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Forecast based Pricing of Weather Derivatives*

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Abstract

Forecasting based pricing of Weather Derivatives (WDs) is a new approach in valuation of contingent claims on nontradable underlyings. Standard techniques are based on historical weather data. Forward-looking information such as meteorological forecasts or the implied market price of risk (MPR) are often not incorporated. We adopt a risk neutral approach (for each location) that allows the incorporation of meteorological forecasts in the framework of WD pricing. We study weather Risk Premiums (RPs) implied from either the information MPR gain or the meteorological forecasts. The size of RPs is interesting for investors and issuers of weather contracts to take advantages of geographic diversification, hedging effects and price determinations. By conducting an empirical analysis to London and Rome WD data traded at the Chicago Mercantile Exchange (CME), we find out that either incorporating the MPR or the forecast outperforms the standard pricing techniques.

Keywords: Weather derivatives, seasonal variation, temperature, risk premia
JEL classification: G19, G29, G22, N23, N53, Q59

1 Introduction

Weather Derivatives (WDs) are financial instruments to hedge against the random nature of weather variations that constitute weather risk (the uncertainty in cash flows caused by weather events). Two years after the first over the counter (OTC) trade of a WD in 1997, the formal exchange Chicago Mercantile Exchange (CME) introduced derivative contracts on weather indices in 1999. Both exchange traded and OTC derivatives are now written on a range of weather indices, including temperature, hurricanes, frost and precipitation. WDs differ from insurances, first because insurances cover low probability

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extreme events, whereas WDs cover lower risk high probability events such as colder winters than expected. Second, a buyer of a WD will receive its payoff at settlement period no matter the loss caused by weather conditions. For insurances, the payoff depends on the proof of damages. Third, from the seller's point of view, WDs eliminate moral hazard and avoid the higher administrative and the loss adjustment expenses of insurance contracts. The WD market is a typical example of an incomplete market in the sense that the underlying weather indexes are non-tradable assets and cannot be replicated by other underyling instruments, like there are in the equity market. Furthermore, the market is relatively illiquid. Campbell and Diebold (2005) argued that this illiquidity is due to non-standardisation of the weather. Given this, one might expect some inefficiencies in the WD market. The protection is achieved, when two counterparties in the transaction of a WD meet: a hedger (e.g. a farmer) who wants to hedge his weather risk exposure and a speculator, to whom the risk has been transferred in return for a reward.

The pricing of WDs is challenging because in contrast to complete markets the assumption of no arbitrage does not assure the existence of a unique risk neutral measure. Many valuation techniques of WDs have overcome this problem: under an equilibrium representative framework (Cao and Wei (2004)), under the Equivalent Martingale Approach EMM (Alaton et al. (2002); Benth (2003)), using marginal utility approach (Davis (2001)) or, more generally, with the principle of equivalent utility (Brockett et al. (2010)). Standard pricing approaches for weather derivatives are based on historical weather data, estimate the physical measure by time series analysis and then calibrate the Market Price of Risk (MPR) in such a way that the traded WDs are martingales under the risk neutral measure. Forward-looking information such as meteorological forecasts or the MPR are often not incorporated in usual pricing approaches. Hence, important market information is not considered in an informational efficient markets, where futures prices reflect all publicity available information.

The literature on how to calibrate the MPR or how to incorporate meteorological weather forecast into the price of weather derivatives is limited. From one side, we have the studies from Härdle and López-Cabrera (2011) and Benth et al. (2011), who use inverse techniques to imply the MPR from the temperature futures traded at CME and suggest a seasonal stochastic behaviour of the non-zero MPR. On the other side, the work from Jewson and Caballero (2003) describes how probabilistic weather forecasts, via single and ensemble forecasts up to 12 days in advance, can be used for the pricing of weather derivatives. Yoo (2004) incorporates seasonal meteorological forecasts into a temperature model, which predicts one of three possible future temperature states. A new perspective on the commodities pricing literature is given in Benth and Meyer-Brandis (2009), who suggest the enlargement of the filtration information set and argue that the stochasticity behaviour of the MPR is due to the misspecified information set in the model. Dorfleitner and Wimmer (2010) include meteorological forecast in the context of WD based index modelling. Ritter et al. (2011) combine historical data with meteorological forecast in a daily basis to price WDs. In this paper, we adopt the risk neutral approach (for each location) that allows the incorporation of meteorological forecasts in the framework of WD pricing and compare it with the information gained by the calibrated MPR. The aim is to study weather Risk Premiums (RP), a central issue in empirical finance, implied from either the information MPR gain or the meteorological forecasts. The size of RPs is interesting for investors and issuers of weather contracts to take advantages of geographic diversification, hedging effects and price determinations. We quantify the RPs of weather

risk by looking at the risk factor under different pricing measures and under different filtration information sets.

We analyse the RPs for temperature derivatives, which constitute the majority of trading volume in the weather market, in London and Rome. Our main goal is to determine the nature of the risk factor embedded in temperature future prices. We find that the seasonal variance of temperature explains a significant proportion of the variation in RPs. The estimated forecast based prices reflect market prices much better than price without the use of forecast. In both approaches, the RPs of futures are different from zero, negative in winters and positive in summers.

The findings of this paper are presented as follows. In Section 2, we present the fundamentals of temperature index derivatives (futures and options) traded at CME and review the stochastic pricing model for average daily temperature and study its properties. Section 3 introduces the concept of RPs across different risk measures and under different filtration information set. In the latter approach, meteorological weather forecasts are incorporated into the WD pricing. In Section 4, we conduct the empirical analysis to temperature futures referring to London and Rome, with meteorological forecast data for London 13 days in advance. Despite this relatively short forecast horizon, the models using meteorological forecasts outperform the classical approach and more accurately forecast the market prices of the temperature futures traded at the Chicago Mercantile Exchange (CME). Section 5 concludes. All computations were carried out in Matlab version 7.6 and `R`.

2 Weather Derivatives

The most commonly weather instruments traded at the CME are futures, call and put options written on weather indices. The CME traded futures can be thought as a swap, such that one party gets paid if the realized index value is greater than a predetermined strike level and the other party benefits if the index value is below. Typically, futures are entered without a payment of premium. In exchange for the payment of the premium, the call option gives the buyer a linear payoff based upon the difference between the realized index value and the strike level. Below this level there is no payoff. On the other hand, the put option gives the buyer a linear payoff based upon the difference between the strike level and the realized index value.

The most popular underlying weather indices are temperature related. The reason is the abundance of historical temperature data and the demand for a weather product coming from end-users with temperature exposure. The weather indices most commonly used in the market are the Heating Degree Days (HDD), Cooling Degree Days (CDD), Cumulative Average Temperature (CAT) and the Cumulative total of 24-hour Average Temperatures (C24AT). The HDD index is computed as the maximum of zero and 65°F (or 18°C) minus the average temperature of the day, accumulated over every day of the corresponding contract period. Equivalently, the CDD index is the accumulation of the maximum of zero and the average temperature minus 65°F (or 18°C). CAT and C24AT cumulate the daily average temperature (average of maximal and minimal temperature) and the 24-hour average temperature of each day respectively. The corresponding trading months for CDD and CAT contracts are April to October, for HDD October to April and for C24AT contracts all months of the year. Temperature derivatives are offered for 24

cities in the USA, 11 in Europe, six in Canada, three in Japan and three in Australia. The notional value of a temperature contract, according to the product specification, is 20 USD, 20 AUD, 20 EUR, 20 GBP or 2500 JPY per index point. In addition to monthly HDD, CAT and HDD futures and options, there are also HDD and CDD seasonal strips futures for multiple months. This study will focus only on monthly temperature future contracts.

2.1 Pricing Temperature Derivatives

The weather market is an example of an incomplete market, i.e. temperature cannot be hedged by other tradeable assets. However, the dynamics of temperature futures should be free of arbitrage. Therefore, a unique equivalent martingale measure does not exist and standard pricing approaches cannot be applied. We assume that a pricing measure $Q = Q_{\theta(t)}$ exists and can be parametrized via the Girsanov transform, where $\theta(t)$ denotes the market price of risk. Then the arbitrage free temperature futures price is:

$$F_{(t,\tau_1,\tau_2)} = \mathsf{E}^{Q_\theta}\left[Y_T\left\{T(t)\right\}|\mathcal{F}_t\right] \tag{1}$$

with $0 \leq t \leq T$. $Y_T\left\{T(t)\right\}$ refers to the payoff at $T > t$ from the (CAT/HDD/CDD) temperature index with measurement period $[\tau_1, \tau_2]$ and $\mathcal{F}_t$ refers to the filtration information set at time t.

The price of a put P_s or call option C_s written on temperature futures $F_{(t,\tau_1,\tau_2)}$ with strike K at exercise price K at exercise time $\tau < \tau_1$ is:

$$\begin{aligned} C_{(t,\tau_1,\tau_2)} &= \mathsf{E}^{Q_\theta}\left[\max\left\{F_{(t,\tau_1,\tau_2)} - K, 0\right\}\right] \\ P_{(t,\tau_1,\tau_2)} &= \mathsf{E}^{Q_\theta}\left[\max\left\{K - F_{(t,\tau_1,\tau_2)}, 0\right\}\right] \end{aligned} \tag{2}$$

Observe that although the payoff is not linked directly to the temperature but to a temperature index, one needs first to model the temperature dynamics $T(t)$ to solve Eq. (1).

2.1.1 Temperature dynamics in discrete time

Most of the models for daily average temperature discussed in the literature capture a linear trend and mean reversion with pronounced cyclical dynamics and strong correlations (long memory). Daily average temperature reflects not only a seasonal pattern from calendar effects (peaks in cooler winter and warmer summers) but also a variation that varies seasonally.

For a particular location, we propose the following model that captures seasonality effects in mean and variations, as well as inter-temporal correlations:

1. Let T_t be the average temperature in discrete time with $t = 1, \ldots M$. A conventional model for T_t is a model with linear trend and a seasonal pattern $T_t = \Lambda_t + X_t$.

2. Λ_t is a bounded and deterministic function denoting the seasonal effect and it is the mean reversion level of temperature at day t. The seasonality function might be

modelled by using the next least squares fitted seasonal function with trend:

$$\Lambda_t = a + bt + \sum_{k=1}^{K} c_k \cos\left\{\frac{2\pi(t-d_k)}{k\cdot 365}\right\} \tag{3}$$

where the coefficients a and b indicate the average temperature and global warming, urban heating effects or air pollution, Campbell and Diebold (2005). The series expansion in (3) with more and more periodic terms provides a fine tuning but this will increase the number of parameters. An alternative is modelling Λ_t by means of a local smoothing approach:

$$\arg\min_{e,f} \sum_{t=365}^{1} \left\{\bar{T}_s - e_s - f_s(t-s)\right\}^2 K\left(\frac{t-s}{h}\right) \tag{4}$$

where $\bar{T}_s$ is the mean of average daily temperature in J years and $K(\cdot)$ is a kernel. Asymptotically, they can be approximated by Fourier series estimators.

3. X_t is a stationary process $I(0)$ that can be checked by using the well known Augmented Dickey-Fuller test (ADF) or the KPSS Test. Empirical analysis of the Partial Autocorrelation Function (PACF) in Diebold and Inoue (2001), Granger and Hyung (2004) and Benth et al. (2011) reveal that the persistence (pronounced cyclical dynamics and strong intertemporal correlation) of daily average is captured by autoregressive processes of higher order AR(p):

$$X_{t+p} = \sum_{i=1}^{p} \beta_i X_{t-l} + \varepsilon_t,\ \varepsilon_t = \sigma_t e_t,\ e_t \sim \mathsf{N}(0,1) \tag{5}$$

The order of the appropriate AR(p) is chosen via the Box-Jenkins analysis and empirical evidence shows that a simple AR(3), suggested by Benth et al. (2007), holds for many cities and explained well the stylised facts of average daily temperature.

4. σ_t is a bounded and deterministic function, representing the smooth seasonal variation of daily average temperature at time t. This can be calibrated with the 2-step GARCH(1,1) model of Campbell and Diebold (2005) ($\hat{\sigma}^2_{t,FTSG}$):

$$\begin{aligned}\hat{\sigma}^2_{t,FTSG} = c_1 + \sum_{l=1}^{L}\left\{c_{2l}\cos\left(\frac{2l\pi t}{365}\right) + c_{2l+1}\sin\left(\frac{2l\pi t}{365}\right)\right\} \\ + \alpha_1(\sigma^2_{t-1}e_{t-1})^2 + \beta_1\sigma^2_{t-1}, e_t \sim \mathsf{N}(0,1)\end{aligned} \tag{6}$$

or via Local Linear Regression $\hat{\sigma}^2_{t,LLR}$:

$$\arg\min_{g,h} \sum_{t=1}^{365} \left\{\hat{\varepsilon}^2_t - g_s - h_s(t-s)\right\}^2 K\left(\frac{t-s}{h}\right) \tag{7}$$

with $K(\cdot)$ being a kernel.

2.1.2 Temperature dynamics in continuous time

Since pricing is done in continuous time, it is convenient to switch to modelling in continuous time. The literature in the last years has focused on the modelling and forecasting of time series trend, seasonal and noisy components, which are exactly the elements that characterize weather risk. Brody et al. (2002) suppose that the process T_t is modelled with a fractional Brownian Motion (FBM). However it is not a semi-martingale, which is a requirement to work under the incomplete market setting. Alaton et al. (2002) show that an Ornstein-Uhlenbeck Model driven by a Brownian motion is enough to capture the stylized facts of temperature. Benth et al. (2007) and Härdle and López-Cabrera (2011) demonstrate that the dynamics of temperature X_t in (5) can be approximated in continuous time with a Continuous-time AutoRegressive process of order p (CAR(p)) for $p \geq 1$:

$$d\mathbf{X}_t = \mathbf{A}\mathbf{X}_t dt + \mathbf{e}_p \sigma_t dB_t \tag{8}$$

where $\mathbf{e}_k$ denotes the k'th unit vector in $\mathbb{R}^p$ for $k = 1, ...p$, $\sigma_t > 0$ states the volatility, B_t is a Brownian motion and $\mathbf{A}$ is a $p \times p$-matrix:

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & & \ddots & & 0 & \vdots \\ 0 & \dots & \dots & 0 & 0 & 1 \\ -\alpha_p & -\alpha_{p-1} & \dots & & 0 & -\alpha_1 \end{pmatrix} \tag{9}$$

with positive constants α_k. The proof is by linking the states $X_{1(t)}, X_{2(t)}, \dots, X_{p(t)}$ with the lagged temperatures up to time $t - p$. Thus, for $p = 3$ and $dt = 1$ we get:

$$X_{1(t+3)} \approx (3 - \alpha_1)X_{1(t+2)} + (2\alpha_1 - \alpha_2 - 3)X_{1(t+1)} + (-\alpha_1 + \alpha_2 - \alpha_3 + 1)X_{1(t)} \tag{10}$$

2.1.3 Pricing temperature models

Several authors have dealt with the pricing problem. Davis (2001) models HDD indices $Y_T\{T(t)\}$ with a geometric Brownian motion and then price by utility maximization theory. Alaton et al. (2002) price WDs as in (1) but with a constant MPR. Benth (2003) derived no arbitrage prices of FBM using quasi-conditional expectations and fractional stochastic calculus. However, there is a discussion in the literature about the arbitrage opportunities of this model. Others like Benth and Saltyte-Benth (2005) assume that the process X_t follows a Lévy process, rather than a Brownian process, and get non-arbitrage prices under a martingale measures determined via the Esscher transform.

Following Benth et al. (2007), by considering the CAR(p) model (8) for the deseasonalised temperatures and by inserting the temperature indices (CAT/HDD/CDD) in (1), the risk neutral futures prices are:

$$F_{\mathrm{HDD}(t,\tau_1,\tau_2)} = \int_{\tau_1}^{\tau_2} \upsilon_{t,s} \psi \left[\frac{c - m_{\left\{t,s,\mathbf{e}_1^\top \exp\{\mathbf{A}(s-t)\}\mathbf{X}_t\right\}}}{\upsilon_{t,s}} \right] ds \tag{11}$$

$$F_{\mathrm{CDD}(t,\tau_1,\tau_2)} = \int_{\tau_1}^{\tau_2} \upsilon_{t,s} \psi \left[\frac{m_{\left\{t,s,\mathbf{e}_1^\top \exp\{\mathbf{A}(s-t)\}\mathbf{X}_t\right\}} - c}{\upsilon_{t,s}} \right] ds \tag{12}$$

$$F_{\mathrm{CAT}(t,\tau_1,\tau_2)} = \int_{\tau_1}^{\tau_2} \Lambda_u du + \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{X}_t + \int_t^{\tau_1} \theta_u \sigma_u \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{e}_p du + \int_{\tau_1}^{\tau_2} \theta_u \sigma_u \mathbf{e}_1^\top \mathbf{A}^{-1}\left[\exp\left\{\mathbf{A}(\tau_2-u)\right\} - I_p\right]\mathbf{e}_p du \tag{13}$$

with $\mathbf{a}_{t,\tau_1,\tau_2} = \mathbf{e}_1^\top \mathbf{A}^{-1}\left[\exp\left\{\mathbf{A}(\tau_2-t)\right\} - \exp\left\{\mathbf{A}(\tau_1-t)\right\}\right]$, the $p\times p$ identity matrix I_p,

$$m_{\{t,s,x\}} = \Lambda_s + \int_t^s \sigma_u \theta_u \mathbf{e}_1^\top \exp\left\{\mathbf{A}(s-t)\right\}\mathbf{e}_p du + x,$$
$$\upsilon_{t,s}^2 = \int_t^s \sigma_u^2 \left[\mathbf{e}_1^\top \exp\left\{\mathbf{A}(s-t)\right\}\mathbf{e}_p\right]^2 du \tag{14}$$

and $\psi(x) = x\Phi(x) + \varphi(x)$ with $x = \mathbf{e}_1^\top \exp\left\{\mathbf{A}(s-t)\right\}\mathbf{X}_t$.

The explicit formulae for the CAT call option written on a CAT future with strike K at exercise time $\tau < \tau_1$ during the period $[\tau_1, \tau_2]$ is given by:

$$C_{\mathrm{CAT}(t,\tau,\tau_1,\tau_2)} = \exp\left\{-r(\tau-t)\right\} \times \left[\left(F_{\mathrm{CAT}(t,\tau_1,\tau_2)} - K\right)\Phi\left\{d\left(t,\tau,\tau_1,\tau_2\right)\right\} + \int_t^\tau \Sigma^2_{\mathrm{CAT}(s,\tau_1,\tau_2)} ds \phi\left\{d\left(t,\tau,\tau_1,\tau_2\right)\right\}\right] \tag{15}$$

where $d\left(t,\tau,\tau_1,\tau_2\right) = \frac{F_{\mathrm{CAT}(t,\tau_1,\tau_2)} - K}{\sqrt{\int_t^\tau \Sigma^2_{\mathrm{CAT}(s,\tau_1,\tau_2)} ds}}$ and $\Sigma_{\mathrm{CAT}(s,\tau_1,\tau_2)} = \sigma_t \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{e}_p$ and Φ denotes the standard normal cdf. The option can be perfectly hedged once the specification of the risk neutral probability Q^θ determines the complete market of futures and options. Then, the option price will be the unique cost of replication.

To replicate the call option with CAT futures, one should compute the number of CAT futures held in the portfolio, which is simply computed by the option's delta:

$$\Phi\left\{d\left(t,T,\tau_1,\tau_2\right)\right\} = \frac{\partial C_{\mathrm{CAT}(t,\tau,\tau_1,\tau_2)}}{\partial F_{\mathrm{CAT}(t,\tau_1,\tau_2)}} \tag{16}$$

The strategy holds close to zero CAT futures when the option is far out of the money, close to 1 otherwise.

2.1.4 Calibrating the implied Market Price of Risk

Note that the advantage of the latter pricing approach is that it provides a closed form solution for temperature futures. Hence, the calibration of the MPR θ_t from market data turns out to be an inverse problem. Härdle and López-Cabrera (2011) infer the MPR from temperature futures. From a parametric specification of the MPR, one checks consistency with different contracts every single date. One finds the MPR by fitting the data:

$$\arg\min_{\hat{\theta}} \Sigma_{i=1}^I \left(F_{(\theta,t,\tau_1^i,\tau_2^i)} - F_{(t,\tau_1^i,\tau_2^i)}\right)^2 \tag{17}$$

with $t \le \tau_1^i < \tau_2^i$, $i = 1, \cdots, I$ contracts, $F_{(\theta,t,\tau_1^i,\tau_2^i)}$ denote the observed market prices and $F_{(t,\tau_1^i,\tau_2^i)}$ are the model specified prices given in (11), (12) and (13).

2.1.5 Meteorological weather forecasts

Equation (1) prices temperature futures based on the filtration $\mathcal{F}_t$, which contains the historical temperature evolution until time t. Benth and Meyer-Brandis (2009) state that the main reason for the irregular pattern of the market price of risk is an inappropriate choice of $\mathcal{F}_t$. There is more information available in the market, such as forward-looking information. Hence, $\mathcal{F}_t$ may be enlarged to a filtration $\mathcal{G}_t$, which contains all relevant information available at time t.

Ritter et al. (2011) enlarge the filtration by adding meteorological forecast values up to k days in advance. These new filtrations are denoted by $\mathcal{G}_t^{\mathrm{MF}k}$ with $k = 0, 1, 2, \ldots$ being the number of days in advance where meteorological forecast data are available. It follows:

$$\mathcal{F}_t \subset \mathcal{G}_t^{\mathrm{MF0}} \subset \mathcal{G}_t^{\mathrm{MF1}} \subset \mathcal{G}_t^{\mathrm{MF2}} \subset \ldots \subset \mathcal{G}_t$$

In an extended model, these meteorological forecast values are added to the historical temperature data as if they were actually realized temperature observations. Then, a discrete-time temperature model (see Section 2.1.1) is fitted to the to the "future" extended time series. The orders K and L of the Fourier series of the seasonality and seasonal variance, see (3), (6), as well as the lag p of the autoregressive process (5) are set beforehand. All other parameters, however, are estimated newly for every day t, according to the data available on that day (historical temperatures up to day $t-1$, meteorological forecasts calculated on day t for the days t, $t+1$, ...). By using Monte Carlo simulation and the simplifying assumption of an $\mathrm{MPR} = 0$, theoretical futures prices with no meteorological forecast data (NMF) and theoretical prices including meteorological forecasts k days in advance (MFk) can be calculated:

$$\begin{aligned} \hat{F}^{\mathrm{NMF}}_{(t;\tau_1,\tau_2)} &= \mathsf{E}[Y_T(T(t))|\mathcal{F}_t], \\ \hat{F}^{\mathrm{MF}k}_{(t;\tau_1,\tau_2)} &= \mathsf{E}[Y_T(T(t))|\mathcal{G}_t^{\mathrm{MF}k}] \end{aligned} \tag{18}$$

where $\mathsf{E}(\cdot)$ is the objective or physical risk measure. For every day t in the trading period, these theoretical prices can be calculated and then compared with the actual market prices to find out if the models using meteorological forecasts predict market prices better than the standard model.

3 Risk premium

Another way to think about future prices is in terms of Risk Premiums (RPs). RP effects are important in practice since issuers of weather contracts like to take advantages of geographic diversification, hedging effects and price determination. We adopt two ways for measuring the RP of weather risk. One is by looking at the risk factor under different pricing measures and the other one is by considering different filtrations.

3.1 Different pricing measures

The RPs in future markets are defined as the difference between the future prices computed with respect to the risk neutral measure and with respect to the objective measure,

Geman (2005):

$$\mathrm{RP}(t,\tau) = \mathsf{E}^{Q_\theta}\left[Y_T\left\{T(t)\right\}|\mathcal{F}_t\right] - \mathsf{E}\left[Y_T\left\{T(t)\right\}|\mathcal{F}_t\right] \tag{19}$$

The first term denotes the future price calculated from the risk neutral dynamics and the second one is calculated from the objective dynamics. In other words, the RP is defined as a drift of the temperature dynamics or a Girsanov type change of probability. Putting (13) in (19) we obtain an expression for the RP for CAT temperature derivatives:

$$\mathrm{RP}_{\mathrm{CAT}(t,\tau_1,\tau_2)} = \int_t^{\tau_1} \theta_u \sigma_u \mathbf{a}_{t,\tau_1,\tau_2} \mathbf{e}_p du + \int_{\tau_1}^{\tau_2} \theta_u \sigma_u \mathbf{e}_1^\top \mathbf{A}^{-1}\left[\exp\left\{\mathbf{A}(\tau_2 - u)\right\} - I_p\right] \mathbf{e}_p du$$

3.2 Information premium

In the previous section, the pricing measure was changed. Incorporating meteorological forecasts changes the filtration. To measure the influence the enlargement of the filtration has on the theoretical prices, Benth and Meyer-Brandis (2009) introduce the term "information premium (IP)". They define it as the difference between the theoretical prices calculated with and without using additional information such as weather forecasts:

$$\mathrm{IP}_t^{\mathcal{G}} = \hat{F}^{\mathcal{G}}_{(t;\tau_1,\tau_2)} - \hat{F}^{\mathcal{F}}_{(t;\tau_1,\tau_2)} = \mathsf{E}\left[Y_T\left\{T(t)\right\}|\mathcal{G}_t\right] - \mathsf{E}\left[Y_T\left\{T(t)\right\}|\mathcal{F}_t\right]. \tag{20}$$

The IP measures how theoretical prices change over time when meteorological forecasts are considered. A non-zero information premium indicates that the meteorological forecasts differ on average from the predictions made by the temperature model without meteorological forecasts. The information premium is positive (negative) if the prices based on $\mathcal{G}_t$ are higher (lower) than those based on the filtration $\mathcal{F}_t$.

4 Empirical Analysis

4.1 Data

The temperature data used in this study for London and Rome are the daily average temperatures from 19730101 (yyyymmdd) to 20100201 and are provided by Bloomberg. To obtain years of equal length, February 29 is removed from the data.

Meteorological forecast data is derived from WeatherOnline. These data consist of point forecasts of the minimal and maximal temperatures for London from 0 to 13 days in advance, calculated every day between 20081229 and 20100201. The forecasts of the daily average temperature are calculated as the average of the forecasted minimal and maximal temperature.

The prices used in this study are the market prices of the London and Rome HDD and CAT futures contracts reported at CME as "last price" for every weekday in the trading period as well as the daily traded volume "last volume". The futures temperature data was extracted from Bloomberg (20020101-20100201). A detailed description of the HDD and CAT contracts for London can be found in Table 1.

London		Trading days	Traded volume	Days with vol>0	Payoff
Feb09	HDD	38/247	1430	11/12	366.0
Mar09	HDD	61/217	13800	18	300.0
Apr09	CAT	82/143	0	0	313.0
May09	CAT	102/249	200	4	441.0
Jun09	CAT	124/185	0	0	518.7
Jul09	CAT	145/206	250	4	570.0
Aug09	CAT	166/228	50	1	589.1
Sep09	CAT	187/249	0	0	487.1
Oct09	HDD	66/68	1270	5	160.4
Nov09	HDD	172/177	1650	1	241.3
Dec09	HDD	185/189	3250	10/11	429.5
Jan10	HDD	205/209	250	3	493.5

Table 1: Futures contracts for London used in this study overlapping with the period of the meteorological forecast data; the number of trading days, the traded volume (number of cleared trades), the number of days with volume>0 and the payoffs (in index points) are shown. If two numbers are depicted, this indicates that less data than available were used because of missing meteorological forecast data.

4.2 Results

We first conduct an empirical analysis of the average daily temperature data for London and Rome. Figure 1 displays the seasonality Λ_t modelled with Fourier truncated series and the Local linear regression. The latter estimator smooths the seasonal curve and captures peak seasons. The inter-correlations of the detrended temperature are well modelled with a simple autoregressive model of order $p = 3$. However, there is still seasonality remained in the residuals, as the ACFs of detrended (squared) residuals show in Figure 2. The empirical FTSG and LLR seasonal variations are displayed in Figure 3, which reveal high variations for both cities in winter times. After removing the seasonal variation of the residuals (corrected residuals), the ACFs of (squared) residuals in Figure 4 are close to zero indicating that we sufficiently reduced the seasonal effect. The result is are displayed with the log of a normal density in Figure 5 (adequate for the Ornstein-Uhlenbeck pricing discussed in Section 2.1.3). The descriptive statistics given in Table 2 indicate the goodness of fit of the Local Linear (LLR) over the Fourier Truncated Series-GARCH (FTS-GARCH) estimator.

4.2.1 Implied Market Price of Risk

In Rome and London, HDD futures are traded from Nov-April (i.e. $i = 7$ calendar months) and CAT futures from April-Nov ($i = 7$). Our results for the implied MPR are given in Table 3 and 4. Table 3 presents the descriptive statistics of different MPR specifications for London-CAT and Rome-CAT daily futures contracts traded before measurement period $t \leq \tau_1^i < \tau_2^i$ during 20031006-20101118 (6247 contracts in 1335 trading dates and 38 measurement periods) and 20050617-20090731 (2976 contracts corresponding to 891 trading dates and 22 measurement periods) respectively. The ranges for the MPR specifications values of London-CAT and Rome-CAT futures are [-69.13,43.93] and

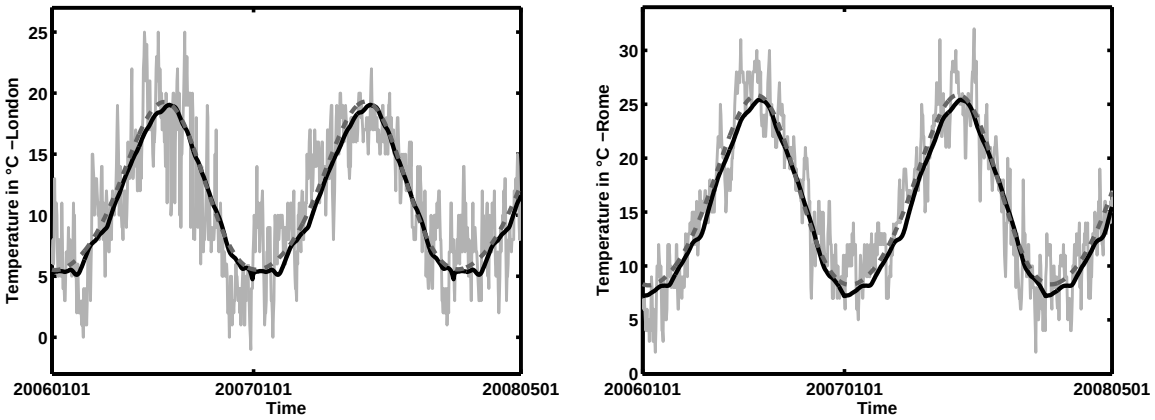

Figure 1: An stretch of eight years plot of the average daily temperatures (gray line), the seasonal component modelled with a Fourier truncated series (dashed line) and the local linear regression (black line) using Epanechnikov Kernel.

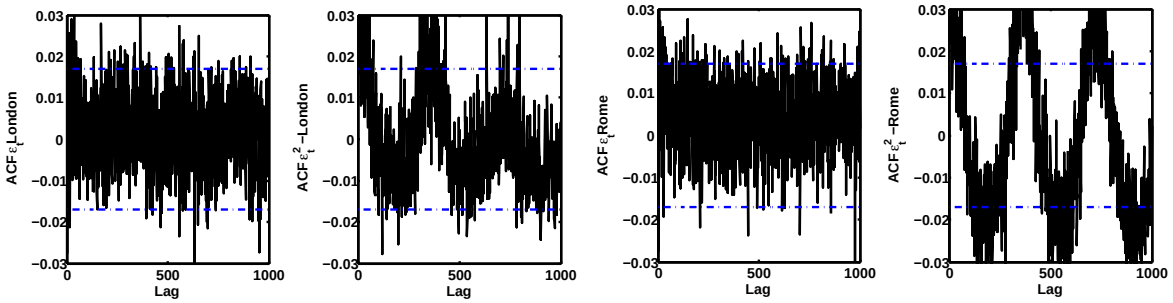

Figure 2: The ACF of Residuals of daily temperatures ε_t (left panels) and Squared residuals ε_t^2 (right panels) of detrend daily temperatures for London (left) and Rome (right).

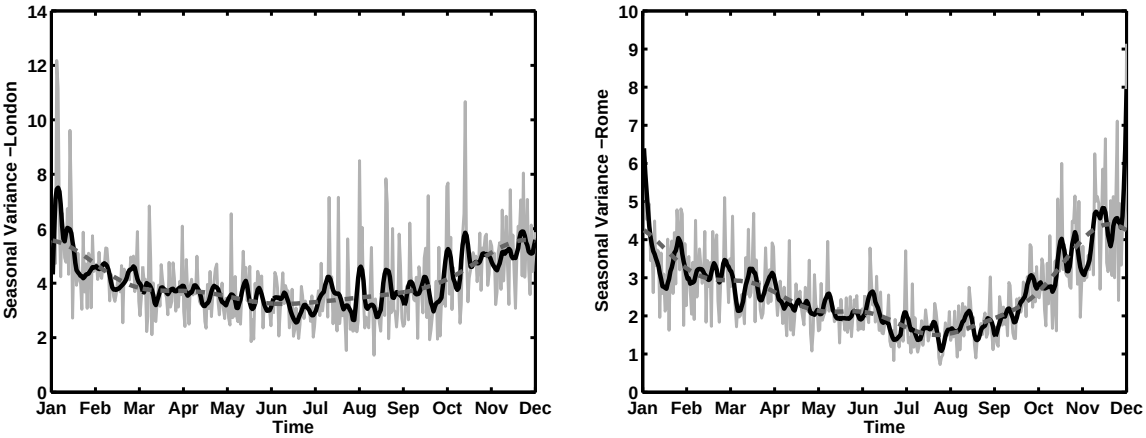

Figure 3: The daily empirical variance (black line), the Fourier truncated (dashed line) and the local linear smoother seasonal variation using Epanechnikov kernel (gray line) for London (left) and Rome (right)

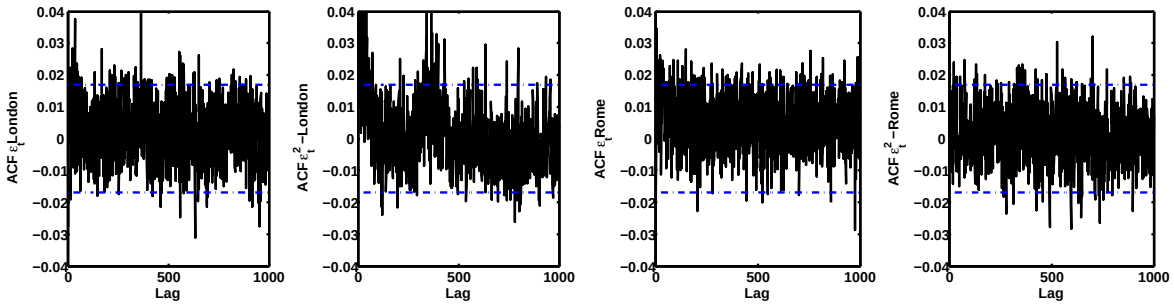

Figure 4: The ACF of Residuals e_t (left panels) and Squared residuals e_t^2 (right panels) of detrended daily temperatures after dividing out the local linear seasonal variance for London (left) and Rome (right).

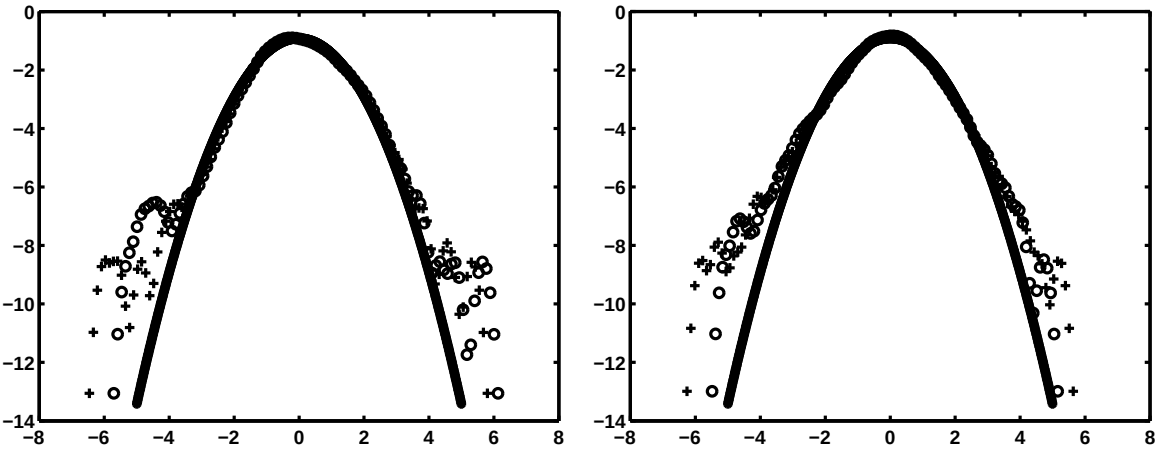

Figure 5: The log of Normal Kernel (*) and log of Kernel smoothing density estimate of residuals after correcting FTS (+) and local linear (o) seasonal variance for London (left) and Rome (right).

		London	Rome
Period		19730101-20091019	19730101-20091019
Seasonality	$\hat{a}$(CI)	10.75(10.62,10.89)	14.74(14.63,14.86)
	$\hat{b}$(CI)	0.0001(0.00005,0.00009)	0.0001(0.00010,0.00013)
	$\hat{c}_1$(CI)	7.88(7.87,7.89)	8.81(8.80,8.82)
	$\hat{d}_1$(CI)	-157.27(157.26,157.28)	-154.24(154.23,154.25)
ADF	$\hat{\tau}$	-33.41*	-37.62*
KPSS	$\hat{k}$	0.17***	0.06***
AR(3)	β_1	0.75	0.82
	β_2	-0.07	-0.08
	β_3	0.04	0.03
CAR(3)	α_1	-2.24	-2,17
	α_2	-1.55	-1.44
	α_3	-0.26	-0.22
	λ_1	-0.25	-0.22
	$\lambda_{2,3}$	-0.99	-0.97
Coefficients of the FTS	$\hat{c}_1$	4.02	2.64
	$\hat{c}_2$	0.94	1.07
	$\hat{c}_3$	-0.07	0.21
	$\hat{c}_4$	0.34	0.35
	$\hat{c}_5$	-0.11	-0.25
	$\hat{c}_6$	0.21	0.07
	$\hat{c}_7$	-0.06	-0.14
	$\hat{c}_8$	0.04	0.11
	$\hat{c}_9$	0.01	-0.12
$\frac{\hat{e}_t}{\hat{\sigma}_t}$ with FTS	JB	190.60	637.26
	Kurt	3.50	4.04
	Skew	0.14	-0.10
$\frac{\hat{e}_t}{\hat{\sigma}_t}$ with LLR	JB	274.05	461.51
	Kurt	3.67	3.88
	Skew	0.09	-0.11

Table 2: Coefficients of the Fourier Truncated Seasonal series (FTS), ADF and KPSS-Statistics, the autoregressive process AR(3), continuous autoregressive model CAR(3), eigenvalues $\lambda_{1,2,3}$, of the matrix $\mathbf{A}$ of the CAR(3) model, seasonal variance $\{c_i\}_{i=1}^{9}$ fitted with a FTS, Skewness (Skew), kurtosis (Kurt), Jarque Bera (JB) test statistics of the corrected residuals with seasonal variances fitted with FTS-GARCH and with local linear regression (LLR) for Rome and London. Confidence Intervals (CI) are given in parenthesis. Dates given in yyymmdd format. Coefficients are significant at 1% level. +0.01 critical values, * 0.1 critical value, **0.05 critical value, ***0.01 critical value.

[-64.55,284.99], whereas the MPR averages are $(0.06, 0.0232)$ for constant MPR for different contracts, $(0.66, -0.23)$ for one piecewise constant, $(0.05, -0.31)$ for two piecewise constant, $(0.06, 0.02)$ for spline and $(0.08, 0.00)$ when bootstrapping the MPR.

We conduct the Wald statistical test to check whether the MPR derived from CAT/HDD futures is different from zero. We reject $H_0 : \hat{\theta}_t = 0$ under the Wald statistic that the MPR is different from zero for Rome and London-CAT futures, see Table 3, it changes over time and changes signs. These results suggest us that the weather market offers the possibility to have different risk adjustments for different times of the year.

Table 4 describes the root mean squared errors (RMSE) of the differences between market prices and the estimated futures prices with implied MPR values. Similar to Härdle and López-Cabrera (2011), the RMSE estimates in the case of the constant MPR for different CAT futures contracts are statistically significant enough to know CAT futures prices. When the MPR is equal to zero, we speak about the existence of additional risk premium revealing the evidence of buyers willing to pay for price protection.

4.2.2 Meteorological forecasts

Section 2.1.5 argues that additional forward-looking information should be included in the pricing model. First, we compare the meteorological forecast data for 2009 and predictions from the statistical model without any meteorological forecast data with the realized temperatures in London in 2009. Figure 6 depicts the deviation in dependence of the number of days in advance the forecasts were calculated. The short-term meteorological forecasts clearly outperform those from the statistical model. The longer the forecast horizon gets, however, the smaller the difference becomes, and for more than 10 days ahead, the meteorological forecasts get worse than the statistical model. This supports the assumption that meteorological forecasts contain additional information which can be used for pricing weather derivatives.

The extended model from Section 2.1.5 computes theoretical prices for every contract on every day t based on different filtrations, from not using any meteorological forecasts to using forecasts 13 days in advance. As an example, Figure 7 shows the results for an HDD contract for December 2009 with reference station London. This contract is offered starting April 2009, but all prices remain constant for a long period. The theoretical prices with and without meteorological forecasts equal as the accumulation period is too far away for an influence of the forecasts on the expected temperature. This changes in the last two months where there are higher fluctuations in all prices and bigger differences between the theoretical prices. In this example, the theoretical prices with meteorological forecasts seem to predict the market prices much better than the theoretical price without using any forecast data.

The IP defined in (20) measures the influence the additional information has on the theoretical prices. Figure 8 shows the IP for the same example as above, the London HDD contract for December 2009. It can be seen that it is zero for a long time, but then it fluctuates and changes its sign several times. Figure 8 also depicts the average IP for all twelve contracts used in this study in absolute value. As expected, the meteorological forecasts have the biggest influence in the last two months before maturity.

The RP (19) describes the difference between the prices under the risk-neutral measure

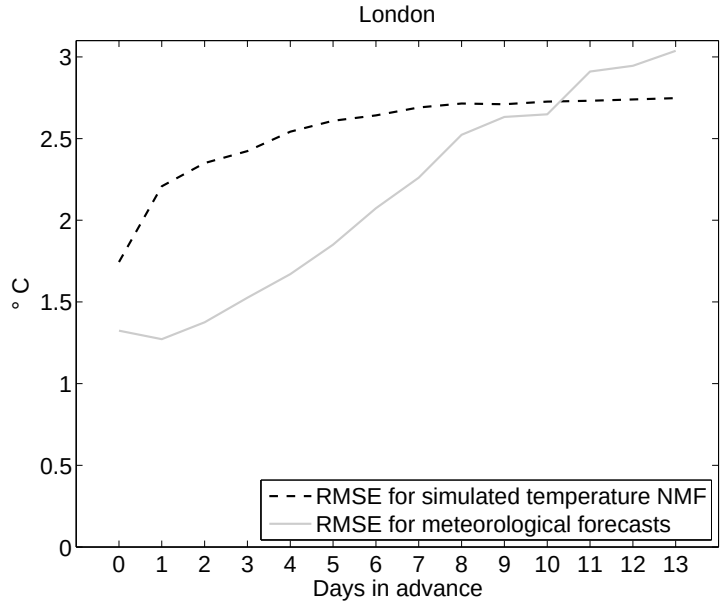

Figure 6: RMSE of the meteorological forecasts and the statistical model (NMF) compared with the observed temperature in London in 2009

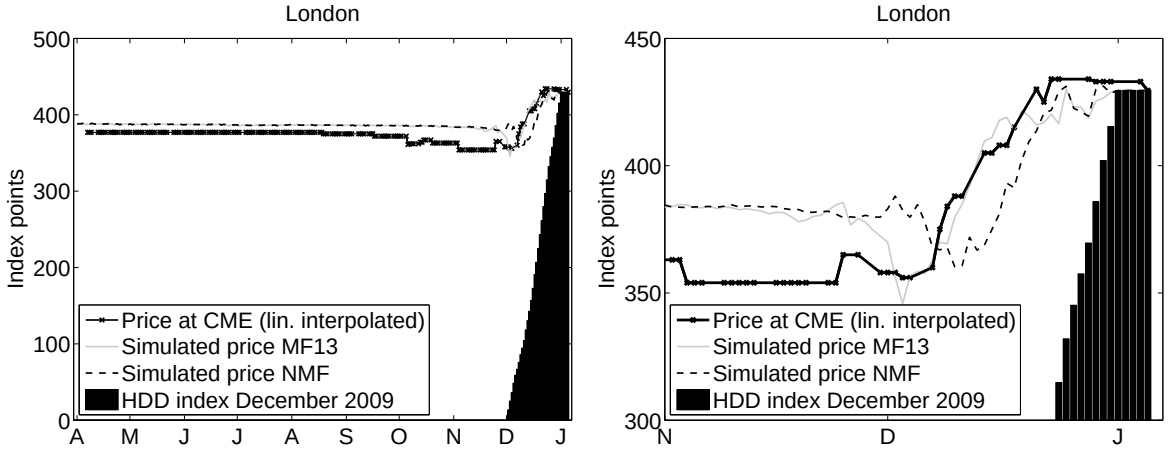

Figure 7: Observed and simulated prices of an HDD contract for December, 2009 in London

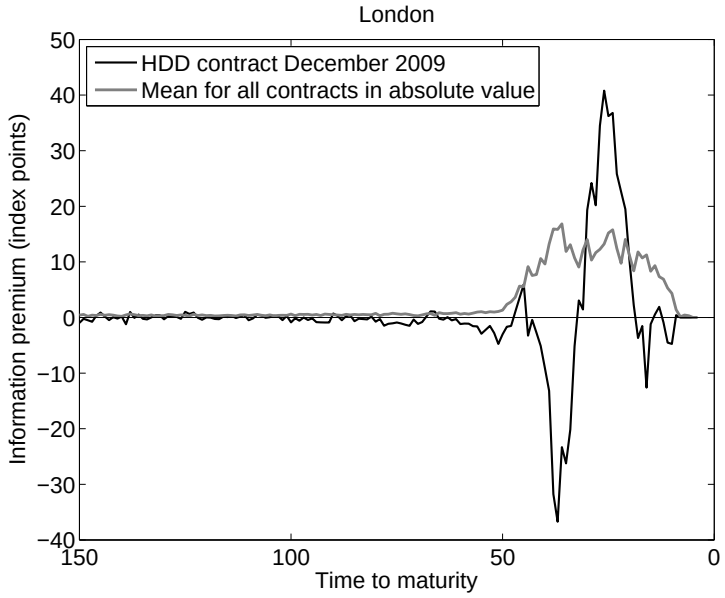

Figure 8: Information premium for the HDD contract for December 2009 in London and the mean in absolute value for all twelve London contracts

and the physical measure. Hence, it can be calculated as the difference between the observed market prices and the theoretical prices with an MPR = 0. Figure 9 shows the RP for the HDD contract for December 2009 with reference station London, where the theoretical prices are calculated with and without using meteorological forecasts. The RP stays almost constant and is equal for both filtrations for the major part of the trading period. When approaching the measurement period, however, the RP with meteorological forecasts differs and is fluctuating closer around zero. This means that the RP declines in absolute value when incorporating meteorological forecasts. Similar results are obtained when depicting the average RP for all contracts in absolute value (Figure 9). For the major part of the trading period, there is no difference between the models. In the last two months, however, the RP is generally lower in absolute value with meteorological forecasts. Consequently, enlarging the filtration helps to better control the RP.

To compare the difference between the models, the RMSE between the theoretical and the observed market prices is calculated for every model and every contract separately. The results in Table 5 show that the error decreases for most of the contracts if a model with meteorological forecasts is used. The mean of the RMSE decreases from 19.1 to about 18 index points when meteorological forecasts are used. On average, the prediction of the market prices with forecasts is much better for the winter months with the HDD contracts. The normalized RMSE (i.e. the RMSE for the model without forecasts is set to 1) is shown in Figure 10.

So far, the RMSE was calculated for the whole trading period of each contract. The results of the information premium, however, show that the influence of the forecasts on the theoretical prices is almost zero for a long time of the trading period and increases significantly for the last two months until maturity. The RMSE restricted on the last two months of the trading period of each contract shows that the meteorological forecasts stronger influence the pricing in that period (see Figure 10).

Although market prices are reported by the CME for every weekday in the trading period,

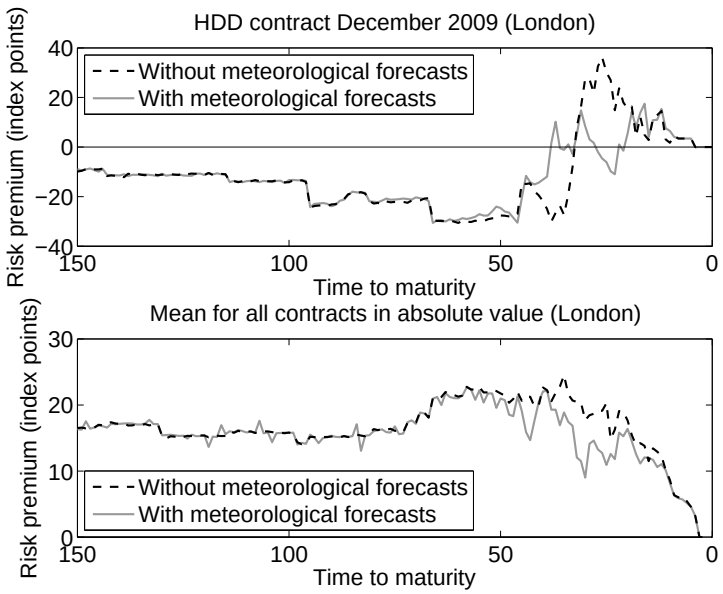

Figure 9: Risk premium for the HDD contract for December 2009 (top) and the mean in absolute value for all twelve London contracts (bottom) without and with including meteorological forecasts (MF13)

actual trading takes place only on a few days in the trading period (compare Table 1). Only if the contract was actually traded on that day, however, the reported price is a real market price and can be assumed to capture all relevant information. The RMSE restricted on those days where the trading volume is larger than zero is also shown in Figure 10. It shows a clear decline of up to 25 % for those models, where meteorological forecasts are included.

All graphs in Figure 10 have in common that they decrease in the beginning, but turn upwards in the end. This means that including all forecast data into the pricing is worse than using just forecasts a few days ahead. A possible reason could be that the market participants are aware of the unreliability of long-term forecasts, which could also be seen in Figure 6.

5 Conclusions

In this paper, we examine the RPs of weather derivatives. Two ways for measuring these RPs are proposed: one is by studying the stochastic behaviour of the temperature underlying under different risk pricing measures and the second one is by using different filtration information sets. The latter IP approach is incorporating weather forecast into the pricing model.

We conduct empirical data analysis for Rome and London temperature futures traded at the CME. The goal is to determine the nature of the risk factor embedded in temperature option and future prices. We find that the seasonal variance of temperature explains a significant proportion of the RP variation. In both approaches, the RPs and IPs of futures contracs are different from zero, negative or positive.

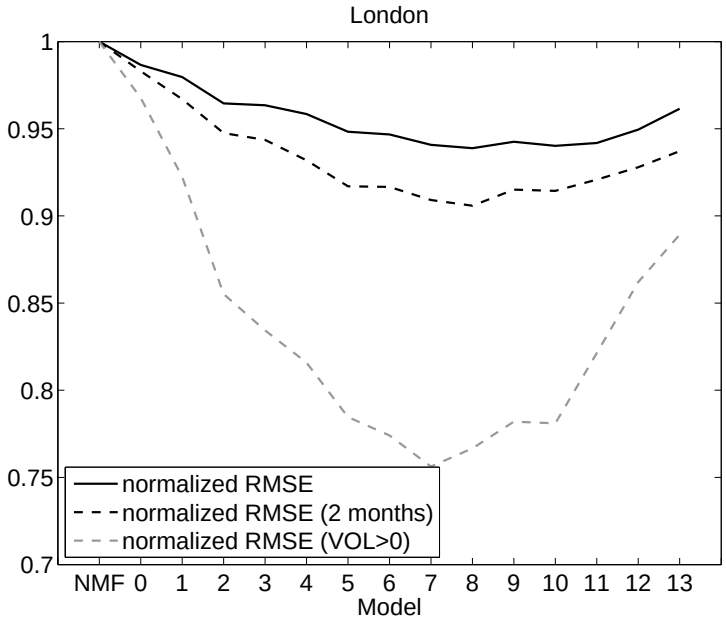

Figure 10: Average nRMSE (whole period, last 2 months, days with volume>0) for London for different models (NMF, MF0–MF13)

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Table 3: Statistics of MPR specifications for London-CAT, Rome-CAT futures contracts traded during 20031006-20090529 (6247 observations corresponding to 1335 trading dates and 38 measurment periods), (20050617-20090731) respectively with trading date before measurement period $t \leq \tau_1^i < \tau_2^i, i = 1 \ldots I$ (where $i = 1$ (30 days), $i = 2$ (60 days),.., $i = I$ (210 days)): the Wald statistics (WS), the WS probabilities (Prob), Minimum (Min), Maximum (Max), Median (Med) and Standard deviation (Std). MPR specifications: Constant for different contracts per trading date (Constant), one piecewise constant, 2 piecewise constant ($\xi = 150$ days), Bootstrap and Spline.

Type	Nr. contracts	Statistic	Constant	1 piecewise	2 piecewise	Bootstrap	Spline
London-CAT		WS(Prob)	0.44(0.49)	0.12(0.27)	0.00(0.02)	0.44(0.49)	1.32(0.75)
30days	589	Min(Max)	-0.49(0.54)	-4.52(4.43)	-4.52(4.48)	-0.49(0.54)	0.02(0.23)
(i=1)		Med(Std)	0.05(0.16)	0.12(1.41)	0.12(1.64)	0.05(0.16)	0.15(0.06)
60days	1215	Min(Max)	-1.70(0.86)	-10.92(16.83)	-69.13(43.93)	-1.70(0.86)	-0.00(0.03)
(i=2)		Med(Std)	0.07(0.19)	0.28(2.02)ï¿½	0.13(5.71)	-0.07(5.03))	0.00(0.01)
90days	1168	Min(Max)	-0.40(0.12)	-20.63(27.39)	-1.42(0.12)	-0.40(0.12)	0.01(0.23)
(i=3)		Med(Std)	0.02(0.06)	-0.11(29.97)	-0.00(0.08)	0.07(0.19)	0.06(0.06)
120days	979	Min(Max)	-2.34(0.85)	-10.92(16.83)	-69.13(43.93)	-2.34(0.85)	0.00(0.23)
(i=4)		Med(Std)	0.07(0.22)	0.29(2.11)	0.14(5.84)	0.07(80.22)	0.13(0.07)
150days	876	Min(Max)	-2.89(0.84)	-10.92(16.83)	-18.26(36.78)	-2.89(0.84)	0.01(0.23)
(i=5)		Med(Std)	0.06(0.32)	0.48(2.14)	0.47(3.71)	0.06(0.32)	0.13(0.09)
180days	815	Min(Max)	-0.61(0.86)	-4.52(11.84)	-65.95(36.78)	-0.61(0.86)	-0.00(0.22)
(i=6)		Med(Std)	0.14(0.09)	0.52(1.76)	0.44(4.60)	0.14(0.09)	0.02(0.08)
210days	605	Min(Max)	-0.61(0.84)	-2.39(11.84)	-63.12(36.78)	-0.61(0.84)	-0.02(0.03)
(i=7)		Med(Std)	0.06(0.08)	0.84(1.55)	0.12(3.26)	0.06(0.08)	-0.01(0.01)
Rome-CAT		WS(Prob)	0.06(0.20)	0.00(0.01)	0.00(0.02)	0.00(0.06)	0.28(0.04)
30days	281	Min(Max)	-1.66(0.89)	-5.86(11.74)	-7.90(11.74)	-1.66(0.89)	0.00(0.01)
(i=1)		Med(Std)	0.02(0.76)	-0.66(2.52)	-0.66(2.88)	0.02(0.76)	0.00(0.02)
60days	583	Min(Max)	-2.38(1.39)	-64.55(284.99)	-64.55(284.99)	-2.38(1.39)	0.02(0.01)
(i=2)		Med(Std)	0.11(0.30)	-0.06(13.66)	-0.06(13.86)	0.11(0.30)	0.00(0.02)
90days	641	Min(Max)	-3.20(1.07)	-64.55(284.99)	-64.55(284.99)	-3.20(1.07)	0.00(0.43)
(i=3)		Med(Std)	0.17(0.36)	0.29(13.11)	0.18(13.25)	0.17(0.36)	0.00 (0.05)
120days	476	Min(Max)	-3.40(1.09)	-11.32(45.78)	-11.32(3.33)	-3.40(1.09)	0.00(0.00)
(i=4)		Med(Std)	0.05(0.42)	0.50(3.67)	0.53(2.90)	0.05(0.42)	0.00(0.00)
150days	413	Min(Max)	-0.59(0.93)	-19.96(3.03)	-19.96(56.90)	-0.59(0.93)	0.01(0.02)
(i=5)		Med(Std)	0.08(0.08)	0.69(2.74)	0.71(4.30)	0.08(0.08)	0.00(0.02)
180days	373	Min(Max)	-0.95(0.18)	-19.96(3.03)	-19.96(56.90)	-0.95(0.18)	0.01(0.02)
(i=6)		Med(Std)	0.01(0.07)	0.91(2.96)	0.71(4.37)	0.01(0.07)	0.00(0.01)
210days	208	Min(Max)	-0.03(1.07)	-19.96(3.03)	-19.96(3.03)	-0.03(1.07)	0.02(0.01)
(i=7)		Med(Std)	0.17(0.10)	0.91(1.64)	0.91(1.68)	0.17(0.10)	0.00(0.00)

Table 4: Root mean squared error (RMSE) of the differences between observed CAT/HDD/CDD futures prices with $t \leq \tau_1^i < \tau_2^i$ and the estimated futures with extracted MPR from different MPR parametrizations (MPR=0, constant MPR for different contracts (Constant), 1 piecewise constant MPR, 2 piecewise constant MPR , bootstrap MPR and spline MPR). Computations with MPR implied directly from specific futures contract types (+) and through the parity HDD/CDD/CAT parity method(*).

Contract type	Measurement Period			RMSE between estimated with MPR (θ_t) and CME prices					
	τ_1	τ_2	No. contracts	MPR=0	Constant	1 piecewise	2 piecewise	Bootstrap	Spline
London-CAT+	20080501	20080531	22	28.39	10.37	196.09	196.09	10.37	27.67
London-CAT+	20080601	20080630	43	5.51	27.93	102.23	102.23	27.93	4.91
London-CAT+	20080701	20080731	64	12.85	61.41	688.99	688.99	61.41	12.44
London-CAT+	20080801	20080831	86	29.94	4.72	99.59	99.59	0.00	29.76
London-CAT+	20080901	20080930	107	41.57	45.97	646.49	646.49	45.97	41.18
London-CAT+	20090401	20090430	120	73.59	77.83	156.44	156.44	77.83	73.75
London-CAT+	20090501	20090531	141	93.51	96.77	96.74	96.747	96.77	93.60
London-CAT+	20090601	20090630	161	100.32	103.56	103.98	103.56	103.56	101.31
Rome-CAT+	20080501	20080531	22	19.10	8.92	103.55	103.55	8.92	18.58
Rome-CAT+	20080601	20080630	43	26.88	16.13	141.82	141.82	16.13	26.18
Rome-CAT+	20080701	20080731	64	13.46	17.27	324.18	324.18	17.27	13.22
Rome-CAT+	20080801	20080831	86	23.66	36.21	761.63	761.63	36.21	23.26
Rome-CAT+	20080901	20080930	107	18.53	45.43	718.83	718.83	45.43	18.51
Rome-CAT+	20090401	20090430	120	97.99	127.62	575.88	575.88	127.62	98.31
Rome-CAT+	20090501	20090531	141	117.13	121.90	117.17	117.17	121.91	117.49
Rome-CAT+	20090601	20090630	141	117.07	120.21	102.34	123.49	120.21	112.95

RMSE	Model														
	NMF	MF0	MF1	MF2	MF3	MF4	MF5	MF6	MF7	MF8	MF9	MF10	MF11	MF12	MF13
Feb09	27.9	26.9	26.1	23.7	23.6	23.7	22.1	21.2	20.9	20.0	20.3	19.4	18.4	18.6	18.9
Mar09	6.7	6.1	6.2	5.8	5.5	5.1	5.5	6.3	6.5	6.5	6.9	6.7	7.7	9.0	10.7
Apr09	16.2	16.1	16.7	16.7	16.8	17.0	17.1	17.1	16.9	17.0	17.2	17.4	17.6	18.0	18.6
May09	17.3	17.4	17.1	17.1	17.2	17.2	17.1	17.0	17.0	16.8	16.9	16.7	16.9	17.0	17.0
Jun09	14.8	15.0	15.3	15.6	16.0	16.0	16.1	16.1	16.3	16.5	16.5	16.6	16.5	16.7	16.7
Jul09	14.7	15.0	15.1	15.2	15.2	15.3	15.1	15.0	15.4	15.6	15.7	15.8	16.2	16.4	16.6
Aug09	18.4	18.5	18.4	18.3	18.5	18.5	18.3	18.3	18.3	18.4	18.6	18.6	18.8	18.8	19.1
Sep09	17.1	17.0	17.1	17.3	17.0	16.8	16.6	16.4	16.3	16.2	16.1	15.7	15.5	15.5	15.6
Oct09	35.6	34.3	33.8	33.6	33.7	33.2	32.7	32.8	31.8	32.2	31.9	32.3	31.8	31.3	31.3
Nov09	28.3	28.1	27.6	27.3	27.1	26.9	26.9	26.9	26.6	26.3	26.2	26.2	26.1	25.9	25.9
Dec09	15.8	15.4	15.1	14.8	14.7	14.5	14.3	14.2	13.9	13.8	13.7	13.8	13.9	13.8	13.8
Jan10	16.2	15.9	15.7	15.5	15.3	15.3	15.4	15.4	15.6	15.7	15.8	16.0	16.1	16.1	16.0
Mean	19.1	18.8	18.7	18.4	18.4	18.3	18.1	18.1	17.9	17.9	18.0	17.9	18.0	18.1	18.3
Mean HDD	21.8	21.1	20.8	20.1	20.0	19.8	19.5	19.5	19.2	19.1	19.1	19.1	19.0	19.1	19.4
Mean CAT	16.4	16.5	16.6	16.7	16.8	16.8	16.7	16.6	16.7	16.7	16.8	16.8	16.9	17.1	17.3

Table 5: RMSE in index points for monthly contracts and different models for London (whole trading period)

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