

Strohsal, Till; Weber, Enzo

**Working Paper**

## Mean-variance cointegration and the expectations hypothesis

SFB 649 Discussion Paper, No. 2011-007

**Provided in Cooperation with:**

Collaborative Research Center 649: Economic Risk, Humboldt University Berlin

*Suggested Citation:* Strohsal, Till; Weber, Enzo (2011) : Mean-variance cointegration and the expectations hypothesis, SFB 649 Discussion Paper, No. 2011-007, Humboldt University of Berlin, Collaborative Research Center 649 - Economic Risk, Berlin

This Version is available at:

<https://hdl.handle.net/10419/56672>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Mean-Variance Cointegration and the Expectations Hypothesis

Till Strohsal\*  
Enzo Weber\*\*

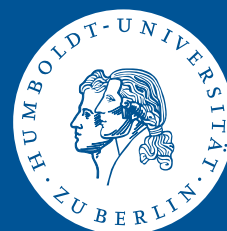


\* Freie Universität Berlin, Germany  
\*\* Universität Regensburg, Germany

This research was supported by the Deutsche  
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

<http://sfb649.wiwi.hu-berlin.de>  
ISSN 1860-5664

SFB 649, Humboldt-Universität zu Berlin  
Spandauer Straße 1, D-10178 Berlin



# Mean-Variance Cointegration and the Expectations Hypothesis<sup>1</sup>

Till Strohsal<sup>2</sup>

*Department of Economics  
Free University Berlin*

Enzo Weber<sup>3</sup>

*Department of Economics and Econometrics  
University of Regensburg*

## Abstract

The present paper sheds further light on a well-known (alleged) violation of the expectations hypothesis of the term structure (EHT) - the frequent finding of unit roots in interest rate spreads. We show that the EHT implies (i) that the nonstationarity stems from the holding premium, which is hence (ii) cointegrated with the spread. In a stochastic discount factor framework we model the premium as being driven by the integrated variance of excess returns. Introducing the concept of mean-variance cointegration we actually find cointegration relations between spreads and premia in US data.

*Keywords:* Expectations Hypothesis, Holding Premium, Persistence, Cointegration, GARCH

*JEL classification:* E43, C32

---

<sup>1</sup>This research was supported by the Deutsche Forschungsgemeinschaft through the CRC 649 "Economic Risk". We are grateful to participants of the Econometric Seminar at the University of Regensburg, the Macroeconometric Workshop 2010 at the DIW Berlin and the 11<sup>th</sup> IWH-CIREQ Macroeconometric Workshop at the IWH Halle for their comments. Of course, all remaining errors are our own.

<sup>2</sup>Free University Berlin, Department of Economics, Institute of Statistics & Econometrics, D-14195 Berlin, Germany, till.strohsal@wiwiss.fu-berlin.de, phone: +49 (0)30 838-53399.

<sup>3</sup>University of Regensburg, Department of Economics and Econometrics, D-93040 Regensburg, Germany, enzo.weber@wiwi.uni-regensburg.de, phone: +49 (0)941 943-1952, fax: +49 (0)941 943-4917, and Institute for Employment Research (IAB), Osteuropa-Institut Regensburg.

# 1 Introduction

The relation between interest rates of different maturities plays a key role in macroeconomics and finance. For monetary policy the transmission mechanism from short rates to long rates is of particular importance. An obvious and plausible approach is given by the expectations hypothesis of the term structure (EHT) which remains both one of the most examined and rejected theories.<sup>4</sup> The present paper focuses on the common implication of the EHT that interest rate spreads should be stationary and provides an explanation why this property is so often not found in empirical studies. We show that this notorious lack of evidence can be attributed to a nonstationary term premium modeled by means of a stochastic discount factor model. Using our newly introduced *mean-variance cointegration* test this explanation is verified by econometric results from unit root and cointegration analysis.

The implication of stationary spreads was first shown by CAMPBELL and SHILLER (1987). A popular linearized version of the EHT states that the spread equals expected future short rate changes plus a *constant term premium*,  $\theta$ . Considering the two-period case it is easy to see that for interest rates integrated of order one ( $I(1)$ ), stationarity of the right-hand side in  $Y_t^{(2)} - Y_t^{(1)} = 1/2 E[\Delta Y_{t+1}^{(1)} | I_t] + \theta$  goes hand in hand with cointegration on the left-hand side. Here,  $Y_t^{(n)}$  denotes the yield on an  $n$ -period bond and  $E[\cdot | I_t]$  is the expectations operator conditioning on the information available up to time  $t$ ,  $I_t$ .

However, much evidence contradicts the implication of mean-reverting spreads. Among many others, HALL et al. (1992), PAGAN et al. (1996) and HANSEN (2003), find that stationarity of the spreads is often not reflected in US data. The larger the difference in maturity the more often this outcome occurs. WOLTERS (1995, 1998) and CARSTENSEN (2003) come to the same result for German bond data. A number of authors argued that the assumption of a constant term premium may be inappropriate. Evidence for a time-varying premium is provided by MANKIW and MIRON (1986), ENGLE et al. (1990) and EVANS and LEWIS (1994), to name just a few. However, the term premium is unobservable and the EHT does not provide any guidance of how such a time-varying premium should be modeled.

By now, a great deal of literature has been produced that is concerned with the question of what exactly drives the commonly accepted time-variation in the term premium. One way of summing up the ongoing academic effort is to classify the different approaches

---

<sup>4</sup>Comprehensive surveys covering early work are provided by MELINO (1988), SHILLER (1990) and CAMPBELL and SHILLER (1991).

within the broad class of stochastic discount factor (SDF) models. Detailed discussion of SDF models is provided by COCHRANE (2001) and BALFOUSSIA and WICKENS (2007). In essence, assets are priced as the expected discounted value of their future pay-offs. Yet, we emphasize that a time-varying but *stationary* premium that may be modeled by any particular SDF model does not change the EHT implication of stationary spreads.

The present paper argues that violation of this implication can only be reconciled with the EHT if integrated spreads come along with *integrated* term premia (Hypothesis i). In that case the nonstationarity puzzle would be rationalized if *spreads* and *premia* were cointegrated; that is what we label *mean-variance cointegration* (Hypothesis ii).

In our analysis we apply the most simple observable one-factor SDF model that is able to describe such an extremely persistent premium: the Sharpe-Lintner CAPM. The term premium is specified as the product of risk and its market price, equalling the expected excess return. Thereby, the conditional second moment of excess returns serves as risk measure.<sup>5</sup> We estimate the term premium via a generalized autoregressive conditional heteroskedasticity (GARCH) model (ENGLE 1982, BOLLERSLEV 1986) and show that the null of integrated conditional variance cannot be rejected. This result survives the inclusion of structural breaks under the alternative hypothesis. Finally, we propose a cointegration test and simulate the appropriate distribution of the test statistic. Empirically, we actually find cointegration relations between premia and spreads in US interest rate data. This explains the (seeming) violation of the necessary condition for the EHT to be valid - the frequent finding of nonstationary spreads.

The paper proceeds as follows. Section 2 discusses stochastic discount factor models for term premia, looks at the EHT and derives two testable hypotheses. In section 3 we introduce the econometric methodology. In particular, we propose a procedure to test for mean-variance cointegration. This is followed by the presentation of the empirical results and several robustness checks. The final section summarizes and contains concluding remarks.

## 2 Term Premium Models and the Expectations Hypothesis

In this section, firstly, the general framework of the SDF approach for modeling term premia is briefly outlined. This is followed by the presentation of the specific SDF model

---

<sup>5</sup>Common examples for that approach are ENGLE et al. (1987) and BOLLERSLEV et al. (1988).

that we employ. In a second step, we turn to the relation between term premium and interest rate spread. Showing that the empirical finding of unit-root behavior in spreads can only be explained by integrated term premia we derive two testable hypotheses. In the third part, it is illustrated how this explanation carries over from the SDF model to a (linearized) version of the EHT, which is prevalent in a large strand of literature.

## 2.1 The Stochastic Discount Factor Model and a CAPM-motivated Pricing Kernel

The SDF model relates the price of an asset to the expected present value of the future pay-off

$$P_t = E[M_{t+1}X_{t+1}|I_t], \quad (1)$$

where  $P_t$  denotes the price at time  $t$ .  $X_{t+1}$  represents the pay-off at  $t+1$ ,  $M_{t+1}$  is the discount factor or pricing kernel ( $0 \leq M_{t+1} \leq 1$ ) and  $E[\cdot|I_t]$  indicates the conditional expectation operator where the information set  $I_t$  contains all information available up to time  $t$ . As we are interested in the return  $R_{t+1} = X_{t+1}/P_t - 1$  it is noted that

$$1 = E[M_{t+1}(1 + R_{t+1})|I_t]. \quad (2)$$

By definition

$$E[M_{t+1}(1 + R_{t+1})|I_t] = E[M_{t+1}|I_t]E[1 + R_{t+1}|I_t] + Cov[M_{t+1}, (1 + R_{t+1})|I_t]$$

holds. Applying equation (2), the expected future gross return can be expressed as

$$E[1 + R_{t+1}|I_t] = \frac{1 - Cov[M_{t+1}, (1 + R_{t+1})|I_t]}{E[M_{t+1}|I_t]}.$$

The return at  $t+1$  from a riskless investment, denoted by  $r_t$ , is known at  $t$  and is hence included in the information set  $I_t$ . Therefore, regarding (2), this return produces the relation

$$E[M_{t+1}|I_t] = \frac{1}{1 + r_t}.$$

The latter equation now allows us to write the *expected excess return* over the risk-free rate as

$$E[R_{t+1}|I_t] - r_t = -(1 + r_t)Cov[M_{t+1}, (1 + R_{t+1})|I_t]. \quad (3)$$

Equation (3) represents the characteristic relation between risk and return. In SDF models risk is measured as the covariance of the return with the variables that represent the discount factor  $M_{t+1}$ , or, in other words, the factors that enter the pricing kernel.

SMITH and WICKENS (2002) show in their survey that the SDF model can be seen as the umbrella framework that includes the most prominent asset pricing models. The SDF models proposed and investigated in the literature greatly differ in the specification of the discount factor. One possible classification relates to the nature of the factors as either observable or latent variables.

In bond pricing, a recently widely used class is given by affine factor models. They assume the discount factor to be a linear function of the observable or unobservable factors. The VASICEK (1977) and the COX, INGERSOLL and ROSS (1985) (CIR) models represent two of the most popular latent variable affine factor approaches. DAI and SINGLETON (2000) compare several multi-factor CIR models. In their influential study ANG and PIAZZESI (2003) augment a multi-factor Vasicek model by additional observable macroeconomic factors, thereby highlighting the importance of macroeconomic sources of risk for the short end and that of latent factors for the long end of the term structure. COCHRANE and PIAZZESI (2005) show that one observable factor, a linear function of certain forward rates, can account for a huge part of the term premium. In the recent literature on affine factor models the intersection of macroeconomics and finance plays a prominent role; see GÜRKAYNAK and WRIGHT (2010) for a survey.

Moreover, there are two prime examples of implicit observable one-factor models: the CAPM (SHARPE 1964, LINTNER 1965) and the CCAPM (RUBINSTEIN 1976, LUCAS 1978). Both models have a long tradition in finance capturing the risk-return trade-off (see, e.g., GHYSELS et al. 2005, LUNDBLAD 2007 and BALI and ENGLE 2010). The CAPM represents the model of choice in the present paper. It implicitly assumes the factor to be the return on the market. The CAPM allows for an appealing economic interpretation due to the connection of risk as non-diversifiable return volatility. We will show that it fits well the purpose of the underlying study, i.e. explaining nonstationarity of spreads and introducing the concept of mean-variance cointegration. Combining this approach with more comprehensive risk models represents an attractive path for future research.

The CAPM can be classified as an implicit observable one-factor model and represents a very parsimonious choice. However, as will be seen below, it is well suited to account for the phenomenon of extremely persistent premia. The CAPM states that excess returns are described by

$$E[R_{t+1} - r_t | I_t] = b \cdot Cov[R_{t+1}^m, R_{t+1} | I_t] . \quad (4)$$

In (4)  $R_{t+1}^m$  indicates the return on the market and

$$b = \frac{E[R_{t+1}^m - r_t | I_t]}{\text{Var}[R_{t+1}^m | I_t]} .$$

Comparing the well-known equation (4) to (3) it becomes obvious that the CAPM may be understood as an implicit one-factor model with the discount factor

$$M_{t+1} = -\frac{b}{1+r_t}(1+R_{t+1}^m) . \quad (5)$$

## 2.2 Stationarity Properties of Spreads and Premia: Testable Hypotheses

The present work is concerned with the term structure of interest rates and the explanation of nonstationary spreads. Hence we shall proceed by deriving theoretical implications and by taking a closer look at the exact form of (4) in case of bond pricing and for the specific type of interest rate data that we investigate. In order to focus primarily on the nonstationarity puzzle, we initially abstract from cross-asset and cross-market dependencies. Therefore, we consider a stylized financial market that comprises only two assets, i.e. one risky asset, a coupon-carrying  $n$ -period bond with a yield to maturity (interest rate) of  $Y_t^{(n)}$  and one riskless asset, a one-period bond offering  $Y_t^{(1)}$ . Let  $H_{t+1}^{(n)}$  denote the return that one realizes at  $t+1$  from holding the  $n$ -period bond for one period, i.e. from  $t$  to  $t+1$

$$H_{t+1}^{(n)} = \frac{P_{t+1}^{(n-1)} - P_t^{(n)} + C}{P_t^{(n)}} . \quad (6)$$

Here  $P_t^{(n)}$  denotes the price that was paid at  $t$  and  $P_{t+1}^{(n-1)}$  refers to the price of the bond at  $t+1$ , which has now existed for one period and hence has only  $n-1$  periods left till maturity.  $C$  is the coupon payment. Since later we investigate holding returns constructed from yield data on bonds that are sold at par, we note that for these bonds by definition  $P_t^{(n)} = 1$  and  $Y_t^{(n)} = C = \text{yield to maturity}$ . For these data in view of (6) the definition of excess holding returns takes the form

$$H_{t+1}^{(n)} - Y_t^{(1)} = P_{t+1}^{(n-1)} - 1 + Y_t^{(n)} - Y_t^{(1)} = c_{t+1} + s_t , \quad (7)$$

with the capital gain (loss) that accrues during the holding-period as  $c_{t+1} = P_{t+1}^{(n-1)} - 1$  and the interest rate spread as  $s_t = Y_t^{(n)} - Y_t^{(1)}$ . The *expected excess holding return* is also



referred to as the *holding premium*<sup>6</sup>, that we denote by

$$\mathbb{E}[H_{t+1}^{(n)} - Y_t^{(1)} | I_t] = \underbrace{\phi_{t+1}^{(n)}}_{I(d)} = \underbrace{\mathbb{E}[c_{t+1} | I_t]}_{I(0)} + \underbrace{s_t}_{I(d)}. \quad (8)$$

Usually, the econometrician cannot observe expectations. However, from the right-hand side of (4) we know how they can be modeled. For the above described two-asset case, the conditional covariance with the market becomes the conditional variance of the excess holding return on the  $n$ -period bond itself. Hence, applying (7) to the SDF model (4) yields

$$\underbrace{\phi_{t+1}^{(n)}}_{I(d)} = b \cdot \text{Var}[c_{t+1} + s_t | I_t] = b \cdot \underbrace{\text{Var}[c_{t+1} | I_t]}_{I(d)}, \quad (9)$$

the SDF-CAPM model. From (8) and (9) we draw two testable hypotheses.

**Hypothesis (i): Equal Degree of Integration** Given  $\mathbb{E}[c_{t+1} | I_t] \sim I(0)$  and interest rate levels are integrated of order one, the spread  $s_t$  and the holding premium  $\phi_{t+1}^{(n)}$  are either both stationary ( $d = 0$ ) or both nonstationary ( $d = 1$ ).

**Hypothesis (ii): Mean-Variance Cointegration** If spread and holding premium are nonstationary ( $d = 1$ ) they must be cointegrated. The cointegrating vector of  $s_t$  and  $\text{Var}[c_{t+1} | I_t]$  equals  $(1, -b)$ .

Hypothesis (i) follows from (8). If  $\mathbb{E}[c_{t+1} | I_t] \sim I(0)$ <sup>7</sup> for the equation to be balanced the degrees of integration of spread and holding premium must be equal. Hypothesis (ii) follows from (9). According to the SDF-CAPM model the holding premium equals  $b \cdot \text{Var}[c_{t+1} | I_t]$  and hence it is the conditional second moment of excess returns that must be cointegrated with the conditional first moment of the spread. We emphasize that the interest rate spread as the second component of excess holding returns plays no role for the conditional variance, as it is included in the information set  $I_t$ . Thus, if the interest rate spread is integrated of order one, it must in fact be cointegrated with the conditional variance of the corresponding capital gain series. This is what we label *mean-variance cointegration*. The cointegrating vector of  $s_t$  and  $\text{Var}[c_{t+1} | I_t]$  is  $(1, -b)$ . In the present model  $b$  may be interpreted as the market price of risk (PoR).

<sup>6</sup>The literature sometimes confusingly uses "term premium" as an umbrella term for forward, holding and rollover premium. Since for the following work it is important to use exact definitions we apply those from the notes of SHILLER (1990).

<sup>7</sup>Theoretically,  $c_{t+1}$  can be considered as a series of price changes. Since prices normally behave like random walks or slightly more general  $I(1)$  processes, agents would expect  $c_{t+1}$  to be  $I(0)$ . Indeed, as will be seen later, this property of capital gains is found in the data.

### 2.3 Linkage to the Linearized Expectations Hypothesis

It is now briefly outlined how Hypotheses (i) and (ii) just derived from the SDF-CAPM model carry over to the frequently used linearized version of the EHT. The reasoning that nonstationary spreads can be explained by nonstationary holding premia is shown to be consistent with the EHT.

The well-known form of the EHT is essentially only a linearization of stochastic equations that define returns (prices) in a financial market in the absence of arbitrage. Following the considerations of SHILLER (1979), the holding return - expressed in terms of yields to maturity - can be linearized by means of a Taylor expansion of order one. The linearized holding return is then simply substituted for  $H_{t+1}^{(n)}$  in the definition of the *holding premium*  $E[H_{t+1}^{(n)} - Y_t^{(1)} | I_t] = \phi_{t+1}^{(n)}$ . The solution of the resulting first order difference equation yields the familiar expression that relates the interest rate spread to expected future short rate differences plus a *rollover premium* (for details see Appendix A):

$$\underbrace{Y_t^{(n)} - Y_t^{(1)}}_{I(1)} \stackrel{\equiv s_t}{=} \underbrace{\sum_{k=1}^{n-1} \omega'(k) E[\Delta Y_{t+k}^{(1)} | I_t]}_{I(0)} + \underbrace{\theta_t}_{I(1)} \quad , \quad (10)$$

$$\theta_t = \sum_{k=0}^{n-1} \omega(k) \phi_{t+k+1}^{(n-k)} \quad (11)$$

and  $\omega(k) = \gamma^k \frac{1-\gamma}{1-\gamma^n}$  respectively  $\omega'(k) = \gamma^k \frac{1-\gamma^{n-k}}{1-\gamma^n}$  with  $\gamma = 1/(1+\bar{r})$ ,  $0 < \gamma < 1$ .

Equation (10), the linearized expectations model, was the theoretical starting point of numerous empirical investigations of the expectations hypothesis of the term structure of interest rates. The conclusion that spreads should be stationary can directly be drawn from the above representation of the spread as a weighted average of expected future short rate changes in (10). Given interest rate series are integrated of order one, agents would expect the changes in  $\sum_{k=1}^{n-1} \omega'(k) E[\Delta Y_{t+k}^{(1)} | I_t]$  to be  $I(0)$ . If furthermore the rollover premium  $\theta_t$  is assumed to be stationary, the same holds for the spread (CAMPBELL and SHILLER 1991).

As can be seen from (11), the rollover premium  $\theta_t$  can be written as a weighted sum of successive holding premia, where the first summand equals  $\phi_{t+1}^{(n)}$  from (9); see also SHILLER (1990). Therefore, theoretically, the orders of integration of the two different kinds of premia,  $\phi_{t+1}^{(n)}$  and  $\theta_t$ , are equal. The conclusion drawn from (8) that nonstationary spreads can be explained by nonstationary holding premia is consistent with the linearized expectations model in (10) that would include an integrated rollover premium. Following the CAPM-motivated SDF model (9) allows us to derive an estimable specification for the premium depending on the conditional variance of capital gains,  $\text{Var}[c_{t+1}|I_t]$ . The results then carry over to the linearized expectations model in (10) which takes no independent stance on how  $\theta_t$  might be measured.

### 3 Econometric Modeling

The methodology to be introduced follows three steps designed to empirically investigate Hypotheses (i) and (ii):

#### Equal Degree of Integration

- (a) We determine the order of integration of interest rate spreads (respectively, conditional means) to obtain evidence of whether assuming stationary premia is appropriate.
- (b) If spreads are  $I(1)$ , testing for integrated premia (respectively, conditional variances) will follow.

#### Mean-Variance Cointegration

- (c) If premia are actually found to be nonstationary<sup>8</sup>, too, we will test for cointegration with the spreads and estimate the proportionality coefficient as well as the adjustment speed.

Hence, at first, we discuss how to test for unit roots in the conditional mean of a time series (the spread) that potentially exhibits heteroskedasticity. Secondly, the same will be done with respect to nonstationarity of the conditional variance of a time series (the capital gain). At last, we introduce the mean-variance cointegration approach to test for cointegration between spread and premium.

---

<sup>8</sup>The term stationarity always refers to *weak covariance stationarity*.

### 3.1 Testing for Integrated Interest Rate Spreads

Herein we conduct step (a). Whether there can be unit roots in interest rates is debatable due to the zero lower bound and some upper bound that applies under regular circumstances. Nonetheless, in limited samples the  $I(1)$  property is often found to be empirically reasonable. Eventually, irrespective of the persistence of the true data generating process (DGP) of interest rates, the conclusion of CAMPBELL and SHILLER (1987) holds: spreads must be stationary.

Since the present data exhibits heteroskedasticity, a usual property of financial time series, we allow innovations to follow GARCH processes. However, unit root tests under conditional heteroskedasticity should be carried out with caution. With regard to the impact of neglected GARCH on the (augmented) Dickey-Fuller (ADF, DICKEY and FULLER 1979) test see, e.g., KIM and SCHMIDT (1993), LING and MCALEER (2003) and the literature surveyed therein. Due to the invariance principle, the ADF test proves to be asymptotically robust to covariance stationary GARCH errors. Yet, small-sample properties were conjectured to be affected in case of very persistent variance processes. SEO (1999), for instance, proposes a more powerful test<sup>9</sup>. The distribution in his test depends on a nuisance parameter, the relative weight  $0 \leq \tau \leq 1$ , and is bounded between the DF distribution ( $\tau = 1$ ) and the standard normal ( $\tau = 0$ ). We will later double-check the standard least squares ADF test results by the Seo test. The well-known ADF test equation (respectively the mean equation in the Seo test) is given by

$$\Delta x_{t+1} = \delta + \psi x_t + \sum_{i=1}^q \delta_i \Delta x_{t+1-i} + u_{t+1} , \quad (12)$$

where  $q$  denotes the lag length and  $u_{t+1}$  is (possibly heteroskedastic) white noise. Under the null of a unit root, the lagged level in (12) has no effect on  $\Delta x_{t+1}$ . The test statistic is given by the  $t$  value of  $\hat{\psi}$ .

---

<sup>9</sup>The test from SEO (1999) uses the information arising from conditional heteroskedasticity by means of joint maximum likelihood estimation (MLE) of the autoregressive and the GARCH parameters. Yet, CHARLES and DARNÉ (2008) find that for many practically relevant GARCH parameter values (i.e. for a sum of the ARCH and GARCH coefficients between 0.8 and 1 and for a GARCH parameter larger than the ARCH parameter) the DF test performs better than the Seo test with respect to power and size. Recent work from KOUROGENIS and PITTIS (2008) explicitly analyzes integrated GARCH (IGARCH) innovations in the context of standard unit root tests. In their Monte Carlo simulations the DF test is included as the special case of uncorrelated innovations and appears to perform surprisingly well in the IGARCH case.

### 3.2 Testing for Integrated Holding Premia

Spreads found to be  $I(1)$  in unit root tests could only be consistent with EHT if the conditional variance of capital gains was nonstationary (Hypothesis i, step b). To set up an according test procedure, the conditional mean of capital gains is specified as an  $AR(p_c)$  process with GARCH(1,1) errors  $\varepsilon_{c,t+1}$ :

$$\begin{aligned} c_{t+1} &= a_c + \sum_{i=1}^{p_c} a_{c,i} c_{t+1-i} + \varepsilon_{c,t+1} , \\ h_{c,t+1} &= \omega_c + \alpha_c \varepsilon_{c,t}^2 + \beta_c h_{c,t} , \end{aligned} \tag{13}$$

where  $\text{Var}[c_{t+1}|I_t] = E[\varepsilon_{c,t+1}^2 | I_t] \equiv h_{c,t+1}$ . The parsimonious (I)GARCH(1,1) specification is known to capture variance dynamics of most financial time series fairly well. This is also true for the present data. The IGARCH(1,1) hypothesis (see ENGLE and BOLLERSLEV 1986), meaning that the slope coefficients in the conditional variance equation sum up to one, is usually checked by likelihood ratio (LR) tests. Yet, LUMSDAINE (1995) shows within a Monte Carlo investigation that the LR test is quite oversized in small samples. BUSCH (2005) proposes a robust LR test based on quasi MLE (QMLE). His test statistic proves to be well behaved in small samples. The correction term  $k = 0.5(E[\xi_t^4] - 1)$  with  $\xi_t = \varepsilon_t/\sqrt{h_t}$  is calculated under the alternative of a covariance stationary GARCH process. In that case the test statistic

$$\lambda = -\frac{2}{k}(l(\hat{\boldsymbol{\theta}}_r) - l(\hat{\boldsymbol{\theta}}_u)) \tag{14}$$

has actual size close to nominal size, even for skewed disturbances.<sup>10</sup> In (14),  $\hat{\boldsymbol{\theta}}_r$  and  $\hat{\boldsymbol{\theta}}_u$  are the restricted and the unrestricted QMLEs for the parameter vector  $\boldsymbol{\theta}$ . However, since persistence in variance is a central question in the present work, we additionally conducted a small-sample simulation experiment. That is, we simulated the respective distribution of  $\lambda$  under the null of a DGP according to (13) with parameter vector  $\hat{\boldsymbol{\theta}}_r$  ( $\alpha_c + \beta_c = 1$ ). The resulting critical values will be applied in addition to the  $\chi^2$  quantiles. Moreover, we will provide evidence from test variants allowing for structural breaks under the alternative hypothesis.

---

<sup>10</sup>For further details see BUSCH (2005).

### 3.3 Testing for Mean-Variance Cointegration

We continue by discussing the methodological approach to examine Hypothesis (ii): If spreads and holding premia are both nonstationary they must be cointegrated (step c). Thus, a test for cointegration between mean of the spread series and variance of the capital gain series is presented.

While one might test for cointegration by checking the residuals from a static regression for stationarity, this approach is known to produce biased estimates and lack efficiency. In order to overcome these problems, we proceed from the dynamic cointegration test proposed by STOCK (1987). The test equation naturally follows from our approach in the previous subsection by augmenting the above ADF test equation (12) for  $s_t$  by the integrated variance series  $h_{c,t+1}$  from (13) (i.e., under the restriction that  $\alpha_c + \beta_c = 1$ ):

$$\Delta s_{t+1} = a + \rho s_t + \gamma h_{c,t+1} + \sum_{i=1}^p a_i \Delta s_{t+1-i} + \varepsilon_{t+1} . \quad (15)$$

Additionally, we control for GARCH effects in  $\varepsilon_{t+1}$ . Hence, (15) and the process for  $\varepsilon_{t+1}^2$  are estimated simultaneously by (Q)ML. Relation (15) describes an ECM for the interest rate spread. Note that the capital gain variance-in-mean of (15) is conditional on the information available at  $t$ . In view of (8) and (9), that is exactly what follows from economic theory - cointegration between  $s_t$  and  $h_{c,t+1} \equiv \text{Var}[c_{t+1}|I_t]$ . For simplicity, lagged differences of  $h_{c,t+1}$  are not included since they turn out to be insignificant. The established reasoning when testing for cointegration in an error correction framework applies (STOCK 1987): In case of cointegration, the common nonstationary factor of the two variables cancels out so that the linear combination  $z_t \equiv (1, \gamma/\rho)(s_t, h_{c,t+1})'$  represents a stationary time series. If so, the relation in levels should significantly contribute to the explanation of  $\Delta s_{t+1}$ . On the contrary, under the null  $z_t$  is nonstationary and thus  $\rho$  is zero.

As concerns the critical values to be applied, one might first think of those provided by BANERJEE et al. (1998) for the case of one exogenous variable. Yet, in contrast to usual cointegration testing in an error correction framework, the proposed test equation (15) contains a latent regressor, the IGARCH series  $h_{c,t+1}$  (the capital gain variance) estimated in a preceding step. Hence, (15) can be seen as a quasi IGARCH-in-Mean cointegration test as the variance that enters the mean equation is driven by the squared residuals  $\varepsilon_{c,t+1}^2$  that originate from a different mean equation - relation (13). Although

$h_{c,t+1}$  is a martingale, it is known that the properties of such a series deviate in many aspects from those of a random walk (see NELSON 1990). Besides, the innovations  $\varepsilon_{t+1}$  in (15) themselves are highly heteroskedastic<sup>11</sup>, as it turned out in specification tests. To the best of our knowledge, a theory on testing for cointegration between a conditional mean and an estimated conditional variance series has not yet been developed. In order to fill this gap we derive the distribution of the test statistic (the  $t$ -value of  $\hat{\rho}$ ) via simulation (see Appendix B).

At this point a further property of realized excess holding returns on par bonds is noteworthy. As can be seen from their definition in (7), excess returns on coupon-carrying bonds consist of two components, the capital gain  $c_{t+1}$  and the interest rate spread  $s_t$ . Excess returns are generally known to exhibit only very slight autocorrelation. However, as follows from EVANS and LEWIS (1994), this empirical result may also occur in case spreads are nonstationary since the variation of capital gains is usually very high relative to  $s_t$ . Then, the persistence of  $c_{t+1}$  and  $c_{t+1} + s_t$  is statistically indistinguishable. Put differently, due to a very low signal-to-noise ratio, statistical tests fail to detect the true order of integration of excess returns in case of  $s_t$  being  $I(1)$ . This fact may also underlie the general difficulties of empirical finance to find a significant risk-return trade-off when trying to explain (statistically) strongly mean reverting excess returns by highly persistent second moments. Importantly, our theoretical result remains: the orders of integration of  $s_t$  and  $\phi_{t+1}$  must be equal. Therefore, if there is cointegration, the ECM (15) estimates the PoR  $b = \frac{\gamma}{\rho}$  superconsistently, determining the long-run relation between the nonstationary component of excess returns,  $s_t$ , on the conditional variance of excess returns,  $h_{c,t+1} \equiv \text{Var}[H_{t+1}^{(n)} - Y_t^{(1)} | I_t] = \text{Var}[c_{t+1} | I_t]$ .

## 4 Empirical Results

### 4.1 Data

The subsequent analysis is based on weekly yields from 1/03/1992 to 12/29/2006 provided by the US Federal Reserve Statistical Release. The 15 years of US interest rate data should ensure a sufficient number of observations (783). All series are taken from the Treasury Constant Maturity data which allows to directly compare these rates.<sup>12</sup> The sample period includes the timespan after the early 1990s' recession and cuts off the

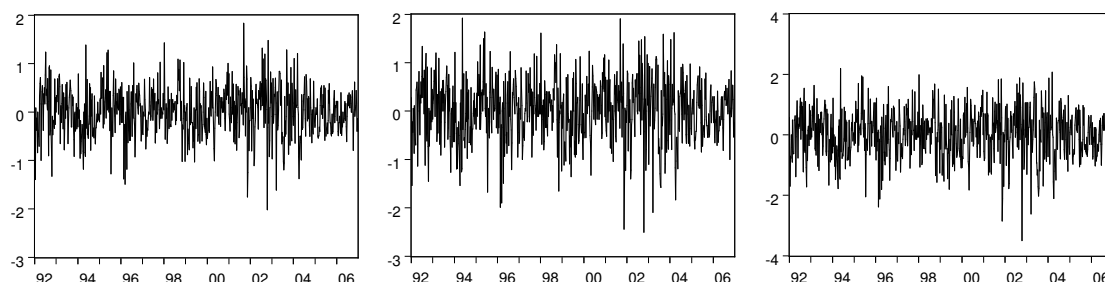
<sup>11</sup>For a discussion of potential consequences of GARCH effects on standard cointegration tests see, e.g., SEO (2007) and the literature cited therein.

<sup>12</sup>All yields represent bond equivalent yields for securities that pay semi-annual interest.

ongoing financial crisis. We chose this period to reduce the probability of breaks in conditional first and second moments often leading to artificial persistence (see LAMOUREUX and LASTRAPES 1990).<sup>13</sup> Excess holding returns and capital gains are calculated as, e.g., in JONES et al. (1998) or CHRISTIANSEN (2000).<sup>14</sup> The calculation method is also described in IBBOTSON and ASSOCIATES (1994) and applied in ENGLE et al. (1987) for the case of effectively infinitely-lived bonds.

Excess returns are defined as the return on holding a longer term bond over one period in excess of the riskless rate. Longer term bonds have maturities of 5, 7 and 10 years. The riskless rate is assumed to equal the standard 3-month Treasury rate, which is from now on referred to as the short rate. This is a common assumption and considered to be the best alternative against using a one week money market rate, which would imply, amongst other issues, discontinuities or outliers on settlement days (see NELSON 1991 or JONES et al. 1998)

As expected, mean excess holding returns increase with maturity of the long term bond (0.032, 0.044 and 0.055). The same holds for the empirical standard deviations (0.488, 0.628 and 0.786). Capital gains as part of the excess returns are denoted by  $C5$ ,  $C7$  and  $C10$ . They equal the change in the present value (price) from one week to another (see Figure 1).



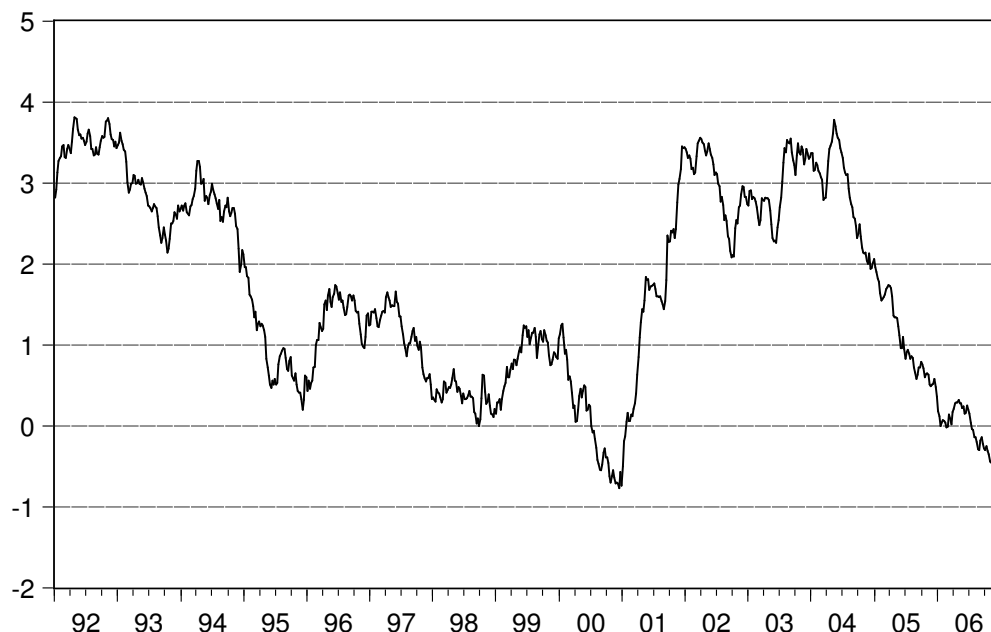
**Figure 1:** Weekly Capital Gain Series  $C5$ ,  $C7$  and  $C10$  for 5-, 7- and 10-Year Bonds

<sup>13</sup>TZAVALLIS and WICKENS (1995) argued that the regime of strong volatility of the very high interest rates in the early and mid-1980s was the cause of persistence in second moments of excess returns. In 1992 interest rates have returned to their pre-early 1980s level and hence our sample should not be affected by that peak.

<sup>14</sup>For the present yield data on par bonds and on semi-annual basis capital gains are defined as  $c_{t+1} = P_{t+1}^{(n-1)} - 1 = \frac{1}{(1 + \frac{1}{2}Y_{t+1}^{(n-1)})^{2n}} + \sum_{i=1}^{2n} \frac{\frac{1}{2}Y_t^{(n)}}{(1 + \frac{1}{2}Y_{t+1}^{(n-1)})^i} - 1$ ; compare equations (6) and (7). The first term represents the present value of the principle and the second term those of the coupon payments. Since  $Y_{t+1}^{(n-1)}$  is not available  $Y_{t+1}^{(n)}$  can be used instead. There should be no measurable difference between the yield of 10-year bond and that of a bond with 9 years and 51 weeks to maturity as pointed out by SHILLER (1979), page 1197, footnote 8.



Since we use weekly observations of annualized interest rates, spreads are calculated as the fraction  $(1/52)$  of the difference between the respective long rate and the short rate, that corresponds to a holding period of one week. Spread series are labeled  $S5$ ,  $S7$  and  $S10$  and. Exemplarily,  $S10$  is depicted in annualized form in Figure 2 ( $S5$  and  $S7$  exhibit a very similar shape).



**Figure 2:** Annualized Interest Rate Spread  $S10$  between the 10-Year and the 3-Month Bond

## 4.2 Unit Root Tests for Interest Rate Spreads

In order to determine the integration order of interest rate spreads (Hypothesis i), the ADF test is applied. Note that as far as levels are concerned the test equation includes a constant whereas for the first differences there is no deterministic part. A linear trend would not be meaningful for interest rate spreads and is also not supported by the data. The number of lagged differences is chosen according to the Schwarz information criterion (SIC). HECQ (1996) shows that even in the IGARCH case standard information criteria can be applied and that the SIC performs best compared to the Hannan Quinn criterion (HQC) and the final prediction error (FPE). Yet, since ADF test results are known to be sensitive to the number of lagged differences in the test equation we double-checked our results by using HQC and FPE. Table 1 summarizes the unit root test results.

**Table 1:** ADF Tests for Interest Rate Spreads

Variable	Levels		First differences	
	$\hat{t}$	$q$	$\hat{t}$	$q$
<i>S5</i>	−1.651 (0.456)	1	−22.916 (0.000)	0
<i>S7</i>	−1.466 (0.551)	1	−22.430 (0.000)	0
<i>S10</i>	−1.275 (0.643)	1	−22.021 (0.000)	0

*Note:* Test statistics are denoted by  $\hat{t}$ .  $q$  refers to the number of lagged differences and  $p$ -values are given in parentheses.

It can be seen that nonstationarity is far from being rejected. Thus, all three spread series should be considered as integrated of order one.<sup>15</sup> When we apply HQC and FPE, both criteria suggest to include more lags but test statistics do barely change. Performing Seo tests we obtain the same result; the null of a unit root cannot be rejected.<sup>16</sup>

### 4.3 IGARCH Tests for Holding Premia

So far it turned out that all spreads should be considered  $I(1)$ . Since capital gains levels are clearly  $I(0)$ <sup>17</sup>, following Hypothesis (i) - equal degree of integration - the EHT can only be valid in presence of nonstationary holding premia. Since the SDF-CAPM model defines the holding premium as  $\phi_{t+1}^{(n)} = b \cdot \text{Var}[c_{t+1}|I_t]$ , testing Hypothesis (i) translates into testing for integrated capital gain variances; step (b).

We fit  $\text{AR}(p_c)\text{-IGARCH}(1,1)$  models to capital gain series. Thereafter, we test these models against the alternative hypothesis of autoregressive processes with covariance stationary variance series, that is against  $\text{AR}(p_c)\text{-GARCH}(1,1)$  models. Table 2 summarizes the test results.

<sup>15</sup>The integration order of the interest rate series has been checked, too. According to ADF test results there is very strong evidence that all interest rate series can be considered as integrated of order one.

<sup>16</sup>As the stationarity properties of interest rate spreads are crucial for the following analysis, we additionally conducted the KPSS test (Bartlett kernel and Newey-West bandwidth selection) with the null hypothesis of stationarity. This is to assure that non-rejections of nonstationarity are not simply due to the possible power problem of ADF-type unit root tests. Yet, for the present data this seems not to be the case since the KPSS test clearly rejects stationarity. All test results being not reported can be obtained upon request.

<sup>17</sup>ADF test statistics of −21.727, −13.363 and −13.290 for *C5*, *C7* and *C10* allowed for a strong rejection of the null.

**Table 2:** Robust Likelihood Ratio Tests for Integrated Holding Premia

Variable	$p_c$	$\hat{\alpha}_c$	$\hat{\beta}_c$	$\hat{k}$	$\hat{\lambda}$	$\chi^2_{0.90}(1)$
<i>C5</i>	1	0.035 [0.014]	0.951 [0.022]	1.598	2.648	2.706
<i>C7</i>	1	0.037 [0.016]	0.948 [0.025]	1.406	2.699	2.706
<i>C10</i>	1	0.033 [0.015]	0.953 [0.024]	1.241	2.163	2.706

*Note:* Unrestricted model:

$$c_{t+1} = a_c + \sum_{i=1}^{p_c} a_{c,i} c_{t+1-i} + \varepsilon_{c,t+1} ,$$

$$h_{c,t+1} = \omega_c + \alpha_c \varepsilon_{c,t}^2 + \beta_c h_{c,t} .$$

We tested for nonstationary holding premia  $\phi_{t+1}^{(n)} = b \cdot \text{Var}[c_{t+1}|I_t]$ , i.e. for integrated conditional variances,  $\text{Var}[c_{t+1}|I_t]$ , of the respective capital gains on 5-, 7- and 10-year bonds; *C5*, *C7* and *C10*.  $\hat{\alpha}_c$  and  $\hat{\beta}_c$  refer to the unrestricted coefficient estimates. Under the null that  $\alpha_c + \beta_c = 1$  the robust LR test statistic  $\lambda = \frac{2}{k} (l(\hat{\theta}_u) - l(\hat{\theta}_r))$  is  $\chi^2(1)$ . The estimated correction term is denoted by  $\hat{k}$ .

Since the test statistic  $\lambda$  is  $\chi^2(1)$  and  $\chi^2_{0.90}(1) \approx 2.706$  the null of integrated variances cannot be rejected even at the 10% level. In all three cases we choose  $p_c = 1$  following the SIC. Additionally, we conducted a small-sample experiment: We simulated the distribution of  $\lambda$  for *C5*, *C7* and *C10* with DGPs under the null equal to our estimated  $\text{AR}(p_c)\text{-IGARCH}(1,1)$  models. For conditional normal distribution, sample length of 783 observations and 100,000 replications the 90% quantiles turned out to be 3.418, 3.470 and 3.448. Thus, these results strengthened the test decision not to reject the null of IGARCH. Yet, it is well known that spurious persistence can be caused by structural breaks neglected in the model specification (LAMOUREUX and LASTRAPES 1990). In our robustness section below we account for possible breaks in the unconditional variance, too.

#### 4.4 Mean-Variance Cointegration Tests

So far, our results have shown that interest rate spreads are  $I(1)$ . The last section demonstrated that the three corresponding holding premia are nonstationary, too. In order to test for Hypothesis (ii) - cointegration between spreads and premia, we apply the mean-variance cointegration test introduced in Section 3.3.

The following three ECMs were estimated:

$$\Delta S5_{t+1} = \underset{[-3.347]}{-0.0007} - \underset{[-3.967]}{0.022} S5_t + \underset{[4.047]}{0.005} \hat{h}_{c,t+1} + \underset{[5.685]}{0.192} \Delta S5_t + \hat{\varepsilon}_{t+1} , \quad (16)$$

$$\Delta S7_{t+1} = \underset{[-2.821]}{-0.0006} - \underset{[-3.455]}{0.016} S7_t + \underset{[3.430]}{0.003} \hat{h}_{c,t+1} + \underset{[6.001]}{0.216} \Delta S7_t + \hat{\varepsilon}_{t+1} , \quad (17)$$

$$\Delta S10_{t+1} = \underset{[-3.898]}{-0.0008} - \underset{[-4.116]}{0.017} S10_t + \underset{[4.435]}{0.002} \hat{h}_{c,t+1} + \underset{[6.025]}{0.222} \Delta S10_t + \hat{\varepsilon}_{t+1} , \quad (18)$$

Compared to the test statistics from ADF tests in Table 1,  $t$ -values of the lagged level  $s_t$  considerably increased in (16), (17) and (18). Again, the lag length is chosen according to the SIC<sup>18</sup> and supported by specification tests for no residual autocorrelation. Furthermore, we allowed the residuals in (16), (17) and (18) to be GARCH(1,1), too. Q statistics of standardized squared residuals as well as LM tests for remaining GARCH show that the parsimonious GARCH(1,1) specification proves to be reasonable.

Table 3 includes individual 1%, 5% and 10% simulated critical values for each of the three models.<sup>19</sup> There is only slight variation as the DGPs are very similar. Test results are clear-cut: In models (16) and (18) the null of no cointegration can be rejected at the 1% level. In (17) we reject at the 5% level. This is considered as strong evidence in favor of the existence of a cointegration relation. Economically, this means that there does exist a long-run equilibrium between US interest rate spreads and the corresponding one-period holding premia, as implied by EHT.

**Table 3:** Critical Values - Mean-Variance Cointegration Test

Model	$\hat{t}$	1%	5%	10%
$\Delta S5$	-3.967	-3.842	-3.247	-2.946
$\Delta S7$	-3.455	-3.838	-3.249	-2.939
$\Delta S10$	-4.116	-3.830	-3.247	-2.949

*Note:*  $\hat{t}$  refers to the estimated  $t$  value of the lagged level  $s_t$  in (16), (17) and (18).

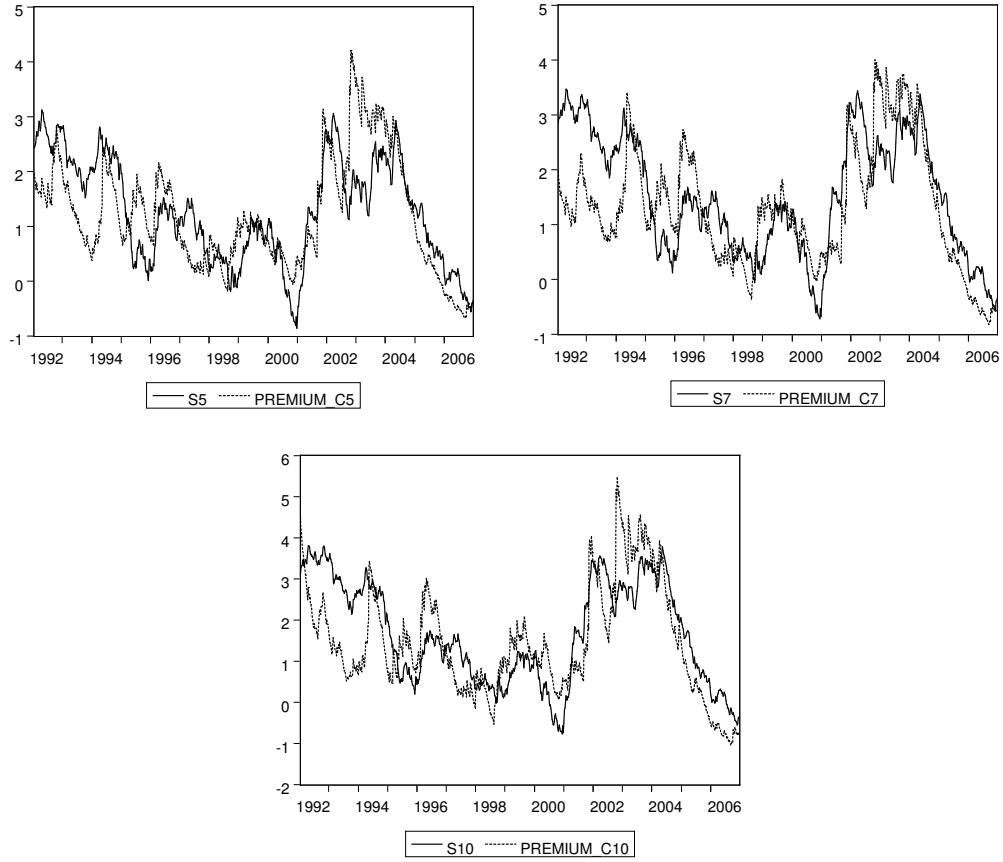
<sup>18</sup>Using different information criteria, all suggesting more lags, does not change cointegration test results.

<sup>19</sup>As concerns the simulated critical values, some experiments made clear that these depend on the conditional variance parameters of the DGPs of the capital gain variance and the spread series; see Appendix B.

Moreover, following our discussion in section 2, the coefficient  $b = \frac{\gamma}{\rho}$  in the respective attractor can be interpreted as the PoR. Recall that we do not estimate the PoR via the standard GARCH-in-Mean model on the basis of excess return data but through a cointegration relation between the nonstationary component of the excess return - the spread - and the integrated variance of the capital gain - the component of excess returns not included in the conditioning information set. Thereby, (super) consistency of the estimator is guaranteed by the existence of cointegration. In (16), (17) and (18) we estimated PoRs of 0.228, 0.164 and 0.127. Compared to other findings these coefficients are relatively, but not implausibly small (see, e.g., TZAVALLIS and WICKENS 1995, BALI and ENGLE 2010). For example, investors holding 7-year Treasury bonds expect at  $t$  that, on average, the excess return they will realize at  $t + 1$  equals about one sixth of its variance.

Regarding the above ECMs, adjustment coefficients may at first glance appear quite small. If, for instance, the spread  $S_{10t}$  exceeds its equilibrium value by one unit (one percentage point), then the spread decreases over the next week by 0.017 units (percentage points). However, after 13 weeks (one quarter) the initial equilibrium error has reduced to 0.771 and the half-life of the shock implied by system (18) equals just 33 weeks.

Figure 3 illustrates the annualized attractor, i.e. the long-run relation  $s = \frac{a}{\rho} + \frac{\gamma}{\rho}h_c$ , in the respective ECMs. Graphical analysis supports the statistical results: The general impression from Figure 3 is quite a strong co-movement between the spreads and the corresponding one-period holding premia. From the beginning of the sample till the New Economy boom the average level of all spreads decreases along with the volatilities. We see three noticeable peaks during that period, in 1994, between 1996 and 1997 as well as around the turn of the millennium. Whereas the first one results from long rates rising faster than the short rate, the second and third ones are triggered by increasing long rates when the short rate remained roughly constant. In view of equation (8), rising spreads are associated with growing holding premia and hence with rising capital gain variances. Indeed, we clearly see that the variance movement features similar peaks. However, most eye-catching is the period after 2001 when the short rate fell steeply for several years and, accordingly, spreads went up. In support of the cointegration test results, this timespan is also characterized by high volatility.



**Figure 3:** Spreads and Corresponding One-Period Holding Premia

## 5 Robustness Checks

Finally, we conducted several robustness checks. Among other things we were concerned with spurious persistence in the conditional variance due to neglected structural breaks, with initial value issues and with the conditional normal distribution assumption.

### Endogenous Structural Breaks and Persistence in Variance

At first, we would like to stress the point that persistence in conditional second moments can be an artifact of neglected structural change in the variance. LAMOUREUX and

LASTRAPES (1990) provide examples of that phenomenon. A specific example is also found in TZAVALIS and WICKENS (1995) who show that the persistence in volatility of US holding premia between 1970 and 1986 is the result of a structural shift during a period of exceptionally high variances (October 1979 - September 1982). In general, the timing of structural breaks is quite difficult. In order to avoid the arbitrariness of choosing break dates exogenously, we conducted an endogenous break search. We therefore augmented the unrestricted conditional variance equation by a shift dummy and selected the date where the dummy has the highest  $t$ -value. As shown in the unit root literature (e.g. ZIVOT and ANDREWS 1992), the additional step of estimating the break date affects the distribution of the test statistic. Therefore, we simulated the distribution allowing for a break in the GARCH constant under the alternative hypothesis. The results show that for none of the three capital gain series the null of IGARCH can be rejected at the 10% level. We also allowed for two level shifts with endogenous break dates. This improved the likelihood under the alternative hypothesis only very little so that nonstationarity was not rejected in this case, either.

### Initial Values and the Shape of the Variance Series

The choice of initial values has no impact on our simulation results. However, the shape of the estimated capital gain variance series varies slightly - particularly during the first year (about 52 observations) when we initialize GARCH models using, for instance, backcast exponential smoothing (where  $h_0 = \kappa^N/N \sum_{t=0}^N \hat{\epsilon}_t^2 + (1 - \kappa) \sum_{j=0}^N \kappa^{N-j-1} \hat{\epsilon}_{N-j}^2$ ,  $0 < \kappa \leq 1$ ) instead of simply the mean of squares of residuals. Since after about one year the initial value impact has essentially vanished, we re-estimated the ECMs starting the sample at the beginning of 1993 using 731 observations.<sup>20</sup> Compared to the estimates in (16), (17) and (18), test statistics decreased (i.e. increased in absolute value) to  $-4.512$ ,  $-4.476$  and  $-4.402$ . The lower number of observations affects critical values only at the second decimal place. We therefore reject the null of no cointegration at the 1% level in all ECMs.

### Distributional Assumption

At last, we investigated how the distributional assumption affects our test and simulation results. First of all, we drew random samples for  $\xi_{c,t+1} = \epsilon_{c,t+1}/\sqrt{h_{c,t+1}}$  and  $\xi_{s,t+1} = \epsilon_{s,t+1}/\sqrt{h_{s,t+1}}$  from Student  $t$ -distributions in both simulation experiments - the LR test for IGARCH (section 3.2) and the mean-variance cointegration test (section 3.3). Degrees of

---

<sup>20</sup>Persistence in variances proves not to be affected by initial conditions.

freedom are set equal to estimated values under the null and lie between 8 and 18. Most of the estimated values are clearly larger than 10 indicating that the initial assumption of normality is not violated too strongly for the present data. Since sample excess kurtosis of all series is relatively small (between 0.8 and 1.5), this is not surprising. So as to analyze the effect of possibly incorrectly specified innovations our estimates become actually QMLEs in the sense that Gaussian likelihoods are maximized even though we have generated  $t$ -innovations. As expected, the smaller the number of degrees of freedom the more the distribution of  $\lambda$  shifts to the right. Hence, the test decision not to reject the null of integrated variances in section 4.3 is strengthened. Similarly, the distribution of the  $t$ -value of  $\hat{\rho}$  also shifts to the right so that we can reject the null of no cointegration in section 4.4 at an even higher significance level (since the  $t$ -values are negative).

## 6 Conclusion

The present paper empirically examines a well-known implication of the expectations hypothesis of the term structure (EHT): interest rate spreads should be stationary. We shed more light on the question why there is so much evidence that contradicts the implication of mean-reverting spreads; see HALL et al. (1992) or HANSEN (2003).<sup>21</sup> This implication has also been the pivotal element in many studies that analyze the spread's predictive power for short rate changes or other macroeconomic variables like inflation and GDP growth (MANKIW and MIRON 1986, KUGLER 1988 or CAPOREALE and CAPOREALE 2008). The consequences of theoretically implied stationarity properties of interest rate spreads are obviously far-reaching. We are therefore concerned with the question why they are so often not met and argue that an answer can be provided by nonstationary holding premia.

The theoretical starting point is the one-period holding premium defined as the sum of interest rate spread and expected capital gain. We derive two testable hypotheses. Hypothesis (i): Given stationary capital gains, spread and holding premium must exhibit the same order of integration. Hypothesis (ii): If this order equals one, spread and holding premium must be cointegrated. With respect to the economic and econometric modeling of spread and premium we refer to mean-variance cointegration.

---

<sup>21</sup>Further cointegration studies as, e.g., SHEA (1992), ZHANG (1993), ENGSTED and TANGGAARD (1994), JOHNSON (1994), WOLTERS (1995), PAGAN et al. (1996), WOLTERS (1998), CARSTENSEN (2003) make the empirical finding of unit roots in interest rate spreads an almost stylized fact.



We show that explaining a nonstationary spread by integrated holding premia is consistent with the frequently used linearized version of the EHT. The latter would include a nonstationary rollover premium that we explicitly link to the holding premium. When it comes to modeling and estimating the holding premium we employ an observable single-factor SDF model with a CAPM-motivated pricing kernel. The holding premium is proportional to the conditional variance of excess returns. In order to test for Hypothesis (i) unit root tests are applied for spreads and robust LR tests for IGARCH variances of excess returns. So as to test for Hypothesis (ii) a mean-variance cointegration test in an error correction framework is proposed and the small-sample distribution of the test statistic is derived through simulation. Our approach may be seen as a quasi IGARCH-in-Mean cointegration test as the variance that enters the mean equation is estimated in a preceding step and is driven by the squared residuals from a different mean equation.

The empirical analysis is based on weekly observations of US Treasury Constant Maturity data. We examine three different spreads between the short rate (3-month Treasury rate) and long rates with maturities of 5, 7 and 10 years. Following the ADF test results, all spreads should be considered nonstationary. Further unit root tests unanimously confirm nonstationarity of the spreads, which is a result not uncommon in the empirical literature. Subsequently, estimating conditional variances of excess returns, it turns out that the null hypothesis of IGARCH cannot be rejected. Additionally, this result holds when incorporating endogenous structural breaks. Hence, we conclude that holding premia are also integrated. The most important step follows: Testing for cointegration between premia and spreads. As the main result of the present work, we actually find highly significant long-run relations between all spreads and corresponding premia.

Following the idea of arbitrage-free financial markets and rational expectations, the EHT provides a simple and appealing description of the relation between interest rates of different maturities. Long rates embody information on expected future short rates and both rates are tied together within a long-term equilibrium relation. This equilibrium can be captured by a cointegration relation. However, the modeling of the term premium plays a key role. This third variable should be modeled carefully and sometimes - as in the present case - even be included in the cointegration relation. The present paper has shown that nonstationary spreads are not necessarily at odds with the EHT. If spreads and premia are cointegrated, i.e. in case of mean-variance cointegration, this apparent contrariness can be rationalized. The basic statement of the EHT on the relation between interest rates of different maturities remains applicable when modeling the premium by means of approaches from finance theory. While the present paper mainly focuses on

the aspect of cointegration, it also underlines the relevance of the vast and still evolving literature on identifying economically interpretable driving forces of the premium (as e.g. ANG and PIAZZESI 2003 or GÜRKAYNAK and WRIGHT 2011).

Two extensions of the present approach appear interesting: First, our approach could be generalized to non-diversifiable risk (e.g. BOLLERSLEV et al. 1988 or BALFOUSSIA and WICKENS 2007): The frequent failure of the EHT would be explained by integrated covariance series that could be obtained from multivariate GARCH models. This would allow to control for cross-asset and cross-market dependencies. Second, since appropriate modeling of the persistence of the premium proved to be crucial, a further possible extension would be to allow for fractional integration in interest rates (CONNOLLY et al. 2007) and conditional variances (BAILLIE et al. 1996). If, for example, the order of integration of spreads and premia is equal but appears to be less than one, cointegration tests may be carried out in a fractionally (co)integrated framework. We leave these issues for future research.

## Appendix

### Appendix A

#### The Linkage from the Holding Premium in the SDF-CAPM Model to the Rollover Premium in the Linearized Expectations Model (section 2.3)

The conclusion drawn from the SDF-CAPM model that nonstationary spreads can be explained by nonstationary *holding premia* is consistent with the familiar linearized version of the EHT that would include an integrated *rollover premium*. The well-known form of the EHT that will be derived now is essentially a linearization of equations that define returns (prices) in a financial market in the absence of arbitrage.

To begin with, consider the definition of the yield to maturity of an  $n$ -period bond

$$P_t^{(n)} = \frac{C}{(1+Y_t^{(n)})} + \frac{C}{(1+Y_t^{(n)})^2} + \cdots + \frac{1+C}{(1+Y_t^{(n)})^n}.$$

Most compactly, this can be written as:

$$P_t^{(n)} = \frac{C}{Y_t^{(n)}} + \frac{Y_t^{(n)} - C}{Y_t^{(n)}(1+Y_t^{(n)})^n} \quad (19)$$

The one-period holding return defined in (6) can be expressed in terms of yields to

maturity by using (19) so that

$$H_{t+1}^{(n)} = \frac{\frac{C}{Y_{t+1}^{(n-1)}} + \frac{Y_{t+1}^{(n-1)} - C}{Y_{t+1}^{(n-1)}(1+Y_{t+1}^{(n-1)})^{n-1}} + C}{\frac{C}{Y_t^{(n)}} + \frac{Y_t^{(n)} - C}{Y_t^{(n)}(1+Y_t^{(n)})^n}} - 1. \quad (20)$$

According to the considerations of SHILLER (1979) we linearize (20) via a Taylor expansion of order one. Considering  $H_{t+1}^{(n)}(Y_t^{(n)}, Y_{t+1}^{(n-1)}, C)$  as a function of three variables, we know from Taylor's theorem that in the neighborhood of  $Y_t^{(n)} = Y_{t+1}^{(n-1)} = C = \bar{Y}$  it holds that

$$\begin{aligned} H_{t+1}^{(n)} &\approx H_{t+1}^{(n)}(\bar{Y}, \bar{Y}, \bar{Y}) + \frac{\partial H_{t+1}^{(n)}}{\partial Y_t^{(n)}} \Big|_{Y_t^{(n)}=Y_{t+1}^{(n-1)}=C=\bar{Y}} \cdot (Y_t^{(n)} - \bar{Y}) \\ &\quad + \frac{\partial H_{t+1}^{(n)}}{\partial Y_{t+1}^{(n-1)}} \Big|_{Y_t^{(n)}=Y_{t+1}^{(n-1)}=C=\bar{Y}} \cdot (Y_{t+1}^{(n-1)} - \bar{Y}) \\ &\quad + \frac{\partial H_{t+1}^{(n)}}{\partial C} \Big|_{Y_t^{(n)}=Y_{t+1}^{(n-1)}=C=\bar{Y}} \cdot (C - \bar{Y}) \equiv H_{t+1}'^{(n)}. \end{aligned}$$

Plugging in (20) and evaluating the derivatives finally yields

$$H_{t+1}'^{(n)} = \delta_n Y_t^{(n)} - (\delta_n - 1) Y_{t+1}^{(n-1)}, \quad (20')$$

where  $\delta_n = 1 + \bar{Y}^{-1} - (\bar{Y}(1 + \bar{Y})^{n-1})^{-1}$ . If one applies  $H_{t+1}'^{(n)} \approx H_{t+1}^{(n)}$  to the definition of the holding premium,  $E[H_{t+1}^{(n)} - Y_t^{(1)} | I_t] = \phi_{t+1}^{(n)}$ , the resulting first order difference equation with variable coefficients is

$$Y_t^{(n)} = \gamma_n E[Y_{t+1}^{(n-1)} | I_t] + (1 - \gamma_n)(\phi_{t+1}^{(n)} + Y_t^{(1)}), \quad (21)$$

where  $\gamma_n = (\delta_n - 1)/\delta_n$ . The solution of (21) can be derived by recursive substitution. Therefore, we initially use the law of iterated expectations and note that

$$\begin{aligned} E[Y_{t+1}^{(n-1)} | I_t] &= \gamma_{n-1} E[Y_{t+2}^{(n-2)} | I_t] + (1 - \gamma_{n-1}) E[\phi_{t+2}^{(n-1)} + Y_{t+1}^{(1)} | I_t] \\ E[Y_{t+2}^{(n-2)} | I_t] &= \gamma_{n-2} E[Y_{t+3}^{(n-3)} | I_t] + (1 - \gamma_{n-2}) E[\phi_{t+3}^{(n-2)} + Y_{t+2}^{(1)} | I_t] \\ &\vdots \\ E[Y_{t+n-1}^{(1)} | I_t] &= \gamma_1 E[Y_{t+n}^{(0)} | I_t] + (1 - \gamma_1) E[\phi_{t+n}^{(1)} + Y_{t+n}^{(1)} | I_t]. \end{aligned}$$

Recursive substituting of the above expressions into (21) yields

$$\begin{aligned}
Y_t^{(n)} &= \overbrace{\gamma_n \cdot \gamma_{n-1} \cdot \dots \cdot \gamma_1 Y_{t+n}^{(0)}}^{=0} + (1 - \gamma_n) E[\phi_{t+1}^{(n)} + Y_t^{(1)} | I_t] \\
&\quad + \gamma_n \cdot (1 - \gamma_{n-1}) E[\phi_{t+2}^{(n-1)} + Y_{t+1}^{(1)} | I_t] \\
&\quad + \gamma_n \cdot \gamma_{n-1} \cdot (1 - \gamma_{n-2}) E[\phi_{t+3}^{(n-2)} + Y_{t+2}^{(1)} | I_t] \\
&\quad \vdots \\
&\quad + \gamma_n \cdot \gamma_{n-1} \cdot \gamma_{n-2} \cdot \dots \cdot \gamma_2 (1 - \gamma_1) E[\phi_{t+n}^{(1)} + Y_{t+n}^{(1)} | I_t] .
\end{aligned}$$

In more compact form this can be written as

$$Y_t^{(n)} = \sum_{k=0}^{n-1} \omega(k) E[Y_{t+k}^{(1)} | I_t] + \sum_{k=0}^{n-1} \omega(k) \phi_{t+k+1}^{(n-k)} ,$$

with the weighting scheme  $\omega(k) = \gamma^k \frac{1-\gamma}{1-\gamma^n}$ . Via the identity  $Y_{t+k}^{(1)} = Y_t^{(1)} + \sum_{i=1}^k \Delta Y_{t+i}^{(1)}$  we obtain

$$Y_t^{(n)} - Y_t^{(1)} = \sum_{k=1}^{n-1} \omega(k) \sum_{i=1}^k E[\Delta Y_{t+i}^{(1)} | I_t] + \sum_{k=0}^{n-1} \omega(k) \phi_{t+k+1}^{(n-k)} .$$

Rearranging terms produces the well-known expression for the interest rate spread, i.e.

$$\overbrace{Y_t^{(n)} - Y_t^{(1)}}^{\equiv s_t^{(n)}} = \sum_{k=1}^{n-1} \omega'(k) E[\Delta Y_{t+k}^{(1)} | I_t] + \overbrace{\theta_t}^{\text{Rollover Premium}} ,$$

where  $\omega'(k) = \gamma^k \frac{1-\gamma^{n-k}}{1-\gamma^n}$  and  $\theta_t = \sum_{k=0}^{n-1} \omega(k) \phi_{t+k+1}^{(n-k)}$  with  $\gamma = 1/(1+\bar{r})$ ,  $0 < \gamma < 1$ . This shows (10) and (11).

## Appendix B

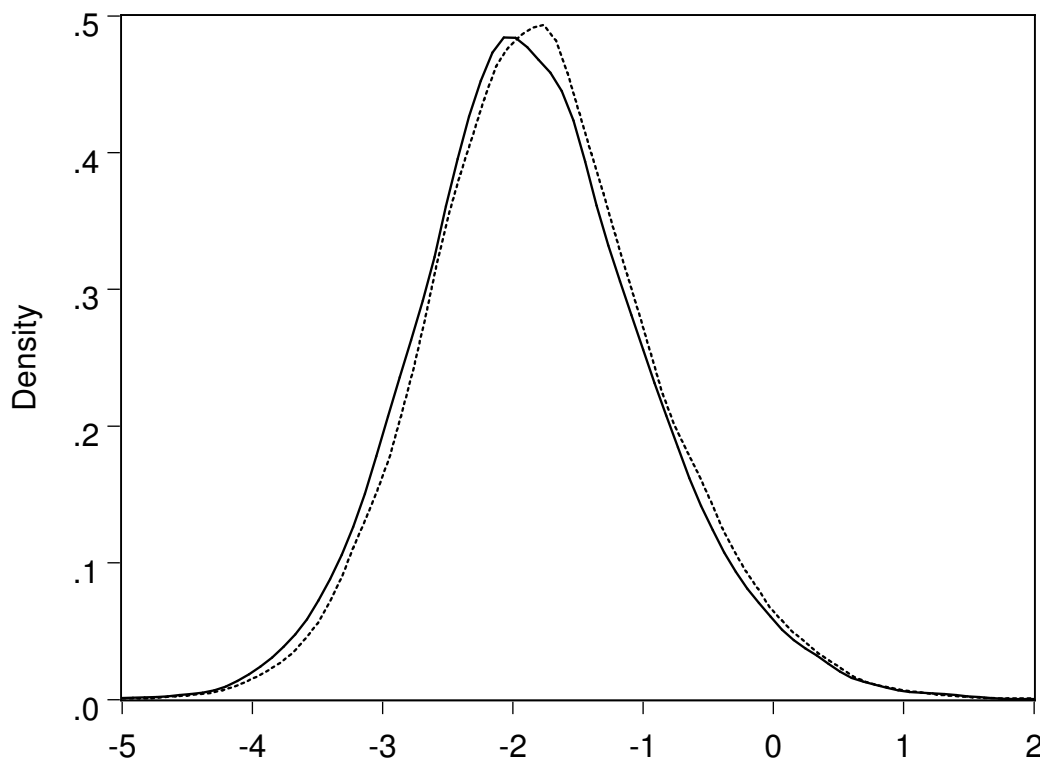
### The Mean-Variance Cointegration Test: Simulating the Distribution of the Test Statistic (section 3.3)

As concerns the simulated critical values, some experiments made clear that these depend on the parameters of the DGPs of the capital gain variance and the spread series. Under

the null, these are the equations in (13) with  $\alpha_c + \beta_c = 1$  and

$$\begin{aligned}\Delta s_{t+1} &= \sum_{i=1}^{p_s} a_{s,i} \Delta s_{t+1-i} + \varepsilon_{s,t+1} , \\ h_{s,t+1} &= \omega_s + \alpha_s \varepsilon_{s,t}^2 + \beta_s h_{s,t} ,\end{aligned}\tag{22}$$

where the first-difference autoregression is implied by the unit root in the level of the spread series;  $\text{Var}[s_{t+1}|I_t] = E[\varepsilon_{s,t+1}^2 | I_t] \equiv h_{s,t+1}$ . Hence, we simulated three different distributions of the test statistic in the error correction models (16), (17) and (18) with parameters in (13) and (22) according to our empirical estimates. Figure 4 exemplarily illustrates the dependence on the parameters of the variance process  $h_{c,t+1}$ ; the DGP (13).



**Figure 4:** Kernel Density Estimates of the Small-Sample Distribution of the Cointegration Test Statistic

Note: The solid line shows the density of the test statistic in model (18) with  $\alpha_c = 0.033$ . The dotted line describes the density for  $\alpha_c = 0.3$  with all other parameters unchanged. Changing  $\alpha_c$  moves the 5% quantile from  $-3.247$  to  $-3.149$ .

It contains two Epanechnikov kernel density estimates<sup>22</sup> of the distributions of the test statistics for  $\alpha_c = 0.033$  and  $\alpha_c = 0.3$  ( $\beta_c = 1 - \alpha_c$ ) with all other parameters unchanged and equal to our estimates for  $C10$  and  $\Delta S10$  in (13) and (22). It can be seen that the increase in  $\alpha_c$  shifts the distribution to the right. The mean (variance) changes from  $-1.841$  ( $0.791$ ) to  $-1.771$  ( $0.765$ ). Both distributions are slightly skewed ( $0.244$  and  $0.226$ ) and exhibit kurtosis of  $3.404$  and  $3.359$ . For  $\alpha_c \rightarrow 0$  the distribution moves leftwards but critical values change only at the second decimal place.

The subsequent steps sketch the simulation of the test statistic of the mean-variance cointegration test proposed in section 3.3.

- Step 1.* Set initial values  $h_{c,0}$  and  $h_{s,0}$  in (13) and (22) equal to the mean of squares of  $\hat{\varepsilon}_{c,t+1}$  and  $\hat{\varepsilon}_{s,t+1}$ , respectively.<sup>23</sup>
- Step 2.* Draw two random samples of size  $N = 783$  (equal to the number of observations in the present analysis) from a standard normal distribution. These random shocks are denoted by  $\xi_{c,t+1}$  and  $\xi_{s,t+1}$ .
- Step 3.* Generate data recursively according to (13) and (22) with  $\varepsilon_{c,t+1} = \xi_{c,t+1} \sqrt{h_{c,t+1}}$  and  $\varepsilon_{s,t+1} = \xi_{s,t+1} \sqrt{h_{s,t+1}}$ .
- Step 4.* Estimate model (13) via ML (BHHH algorithm) and save  $\hat{h}_{c,t+1}$ .
- Step 5.* Estimate model (15) via ML using the generated spread series from *Step 3* and the estimated capital gain variance from *Step 4* and save the  $t$ -value of  $\hat{\rho}$  based on robust standard errors following BOLLERSLEV and WOOLDRIDGE (1992).
- Step 6.* Repeat *Step 1* to *Step 5* 100,000 times.
- Step 7.* Calculate the 1.00, 5.00 and 10.00 percentiles from the distribution of the  $t$ -value of  $\hat{\rho}$ .

## References

- ANG, A. and M. PIAZZESI (2003). A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables. *Journal of Monetary Economics*, 50(4):745–787.
- BAILLIE, R.T., T. BOLLERSLEV, and H.O. MIKKELSEN (1996). Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, 74(1):3–30.

---

<sup>22</sup>We use data-based bandwidth selection according to SILVERMAN (1986).

<sup>23</sup>Dependence on the initial values turned out to be negligible. The initial values of  $s_t$ ,  $t = 0, \dots, p_s$  are arbitrarily chosen in the sense that they are realizations of two standard normally distributed random variables.

- BALFOUSSIA, H. and M.R. WICKENS (2007). Macroeconomic Sources of Risk in the Term Structure. *Journal of Money, Credit and Banking*, 39(1):205–236.
- BALI, T.G. and R.F. ENGLE (2010). The Intertemporal Capital Asset Pricing Model with Dynamic Conditional Correlations. *Journal of Monetary Economics*, 57(4):377–390.
- BANERJEE, A., J. DOLADO, and R. MESTRE (1998). Error-Correction Mechanism Tests for Cointegration in a Single-Equation Framework. *Journal of Time Series Analysis*, 19(3):267–283.
- BOLLERSLEV, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, 31(3):307–327.
- BOLLERSLEV, T. and J. WOOLDRIDGE (1992). Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances. *Econometric Reviews*, 11(2):143–172.
- BOLLERSLEV, T., R.F. ENGLE, and J.M. WOOLDRIDGE (1988). A Capital Asset Pricing Model with Time-Varying Covariances. *Journal of Political Economy*, 96(1):116–131.
- BUSCH, T. (2005). A Robust LR Test for the GARCH Model. *Economics Letters*, 88(3):358–364.
- CAMPBELL, J.Y. and R.J. SHILLER (1987). Cointegration and Tests of Present Value Models. *Journal of Political Economy*, 95(5):1062–1088.
- CAMPBELL, J.Y. and R.J. SHILLER (1991). Yield Spreads and Interest Rate Movements: A Bird’s Eye View. *Review of Economic Studies*, 58(3):495–514.
- CAPORALE, B. and T. CAPORALE (2008). Political Risk and the Expectations Hypothesis. *Economics Letters*, 100(2):178–180.
- CARSTENSEN, K. (2003). Nonstationary Term Premia and Cointegration of the Term Structure. *Economics Letters*, 80(3):409–413.
- CHARLES, A. and O. DARNÉ (2008). A Note on Unit Root Tests and GARCH Errors: A Simulation Experiment. *Communications in Statistics - Simulation and Computation*, 37(2):314–319.
- CHRISTIANSEN, C. (2000). Macroeconomic Announcement Effects on the Covariance Structure of Government Bond Returns. *Journal of Empirical Finance*, 7(5):479–507.
- COCHRANE, J. (2001). *Asset Pricing*. Princeton University Press, Princeton, New Jersey.
- COCHRANE, J.H. and M. PIAZZESI (2005). Bond Risk Premia. *The American Economic Review*, 95(1):138–160.
- CONNOLLY, R.A., Z.N. GÜNER, and K.N. HIGHTOWER (2007). Evidence on the Extent and Potential Sources of Long Memory in U.S. Treasury Security Returns and Yields. *Journal of Money, Credit and Banking*, 39(2–3):689–702.

- COX, J.C., J.E. INGERSOLL, and S.A. ROSS (1985). A Theory of the Term Structure of Interest Rates. *Econometrica*, 53(2):385–407.
- DAI, Q. and K.J. SINGLETON (2000). Specification Analysis of Affine Term Structure Models. *The Journal of Finance*, 55(5):1943–1978.
- DICKEY, D.A. and W.A. FULLER (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association*, 74(366):427–431.
- ENGLE, R.F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 50(4):987–1007.
- ENGLE, R.F. and BOLLERSLEV, T. (1986). Modelling the Persistence of Conditional Variances. *Econometric Reviews*, 5(1):1–50.
- ENGLE, R.F., D.M. LILIEN, and R.P. ROBINS (1987). Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model. *Econometrica*, 55(2):391–407.
- ENGLE, R.F., V.K. NG, and M. ROTHSCILD (1990). Asset Pricing with a Factor-ARCH Covariance Structure: Empirical Estimates for Treasury Bills. *Journal of Econometrics*, 45(1-2):213–237.
- ENGSTED, T. and C. TANGGAARD (1994). Cointegration and the US Term Structure. *Journal of Banking & Finance*, 18(1):167–181.
- EVANS, M.D.D. and K.K. LEWIS (1994). Do Stationary Risk Premia Explain It All?: Evidence from the Term Structure. *Journal of Monetary Economics*, 33(2):285–318.
- GHYSELS, E., P. SANTA-CLARA, and R. VALKANOV (2005). There is a Risk-Return Trade-Off After All. *Journal of Financial Economics*, 76(3):509–548.
- GÜRKAYNAK, R.S. and J.H. WRIGHT (2011, forthcoming). Macroeconomics and the Term Structure. *Journal of Economic Literature*.
- HALL, A.D., H.M. ANDERSON, and C.W.J. GRANGER (1992). A Cointegration Analysis of Treasury Bill Yields. *The Review of Economics and Statistics*, 74(1):116–126.
- HANSEN, P.R. (2003). Structural Changes in the Cointegrated Vector Autoregressive Model. *Journal of Econometrics*, 114(2):261 – 295.
- HECQ, A. (1996). IGARCH Effect on Autoregressive Lag Length Selection and Causality Tests. *Applied Economics Letters*, 3(5):317–23.
- IBBOTSON and ASSOCIATES (1994). *Stocks, Bonds, Bills, and Inflation Yearbook*. Ibbotson Associates, Chicago.
- JOHNSON, P.A. (1994). On the Number of Common Unit Roots in the Term Structure of Interest Rates. *Applied Economics*, 26(8):815–20.
- JONES, C.M., O. LAMONT, and R.L. LUMSDAINE (1998). Macroeconomic News and Bond Market Volatility. *Journal of Financial Economics*, 47(3):315–337.



- KIM, K. and P. SCHMIDT (1993). Unit Root Tests with Conditional Heteroskedasticity. *Journal of Econometrics*, 59(3):287–300.
- KOUROGENIS, N. and N. PITTIS (2008). Testing for a Unit Root Under Errors with Just Barely Infinite Variance. *Journal of Time Series Analysis*, 29(6):1066–1087.
- KUGLER, P. (1988). An Empirical Note on the Term Structure and Interest Rate Stabilization Policies. *The Quarterly Journal of Economics*, 103(4):789–792.
- LAMOUREUX, C.G. and W.D. LASTRAPES (1990). Persistence in Variance, Structural Change, and the GARCH Model. *Journal of Business & Economic Statistics*, 8(2):225–234.
- LING, S., t. and M. MCALEER (2003). Estimation and Testing for Unit Root Processes with GARCH (1, 1) Errors: Theory and Monte Carlo Evidence. *Econometric Reviews*, 22(2):179–202.
- LINTNER, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics*, 47(1):13–37.
- LUCAS, R.E. (1978). Asset Prices in an Exchange Economy. *Econometrica*, 46(6):1429–1445.
- LUMSDAINE, R.L. (1995). Finite-Sample Properties of the Maximum Likelihood Estimator in GARCH(1,1) and IGARCH(1,1) Models: A Monte Carlo Investigation. *Journal of Business & Economic Statistics*, 13(1):1–10.
- LUNDBLAD, C. (2007). The Risk Return Tradeoff in the Long Run: 1836-2003. *Journal of Financial Economics*, 85(1):123–150.
- MANKIW, N.G. and J.A. MIRON (1986). The Changing Behavior of the Term Structure of Interest Rates. *The Quarterly Journal of Economics*, 101(2):211–228.
- MELINO, A. (1988). The Term Structure of Interest Rates: Evidence and Theory. *Journal of Economic Surveys*, 2(4):335–366.
- NELSON, D.B. (1990). Stationarity and Persistence in the GARCH(1,1) Model. *Econometric Theory*, 6(3):318–334.
- NELSON, D.B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, 59(2):347–370.
- PAGAN, A.R., A.D. HALL, and V. MARTIN (1996). Modelling the Term Structure. *Handbook of Statistics*, (14):91–118.
- RUBINSTEIN, M (1976). The Valuation of Uncertain Income Streams and the Pricing of Options. *The Bell Journal of Economics*, 7(2):407–425.
- SEO, B. (1999). Distribution Theory for Unit Root Tests with Conditional Heteroskedasticity. *Journal of Econometrics*, 91(1):113–144.

- SEO, B. (2007). Asymptotic Distribution of the Cointegrating Vector Estimator in Error Correction Models with Conditional Heteroskedasticity. *Journal of Econometrics*, 137(1):68–111.
- SHARPE, W.F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance*, 19(3):425–442.
- SHEA, G.S. (1992). Benchmarking the Expectations Hypothesis of the Interest-Rate Term Structure: An Analysis of Cointegration Vectors. *Journal of Business & Economic Statistics*, 10(3):347–366.
- SHILLER, R.J. (1979). The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure. *The Journal of Political Economy*, 87(6):1190–1219.
- SHILLER, R.J. (1990). The Term Structure of Interest Rates. In B.M. FRIEDMAN and F.H. HAHN, editors, *Handbook of Monetary Economics*, pages 627–722. North-Holland, Amsterdam.
- SILVERMAN, B.W. (1986). *Density Estimation for Statistics and Data Analysis*. Chapman and Hall, London, UK.
- SMITH, P. and M.R. WICKENS (2002). Asset Pricing with Observable Stochastic Discount Factors. *Journal of Economic Surveys*, 16(3):397–446.
- STOCK, J.H. (1987). Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors. *Econometrica*, 55(5):1035–1056.
- TZAVALLIS, E. and M.R. WICKENS (1995). The Persistence in Volatility of the US Term Premium 1970-1986. *Economics Letters*, 49(4):381–389.
- VASICEK, O. (1977). An Equilibrium Characterization of the Term Structure. *Journal of Financial Economics*, 5(2):177–188.
- WOLTERS, J. (1995). On the Term Structure of Interest Rates – Empirical Results for Germany. *Statistical Papers*, 36(1):193–214.
- WOLTERS, J. (1998). Cointegration and German Bond Yields. *Applied Economics Letters*, 5(8):497–502.
- ZHANG, H. (1993). Treasury Yield Curves and Cointegration. *Applied Economics*, 25(3):361–67.
- ZIVOT, E. and W.K. ANDREWS (1992). Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis. *Journal of Business & Economic Statistics*, 20(1):25–44.

# SFB 649 Discussion Paper Series 2011

For a complete list of Discussion Papers published by the SFB 649, please visit <http://sfb649.wiwi.hu-berlin.de>.

- 001 "Localising temperature risk" by Wolfgang Karl Härdle, Brenda López Cabrera, Ostap Okhrin and Weining Wang, January 2011.
- 002 "A Confidence Corridor for Sparse Longitudinal Data Curves" by Shuzhuan Zheng, Lijian Yang and Wolfgang Karl Härdle, January 2011.
- 003 "Mean Volatility Regressions" by Lu Lin, Feng Li, Lixing Zhu and Wolfgang Karl Härdle, January 2011.
- 004 "A Confidence Corridor for Expectile Functions" by Esra Akdeniz Duran, Mengmeng Guo and Wolfgang Karl Härdle, January 2011.
- 005 "Local Quantile Regression" by Wolfgang Karl Härdle, Vladimir Spokoiny and Weining Wang, January 2011.
- 006 "Sticky Information and Determinacy" by Alexander Meyer-Gohde, January 2011.
- 007 "Mean-Variance Cointegration and the Expectations Hypothesis" by Till Strohsal and Enzo Weber, February 2011.

**SFB 649, Ziegelstraße 13a, D-10117 Berlin**  
**<http://sfb649.wiwi.hu-berlin.de>**

This research was supported by the Deutsche  
Forschungsgemeinschaft through the SFB 649 "Economic Risk".

