(i) = \frac{\psi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2, \quad \psi \geq 0 \quad (9)$$

The parameter ψ denotes the marginal adjustment cost reaction on deviations of price relations $\frac{P_t}{P_{t-1}}$ from steady state gross inflation π . An intermediate firm chooses a price $P_t(i)$ that maximizes real profits

$$E_t \sum_{k=0}^{\infty} \Delta_{t,t+k} \left[\frac{P_{t+k}(i) Y_{t+k}(i)}{P_{t+k}} - MC_{t+k}(i) Y_{t+k}(i) - Q_{t+k}(i) \right] \quad (10)$$

subject to (4) and (9), where $\Delta_{t,t+k} = \beta^k \frac{\partial U_{t+k} / \partial C_{t+k}}{\partial U_t / \partial C_t}$ is the stochastic discount factor for real profit income flows to shareholders (households). The first-order condition reads:

$$\begin{aligned} \psi \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right) \frac{P_t(i)}{P_{t-1}(i)} &= \epsilon_t Y_t(i) \left(MC_t(i) - \frac{1}{\mu_t} \frac{P_t(i)}{P_t} \right) \\ &+ E_t \left[\psi \frac{\Delta_{t,t+1}}{\Delta_{t,t}} \left(\frac{P_{t+1}(i)}{P_t(i)} - \pi \right) \frac{P_{t+1}(i)}{P_t(i)} \right] \end{aligned} \quad (11)$$

The variable μ_t is the time varying monopolistic markup of intermediate firms, which is given by:

$$\mu_t = \left(\frac{\epsilon_t}{\epsilon_t - 1} \right) \quad (12)$$

Note that $\psi = 0$ gives the monopolistic price setting without costs of adjustment:

$$P_t(i) = \mu_t P_t MC_t(i) \quad (13)$$

2.3 Households

There is a continuum of households $j \in [0, 1]$ maximizing the following discounted sum of expected utility streams:

$$E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}(j)}{1-\sigma} - \frac{N_{t+k}^{1+\eta}(j)}{1+\eta} \right) \quad (14)$$

where $\frac{1}{\sigma} > 0$ is the intertemporal elasticity of substitution in consumption and $\frac{1}{\eta} > 0$ is the real wage elasticity of labor supply. The household saves in one-period nominal bonds $B_t(j)$ at the gross deposit rate R_t and receives real dividends $Div_t^r(j)$ from intermediate firms. The period-by-period real budget constraint is given by

$$C_t(j) + \frac{B_t(j)}{P_t} = \frac{W_t(j)}{P_t} N_t(j) + R_{t-1} \frac{B_{t-1}(j)}{P_t} + Div_t^r(j) \quad (15)$$

Differentiation of (14) with respect to C_t , N_t , and B_j subject to (15) gives

$$\lambda_t(j) = C_t(j)^{-\sigma} \quad (16)$$

$$\frac{W_t}{P_t} = \frac{N_t(j)^\eta}{\lambda_t(j)} \quad (17)$$

and

$$\lambda_t(j) = \beta E_t \left[\lambda_{t+1}(j) R_t \frac{P_{t+1}}{P_t} \right] \quad (18)$$

where $\lambda_t(j)$ is the Lagrange multiplier of agent j .

2.4 Equilibrium and Aggregation

We assume a symmetric equilibrium. The market clearing for each intermediate good implies

$$Y_t^d(i) = Y_t^s(i) = Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_t} Y_t \quad (19)$$

and the symmetry assumption implies that all intermediate firms face the same price setting problem. Therefore, they set the same price, which implies $P_t(i) = P_t$ and $Y_t(i) = Y_t$. Moreover, we neglect the index j since all households are assumed to be identical. Inserting (16) into (18) gives the consumption Euler equation

$$C_t^{-\sigma} = \beta E_t [C_{t+1}^{-\sigma} R_t \pi_{t+1}^{-1}] \quad (20)$$

where $\pi_t = P_t/P_{t-1}$ denotes gross inflation. Combining (16) and (17) leads to the following labor supply:

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\eta \quad (21)$$

Using $\Delta_{t,t+1}/\Delta_{t,t} = (C_{t+1}/C_t)^{-\sigma}$, the first-order condition of the intermediate firm (11) can be rewritten as

$$\pi_t (\pi_t - \pi) = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1} - \pi) \pi_{t+1} \right] + \frac{Y_t \epsilon_t}{\psi} \left(MC_t - \frac{1}{\mu_t} \right) \quad (22)$$

which represents a nonlinear New Keynesian Phillips curve. Aggregate real marginal costs and real output are given by

$$MC_t = \left(\frac{1}{1-\alpha} \right) \frac{W_t}{P_t} Y_t^{\frac{\alpha}{1-\alpha}} \quad (23)$$

and

$$Y_t = N_t^{1-\alpha} \quad , \quad 0 \leq \alpha < 1 \quad (24)$$

The economy-wide resource utilization is given by the aggregate budget constraint of the household (15), together with the bond market clearing condition $B_t = B_{t-1} = 0$

as

$$C_t = \frac{W_t}{P_t} N_t + Div_t^r \quad (25)$$

where aggregate real profits of intermediate goods producers are

$$Div_t^r = Y_t - \frac{W_t}{P_t} N_t - Q_t \quad (26)$$

Inserting (26) into (25) leads to the overall resource constraint

$$Y_t = C_t + \frac{\psi}{2} (\pi_t - \pi)^2 \quad , \quad \psi \geq 0 \quad (27)$$

As can be seen, $\psi > 0$ limits the resources available for aggregate consumption.

2.5 Monetary Policy

The monetary authority is either committed to a Ramsey policy under a timeless perspective or to a simple interest rate rule. The Ramsey planner aims to maximize household utility subject to the model economy¹. In our setting, the constraints to be taken into consideration are the New Keynesian Phillips curve (22) and the aggregate resource constraint (27). Substitution of W_t/P_t , MC_t , and Y_t with (21), (23), and (24) gives the constraints only in terms of the control variables C_t , N_t , and π_t :

$$\begin{aligned} \pi_t (\pi_t - \pi) - \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1} - \pi) \pi_{t+1} \right] \\ - \frac{N_t^{1-\alpha} \epsilon_t}{\psi} \left(\left(\frac{1}{1-\alpha} \right) C_t^\sigma N_t^{\alpha+\eta} - \frac{1}{\mu_t} \right) = 0 \end{aligned} \quad (28)$$

¹ See Kahn et al. (2003).

$$N_t^{1-\alpha} - C_t - \frac{\psi}{2} (\pi_t - \pi)^2 = 0 \quad (29)$$

The Lagrangian for a given shock process (3) then reads

$$\begin{aligned} \mathcal{L}_t = & E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma} - \frac{N_{t+k}^{1+\eta}}{1+\eta} \right) \\ & + E_t \sum_{k=0}^{\infty} \beta^k \lambda_{1,t+k+1} \left(N_{t+k}^{1-\alpha} - C_{t+k} - \frac{\psi}{2} (\pi_{t+k} - \pi)^2 \right) \\ & + E_t \sum_{k=0}^{\infty} \beta^k \lambda_{2,t+k+1} \left((\pi_{t+k} - \pi) \pi_{t+k} - \beta \left(\frac{C_{t+k+1}}{C_{t+k}} \right)^{-\sigma} (\pi_{t+k+1} - \pi) \pi_{t+k+1} \right. \\ & \quad \left. - \frac{\epsilon_{t+k}}{\psi} \left(\left(\frac{1}{1-\alpha} \right) C_{t+k}^{\sigma} N_{t+k}^{1+\eta} - \frac{1}{\mu_{t+k}} N_{t+k}^{1-\alpha} \right) \right) \end{aligned} \quad (30)$$

where $\lambda_{1,t}$ and $\lambda_{2,t}$ are the respective Lagrange multipliers. The first-order conditions with respect to C_t , N_t , and π_t are given by

$$\begin{aligned} E_t \left[C_t^{-\sigma} - \lambda_{1,t+1} - \lambda_{2,t+1} \beta \sigma (\pi_{t+1} - \pi) \pi_{t+1} \frac{1}{C_t} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \right. \\ \left. - \lambda_{2,t+1} C_t^{\sigma-1} N_t^{1+\eta} \epsilon_t \left(\frac{\sigma}{1-\alpha} \right) \frac{1}{\psi} + \lambda_{2,t} \sigma (\pi_t - \pi) \pi_t \frac{1}{C_t} \left(\frac{C_{t-1}}{C_t} \right)^{\sigma} = 0 \right] \end{aligned} \quad (31)$$

$$\begin{aligned} E_t \left[-N_t^{\eta} + \lambda_{1,t+1} N_t^{-\alpha} (1-\alpha) - \lambda_{2,t+1} C_t^{\sigma} N_t^{\eta} \epsilon_t \left(\frac{1+\eta}{1-\alpha} \right) \frac{1}{\psi} \right. \\ \left. + \lambda_{2,t+1} N_t^{-\alpha} (\epsilon_t - 1) \left(\frac{1-\alpha}{\psi} \right) = 0 \right] \end{aligned} \quad (32)$$

$$E_t \left[-\lambda_{1,t+1} \psi (\pi_t - \pi) + \lambda_{2,t+1} (2\pi_t - \pi) - \lambda_{2,t} (2\pi_t - \pi) \left(\frac{C_{t-1}}{C_t} \right)^{\sigma} = 0 \right] \quad (33)$$

Note that $\lambda_{1,t}$ is a jump variable, while $\lambda_{2,t}$ is predetermined. Since the latter is non-zero and equal to its steady state value, the Ramsey policy is of a timeless perspective nature. This implies that the policy maker credibly commits to a time-invariant policy strategy with the disadvantage that aggregate utility is not at its globally optimal level. The alternative strategy is to commit to an interest rate rule

following Taylor (1993), where the gross interest rate R_t is the control instrument:

$$\left(\frac{R_t}{R}\right) = \left(\left(\frac{\pi_t^p}{\pi} \right)^{\delta_\pi} \left(\frac{Y_t}{Y} \right)^{\delta_y} \right)^{1-\phi} \left(\frac{R_{t-1}}{R} \right)^\phi \quad (34)$$

$\delta_\pi > 1$ gives the weight on inflation and $\delta_y > 0$ on output deviations from the steady state. The parameter $\phi > 0$ generates interest rate smoothing behaviour.

2.6 Welfare Measure

Absolute (*abs.*) welfare at period t is simply measured by

$$V_t^{abs.} = \frac{N_t^{1+\eta}}{1+\eta} - \frac{C_t^{1-\sigma}}{1-\sigma} + \beta E_t [V_{t+1}^{abs.}] \quad (35)$$

where $V_t^{abs.} > 0$ for $\sigma > 1$ and $\eta > 0$. We compare absolute welfare losses between the two policy strategies by computing

$$V_t^{rel.} = \left(\frac{V_t^{abs.} (Taylor)}{V_t^{abs.} (Ramsey)} \right) \quad (36)$$

Therefore, the Taylor rule is superior to the Ramsey policy under timeless perspective commitment if $V_t^{rel.} < 100\%$.

3 Approximations

We take a first-order Taylor approximation in logarithms around the non-stochastic steady state. In the following, we provide the steady state relationships and the approximated model equations.

3.1 Steady State

We derive the non-stochastic steady state by neglecting all time indices. The optimality condition (21), the production function (24), and the aggregate resource

constraint (27) are then:

$$\frac{W}{P} = C^\sigma N^\eta \quad (37)$$

$$N = Y^{\frac{1}{1-\alpha}} \quad (38)$$

$$C = Y \quad (39)$$

Inserting (38) and (39) into (37) gives

$$\frac{W}{P} = Y^{\frac{\sigma(1-\alpha)+\eta}{1-\alpha}} \quad (40)$$

From the inflation curve (22) and the aggregate real marginal costs (23) we obtain

$$MC = \frac{1}{\mu} \quad (41)$$

$$MC = \left(\frac{1}{1-\alpha} \right) \frac{W}{P} Y^{\frac{\alpha}{1-\alpha}} \quad (42)$$

Using (40) and (41) to eliminate W/P and MC in (42) and solving for Y delivers

$$Y = C = \left(\frac{1-\alpha}{\mu} \right)^{\frac{1-\alpha}{\sigma(1-\alpha)+\eta+\alpha}} \quad (43)$$

Inserting (43) back into (38) and (40) gives the steady state levels

$$N = \left(\frac{1-\alpha}{\mu} \right)^{\frac{1}{\sigma(1-\alpha)+\eta+\alpha}} \quad (44)$$

$$\frac{W}{P} = \left(\frac{1-\alpha}{\mu} \right)^{\frac{\sigma(1-\alpha)+\eta}{\sigma(1-\alpha)+\eta+\alpha}} \quad (45)$$

We assume no trend inflation, which implies $\pi = 1$. From the consumption Euler equation (20), one obtains the steady state gross interest rate

$$R = \frac{1}{\beta} \quad (46)$$

Using (31), (32), and (33), the steady state values of the costate variables can be computed as

$$\lambda_1 = \left(C^{-\sigma} + \frac{a}{b} N^\eta \right) \left(1 + \frac{a}{b} (1 - \alpha) N^{-\alpha} \right)^{-1} \quad (47)$$

$$\lambda_2 = \frac{1}{a} (C^{-\sigma} - \lambda_1) \quad (48)$$

where

$$a = \frac{\epsilon}{\psi} \left(\frac{\sigma}{1 - \alpha} \right) C^{\sigma-1} N^{1+\eta} \quad (49)$$

$$b = \frac{\epsilon}{\psi} \left(\left(\frac{1 + \eta}{1 - \alpha} \right) C^\sigma N^\eta - \left(\frac{1 - \alpha}{\mu} \right) N^{-\alpha} \right) \quad (50)$$

3.2 Model Approximation

The first-order approximation of the model is of the form $\left(\frac{X_t - X}{X} \right) \approx \log(X_t) - \log(X) \equiv \hat{X}_t$. The core equations (20), (21), (22), (23), (24), (27), and the autoregressive process (3) are then:

$$\hat{C}_t = E_t[\hat{C}_{t+1}] - \frac{1}{\sigma} (\hat{R}_t - E_t[\hat{\pi}_{t+1}]) \quad (51)$$

$$\hat{W}_t - \hat{P}_t = \sigma \hat{C}_t + \eta \hat{N}_t \quad (52)$$

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \frac{Y(\epsilon - 1)}{\psi} (\widehat{MC}_t + \hat{\mu}_t) \quad (53)$$

$$\widehat{MC}_t = \hat{W}_t - \hat{P}_t + \left(\frac{\alpha}{1 - \alpha} \right) \hat{Y}_t \quad (54)$$

$$\hat{Y}_t = (1 - \alpha) \hat{N}_t, \quad 0 \leq \alpha < 1 \quad (55)$$

$$\hat{Y}_t = \hat{C}_t \quad (56)$$

$$\hat{\epsilon}_t = \rho \hat{\epsilon}_{t-1} + e_t \quad , \quad 0 \leq \rho < 1 \quad (57)$$

where $\hat{\mu}_t = -\hat{\epsilon}_t (\epsilon - 1)^{-1}$. The first-order conditions of the Ramsey planner (31), (32), and (33) are now given by

$$\begin{aligned} & \sigma (\epsilon (1 - \sigma) \lambda_2 N^{1+\eta} - \psi (1 - \alpha) C^{1-2\sigma}) (\psi (1 - \alpha) \lambda_1 C^{1-\sigma})^{-1} \hat{C}_t - E_t [\hat{\lambda}_{1,t+1}] \\ & + \frac{\lambda_1 \sigma}{\lambda_2 C} \left(\hat{\pi}_t - \beta E_t [\hat{\pi}_{t+1}] \right) - \frac{\lambda_1 \epsilon N^{1+\eta}}{\lambda_2 \psi C^{1-\sigma}} \left(E_t [\hat{\lambda}_{2,t+1}] + \hat{\epsilon}_t + (1 + \eta) \hat{N}_t \right) \left(\frac{\sigma}{1 - \alpha} \right) = 0 \end{aligned} \quad (58)$$

$$\begin{aligned} & - \left(\frac{\alpha (1 - \alpha)^2 N^{-\alpha} (\lambda_1 \psi + \lambda_2 (\epsilon - 1)) + \eta N^\eta (\psi (1 - \alpha) + \lambda_2 C^\sigma (1 + \eta) \epsilon)}{\psi (1 - \alpha)} \right) \hat{N}_t \\ & - \left(\frac{\lambda_2 (1 + \eta) C^\sigma N^\eta \sigma \epsilon}{\psi (1 - \alpha)} \right) \hat{C}_t + \left(\frac{1 - \alpha}{\lambda_1^{-1} N^\alpha} \right) E_t [\hat{\lambda}_{1,t+1}] \\ & + \left(\frac{\lambda_2 ((\epsilon - 1) (1 - \alpha)^2 - C^\sigma N^{\eta+\alpha} (1 + \eta) \epsilon)}{\psi (1 - \alpha) N^\alpha} \right) E_t [\hat{\lambda}_{2,t+1}] \\ & + \left(\frac{\lambda_2 ((1 - \alpha)^2 - C^\sigma N^{\eta+\alpha} (1 + \eta) \epsilon)}{\psi (1 - \alpha) N^\alpha} \right) \hat{\epsilon}_t = 0 \end{aligned} \quad (59)$$

$$-\lambda_1 \psi \hat{\pi}_t + \lambda_2 (E_t [\hat{\lambda}_{2,t+1}] - \hat{\lambda}_{2,t}) + \lambda_2 (\hat{C}_t - \hat{C}_{t-1}) \sigma = 0 \quad (60)$$

The interest rate rule (34) now reads

$$\hat{R}_t = (1 - \phi) (\delta_\pi \hat{\pi}_t + \delta_y \hat{Y}_t) + \phi \hat{R}_{t-1} \quad (61)$$

3.3 Welfare Measure Approximation

According to Benigno and Woodford (2004, 2006), Kim and Kim (2006), and Woodford (1999, 2003), welfare could be measured by a quadratic approximation of the aggregate utility function. The absolute welfare measure is then given by

$$J_t^{abs.} = \Omega (\hat{Y}_t - \gamma)^2 + \Gamma \hat{\pi}_t^2 + \beta E_t [J_{t+1}^{abs.}] \quad (62)$$

where the output and inflation weights are determined by

$$\Omega = \frac{Y^{1-\sigma}}{2} \frac{\beta(1+\eta) - (1-\sigma)(1-\alpha)}{(1-\alpha)} \quad (63)$$

$$\gamma = \frac{(1-\alpha)(1-\beta(1-\varphi))}{\beta(1+\eta) - (1-\sigma)(1-\alpha)} \quad (64)$$

$$\Gamma = \frac{Y^{-\sigma}\psi}{2} \quad (65)$$

and the parameter $\varphi = 1 - \frac{1}{\mu}$ gives the monopolistic distortions in the economy at the steady state. Relative welfare between the two policies is compared with

$$J_t^{rel.} = \left(\frac{J_t^{abs.} (Taylor)}{J_t^{abs.} (Ramsey)} \right) \quad (66)$$

Since $J_t^{abs.} > 0$, the Taylor rule generates a lower welfare loss than the Ramsey policy if $J_t^{rel.} < 100\%$.

4 Simulation

The model parameters are fixed according to a quarter time unit. We set the households' subjective discount factor β to 0.99, implying an annualized steady state real interest rate of 4 percent. The steady state value of the substitution elasticity between intermediate goods is $\epsilon = 6$. This implies a steady state markup on firms' marginal costs of 20 percent. Concerning the Taylor rule (34) (or (61)), the parameters are $\delta_\pi = 1.5$ and $\delta_y = 0.5$. The remaining parameters are varied along the values of the calibration literature but such that the parameter regions fulfill the

Blanchard Kahn stability conditions (see Blanchard and Kahn (1980)). We let $\alpha \in [0, 0.3]$ in order to explore the consequence of diminishing returns to labor. Similarly, $\rho \in [0, 0.9]$ represents an increasing degree of persistence in the shock process. We also let $\phi \in [0, 0.8]$ (Increasing interest rate smoothing), $\psi \in [250, 550]$ (Increasing price persistence), $\sigma \in [2, 2.3]$ (Increasing risk aversion), and $\eta \in [2, 2.9]$ (Increasing labor supply elasticity). The shock impulse e in the autoregressive process (3) (or (57)) leads to a decrease of the elasticity of substitution ϵ (or $\hat{\epsilon}$) and therefore to an increase of the markup μ (or $\hat{\mu}$).

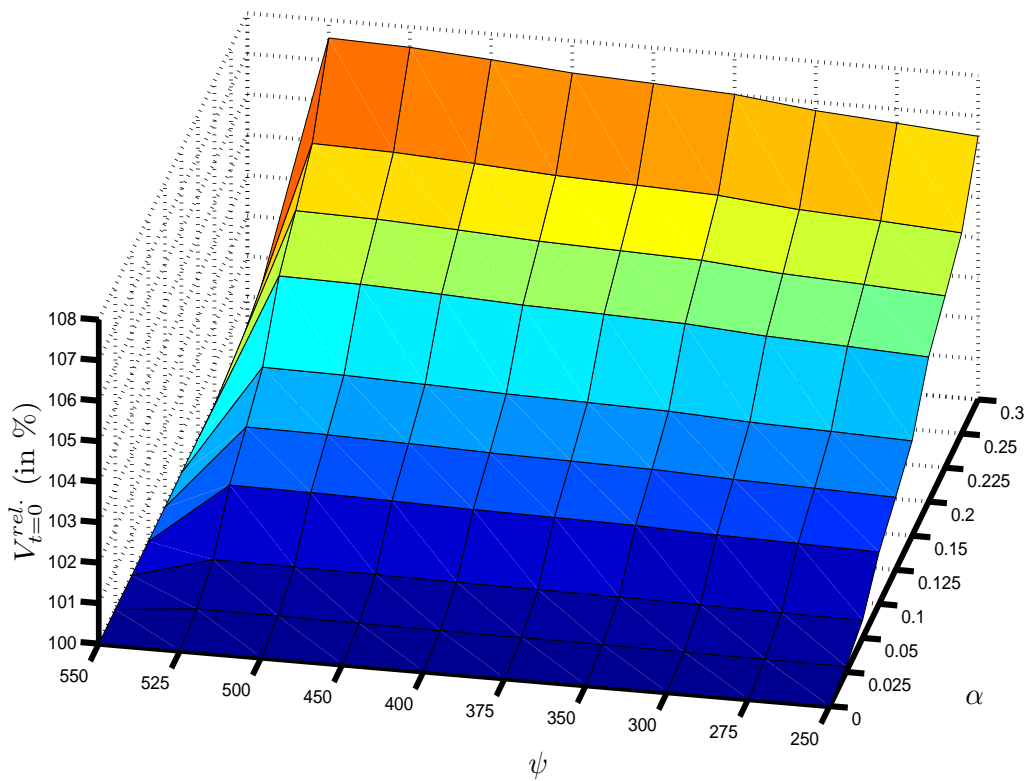


Figure 1: Relative welfare in the *nonlinear* model, where $V_{t=0}^{rel.} = \left(\frac{V_{t=0}^{abs.}(Taylor)}{V_{t=0}^{abs.}(Ramsey)} \right) 100$ for $\psi \in [250, 550]$ and $\alpha \in [0, 0.3]$. Parametrization: $\beta = 0.99$, $\sigma = 2$, $\eta = 2$, $\phi = 0$, $\rho = 0$, $\delta_y = 0.5$, $\delta_\pi = 1.5$, $\epsilon = 6$.

Thus, we simulate a stagflationary cost-push shock of one percent at $t = 0$. Note that the nonlinear model is deterministic, while the approximated framework is stochastic with $e \sim N(0, 1)$ in (57). We simulate the nonlinear model version (3),(20),(21),(22),(23),(24), and (27) with the Taylor rule (34). In a second step, we

replace (34) by the Ramsey policy (31),(32),(33), and compute $V_{t=0}^{rel.}$ according to (36). These two steps are repeated in the approximated model (51)-(61) and $J_{t=0}^{rel.}$ is computed according to (66). A welfare reversal occurs if $V_{t=0}^{rel.}$ contradicts $J_{t=0}^{rel.}$. Figure 1 plots relative welfare in the nonlinear model against parameter combinations of α and ψ . The relative welfare performance of the Taylor rule worsens with an increase of both parameters. However, the effect of α is stronger. A glance at figure 2 reveals that changes in relative welfare are much greater in the approximated environment.

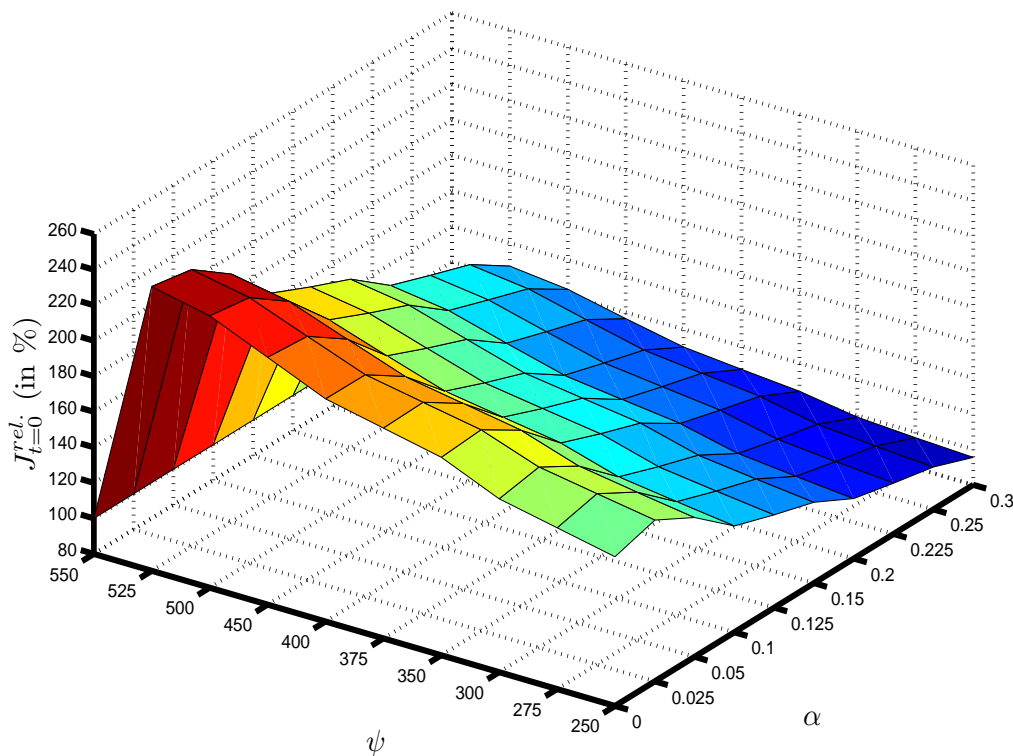


Figure 2: Relative welfare in the *approximated* model, where $J_{t=0}^{rel.} = \left(\frac{J_{t=0}^{abs.}(Taylor)}{J_{t=0}^{abs.}(Ramsey)} \right) 100$ for $\psi \in [250, 550]$ and $\alpha \in [0, 0.3]$. Parametrization: $\beta = 0.99$, $\sigma = 2$, $\eta = 2$, $\phi = 0$, $\rho = 0$, $\delta_y = 0.5$, $\delta_\pi = 1.5$, $\epsilon = 6$.

While in the nonlinear model $V_{t=0}^{rel.} \in [100\%, 106\%]$, we have $J_{t=0}^{rel.} \in [98\%, 223\%]$ in the approximated case. Notably, $J_{t=0}^{rel.} < 100\%$, which was not observable in the nonlinear case. Therefore, we obtain a welfare reversal (i.e. the Taylor rule dominates) for low degrees of price persistence $\psi \in [250, 300]$ and accentuated degrees of diminishing

returns to labor $(1 - \alpha) \in [0.7, 0.75]$. An important assessment is the importance of both parameters in driving this result. Now consider figure 3:

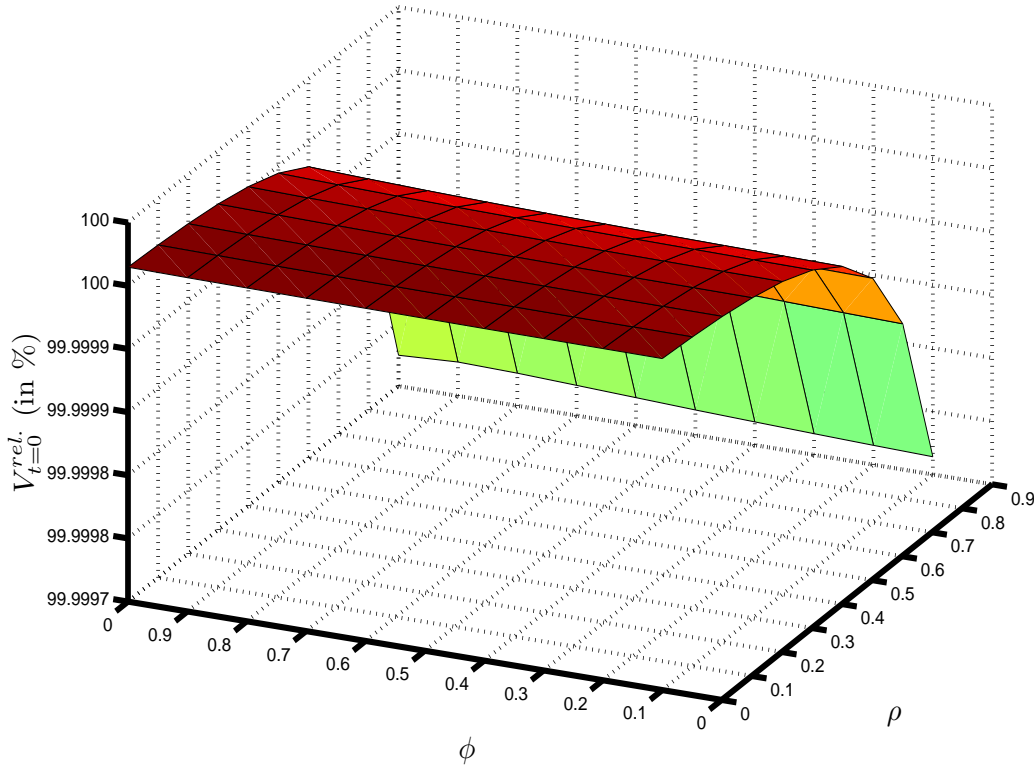


Figure 3: Relative welfare in the *nonlinear* model, where $V_{t=0}^{rel.} = \left(\frac{V_{t=0}^{abs.}(Taylor)}{V_{t=0}^{abs.}(Ramsey)} \right) 100$ for $\phi \in [0, 0.9]$ and $\rho \in [0, 0.9]$. Parametrization: $\alpha = 0$, $\psi = 350$, $\beta = 0.99$, $\sigma = 2$, $\eta = 2$, $\delta_y = 0.5$, $\delta_\pi = 1.5$, $\epsilon = 6$.

By letting ϕ increase at a given value of ρ , relative welfare increases. The opposite is observed when ρ increases at a given value of ϕ . Starting from $\rho = 0.4$ upwards, $V_{t=0}^{rel.}$ drops below 100%. Even though this drop is almost unperceivable, the ordinal ranking is reversed. The Taylor rule should therefore be preferred in the nonlinear model, when $\rho \in [0.4, 0.9]$ and $\phi \in [0, 0.9]$. A glance at figure 4 reveals that no such reversal occurs in the approximated model as $J_{t=0}^{rel.} \geq 200\%$ throughout the defined parameter ranges. The relative importance of ρ for movements in $J_{t=0}^{rel.}$ is preserved, but in the opposite direction: The Taylor rule worsens in terms of relative welfare as ρ increases. We assess the importance of ρ and the irrelevance of ϕ for welfare reversals.

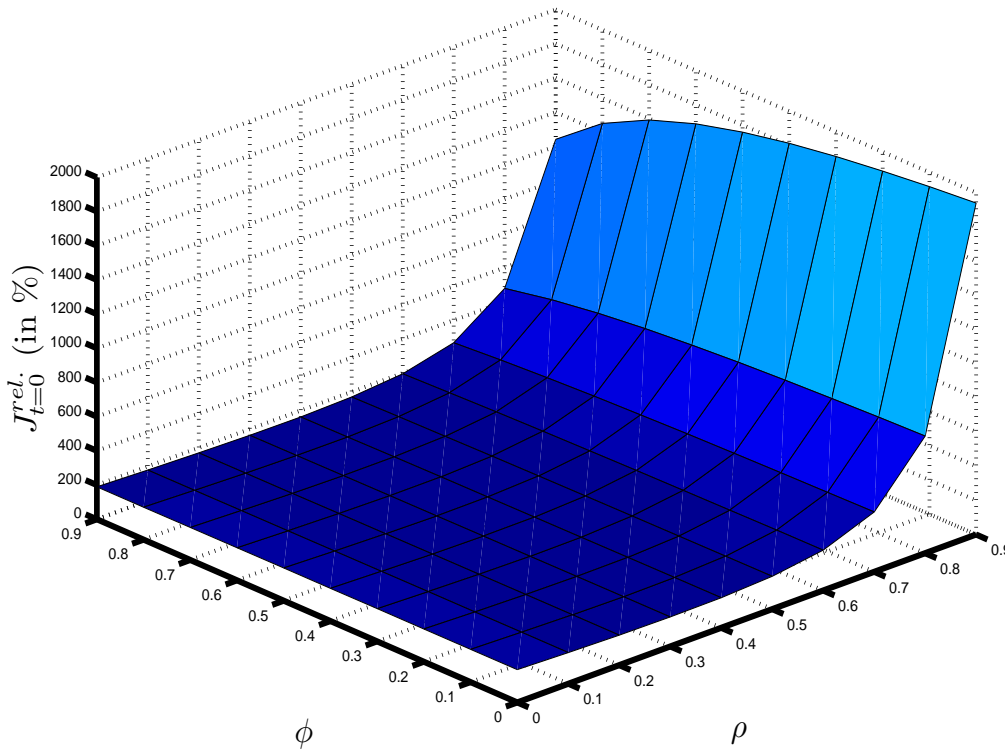


Figure 4: Relative welfare in the *approximated* model, where $J_{t=0}^{rel.} = \left(\frac{J_{t=0}^{abs.}(Taylor)}{J_{t=0}^{abs.}(Ramsey)} \right) 100$ for $\phi \in [0, 0.9]$ and $\rho \in [0, 0.9]$. Parametrization: $\alpha = 0$, $\psi = 350$, $\beta = 0.99$, $\sigma = 2$, $\eta = 2$, $\delta_y = 0.5$, $\delta_\pi = 1.5$, $\epsilon = 6$.

The remaining parameters to be analyzed are σ and η . Figure 5 shows that relative welfare losses of the Taylor rule increase marginally (0.28 percent points) as η decreases and σ rises. A similar effect can be stated in the approximated model for an increasing η and a *decreasing* value of σ . There is no welfare reversal in our predefined parameter ranges, although changes in $J_{t=0}^{rel.}$ are stronger than in $V_{t=0}^{rel.}$.

5 Conclusions

The aim of this paper was to determine parameters driving welfare reversals when approximating a basic New Keynesian model. We find that the assumption of diminishing returns to labor at an empirically relevant value and a moderate price rigidity leads to counterfactual policy recommendations.

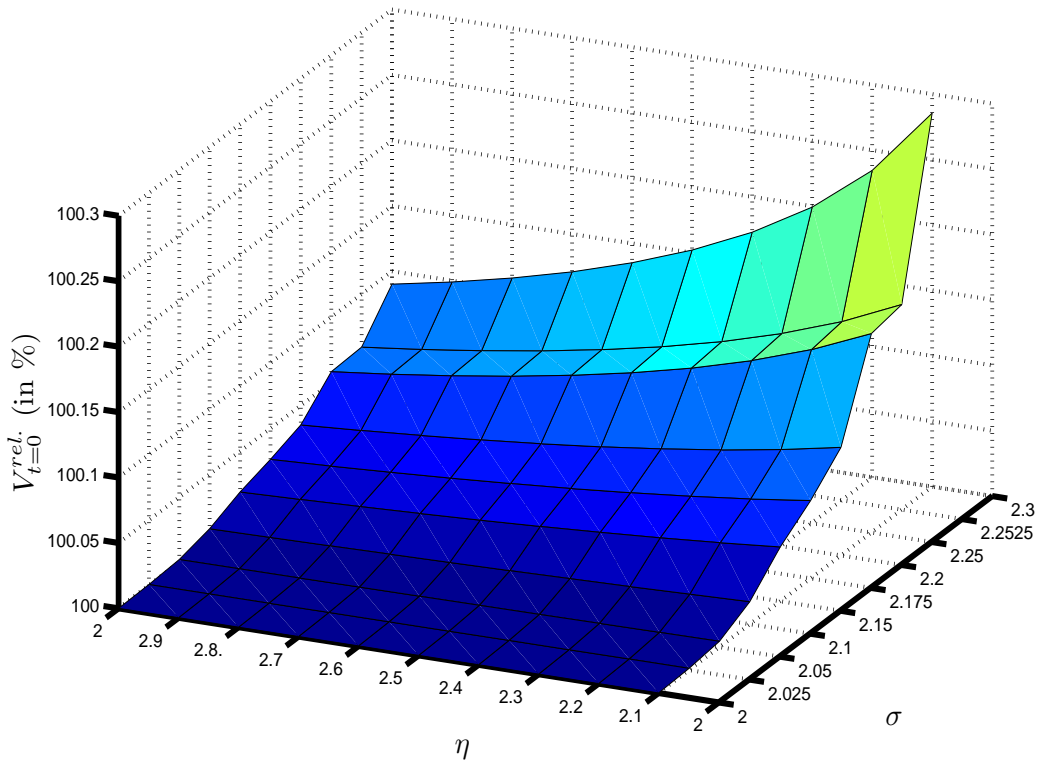


Figure 5: Relative welfare in the *nonlinear* model, where $V_{t=0}^{rel.} = \left(\frac{V_{t=0}^{abs.}(Taylor)}{V_{t=0}^{abs.}(Ramsey)} \right) 100$ for $\eta \in [2, 2.9]$ and $\sigma \in [2, 2.3]$. Parametrization: $\alpha = 0$, $\psi = 350$, $\rho = 0$, $\phi = 0$, $\beta = 0.99$, $\delta_y = 0.5$, $\delta_\pi = 1.5$, $\epsilon = 6$.

That is, the Ramsey monetary policy under timeless perspective commitment generates lower welfare losses than the Taylor rule in the nonlinear framework, but not in the approximated model. This may result from the increased curvature in the nonlinear utility function, whose quadratic approximation would fail to account for. However, empirically relevant values of the preference parameters do not generate this result because the range of values is relatively tight due to the Blanchard Kahn stability requirement. The degree of smoothing in the interest rate rule does not play any role since the welfare ranking remains intact. However, an interesting observation is that relative welfare losses increase in the nonlinear model as the degree of interest rate smoothing increases. This is not the case in the approximated model, in which a past-oriented interest rate rule tends to stabilize expectations and therefore reduce welfare losses. Another key parameter driving welfare reversals is

the persistence degree of the autocorrelated shock. While a long-lived stagflationary shock calls for the Taylor rule in the nonlinear setting, it favors the timeless Ramsey policy in the approximated model. Our results imply that the informational loss due to the neglect of nonlinearities could be substantial, especially when considering diminishing returns and persistent cost-push shocks.

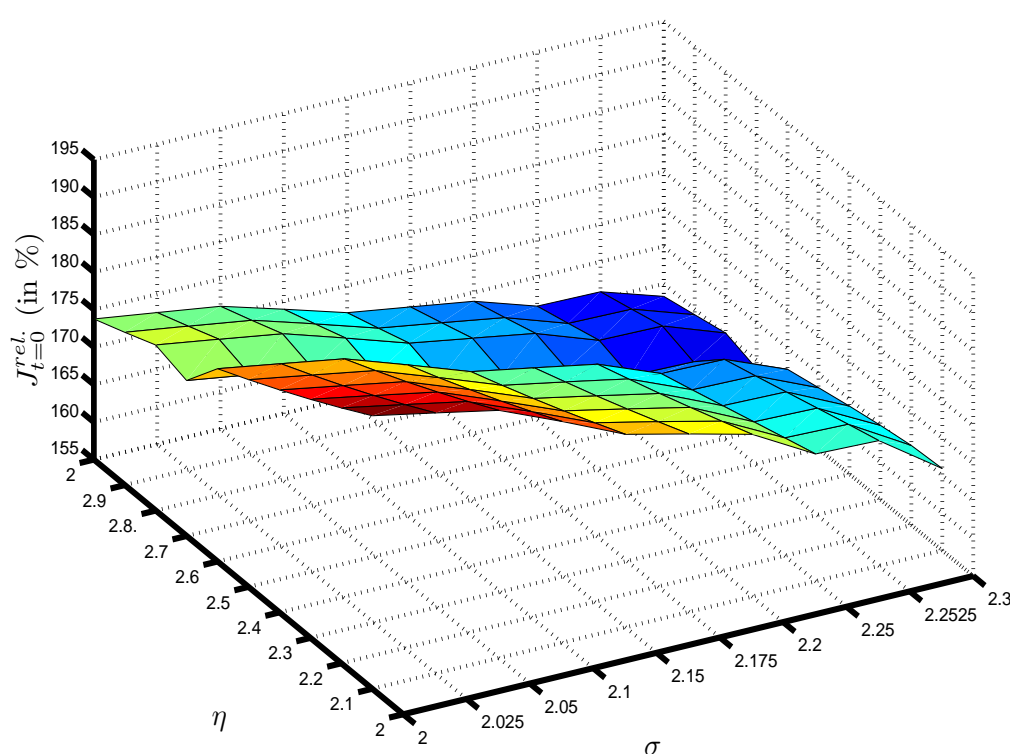


Figure 6: Relative welfare in the *approximated* model, where $J_{t=0}^{rel.} = \left(\frac{J_{t=0}^{abs.}(Taylor)}{J_{t=0}^{abs.}(Ramsey)} \right) 100$ for $\eta \in [2, 2.9]$ and $\sigma \in [2, 2.3]$. Parametrization: $\alpha = 0$, $\psi = 350$, $\rho = 0$, $\phi = 0$, $\beta = 0.99$, $\delta_y = 0.5$, $\delta_\pi = 1.5$, $\epsilon = 6$.

6 References

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