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by

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# Persistence Endogeneity Via Adjustment Costs: An Assessment based on Bayesian Estimations ☆

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## Abstract

This paper estimates a dynamic stochastic general equilibrium (DSGE) model for the European Monetary Union by using Bayesian techniques. A salient feature of the model is an extension of the typically postulated quadratic cost structure for the monopolistic choice of price variables. As shown in Sienknecht (2010a), the enlargement of the original formulation by Rotemberg (1983) and Hairault and Portier (1993) leads to structurally more sophisticated inflation schedules than in the staggering environment by Calvo (1983) with rule-of-thumb setters. In particular, a desired lagged inflation term always arises together with a two-period-ahead expectational expression. The two terms are directly linked by a novel structural parameter. We confront the relationships obtained by Sienknecht (2010a) against European data and compare their data description performance against the widespread extension of the Calvo setting with rule-of-thumb behavior.

***JEL classification:*** C11; C15; E31; E32

***Key words:*** Bayesian; Simulation; Indexation; Model Comparison

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# 1 Introduction

The Bayesian estimation methodology is the most common choice when it comes to evaluate a dynamic stochastic general equilibrium (DSGE) model with empirical data. It allows for an estimation of model parameters taking all economic relationships simultaneously into consideration. Apart from this desirable full characterization of observed data, the Bayesian approach allows for an intuitive comparison between different models with respect to their empirical fit. We complement previous theoretical work by Sienknecht (2010a) by using Bayesian techniques. The novelty of the model introduced in that paper is an extension of the quadratic adjustment cost structure, which was originally postulated by Rotemberg (1982) and Hairault and Portier (1993). On the aggregate level, this modified cost structure leads to a lagged inflation term connected with an additional and unavoidable two-period-ahead inflation expectation. This connection is direct, through a novel structural parameter. Therefore, Sienknecht (2010a) obtains more sophisticated hybrid New Keynesian Phillips curves than in the staggering environment of Calvo (1983) with rule-of-thumb setters. Naturally, the question about the plausibility of this inflation schedule structure arises because an additional two-period-ahead expectation term is rather unusual. The argument is clarified by Sienknecht (2010a). Accordingly, the importance of intertemporal adjustment cost amounts could be taken into consideration, along with a quarterly parameter calibration. The latter makes the two-period-ahead expectational time horizon not questionable at least on theoretical grounds. However, these theoretical foundations have to be complemented by sensible econometric results in order to verify the practical usefulness of the model. We pursue two main objectives taking into account empirical series for the European Monetary Union. Firstly, the hybrid adjustment cost model by Sienknecht (2010a) is estimated using Bayesian methods in order to obtain empirical values for the newly introduced parameters. Secondly, another two models are estimated as a point of reference (as in Rabanal and Rubio-Ramírez (2005)). One of them is the framework with purely forward-looking New Keynesian Phillips curves postulated by Rotemberg (1982) and Hairault and Portier (1993). The remaining reference model entails hybrid inflation schedules resulting from a standard Calvo environment with rule-

of-thumb setters. We estimate the reference models in order to rank the adjustment cost model by Sienknecht (2010a) against them. The ranking criterion is the ability of a model to fit the observed data series.

The rest of the paper is organized as follows. Section 2 presents our linearized DSGE model versions. Section 3 describes the underlying dataset for the European Monetary Union and reviews the basic idea behind the Bayesian estimation methodology. Section 4 presents our estimation results. Section 5 concludes and points to further areas of research.

## 2 The Linearized DSGE Models

This section outlines three different linear equation systems. Each collection of equations is characterized by a specific type of New Keynesian Phillips curve, while all relationships other than inflation schedules remain identical across the models (as in Rabanal and Rubio-Ramírez (2005)). The baseline model (BSAC) contains a purely forward-looking inflation curve following Rotemberg (1982) and Hairault and Portier (1993). We compare the empirical performance of this baseline specification to the hybrid alternative (HYAC) postulated by Sienknecht (2010a). Most importantly, we pursue an empirical comparison of these hybrid inflation curves against the standard Calvo (1983) schedules with rule-of-thumb setters (HYCC). The latter are derived according to Galí and López-Salido (2001). Note that the only disturbance considered in Sienknecht (2010a) is a stochastic shock on the interest rate instrument. However, we have to increase the number of shocks in order to rule out a stochastic singularity in Bayesian estimation. Since we employ five time series of European data, the stochastic singularity problem is avoided by introducing six exogenous stochastic shocks.

## 2.1 Common Equations

Several relationships are general to the three models considered here and can be derived as in Sienknecht (2010a, 2010b). Concerning the representative household, we assume internal habit formation in consumption (Casares (2006)). Moreover, we introduce a variable  $\varpi_{B,t}$  which represents a general shock to preferences affecting intertemporal substitution (Smets and Wouters (2003)). The detailed optimization problem of the household can be inspected in the appendix. From the first-order conditions, we derive the linearized Euler equation for intertemporal consumption. Since in our model aggregate demand is composed only by consumption ( $\hat{Y}_t = \hat{C}_t$ ), we obtain:

$$\begin{aligned} \hat{Y}_t = & \Theta_1 \hat{Y}_{t-1} + \Theta_2 E_t [\hat{Y}_{t+1}] - \Theta_3 E_t [\hat{Y}_{t+2}] - \Theta_4 (\hat{R}_t^d - E_t [\hat{\pi}_{t+1}^p]) \\ & + \Theta_5 \hat{\varpi}_{B,t} - \Theta_6 E_t [\hat{\varpi}_{B,t+1}] + \Theta_7 E_t [\hat{\varpi}_{B,t+2}] \end{aligned} \quad (1)$$

Throughout the paper, a hat over a variable represents its logarithmic deviation from its steady state. Here,  $\hat{R}_t^d$  gives the gross deposit interest rate at commercial banks, and  $\hat{\pi}_t^p$  represents the gross price inflation rate. The variables  $\Theta_1 \dots \Theta_7$  are functions of deep model parameters and can be examined in detail in the appendix. From the constrained utility maximization with respect to the real money stock  $\frac{M_t}{P_t}$ , one obtains the following aggregate real money demand schedule:

$$\begin{aligned} \hat{M}_t - \hat{P}_t = & \Phi_1 \hat{Y}_t - \Phi_2 \hat{Y}_{t-1} - \Phi_3 E_t [\hat{Y}_{t+1}] - \Phi_4 \hat{R}_t^d \\ & + \Phi_5 \hat{\varpi}_{M,t} - \Phi_6 (\hat{\varpi}_{B,t} - E_t [\hat{\varpi}_{B,t+1}]) \end{aligned} \quad (2)$$

where  $\Phi_1 \dots \Phi_6$  are, again, composite parameters. The same applies for  $\iota_1 \dots \iota_4$  when considering the household marginal rate of substitution with an inverse real wage elasticity of labor supply  $\eta$ :

$$\widehat{MRS}_t = \eta \hat{N}_t + \iota_1 \hat{Y}_t - \iota_2 \hat{Y}_{t-1} - \iota_3 E_t [\hat{Y}_{t+1}] - \iota_4 (\hat{\varpi}_{B,t} - E_t [\hat{\varpi}_{B,t+1}]) \quad (3)$$

Intermediate firms are assumed to prefinance labor costs by using bank loans (Henzel et. al (2009) and Hülsewig et. al (2009)). The resulting real marginal costs can be shown to evolve as:

$$\widehat{MC}_t = \hat{W}_t - \hat{P}_t + \hat{R}_t^L + \left( \frac{\alpha}{1-\alpha} \right) \hat{Y}_t - \left( \frac{1}{1-\alpha} \right) \hat{\omega}_{A,t} \quad (4)$$

where  $\hat{R}_t^L$  is the aggregate loan rate,  $\hat{W}_t$  is the aggregate nominal wage rate, and  $\hat{P}_t$  is the aggregate price level. The variable  $\hat{\omega}_{A,t}$  represents a technology shock augmenting the marginal product of labor. Aggregate employment  $\hat{N}_t$  and real output are linked through the following technology with decreasing returns to labor:

$$\hat{N}_t = \frac{1}{1-\alpha} (\hat{Y}_t - \hat{\omega}_{A,t}) \quad , \quad 0 < \alpha < 1 \quad (5)$$

Price inflation and nominal wage inflation rates are linked to one another by a standard inflation identity equation:

$$\hat{W}_t - \hat{P}_t = \hat{W}_{t-1} - \hat{P}_{t-1} + \hat{\pi}_t^w - \hat{\pi}_t^p \quad (6)$$

The monetary policy authority is assumed to follow a Taylor rule (see Taylor (1993)) with interest rate smoothing:

$$\hat{R}_t = (1-\phi) (\delta_\pi \hat{\pi}_t^p + \delta_y (\hat{Y}_t - \hat{Y}_t^{pot.})) + \phi \hat{R}_{t-1} + \hat{\omega}_{R,t} \quad (7)$$

where the shock variable  $\hat{\omega}_{R,t}$  captures an unsystematic deviation from the instrument rule and  $\phi$  denotes the degree of interest rate smoothing. In contrast to Sienknecht (2010a), the output gap  $\hat{Y}_t - \hat{Y}_t^{pot.}$  is taken into account and not just simply real output  $\hat{Y}_t$ . The reason is the appearance of shock variables  $\hat{\omega}_{A,t}$  and  $\hat{\omega}_{B,t}$  that affect real output and its potential level  $\hat{Y}_t^{pot.}$  simultaneously.

## 2.2 Baseline Adjustment Cost Model (BSAC)

The first model version is given by the equations declared so far together with purely forward-looking New Keynesian Phillips curves. Nominal rigidities stem from costs of price adjustment for the monopolistic agent (Rotemberg (1982) and Hairault

and Portier (1993)). Detailed derivations can be found for example in Sienknecht (2009a). The development of price inflation is derived from the first-order condition for the monopolistic intermediate firm. On the aggregate level, we obtain the following inflation schedule:

$$\hat{\pi}_t^p = \beta E_t [\hat{\pi}_{t+1}^p] + \gamma (\widehat{MC}_t + \hat{\mu}_{p,t}) \quad (8)$$

where  $\beta$  is the household discount factor and  $\hat{\mu}_{p,t}$  denotes the time-varying monopolistic markup of the intermediate firm. It is inversely related to the price elasticity of demand  $\hat{\epsilon}_{p,t}$ , with  $\epsilon_p$  as its steady state counterpart:

$$\hat{\mu}_{p,t} = -\frac{1}{\epsilon_p - 1} \hat{\epsilon}_{p,t} \quad (9)$$

Therefore, an unexpected decrease in  $\hat{\epsilon}_{p,t}$  implies an increase in the intermediate firm's monopolistic power. Thus, it represents an elasticity-driven cost-push shock. The degree of inflation reactivity  $\gamma$  is a function of deep parameters (see the appendix). Nominal wage inflation results from the first-order condition for the wage-setting household:

$$\hat{\pi}_t^w = \beta E_t [\hat{\pi}_{t+1}^w] + \kappa (\widehat{MRS}_t - (\hat{W}_t - \hat{P}_t) + \hat{\omega}_{B,t} + \hat{\mu}_{w,t}) \quad (10)$$

where  $\kappa$  comprises again deep model parameters stated in the appendix. Similarly,  $\hat{\mu}_{w,t}$  denotes the time-varying monopolistic markup of the representative household inversely related to the substitution elasticity between labor types  $\hat{\epsilon}_{w,t}$ :

$$\hat{\mu}_{w,t} = -\frac{1}{\epsilon_w - 1} \hat{\epsilon}_{w,t} \quad (11)$$

We introduce monopolistic commercial banks as in Henzel et. al (2009) and Hülsewig et. al (2009). The resulting equation for the evolvement of the aggregate loan rate can be written as:

$$\hat{R}_t^L = \varphi_1 \hat{R}_{t-1}^L + \varphi_2 E_t [\hat{R}_{t+1}^L] + \varphi_3 (\hat{R}_t^M + \hat{\mu}_{r,t}) \quad (12)$$



Analogously, the pertinent relationship for the monopolistic markup in the banking sector is given by:

$$\hat{\mu}_{r,t} = -\frac{1}{\zeta - 1} \hat{\epsilon}_{r,t} \quad (13)$$

where a sudden decrease of  $\hat{\epsilon}_{r,t}$  represents an inflationary shock as the resulting increase in  $\hat{R}_t^L$  feeds into the real marginal cost of the firm  $\widehat{MC}_t$  (see equation (4)). The market interest rate  $\hat{R}_t^M = \hat{R}_t$  drives the aggregate loan rate as it constitutes the marginal cost of commercial banks. Note again that  $\varphi_1 \dots \varphi_3$  are functions of deep parameters listed in the appendix.

### 2.3 Hybrid Adjustment Cost Model (HYAC)

The second model version consists of the common equations and the hybrid New Keynesian Phillips curves derived by Sienknecht (2010a). The latter result from a modification of the nonlinear adjustment cost structure given by Rotemberg (1983) and Hairault and Portier (1993) :

$$Q_t^x = \frac{\psi_x}{2} \left( \frac{X_t}{X_{t-1}} - \frac{X}{X} \right)^2 + \frac{\nu_x}{2} \left( \frac{X_t}{X_{t-1}} - \frac{X_{t-1}}{X_{t-2}} \right)^2, \quad \psi_x, \nu_x > 0 \quad (14)$$

where  $X_t$  is the choice variable of a monopolistic agent not explicitly indexed and  $X$  denotes the steady state level of this variable. The extension by Sienknecht (2010b) is given by the second term with a novel rigidity parameter  $\nu_x > 0$ . Whenever  $X_t$  is adjusted, not only the absolute level of adjustment costs  $\frac{X_t}{X_{t-1}}$  is a matter of concern, but also its change relative to the last period. The agent is agnostic with respect to the last period's adjustment cost amount if  $\nu_x = 0$ . The representative intermediate firm maximizes profits with respect to its own price taking into account the cost structure (14) and its specific product demand schedule (see the appendix). Aggregation and linearization in logarithms leads to the following hybrid New Keynesian Phillips curve for price inflation:

$$\hat{\pi}_t^p = \gamma_1 \hat{\pi}_{t-1}^p + \gamma_2 E_t [\hat{\pi}_{t+1}^p] - \gamma_3 E_t [\hat{\pi}_{t+2}^p] + \gamma_4 (\widehat{MC}_t + \hat{\mu}_{p,t}) \quad (15)$$

where  $\gamma_1 \dots \gamma_4$  are functions of deep parameters stated explicitly in the appendix. The cost structure (14) can also be assumed for the monopolistic adjustment of nominal wages. From the assumption that households supply labor in a monopolistic manner, one can derive the nominal wage inflation schedule as:

$$\begin{aligned} \hat{\pi}_t^w = & \kappa_1 \hat{\pi}_{t-1}^w + \kappa_2 E_t[\hat{\pi}_{t+1}^w] - \kappa_3 E_t[\hat{\pi}_{t+2}^w] \\ & + \kappa_4 (\overline{MRS}_t - (\hat{W}_t - \hat{P}_t) + \hat{\omega}_{B,t} + \hat{\mu}_{w,t}) \end{aligned} \quad (16)$$

Again,  $\kappa_1 \dots \kappa_4$  are deep parameter containers. Our inflation equations (15) and (16) differ structurally from the baseline model since a lagged term arises in conjunction with a two-period-ahead expectational term. Both are directly linked through the deep parameters  $v_p$  and  $v_w$ , respectively. Setting these parameters equal to zero ( $v_p = v_w = 0$ ) eliminates the corresponding composite parameters ( $\gamma_1 = \gamma_3 = \kappa_1 = \kappa_3 = 0$ ) and renders New Keynesian Phillips curves structurally equivalent to equation (8)<sup>1</sup>. A glance in the appendix reveals a similar optimization problem for a commercial bank. However, we only impose the standard non-linear adjustment cost structure (For the general case (14), this implies  $v_x = 0$ ). The reason is that a persistence endogeneity of loan rate changes is not expected to be found in the data (Henzel et. al (2009) and Hülsewig et. al (2009)). Therefore, the relevant loan rate schedule in this model version is identical to equation (12).

## 2.4 Hybrid Calvo Model (HYCC)

The most widespread approach of nominal rigidity modelling is the staggered price setting environment of Calvo (1983). Standardly, purely forward-looking New Keynesian Phillips curves are transformed into hybrid versions by assuming rule-of-thumb setters. We formulate this behavior in the spirit of Galí, Gertler, and López-

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<sup>1</sup> Note that the BSAC model does not contain an inflation schedule for nominal wages since the latter are completely flexible in that setting.

Salido (2001). In our setting, the following hybrid New Keynesian Phillips curve for price inflation is obtained:

$$\hat{\pi}_t^p = \gamma_1^c \hat{\pi}_{t-1}^p + \gamma_2^c E_t [\hat{\pi}_{t+1}^p] - \gamma_3^c E_t [\hat{\pi}_{t+2}^p] + \gamma_4^c (\widehat{MC}_t + \hat{\mu}_{p,t}) \quad (17)$$

where the superscript ‘c’ denotes reaction parameters under the Calvo pricing assumption. Their dependence on deep model parameters can be confirmed in the appendix. Note that in this well-known approach an expectational term of two periods ahead does not arise. We included the term  $\gamma_3^c E_t [\hat{\pi}_{t+2}^p]$  with  $\gamma_3^c = 0$  merely for the sake of comparability against the hybrid adjustment cost case (HYAC). Similarly, nominal wage inflation evolution á la Calvo (1983) can easily be shown to be:

$$\begin{aligned} \hat{\pi}_t^w = & \kappa_1^c \hat{\pi}_{t-1}^w + \kappa_2^c E_t [\hat{\pi}_{t+1}^w] - \kappa_3^c E_t [\hat{\pi}_{t+2}^w] \\ & + \kappa_4^c (\widehat{MRS}_t - (\hat{W}_t - \hat{P}_t) + \hat{\omega}_{B,t} + \hat{\mu}_{w,t}) - \kappa_5^c \hat{\pi}_t^p \end{aligned} \quad (18)$$

where, again,  $\kappa_3^c = 0$ . The aggregate loan rate change schedule is calculated once more as being purely forward-looking in order to keep consistence with loan rate data:

$$\hat{R}_t^L = \varphi_1^c \hat{R}_{t-1}^L + \varphi_2^c E_t [\hat{R}_{t+1}^L] + \varphi_3^c (\hat{R}_t^M + \hat{\mu}_{r,t}) \quad (19)$$

## 2.5 Shock Processes

The elasticities  $\hat{\epsilon}_{p,t}$ ,  $\hat{\epsilon}_{w,t}$ ,  $\hat{\epsilon}_{r,t}$  and the instrument interest rate shock variable  $\hat{\epsilon}_{R,t}$  are assumed to be subject to a one-off white noise disturbance with a constant variance:

$$\hat{\epsilon}_{z,t} = e_{z,t} \quad , \quad e_{z,t} \sim N(0, \sigma_z^2) \quad , \quad \text{where } z = p, w, r, R \quad (20)$$

In contrast, we assume the real shocks to be persistent by means of AR(1) processes:

$$\hat{\omega}_{s,t} = \rho_s \hat{\omega}_{s,t-1} + e_{s,t} \quad , \quad \rho_s \in [0, 1) \quad , \quad e_{s,t} \sim N(0, \sigma_s^2) \quad , \quad \text{where } s = A, B \quad (21)$$

### 3 Data and Preliminaries

We proceed to estimate the presented theoretical models and to assess their empirical fit. The ranking strategy is closely related to Rabanal and Rubio-Ramírez (2005). That is, we compare the probabilities (likelihoods) of the models to describe the empirical data. Apart from presenting the set of time series used, the following sections explain the rationale behind our fixed parameter values and the parameter priors. They are crucial for the Bayesian estimation procedure, which is also briefly reviewed.

#### 3.1 The Data

The underlying dataset includes five quarterly and seasonally adjusted series. Four series are from the Area-Wide-Model (AWM) for the time period 1990Q1-2002Q4<sup>2</sup>:

1. Log of real GDP, seasonally adjusted (**AWM code: YER**).
2. Inflation rate, annualized quarterly change of GDP deflator in percent, seasonally adjusted (**AWM code: YED**).
3. Nominal wage inflation, annualized quarterly change of wage rate in percent (**AWM code: WRN**).
4. Short-term nominal interest rate, in percent (**AWM code: STN**).

In addition, we use the following series for the aggregate loan rate:

6. Retail bank lending rates for loan to enterprises with maturities up to one year, nominal in percent<sup>3</sup>.

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<sup>2</sup> The data was obtained from the website <http://www.eabcn.org/area-wide-model>. See Fagan, Henry and Maestre (2001) for an overview of the AWM and the underlying historical series. We extract the trend of real output by passing the series through a Hodrick-Prescott filter for a quarterly time basis. We also applied a linear quadratic trend filter in order to check the robustness of estimate results. We do not assess any significant difference for our results across the two filtering methods.

<sup>3</sup> This is the same series used by Hülsewig, et al. (2009), which was kindly provided by one of the corresponding authors.

### 3.2 Estimation Methodology

Instead of reviewing the vast amount of literature on Bayesian estimation methods, we provide this section with the very essential idea. The interested reader is referred to core contributions such as Fernández-Villaverde and Rubio-Ramírez (2004) and An and Schorfheide (2007). Fernández-Villaverde (2009) and Almeida (2009) have recently provided very comprehensive explanations of the topic. Since we use the Matlab preprocessor Dynare for solving and estimating our three DSGE models, the following explanations rely heavily on the pertinent instruction manuals<sup>4</sup>. The Bayesian procedure estimates a subset of model parameters with a weighted Maximum Likelihood approach. More precisely, priors for the parameters to be estimated (mean, variance, and type of distribution) are prespecified and combined with the model-specific likelihood function. This gives the target function to be maximized with an optimization routine<sup>5</sup>. As an outcome, one obtains the combination of posterior parameter estimates that renders the dataset and the imposed a-priori beliefs “most likely”. The following lines should give a sensible idea of the Bayesian estimation procedure. All steps explained below are ultimately targeted towards a posterior density function of the form  $p(\boldsymbol{\theta}|\mathbf{Y}_T)$ , where  $\boldsymbol{\theta}$  denotes the vector of parameters to be estimated and  $\mathbf{Y}_T$  is the vector of observables up to period  $T$ . We start with general explanations on the computation of the rational expectations equilibrium and the likelihood function.

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<sup>4</sup> We use Matlab Version 7.7.0.471 (R2008b) and Dynare Version 4.1.2. See for a reference manual Griffoli (2007). More information on Dynare can be retrieved from the website <http://www.dynare.org/>

<sup>5</sup> A well-known advantage of Bayesian estimation is the avoidance of several problems connected with a stand-alone Maximum Likelihood estimation of medium-scale DSGE models. In particular, the reweighting of the likelihood function increases the curvature of the target function. This avoids common problems, such as a flat likelihood over large parameter subspaces and the so-called “dilemma of absurd parameter estimates”.

Note first that any DSGE model is simply a collection of non-linear first-order and equilibrium conditions that takes the following general form:

$$E_t \{f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{e}_t)\} = 0 \quad , \quad \mathbf{e}_t \sim N(0, \Sigma_e) \quad , \quad E_t(\mathbf{e}_t \mathbf{e}'_s) = 0 \quad , \quad t \neq s \quad (22)$$

where  $\mathbf{y}$  is the vector of endogeneous variables and  $\mathbf{e}$  is the vector of shock innovations assumed to be Gaussian white noise processes. The set of endogeneous variables  $\mathbf{y}_{t-1}$  denote predetermined variables while the remaining quantities are only known at time  $t$ . The solution to this system is called the policy function  $g(\cdot)$ . It is a set of equations relating variables in the current period to the past state of the system and to current shocks:

$$\mathbf{y}_t = g(\mathbf{y}_{t-1}, \mathbf{e}_t) \quad (23)$$

Our three models are systems of log-linearized equations. A first-order Taylor expansion in logs around the deterministic steady state of (22) and (23) yields the approximated system:

$$E_t \{f_{y+1} \hat{\mathbf{y}}_{t+1} + f_y \hat{\mathbf{y}}_t + f_{y-1} \hat{\mathbf{y}}_{t-1} + f_{e+1} \mathbf{e}_{t+1} + f_e \mathbf{e}_t\} = 0 \quad (24)$$

$$\hat{\mathbf{y}}_t = g_{y-1} \hat{\mathbf{y}}_{t-1} + g_e \mathbf{e}_t \quad (25)$$

where  $f_{y+1}$ ,  $f_y$ , and  $f_{y-1}$  are the matrix derivatives evaluated at the steady state with respect to  $\mathbf{y}_{t+1}$ ,  $\mathbf{y}_t$ , and  $\mathbf{y}_{t-1}$ , respectively. The same reasoning applies for the remaining derivative terms  $g_{y-1}$  and  $g_e$ . Vectors denoted as  $\hat{\mathbf{y}}_{t+1}$ ,  $\hat{\mathbf{y}}_t$ , and  $\hat{\mathbf{y}}_{t-1}$  contain the logarithmic deviation of endogenous variables from their steady state. The system (24) and (25) can be rewritten in compact matrix notation as:

$$\mathbf{A} \hat{\mathbf{y}}_t = \mathbf{B} \hat{\mathbf{y}}_{t-1} + \mathbf{C} \mathbf{e}_t \quad (26)$$

where the matrices  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  contain matrix derivatives evaluated at the steady

state. The system can be solved with a generalized Schur decomposition along with a verification of the Blanchard-Kahn condition fulfillment, namely a number of generalized eigenvalues outside the unit circle equalized to the number of forward-looking variables. We need to establish a relationship between the observables  $\hat{\mathbf{y}}^*$  in the set of observable variables  $\mathbf{Y}_T$  and the model variables  $\hat{\mathbf{y}}$ . This is done in terms of a measurement equation with a measurement error  $\tilde{\mathbf{e}}$ . We can therefore rewrite the solution to a DSGE model as a system in the following manner:

$$\hat{\mathbf{y}}_t^* = \mathbf{F}\hat{\mathbf{y}}_t + \mathbf{G}\tilde{\mathbf{e}}_t \quad (27)$$

$$\hat{\mathbf{y}}_t = \mathbf{D}\hat{\mathbf{y}}_{t-1} + \mathbf{E}\mathbf{e}_t \quad (28)$$

where the first equation is the measurement equation with a linking matrix  $\mathbf{F}$  and a weighting matrix  $\mathbf{G}$ . The measurement error vector  $\tilde{\mathbf{e}}$  is assumed to be a Gaussian white noise process in the same manner as  $\mathbf{e}$ . The second equation corresponds to (25). The log-likelihood of a model is retrieved by using the Kalman filter recursion<sup>6</sup>. Its application leads to the expression:

$$\begin{aligned} \ln \mathcal{L}(\mathbf{Y}_T|\boldsymbol{\theta}) = & -\frac{Tk}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \log |\Sigma_{\hat{\mathbf{y}}_{t|t-1}^*}| \\ & - \frac{1}{2} \sum_{t=1}^T (\hat{\mathbf{y}}_t^* - \hat{\mathbf{y}}_{t|t-1}^*)' (\Sigma_{\hat{\mathbf{y}}_{t|t-1}^*})^{-1} (\hat{\mathbf{y}}_t^* - \hat{\mathbf{y}}_{t|t-1}^*) \end{aligned} \quad (29)$$

where the vector  $\boldsymbol{\theta}$  contains the parameters to be estimated and, again,  $\mathbf{Y}_T$  gives the set of observable endogenous variables  $\hat{\mathbf{y}}_t^*$  in the measurement equation until period  $T$  (see Almeida 2010). Note that  $\hat{\mathbf{y}}_{t|t-1}^*$  is a predictor of  $\hat{\mathbf{y}}_t^*$  using information up to  $t-1$ ,  $\Sigma_{\hat{\mathbf{y}}_{t|t-1}^*}$  is a predictor of the variance-covariance matrix of  $\hat{\mathbf{y}}_t^*$  using information up to  $t-1$ , and both predictors depend on the parameter vector  $\boldsymbol{\theta}$ . Given the likelihood

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<sup>6</sup> See for example Fernández-Villaverde (2009) and Hamilton (1994), ch. 13.

function (29) and having defined a prior density  $p(\boldsymbol{\theta})$ , the computation of the posterior density  $p(\boldsymbol{\theta}|\mathbf{Y}_T)$  as an update of the priors is straightforward. Using the Bayes theorem, we can write the posterior density as:

$$p(\boldsymbol{\theta}|\mathbf{Y}_T) = \frac{p(\boldsymbol{\theta}; \mathbf{Y}_T)}{p(\mathbf{Y}_T)} \quad (30)$$

where  $p(\boldsymbol{\theta}; \mathbf{Y}_T)$  is the joint density of the parameters and the data and  $p(\mathbf{Y}_T)$  is the marginal density of the data. At the same time we can write for the density of the data conditional on the parameters (the likelihood function):

$$\begin{aligned} p(\mathbf{Y}_T|\boldsymbol{\theta}) &= \frac{p(\boldsymbol{\theta}; \mathbf{Y}_T)}{p(\boldsymbol{\theta})} \\ \Leftrightarrow p(\boldsymbol{\theta}; \mathbf{Y}_T) &= p(\mathbf{Y}_T|\boldsymbol{\theta})p(\boldsymbol{\theta}) \end{aligned} \quad (31)$$

The combination of the last two identities gives the posterior density as:

$$p(\boldsymbol{\theta}|\mathbf{Y}_T) = \frac{p(\mathbf{Y}_T|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathbf{Y}_T)} \quad (32)$$

Since the marginal density  $p(\mathbf{Y}_T)$  is independent of the parameter vector  $\boldsymbol{\theta}$ , it can be treated as a constant. The posterior kernel  $\mathcal{K}(\boldsymbol{\theta}|\mathbf{Y}_T)$  or the unnormalized posterior density corresponds to the numerator of the last expression:

$$p(\boldsymbol{\theta}|\mathbf{Y}_T) \propto p(\mathbf{Y}_T|\boldsymbol{\theta})p(\boldsymbol{\theta}) \equiv \mathcal{K}(\boldsymbol{\theta}|\mathbf{Y}_T) \quad (33)$$

Taking logs of the kernel expression delivers:

$$\ln \mathcal{K}(\boldsymbol{\theta}|\mathbf{Y}_T) = \ln p(\mathbf{Y}_T|\boldsymbol{\theta}) + \ln p(\boldsymbol{\theta}) = \ln \mathcal{L}(\mathbf{Y}_T|\boldsymbol{\theta}) + \ln p(\boldsymbol{\theta}) \quad (34)$$

The next step is to maximize the log kernel with respect to  $\boldsymbol{\theta}$  in order to find estimates for the posterior mode  $\boldsymbol{\theta}^m$ . However, the analytical intractability of the log kernel (as a non-linear and complicated function of the deep parameters  $\boldsymbol{\theta}$ ) requires the use of a numerical optimization algorithm. We use Marco Ratto's Matlab routine `newrat` for the purpose of obtaining the mode  $\boldsymbol{\theta}^m$  and the Hessian matrix  $\mathbf{H}(\boldsymbol{\theta}^m)$



evaluated at the mode. Finally, the random-walk Metropolis-Hastings algorithm is called in order to generate the posterior distribution (mean and variance) of our parameters around the mode<sup>7</sup>. Having estimated our three model versions in the way described so far, it is possible to undertake a model comparison using posterior distributions. See Schorfheide (2000) and Rabanal and Rubio-Ramírez (2005) for a comprehensive overview. First, define the prior distribution over two competing models  $\mathbf{A}$  and  $\mathbf{B}$  as  $p(\mathbf{A})$  and  $p(\mathbf{B})$ . Using the Bayes rule, one can compute the posterior probability of each model as:

$$p(\mathbf{A}|\mathbf{Y}_T) = \frac{p(\mathbf{A})p(\mathbf{Y}_T|\mathbf{A})}{p(\mathbf{A})p(\mathbf{Y}_T|\mathbf{A}) + p(\mathbf{B})p(\mathbf{Y}_T|\mathbf{B})} \quad (35)$$

$$p(\mathbf{B}|\mathbf{Y}_T) = \frac{p(\mathbf{B})p(\mathbf{Y}_T|\mathbf{B})}{p(\mathbf{A})p(\mathbf{Y}_T|\mathbf{A}) + p(\mathbf{B})p(\mathbf{Y}_T|\mathbf{B})} \quad (36)$$

The expressions above describe the probability of a model being true after observing the data. Therefore, a natural way to compare the empirical fit of two models is given by the posterior odds ratio:

$$PO_{\mathbf{A},\mathbf{B}} = \frac{p(\mathbf{A}|\mathbf{Y}_T)}{p(\mathbf{B}|\mathbf{Y}_T)} = \frac{p(\mathbf{A})p(\mathbf{Y}_T|\mathbf{A})}{p(\mathbf{B})p(\mathbf{Y}_T|\mathbf{B})} \quad (37)$$

where  $\frac{p(\mathbf{Y}_T|\mathbf{A})}{p(\mathbf{Y}_T|\mathbf{B})}$  is the Bayes factor describing the evidence in the data favoring model  $\mathbf{A}$  over model  $\mathbf{B}$  and  $\frac{p(\mathbf{A})}{p(\mathbf{B})}$  is the prior odds ratio giving the relative probability subjectively assigned to the models. The terms entering the numerator and denominator of the Bayes factor are marginal densities of the data conditional on the respective model (the marginal likelihood of a model).

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<sup>7</sup> We opt for 500,000 draws from each model's posterior distribution with 4 distinct chains. For details on the Metropolis-Hastings algorithm, see An and Schorfheide (2007) and Fernández-Villaverde (2009).

They can be obtained, at least theoretically, by integrating out the deep parameters from the posterior kernel:

$$\begin{aligned}
 p(\mathbf{Y}_T|\mathbf{A}) &= \int_{\boldsymbol{\theta}_A} p(\mathbf{Y}_T, \boldsymbol{\theta}_A|\mathbf{A}) d\boldsymbol{\theta}_A \\
 &= \int_{\boldsymbol{\theta}_A} p(\mathbf{Y}_T|\boldsymbol{\theta}_A, \mathbf{A}) p(\boldsymbol{\theta}_A|\mathbf{A}) d\boldsymbol{\theta}_A \\
 &= \int_{\boldsymbol{\theta}_A} \mathcal{K}(\boldsymbol{\theta}_A|\mathbf{Y}_T, \mathbf{A}) d\boldsymbol{\theta}_A
 \end{aligned} \tag{38}$$

$$\begin{aligned}
 p(\mathbf{Y}_T|\mathbf{B}) &= \int_{\boldsymbol{\theta}_B} p(\mathbf{Y}_T, \boldsymbol{\theta}_B|\mathbf{B}) d\boldsymbol{\theta}_B \\
 &= \int_{\boldsymbol{\theta}_B} p(\mathbf{Y}_T|\boldsymbol{\theta}_B, \mathbf{B}) p(\boldsymbol{\theta}_B|\mathbf{B}) d\boldsymbol{\theta}_B \\
 &= \int_{\boldsymbol{\theta}_B} \mathcal{K}(\boldsymbol{\theta}_B|\mathbf{Y}_T, \mathbf{B}) d\boldsymbol{\theta}_B
 \end{aligned} \tag{39}$$

However, this function is analytically intractable and has to be substituted by the Laplace or the Harmonic Mean approximation. Our estimation results give the (positive) logarithmic value of the approximated marginal likelihood of each model. By assuming an uniform distribution across two models ( $p(\mathbf{A}) = p(\mathbf{B})$ ), we compare them by simply taking the logarithmic difference of the posterior odds ratio:  $\ln p(\mathbf{Y}_T|\mathbf{A}) - \ln p(\mathbf{Y}_T|\mathbf{B})$ . A positive value of this difference is interpreted as an outperformance of model  $\mathbf{A}$  over model  $\mathbf{B}$  in the description of the dataset  $\mathbf{Y}_T$ .

### 3.3 Parameters and Priors

This section presents the subset of fixed parameter values and the priors<sup>8</sup>. The calibration of some parameters is due to identification problems and resulting difficulties in estimating them. As pointed out in Rabanal and Rubio-Ramírez (2005), the absence of capital services in our model hinders the estimation of the household discount factor  $\beta$  and the capital share of output  $\alpha$ . Therefore, we set  $\beta = 0.99$  and  $\alpha = 0.3$ , which are standardly used values. Moreover, identification problems arise between the steady state elasticities  $\epsilon_p$ ,  $\epsilon_w$ , and  $\epsilon_r$  and the corresponding rigidity parameters  $\psi_p$ ,  $\psi_w$ , and  $\psi_r$  (BSAC and HYAC) or  $\theta_p$ ,  $\theta_w$ , and  $\theta_r$  (HYCC). For this reason, we assume  $\epsilon_p = 11$ ,  $\epsilon_w = 6$ , and  $\epsilon_r = 3.5$ , which are also commonly used values<sup>9</sup>. We also encountered identification problems concerning the inverse of the interest rate elasticity of money demand  $\nu$ . This parameter is set as  $\nu = 4$ , which is also in the range of commonly assumed values. Concerning the parameter priors, we opt to choose prior mean values that are mostly found in the standard literature and microeconomic studies. Their tightness (prior standard deviation) was set as loosely as possible in order to let the data drive our posterior results as much as possible. However, the looseness degree is restricted by the success of the numerical optimization of the posterior kernel across the three model versions<sup>10</sup>. This leads to prior densities for some parameters in our models that are somewhat tighter than in the prevailing literature. An overview of the parameter priors is given by the third column of tables 1 and 2.

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<sup>8</sup> The total numbers of parameters are given by: 15 (BSAC), 19 (HYAC), and 19 (HYCC). We fix a subset of 6 parameters contained in all model versions. Additionally, we have 6 identically and independently distributed shock impulses whose priors have also to be prespecified.

<sup>9</sup> This implies steady state markups of  $\mu_p = 1.1$  (ten percent),  $\mu_w = 1.2$  (twenty percent), and  $\mu_r = 1.4$  (forty percent)

<sup>10</sup> In fact, we checked by using the Dynare command “mode\_check” that the maximum of the posterior log kernel is unique for most parameters across the models.

We start with the characterization of priors for the shock processes (table 2). Concerning the prior standard deviation of the shocks, we used the inverse gamma distribution with relatively low prior means of 0.08 for the price and wage markup shocks and of 0.01 for the remaining impulses<sup>11</sup>. The prior for the autoregressive shock parameters  $\rho_A$  and  $\rho_B$  were assumed to follow a beta distribution with a mean and a standard deviation (henceforth in brackets) of 0.7 (0.01) and 0.5 (0.01), respectively. The remaining priors are summarized in the third column of table 1. Across both adjustment cost models (BSAC and HYAC), the prior price rigidity parameter  $\psi_p$  is assumed to be normally distributed with a mean of 400 and a standard deviation of 100. This corresponds approximately to a prior for the Calvo parameter in the HYCC model given by  $\theta_p$  given by 0.70 (0.01). The BSAC model is characterized by flexible wages ( $\psi_w = \theta_w = 0$ ) and there is no relative cost amount consideration or rule-of-thumb behavior in the price and nominal wage setting ( $v_p = v_w = \omega_p = \omega_w = 0$ ). In contrast, we estimate these parameters for the HYAC and the HYCC model. We arbitrarily set the prior for  $\psi_w$  and  $\psi_r$  to 300 (50). These prior values correspond approximately to priors for  $\theta_w$  and  $\theta_r$  in the HYCC model given by 0.60 (0.01) and 0.4 (0.01), respectively. Similar rationales underlie our choice of (partially) backward-oriented parameter priors. In the HYAC model, we choose a very loose prior for  $v_p$  and a somewhat tighter prior for  $v_w$ . We arbitrarily set 5000 (2500) for  $v_p$  and 3000 (300) for  $v_w$ . Since we don't have any prior knowledge about these parameters, the imposition of a loose prior density distribution is very important. In contrast, the existing literature enables us to assume standard values for the beta-distributed priors of  $\omega_p$  and  $\omega_w$ . The latter are given by 0.75 (0.015) and 0.45 (0.015), respectively. Concerning the Taylor rule coefficients, we assume priors  $\delta_\pi = 1.6$  and  $\delta_y = 0.5$ , which are close to the original estimates by Taylor (1993). We restrict those parameters to have a positive support by imposing a gamma distribution with standard deviations being equal to

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<sup>11</sup> We did not have any hint on the prior dimension of these parameters, but chose them as loosely and conveniently with respect to the likelihood maximization as possible.

0.05 and 0.01, respectively. The smoothing parameter  $\phi$  follows a beta distribution with a mean equal to 0.5 and a standard deviation of 0.05. Finally, priors for the household preference parameters have to be chosen. The parameters  $\sigma$  and  $\eta$  follow a normal distribution with mean 2.0. While these prior mean values are commonly assumed in the literature (See for example Smets and Wouters (2003)), we impose a somewhat tighter density for  $\eta$  of 0.05. Similarly, the prior of the habit persistence parameter  $h$  is assumed to follow a beta distribution with 0.5 (0.01).

## 4 Empirical Findings

### 4.1 Parameter Estimates

Apart from reporting posterior parameter distributions, we also present graphical estimation results for the HYAC model. The emphasis on the model by Sienknecht (2010a) is because of two reasons. Firstly, analogous figures were obtained across all models but the outcomes for the HYAC model are used in order to exemplify the underlying interpretations. Secondly, these graphical results are not very well known because estimated adjustment cost models (BSAC and HYAC) are somewhat rare in the literature. In contrast, most theoretical and empirical studies have concentrated on the (hybrid) Calvo environment. Figures 1 and 2 display results in the HYAC model for the posterior kernel maximization that confirm the unique and robust maximum for almost all model parameters. Figure 3 allows for a statement on the overall convergence of the Metropolis-Hastings sampling algorithm. Following for example Almeida (2009), the information is summarized in three graphs and each one of them represents a specific global convergence measure. The two distinct lines display the results within and between the chains of the Metropolis-Hastings algorithm. These measures are related to the analysis of the parameters' mean ("interval"), of the variances ("m2") and of the third moments ("m3"). Ideally, the two lines converge to each other and become relatively stable for each of the

three measures. Figure 3 shows that convergence was obtained<sup>12</sup>. The last three columns of table 1 contain the posterior mean and the posterior standard deviation of estimated parameters. Table 2 gives posterior standard deviations for the shocks and the posterior distributions of the shock persistence parameters. Estimation results for the BSAC model are presented in the fourth column of the tables 1 and 2. Accordingly, the degree of price rigidity remains roughly at  $\psi_p = 400$ , with a lower degree of uncertainty (SD=11.403). The BSAC model implies full nominal wage and loan rate flexibility. Therefore, we do not estimate these parameters and set  $\psi_w = \psi_r = \nu_p = \nu_w = 0$ . Most of the remaining parameter estimates in the BSAC model remain close to their priors and show a lower degree of uncertainty. The coefficients of the Taylor rule are very close to the values obtained by Clarida et al. (2000) and Rabanal and Rubio-Ramírez (2005). We obtain a posterior for the inverse of the intertemporal elasticity of substitution which is lower than in the standard literature, namely  $\sigma = 0.60$ <sup>13</sup>. A glance at the fourth column of table 2 reveals a high posterior volatility of price markups and confirms the important role of price markup shocks in DSGE models. At the same time, the importance of the remaining shocks for the explanation of fluctuations is reduced. The posterior autoregressive parameters show little deviation from our prior distribution beliefs. Posterior parameter estimates for the HYAC model can be inspected in the fifth column of tables 1 and 2. They are complemented by the figures 4 and 5. Again, the posterior degree of uncertainty is lower for all parameters than our prior subjective beliefs. We obtain a higher degree of price and nominal wage rigidity in the HYAC model, namely  $\psi_p = 476.15$  and  $\psi_w = 333.00$ . However, the degree of loan rate stickiness is somewhat lower ( $\psi_r = 239.75$ ). The most important posterior

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<sup>12</sup> We also confirm this diagnostic result for each parameter in the HYAC model. The aggregate and parametrical convergence was also obtained in the BSAC and in the HYCC model.

<sup>13</sup> All posterior parameter estimates in the BSAC model fulfill the Blanchard-Kahn stability condition. The same is true for the HYAC and the HYCC model. The relatively low degree of risk aversion (or high intertemporal elasticity of substitution in consumption) leads to qualitatively unchanged impulse responses after shocks.

results are those for the novel parameters in the model by Sienknecht (2010a). We obtain posterior estimates for  $v_p$  as 917.38 (293.542) and 2745 (253.437) for  $v_w$ . This result clearly indicates the relevance of these parameters for the description of the underlying data. In order to reject the hybrid specification by Sienknecht (2010a), one would need to obtain values of these parameters in the neighborhood of zero or a highly increased degree of uncertainty. Of course, the posterior positive values of  $v_p$  and  $v_w$  could be the result of the lagged inflation components alone. However, the theoretical result is an additional two-period-ahead expectation term that cannot be disentangled from the lagged term. Our posterior parameter values clearly point at the relevance of two-period-ahead expectations. The remaining posterior estimates in the HYAC model are similar to those obtained in the BSAC environment. The only exception is the inverse of the intertemporal substitution in consumption  $\sigma = 1.24$  (0.274), which is close to the values obtained in standard empirical studies. The fourth column of table 2 shows a high posterior volatility in all markups whereas the relevance of the remaining shocks is rather limited. Again, the posterior autoregressive parameters show little deviation from our prior distribution beliefs.

The estimation outcomes for the hybrid Calvo-type model (HYCC) are reported in the sixth column of the tables 1 and 2. All posterior parameter estimates are very close to their priors due to the high prior tightness degree. Since the parameters of the HYCC model have been subject to many empirical studies in the past, our choice of priors with a low degree of uncertainty appears to be justifiable. The drop in the price markup volatility (see table 2) relative to the BSAC model has also been obtained by Rabanal and Rubio-Ramírez (2005). Nonetheless, other results concerning the posterior variability of shocks are mixed.

## 4.2 Model Comparison

The empirical performance of a model is assessed by means of its marginal likelihood. As a starting point, we compare the hybrid adjustment cost model (HYAC) against the baseline adjustment cost model (BASC) and determine which of the two dominates in terms of data description performance. The resulting outperformer is checked against the widely applied hybrid Calvo-type model (HYCC). In each comparison, we assume a uniform prior probability across the two models involved, such that the prior odds ratio is equal to one. The posterior odds ratio is in logs, which implies differences in log marginal likelihoods as our instrument for ranking models<sup>14</sup>.

The last two rows of table 1 give the (Laplace- and Metropolis-Hastings-) approximated marginal likelihood in logs. Note first that the HYAC model clearly outperforms the BSAC model since the differences in log marginal likelihoods, namely 80.36 (Laplace) and 39.61 (Harmonic mean), are positive and large. This result is not surprising, as the BSAC model fails to describe the high degree of price and wage inflation persistence present in the data. In contrast, the HYAC model performs better since the hybrid inflation curves display lagged terms. Note again, the lagged price and wage inflation terms in the HYAC model are linked directly to the two period-ahead expectation through parameters  $v_p$  (price inflation) and  $v_w$  (wage inflation). As stated in Sienknecht (2010a), this relationship is unavoidable when trying to induce a lagged inflation term in the adjustment cost environment. Therefore, we cannot assess the empirical contribution of this additional forward-looking term alone.

Having established the superiority of the HYAC model against the baseline adjustment cost framework, we now ask if there is also an outperformance against the HYCC model<sup>15</sup>. From the point of view of the HYAC model, the differences in log

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<sup>14</sup> This practice of ranking models is the same as in Rabanal and Rubio-Ramírez (2005)

<sup>15</sup> Needless to say that the HYCC model is also superior against the BASC model since the former implies a lagged inflation term that captures the inflation persistence present in the data.



marginal likelihood are  $-41.96$  (Laplace) and  $-48.40$  (Metropolis-Hastings). Therefore, we find a marginal likelihood in the HYCC model which is higher than in the HYAC model. The better empirical fit of the HYCC could be interpreted as a rather hindering feature of the two-period-ahead expectational term in the HYAC model. However, note again that the lagged and the two-period-ahead inflationary term in the HYAC model are theoretically linked together and this link is unavoidable when widening the traditional adjustment cost structure of Rotemberg (1982) and Hairault and Portier (1993). We conclude on the one hand that the HYAC model should be preferred over the BSAC model, whenever the adjustment cost environment is considered for empirical tasks. On the other hand, one should be aware of the potential ability of the hybrid specification of the Calvo model to describe the data with a higher degree of accuracy.

Finally, we generate impulse responses in the HYAC model based on our parameter estimates and with one percent standard deviation shocks. As can be seen in figure 6, the posterior parameter values are such that the Blanchard-Kahn stability condition is met. Sienknecht (2010a) shows impulse responses in the HYAC model after an interest rate shock with calibrated parameter values. The last three graphs of figure 6 give responses after a non-persistent interest rate shock with our estimated parameter values. As can be seen, the loan rate increases only weakly after the interest rate shock. This low interest rate pass-through leads to a small decrease in real marginal cost since the (also weak) drop in real wages dominates. The latter is the result of an increase in the real interest rate and the following decrease in consumption, output and labor demand. This small decrease of real marginal costs translates to price and wage inflation rates. All responses for output and inflation rates display a high degree of persistence across all shocks, independently of the degree of autocorrelation in the shock process. The remaining shock impulses generate very similar responses as those in the standard literature with other rigidity mechanisms.

## 5 Conclusions

The scope of this paper was to complement previous theoretical work by Sienknecht (2010a) with empirical results. Since Bayesian estimation methods are becoming increasingly popular in the DSGE literature, we opted for them to carry theoretical results towards European data. First, we provided evidence that the hybrid inflation schedules by Sienknecht (2010a) perform better than purely forward-looking specifications. This is by no means a surprising result, as it is well known that the ordinary New Keynesian Phillips curve is not able to capture the high degree of inflation persistence found in the data. The appearance of a lagged inflation term does always improve the empirical performance of any purely forward-looking inflation relationship. However, the inducement of such a lagged term in the adjustment cost environment of Hairault and Portier (1993) and Rotemberg (1982) is always connected with a two-period-ahead additional expectational term (Sienknecht (2010a)). We showed that the parameter linking this two terms adopts meaningful empirical values unequal to zero, which highlights the importance of expectations that look deeper into the future, namely two periods ahead. This is by no means unreasonable, taking the quarterly timing into consideration. However, the log marginal likelihood value of the Sienknecht (2010a) model is not as high as the marginal density of the Calvo model with indexation. We conclude from our results that the model by Sienknecht (2010a) is a reasonable alternative, when considering DSGE models that rely on adjustment costs only. Fruitful areas of research could be targeted towards estimations for other countries in order to establish a potential generality of our results. At the theoretical end, an assessment on optimal policy strategies under two-period-ahead expectations could be interesting as well.

## A Agent Optimization Problems

### A.1 Households

$$\max_{C_t(j), M_t(j), d_t(j), W_t(j)} \left\{ E_t \sum_{k=0}^{\infty} \beta^k \varpi_{t+k}^B \left[ \frac{(C_{t+k}(j) - h C_{t+k-1}(j))^{1-\sigma}}{1-\sigma} + \frac{\left(\frac{M_{t+k}(j)}{P_{t+k}}\right)^{1-\nu}}{1-\nu} - \frac{\left(\frac{W_{t+k}(j)}{W_{t+k}}\right)^{-(1+\eta)\epsilon_{w,t+k}} N_{t+k}^{1+\eta}}{1+\eta} \right] \right\} \quad (40)$$

s.t.

$$\begin{aligned} E_t \sum_{k=0}^{\infty} \beta^k \left[ C_{t+k}(j) + \frac{d_{t+k}(j)}{P_{t+k}} + \frac{M_{t+k}(j)}{P_{t+k}} \right] &= E_t \sum_{k=0}^{\infty} \beta^k \left[ \frac{W_{t+k}(j)}{P_{t+k}} \left(\frac{W_{t+k}(j)}{P_{t+k}}\right)^{-\epsilon_{w,t+k}} N_{t+k} \right. \\ &+ R_{t+k-1}^d \frac{d_{t+k-1}(j)}{P_{t+k}} + \frac{M_{t+k-1}(j)}{P_{t+k}} + Div_{t+k}^r(j) \\ &\left. - \frac{\psi_w}{2} \left(\frac{W_{t+k}(j)}{W_{t+k-1}(j)} - 1\right)^2 - \frac{\nu_w}{2} \left(\frac{W_{t+k}(j)}{W_{t+k-1}(j)} - \frac{W_{t+k-1}(j)}{W_{t+k-2}(j)}\right)^2 \right] \end{aligned} \quad (41)$$

### A.2 Final Good Producers

$$\max_{Y_t(i)} \left\{ \left( \int_0^1 Y_t(i)^{\frac{\epsilon_{p,t}}{\epsilon_{p,t-1}}} di \right)^{\frac{\epsilon_{p,t-1}}{\epsilon_{p,t}}} - \int_0^1 \frac{P_t(i) Y_t(i)}{P_t} di \right\} \quad (42)$$

### A.3 Intermediate Good Producers

$$\max_{P_t(i)} \left\{ E_t \sum_{k=0}^{\infty} \Delta_{t,t+k} \left[ \left( \frac{P_{t+k}(i)}{P_{t+k}} \right)^{1-\epsilon_{p,t+k}} Y_{t+k} - MC_{t+k}(i) \left( \frac{P_{t+k}(i)}{P_{t+k}} \right)^{-\epsilon_{p,t+k}} Y_{t+k} - \frac{\psi_p}{2} \left( \frac{P_{t+k}(i)}{P_{t+k-1}(i)} - 1 \right)^2 - \frac{v_p}{2} \left( \frac{P_{t+k}(i)}{P_{t+k-1}(i)} - \frac{P_{t+k-1}(i)}{P_{t+k-2}(i)} \right)^2 \right] \right\} \quad (43)$$

### A.4 Commercial Banks

$$\max_{R_t^L(l)} \left\{ E_t \sum_{k=0}^{\infty} \Delta_{t,t+k} \left[ \frac{R_{t+k}^L(l)}{P_{t+k}} \left( \frac{R_{t+k}^L(l)}{R_{t+k}^L} \right)^{-\epsilon_{r,t+k}} L_{t+k} - MC_{t+k}(l) \left( \frac{R_{t+k}^L(l)}{R_{t+k}^L} \right)^{-\epsilon_{r,t+k}} L_{t+k} - \frac{\psi_r}{2} \left( \frac{R_{t+k}^L(l)}{R_{t+k-1}^L(l)} - 1 \right)^2 \right] \right\} \quad (44)$$

### A.5 Structural Parameters

$$\Theta_1 = \frac{h}{1+h(1+\beta h)} \quad (45)$$

$$\Theta_2 = \frac{1+\beta h(1+h)}{1+h(1+\beta h)} \quad (46)$$

$$\Theta_3 = \frac{\beta h}{1+h(1+\beta h)} \quad (47)$$

$$\Theta_4 = \frac{(1-h)(1-\beta h)}{\sigma(1+h(1+\beta h))} \quad (48)$$

$$\Theta_5 = \frac{1-h}{\sigma(1+h(1+\beta h))} \quad (49)$$

$$\Theta_6 = \frac{(1-h)(1+\sigma\beta h)}{\sigma(1+h(1+\beta h))} \quad (50)$$

$$\Theta_7 = \frac{(1-h)\beta h}{\sigma(1+h(1+\beta h))} \quad (51)$$

$$\Phi_1 = \frac{\sigma(1+\beta h^2)}{\nu(1-h)(1-\beta h)} \quad (52)$$

$$\Phi_2 = \frac{\sigma h}{\nu(1-h)(1-\beta h)} \quad (53)$$

$$\Phi_3 = \frac{\sigma\beta h}{\nu(1-h)(1-\beta h)} \quad (54)$$

$$\Phi_4 = \frac{\beta}{\nu(1-\beta)} \quad (55)$$

$$\Phi_5 = \frac{1}{\nu} \quad (56)$$

$$\Phi_6 = \frac{\beta h}{\nu(1-\beta h)} \quad (57)$$

$$\iota_1 = \frac{\sigma(1 + \beta h^2)}{(1 - h)(1 - \beta h)} \quad (58)$$

$$\iota_2 = \frac{h\sigma}{(1 - h)(1 - \beta h)} \quad (59)$$

$$\iota_3 = \frac{\sigma\beta h}{(1 - h)(1 - \beta h)} \quad (60)$$

$$\iota_4 = \frac{\beta h}{1 - \beta h} \quad (61)$$

$$\gamma = \frac{Y(\epsilon_p - 1)}{\psi_p} \quad (62)$$

$$\kappa = \frac{N(\epsilon_w - 1)}{\psi_w} \frac{W}{P} \quad (63)$$

$$Y = \left( \frac{(1 - \alpha)(1 - \beta h)\beta}{\mu_p \mu_w \Xi (1 - h)^\sigma} \right)^{\frac{1 - \alpha}{\alpha + \eta + \sigma(1 - \alpha)}} \quad (64)$$

$$N = \left( \frac{(1 - \alpha)(1 - \beta h)\beta}{\mu_p \mu_w \Xi (1 - h)^\sigma} \right)^{\frac{1}{\alpha + \eta + \sigma(1 - \alpha)}} \quad (65)$$

$$\frac{W}{P} = \frac{\mu_w(1 - h)^\sigma}{1 - \beta h} \left( \frac{(1 - \alpha)(1 - \beta h)\beta}{\mu_p \mu_w (1 - h)^\sigma} \right)^{\frac{1}{\eta + \sigma(1 - \alpha)}} \quad (66)$$

$$\varphi_1 = \frac{\psi_r}{\psi_r(1 + \beta) + \zeta N(W/P)R^M} \quad (67)$$

$$\varphi_2 = \frac{\psi_r \beta}{\psi_r(1 + \beta) + \zeta N(W/P)R^M} \quad (68)$$

$$\varphi_3 = \frac{\zeta N(W/P)R^M}{\psi_r(1 + \beta) + \zeta N(W/P)R^M} \quad (69)$$

$$\gamma_1 = \frac{v_p}{v_p(1 + 2\beta) + \psi_p} \quad (70)$$

$$\gamma_2 = \frac{v_p(2 + \beta)\beta + \psi_p\beta}{v_p(1 + 2\beta) + \psi_p} \quad (71)$$

$$\gamma_3 = \frac{v_p\beta^2}{v_p(1 + 2\beta) + \psi_p} \quad (72)$$

$$\gamma_4 = \frac{(\epsilon_p - 1)Y}{v_p(1 + 2\beta) + \psi_p} \quad (73)$$

$$\kappa_1 = \frac{v_w}{v_w(1 + 2\beta) + \psi_w} \quad (74)$$

$$\kappa_2 = \frac{v_w(2 + \beta)\beta + \psi_w\beta}{v_w(1 + 2\beta) + \psi_w} \quad (75)$$

$$\kappa_3 = \frac{v_w\beta^2}{v_w(1 + 2\beta) + \psi_w} \quad (76)$$

$$\kappa_4 = \frac{(\epsilon_w - 1)}{v_w(1 + 2\beta) + \psi_w} \frac{W}{P} N \quad (77)$$

$$R^M = \frac{\pi^p}{\beta} = \frac{\pi^w}{\beta} = \frac{1}{\beta} \quad (78)$$

$$\gamma_1^c = \frac{[1 + \alpha (\epsilon_p (1 - \beta \theta_p) - 1)] \omega_p}{[1 + \alpha (\epsilon_p (1 - \beta \theta_p) - 1)] (\theta_p + \omega_p (1 - \theta_p)) + (1 - \alpha) \beta \theta_p \omega_p} \quad (79)$$

$$\gamma_2^c = \frac{(1 - \alpha) \beta \theta_p}{[1 + \alpha (\epsilon_p (1 - \beta \theta_p) - 1)] (\theta_p + \omega_p (1 - \theta_p)) + (1 - \alpha) \beta \theta_p \omega_p} \quad (80)$$

$$\gamma_3^c = 0 \quad (81)$$

$$\gamma_4^c = \frac{(1 - \alpha) (1 - \beta \theta_p) (1 - \theta_p) (1 - \omega_p)}{[1 + \alpha (\epsilon_p (1 - \beta \theta_p) - 1)] (\theta_p + \omega_p (1 - \theta_p)) + (1 - \alpha) \beta \theta_p \omega_p} \quad (82)$$

$$\kappa_1^c = \frac{\theta_w \omega_w}{\theta_w (1 - \omega_w (1 + \beta (\theta_w)^2)) + \omega_w} \quad (83)$$

$$\kappa_2^c = \frac{\beta \theta_w}{\theta_w (1 - \omega_w (1 + \beta (\theta_w)^2)) + \omega_w} \quad (84)$$

$$\kappa_3^c = 0 \quad (85)$$

$$\kappa_4^c = \frac{(1 - \theta_w) (1 - \omega_w) (1 - \beta \theta_w)}{[\theta_w (1 - \omega_w (1 + \beta (\theta_w)^2)) + \omega_w] (1 + \eta \epsilon_w)} \quad (86)$$

$$\kappa_5^c = \frac{\beta \theta_w (1 - \theta_w) \omega_w}{\theta_w (1 - \omega_w (1 + \beta (\theta_w)^2)) + \omega_w} \quad (87)$$

$$\varphi_1^c = \frac{\theta_r}{1 + \beta (\theta_r)^2} \quad (88) \quad \varphi_2^c = \frac{\beta \theta_r}{1 + \beta (\theta_r)^2} \quad (89)$$

$$\varphi_3^c = \frac{(1 - \theta_r) (1 - \beta \theta_r)}{1 + \beta (\theta_r)^2} \quad (90)$$

## B Tables

**Table 1:** Prior and posterior distribution of behavioral parameters (1990Q1-2002Q4).

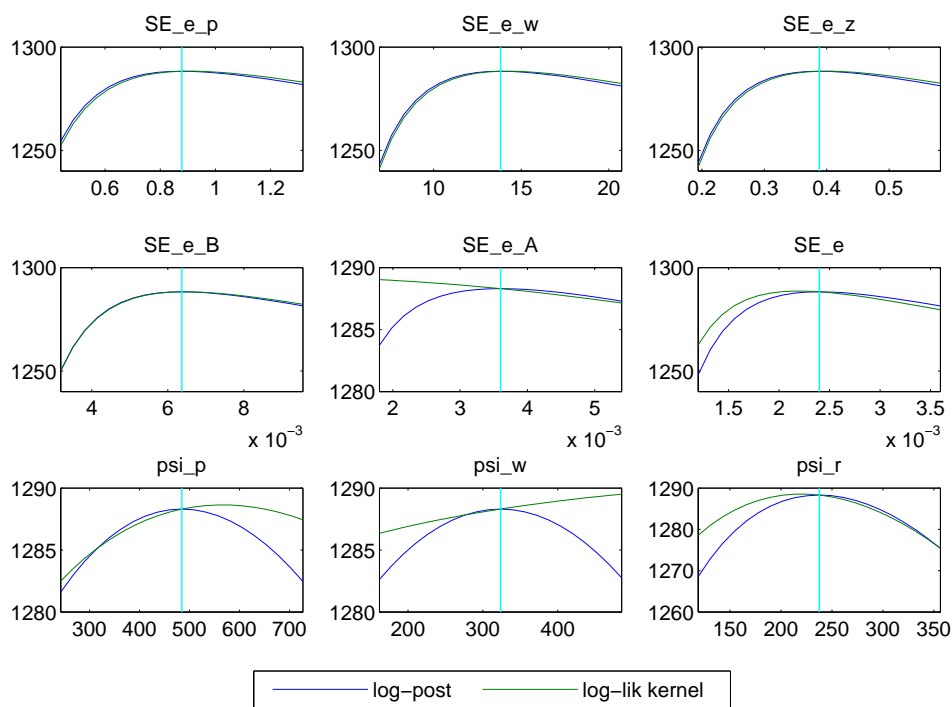
	Prior distribution	Posterior distribution			
		Mean (SD)	BSAC Mean (SD)	HYAC Mean (SD)	HYCC Mean (SD)
$\psi_p$ Price rigidity	normal	400 (100)	400.91 (11.403)	476.15 (83.115)	— (—)
$\psi_w$ Wage rigidity	normal	300 (50)	— (—)	333.00 (49.277)	— (—)
$\psi_r$ Loan rate rigidity	normal	300 (50)	— (—)	239.75 (32.891)	— (—)
$v_p$ Price inflation end.	normal	5000 (2500)	— (—)	917.38 (293.542)	— (—)
$v_w$ Wage inflation end.	normal	3000 (300)	— (—)	2745 (253.437)	— (—)
$\theta_p$ Price rigidity	normal	0.70 (0.010)	— (—)	— (—)	0.70 (0.009)
$\theta_w$ Wage rigidity	normal	0.60 (0.010)	— (—)	— (—)	0.60 (0.009)
$\theta_r$ Loan rate rigidity	normal	0.40 (0.010)	— (—)	— (—)	0.39 (0.009)
$\omega_p$ Price inflation end.	normal	0.75 (0.015)	— (—)	— (—)	0.75 (0.014)
$\omega_w$ Wage inflation end.	normal	0.45 (0.010)	— (—)	— (—)	0.44 (0.015)
$\sigma$ Consumption utility	normal	2.0 (1.0)	0.60 (0.038)	1.24 (0.274)	0.89 (0.277)
$\eta$ Labor utility	normal	2.0 (0.050)	1.97 (0.004)	1.99 (0.049)	2.0 (0.049)
$h$ Consumption habit	beta	0.50 (0.010)	0.51 (0.030)	0.51 (0.010)	0.51 (0.009)
$\delta_\pi$ Taylor inflation	normal	1.60 (0.050)	1.61 (0.049)	1.58 (0.050)	1.63 (0.049)
$\delta_y$ Taylor output	normal	0.50 (0.010)	0.50 (0.006)	0.51 (0.010)	0.49 (0.010)
$\phi$ Taylor smoothing	normal	0.50 (0.050)	0.48 (0.006)	0.52 (0.045)	0.52 (0.043)
$\log(L)$ : Laplace			<b>1189.82</b>	<b>1270.18</b>	<b>1312.14</b>
$\log(L)$ : Harmonic Mean			<b>1207.45</b>	<b>1247.06</b>	<b>1295.46</b>

**Table 2:** Prior and posterior distribution of shock innovations and persistence parameters.

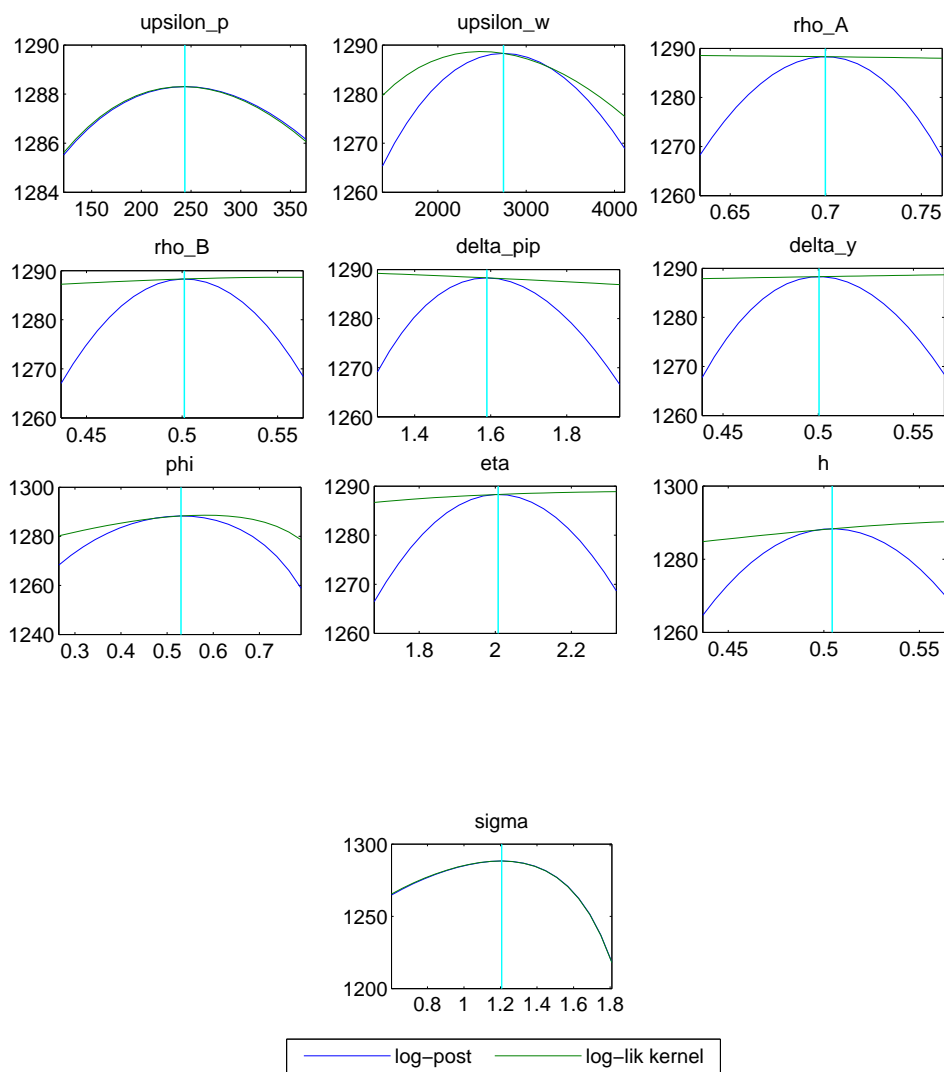
	Prior distribution	Posterior distribution			
		Mean (SD)	BSAC Mean (SD)	HYAC Mean (SD)	HYCC Mean (SD)
$\sigma_p$ Price markup	inv. gamma	0.080 (0.080)	0.43 (0.43)	1.86 (1.86)	0.37 (0.37)
$\sigma_w$ Wage markup	inv. gamma	0.080 (0.080)	0.020 (0.020)	14.32 (14.32)	0.060 (0.060)
$\sigma_r$ Loan markup	inv. gamma	0.010 (0.010)	0.004 (0.004)	0.407 (0.407)	0.004 (0.004)
$\sigma_R$ Monetary policy	inv. gamma	0.010 (0.010)	0.003 (0.003)	0.002 (0.002)	0.002 (0.003)
$\sigma_A$ Productivity	inv. gamma	0.010 (0.010)	0.003 (0.003)	0.004 (0.004)	0.004 (0.004)
$\sigma_B$ Preference	inv. gamma	0.010 (0.010)	0.003 (0.003)	0.007 (0.007)	0.004 (0.004)
$\rho_A$ Productivity pers.	beta	0.70 (0.010)	0.69 (0.010)	0.70 (0.010)	0.69 (0.010)
$\rho_B$ Preference pers.	beta	0.50 (0.010)	0.50 (0.010)	0.50 (0.010)	0.50 (0.010)



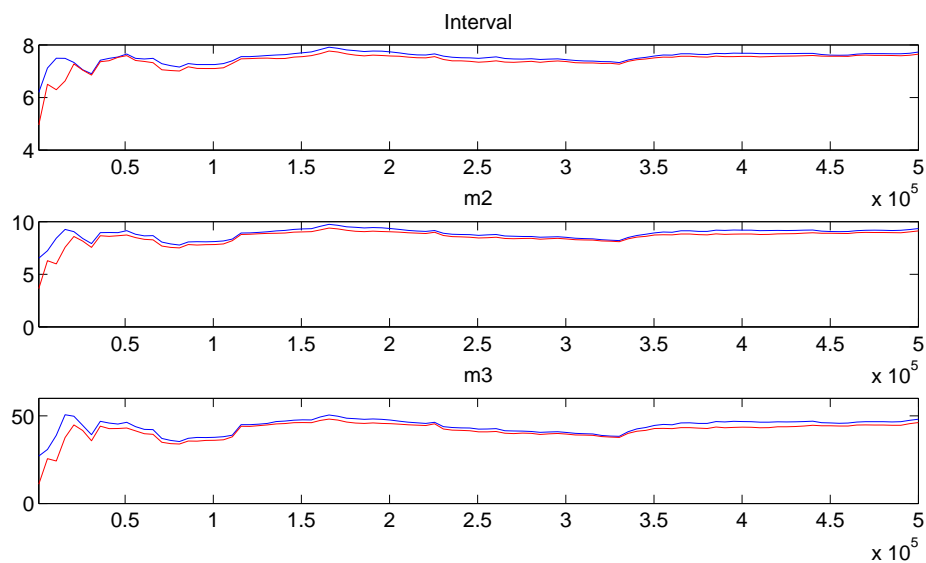
## C Figures



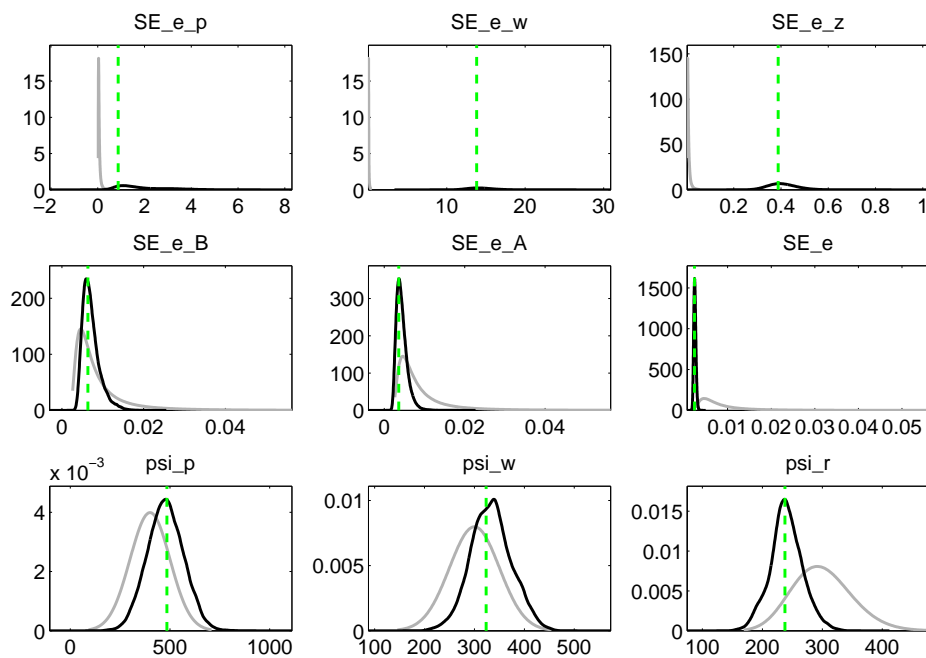
**Figure 1:** Results from the posterior likelihood function maximization in the hybrid adjustment cost model (HYAC).



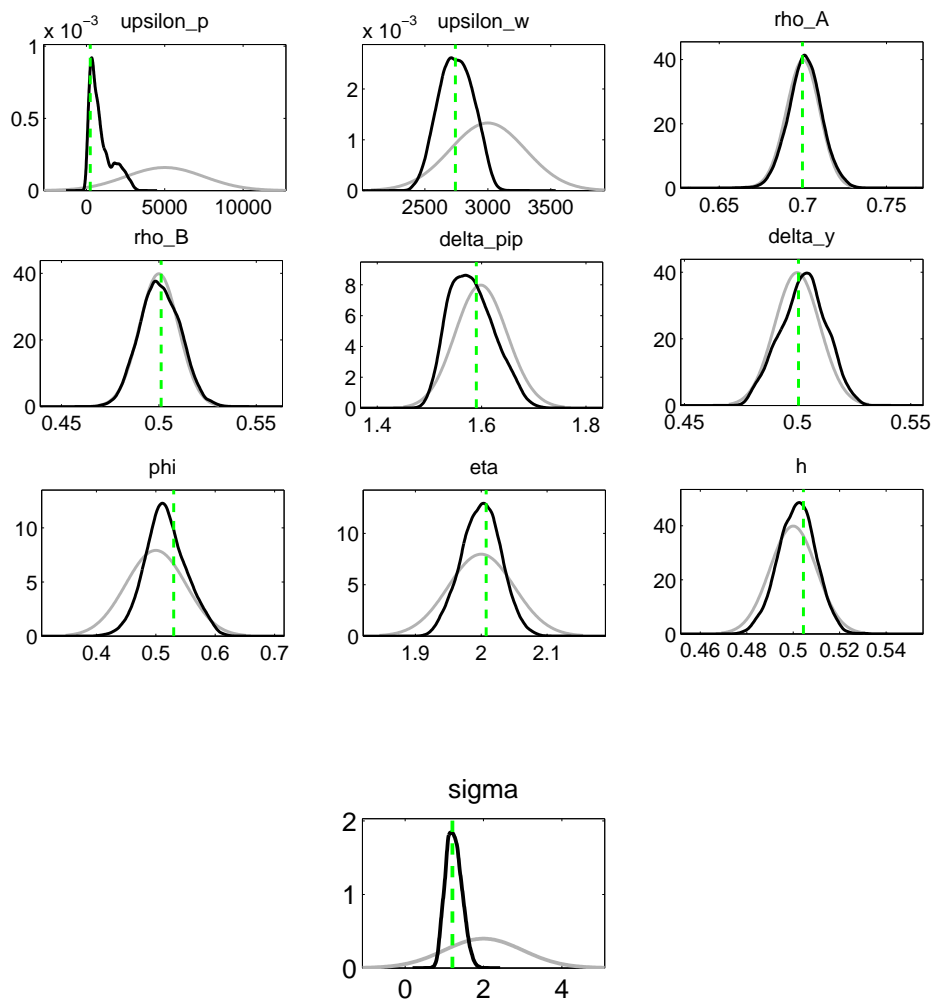
**Figure 2:** Results from the posterior likelihood function maximization in the hybrid adjustment cost model (HYAC).



**Figure 3:** Multivariate Metropolis-Hastings convergence diagnosis in the hybrid adjustment cost model (HYAC).



**Figure 4:** Prior and posterior distributions in the hybrid adjustment cost model (HYAC).



**Figure 5:** Prior and posterior distributions in the hybrid adjustment cost model (HYAC).

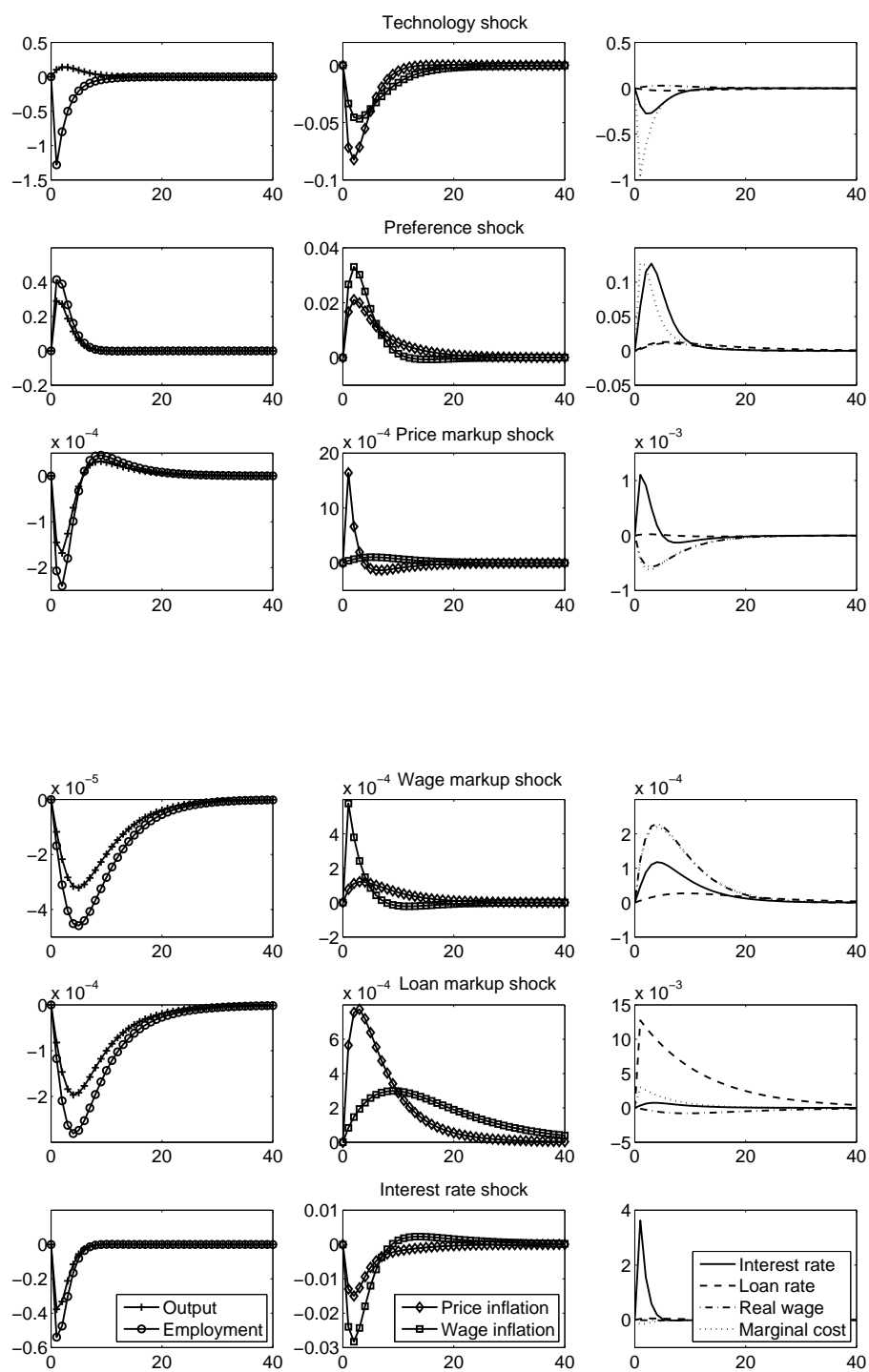


Figure 6: Impulse responses (quarterly timing in %) in the HYAC model.

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