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## **Losing Face**

**by**

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# Losing Face

David Hugh-Jones and David Reinstein\*

September 27, 2010

## Abstract

When person A makes an offer to person B and B rejects it, then A may “lose face.” This loss of face is assumed to occur *only* if B knows for sure of A’s offer. While under some circumstances loss of face can be rationalized by the consequences for future reputation, it may also enter *directly* into the utility function. Loss of face concerns can lead to fewer offers and inefficiency in markets that involve matching, discrete transactions, and offers/proposals in both directions, such as the marriage market, certain types of labor markets, admissions to colleges and universities, and joint ventures and collaborations. We offer a simple model of this, and show that under some circumstances welfare can be improved by a mechanism that *only* reveals offers when both parties say “yes.”

Keywords: Matching, marriage markets, anonymity, reputation, adverse selection, Bayesian games, emotions. JEL codes: D83, D03

## 1 Introduction

In a market that involves two-sided matching (as surveyed in Burdett and Coles, 1999) the fear of rejection can lead to inefficiency. A proposer may not ask someone out on a date, ask for a study partner, apply for a job, make a business proposition, or propose a paper co-authorship, because she does not want the other party to *know* of her interest and then turn her down. We call the disutility from this potential outcome “loss of face” (LoF).<sup>12</sup>

Consider a matching game where each player can choose “accept” or “reject,” there is asymmetric information about players’ types, and the outcome of the game (actions and payoffs) becomes common knowledge after all actions have been taken. Under reasonable assumptions over the gains to a match and the loss of face cost, LoF will worsen the set of Nash equilibria, ruling out the most efficient outcomes and allowing less efficient ones. Assuming coordination on the most (or least) efficient Nash equilibrium, LoF will make equilibrium play less efficient. In this case there will be a set of mutually beneficial transactions that would occur without LoF, but do not occur with LoF because:

1. the proposer does not know for sure whether the other party will accept or reject *and*
2. a high enough probability of rejection can outweigh the expected gains to a successful transaction.

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<sup>1</sup>In common parlance, a loss of face can have other causes and interpretations (e.g., a loss of face from acceding to an enemy’s demands, or from being proved wrong); in our paper, the operational definition of LoF is limited and specific. In particular it must be distinguished from a model where a proposer gets utility in her self image – as in Benabou and Tirole (2003) and Koszegi (2006) – and fears that learning of her own rejection will lower her image of herself.

In some contexts the LoF concern can be justified instrumentally. For example, suppose each individual accurately knows her own quality, but not that of others. Then in many types of dynamic matching, sorting/screening, and exclusion games, her willingness to match with, cooperate with, or not exclude another individual (henceforth, to “accept” them) may be taken as a negative signal of her type, as in the equilibrium of Chade (2006). As a consequence, the player she accepts (and perhaps others, if this is publicly observed) will downgrade his beliefs about her worth.<sup>3</sup> If we change the information structure so that *others* have private signals of an individual’s quality, an individual’s *being rejected* will lead third parties to downgrade their beliefs about her worth. Her reputation, in turn, may impact her utility either directly (as in Andreoni and Bernheim, 2008, Ellingsen and Johannesson, and Grossman (2008), where reputation is a primal concern) or via her expected future *material* payoffs, as in Spence (1973) and Cho and Kreps (1987).

However, as we discuss in section 2, there is evidence that a desire not to lose face is also a *primal* human concern, perhaps a product of evolutionary factors, or perhaps an internalization of the reputation motive. In such a case the disutility of openly accepting someone and being rejected by this person is not merely the cumulative effect of the two types of signaling mentioned above; there is a special loss from the combined knowledge that you accepted somebody, but they rejected you. The assumption of a primal LoF leads to a model that bears some resemblance to a “psychological game,” (Geanakoplos et al. (1989)), in which payoffs depend on beliefs as well as actions. However, in our model, payoffs depend not precisely on beliefs, but on information sets, specifically, the information sets at the end nodes of the game.<sup>4</sup> When (e.g.) a woman accepts a man and he rejects her, her material payoff is the same no matter what beliefs or information set either party has. However, because of LoF, her psychic payoffs are lower when *it is common knowledge* that she accepted him and he rejected her. In other words, what the other player knows for sure – the other player’s information set – is a component of a player’s utility function (this is similar to Battigalli and Dufwenberg (2007)). In particular, we assume that a probabilistic inference by the other player, that he may have been accepted, does not cause loss of face. Unlike in models of fairness (Rabin (1993); Moreno, 2008) the payoffs *do not* rely on a set of beliefs that are external to the game’s structure. Thus, as long as we know the (terminal) information structure, LoF transforms material payoffs into psychic payoffs in a straightforward way.<sup>5</sup> We will focus on the primal LoF interpretation. We see this as particularly relevant in some cases, such as a one-shot game where no outside parties observe the results.

If loss of face depends on the information sets in this way, then it can be avoided by changing the information sets so that players only learn about each other’s behaviour if they both play Accept. (In technical terms, instead of final information sets being  $\{\{AA\}, \{AR\}, \{RA\}, \{RR\}\}$ , they become  $\{\{AA\}, \{AR,RA,RR\}\}$ .) For example, a speed dating agency may ask men and women to mark the partners who they are interested in, and then inform only those couples who both marked each other. Now, after playing Accept, you will still be able to infer if you have been Rejected; but the other person will not know that you Accepted them, and, knowing this, you will not suffer a loss of face. We call such setups Conditionally Anonymous Environments or CAEs. The setup where all actions are known is the Full Revelation Environment, or FRE.

Our paper’s main insights are the following. There are circumstances in which LoF occurs even where it can not be justified by a reputation motive; i.e., there is a “primal” LoF. It is well-known that with asymmetric information, players may end up rejecting partners whom they would match with if actual types were known. However, if one or both players face the possibility of LoF, this inefficiency may become worse: players may reject potential partners even where their *expected* utility from a match is positive. This can lead to less efficient equilibrium outcomes in

<sup>3</sup>To reinterpret Groucho Marx “if I am willing to be part of this club, how good can I be?”

<sup>4</sup>We assume the structure of information sets is common knowledge.

<sup>5</sup>Furthermore, LoF has no obvious interpretation in terms of fairness; since revelation of “who proposed to whom” or “who was kind to whom” occurs after these decisions were made it should have no impact on beliefs about whether a player knew his play was “fair” in the sense of being congruent with the other player’s kindness or unkindness.

economies such as the marriage market (Becker (1973)).

Changing the information structure of the game's end nodes – in particular, only revealing mutual matches where both players played Accept – can improve outcomes by eliminating the possibility of LoF.

Our paper proceeds as follows. In section 2 we discuss our concept of in more detail, offer intuitive arguments, anecdotal evidence, and previous academic support for our model, and discuss situations where LoF is particularly relevant. We also briefly survey the related economic literature. In section 3.1 we describe our baseline setup, resembling that of earlier authors, and yielding only monotonic equilibria. In section 3.2 we introduce “end node histories” and LoF to the model, and illustrate how this can cause a welfare loss, and how anonymity can improve this outcome. We conclude in section 4 with a discussion of ways our model can be extended and tested empirically.

## 2 Background

There is abundant psychological evidence that “rejection hurts” (Eisenberger and Lieberman, 2004) and that social ostracism can cause a neurochemical effect that resembles physical pain Williams (2007). However, these studies do not distinguish between cases where it is common knowledge that the rejected party has expressed an interest from cases where this is private information. We claim that people fear proposing, and they fear it more when proposals are known. We ask the reader: which of scenarios below would likely cause you more psychological pain?

1. A friend or colleague, in whom you have an unexpressed romantic interest, informs you while talking about his/her tastes in men/women that s/he wouldn't go out with you because you are not “his/her type.” You have no reason to believe that s/he knows you are interested in him/her.
2. Without having the conversation in scenario 1, you ask this same person out on a date and s/he refuses because you are not “his/her type.”

We speculate that the second scenario would be more painful: you now both know that you have asked him/her out and s/he has refused. Although s/he may have tried to soften the blow by posing this as a matter of idiosyncratic preference rather than quality, you have lost face, and you are established as his/her inferior in one sense. In the first case, although you can presume s/he is not interested in you, and this may hurt your self esteem, s/he doesn't know you like him/her, and you have not lost face.

The fear of LoF is closely related to what psychologists call “rejection sensitivity”. For example, London et al provide evidence from a longitudinal study of middle school students that, for boys, “peer rejection at Time one predicted an increase in anxious and angry expectations of rejection at Time 2.” They also find that anxious and angry expectations of rejection are positively correlated to later social anxiety, social withdrawal, and loneliness. In explaining the connection to loneliness, they posit that the rejection sensitive may exhibit “‘flight’ (social anxiety/withdrawal) or ‘fight (aggression)’” reactions. Each of these “behavioral overreactions ... can reduce possibilities for the type of social relations that can combat loneliness.” It is easy to interpret either of these as a way to choose “reject” in our matching game in order to avoid further loss of face. The psychological evidence motivates us to consider heterogeneity: subjects with observable (e.g., sex, race) and unobservable (e.g., popularity) differences may be more or less concerned with LoF.

Erving Goffman (2005) has written extensively about losing and preserving *face*:

The term *face* may be defined as the positive social value a person effectively claims for himself by the line others assume he has taken during a particular contact ... A person tends to experience an immediate emotional response to the face which a contact with others allows him.

While each side wants to preserve their *own* face, they generally act to maintain the face of others as well (a concept we discuss later), and institutions and norms are usually structured to allow this:

...the combined effect of the rule of self-respect and the rule of considerateness is that the person tends to conduct himself during an encounter so as to maintain both his own face and the face of the other participants ... Each culture has its own “repertoire of face-saving practices.

Individuals undertake various forms of “face-work” to preserve their face:

*The avoidance process.*— The surest way for a person to prevent threats to his face is to avoid contact in which these threats are likely to occur. In all societies one can observe this in the avoidance relationship and in the tendency for certain delicate transactions to be conducted by go-betweens ...

In the context of this paper, Goffman’s “avoidance” is essentially preemptive rejection: you cannot be matched with a partner if you don’t show up.

According to a 2006 report, 37% of internet users who say they are single and seeking a romantic partner have gone on online-dating websites;<sup>6</sup> this industry earned an estimated \$642 million in the US in 2007. Internet dating *itself* can be seen as an institution designed to minimize the LoF that comes with face-to-face transactions, allowing people to access a network of potential partners who they are not likely to run into again at the office or on the street. Furthermore, several sites and applications on the social networking site “Facebook” have introduced the CAE environment, where member A can express interest in member B and member B only finds out about this if B also expresses an interest in A. However, there is a trade-off between preserving face and getting noticed: with thousands of members, each member may only view a fraction of eligible dates, and if A expresses anonymous interest there is no guarantee that B will even see A’s profile.<sup>7</sup>

“Speed dating” events are another recent innovation in the singles scene. These events usually attract an equal number customers of each gender; men rotate from one woman to another, spending a few minutes in conversation with each. Here there is also an effort to minimize the possibility of LoF. In fact, speed dating agencies often promote themselves on these grounds, e.g., Xpress dating advertises “Rejection free dating in a non-pressurized environment.”<sup>8</sup> Typically, participants are asked to select who they would like to go on “real dates” with only *after* the event is over. In most cases the agency will only reveal these “proposals” where there is a mutual match, i.e., where both participants have selected each other.<sup>9</sup>

LoF is not limited to the dating world. These concerns are also present on both sides of the job market. Within an employment relationship, LoF can affect decisions over how much effort to put in, and whether to identify with the firm, as well as whether to apply for promotions. Akerlof and Kranton (2005) consider a worker’s identity as an “outsider” or “insider,” and claim that a firm can take steps to change this identity. In their model, “when an employee has [insider] identity, she loses some utility insofar as she deviates from the ideal action of [the firm],” and for an outsider this is reversed. This has a LoF interpretation. Suppose that my putting in visible effort is a clear sign that I have identified with the firm, and not putting in effort is clear sign that I prefer to be an “outsider.” If I identify with the firm and try hard I am playing “accept” in the game with the firm. If the firm observes my effort and my success it

<sup>6</sup>Online Dating. Pew Internet & American Life Project. March 5, 2006. Available at [www.pewinternet.org/pdfs/PIP\\_Online\\_Dating.pdf](http://www.pewinternet.org/pdfs/PIP_Online_Dating.pdf)

<sup>7</sup>Online dating has been portrayed as a modern analogue to the traditional “matchmaker,” who was able to separately interview prospective mates and their families about their likes and preferences, helping arrange marriages while preserving anonymity.

<sup>8</sup>[http://www.xpressdating.co.uk/speed\\_intro.htm](http://www.xpressdating.co.uk/speed_intro.htm)

<sup>9</sup>“Speed dating” institutions have been extended outside the realm of romance and marriage, into forming study groups, “speed networking”, “industrial speed dating,” and business partnering; these may be established (in part) to minimize LoF. See: Collins and Goyder (2008), <http://edition.cnn.com/2005/BUSINESS/01/27/speed.networking/index.html>, <http://www.earthwisesociety.bc.ca/TEIP/TEIP/events/08-11-BPS/Speed%20dating%20checklist.pdf>, and [http://www.businesswire.com/portal/site/home/permalink/?ndmViewId=news\\_view&newsId=20090622005588&newsLan](http://www.businesswire.com/portal/site/home/permalink/?ndmViewId=news_view&newsId=20090622005588&newsLan)

will reward me (“accept” me) if it thinks I did well enough/am good enough. If not, the firm will “reject” me. There is a signaling issue: the fact that I was willing to work hard and identify with the firm implies that I think success at this firm is a prize worth having, which may lead them (or others) to downgrade my status. But the firm’s rejection also hurts a lot more if I have played “accept”, i.e., I have identified with the firm and tried hard: this is the primal LoF. Because of this, employees who fear LoF may only try hard on tasks that they know they will succeed in. These LoF-prone employees might also try hard on tasks where their effort will only be discovered if they are successful, e.g., projects they can work on in relative obscurity. The same authors’ (2000) model of social exclusion can be adapted to a LoF interpretation: if being seen “acting white” involves sacrificing Black identity, a Black person may choose not to attempt “admission to the dominant culture” because she is uncertain about the “level of social exclusion” she will face; e.g., whether she will be accepted by a school, employer, or white social group. On the other hand, if she can make this attempt anonymously, she will do so without risking these identity losses.

The employer too may be vulnerable to LoF. Cawley (2003), in his guide for economists on the junior job market, writes that he has “heard faculty darkly muttering about job candidates from years ago who led them on for a month before turning them down.” This aggravation surely involves LoF in addition to the loss of time. This LoF is recognized by professional recruiters as well: “recruiters lose face when candidates pull out of accepted engagements at the last minute.”<sup>10</sup>

For the rejection-sensitive, any economic transaction that involves an “ask” may risk a LoF. This may explain the prevalence of posted prices, and aversion to bargaining in certain countries. Rejection sensitivity is particularly disabling for sales personnel.<sup>11</sup>

A prominent case where the desire to preserve face, driven by instrumental concerns for reputation, may be leading to inefficiency is the university admissions process at selective institutions. One particularly important measure of a program’s quality and desirability is the “admit yield”: the share of admissions offers that are taken up by students. Indeed, this has been used by the US News and World Report in their college rankings. Some insiders claim that as a result, admissions officers specifically target admit yield, and “prefer a well-qualified applicant who they consider likely to attend over an exceptionally qualified candidate who they believe would probably choose not to enroll.”<sup>12</sup> In other words, a university may reject its strongest applicants in order to avoid being rejected by them. As a result, an unlucky student may “fall between the cracks,” rejected by higher ranking schools for not being good enough, and rejected by others for being “too good,” even though the match would be mutually beneficial. Efficiency might be improved by an admissions process with two-way ranking that only reveals a single “mutual match” between a student and a college.<sup>13</sup>

Our model may also be important in an archetypal situation where “preserving face” is valued the resolution of personal and political disagreements; essentially peacemaking. Neither side may want to make a peaceful overture unilaterally – this can be seen as evidence of admission of guilt or weakness, and may be psychologically painful in itself. Again, the double-blind mechanism can resolve this dilemma. Often, peace talks are made in secret, and only announced if a successful agreement has been reached.

While previous economists have studied related concepts, to our knowledge no authors have considered the consequences of common-knowledge rejection, as distinguished from rejection where only one side knows she was rejected.

<sup>10</sup>Leslie Merrow, Staffing Consultancy Principal, “How to Avoid the Counter-Offer Trap,” Friday, August 17, 2007. <<http://directsearchalliance.blogspot.com/2007/08/how-to-avoid-counter-offer-trap.html>>

<sup>11</sup>“Call reluctance, which strikes both individuals and teams, develops in many forms. Representatives may be ‘gun shy’ from an onslaught of rejection or actively avoid certain calling situations such as calling high-level decision makers or asking for the order. Call reluctance is the product of fear; fear of failure, fear of losing face, fear of rejection or fear of making a mistake. If the fear perpetuates, productivity suffers.” – “Business Services Industry Contests combat call reluctance,” Telemarketing & Call Center Solutions, Oct 1996 by Brian J. Geery.

<sup>12</sup>Lynn Radlauer Lubell, Founder of Admission By Design, an educational consultancy. <<http://www.inlikeme.com/apply/early-action-and-early-decision-what-you-need-know.html>> accessed on August 4, 2010.

<sup>13</sup>While such “ranking schemes” are potentially vulnerable to manipulation Gale, D. and Shapley, L. S. (1962), these may be an improvement on the current system.

Becker (1973) introduced a model of equilibrium matching in his “Theory of Marriage.” He considers the total surplus generated from marriage through a household production function that takes into account both husband and wife’s wage rates and productivity in the market and non-market sectors. His model allows the division of output between spouses to be divided ex-ante according to each party’s outside option in an efficient “marriage market.” In this context the interaction between a potential husband and wife would involve not just each party’s decision to “accept” or “reject” but a negotiation over who will get what out of the marriage (perhaps resembling a marriage contract). Anderson and Smith brought reputation into this context, noting “matches yield not only output but also information about types,” Anderson and Smith (Anderson and Smith). Chade (2006) explored a search and matching environment where participants observe “a noisy signal of the true type of any potential mate.” He noted “as in the winner’s curse in auction theory – information about a partner’s type [is] contained in his or her acceptance decision.” However, in Chade’s model there is only a single interaction between the same man and woman, and outside parties do not observe the results; thus there is no scope for either party’s actions to affect their future reputations (nor any direct cost of being rejected). Our work shares some features of models of directed search but there are some important differences. In the previous employment models, the cost of submitting a job application is unrelated to the probability of success, as well as unrelated to whether the employer knows a candidate has applied. In our model, the job seeker knows that if there is a high probability they will get the job (even if they do not accept the offer), there is a low probability they will lose face, and thus this cost is low in expectation.

### 3 Model

#### 3.1 Basic setup

Consider a simple one-period matching game with two players  $F$  and  $M$ .<sup>14</sup> Action spaces for both players are  $\{A, R\}$  where  $A$  denotes “accept” and  $R$  “reject”. We assume asymmetric information: each player knows only his or her own type.<sup>15</sup> Types are  $x_J \in \mathbb{R}$  for  $J \in \{F, M\}$ , both drawn independently from distributions with cdf’s  $\Phi_F$  and  $\Phi_M$ . We assume both have bounded support; without loss of generality let this be  $[\underline{t}, \bar{t}]$ . Players choose complete strategy functions  $\sigma^J(x_J)$ ,  $x_J \in [\underline{t}, \bar{t}]$ , where  $\sigma^J(x_J)$  specifies a probability of choosing Accept for a particular type. Material payoffs are  $\gamma^F(x_F, x_M)$ ;  $\gamma^M(x_M, x_F)$  for  $(A, A)$  and 0;0 for any other choices, as portrayed in the matrix below.

$\begin{matrix} F & M \end{matrix}$		$A$	$R$
$A$		$\gamma^F(x_F, x_M); \gamma^M(x_M, x_F)$	0,0
$R$		0,0	0,0

(1)

We make the following assumptions for  $J \in \{F, M\}$ .  $\gamma^J$  is continuous and differentiable with partial derivatives

**Asn 1.**  $\gamma'_2 > 0$ ,

,

**Asn 2.**  $\gamma'_1 < 0$ ,

and

<sup>14</sup>The notation loosely follows Chade (2006). For notational simplicity, we state the same distributions over type, matching functions, and LoF functions for M and F. However, we could allow these to differ for each player and all of our results would be preserved.

<sup>15</sup>In a similar game with symmetric information, LoF is less interesting. It does not affect the Pareto-optimal Nash equilibrium. However, if both players get positive utility from a match, “reject” is weakly dominated for both players in the standard game, but if both players have LoF concerns neither actions is dominated.



**Asn 3.**  $\exists x_M s.t. \Phi_M(x_M) > 0$  and  $\gamma^M(\underline{t}, x_M) > 0$  and  $\exists x_F s.t. \Phi_M(x_M) > 0$  and  $\gamma^F(x_F, \underline{t}) > 0$ .

The first assumption embodies agreed-upon preferences over a partner's type (as in Chade (2006) and many other papers) – partner's are “better” or “worse” along a single dimension. The assumption that  $\gamma_1^J < 0$  can be justified if we interpret the payoff as the gain to a match *relative* to remaining single. For example, let each player's gain from not matching be equal to his or her type; this contrasts with Becker (1973), but agrees with Chade's (2006) results in sign. If types represent productivity, it is plausible that those who add a lot of value in a match will be also be productive alone, and those who are unproductive alone have more to gain from matching with someone of a given type.<sup>16</sup> Let  $J$ 's gain from  $(A, A)$  equal  $\hat{\gamma}^J(x_K)$  for  $K \neq J$ . Then for each type we can rewrite his utility, by a shift of the origin, as  $\gamma^J(x_J, x_K) = \hat{\gamma}^J(x_K) - x_J$  from  $(A, A)$  and  $0 = x_J - x_J$  for not trading. The final assumption implies that, on each side of the market, there are some types, occurring with positive probability, that gain even from a match with a partner of the lowest type.

There is always an equilibrium where both players always reject. since if so, both get 0 whatever they play). We focus only on “nontrivial” equilibria, where players always accept with some minimum probability  $\varepsilon > 0$ . We note that the no-matching equilibrium involves weakly dominated strategy profiles (and is thus not trembling-hand perfect) and is not stable – if a single type chooses  $A$  then, given the third assumption above, a positive-probability set of types will also gain by choosing  $A$ .

**Lemma.** *Any nontrivial equilibrium is monotonic, with all types below some cutpoint  $\hat{x}_J$  playing accept for  $J \in \{F, M\}$ .*

*Proof.* If nobody accepts, define  $\hat{x}_J = \underline{t}$ . Otherwise, player  $J$  accepts if  $Pr(A)E(\gamma^J(x_J, x_K)|x_K \in \mathbf{A}^k) \geq 0$  where  $Pr(A)$  is the probability the other type plays accept, and the expectation is taken over the set of types that accept, For a nontrivial equilibrium,  $Pr(A) \geq \varepsilon > 0$  by assumption, and  $\gamma^J$  is strictly decreasing in  $x_J$ , so if type  $x$  accepts so must all types  $x' < x$ .  $\square$

Thus we have shown the existence of a Nash equilibrium in which the strategies are:

$$\sigma^J(x_J) = \begin{cases} A & \text{if } x_J \leq \hat{x}_J \\ R & \text{if } x_J > \hat{x}_J \end{cases} \text{ for } J \in \{F, M\}$$

We cannot yet rule out multiple equilibria, since  $J$ 's best response cutpoint is increasing in  $K$ 's cutpoint.

In this game with material payoffs, sequence is irrelevant.<sup>17</sup> Since both players' actions impact payoffs only in the case that the other player plays accept, it does not change their best response if they learn whether or not the other player *did* in fact accept; the same “acceptance curse”(Chade, 2006) occurs in either case. For concreteness, we offer a parametric example in appendix A.1.

### 3.2 With loss of face and incomplete information

Since LoF results from the *common knowledge* of one party accepting and the other rejecting, to model LoF we need to make payoffs depend not only on actions, but also on information sets. The set of end nodes of the game, defined

<sup>16</sup>This might also be justified through interpreting the payoff to no match as the continuation value in an indefinitely repeated version of the matching game. In Chade (2006) this value increases in type as higher-type players tend to have higher signals and other players accept them more often. However, this does not hold in our model as we have no such signals.

<sup>17</sup>By “sequence is irrelevant” we mean that in a sequential game in which (say) player M observes F's move before making his own, M's action after observing F's acceptance will be  $a$  if and only if  $x_M \leq \hat{x}_M$  as defined above; his action after observing a rejection can be anything. And given this, F will play  $A$  if and only if her type is less than  $\hat{x}_F$ .

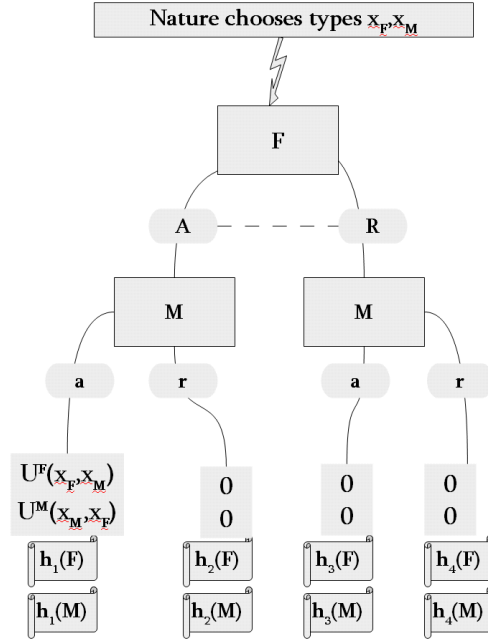


Figure 1: Material payoffs (without LoF), simultaneous game, full revelation environment, extensive form.

by their histories, is  $h = \{h_1, h_2, h_3, h_4\} = \{AA, AR, RA, RR\}$ .<sup>18</sup> Let  $\bar{I}_F$  be the collection of player F's information sets over these endnodes, and  $\bar{I}_M$  be player M's information partition (we will call each element of these "terminal information sets").<sup>19</sup> For example, if there is full revelation, then

$$\bar{I}_F = \bar{I}_M = \{\{AA\}, \{AR\}, \{RA\}, \{RR\}\}. \quad (2)$$

This game is illustrated in the game tree in figure 1 (*material* payoffs are displayed in each of the figures below). Since neither player "has the move" at the terminal node, we give each history two boxes to depict each player's information set;  $h_j(M)$  and  $h_j(F)$  are the same (for  $j \in \{1, 2, 3, 4\}$ ). Note that even though M does not observe F's choice before his move, he does observe it at the terminal node.

<sup>18</sup>We leave nature's move out of these histories; it does not affect our discussion. For completeness we can assume that player's never learn the other player's type. Thus, in our model LoF will only depend on the conditional expectation of the other player's type, not the type itself.

<sup>19</sup>While these interactions could also be depicted in a more standard notation, this terminology is intuitive and convenient for our purposes.

On the other hand, if only mutual acceptances are revealed to the players, terminal information partitions are

$$\bar{I}_F = \{\{AA\}, \{AR\}, \{RA, RR\}\} \text{ and } \bar{I}_M = \{\{AA\}, \{AR, RR\}, \{RA\}\}. \quad (3)$$

Note in figure 1, depicting this simultaneous FRE, that the  $\{AA\}$  information set is a singleton for both players, while the histories where a player played “reject” are connected by an information set (for that player).

We model loss of face as depending directly on the information set, and on inferences each player can make about the other’s information set.

In particular, player  $J$  loses face when

1.  $J$  played accept.
2.  $J$  knows that player  $K$  played reject.
3.  $J$  knows that  $K$  knows that  $J$  played accept.

The second point is always satisfied when  $AR$  or  $RA$  is played, since players who played accept can infer directly from their payoffs whether they were accepted or rejected.<sup>20</sup> Suppose  $F$  arrives at her information set  $\{AR\}$  under full revelation. Then she knows that  $M$  is at his information set  $\{AR\}$  also. On the other hand if only mutual acceptances are revealed then  $F$  only knows that  $M$  is at  $\{AR, RR\}$ . In the first case,  $M$  has learned something about  $F$ ’s type; in the second case not. This is a potential signaling reason for LoF, which would require us to analyze equilibrium strategies to figure out what  $M$  has learned. For the present paper, we just assume that if conditions 1-3 are fulfilled, player  $J$  suffers a loss  $L^J(x_J, \Xi_K(x_k))$ , a composition of functions which may depend on her type  $x_J$  and on her belief over the probability distribution over the other’s type, the cumulative probability density function  $\Xi_K = \Xi_K(x_k) = \Phi_J(x_k | x_k < \hat{x}_k)$ .

When only mutual acceptances are revealed (as in figure 2), neither player can suffer a loss of face, since after accepting and being rejected, player  $J$  knows that the other player does not know whether  $J$  played accept or reject. Thus, the payoffs and the equilibrium are the same as those with material payoffs. However, with full revelation, the game is altered. Now, if either  $AR$  or  $RA$  is played, the accepting player  $J$  loses  $L^J(x_J, \Xi_K)$  where  $\Xi_K$  is the posterior distribution over player  $J$ ’s type, given that they rejected. We can write the (expected) payoff matrix as

$F \backslash M$	$A$	$R$	
$A$	$\gamma^F(x_F, x_M), \gamma^M(x_M, x_F)$	$-L^F(x_F, \Xi_M), 0$	(4)
$R$	$0, -L^M(x_M, \Xi_F)$	$0, 0$	

This is depicted in figure 3.

<sup>20</sup>This assumes that  $\gamma$  is nowhere constant at 0 and that  $f$  has no mass points.

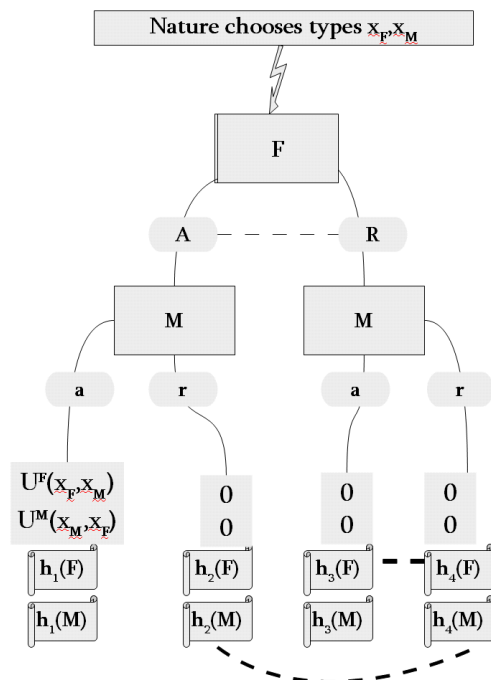


Figure 2: Simultaneous game, conditionally anonymous environment, extensive form.

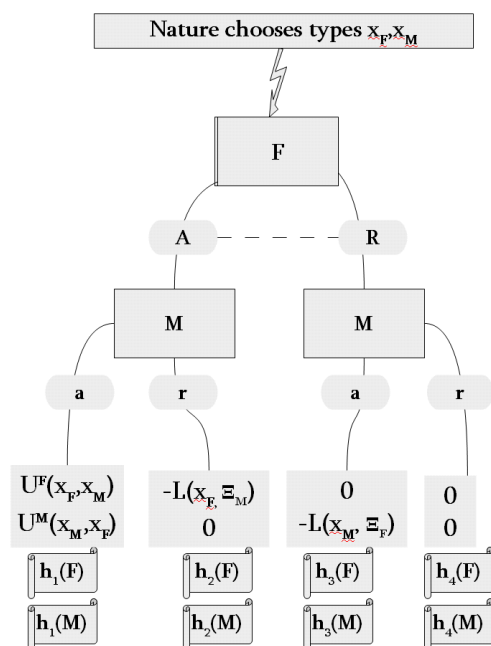


Figure 3: Overall payoffs, simultaneous game, full revelation environment, extensive form.

For concreteness suppose that for  $J \in \{F, M\}$

**Asn 4.**  $L_1^J > 0$

and

**Asn 5.**  $L^J(x_J, \Xi_K) > L^J(x_J, \Xi'_K)$  if  $\Xi'_K$  first order stochastically dominates  $\Xi_K$ .

That is, it hurts more to be (and know that you have been) rejected by someone you think is a lower type, and it hurts more to be (and know that you have been) rejected if you are a higher type (higher types may have more “pride”).<sup>21</sup> In particular, this includes the case  $L^J(x_J, \Xi_K) = E(l^J(x_J, x_K))$  where the expectation is taken over the other player’s types, where  $l^J(x_J, x_K)$  is a “realized loss of face,” and where  $\partial l^J / \partial x_K < 0$ : this may hold when LoF is caused by reputation concerns, for instance.

We also make the natural assumption that

**Asn 6.**  $L^J(x_J, \Xi_K) > 0$  for all  $x_J, \Xi_K$

so that (commonly-known) rejection always hurts to some extent. With these assumptions we can guarantee that an equilibrium exists, is monotonic, and that LoF may cause harm in the sense described below.

**Proof of existence of equilibrium with LoF:** In the Appendix, we use Brouwer’s Fixed Point Theorem to prove the existence of an equilibrium.

**Proposition 1.** *With full revelation and LoF, any equilibrium is monotonic. Furthermore, the lowest and highest equilibrium cutpoints are weakly lower than in the game with material payoffs, and strictly lower if they are within the interior( $\underline{t}, \bar{t}$ ). When LoF implies lower equilibrium cutpoints this implies lower social welfare, even net of LoF.*

We modestly refer to this proportion as “the Fundamental Theorem of Face.” The proof is in the appendix. The intuition is straightforward: loss of face makes playing accept (ex ante) less attractive for each party. Since lower types always play accept, the result of this is a lower cutpoint. This effect is amplified by the interaction between cutpoints: when  $M$ ’s cutpoint becomes lower,  $M$ ’s who play Accept become less attractive and so  $F$  follows suit by lowering her cutpoint. An example is shown in Figure 4 on page 12.

As this figure shows, when Loss of Face becomes an issue, the highest and lowest equilibria move downwards, though intermediate equilibria need not. Thus if we assume coordination on the Pareto-preferred equilibrium (or assume a single-crossing condition), as long as all parties do not already match under the Full Revelation Environment, moving to the Conditionally Anonymous Environment will not only eliminate welfare losses from LoF, but also increase the expected number of matches. These effects must increase welfare: (all types) lower types gain by the “entry” of the new higher types (a better pool playing “accept”), and, as the these better types now decide that playing “accept” is better for them, their welfare must also be improved.

### 3.3 Parametric example

We offer a simple parametric example to illustrate how Loss of Face can cause a loss of welfare. Let:

<sup>21</sup>This second assumption is not necessary for any of our later results; in fact, if LoF decreases in the expected type of the other player, this will diminish the effect mentioned in proposition 1. As LoF causes the equilibrium cutpoint to decline, the expected type of a player who rejects also declines, lessening the expected LoF from accepting and being rejected. We included this to show the robustness of our model to what we see as an intuitive assumption – being rejected by someone you think is of little worth (“even *he* rejects me”) might be more painful. This might also be loosely justified by a reputation story – proposing to the highest type should reveal little about my own type, as nearly everyone would want to match with her.

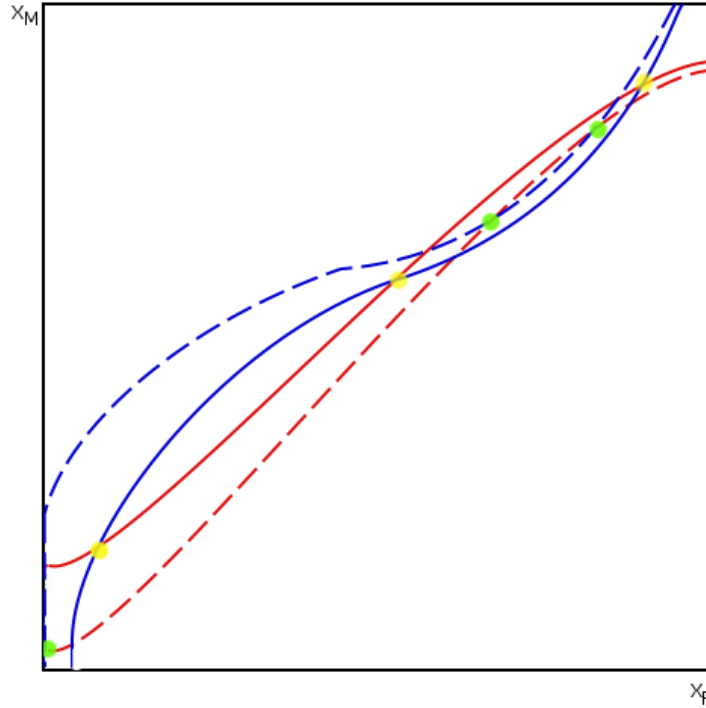


Figure 4: Equilibria with and without loss of face. The lines are best response functions: dashed lines include loss of face, which increases when the other player's cutpoint is lower. Equilibrium points with (without) loss of face are shown in green (yellow).

$$\gamma(x^J, x^K) = \frac{1}{4} + x^J - x^K$$

$$L(x^J, x^K) = \frac{1}{4} + I[E(x^K | \sigma^K = R) > x^J] \times (x^J - E(x^K | \sigma^K = R))$$

and  $x^J$  and  $x^K \sim U(0, 1)$ ,

where  $J, K \in M, F$  and  $J \neq K$ ,  $I$  represents the indicator function and  $\sigma^K = \sigma^K(x^K)$ ,  $k$ 's realized strategy.

In the case without Loss of Face, there are two equilibria; there is an unstable trivial equilibrium, where no one chooses "accept." The probability of accepting is  $\hat{x}^J$  for  $J \in \{M, F\}$ , the cutoff point. The probability of not accepting is  $1 - \hat{x}^J$ . Simple algebra yields the best responding cutpoints  $\hat{x}^J = \frac{1}{4} + \frac{1}{2}\hat{x}^K$ , yielding the nontrivial equilibrium strategy profile,  $\hat{x}^{M*} = \hat{x}^{F*} = \frac{1}{2}$ , implying a 25% probability of a match, and expected utilities  $\frac{1}{2} \frac{1}{2} (\frac{1}{4} + \frac{1}{4} - \frac{1}{4}) = \frac{1}{16}$  per player.

In the LoF case, the utility of rejecting for any individual is zero, while the utility of accepting is given by:

$$U = (\text{probability of Accepting})(\text{Expected Gain of Accepting}) - (\text{Probability Of Rejecting})(\text{Expected Loss Of Face if Lose Face})$$

The expected gain from the match for a player of type  $x^J$  is again  $\frac{1}{4} + \frac{x^K}{2} - x^J$ . The expected Loss of Face (if it occurs) is  $\frac{1}{4} + x^J - E(x^K | x^K > \hat{x}^K)$  where  $x^J > E(x^K | x^K > \hat{x}^K)$  and  $\frac{1}{4}$  otherwise. . Thus the expected utility from a type  $x^J$  player playing accept is given by :  $U^J(x^J | \hat{x}^K) = \hat{x}^K (\frac{1}{4} + \frac{x^K}{2} - x^J) - (1 - \hat{x}^K)(\frac{1}{4})$  if  $x^J < \frac{1 + \hat{x}^K}{2}$  and  $\hat{x}^K (\frac{1}{4} + \frac{x^K}{2} - x^J) - (1 - \hat{x}^K)(\frac{1}{4} + x^J - \frac{\hat{x}^K}{2})$  otherwise. Solving for the  $x^J$  that yields zero utility as a function of  $\hat{x}^K$  yields the equilibrium cutoff point condition  $\hat{x}^J = \hat{x}^K - \frac{1}{4}$  in the latter case, and  $\hat{x}^J = \frac{1}{4} \frac{1}{\hat{x}^K} (2\hat{x}^K + 2(\hat{x}^K)^2 - 1)$  in the former case. By symmetry, we can substitute J and K in each reaction function. Clearly, there can be no equilibrium where both players' cutpoints are above their expectation for the other "rejecting" player's type, and we can show that also no equilibrium where one player's cutpoint is above and the others' below.<sup>22</sup> If both cutpoints are below this expectation, we substitute in

<sup>22</sup>Substituting the "lower" reaction function into the higher one yields  $\hat{x}^J = \frac{1}{4} \frac{1}{\hat{x}^K} (2\hat{x}^K + 2(\hat{x}^K)^2 - 1) - \frac{1}{4}$ , which has no real solution.

to find the unique real fixed point at  $\hat{x}^J = \hat{x}^K = \frac{1}{6}\sqrt{15} - \frac{1}{2} \approx 0.145$ . This implies that each player only plays “accept” below the lower 15% of her types, yielding a  $\frac{2}{3} - \frac{1}{6}\sqrt{15} \approx 2\%$  probability of a match, and expected pre-LoF utilities  $(\frac{2}{3} - \frac{1}{6}\sqrt{15}) \times (\frac{1}{4}) \approx 0.005$  for each player.<sup>23</sup> Thus, the Pareto-preferred LoF equilibrium implies fewer matches and, even leaving out the LoF component, lower expected “material” utility than the Pareto-optimal equilibrium before loss of face.

## 4 Conclusion and suggestions for future work

Our work has potential normative implications. Some mechanisms and policies may be more efficient than others in the presence of LoF effects, and firms and policymakers should take this into account. Although setting up a conditionally anonymous environments may take some administrative effort, and may require a third-party monitor, we suspect that there are many cases in which it will lead to more and better matches and improve outcomes.

This modeling of this paper can be expanded in several ways, relaxing many of our assumptions and generalizing our results. Future research should more directly connect the LoF to the impact of revealing offers on how others estimate a player’s type, in an environment where players have private and public signals of their own and others’ types; however we should bear in mind that in many contexts, LoF may have an impact over and above the effect on reputation. In equilibrium, the effects of revealing offers on reputation and on match efficiency may be complex. If a player is known to be vulnerable to LoF, his making an offer might actually be interpreted as a signal of his *confidence* that he will be accepted, thus a positive signal about his own type. Whenever a player rejects another, there is some possibility that she did so merely to avoid losing face; noting this possibility should presumably “soften the blow” to a player’s reputation when she is rejected. Relaxing the assumptions further, preferences over types may be heterogeneous, or may involve a horizontal component, this may also change the equilibrium reputation effects of revealing offers. It also may be interesting to consider the effects of a player who is either altruistic, suffering when the other player loses face, or spiteful, relishing in making others lose face. Considering a sequential game where only the first mover is vulnerable to a LoF and the second mover is a known altruist. Here the first-mover might manipulate this altruism, playing “accept” and in effect guilting the second-mover into matching with her; this could lead to inefficient matching.

Empirically, our anecdotal and referential evidence for LoF should be supplemented by evidence from laboratory and field experiments. As well as strengthening the evidence for the existence of this motivation, these should examine the causes and correlates of LoF, and its efficiency consequences in various environments. Do people act strategically to minimize their own risk of LoF? Will they be willing to pay to preserve the anonymity of their offers? Who is most affected by loss of face and when? How can these issues be addressed to improve matching efficiency in real-world environments? We hope that future research will offer empirical answers to these questions.

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<sup>23</sup>If we also include LoF in the utility computation, the expected utility is  $(\frac{2}{3} - \frac{1}{6}\sqrt{15}) \times (\frac{1}{4}) - (\frac{2}{3} - \frac{1}{6}\sqrt{15}) \times (1 - (\frac{2}{3} - \frac{1}{6}\sqrt{15})) \times \frac{1}{4} \approx 0.0001$ .

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## A Proofs of results and computations

### Proof of existence of equilibrium with LoF:

*Definition of Brouwer's Fixed Point Theorem:* Suppose that  $A \subset \mathbb{R}^N$  is a nonempty, compact, convex set, and that  $f : A \rightarrow A$  is a continuous function from  $A$  into itself. Then  $f()$  has a fixed point ( $x = f(x)$ ).

First we note that an individual's strategy is an action for each type  $t$ , to accept or reject. If we define as  $f()$  the function that gives the minimum  $t_i$  for which an individual  $i$  plays accept, given that the other individual's strategy is to accept when his type is higher than a value  $t_j$ . Given that an individual will always play accept when her type is higher than the minimum value  $t_i$ , the function  $f()$  represents each player's best response to the other player's strategy.  $f()$  is the value of  $t$  that solves the following equation:

$$Pr(a)E(\gamma(x,y)|a) - \lambda Pr(r)L(x,E(y|r)) = 0$$

$f()$  goes from  $A$  to  $A$ , where  $A$  is the space of types and  $A$  goes from  $\underline{t}$  to  $\bar{t}$  and therefore  $A$  is a non-empty, compact, and convex set. The function  $f()$  is continuous, if  $\gamma(x,y)$ ,  $L(x,y)$  and the distribution of types are all continuous functions. By Brouwer's fixed point theorem there is a value of  $t$  for which  $t = f(t)$ . This is an equilibrium.

### A.1 Parametric example of basic setup

Let  $x_M$  and  $x_F$  be independently uniformly distributed on  $(0, 1)$ , and

$$\gamma'(x_J, x_K) = \log \frac{x_K + \beta}{1 - x_K + \beta} - \log \frac{x_J}{1 - x_J}.$$

where  $\beta$  is small and positive. Now the first component varies from  $\log \frac{\beta}{1+\beta}$  to  $\log \frac{1+\beta}{\beta}$ , and a bit of algebra shows that  $\gamma$  is always positive for

$$x < \frac{\beta}{1+2\beta}$$

and always negative for  $x > \frac{1+\beta}{1+2\beta}$ , so some types always want to accept, and some never do (each occurring with positive probability).

To solve this, we remove all the logs, so the disagreement payoff is 1 and  $\gamma(x,y) = \frac{y+\beta}{1-y+\beta} \frac{1-x}{x}$ . Then  $\hat{x}$  solves

$$E(\gamma(\hat{x}, y)|y < \hat{y}) = 1$$

equivalently:

$$\int_0^{\hat{y}} \frac{y+\beta}{1-y+\beta} \frac{1-\hat{x}}{\hat{x}} \frac{1}{\hat{y}} dy = 1$$

where the final  $1/\hat{y}$  is the posterior pdf; equivalently

$$\int_0^{\hat{y}} \frac{y+\beta}{1-y+\beta} dy = \frac{\hat{x}\hat{y}}{1-\hat{x}}$$

Integrating this yields

$$\hat{x} = \frac{\hat{y} + W}{W} \text{ where } W = (2\beta + 1)[\log(1 + \beta - \hat{y}) - \log(1 + \beta)] < 0.$$

By symmetry (assuming that  $\gamma(y, x)$  is the same as  $\gamma(x, y)$ , transposing the x and y, and have the same  $\beta$ ),

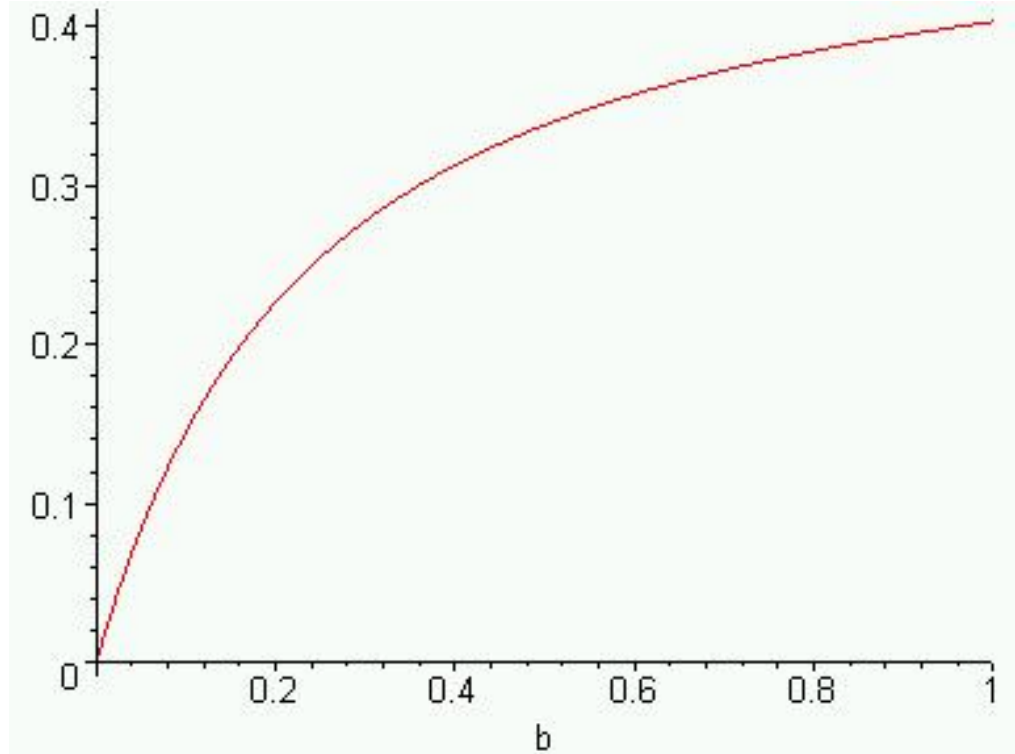
$$\hat{x} = \frac{\hat{y} + W}{W} = \hat{y} = \frac{\hat{x} + W}{W}$$

$$\rightarrow \hat{x} = \frac{\hat{x} + W}{W} \rightarrow \hat{x} = \frac{1}{(1 - \frac{1}{W})} = \frac{W}{W - 1}$$

$$\rightarrow \hat{x} = \frac{(2\beta + 1)[\log(1 + \beta - \hat{x}) - \log(1 + \beta)]}{(2\beta + 1)[\log(1 + \beta - \hat{x}) - \log(1 + \beta)] - 1}.$$

While we do not solve this in general, we plot the critical points for numeric values of  $\beta$ . With  $\beta = 0$ ,  $\hat{x} = \frac{(1)[\log(1 - \hat{x}) - \log(1)]}{(1)[\log(1 - \hat{x}) - \log(1)] - 1} = \frac{[\log(1 - \hat{x})]}{[\log(1 - \hat{x})] - 1}$ , which is only solved by  $\hat{x} = 0$ ; this resembles the extreme “lemons” case of Akerlof (1970). Similarly, with  $\beta = 0.1$ ,  $\hat{x} = 0.1453..$  Plotting  $\hat{x}$  (and hence  $\hat{y}$ ) for  $\beta = 0..1$ :

Figure 5: Plotting ...



Note that with this functional form “higher than average” types will never want to accept for any value of  $\beta$ .

**Proposition 2.** *With full revelation and LoF, any equilibrium is monotonic. Furthermore, the lowest and highest equilibrium cutpoints are weakly lower than in the game with material payoffs, and strictly lower if they are within the interior( $\underline{t}, \bar{t}$ ).*

*Proof.* We write the general form of the game as

$F$	$M$		
		$A$	$R$
$A$		$\gamma^F(x_F, x_M), \gamma^M(x_M, x_F)$	$-\lambda L^F(x_F, x_M), 0$
$R$		$0, -\lambda L^M(x_M, x_F)$	$0, 0$

(5)

where  $\lambda$  parametrizes the importance of loss of face. Player  $J$  (of type  $x_J$ ) plays accept iff

$$Pr(A)E(\gamma^J(x_J, x_K)|x_K \in A^K) - (1 - Pr(A))\lambda L^J(x_J, X_K) \geq 0 \quad (6)$$

where  $Pr(A)$  is the probability that player  $K$  plays Accept. Exactly as before, the left hand side is strictly decreasing in  $x_J$  so that the best response functions are monotonic. We can treat the players' strategies as a choice of cutpoints, with utility from playing  $A$  being  $\int_{\underline{t}}^{\bar{t}} \{Pr(A)E(\gamma^J(x_J, x_K)|x_K > \hat{x}_K) - (1 - Pr(A))\lambda L^J(x_J, X_K)\} d\Phi(x_K)$ .  $\square$

*Proof.* We next show that the lowest and highest equilibrium cutpoints are weakly lower when  $\lambda$  increases (including when it increases from 0 to 1).

First, let  $\hat{x}_J^*(\hat{x}_K)$  be  $J$ 's best response cutpoint when  $K$ 's cutpoint is  $\hat{x}_K$ . An equilibrium is defined by a pair of points  $\hat{x}_J = \hat{x}_J^*(\hat{x}_K)$ ,  $\hat{x}_K = \hat{x}_K^*(\hat{x}_J)$ . We show that  $\hat{x}_J^*(\hat{x}_K)$  is weakly increasing everywhere, and strictly increasing on the interior  $(\underline{t}, \bar{t})$ .

**Proof**

An increase in  $\hat{x}_K$  has three effects:

First, it increases  $E(\gamma^J(x_J, x_K))$  since  $\gamma_2^J > 0$

Second it decreases  $L^J$ , since  $X_K$ , the conditional cdf given cutpoint  $\hat{x}_K$  is

$$X_K = \Phi_{\hat{x}_K} = \Phi_J(x_K | x_K < \hat{x}_K) = \begin{cases} 0, & x < x_K \\ \frac{\Phi(x) - \Phi(x_K)}{1 - \Phi(x_K)} & x \in (x_K, \bar{t}] \end{cases}$$

and thus  $\hat{x}_K' > \hat{x}_K$  implies  $\Phi_{\hat{x}_K'}$  first order stochastically dominates  $\Phi_{\hat{x}_K}$ .

Third, it increases  $Pr(A)$  and decreases the probability of rejection.

The first two effects always increase the left hand side of (6) (strictly for the first effect), and the last effect weakly increases it at the equilibrium point  $\hat{x}_J^*$ , since there  $E(\gamma^J(x_J, x_K))$  must be non-negative. We have just shown that the game is supermodular in  $(x_J, x_K)$ . Also, an increase in  $\lambda$  must decrease  $\hat{x}_J^*(\hat{x}_K)$ , as is immediate from (6); thus the game has increasing differences in  $(x_J, -\lambda)$ . These suffice to show that the highest and lowest cutpoints are weakly decreasing in  $\lambda$  (Milgrom and Roberts, 1990). To show they are strictly decreasing on the interior observe that an increase in  $\lambda$  must strictly decrease the  $\hat{x}_J$  that solves (6) with equality, and that if  $\hat{x}_J$  decreases an interior best response  $\hat{x}_K^*(\hat{x}_J)$  must also strictly decrease.  $\square$