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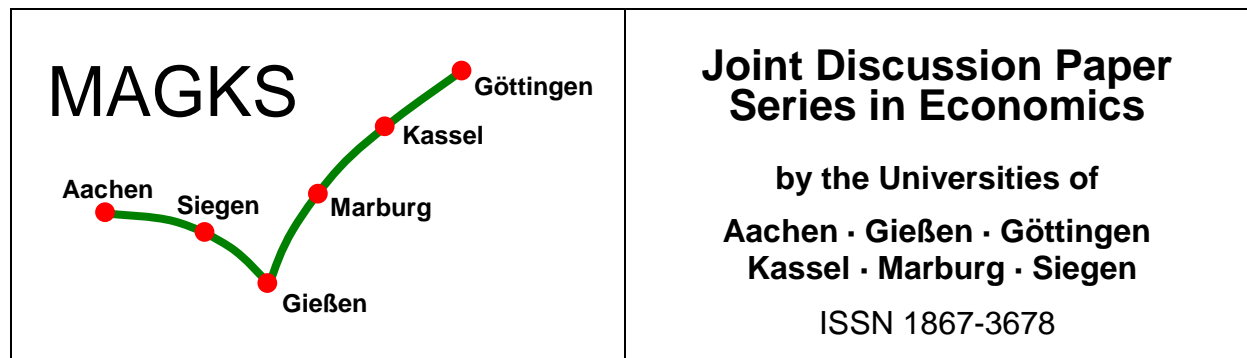
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**No. 29-2011**

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# Threshold effects in the monetary policy reaction function of the Deutsche Bundesbank \*

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(Second version, July 2011)

## Abstract

We estimate monetary policy reaction functions with threshold effects for the Deutsche Bundesbank using a real-time data set. Estimates based on the deviation of inflation from the Bundesbank's inflation target as threshold variable suggest a switch to a stronger output gap response in the reaction function if past inflation was high. The reaction function in the regime with higher inflation implies an overall less contractionary monetary policy than that for the low inflation regime. A modified model with three regimes shows this result to be related to periods of substantial excess inflation. We explore a threshold reaction function with a moving inflation target that captures a gradual adjustment of an intermediate to a long-term inflation target and find the Bundesbank to follow a more restrictive monetary policy stance if inflation is above the intermediate-term inflation target.

*Keywords:* monetary policy reaction function, threshold regression, instrumental variables, real-time data

*JEL Classification:* E52, E58, C24, C26

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# 1 Introduction

This paper presents evidence on threshold effects in the monetary policy reaction function of the Deutsche Bundesbank. We find significant changes to the Bundesbank's reaction function depending on the size of the deviation of actual inflation from the Bundesbank's target.

In the two decades before monetary policy in Germany was finally handed over to the European Central Bank (ECB) the Bundesbank established a reputation for successfully maintaining price stability even when faced with adverse shocks as in the late 1970s and early 1990s. The ECB now has the same primary objective of price stability that guided the Bundesbank's monetary policy. With the intention of enabling the ECB to pursue a similarly successful monetary policy, the objectives and many institutional features of the ECB were set up in close resemblance to those of the Bundesbank. Hence, empirical investigations into how the Bundesbank conducted monetary policy can provide helpful insights for the monetary policy of the ECB.

In this study we focus on nonlinearities in the Bundesbank's monetary policy reaction. We extend a monetary policy reaction based on a standard forward-looking Taylor rule by including shifts in the reaction function coefficients depending on past inflation. Surprisingly, our results show that the Bundesbank's reaction function which is associated with high inflation in the previous quarter implies actually a less restrictive monetary policy than the reaction function that prevails in times of previously low inflation. We show this feature to be robust across a broad range of specifications. Only by specifying a moving intermediate-term inflation target for the Bundesbank similar to the one assumed for the Fed in Bunzel and Enders (2010) we find evidence that an increase in inflation above this moving intermediate inflation target triggers a much more restrictive monetary policy relative to the monetary policy reaction function for inflation rates below the target.

Compared to the estimates for the Fed presented in Bunzel and Enders (2010) the

Bundesbank's response to inflation did not change much across regimes and was larger (smaller) than that of the Fed if inflation was low (high). In all regimes the Bundesbank reacted less than the Fed to the output gap.

This paper adds to the literature on estimated monetary policy reaction functions based on real-time data. While there is an extensive literature on the Federal Reserve's reaction function estimated from real-time data (e.g. Boivin, 2006; Orphanides, 2001) the reaction functions of the ECB and particularly that of the Bundesbank have received less attention. Gerdesmeier and Roffia (2004) and Sauer and Sturm (2007) present real-time estimates for Taylor rules for the ECB while Clausen and Meier (2005) and Gerberding et al. (2005) estimate Taylor rules for the Bundesbank from real-time data. The data set compiled by Gerberding et al. (2005) is more extensive than the one considered by Clausen and Meier (2004) because it includes additional time series for the Bundesbank's implicit inflation target ("price norm") and for the Bundesbank's own estimates of potential output. In this paper we use this data set which the authors kindly made available to us.

The central modification in our empirical models compared to the literature is the inclusion of a threshold effect. Univariate threshold models have been introduced by Tong (1978) and Tong and Lim (1980) and allow for the coefficients in an estimated equation to shift dependent on the value of a threshold variable.<sup>1</sup>

Only few other empirical studies consider threshold effects in a monetary policy reaction function. The most recent and most similar one to this paper is by Bunzel and Enders (2010). They estimate Taylor rules for the Federal Reserve (Fed) with threshold effects but consider only reaction functions in which the Fed reacts to current inflation and output gaps. The empirical literature, however, suggests that forward-looking monetary policy reaction functions in which the central bank responds to forecasts of future inflation and output gaps are more appropriate descriptions of monetary policy (e.g. Clarida et al. 1998). In our paper we apply the threshold instrumental vari-

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<sup>1</sup>See Tong (1990) for an extensive survey.

able estimator suggested by Caner and Hansen (2004) which enables us to investigate threshold effects in forward-looking monetary policy reaction functions.

Bec et al. (2002) estimate Taylor rules with threshold effects for the Banque de France, the Deutsche Bundesbank and the U.S. Fed. Our paper differs from theirs in important ways. First, we use a real-time data set that approximates more closely the information policymakers at the Bundesbank responded to than the ex-post revised data used in their study. Second, they assume the central banks' reaction function to switch depending on the sign of the output gap. In contrast, our focus is on inflation as the variable triggering regime changes in monetary policy and we estimate the numerical threshold value instead of just assuming it.

As in our paper Castro (2008) focuses on regime changes in monetary policy caused by inflation. Similarly to Petersen (2007) he estimates smooth transition models for the monetary policy reaction functions of the U.S. Fed and the ECB. He shows that Euro area inflation in excess of the ECB's inflation target of 2% leads to a stronger response of the ECB to inflation and the output gap but does not derive his results from real-time data. Martin and Milas (2004) and Taylor and Davradakis (2006) use threshold models to study the Bank of England's monetary policy reaction function. Both studies present evidence for the Bank of England to tighten monetary policy in a non-linear way if inflation moves out of a zone around the inflation target.

Generally, this literature finds evidence for significant threshold effects in the monetary policy reaction functions of the various central banks. Above the inflation threshold the central banks tend to react more aggressively to both inflation and to the output gap. While we present similar results for the Bundesbank our findings differ from those in the literature in one important aspect: We do not find a uniformly and significantly stronger response to inflation if inflation exceeds the estimated threshold.

Our paper proceeds as follows: Section 2 describes the structure of the monetary policy reaction function and explains how to estimate a threshold version of it. Section 3 provides some information on the data. Section 4 presents the results for various

versions of our threshold monetary policy reaction function and Section 5 concludes.

## 2 The threshold model for the Bundesbank's reaction function

As a starting point for investigating the Bundesbank's monetary policy reaction function we use a forward-looking Taylor rule with partial adjustment of the actual interest rate (Clarida et al., 1998). The Taylor rule specifies how the short-term interest rate controlled by the central bank responds to forecasts of inflation and of the output gap and can be written in reduced form as

$$\dot{i}_t = \gamma_0 + \gamma_\pi E_t \pi_{t+n} + \gamma_y E_t y_{t+m} + \gamma_i \dot{i}_{t-1} + \nu_t, \quad (1)$$

with  $\gamma_0$ ,  $\gamma_\pi$ ,  $\gamma_y$  and  $\gamma_i$  as coefficients.  $n$  and  $m$  are the central bank's forecast horizons for the inflation rate  $\pi$  and for the output gap  $y$ . The autoregressive term captures the gradual adjustment of the interest rate to the level desired by the central bank.

If the central bank's own internal forecasts for inflation and for the output gap are not observable the standard approach is to replace them by their initial (unrevised) estimates in quarters  $t+n$  and  $t+m$ ,  $(\pi_{t+n|t+n}, y_{t+m|t+m})$

$$\dot{i}_t = \gamma_0 + \gamma_\pi \pi_{t+n|t+n} + \gamma_y y_{t+m|t+m} + \gamma_i \dot{i}_{t-1} + \epsilon_t. \quad (2)$$

The error term  $\epsilon_t$  summarizes both the approximation error  $\nu_t$  and the forecast errors for the inflation rate and for the output gap,

$$\epsilon_t = \gamma_\pi (E_t \pi_{t+n} - \pi_{t+n|t+n}) + \gamma_y (E_t y_{t+m} - y_{t+m|t+m}) + \nu_t. \quad (3)$$

Accounting for the correlation between the explanatory variables  $(\pi_{t+n|t+n}$  and  $y_{t+m|t+m})$

and the error term, the parameters of the monetary policy reaction function (2) can be estimated using the generalized methods of moments (GMM) and appropriate instruments.

Estimating a Bundesbank reaction function like equation (2) possibly conceals important nonlinearities. As discussed in the introduction, some empirical studies have found evidence for threshold effects in the monetary policy reaction function of many central banks, i.e. for endogenous regime-shifts in the reaction coefficients depending on the state of the economy. One cause of such nonlinearities might be asymmetries in the central bank's loss function, such as e.g. the central bank attaching different importance to positive deviations of inflation from its target compared to negative deviations of the same size (e.g. Bunzel and Enders, 2010, pp. 936). Another explanation for a nonlinear monetary policy reaction function is explored by Aksoy et al. (2006) and Orphanides and Wilcox (2003). They propose a loss function for the central bank that implies a target zone for the inflation rate. As long as inflation remains within the target zone monetary policy remains passive. If a shock, however, pushes inflation above this range the central bank responds vigorously. Cukierman (1992) and Cukierman and Meltzer (1992) argue that concerns of the central bank about a loss of public confidence in its commitment to the inflation target cause a more aggressive central bank response to sizable inflationary excesses than to small ones.

One way to model empirically the dependence of the central bank reaction function on the state of the economy is a threshold model. Using (2) as a starting point a threshold reaction function with two regimes can be written as

$$\begin{aligned}
 i_t = & (\alpha_0 + \alpha_1\pi_{t+n|t+n} + \alpha_2y_{t+m|t+m} + \alpha_3i_{t-1})I_t(x_{t-d} \leq \tau) \\
 & + (1 - I_t(x_{t-d} \leq \tau))(\beta_0 + \beta_1\pi_{t+n|t+n} + \beta_2y_{t+m|t+m} + \beta_3i_{t-1}) + \eta_t. \quad (4)
 \end{aligned}$$

$I_t$  is an indicator which takes on the value of one if the threshold variable  $x_{t-d}$  does not exceed the threshold value  $\tau$  in period  $t - d$  and zero otherwise. The model (4)



implies two piecewise linear reaction functions. If the threshold variable is less than or equal to  $\tau$  the reaction function is given by  $\alpha_0 + \alpha_1\pi_{t+n|t+n} + \alpha_2y_{t+m|t+m} + \alpha_3i_{t-1} + \eta_t$  otherwise it is given by  $\beta_0 + \beta_1\pi_{t+n|t+n} + \beta_2y_{t+m|t+m} + \beta_3i_{t-1} + \eta_t$ .

A test for the presence of threshold effects is a test of the hypothesis  $H_0 : \alpha_0 = \beta_0, \alpha_1 = \beta_1, \alpha_2 = \beta_2, \alpha_3 = \beta_3$ . Since the threshold value  $\tau$  is an unidentified nuisance parameter under the null hypothesis it is not possible to employ a standard F-test. Following Hansen (1996, 1997) and Caner and Hansen (2004) we construct a Wald statistic ( $supW$ ) for the null hypothesis of no threshold effects as the supremum of Wald statistics for  $H_0$  for each potential threshold value. P-values are obtained from the empirical distribution of  $supW$  constructed by Monte-Carlo simulation (Caner and Hansen, 2004, pp. 823).

For a given threshold variable  $x_{t-d}$  the threshold estimate  $\hat{\tau}$  is selected by a grid search over all potential thresholds using the sum of squared residuals as selection criterion. An adequate number of observations on each side of the threshold is ensured by considering only those candidate values which leave at least 20% of the observations in each regime.

Bunzel and Enders (2010) estimate such a model for the Fed with real-time observations for inflation and for the output gap. They try to avoid the problem of correlation between the explanatory variables and the error term by using current inflation rate and output gap as right-hand-side variables. However, information on the output gap is not available within the current quarter. Using its current observation as an explanatory variable overstates the central bank's information set and still causes correlation between the explanatory variables and the error term. A similar but less severe problem applies to using the current quarter's inflation rate for which at least observations on the first two months in the quarter are available.

In contrast, in this paper we estimate the forward-looking version of the threshold model as shown in (4). Using the threshold instrumental variable estimation approach by Caner and Hansen (2004) we can account for the correlation between the explanatory

variables and the error term and achieve consistent estimates of the coefficients in the monetary policy reaction function.

### 3 Data

Most of the first estimates of monetary policy reaction functions (e.g. Clarida et al., 1998) relied on ex-post revised data, i.e. on the latest data vintage available to the researchers. Orphanides (2001, 2003) and others showed the estimates of the reaction coefficients of the Fed to change considerably if the estimation was based on real-time data, i.e. the estimates of macroeconomic variables which were available at the point in time the monetary policy decisions were taken.

Estimates of monetary policy reaction functions for the Bundesbank based on real-time data were presented by Clausen and Meier (2005) and Gerberding et al. (2005). Clausen and Meier constructed a real-time data set with observations on GDP retrieved from Bundesbank publications and derived real-time output gap estimates by filtering these series. Gerberding et al. augmented this data set by real-time observations of potential output, consumer prices and money growth rates from the Bundesbank's own publications and internal briefing documents. The data set for potential output enabled them to construct real-time observations of the output gap as perceived by the Bundesbank.<sup>2</sup> Using the very same data set we study the Bundesbank's reaction function including the possibility of threshold effects. We estimate the monetary policy reaction function using quarterly observations since information on the output gap is only available at this frequency. Following Gerberding et al. (2005) we use for the right-hand side variables quarterly averages of the annual percentage change in the consumer price index and quarterly estimates of the output gap. The dependent variable is the end of quarter observation of the three-month interest rate. This ensures that the

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<sup>2</sup>For details, see Gerberding et al. (2005), pp. 279. This data set is available at the Bundesbank's website.

information on the right-hand side variables was indeed available to policymakers at the time of the interest rate decision. Our sample period is 1979Q1 to 1998Q4. This excludes the turbulent period before the introduction of the European Exchange Rate Mechanism and the years in which the Bundesbank’s strategy of monetary targeting had not settled down yet.

As mentioned before, the data set also includes observations on the Bundesbank’s target rate of inflation. This allows us to estimate a threshold model with switches in the reaction function dependent on the deviation of inflation from the Bundesbank’s target instead of using only the level of inflation as in previous studies. We represent the inflation target by the Bundesbank’s “price norm” which entered the derivation of the Bundesbank’s growth target for the money stock. Based on the quantity theory the money growth target for year  $t$  ( $\hat{M}_t^*$ ) was derived as

$$\hat{M}_t^* = \pi_t^* + \hat{Y}_t^{pot} - \hat{V}_t^{trend}.$$

$\hat{Y}_t^{pot}$  is the expected growth rate of potential output over year  $t$ ,  $\hat{V}_t^{trend}$  is the long-run (trend) change in the velocity of circulation and  $\pi_t^*$  is the “price norm”, i.e. the change in the price level that is considered to be consistent with maintaining price stability (Deutsche Bundesbank, 1995, p. 83).

In order to account for the correlation between the future observations of the inflation rate and the output gap and the error term in the reaction function we need a set of instruments which is uncorrelated with the forecast errors but correlated with inflation and output gap forecasts. Assuming rational forecasts the forecast errors are uncorrelated with any information available to policymakers at time  $t$ . Hence, we use as instruments four lags of the interest rate, the period  $t$  (first) estimate of inflation, the period  $t$  (revised) estimates of inflation and of the output gap in the previous four quarters and the period  $t$  value of the inflation target. We cannot use the current quarter’s output gap as an instrument because this information only becomes available with a lag of one quarter. The overidentifying restrictions imposed by the instruments

are tested with Hansen's J-statistic.

## 4 Results

### 4.1 A threshold model with two regimes

Table 1 presents the results of estimating a threshold monetary policy reaction function as a Taylor rule in inflation and the output gap as in equation (4). The results shown are for a version of equation (4) augmented by second autoregressive terms in the interest rate since for many cases we found evidence of significant autocorrelation in the residuals with just one lag of  $i_t$ . An AR(2) specification is also used in Bunzel and Enders (2010) for the Fed and in Beyer et al. (2009) for the Bundesbank, although the latter study does not consider threshold effects. If the second autoregressive term turned out to be not statistically significantly different from zero in a regime we imposed this zero restriction for the coefficient estimates within this regime only.

The threshold variable in Table 1 is the quarter  $t$  estimate of last quarter's inflation deviation from the Bundesbank's inflation target ( $\pi_{t-1|t} - \pi_{t-1}^*$ ). Figure 1 shows the time series of these variables. From the late 1970s on the Bundesbank has gradually lowered its inflation target to two percent from the mid 1980s onwards. Actual inflation exceeded this target strongly in the late 1970s up to the early 1980s and again from the late 1980s to the mid 1990s. From 1986 to 1988 inflation fell considerably short of the target.

Table 1 presents estimates for six versions of forward-looking Taylor rules with different forecast horizons for inflation ( $n$ ) and for the output gap ( $m$ ). As shown in the last two columns there is evidence for significant threshold effects across all rows. The threshold estimates for all specifications are identical and imply a break in the reaction function if inflation is more than 1.3 percentage points above the Bundesbank's inflation target. Comparison of the sum of squared residuals shows the threshold model's fit

$\hat{\tau}$	$\alpha_1$	$\beta_1$	$\alpha_2$	$\beta_2$	$\alpha_3 + \alpha_4$	$\beta_3 + \beta_4$	$P(J_\alpha)$	$P(J_\beta)$	$SSR$	$SSR_f$	$supW$	prob
$n = 2, m = 0$	0.288 (0.048) (***)	0.292 (0.058) (***)	-0.027 (0.025)	0.187 (0.013) (***)	0.927 (0.032) (***)	0.839 (0.044) (***)	0.57	0.06	18.84	25.57	82.12	0.01
$n = 3, m = 0$	0.293 (0.047) (***)	0.273 (0.061) (***)	-0.049 (0.027) (**)	0.166 (0.019) (***)	0.936 (0.030) (***)	0.860 (0.048) (***)	0.47	0.17	18.10	23.25	72.14	0.01
$n = 4, m = 0$	0.337 (0.047) (***)	0.263 (0.047) (***)	-0.041 (0.025) (*)	0.159 (0.024) (***)	0.927 (0.034) (***)	0.862 (0.044) (***)	0.69	0.15	19.92	25.21	71.89	0.01
$n = 2, m = 1$	0.279 (0.049) (***)	0.356 (0.099) (***)	-0.037 (0.027) (*)	0.198 (0.023) (***)	0.931 (0.033) (***)	0.813 (0.057) (***)	0.66	0.12	19.11	26.60	89.790	0.01
$n = 3, m = 1$	0.287 (0.047) (***)	0.263 (0.076) (***)	-0.059 (0.028) (**)	0.191 (0.022) (***)	0.935 (0.031) (***)	0.863 (0.054) (***)	0.54	0.17	18.92	23.71	80.25	0.00
$n = 4, m = 1$	0.336 (0.048) (***)	0.220 (0.059) (***)	-0.050 (0.025) (**)	0.178 (0.026) (***)	0.931 (0.035) (***)	0.870 (0.049) (***)	0.74	0.09	20.70	25.45	63.20	0.01

Standard errors in parentheses. \* / \*\* / \*\*\* denotes significance at the 10%/5%/1% level. Estimation period 1979:1-1998:4. Estimation by threshold IV. Threshold variable is inflation gap in preceding quarter.  $\hat{\tau}$ : estimated threshold value.  $P(J)$ : P-value of J-statistic.  $SSR$ : sum of squared residuals (AR2 model).  $SSR_f$ : sum of squared residuals of linear reaction function (AR2 model).  $supW$ : Wald statistic for null of no threshold effect.  $prob$ : Simulated P-value of  $supW$ .

Table 1: Estimated threshold models (Taylor rules) with two regimes using the inflation gap as threshold variable

( $SSR$ ) to be far superior to that of the linear Taylor rule ( $SSR_f$ ). For the best fitting specification ( $m = 3, n = 0$ ) Figure 2 shows the result for the LR-test from Caner and Hansen (2004) for each possible threshold value in the estimation of model (4). It tests the null hypothesis that the nonlinearity in the monetary policy reaction function can be modelled equally well by the threshold value on the horizontal axis as by the one that minimizes the sum of squared residuals. The two horizontal lines represent the critical values with the dashed line applying to the LR-test corrected for heteroscedastic residuals. Figure 2 shows that the threshold value is estimated very precisely. Only values between one and 1.3 percent turn out to be acceptable as threshold estimates.<sup>3</sup>

Considering the time series of the threshold variable in Figure 1 the second regime clearly is associated with the high inflationary episodes in the late 1970s/early 1980s and in the early 1990s. The point estimates for the inflation coefficients ( $\alpha_1, \beta_1$ ) show a slight decrease after crossing the threshold from below ( $\alpha_1 > \beta_1$ ), except for the specifications with a two-quarter forecast horizon for the inflation rate. However, the changes in the Taylor rule's inflation coefficient mostly do not exceed two standard deviations. In contrast, the coefficients on the output gap are markedly higher (more than two standard deviations) in the above threshold regime ( $\beta_2 > \alpha_2$ ) and always significantly different from zero whereas they are slightly negative in the below threshold regime. The sum of the autoregressive coefficients which represents the extent of interest rate smoothing is smaller in the above threshold regime but mostly drops by less than two standard deviations.

The best fitting specification assumes a forecast horizon of three quarters for the inflation rate. If the inflation rate in the preceding quarter was less than 1.3 percentage points above the Bundesbank's inflation target the reaction function is estimated as

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<sup>3</sup>The results for the other specifications are very similar.

(with standard errors in parentheses)

$$i_t = -0.306 + 0.293E_t\pi_{t+3} - 0.049E_t y_t + 1.316i_{t-1} - 0.380i_{t-2}. \quad (5)$$

(0.204)      (0.047)      (0.027)      (0.121)      (0.122)

The estimated reaction function for the inflation rate in the preceding quarter in excess of the inflation target of more than 1.3 percentage points is

$$i_t = 0.026 + 0.273E_t\pi_{t+3} + 0.166E_t y_t + 0.860i_{t-1}. \quad (6)$$

(0.360)      (0.061)      (0.019)      (0.048)

The estimate of the corresponding linear Taylor rule without threshold effects with a sum of squared residuals of 27.43 is<sup>4</sup>

$$i_t = 0.480 + 0.304E_t\pi_{t+3} + 0.037E_t y_t + 0.815i_{t-1}. \quad (7)$$

(0.194)      (0.057)      (0.030)      (0.027)

The coefficient on the output gap is not significantly different from zero and falls in between the coefficients of the two regimes while the coefficient on the inflation forecast is slightly higher than the one shown for the high inflation regime.

Compared to the results for the Fed presented in Bunzel and Enders (2010), which were obtained using a similar threshold model specification, some interesting differences between the Bundesbank's and the Fed's reaction coefficients emerge in the threshold model.<sup>5</sup> Below the inflation threshold, the Bundesbank's response to inflation is stronger than that of the Fed while the reverse is true for inflation exceeding the threshold estimate. In both regimes the Bundesbank responds less to the output gap than the Fed. While the estimated extent of interest rate smoothing in the below threshold regime is very similar to the estimates in Bunzel and Enders (2010) the sum

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<sup>4</sup>The results are obtained from GMM estimation using an optimal weighting matrix and robust errors with respect to heteroskedasticity and autocorrelation

<sup>5</sup>We compare our results in Table 1 to those in Table 3 in Bunzel and Enders (2010), p. 940. Note, that their estimated reaction function is not forward looking but in current inflation and output gap.

of the autoregressive coefficients is much higher for the Bundesbank than for the Fed if inflation is above the threshold.

How do the two reaction functions from the threshold model compare in terms of monetary policy tightness? Figure 3 presents the fitted interest rates for both the below and above threshold reaction functions. The shaded area indicates the time periods in which the high inflation regime prevailed. It is obvious that the Bundesbank reaction function estimated for the low inflation regime almost always implied interest rates at least as high as the reaction function for the high inflation regime and therefore represents a more restrictive monetary policy. Even in the high inflation periods indicated by the shaded areas the interest rate would have been set higher if the Bundesbank had stuck to the first of the two reaction functions. This result which is robust across all model specifications in Table 1 is in contrast to the results of many other studies for other central banks such as the Fed (Bunzel and Enders, 2010; Castro, 2008), the ECB (Castro, 2008), and the Bank of England (Martin and Milas, 2004; Taylor and Davradikis, 2006). These studies find that the coefficient estimates from the high inflation regime implied a significantly tighter monetary policy than those from the low inflation regime.

## 4.2 A threshold model with three regimes

To explore this issue further we study a threshold model with three regimes. It is possible that the time period in the mid 1980s with inflation well below the Bundesbank's target but an interest rate only slowly trending downward might indicate a third set of reaction coefficients that gets mixed up with the other two sets of coefficients if we restrict ourselves to a model with two regimes only.

The threshold estimates in the second column of Table 2 show that the new low inflation regime is relevant if the inflation rate in the preceding quarter was less than the inflation target plus 0.2 percentage points. It dominates the mid 1980s and is also relevant in the



$\hat{\tau}_{1,2}$	$\alpha_1$	$\beta_1$	$\gamma_1$	$\alpha_2$	$\beta_2$	$\gamma_2$	$\alpha_3 + \alpha_4$	$\beta_3 + \beta_4$	$\gamma_3 + \gamma_4$	$P(J_\alpha)$	$P(J_\beta)$	$P(J_\gamma)$	$SSR$	$SSR_f$	prob
$n = 2, m = 0$	0.325 (0.042)	0.327 (0.091)	0.292 (0.058)	0.004 (0.027)	0.060 (0.051)	0.187 (0.013)	0.952 (0.069)	0.968 (0.053)	0.839 (0.044)	0.33	0.98	0.06	16.51	25.57	0.00
$n = 3, m = 0$	0.265 (0.041)	0.318 (0.079)	0.273 (0.061)	-0.037 (0.034)	0.059 (0.039)	0.166 (0.019)	0.931 (0.069)	0.971 (0.056)	0.860 (0.048)	0.49	0.98	0.15	16.86	23.25	0.00
$n = 4, m = 0$	0.309 (0.041)	0.316 (0.079)	0.263 (0.047)	-0.061 (0.034)	0.026 (0.039)	0.159 (0.024)	0.890 (0.069)	1.008 (0.056)	0.862 (0.444)	0.45	0.92	0.15	19.09	25.21	0.00
$n = 2, m = 1$	0.331 (0.043)	0.361 (0.099)	0.356 (0.099)	0.004 (0.034)	0.025 (0.076)	0.198 (0.023)	0.955 (0.073)	0.969 (0.055)	0.814 (0.057)	0.38	0.97	0.12	16.27	26.60	0.00
$n = 3, m = 1$	0.260 (0.045)	0.392 (0.094)	0.263 (0.076)	-0.043 (0.035)	0.010 (0.053)	0.191 (0.022)	0.930 (0.075)	0.948 (0.068)	0.864 (0.054)	0.54	0.83	0.17	16.78	23.71	0.00
$n = 4, m = 1$	0.312 (0.043)	0.375 (0.093)	0.220 (0.059)	-0.071 (0.030)	0.008 (0.031)	0.178 (0.026)	0.857 (0.081)	0.966 (0.065)	0.870 (0.049)	0.56	0.94	0.10	19.64	25.45	0.00

Standard errors in parentheses. \*/\*\*/\*\* denotes significance at the 10%/5%/1% level. Estimation period 1979:1-1998:4. Estimation by threshold IV. Threshold variable is inflation gap in preceding quarter.  $\hat{\tau}_{1,2}$ : estimated threshold values.  $P(J)$ : P-value of J-statistic.  $SSR$ : sum of squared residuals (AR2 model).  $SSR_f$ : sum of squared residuals of linear reaction function (AR2 model). prob: Simulated P-value of  $supW$ -statistic.

Table 2: Estimated threshold models (Taylor rules) with three regimes using the inflation gap as threshold variable

final years before the introduction of the Euro (Figure 4). The second regime applies to inflation rates that deviate more than 0.2 but less than 1.3 percentage points from the Bundesbank's target and the third one applies to inflation rates even higher. All model specifications in Table 2 lead to identical threshold estimates. From the low to the medium and then to the high inflation regime Table 2 shows an inverted U-shaped pattern for the inflation coefficient which is generally highest in the medium inflation regime. However, the size of most of these changes is less than two standard deviations. The response coefficients to the output gap are mostly not significantly different from zero in both the low and the medium inflation regime and increase strongly in the high inflation regime - a result which carries over from the two-regime threshold model. Similarly to Table 1 we also find less interest rate smoothing, i.e. a lower sum of AR coefficients in the high inflation regime.

Figure 5 shows the fitted values for the interest rate that are implied by the three reaction functions for the threshold model with a three-quarter forecast horizon for inflation and a one-quarter forecast horizon for the output gap ( $n = 3, m = 1$ ) which among the models in Table 2 displays some of the most sizable changes in the coefficients. The three panels differ only by the shading which indicates the time periods in which the low inflation (top), medium inflation (middle) and high inflation regime (bottom) prevailed. Again, the reaction coefficients for the high inflation regime (dash-dotted line) imply the least restrictive monetary policy and the lowest interest rates throughout the sample period. Although the fitted interest rates from the two other sets of reaction coefficients are very close to each other the reaction function for the low inflation regime (dotted line) generally implies a higher interest rate than the other two reaction functions if the low inflation regime is relevant.

Since growth rates of monetary aggregates were very important in the communication strategy of the Bundesbank (e.g. Deutsche Bundesbank, 1995; von Hagen, 1999) we also estimated threshold models of monetary policy reaction functions augmented by real-time estimates of the deviation of the growth rate of  $M3$  from the Bundesbank's

announced target growth rate and found our main results to be unaffected.<sup>6</sup>

Our results from the three-regimes threshold models so far indicate that the Bundesbank switched to a more restrictive monetary policy stance when inflation exceeded its inflation target by about 0.2 percentage points. However, we also found evidence for a less restrictive monetary policy if inflation was strongly above the target, i.e. in the third regime. One possible explanation for this puzzling result is that the inflation gap in Figure 1 actually overstates the true inflation deviation from target during episodes of drastically higher inflation. Such a mismeasurement of the inflation gap could result from unobserved (temporary) shifts in the inflation target.

### 4.3 A threshold model with a moving average inflation target

The prevalence of the high inflation regime in the early 1980s and in the early 1990s was associated with relatively persistent inflationary shocks hitting the German economy - in the first episode the second oil price shock, in the second episode the German re-unification. Figure 1 shows that during the first inflationary surge the Bundesbank's inflation target was initially at 4%, significantly above the 2% on average in the later sample, and then was revised slowly downwards. In the high inflation period following the re-unification of Germany the inflation target remained constant at 2%. One possibility which might distort our results is that this officially announced inflation target of the Bundesbank which was almost constant from 1983 onwards does not convey the actual intermediate term policy objective.

Bunzel and Enders (2010) replace an implicit constant long-run inflation target in the threshold variable by a moving average intermediate (or *interim*) inflation target which is defined as a moving average of past inflation rates.<sup>7</sup> They propose this specification

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<sup>6</sup>The effect of this modification is that the coefficient on inflation becomes statistically insignificantly different from zero in the high inflation regime. Furthermore, the money growth deviation turns out to be either insignificant in the low inflation regime or enters with the wrong, i.e. negative sign in the high inflation regime. The results are available from the author by request.

<sup>7</sup>Specifically, they define the inflation target as  $\pi_t^* = (\pi_{t-5} + \pi_{t-9})/2$ .

to represent empirically an opportunistic approach to monetary policy (Bomfin and Rudebusch, 2000). In the opportunistic approach to disinflation the central bank accepts temporarily a rise in inflation and remains relatively inactive waiting for favorable shocks or the adjustment of the economy to the steady state to let inflation decline gradually. Bunzel and Enders (2010) threshold specification conditions the inflation target on past inflation. A regime shift in the monetary policy reaction function occurs if inflation is high relative to the value of inflation “inherited” from the past. Bunzel and Enders (2010, p. 943) interpret their estimates of a significant increase in the Fed’s reaction coefficients to inflation and the output gap if inflation is above the intermediate target as evidence for opportunistic behavior of the Fed. However, according to the theoretical models of Bomfin and Rudebusch (2000), Orphanides and Wilcox (2002), and Aksoy et al. (2006) the dependence of the intermediate target on past inflation can be, on its own, already interpreted as evidence in favor of opportunistic behavior.

A specification with an inflation target depending on past inflation is useful for our study as well: First, it allows us to study whether we can find evidence for opportunistic behavior of the Bundesbank. Second, assuming a moving intermediate inflation target can explain some of our puzzling results: Using the three-regimes threshold models we found the Bundesbank’s monetary policy to become less restrictive after very high deviations of inflation from the announced long-term inflation target. Comparing Figures 1 and 3 the coefficient estimates of the high inflation regime are determined by two episodes in which inflation rose quickly and (particularly in the early 1990s) came down only slowly. Both episodes were caused by persistent inflationary shocks (oil price shock and German re-unification) of which the inflationary effects could not be undone quickly. It is possible that the Bundesbank accepted a temporary increase in inflation and raised its intermediate term inflation target without changing the long-run inflation target. This would imply that, particularly during the slow decline in inflation, inflation was actually closer to the (temporary intermediate) inflation target than to the long-run inflation target (the price norm) and, hence, our models from the pre-

vious subsection overstate the deviation in the threshold variable over these episodes and might lead to distorted results. In fact, the Bundesbank emphasized that its price norm of 2% after 1984 applied to the medium-term perspective (Deutsche Bundesbank, 1995, p. 83).

Table 3 presents results for various model specifications with moving average inflation targets. The monetary policy reaction function is specified as in Table 1 with the inflation rate and the output gap as explanatory variables. Specification (1) uses as threshold variable a weighted average of the Bundesbank's official inflation target and of the actually observed inflation rates 5 and 9 quarters ago, i.e.  $\pi_t^* = (\bar{\pi}_t^* + \pi_{t-5} + \pi_{t-9})/3$ , with  $\bar{\pi}_t^*$  as the official inflation targets. This specification combines the announced inflation target with the threshold variable used in Bunzel and Enders (2010). Including the announced inflation target in the weighted average conforms to the suggestion in Bomfim and Rudebusch (2000).

In terms of fit the models are inferior to those based on the long-run inflation target, a result that mirrors the evidence in Bunzel and Enders (2010) for the Fed. The threshold estimate  $\hat{\tau}$  is relatively close to zero for the reaction function with a three quarter forecast horizon for inflation. The difference between the two inflation reaction coefficients is always less than one standard deviation indicating no significant change in the Bundesbank's reaction to inflation. Compared to Bunzel and Enders' (2010) estimate of the Fed's response coefficient to inflation the Bundesbank's response is stronger in the low inflation regime.<sup>8</sup> The estimated coefficient on the output gap is significantly negative in the below threshold regime but significantly positive in the above threshold regime. Finally, the sum of the AR coefficients does not differ by much across regimes but is much smaller than Bunzel and Enders' estimates for the Fed. The constant term in the reaction function which is not shown in the table also increases drastically in the above relative to the below threshold regime.

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<sup>8</sup>See Bunzel and Enders (2010), Table 4, p. 944. Relative to some specifications in their paper the Bundesbank reacted more aggressively to inflation in the above threshold regime as well.

	$\hat{\tau}$	$\alpha_1$	$\beta_1$	$\alpha_2$	$\beta_2$	$\alpha_3 + \alpha_4$	$\beta_3 + \beta_4$	$P(J_\alpha)$	$P(J_\beta)$	$SSR$	$supW$	prob
(1)	$n = 3, m = 0$	-0.128 (0.055)	0.367 (0.076)	-0.119 (0.036)	0.159 (0.032)	0.695 (0.059)	0.701 (0.039)	0.68	0.63	24.60	71.31	0.00
	$n = 4, m = 0$	-0.411 (0.063)	0.399 (0.059)	-0.111 (0.035)	0.132 (0.031)	0.727 (0.055)	0.714 (0.031)	0.79	0.36	26.06	81.37	0.00
(2)	$n = 3, m = 0$	0.000 (0.058)	0.370 (0.075)	-0.117 (0.034)	0.167 (0.030)	0.709 (0.057)	0.708 (0.040)	0.63	0.55	25.16		
	$n = 4, m = 0$	0.000 (0.053)	0.379 (0.063)	-0.159 (0.024)	0.130 (0.034)	0.690 (0.056)	0.710 (0.032)	0.48	0.53	29.11		
(3)	$n = 3, m = 0$	-0.278 (0.066)	0.356 (0.065)	-0.161 (0.032)	0.147 (0.027)	0.736 (0.051)	0.730 (0.034)	0.69	0.30	23.59	89.43	0.00
	$n = 4, m = 0$	-0.278 (0.058)	0.368 (0.056)	-0.156 (0.029)	0.128 (0.031)	0.750 (0.057)	0.722 (0.030)	0.69	0.33	25.32	74.02	0.00
(4)	$n = 3, m = 0$	0.000 (0.059)	0.332 (0.076)	-0.109 (0.033)	0.094 (0.042)	0.625 (0.048)	0.712 (0.042)	0.34	0.78	24.27		
	$n = 4, m = 0$	0.000 (0.062)	0.336 (0.064)	-0.132 (0.032)	0.070 (0.039)	0.633 (0.048)	0.712 (0.034)	0.28	0.67	28.11		

Standard errors in parentheses. \*/\*\*/\*\*\*/ denotes significance at the 10%/5%/1% level. Estimation period 1979:1-1998:4. Estimation by threshold IV. Threshold variable is inflation gap in preceding quarter.  $\hat{\tau}$ : estimated threshold value.  $P(J)$ : P-value of J-statistic.  $SSR$ : sum of squared residuals.  $supW$ : Wald statistic for null of no threshold effect.  $prob$ : Simulated P-value of  $supW$ .

Table 3: Estimated threshold models with two regimes using the inflation gap with a moving average inflation target as threshold variable

Figure 6 shows the fitted interest rate paths for both the above and below threshold reaction function in the first row in Table 6. Clearly, the above threshold reaction function implies a tighter monetary policy stance, i.e. a higher interest rate if inflation is above the estimated threshold. In particular, the interest rate from the above threshold reaction function exceeds the one from the below threshold reaction function in those parts of the shaded regions where interest rates are increasing. Interestingly, the reaction function estimated for the below threshold regime does not always lead to a lower interest rate compared to the other reaction function if inflation is below the threshold. In the 1982-1984 period when the output gap was strongly negative the below threshold reaction function with its negative output gap coefficient led to a higher interest rate than would have resulted from the above threshold reaction function. In this episode the estimated reaction function captures the Bundesbank's reluctance in pushing down the interest rate quickly in response to the deteriorating output conditions. Probably, the negatively estimated output gap coefficient in the below threshold reaction function is partly due to this episode.

The next rows in Table 3 show results for various modifications of the threshold model with a moving average inflation target in order to study the robustness of the results. Specification (2) is based on the same threshold variable but imposes a threshold value of zero. Model (3) shifts the moving average of past inflation rates used to construct the moving inflation target to observations to  $t - 4$  and  $t - 8$ . Finally, the results shown under (4) are those using a threshold variable identical to the one in Bunzel and Enders (2010), i.e. a simple moving average of  $\pi_{t-5}$  and  $\pi_{t-9}$ . These modifications have very little effects on the results. Note that the estimated threshold values for both models in (1) and (3) are negative. Although imposing a zero threshold value in model (2) leaves the results mostly unchanged (compare specifications (1) and (2)) these negative estimates might indicate that our intermediate inflation target overstates the true intermediate inflation target to some extent and that the opportunistic behaviour of the Bundesbank as far as its intermediate inflation target is concerned is actually less

pronounced than assumed by our weighted average of past inflation and the long-run inflation target.

## 5 Conclusions

This paper presented evidence on significant threshold effects in the monetary policy reaction function of the Deutsche Bundesbank with a special focus on using appropriate real-time data to model the information set available to the Bundesbank's policy-makers. Using past deviations of inflation from the Bundesbank's inflation target as threshold variable we showed systematic shifts in the reaction functions across regimes. Specifically, we found that the reaction function triggered by high deviations of inflation from target implied a much stronger response to the output gap. Comparing the implied interest rate paths across the reaction functions in the two regimes we found the coefficient estimates in the high inflation regime to imply a less restrictive monetary policy, i.e. lower interest rates, for almost all historically observed states of the economy. These results were robust across a range of forecast horizons for inflation and the output gap in the monetary policy reaction function. Introducing a third regime associated with the very low inflation rates in the mid 1980s also left our results intact.

These surprising results led us to consider a time-varying intermediate inflation target as an average of the announced inflation target and past inflation rates. Using the deviation of inflation from this moving target we found evidence for the Bundesbank switching to a more restrictive monetary policy regime when inflation was above the intermediate target. This shift in the monetary policy reaction function was associated with a significantly positive response of the Bundesbank to the output gap and an increase in the reaction function's constant. In contrast, the reaction coefficient for inflation is very similar for both regimes.

One interesting aspect of our results is that, in contrast to estimates of monetary policy reaction function with inflation thresholds for other central banks, we do not find evi-



dence for a strong increase in the Bundesbank's reaction to inflation if inflation crosses the estimated threshold value from below. In fact, the strength of the Bundesbank's reaction to inflation appears to be the same independent of the size of the deviation of inflation from target. What does change significantly between regimes is the response to the output gap. A second important finding is that the evidence for opportunistic behavior of the Bundesbank is weaker than that for the Fed. We use a similar approach as Bunzel and Enders (2010) and define an intermediate inflation target as a weighted average of past inflation and the announced inflation target and use this time series to construct the inflation gap threshold variable. In contrast to their results for the Fed we find only a significant increase in the response of the Bundesbank to the output gap but no significant change in the inflation response of the Bundesbank when inflation moved above the intermediate target.

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Figure 1: Inflation rate, price norm and inflation gap.

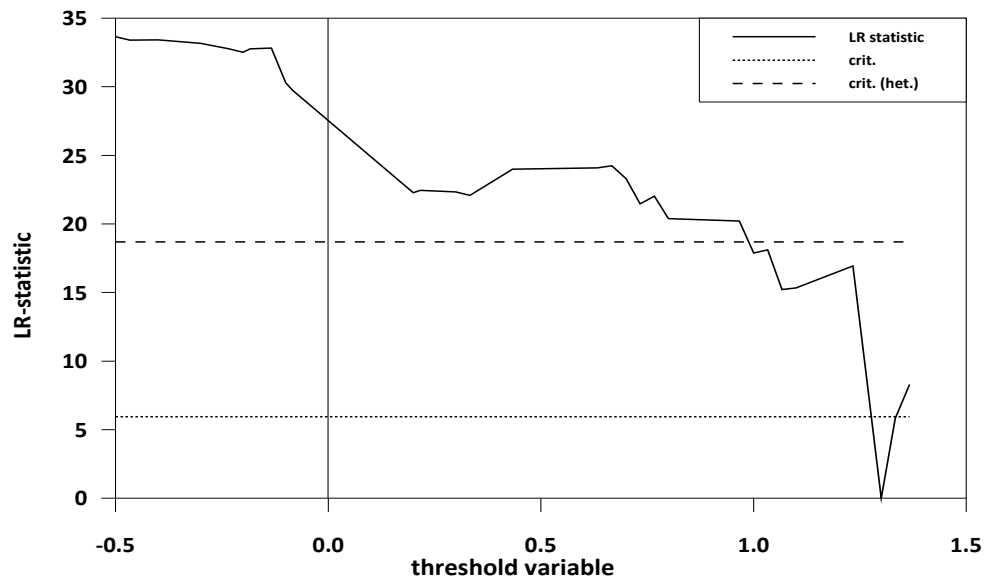


Figure 2: Inflation gap threshold estimation for standard Taylor rule ( $n=3, m=0$ ).

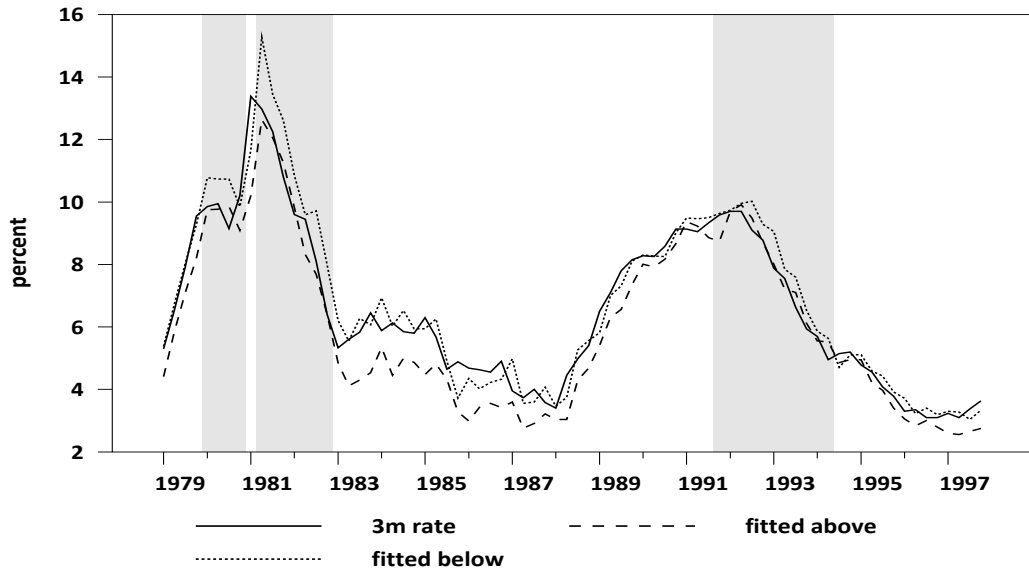


Figure 3: 3-months interest rate and fitted values from both regimes ( $n=3, m=0$ ), inflation gap threshold.

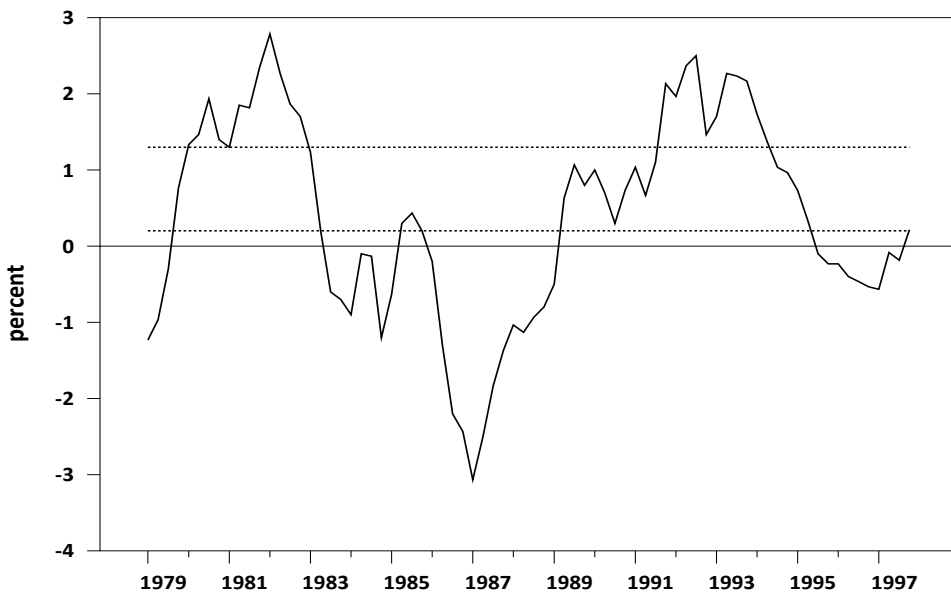


Figure 4: Inflation gap and threshold estimates for three regimes ( $n=3, m=1$ ).

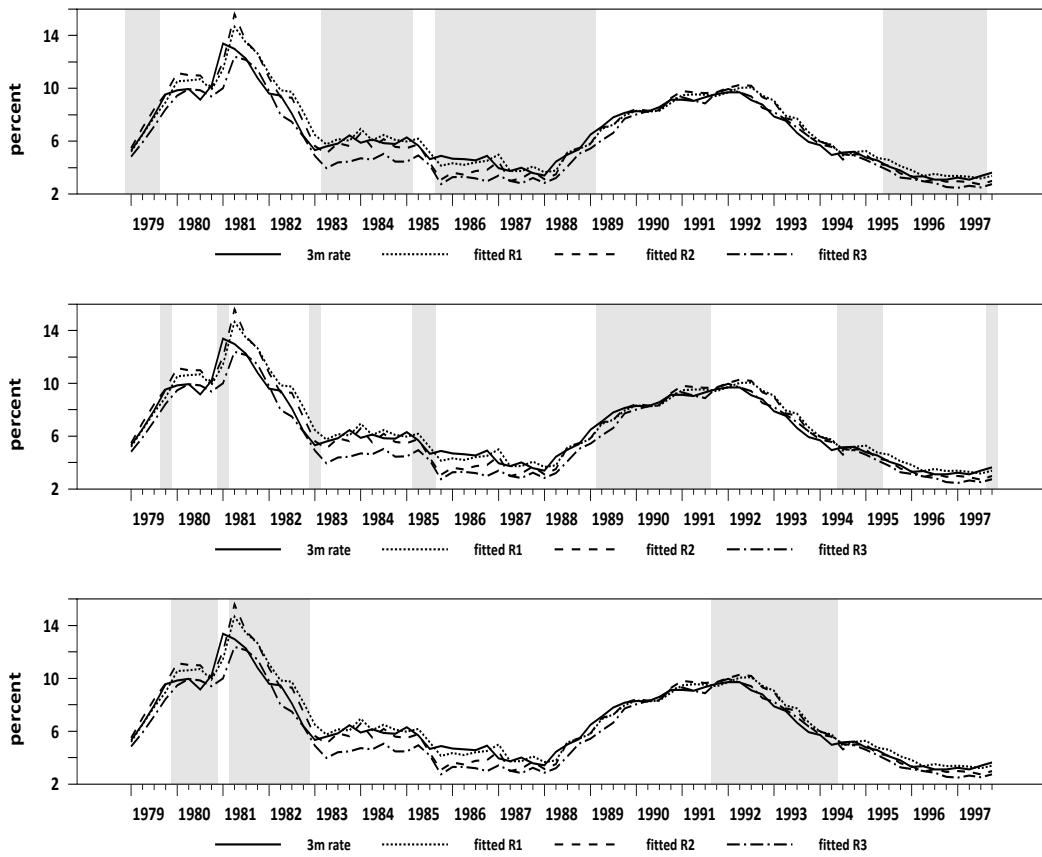


Figure 5: 3-months interest rate and fitted values from three regimes ( $n=3, m=1$ ), inflation gap threshold.

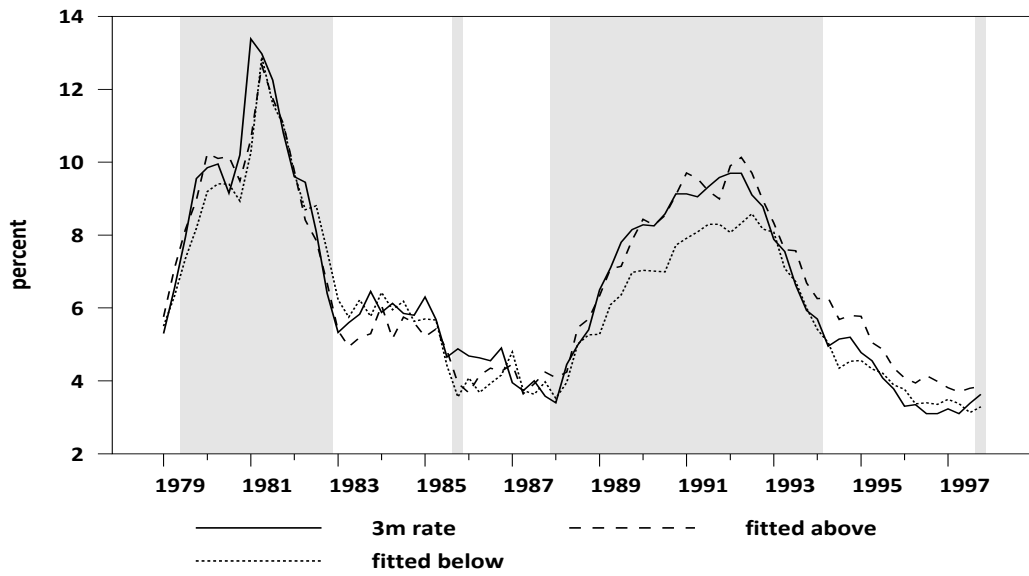


Figure 6: 3-months interest rate and fitted values from two regimes ( $n=3, m=0$ ) with moving average inflation target (Table 3, model (1)).