Bender, Christian M.; Götz, Georg

Working Paper

Coexistence of service- and facility-based competition: The relevance of access prices for "make-or-buy"-decisions

Joint discussion paper series in economics, No. 07-2011

Provided in Cooperation with:
Faculty of Business Administration and Economics, University of Marburg

Suggested Citation: Bender, Christian M.; Götz, Georg (2011) : Coexistence of service- and facility-based competition: The relevance of access prices for "make-or-buy"-decisions, Joint discussion paper series in economics, No. 07-2011, Univ., Dep. of Business Administration & Economics, Marburg

This Version is available at:
http://hdl.handle.net/10419/56526

Standard-Nutzungsbedingungen:
Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden. Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen. Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Coexistence of Service- and Facility-Based Competition: The Relevance of Access Prices for Make-or-Buy"-Decisions
Coexistence of Service- and Facility-Based Competition: The Relevance of Access Prices for “Make-or-Buy”-Decisions

Christian M. Bender*† & Georg Götz*

This Version: February 2011

This paper models competition between two firms, which provide broadband Internet access in regional markets with different population densities. The firms, an incumbent and an entrant, differ in two ways. First, consumers bear costs when switching to the entrant. Second, the entrant faces a make-or-buy decision in each region and can choose between service-based and facility-based entry. The usual trade-off between static and dynamic efficiency does not apply in the sense that higher access fees might yield both, lower retail prices and higher total coverage. This holds despite a strategic effect in the entrant’s investment decision. While investment lowers marginal costs in regions with facility-based entry, it intensifies competition in all regions. We show that the cost-reducing potential of investments dominates the strategic effect: Higher access fees increase facility-based competition, decrease retail prices and increase total demand.

Keywords: Broadband access markets, facility- and service-based entry, investments, economies of density, switching costs

JEL Code: D43, L13, L51, L96

* Justus-Liebig-University Giessen, Chair for Industrial Organization, Regulation and Antitrust (VWL I), Licher Str. 62, D-35394 Giessen, Germany.
† Corresponding author. E-mail: christian.m.bender@wirtschaft.uni-giessen.de

Earlier versions of this paper were presented at the Lisboa Nova Summer School 2009, Vale de Lobo, at the CRESSE conference 2010, Chania, at the EARIE conference 2010, Istanbul, at the European regional ITS conference 2010, Copenhagen, at the Vienna Research Seminar of Regulated Network Industries, Vienna, and at the ICT & Economic Growth conference 2010, Munich. We thank the participants and our colleagues in Giessen for helpful comments and discussions. Bender gratefully acknowledges funding of doctoral studies from Deutsche Telekom AG.
1 Introduction

The effects of regulation on investments in telecommunication infrastructures are object of an ongoing debate due to their importance for economic growth.\(^1\) The access regulation regime is important as it shapes the kind and intensity of competition within the markets and, in particular, the investment incentives.\(^2\) While in the USA mandatory access is on the decline and the Federal Communications Commission (FCC) eliminated unbundling obligations in broadband markets in 2005, European’s broadband markets are still subject to extensive regulatory interventions. The European Commission’s regulatory framework comprise access obligations for the network of the telecommunication incumbents if they have significant market power.\(^3\) A noteworthy difference between the US and most European broadband markets is the level of platform competition between telephony and cable networks which is very intense in the US. In many European countries and, in particular, in Germany competition in DSL markets was the main issue whereas competition between different platforms played a minor role or was restricted to urban areas, respectively. Moreover, a high degree of competition within the DSL broadband access market was based on reselling incumbents’ access products.\(^4\)

The initial objective of access regulation was to promote competition by providing entrants the opportunity to establish themselves in the market and successively invest in own infrastructure facilities to achieve sustainable competition in the long run. Given the experience from the US and Europe, the success of this stepping-stone or, in European terms, this ladder of investment approach\(^5\) is at least questionable. A range of empirical studies showed that strict access regulation decrease the investment incentives of entrants within a technology as well as investments in alternative platforms and suggest that most entrants in telecommunications markets stayed at the lowest rung and did not climb the ladder.\(^6\)

Another issue regarding the broadband access markets is the objective to bridge the digital divide and provide sufficient coverage also in sparsely populated regions. While there are some regions, in particular urban areas, where infrastructure based competition between different technologies takes place, there is concern about regions with only one infrastructure provider

---

\(^1\)Cf. Röller & Waverman (2001), Czemich et al. (2009) or Katz et al. (2010).

\(^2\)Guthrie (2006) provides a comprehensive overview of the effect of different regulation regimes and their impact on investment incentives in network industries. An overview of empirical as well as theoretical studies related to broadband markets is provided by Cambini & Jiang (2009).


\(^6\)For example, Waverman et al. (2007) showed that lower access fees in one technology yields disproportional lower investments in alternative infrastructures. Friedeliszieck, Grajek & Röller (2007) found a negative effect of access regulation on investments of entrants within one infrastructure based on data from 25 European countries in the period 1997-2006. Hausman & Sidak (2005) examined the stepping stone hypothesis empirically with data from five countries and did not find any evidence for the transition from service- to facility-based competition.
and regions without any infrastructure provider, in particular rural areas. The discussion about regulation in dense areas, often referred to as “black” areas, focuses on the question whether access regulation is still necessary and some European regulators, e.g. the British Office of Communications (OFCOM) and the Austrian Rundfunk und Telekom Regulierungs-GmbH (RTR), denied this and made the attempt to relax regulation in some regions. Regarding the regions with only one infrastructure provider, referred to as “gray” areas, access regulation is considered as a necessary intervention by most National Regulatory Authorities (NRA) and the European Commission. The coverage of rural regions with insufficient provision of broadband access infrastructures, often referred to as “white” areas, is a topic of increasing importance. The European Commission announced strategies to bridge the coverage gaps by the end of 2010, including the opportunity of subsidies.\footnote{Cf. BMWi (2009) and Commission of the European Communities (2009).} Hence, the population density plays a crucial role regarding infrastructure investments.

As Laffont & Tirole (2001, p.7) pointed out, there exists a Schumpeterian trade-off between the promotion of (service-based) competition to increase social welfare by lowering prices given investment and encouraging investments ex ante. This trade-off between static efficiency, i.e. intense competition with low prices in a situation with an existing infrastructure, and dynamic efficiency regarding the investment incentives for new infrastructures is a crucial aspect for regulation in general\footnote{Cf. Weisman (2010).} and for the roll-out of Next Generation Access Networks (NGA) in telecommunications in particular. Hence, there exists a strong linkage between both topics, the kind and intensity of competition in the market and the incentives to provide the necessary infrastructure even in less densely populated areas.

In this paper, we try to enlighten the debate by examining this linkage and analyzing the effect of the access fee on investment incentives. Our model examines price competition of two firms, an incumbent and an entrant, in the Internet broadband access market. Both firms use an infrastructure facility to provide services to consumers in regional markets with different population densities. While the incumbent provides broadband access to consumers via its own infrastructure, the entrant faces a decision between service-based and facility-based entry. The entrants might buy access from the incumbent and act as reseller, possibly using alternative providers or own infrastructures at higher network layers. This approach is often discussed especially in the context of bit stream access to fiber-to-the-home (FTTH) networks. Alternatively, the entrants might invest in own infrastructure facilities, e.g. leasing unbundled local loops from the incumbent and building own (V)DSL Access Multiplex (VDSLAM). For instance, this is an approached discussed in Switzerland in the context of NGA where network providers might install several dark fiber to each covered household and offer them to
competitors.

We show that the entrant’s investment decision includes a trade-off. On the one hand, investment yields cost savings in regions with facility-based entry and enables the entrant to attract more consumers with lower prices. On the other hand, as the entrant becomes more aggressive, the incumbent becomes more aggressive, too, and retail competition is fostered in all covered regions. The entrant will take the competition effect of its investments into account and, therefore, act strategically. Hence, it is not sufficient that investment yields average costs below the access fee in a specific region to trigger facility-based competition.

These results are related to Sappington (2005) and Gayle & Weisman (2007). Sappington pointed out that the typically implemented cost-based access fees, like Total Element Long Run Incremental Costs (TELRIC), do not sufficiently account for the impact of the investment decision of the entrant on the subsequent retail price competition. Using an Hotelling retail competition, Sappington showed that the entrant will always make an efficient make-or-buy decision even if the access fee varies significantly from incumbent’s costs to provide the input. Gayle & Weisman (2007) showed that this “irrelevance of input prices” does not hold in general. With reference to Chen (2001), they argued that entrants might choose to purchase the input from the incumbent to soften downstream competition independent from the relation between access price and incumbent’s cost. This strategic effect also holds in a Bertrand setup with differentiated goods, whereas it disappears in a Cournot setting. Mandy (2009) showed in a more general setup that the irrelevance of the input price on the make-or-buy decision only holds for restrictive assumptions on demand and that there exists a latitude of access fees inducing efficient make-or-buy decisions.

In contrast to the cited models, we account for fixed investment costs in infrastructure facilities which yield decreasing instead of constant average costs. Moreover, we take into account the investment incentives of both, the incumbent and the entrant, because innovations in telecommunications comprehend the necessity for upgrades and extensions of the existing infrastructures. Thereby, the population density plays a crucial role as providing infrastructures in less densely populated regions involves higher average costs. In the presence of economies of density, our results show that the entrant will never choose to compete via facility-based entry in all covered regions and that the incumbent will never provide coverage in all regions. Hence, our model includes the mentioned “black”, “gray”, and “white” areas. This result is similar to the findings of Faulhaber & Hogendorn (2000), who showed that infrastructure-based entry will only take place in dense areas whereas the provision of infrastructures in less dense regions is only profitable for the first provider and total coverage is never achieved without additional instruments, e.g. an universal service obligation.

As a main result, we show that higher access fees might yield both, lower retail prices and higher total coverage. Hence, the usual trade-off between static and dynamic efficiency, i.e.
higher retail prices with lower penetration and higher coverage versus lower retail prices with higher penetration and lower coverage, does not apply in general. This is a major difference to the findings of the closely related papers of Valletti, Hoernig & Barros (2002) and Götz (2009). Valletti et al. (2002) analyzed the effect of uniform pricing obligations on coverage and showed that uniform pricing constraints might yield strategic effects between otherwise unrelated markets. As a result, such a policy might yield lower investment by both, the incumbent and the entrant, compared to a situation with geographically differentiated prices. Götz (2009) examined a similar setup and showed that there exist mixed-strategy equilibria in the absence of uniform pricing obligations in which the incumbent alters between high and low prices, i.e. fierce price competition in region with infrastructure competition and monopoly outcomes in regions without infrastructure-based competition. In a nutshell, these models include a trade-off between static and dynamic efficiency because increasing facility-based competition and therefore lower retail prices yield lower investments by the incumbent, i.e. lower total coverage. As we include the opportunity that a competitor enters the market via facility-based as well as via service-based competition, this trade-off does not necessarily apply in our model. As long as an entrant firm provides broadband access via the network of the incumbent in regions without own infrastructures, the effect of increasing facility-based competition, decreasing retail prices and increasing total demand might dominate the negative effects of more intense competition on investments.

Lastly, a major difference to the most papers on the topic of access regulation and investments in broadband access markets is the presence of switching costs in our model. We provide a setup which includes a demand allowing for different switching costs as well as different reservation utilities for each consumer including an outside option, i.e. not buying broadband access at all. The motivation for this shaping is twofold. First, the usual assumption about some sort of product differentiation between incumbent’s and entrants’ products does not explain satisfactorily why entrants offer their products at a lower price. Second, the assumption of switching costs is in line with empirical findings, e.g. Gruber & Verboven (2001) or Krafft & Salies (2008). The existence and magnitude of product differentiation is at least arguable, especially regarding broadband Internet access via the same technology, e.g. VDSL or DSL products based on reselling and local loop unbundling. Hence, assuming the presence of consumer switching costs seems more appropriate in the context of broadband access markets via the same or similar technologies.

The remainder of the paper is as follows. Section 2 proceeds with the basic setup of our model. In Section 3, we derive the equilibrium conditions for a benchmark case with only service-based entry and for the case in which the entrant faces a make-or-buy decision in each region. Section 4 provides a numerical simulation of our findings and a discussion of the results.
Section 6 concludes.

2 The Model

The model examines a market with two firms, an incumbent, denoted by I, and an entrant, denoted by E, in the Internet broadband access market. Both firms use an infrastructure facility to provide services to consumers in regional markets.

In a previous period, denoted as period zero, the incumbent serves the market with a legacy infrastructure. One may think of a telecommunications network which the incumbent owns as a former state-owned monopoly or a regulated monopoly constrained to an universal service obligation. For example, the network might be a public switched telephone network offering access to the Internet via 56K modem or DSL.

In the first period, the incumbent might upgrade the network which involves sunk costs $F_I$ for each region, e.g. for DSL access multiplex at the regional telephone exchange or upgrading the serving area interface for VDSL. Furthermore, the competitor enters the market and decides whether to buy access from the incumbent and resells the access product to the consumers or to invest in own infrastructure facilities. T hereby, consumers who bought from the incumbent in period zero and switch to the entrant exhibit switching costs, e.g. information or search costs.

Demand

The whole market is composed of a continuum of regional markets with uniformly distributed population densities $\delta \in [0, \bar{\delta}]$ and the number of the regions and their density is assumed as a linear relation. Figure 1 illustrates this setup.

In each region exists a continuum of consumer types with uniformly distributed reservation utility $\nu_i \in [0, \nu]$. Total demand in region $\delta$ is given by

$$y(\delta, p) = \delta(\lambda \nu - p),$$

with population density $\delta$, reservation utility $\nu$, utility adjustment parameter due to investment $\lambda$, and price $p$. The demand is positively related with the density of a region and $y(\delta') \geq y(\delta'')$ applies for all $\delta' > \delta''$.

Moreover each consumer type $\nu_i$ incorporates a continuum of mass 1 of consumers with

---

9 An alternative modeling with investment in period zero does not alter the results qualitatively. A setup which allows the incumbent to offer the new service as a monopolist could be interpreted as regulatory holidays. As this is not the objective of this paper - and the mathematics of such a setup is more complex - we assume an upgrade of the existing network in the first period.

10 Götz (2009).
switching costs $\omega \in [0, \overline{\omega}]$ and $\overline{\omega} > 0$. The switching costs are only relevant if a consumer bought from the incumbent in period 0 and switches to the entrant in period 1. Hence, each consumer $i$ is characterized by a combination of a willingness to pay $\nu_i$ and switching costs $\omega_i$, so that the consumer population in a region $\delta$ is on a two-dimensional continuum in the domain $\mathbb{R}_+^2 = [0, \nu] \times [0, \overline{\omega}]$. Note that there exists no correlation between the reservation utility and the switching costs. There exist consumers with a low willingness to pay and high switching costs as well as consumers with a high willingness to pay and low switching costs.

This gives us the opportunity to analyze different reservation utilities as well as different intensities of switching costs. Even though both parameters are exogenous, both might - at least to some extent - be influenced by the firms and the regulatory authority. While firms’ decision towards the underlying technology might affect the utility and willingness to pay, e.g. due to higher bandwidth or lower latency, the regulatory authorities might influence the amount of switching costs with obligation regarding switching duration etc.

Both firms provide a homogeneous good, broadband Internet access, to the consumers and the entrant gets two kinds of demand.

First, there are consumers who bought from the incumbent in the previous period and therefore exhibit switching costs. Consumers switch to the entrant if their net-utility increases and the crucial switching costs of the indifferent consumer $\omega_k$ could be written as

$$\lambda \nu_i - p_E - \omega_i \geq \lambda \nu_i - p_I \quad \Rightarrow \quad \omega_k = p_I - p_E.$$
is independent of the reservation utility \( \nu_i \) and, therefore, demand of the incumbent in period 1 is not subject to the reservation utility as long as buying the product yields a positive net utility. With a mass of 1 for all consumer types, the share of switching consumers is given by

\[
(p_I - p_E) \frac{1}{\omega} = \frac{\omega_k}{\omega}
\]

with \( 0 \leq \frac{\omega_k}{\omega} \leq 1 \). Figure 2b illustrates that higher maximum switching costs yield a lower share of switching consumers if the price difference of both firms does not change. Thus, entrant’s demand from those consumers decreases with the maximum switching costs.

Second, consumers who did not buy in the previous period will always buy from the firm with the lower price. As tie-breaking rule, new consumers will slightly prefer the incumbent if both firms offer the product for the same price and only choose the entrant if \( p_E < p_I \). As the entrant has to undercut the incumbent’s price to get some of the old consumers, the unattached consumers will always buy from the entrant. Demand of those unattached consumers is given by

\[
\delta(\lambda \nu - p_E) - \delta(\nu - p_{I0}) = \delta((\lambda - 1)\nu + p_{I0} - p_E).
\]

with \( \lambda \) normalized to 1 in the previous period and \( \lambda \geq 1 \) as adjustment of the reservation utility due to the upgrade of the network in the first period.

The incumbent’s demand in region \( \delta \) in period 1 is its demand from the previous period minus
the portion of consumers switching to the entrant, i.e.

\[ y_I(\delta, p) = y_{I0} \left( 1 - \frac{\omega_k}{\nu} \right) \]  

(2)

The entrant’s total demand in region \( \delta \) is the sum of switching and new consumers, i.e.\(^{11}\)

\[ y_E(\delta, p) = \left( \frac{\omega_k}{\nu} \right) y_{I0} + \delta((\lambda - 1)\nu + p_{I0} - p_E) \]  

(3)

For simplicity, let us assume that the incumbent acts as a myopic monopolist in period zero.\(^{12}\) Thus, the equilibrium price in period zero is given by \( p_{I0} = \nu/2 \) and equation (1) yields the equilibrium demand \( y_{I0}(\delta, p_{I0}) = \nu/2 \). The demand functions of both firms, Equations (2) and (3) could then be rearranged to

\[ y_I(\delta, p) = \frac{\delta}{2\nu}(\lambda \nu + p_E - p_I) \]  

(4)

\[ y_E(\delta, p) = \frac{\delta}{2\nu}((2\lambda - 1)\nu + p_I) - p_E(2\nu + \nu)) \]  

(5)

Equations (4) and (5) illustrate that both firms face linear demands in each region \( \delta \) subject to the maximum switching costs \( \omega \), the reservation utility \( \nu \) the price of the competitor and the firm’s own price. Obviously, the demand of a firm increases with the price of the competitor and decreases with the firm’s own price. Both demands increase with the reservation utility and the population density. The effect of the switching costs on both demands goes in opposite directions, i.e. for higher switching costs the incumbent’s demand increases and the entrant’s demand decreases. This is straightforward because fewer consumers will switch from the incumbent to the entrant given a constant price difference. The entrant’s demand is also influenced by the utility adjustment parameter \( \lambda \). The higher the utility from the upgrade, the higher the entrant’s demand from new consumers. To ensure that the upgrade does not change the market size, we assume that \( \lambda \) is such that \( y(\delta, p_E) \leq y_0(\delta, 0) \), which holds in equilibrium.

To summarize, we derive continuous and differentiable demand functions which show intuitive features for the different parameters. Moreover, these demand functions allow an analysis of consumer switching costs and different reservation utilities including an outside option, i.e. consumers not buying broadband access at all.

---

\(^{11}\) This demand holds as long as \( \delta(\lambda \nu - p_E) \geq \delta(\nu - p_{I0}) \iff \lambda \geq p_E/\nu + 1/2 \). Otherwise, it is not ensured that all of the incumbent’s previous consumers have a sufficiently high reservation utility.

\(^{12}\) Given that the incumbent anticipates the entry in the first period, the price in previous period will decreasing with increasing switching costs as the incumbent tries to attract more consumers in period zero. Compare e.g. Klemperer (1987).
Firms’ profit maximization

Both firm’s maximize their profits over all covered regions, consisting of regions in which the entrant invested in own infrastructure facilities, i.e. \([\delta_E, \delta_I]\), and regions in which the entrant buys access from the incumbent, i.e. \([\delta_I, \delta_E]\). As simplification, we assume that marginal costs of providing broadband access to the consumers equal zero and therefore the maximization problems of the firms read as

\[
\max_{p_I, \delta_I} \pi_I = \int_{\delta_I}^{\delta_E} p_I y_I - F_I d\delta + \int_{\delta_I}^{\delta_E} \alpha_y E d\delta \\
\max_{p_E, \delta_E} \pi_E = \int_{\delta_E}^{\delta_I} p_E y_E - F_E d\delta + \int_{\delta_E}^{\delta_I} (p_E - \alpha) y_E d\delta
\]

with the prices \(p_j\), the demand functions \(y_j\), the investment costs \(F_j\), and the access fee \(\alpha\).

The profit functions include the assumption that entrant’s investment will always be lower or equal to the incumbent’s investment, i.e. \(\delta_E \geq \delta_I\). As we will see later, this assumption holds in equilibrium as long as both firms face similar investment costs.

Note that the entrant will only buy access from the incumbent if the access fee is below the retail price. Otherwise, the entrant would realize losses and consequently not provide services in regions without own infrastructure. This would yield the same outcome as in Valletti et al. (2002) or Götz (2009) with facility-based competition in regions with investments of both firms. Alternatively, if the entrant has to provide the service in all covered regions even if the access fee is above the equilibrium prices, this might be interpreted as some sort of universal service obligation for entrants. Alternatively, one might think about a product for which the entrant needs to compete in all covered regions, e.g. mobile networks and the entrant uses roaming.

Timing of the game

The model is separated in three stages and the timing is as follows:

1. **Access Stage**
   The regulator sets the access fee \(\alpha\)

2. **Investment Stage**
   Both firms choose their investment level, i.e. the smallest covered regions \(\delta_I\) and \(\delta_E\), and the entrant choose whether to buy access in the remaining covered regions

3. **Pricing Stage**
   Both firms set their prices \(p_I\) and \(p_E\) simultaneously and compete for consumers

Regarding the access stage, we assume that the regulator is able to credibly commit to the announced access fee. If this is not the case, the order of stages 1 and 2 are reversed and a
hold-up problem is probable because the regulator has an incentive to introduce a lower access fee which does not account sufficiently for the irreversible fixed investment costs.

We analyze the investment stage as a simultaneous decision even though the entrant’s decision to buy access is subject to total coverage, i.e. the incumbent’s investment decision. Hence, the investment stage is a sequential game in which the incumbent chooses its coverage and thereafter the entrant decides about its investment. To ensure that an analysis as simultaneous decisions does not alter the results, we assume that the incumbent is not able to credible commit not to reinvest after the entrant has chosen its investment level.\textsuperscript{13}

At the last stage of the game, both firms set their prices and compete in the broadband access retail market.

3 Equilibria

Before we analyze the equilibrium in the case where the entrant might chose to invest in own infrastructure facilities, let us consider the case in which the entrant acts as pure reseller of the incumbent’s product. This case with only service-based competition is a subset of the case with facility-based competition as the opportunity of pure reselling of incumbent’s bit stream access products still applies. However, the case without facility-based entry provides a helpful benchmark and gives some insights into the general processes on one-way access regulation in the context of switching costs and economies of density.

3.1 Benchmark: Service-Based Entry

In this benchmark case, the regulatory authority announces the access fee and thereafter the incumbent decides about its investment, i.e. the last covered region $\delta_I$, which involves investment costs $F_I$ in each covered region. Afterwards, both firms choose their prices $p_I$ and $p_E$ simultaneously. The entrant resells the product in all covered regions and pays an access fee $\alpha$ per consumer.

Pricing stage

Solving recursively, we first derive the optimal price decisions of both firms at the last stage of the game. The profit of the incumbent with service-based competition, denoted by $sbc$, is

\textsuperscript{13}A more detailed discussion of this game structure is available in Götz (2010).
given by

\[
\pi_{I}^{sbc} = \int_{\delta_I}^{\bar{\delta}} p_I y_I + \alpha y_E - F_I d\delta
\]

\[
\begin{aligned}
&= (\bar{\delta}^2 - \delta_I^2) \nu p_I (\bar{\omega} + p_E - p_I) + \alpha \nu (p_I - p_E) + \bar{\omega} (2\lambda - 1) \nu - 2p_E) \\
&\quad \frac{4\bar{\omega}}{4\bar{\omega}} - (\bar{\delta} - \delta_I) F_I
\end{aligned}
\]

The incumbent maximizes its profit over all covered regions. This includes the earnings from old consumers, not switching to the entrant, the earning from selling access to the entrant and the investment costs per region.

The entrant provides the service in all covered regions and pays an access fee per consumer. Hence its profit function is given by

\[
\pi_{E}^{sbc} = \int_{\delta_I}^{\bar{\delta}} (p_E - \alpha) y_E d\delta
\]

\[
= (\bar{\delta}^2 - \delta_I^2) (p_E - \alpha) \nu (2\lambda - 1) \nu + p_I) - p_E (2\bar{\omega} + \nu))
\]

Differentiating the profit functions with respect of the firms prices and rearranging both first order condition yield the firms’ reaction functions, i.e.

\[
\frac{\partial \pi_{I}^{sbc}}{\partial p_I} = 0 \iff p_I = \frac{1}{2} (\bar{\omega} + p_E + \alpha)
\]

\[
\frac{\partial \pi_{E}^{sbc}}{\partial p_E} = 0 \iff p_E = \frac{1}{2} \left( \alpha + \frac{\nu (2\lambda - 1) \nu + p_I)}{2\bar{\omega} + \nu} \right)
\]

Both firms increase their prices with the price of the competitor. Therefore prices are strategic complements. The effects of both, the access fee and the switching costs, are positive. The incumbent’s best response is independent of the reservation utility whereas the entrant’s prices increase with the reservation utility. Intuitively, the dependency of entrant’s best response on the reservation utility is based on the unattached consumers because a higher reservation utility and a higher additional utility from the investment, i.e. a higher \(\lambda\), yields more new consumers in equilibrium. Note that both prices are independent from the coverage decision of the incumbent.

Solving both reaction function with respect to the prices yields the equilibrium prices

\[
p_{I}^{sbc} = \frac{\bar{\omega} ((2\lambda + 1) \nu + 4\bar{\omega})}{3\nu + 8\bar{\omega}} + \frac{\alpha 3(\nu + 2\bar{\omega})}{3\nu + 8\bar{\omega}}
\]

\[
p_{E}^{sbc} = \frac{(4\lambda - 1) \nu \bar{\omega}}{3\nu + 8\bar{\omega}} + \frac{\alpha 3\nu + 4\bar{\omega}}{3\nu + 8\bar{\omega}}
\]

which consist of two parts. The first term is a mark-up due to the switching costs. The second
term represents the marginal costs of the entrant. From the derivation of the equilibrium prices follows that higher maximum switching costs $\omega$ yield higher prices and that the the equilibrium prices increase with the reservation utility $\nu$ and $\lambda$, too. The optimal price difference, $\omega_k^{sbc} = p_I^{sbc} - p_E^{sbc}$, increases with the switching costs. For the case without switching costs, i.e. $\omega = 0$, we get a Bertrand outcome with prices equal access fee which are the marginal costs of the entrant. As long as the switching costs are strictly positive, the tie-breaking rule is irrelevant because the entrant will always choose a price below the incumbent’s equilibrium price. Obviously, the higher the access fee $\alpha$, the higher the entrant’s marginal costs from buying access and the higher both prices. Although this result is straightforward, we capture this statement as a proposition as it will become important in the discussion of the results with investments of the entrant.

**Proposition 1.** If the entrant acts as a pure reseller, i.e. only service-based competition is realized, the equilibrium prices of both firms increase with the access fee.

**Proof.** Follows from the derivation of equations 6 and 7 with respect to the access fee. ■

**Investment stage**

Given the equilibrium prices, we can derive the investment decision of the incumbent. The incumbent will invest as long as the coverage of an additional region yields positive profits. Hence, profits in the last covered region equal zero. Rearranging yields\(^{15}\)

$$
\delta_I^{sbc} = \frac{F_I}{p_I^{sbc}y_I(p^*) + \alpha y_E(p^*)}
$$

whereas $y_j$ is the demand of the firm in one region subject to the equilibrium prices and independent from population density.

Substituting the equilibrium prices in the above equation, or alternatively differentiating incumbent’s reduced profit function with respect to the coverage decision and solving for $\delta_I$, yields the equilibrium investment decision of the incumbent, i.e.

$$
\delta_I^{sbc} = \frac{2F_I(3\nu + 8\overline{\omega})^2}{2\alpha(\nu(\nu\overline{\omega} + \lambda(9\nu^2 + 32\nu\overline{\omega} + 32\overline{\omega}^2))) - \alpha(9\nu^2 + 34\nu\overline{\omega} + 32\overline{\omega}^2)) + \nu\overline{\omega}((2\lambda + 1)\nu + 4\overline{\omega})^2}
$$

\(^{14}\)For the case without switching costs, both firms would set their prices equal marginal costs, i.e. equal the access fee. Given our tie-breaking rule, i.e. a slightly preference of the consumers towards the incumbent, the incumbent would deter entry. As long as the access fee represents the average costs of the incumbent, the equilibrium would be the typical outcome of a contestable market as the incumbent would be disciplined by the possibility of service-based entry without barriers to entry.

\(^{15}\)This approach is similar to Götz (2009), Valletti et al. (2002), and Faulhaber & Hogendorn (2000).
Obviously, the investment costs have a negative impact on the coverage as the nominator decreases and therefore the density of the last covered region increases. Intuitively, if the investment costs increase, more consumers are necessary to cover the investment costs and the density of the regions providing zero profits must be higher. Full coverage is only provided for investment costs equal zero. Otherwise, the entrant will only provide partial coverage and broadband access is not provided in sparsely populated regions. The reservation utility \( v \), the utility adjustment parameter \( \lambda \), the maximum switching costs \( \omega \), and the access fee \( \alpha \) are positive related with the coverage decision of the incumbent. The higher \( v \), \( \lambda \), \( \omega \), or \( \alpha \), the lower the density in the last covered region \( \delta_{sbc}^I \). Note that the investment decision of the incumbent is not subject to the region with the highest density but determined by its profits in the last covered region.

This benchmark provides a difference to the related literature of Valletti et al. (2002) and Götz (2009). As the entrant accede the market in all covered markets via service-based competition, the incumbent is not able to differentiate prices geographically and both firms compete with uniform prices.

**Proposition 2.** If the entrant acts as a pure reseller, i.e. only service-based competition is realized, the equilibrium coverage increases with the access fee as long as the access fee is not too high, i.e. \( \alpha < \frac{\nu v \sigma + \lambda (9 \nu^2 + 32 \nu \sigma + 32 \sigma^2) }{9 \nu^2 + 34 \nu \sigma + 32 \sigma^2} \).

**Proof.** See Appendix A.1. ■

The intuition behind this result is straightforward. The incumbent’s earnings per consumer increase directly with a higher access fee and indirectly due to higher equilibrium prices. Because the demand of switching consumers is not subject to the prices in absolute manner but the price difference and both firms increase their prices, the effect on incumbent’s demand is negligible. The entrant’s demand for access decreases with higher prices as the demand of unattached consumers decreases. However, the entrant’s decreasing demand for access to the incumbent’s network is overcompensated by the positive effect of higher access fees on the incumbent’s profits as long as the access fee is not too high. Hence, the incumbent’s profits in the last covered region increase with the access fee and the last region providing zero profits decreases, i.e. has a lower density.

The incumbent will invest as long as the investment costs are not high, i.e. less or equal

\[
F_{I}^{\text{max}} = \frac{3(2 \nu (v \sigma + \lambda (9 \nu^2 + 32 \nu \sigma + 32 \sigma^2))) - \alpha (9 \nu^2 + 34 \nu \sigma + 32 \sigma^2) + \nu \sigma (3 \nu + 4 \sigma)^2}{2 (3 \nu + 8 \sigma)^2}
\] (9)

For this investment costs, the incumbent’s profit will be zero in the region with the highest density \( \bar{\delta} \) and for \( F_I < F_{I}^{\text{max}} \), the incumbent will invest even if the regulator sets an access fee of zero. At first sight, this might be contra intuitive but could be explained via the existence
of strictly positive switching costs. Even if the entrant get costless access and the incumbent will lose consumers to the entrant, some consumers will stay with the incumbent. Hence the incumbent is able to make positive profits and will invest in some regions. Note that investment might be very low subject to the amount of switching costs.

To summarize our results, i.e. Propositions 1 and 2, there is a (well known) trade-off between static and dynamic efficiency, i.e. lower retail prices for given infrastructure versus higher prices and more investment. Figure 3 illustrates these results. The same applies for the switching costs. Higher maximum switching costs decrease the intense of competition, yield higher prices as well as higher coverage and vice versa.

The effect of higher access fees and, therefore, increasing equilibrium prices and investment incentives is similar to analysis of cost-based access pricing, e.g. the TELRIC, compared to retail-minus, e.g. a price cap. Several papers showed that the investment incentives of an incumbent are always higher if the regulatory authority chooses a retail-minus regulation and allow for higher access fees (Compare Sarmento & Brandão (2007) or Götz (2009)).

Even though, the obtained results just confirm previous results, they provide a useful benchmark for the situation with investments of the entrant.

3.2 Facility- and Service-Based Entry

Let us now consider a case where the entrant faces a make-or-buy decision, i.e. might choose between service-based and facility-based entry in every region $\delta \in [\underline{\delta}, \overline{\delta}]$. For instance, one might think of this setup as facility-based competition from an alternative infrastructure provider, e.g. cable, who resells (V)DSL in regions without own network.\textsuperscript{16} This is a major difference to the

\textsuperscript{16}This would be similar to the business case of CableCom, Swiss’ main cable provider, who provided broadband access via its cable network and via reselling of DSL from the former Swiss’ telecommunications incumbent
related models from Faulhaber & Hogendorn (2000), Valletti et al. (2002), and Götz (2009). Although the assumptions about population densities and a monopolistic infrastructure provider in some regions are basically the same, the incumbent is not able to act as a monopolist in regions where other firms do not provide own infrastructures as there exists service-based competition. Hence, while their models examine infrastructure-based competition between different technologies\(^\text{17}\) and firms might only compete with other firms if they provide their own infrastructure, the entrant is able to compete in all covered regions regardless of its investments in our model.

Furthermore, the setup might be interpreted as an analysis of the ladder of investment. Thereby, we reduce Cave (2006)’s investment ladder to two ladder spokes, i.e. bit stream access and local loop unbundling. This interpretation does not explicitly account for the additional rental costs of the local loop. Instead of an increasing bit stream access fee one might think of a decreasing monthly rental rate for the local loop. Even though the static setup of the model might be seen as contradiction to the dynamic aspects of the ladder concept at first sight, increasing access fees could be interpreted as changes of the access fee or the regulation regime over time.\(^\text{18}\) Let us assume that initial entry is solely service-based and grants the entrant the opportunity to establish itself in the market. In subsequent periods, the entrant has three opportunities. He invests in all regions, he might invest in some regions and loose his consumers in regions without investment, or he has to provide the service in regions without investment via reselling. Given the objective of the ladder approach – entrants’ establishment in the market and successive investment – the third opportunity seems suitable for the modeling of subsequent periods. Thereby, the model offers some insights regarding the investment incentives of entrants without modeling the ladder of investment as a dynamic game explicitly.

In opposite to the benchmark case, the entrant faces a trade-off in this setup. On the one hand, the entrant avoids the access fee in regions with own infrastructure facilities and consequently might lower its costs. On the other hand, prices are strategic complements and therefore a price decrease due to lower costs yields a more intense competition. Both aspects are similar to Sappington (2005) and Gayle & Weisman (2007). The entrant’s efficient make-or-buy decision accounts for the competition effect and therefore higher access fees are implementable. A major difference is the presence of an investment decision of both firms and the question whether and how entrant’s investment affects the incumbent’s coverage decision.

\(^{16}\)Swisscom.

\(^{17}\)E.g. cable and DSL network providers. Note that Faulhaber & Hogendorn (2000) model competition between different cable network providers, i.e. competition with one technology.

\(^{18}\)Even though, the concept of increasing access fee over time implies some legal concerns, e.g. discrimination of later entry (compare Cave (2006, p.233)), the effects of such policies could be analyzed within the model.
Pricing stage

In comparison to the benchmark case, the incumbent earnings from the access fee are reduced to those regions in which the entrant does not invest, i.e. up to the smallest covered region $\delta_E$. Hence, the incumbent’s profit function is given by

$$\pi_I = \int_{\delta_l}^{\delta} p_I y_I - F_I d\delta + \int_{\delta_t}^{\delta_E} ay_E$$

$$= (\delta^2 - \delta_I^2) \left( \frac{\nu p_I (\overline{\omega} + p_E - p_I)}{4\overline{\omega}} \right) - (\delta - \delta_I) F_I + \alpha (\delta_E^2 - \delta_I^2) \frac{\nu (p_I + (2\lambda - 1)\overline{\omega}) - p_E (2\overline{\omega} + \nu)}{4\overline{\omega}}$$

Thereby, incumbent’s profits include the investment costs over all covered regions, the earning from its consumers, and the earnings from selling access to the entrant in regions without facility-based competition. The profit function of the entrant is given by

$$\pi_E = \int_{\delta_E}^{\delta} p_E y_E - F_E d\delta + \int_{\delta_t}^{\delta_E} (p_E - \alpha) y_E d\delta$$

$$= (p_E (\delta^2 - \delta_I^2) - \alpha (\delta_E^2 - \delta_I^2)) \frac{\nu (2\lambda - 1)\overline{\omega} + p_I - p_E (2\overline{\omega} + \nu)}{4\overline{\omega}} - (\delta - \delta_E) F_E$$

and includes the earnings from its consumers in all covered regions, the costs from purchasing access in regions without investment, and the investment costs in regions with facility-based entry. For an access fee above the equilibrium price, the second part of the profit function would disappear if the entrant does not have to resell the incumbent’s access in regions without own infrastructures.

Differentiating both profit functions with respect to the prices and rearranging yields the best response functions

$$\frac{\partial \pi_I}{\partial p_I} = 0 \iff p_I = \frac{1}{2} \left( \frac{\overline{\omega} + p_E + \alpha \frac{\delta_E^2 - \delta_I^2}{\overline{\omega} - \delta_I^2} }{\overline{\omega} - \delta_I^2} \right)$$

$$\frac{\partial \pi_E}{\partial p_E} = 0 \iff p_E = \frac{1}{2} \left( \frac{\nu ((2\lambda - 1)\overline{\omega} + p_I) - p_E (2\overline{\omega} + \nu)}{2\overline{\omega} + \nu} + \alpha \frac{\delta_E^2 - \delta_I^2}{\overline{\omega} - \delta_I^2} \right)$$

and solving for the prices yields the equilibrium prices subject to the investment decisions

$$p_I^*(\delta_I, \delta_E) = \frac{\overline{\omega} (2\lambda + 1)\nu + 4\overline{\omega}}{3\nu + 8\overline{\omega}} + \alpha \frac{3(\nu + 2\overline{\omega}) \delta_E^2 - \delta_I^2}{3\nu + 8\overline{\omega}} \frac{\delta_E^2 - \delta_I^2}{\overline{\omega} - \delta_I^2}$$ (10)

$$p_E^*(\delta_I, \delta_E) = \frac{4\lambda - 1)\nu \overline{\omega}}{3\nu + 8\overline{\omega}} + \alpha \frac{3\nu + 4\overline{\omega} \delta_E^2 - \delta_I^2}{3\nu + 8\overline{\omega}} \frac{\delta_E^2 - \delta_I^2}{\overline{\omega} - \delta_I^2}$$ (11)

Comparing the equilibrium prices with the benchmark case (equations (6) and (7)) shows that
the effect of the access fee on the prices is lowered by a ratio between the regions not covered by the entrant and total coverage, i.e. the ratio of service-based competition. Thus, the benchmark case is a subset of this case in which the weight of the access fee on prices equals 1. As long as the entrant does not invest, Proposition 1 applies. If the entrant invests, the weight is lower and prices decrease with the investment of the entrant, i.e. a decrease of \( \delta_E \). Otherwise, the same effects apply as in the benchmark case, i.e. the equilibrium prices increase with the reservation utility \( \nu \), the utility adjustment parameter \( \lambda \), and the maximum switching costs \( \bar{\omega} \).

Because the entrant’s investment decision is subject to the access fee, too, it is ambiguous which effect of \( \alpha \) dominates. On the one hand, if the access fee increases, prices increase due to the direct effect. On the other side, increasing access fees will increase investment and decrease prices due to the indirect effect. This means, if the entrant invests, the entrant acts more aggressively due to lower average marginal costs. The incumbent reacts and lowers its prices due to strategic complementary. Either the direct or the indirect effect might dominate.

**Proposition 3.** If the entrant invests in own infrastructure facilities, the equilibrium prices of both firms decrease with the extent of facility-based competition.

**Proof.** Follows from the derivation of equations (10) and (11) with respect to the entrant’s investment decision \( \delta_E \). □

Given the equilibrium prices, we can derive the equilibrium price difference of both firms \( \omega_k \), determining the amount of switching consumers.

\[
\omega_k = p^*_I - p^*_E = \frac{2\bar{\omega}}{3\nu + 8\bar{\omega}} \left( 2\bar{\omega} + \alpha \frac{\delta_E^2 - \delta_I^2}{\delta_E^2 - \delta_I^2} + (1 - \lambda)\nu \right)
\]

In equilibrium, the price difference between the incumbent and the entrant increases with the maximum switching costs because the entrant has to undercut incumbent’s price further to attract consumers. As long as the entrant only competes via service-based entry, i.e. \( \delta_E = \bar{\delta} \), the price difference increases with the access fee, whereas facility-based entry leads to a lower price difference. As for the equilibrium prices, the net effect of the access fee is ambiguous. This is a difference to the benchmark case, where both, the equilibrium prices and the price difference were strictly increasing with the access fee. We discuss this finding in more detail below.
Entrant’s investment decision

Substituting the equilibrium prices (10) and (11) into the entrant’s profit function and differentiation this reduced profit function with respect to the investment decisions yields

$$\frac{\partial \pi_E(p^*)}{\partial \delta_E} = F_E - 4\alpha \delta_E \omega \left( (4\lambda - 1)\nu - 4\alpha \delta_E^2 - \delta_I^2 \right) \frac{\nu + 2\omega}{(3\nu + 8\omega)^2} = 0 \quad (12)$$

Solving the first order condition of entrant’s profits with respect to the coverage decision \(\delta_E\) yields a polynomial of degree three and therefore three solutions which are non-trivial.

Proposition 4. A unique and optimal investment decision of the entrant, i.e. a best response to the incumbent’s investment decision, denoted by \(\delta_E^*\), satisfying the first order condition and maximizing the profits exists in the admissible range \(\delta_E^* \in [\delta_I, \delta]\). For sufficiently low investment costs \(F_E < F_E^{\max} = 4\alpha \omega \left( (4\lambda - 1)\nu (\delta^2 - \delta_I^2) + 4\alpha \delta_I^2 \right) \frac{\sqrt{(4\lambda - 1)\nu (\delta^2 - \delta_I^2) + 4\alpha \delta_I^2}}{3\lambda (\delta^2 - \delta_I^2)} \frac{\nu + 2\omega}{(3\nu + 8\omega)^2}\), the entrant invests in own infrastructure facilities in some regions.

Proof. See Appendix A.2. ■

The first order condition (12) gives some insights regarding the optimal investment decision of the entrant.

Figure 4 illustrates the first order condition for different access fees and different investment costs. Note that the intersection of the first order condition and the abscissa represents the optimal investment. The optimal investment, which satisfies equation (12), decreases with the investment costs, i.e. \(\delta_E^*\) increases, and increases with the access fee \(\alpha\), i.e. \(\delta_E^*\) decreases. This is intuitive as higher investment costs lower the investment incentives and buying access becomes cheaper compared to building own facilities and vice versa. Increasing switching costs affects the investment incentives in a positive manner. The higher the switching costs, the higher the equilibrium investments of the entrant. Again, this is intuitive and analog to the benchmark case as higher switching costs soften competition, yield higher equilibrium retail prices and therefore increase the investment incentives. Note that for switching costs equal to zero, the entrant will never invest as long as investment is not costless because the resulting Bertrand-prices would not allow entry for positive fixed costs. The reservation utility and the utility adjustment parameter have a positive impact on investments, i.e. the higher \(\nu\) or \(\lambda\) the lower \(\delta_E^*\) satisfying the first order condition.

The entrant’s decision between investment and reselling via bought access comprehends a decision between two cost structures, namely fixed costs and no marginal costs or no fixed costs and marginal costs equal to the access fee. Let us assume that the entrant’s investment decision is solely based on the costs. The entrant would choose to invest until its average costs equal
marginal costs, i.e.

\[
\frac{F_E}{y_E(p^*, \delta)} = \alpha \Leftrightarrow 2F_E - \alpha \delta_E \left( (4\lambda - 1)\nu - 4\alpha \frac{\delta_E^2 - \delta_I^2}{\delta - \delta_I} \right) \frac{\nu + 2\omega}{3\nu + 8\omega} E = 0
\]

Obviously, this condition differs from the entrant’s first order condition. Hence, the investment decision of the entrant takes more aspects into account than just the trade-off between marginal and average costs. We can show that the optimal investment based on this simplified decision rule always yields higher investment. As the equilibrium prices are decreasing with the range of facility-based competition, the entrant faces a trade-off. On the one hand, the entrant would like to invest as this yields lower costs. On the other hand, investments foster retail price competition because prices are strategic complements and the incumbent becomes more aggressively if the entrant decrease its price.

**Proposition 5.** For positive investment costs and access fees, the entrant acts strategically. In order to soften retail competition, it invests less compared to the case without consideration of the effect of investment on prices.

**Proof.** See Appendix A.3. □

Moreover, the entrant will only invest if the investment costs are less or equal to

\[
F_E^{\text{max}} = 4\alpha \omega \left( (4\lambda - 1)\nu(\delta^2 - \delta_I^2) + 4\alpha \delta_I^2 \right) \frac{\sqrt{(4\lambda - 1)\nu(\delta^2 - \delta_I^2) + 4\delta_I^2} \alpha}{3\sqrt{3}\alpha(\delta^2 - \delta_I^2)} \frac{\nu + 2\omega}{(3\nu + 8\omega)^2}
\]

For investment costs above this threshold, the first order condition is strictly positive and reselling dominates investment. The higher the access fee, the higher the investment costs for which the entrant is willing to invest. Differentiating $F_E^{\text{max}}$ shows that the maximum investment cost for which investment takes place increases with the switching costs, with the density of
region \( \delta \), with the reservation utility, and with the utility adjustment parameter. The effect of the incumbent’s investment might be positive as well as negative subject to the relation between the access fee and the reservation utility. If incumbent’s investment is low, i.e. \( \delta_I \) is close to \( \bar{\delta} \), the maximum investment costs go to infinity.

Given the investment costs are sufficiently low, i.e. \( F_E < F_E^{max} \), the entrant’s first order condition is strictly decreasing in the access fee as long as the access fee is below

\[
\hat{\alpha} = \frac{(4\lambda - 1)\nu(\bar{\delta}^2 - \delta_I^2)}{4(3\delta_E^2 - \delta_I^2)}
\]  

(14)

and convex for higher access fees. As long as the investment costs are below

\[
\hat{F}_E = 2\delta^3((4\lambda - 1)\nu)^2\frac{\delta^2 - \delta_I^2}{(3\delta^2 - \delta_I^2)^2} \frac{\nu + 2\omega}{(3\nu + 8\omega)^2}
\]  

(15)

the entrant’s initial investment takes place in the region with the highest density. For investment costs above \( \hat{F}_E \), a higher access fee is necessary to trigger investment. Due to the convexity of the first order condition for such high \( \alpha \), i.e. for \( \alpha > \hat{\alpha} \), the first order condition does not become zero at the point \( \delta_E = \bar{\delta} \) at first. The entrant’s initial investment does not take place in the region with the highest density \( \delta \) but in the region\(^{19}\)

\[
\hat{\delta}_E = \frac{1}{2} \sqrt{\frac{4\delta_I}{3} + \frac{(4\lambda - 1)\nu(\bar{\delta}^2 - \delta_I^2)}{3\alpha}} < \bar{\delta}
\]  

(16)

Hence, there is a discontinuity in the entrant’s investment function subject to the access fee if the investment costs are above \( \hat{F}_E \). Figure 5 illustrates the initial investment of the entrant in region \( \hat{\delta}_E \) for investment costs \( \hat{F}_E < F_E < F_E^{max} \).

![Figure 5: First order condition of the entrant for investment costs \( \hat{F}_E < F_E < F_E^{max} \)](image)

As long as the access fee is below \( \hat{\alpha} \), the entrant’s first order condition is strictly positive and

\(^{19}\)The formal derivation of this region is part of the proof of Proposition 4 in Appendix A.2.
reselling dominates investment. For an access fee equal $\hat{\alpha}$, the entrant’s first order condition is tangent to the abscissa at the point $\delta_E = \hat{\delta}_E^*$. Thus, the entrant initially invests in all regions with density greater or equal $\hat{\delta}_E^*$ and invests in more regions if the access fee increases further. This investment behavior could be explained via the price effect of investment and entrant’s demand. For high investment costs, the entrant will choose to resell in all regions as long as the access fee is sufficiently low. If the access fee becomes too high, the entrant invests and prices decrease. Initial investment in the densest region $\delta$ would only yield a small price decrease whereas the price effect of investments in all regions is very strong. If the investment incentives are sufficiently strong, i.e. if the access fee is sufficiently high, the entrant invests down to $\hat{\delta}_E^*$ at once which yields a higher price decrease and sufficient new consumers could be attracted to cover the expense of investment. Keep in mind that a price decrease does not yield significantly higher demand from old consumers as the incumbent lowers its price, too.

**Incumbent’s investment decision**

The next step is to analyze the investment decision of the incumbent. As in the benchmark case, it is determined by the profits in the last covered region. Investment will take place down to the region in which the incumbent’s profit equals zero. Hence, a simplified optimal investment decision, as in the benchmark case, is given by

$$\pi_I(\delta_I) = p_I y_I + \alpha y_E - F_I \overset{!}{=} 0$$

$$\Leftrightarrow \quad \delta_I^* = \frac{2\omega F_I}{p_I^* \nu (\bar{\omega} + p_E^* - p_I^*) + \alpha (\nu ((2\lambda - 1)\bar{\omega} + p_I^*) - p_E^* (2\omega + \nu))}$$

Obviously, the higher the investment costs, the lower the investment. The effect of incumbent’s price on investment is ambiguous. Based on equilibrium prices, a decrease of $p_I$ increases the denominator because the decrease in incumbent’s demand does not compensate for the higher earnings per consumer, i.e. $p_I^* \nu (\bar{\omega} + p_E^* - p_I^*)$ decreases. On the other side, a higher price increases the denominator as entrant’s demand increases and therefore earnings from selling access to the entrant increases, too, i.e. $\alpha (\nu ((2\lambda - 1)\bar{\omega} + p_I^*) - p_E^* (2\omega + \nu))$ increases.

Substituting equilibrium prices in above expression and rearranging, or alternatively differentiating the incumbent’s reduced profit function with respect to the investment decision, yields the first order condition of the incumbent’s investment decision

$$\frac{\partial \pi_I(p_I^*)}{\partial \delta_I} = F_I - \delta_I \left( \nu \bar{\omega} (2\lambda + 1)\nu + 4\bar{\omega}^2 \right) - \frac{\alpha (\nu \bar{\omega} + \lambda (9\nu^2 + 32\nu \bar{\omega} + 32\bar{\omega}^2))}{(3\nu + 8\bar{\omega})^2}$$

$$- \frac{\alpha (\nu \bar{\omega} + \lambda (9\nu^2 + 32\nu \bar{\omega} + 32\bar{\omega}^2))}{(3\nu + 8\bar{\omega})^2} \left( \frac{2\omega^2 - \delta_I^2 (2\omega^2 - \delta_I^2) (\nu + 2\bar{\omega}) (9\nu + 16\bar{\omega})}{(\delta - \delta_I^2)^2 (3\nu + 8\bar{\omega})^2} \right) \bigg) \overset{!}{=} 0 \quad (17)$$

This equation is non-trivial and solving for the investment decision $\delta_I$ yields a polynomial of...
Proposition 6. A unique optimal investment decision of the incumbent, i.e. a best response to the investment of the entrant, denoted by $\delta^*_I$, satisfying the first order condition and maximizing the profits $\pi_I$ in the admissible range $\delta^*_I \in [0, \bar{\delta}]$ exists for investment costs $F_I \leq F^*_I = \bar{\delta}(2\alpha(\nu\bar{\varpi} + \lambda(9\nu^2 + 32\nu\bar{\varpi} + 32\bar{\varpi}^2)) - \alpha(9\nu^2 + 34\nu\bar{\varpi} + 32\bar{\varpi}^2)) + \nu\bar{\varpi}((2\lambda + 1)\nu + 4\bar{\varpi})^2}{2(3\nu + 8\bar{\varpi})^2}$ and if the access fee is not too high, i.e. the sufficient condition $\alpha \leq \nu(\bar{\varpi} + \lambda(9\nu^2 + 32\nu\bar{\varpi} + 32\bar{\varpi}^2)) \frac{9\nu^2 + 34\nu\bar{\varpi} + 32\bar{\varpi}^2}{2(3\nu + 8\bar{\varpi})^2}$ holds.

Proof. First, the incumbent will never provide full coverage if the investment costs are positive, i.e. for $F_I > 0$ always follows $\delta^*_I > 0$. This finding is intuitive as demand in the least dense region is approaching zero and the incumbent is not able to cover its investment expense. Second, the incumbent will only invest if the investment costs are not too high, i.e. less or equal

$$F^*_I = \bar{\delta}(2\alpha(\nu\bar{\varpi} + \lambda(9\nu^2 + 32\nu\bar{\varpi} + 32\bar{\varpi}^2)) - \alpha(9\nu^2 + 34\nu\bar{\varpi} + 32\bar{\varpi}^2)) + \nu\bar{\varpi}((2\lambda + 1)\nu + 4\bar{\varpi})^2}{2(3\nu + 8\bar{\varpi})^2}.$$  

(18)

Otherwise, the first order condition is strictly positive and the incumbent will not invest. Again, this is intuitive as the incumbent’s profits in the densest region are not sufficient to cover excessively high investment costs. Note that the maximal investment costs are the same as in the benchmark case.

Given the restriction on the investment costs, the first order condition is strictly decreasing in $\delta_I$ if the access fee satisfies

$$\alpha \leq \nu\frac{(\delta^2 - \delta_I^2)}{(\delta^2 - \delta_I^2)(2\delta^2 - \delta^2_E - \delta_I^2)} \frac{\nu\bar{\varpi} + \lambda(9\nu^2 + 32\nu\bar{\varpi} + 32\bar{\varpi}^2)}{9\nu^2 + 34\nu\bar{\varpi} + 32\bar{\varpi}^2}.$$  

Note that this is a sufficient and no necessary condition because the first order condition might increase for higher access fees. This boundary of $\alpha$ is at minimum, i.e. for $\delta_E = \bar{\delta}$ and $\lambda = 1$, slightly below $\nu$. Hence, this upper boundary is more a technical restriction and states that an access fee around or above the reservation utility, and therefore equilibrium prices above the reservation utility, yield no investment which is straightforward. Because the first order condition (17) has opposite signs in the limits, i.e. $\lim_{\delta_I \to 0} \frac{\partial \pi_I(p^* \delta_I)}{\partial \delta_I} \to F_I$ and $\lim_{\delta_I \to \bar{\delta}} \frac{\partial \pi_I(p^* \delta_I)}{\partial \delta_I} \to -\infty$, and is monotonic decreasing in $\delta_I$, a unique optimal response $\delta^*_I$ exists.

The derivations of the first order condition show that a higher reservation utility yields higher optimal investment of the incumbent, i.e. $\delta^*_I$ decreases, in the reservation utility $\nu$ and the

---

20 For higher access fees, there might be two solutions for the first order condition and an optimal investment decision might still exist.
utility adjustment parameter $\lambda$. This is straightforward as a higher willingness to pay increase both, the equilibrium prices and the amount of new consumers. The effect of higher maximum switching costs is ambiguous. Although the optimal coverage increase with higher switching costs for most parameter combinations, this effect is reversed for some combinations. If the entrant invests more, i.e. $\delta_F$ decrease, the optimal investment of the incumbent increases, too. The intuition of this strategic complementary is as follows. If the entrant invests more, retail prices are decreasing and therefore entrant’s demand from unattached consumers increases. As long as the entrant buys access in the last covered region from the incumbent, the incumbent’s profit in last covered region increases due to the additional demand from unattached consumers and the entrant’s increasing demand for access. Therefore, the incumbent might provide the infrastructure in a less densely populated region.

Moreover, the derivations show that higher access fees increase the optimal investment of the incumbent. The access fee has a doubled effect on total coverage, a first order effect through the incumbent’s first order condition and a second order effect due to the increasing investment of the entrant. This finding might be interpreted as a sign that the indirect coverage effect overcompensates the direct effect of the access fee on prices as the second order effect might be explained by decreasing prices and increasing demand of new consumers. Given this, the profits of the incumbent based on higher access fees would not only increase due to increasing revenues per consumer but also due to the increasing demand for access in the last covered region.

From the analysis of the first order conditions, it becomes clear that both investment incentives increase in the access fee and therefore the ratio of service-based competition $(\delta_E^2 - \delta_I^2)/(\delta_E^2 - \delta_I^2)$ decreases. Although the equilibrium prices are unambiguously increasing with the access fee as long as the entrant does not invest, the net effect of the access fee on prices is ambiguous. Even so, the discussion of both investment decisions indicates that the net effect is negative, i.e. an increasing access fee yields lower prices if the entrant invests, and the simulation below will show that this is in fact the case.

Note that the first order conditions regarding the investment decisions of the entrant (12) and the incumbent (17) could not be solved analytically due to the polynomial equations of high degrees and the equilibrium prices could not be generalized further. Again, we refer the reader to the subsequent simulation. In this section we showed that best investment responses of both firms exist and the numerical simulation confirms the existence of an equilibrium for a range of parameters.
**Effect of the access price**

Lastly, we analyze the effect of the access fee on entrant’s investment decision. To be more precise, we derive the access fees leading to the corner solution, i.e. no investment. Withal, we show that the entrant will never invest in all covered regions. Hence, we can determine ranges of the access fee which yield interior solutions with an interception of both response functions and consequently with partial facility-based competition.

Let $\alpha$ denote the access fee for which the entrant is indifferent between make-or-buy or, in other words, the lower boundary of access fee for which the entrant starts to invest. Solving the first order condition of the entrant (12) for $\alpha$ yields

\[
\alpha = \frac{\delta^2 - \delta_I^2}{\delta_E^2 - \delta_I^2} \left( \frac{(4\lambda - 1)\nu}{8} - \sqrt{\frac{(4\lambda - 1)^2\nu^2}{64} - \frac{F_E(3\nu + 8\omega)^2}{16\delta_E\omega(\nu + 2\omega)}(\delta_E^2 - \delta_I^2)} \right). \tag{19}
\]

For low investment costs $F_E \leq \hat{F}_E$, the entrant starts its investment in the densest region $\delta$ and the access fee inducing investment simplifies to

\[
\alpha = \frac{(4\lambda - 1)\nu}{8} - \sqrt{\frac{(4\lambda - 1)^2\nu^2}{64} - \frac{F_E(3\nu + 8\omega)^2}{16\delta_E\omega(\nu + 2\omega)}}
\]

Hence, for this access fee and investment costs less or equal $\hat{F}_E$, the entrant is indifferent between investment and reselling in region $\delta$. Higher access fees yield investment in some regions whereas the crucial access fee is only subject to the reservation utility, the utility adjustment parameter, the switching costs, and the investment costs and independent from the investment decision of the incumbent.

For high investment costs $F_E > \hat{F}_E$, the entrant starts its initial investment in region $\hat{\delta}_E$, as defined in equation (16). Substituting this region in equation (19) and solving for the access fee yields a non-trivial expression. Moreover, this access fee is subject to incumbent’s optimal investment decision which depends on the access fee, too. Hence, solving this equation analytically is not possible due to the high degree polynomials. In general, we can note that for high investment costs, the entrant initially invests in region $\hat{\delta}_E$ for an access fee equal $\hat{\alpha}$, as defined in equation (14). Given these equations, numerical solutions for the access fee inducing investment as well as as the region up to which the entrant initially invests are deducible even for high investment costs.

---

21Solving the first order condition for the access fee yields a quadratic function and therefore two solutions. Assume investment costs of $F_E = 0$. The entrant would only buy access if it is costless too. If we choose the solution with the addition of the square root, the lower boundary, i.e. the access fee for which the entrant is indifferent between buying access and investment, would be strictly positive. Hence, the solution with subtracting of the square root, which yields $\alpha = 0$ for $F_E = 0$, is the lower boundary.

22Note that $\frac{(4\lambda - 1)^2\nu^2}{64} - \frac{F_E(3\nu + 8\omega)^2}{16\delta_E\omega(\nu + 2\omega)} > 0$ because $\frac{\delta^2((4\lambda - 1)\nu^2\omega^2 - \delta_I^2)(\nu + 2\omega)}{(3\delta^2 - \delta_I^2)^2(3\nu + 8\omega)^2} < \frac{\delta_E((4\lambda - 1)\nu^2\omega^2 - \delta_I^2)(\nu + 2\omega)}{4(3\delta^2 - \delta_I^2)^2(3\nu + 8\omega)^2}$. 

25
Summing up, the minimal access fee inducing investment $\alpha$ is given by

$$
\alpha = \begin{cases} 
\frac{(4\lambda - 1)\nu}{8} - \sqrt{\frac{(4\lambda - 1)^2\nu^2}{64} - \frac{F_E(3\nu + 8\omega)^2}{16\nu(\nu + 2\omega)}} & \text{for } 0 < F_E \leq \hat{F}_E \\
\frac{\nu^2}{\delta_E^2 - \delta_I^2} - \frac{(4\lambda - 1)\nu}{8} - \sqrt{\frac{(4\lambda - 1)^2\nu^2}{64} - \frac{F_E(3\nu + 8\omega)^2}{16\nu(\nu + 2\omega)}\frac{\delta_E^2 - \delta_I^2}{\delta_E^2 - \delta_I^2}} & \text{for } \hat{F}_E < F_E \leq F_{E_{\max}}
\end{cases}
$$

(20)

**Proposition 7.** As long as the investment costs of the entrant are not too high, i.e. $F_E \leq F_{E_{\max}}$, there exists a unique access fee $\alpha$ for which the entrant is indifferent between make-or-buy and for all access fees greater than $\alpha$ the entrant will invest in some regions.

**Proof.** Follows from above arguments.  

Now we can show that in equilibrium the entrant will never invest in all covered regions as long as the difference between the investment costs of both firms is not too high.

**Proposition 8.** As long as the investment costs of the entrant are not too low compared to the investment costs of the incumbent, i.e. $F_E > F_{I_{4(4\lambda - 1)(\nu + 2\omega)}}$, (i) there exists no access fee $\bar{\alpha}$ for which the entrant is willing to invest in all covered regions, i.e. $\delta_E^* = \delta_I^*$, and (ii) the entrant always invests less than the incumbent, i.e. $\delta_E^* > \delta_I^*$ applies.

**Proof.** For a proof of (i) see Appendix A.4. Combined with Proposition 7 follows (ii).

Hence, for all $\alpha < \alpha$ the entrant does not invest and for all $\alpha \geq \alpha$ the entrant invests in dense regions and might buy access in the less dense regions. Figure 6 illustrates the lower boundaries and the set of $\alpha$ which yields partial facility-based competition.

![Figure 6: Lower boundary of the access fee $\alpha$](image)

The lower boundary is increasing with the investment costs which is straightforward. The higher the costs for building own infrastructures, the lower the investment incentives. Hence, the access fee has to increase to induce investment.

Furthermore, using the derivations of $\alpha$ it becomes clear the boundary decrease with the reservation utility, with the utility adjustment parameter, and with the maximum consumer
switching costs, i.e. lower access fees are sufficient to induce investment. For investment costs above $\hat{F}_E$, for which it is never optimal for the entrant to start investment in region $\delta$, the necessary access fee to induce investment could not be derived analytically. However, numerical simulations show that starting at the point $\alpha = \hat{\alpha}$, for which the minimum of entrant’s first order condition equals zero, the access fee increases with the investment costs. Note that as long incumbent’s investment is not too low, i.e. $\delta^*_I$ close to $\bar{\delta}$, the slope of $\alpha$ subject to the investment costs is higher for the second case. Intuitively, if the entrant initially invests in more regions, the access fee has to increase further to induce this investment.

4 Illustration and Discussion of the Results

After deriving the equilibrium conditions, we illustrate and discuss the findings. We will discuss reasonable values and ranges for the parameters to provide demonstrative exemplifications and to adjust the results to empirical findings. Moreover, we will discuss possible extensions and adjustments of the existing model and the robustness of some of the underlying assumptions.

4.1 Parameter Adjustment

In this section, we want to determine the exogenous and endogenous parameters determined by the model. Moreover, the exogenous parameters access fee and maximum switching costs could be influenced by the regulatory authority.

The population density in the densest region $\bar{\delta}$ is assumed as 100. We choose this value for two reasons. First, the average population density in Europe is around 100 people per square kilometer so that the assumed value is not unrealistic high or low. Second, the value could be interpreted as ratio and the region with the densest population is represented with 100% density.

The highest reservation utility $\nu$ is assumed as 100, too. Again, this could be interpreted as ratio and the highest willingness to pay has a value of 100%. Furthermore, a price for a broadband access product in between 0 € and 100 € seems reasonable.

The utility adjustment parameter $\lambda$ is assumed as 1 as the additional benefit from new telecommunications infrastructures are uncertain prior to the investment and a conservative estimation seems reasonable.

As both parameters, the population density in the densest region and the reservation utility, are strictly exogenous and could be interpreted as relative values, both will be seen as fixed and not be varied within the simulation.

The value of the maximum switching costs $\omega$ is assumed to be below the reservation utility to avoid a complete lock-in of consumers. Furthermore, the maximum switching costs have
to be strictly positive to avoid a Bertrand-Equilibrium with price equals marginal costs and no investment at all. For the beginning, let us assume maximum switching costs of \( \bar{\omega} = 40 \), i.e. switching costs below the original price. This assumption will be relaxed later and we will discuss the results for different values. Note, that the maximum switching costs are per se an exogenous parameter but that regulatory guidelines might influence them. An example for such an influence of the regulatory guidelines on the switching costs is the specifications regarding number portability in the Framework Directive (2002/21/EC) but could also apply if the regulator specifies maximum contract durations or switch duration if a consumer changes her provider.

For simplicity, the investment costs of the entrant \( F_E \) and the incumbent \( F_I \) are assumed to be equal. Thereby, we connive at the fact that costs for capital might be higher for entrants or that there might be economies of scale or scope for the incumbent. As investment costs of zero obviously lead to full coverage of both firms, the investment costs are assumed to be strictly positive. Furthermore, there are three values for the investment costs which seems crucial or at least remarkable. First, for investment costs \( F_I > F_{I}^{\text{max}} \), as defined in equation (18), the incumbent’s first order condition (17) does not become zero and therefore the incumbent would never invest. For the given assumptions, the crucial investment costs per region are between 110 094 for \( \alpha = 0 \) and 285 172 for \( \alpha = \frac{\nu}{2} \). Second, the initial investment decision of the entrant is determined by \( \alpha \) which includes two cases and an inner boundary at \( \hat{F}_E \) subject to incumbent’s investment decision, as defined in equation (15). If the incumbent does not invest the investment costs \( \hat{F}_E \) equals 0 and if the incumbent provides full coverage \( \hat{F}_E \) equals 37 461. Third, for investment costs \( F_E > F_{E}^{\text{max}} \), as defined in equation (13), the entrant’s first order condition (12) is strictly positive and the entrant will always prefer reselling in all regions. The maximum investment costs are subject to incumbent’s investment and the access fee. Given the assumptions about the other parameters and an high access fee \( \alpha = \frac{\nu}{2} \), the maximum investment costs goes to infinity if incumbent’s investment goes to \( \delta \) and against \( F_{E}^{\text{max}} = 52 997.8 \) if the incumbent provides full coverage. Given these values, we will simulate results for investment costs between 10 000 and 50 000 to get results for both cases of the lower boundary \( \alpha \). As comparison, recent studies estimated the costs of providing broadband access subject to the underlying technology between 300 € for VDSL and 2 200 € for FTTH per household.\(^{23}\) Taking the capacity of a serving area interface into account yields investment costs of approximately 30 000 € for the VDSL upgrade of one serving area interface.\(^{24}\)

The last parameter to define, the access fee \( \alpha \), will be the independent variable to determine the price and investment decisions. Given the other restrictions and the results from the benchmark case, the access fee will be varied between zero and 50. Note that the entrant will

\(^{23}\)Cf. Katz et al. (2009) and Elixmann et al. (2008).

\(^{24}\)Please note that we might implement higher investment costs if we assume a higher utility adjustment parameter \( \lambda \). Hence, investments in more expensive fiber-to-the-home networks are feasible, too.
only act as reseller in regions without infrastructure if its equilibrium price is greater or equal the access fee. Otherwise, the entrant will only compete via facility-based competition in some regions and the results of Götz (2009) will apply. We do not account for this in our simulation explicitly because the main results do not change qualitatively and our model does not include a benchmark with pure facility-based competition. Alternatively, we might interpret the situation in which the entrant buys access even though the access price is above its equilibrium price such that the provision of service in all covered regions is very important. As an example, one might think of roaming in mobile access technologies.

4.2 Simulation Results

As there exist no closed-form solutions for the investment decisions of the entrant and the incumbent due to the polynomials of high degree, we illustrate the equilibrium results with numerical simulations and discuss them. First, we show how the access fee influences the equilibrium prices. Second, we analyze the effect of the access fee on the investment decisions. Third, we show the impact on demand and profits. Fourth, we provide a reference point for the access fee by simulating the resulting average costs.

Pricing decisions

The equilibrium prices subject to the access fee for low and intermediate investment costs are illustrated in Figure 7. Both prices change in the same manner and the difference between both is based on and increases with the switching costs.

![Figure 7: Equilibrium prices subject to the access fee (for \( \nu = 100, \lambda = 1, \delta = 100, \omega = 40 \)](image_url)

The equilibrium price curve is subdivided in two sections. Prices increase with the access fee as long as the entrant does not invest, i.e. as long as \( \alpha < \alpha \). This yields the same results as in
the benchmark case. The entrant acts as a pure reseller who buys access from the incumbent. For access fees above $\alpha$, the entrant starts to invest and prices decrease with the access fee and with the investment of the entrant. Reconsidering the discussion of the equilibrium prices subject to the investment decisions (equations (10) and (11)), the indirect effect of the access fee via the investment decision dominates the direct effect of the access fee on prices.

The intuition behind the decreasing prices is straightforward. The entrant has to undercut the incumbent’s price to attract consumers from the incumbent and thereby gains new consumers. For access fees above $\alpha$ the entrant starts to invest in the densest areas. The cost decreasing effect of investment dominates the competitive effect of investment, i.e. the cost savings in regions with investment are sufficiently high to overcompensate the decreasing earnings per consumer in all regions. Hence, the entrant becomes more aggressive and the incumbent reacts by lowering its prices, too.

![Figure 8: Equilibrium prices subject to the access fee for high investment costs](image)

In the case with high investment costs $F > \tilde{F}_E$, illustrated in Figure 8, equilibrium prices drop abruptly at $\alpha$. This is based on the higher initial investment of the entrant down to region $\delta^*_E$. The entrant attracts a high amount of new consumers at once and the incumbent decrease its price abruptly, too, to avoid excessive switching of old consumers to the entrant.

**Investment decisions**

Figure 9 illustrates the investment decisions of both firms subject to the access fee for low and intermediate investment costs. The investment of both firms increase with the access fee, i.e. $\delta^*_E$ and $\delta^*_I$ decrease. Though, the effect of the access fee on the incumbent’s investment is not too big compared to the effect on the investment decision of the entrant. The comparison of both figures shows that higher investment costs yield lower investments by both firms and later investment by the entrant, i.e. an higher access fee is necessary to induce facility-based competition. Furthermore, if the access fee is sufficiently high to induce investment, the ratio
of regions with service-based competition, i.e. the difference between $\delta^*_E$ and $\delta^*_I$ and therefore $(\delta^2_E - \delta^2_I)/(\bar{\sigma}^2 - \delta^2_I)$, decreases with the investment costs.

For high investment costs $F > \hat{F}_E$, illustrated in Figure 10, the entrant’s optimal investment decision subject to the access fee has a discontinuity at $\alpha$ and the entrant’s initial investment covers all regions with density greater than $\hat{\delta}^*_E$. Moreover, the incumbent’s investment decision is also discontinuous at this point and total coverage not only increases further but the effect of the access fee becomes stronger. Intuitively, as the prices jump down and therefore a lot of unattached consumers start buying broadband access from the entrant, the incumbent’s profit in last covered region increases due to the additional access sold to the entrant. The region down to which the entrant initially invests $\hat{\delta}^*_E$ decreases with increasing investment costs. Even though a very high access fee is necessary to initially induce investment, the amount of regions with initial facility-based competition increases.

Another result which becomes clear from these illustrations is that the entrant will never
invest in all covered regions. This result does not differ from the findings in other models with economies of density, e.g. Faulhaber & Hogendorn (2000), Valletti et al. (2002) or Götz (2009), and corresponds with situations observable in many European broadband access markets.

Remember that there exists a major difference to these models regarding the competition in the gray areas, the covered regions in which the entrant does not invest. In this setup, the incumbent faces service-based competition in these regions due to an access obligation and is not able to act as a monopoly. The profit of the incumbent in the last covered region is not significantly affected even if the entrant starts to invest because the entrant does not invest in all regions. There are two explanations for this “irrelevance” of entrant’s investment on incumbent’s investment. First, if the entrant starts to invest and lowers its price, the incumbent becomes more aggressive and decreases its price, too. As the incumbent’s demand of old consumers is not subject to prices in an absolute manner but the price difference, the incumbent’s demand stays nearly stable. Second, if the entrant invests and is able to lower its prices, demand of new consumers increases and the entrant purchases more access in the last covered region from the incumbent. Hence, the incumbent realizes additional profits from selling access to the entrant.

The effect of the consumer switching costs on investment is positive. As illustrated in Figure 11 higher maximum switching costs, represented by the solid lines, yield higher investment of the incumbent and lower access fees to induce investment by the entrant. This effect on incumbent’s investment is straightforward as higher switching costs weaken competition and yield higher equilibrium prices. Hence, the incumbent realizes higher profits in each region and the density of the last covered region \( \delta^* \) with zero profits decreases. The effect of the switching costs on incumbent’s investment decision decreases with increasing access fees. As long as the access fees are low, the incumbent invests more whereas the optimal coverage is not significantly affected for high access fees.

Figure 11: Equilibrium investment decisions subject to the access fee with different switching costs (for \( \nu = 100, \lambda = 1, \bar{\delta} = 100, F = 10000, \) and \( \omega = \{10, 40\} \))
Demand

Figure 12a illustrates the equilibrium demands of both firms. Although the demand of the entrant is significantly influenced by the access fee, the effect on incumbent’s demand is negligible. The incumbent’s equilibrium price yields an almost constant demand of its old consumers and total demand is slightly increasing because more regions are covered.

Figure 12b illustrates the two types of the entrant’s demand, i.e. switching consumers and unattached consumers. As long as the entrant does not invest, the demand of switching consumers increases with the access fee as the price difference increases slightly. If the entrant invests, the price difference between both firms decreases and therefore demand of this consumer group decreases, too. As the prices are increasing with the access fee, the demand of new consumers decreases as long as the entrant does not invest and increases if the entrant invests. The demand of new consumers exceeds the demand of switching consumers and, therefore, the effect of entrant’s investment on its total demand is positive.

The increase of total demand due to the attraction of new consumers could be seen as a justification of access regulation. Given a case in which the entrant only attracts switching consumers, the competitive effect of access regulation is negligible.25 In the context of the “ladder of investment”, we might interpret this finding as follows: As the entrant’s incentives to build own infrastructure facilities is mainly driven by the profits based on the additional demand of unattached consumers, the objective of “establishing” entrants in the market is - in our setup - at least questionable.

---


**Profits**

The effect of the access fee on the firms’ profits is illustrated in Figure 13. The entrant’s profit is maximized for costless access to incumbent’s infrastructure whereas the incumbent’s profit is maximized for an access fee which yields indifference between buying access and investment for the entrant.

![Equilibrium profits subject to the access fee](image)

**Figure 13: Equilibrium profits subject to the access fee** (for \( \nu = 100, \lambda = 1, \delta = 100, \omega = 40 \))

Both findings are straightforward. The entrant’s profit decreases with the access fee due to its increasing marginal costs in regions without own infrastructures. If the entrant invests, the slope at which its profit decreases is lowered as this allows the entrant to realize cost savings, i.e., to realize marginal costs equal zero in regions with own infrastructures. Even though the earnings per consumer decrease due to the fiercer retail competition, the entrant’s demand of unattached consumers increases and therefore profits decrease at a lower rate. The profits of the incumbent increases with the access fee as long as the entrant does not invest and decreases with access fees above \( \bar{\alpha} \). This is intuitive because higher access fees increase the incumbent’s profit from selling access in all regions as long as the entrant does not invest. If the entrant starts to invest, the incumbent’s profit decreases due to the lower prices and due to the lost revenues from selling access in the regions with facility-based competition.

The higher the investment costs, the lower the initial profit of the incumbent due to the higher costs of providing the infrastructure and the resulting lower coverage and total demand. The magnitude of the investment costs does not alter the entrant’s profit as long as both firms compete via service-based competition but increases the access fee \( \bar{\alpha} \) which triggers investment. Hence, as the entrant prefers service-based entry for higher access fees, the entrant’s profit decreases further and the incumbent’s profit maximum increases.

The obtained results offer two implications for access regulation and deregulation. First, from the profit functions it becomes obvious why entrants argue against higher access fees. The argumentation that in order to enter the market, entrants have to establish in the market
and to ensure cost coverage with an installed consumer base is not confirmed in our setup. The entrant’s profits are non-negative, even if we allow for higher investment costs of the entrant relative to the incumbent’s investment costs. Second, the possibility of negotiated access does probably yield an inferior solution. Given the optimal access fee for the entrant and the incumbent, the result will be between zero and $\alpha$ and therefore in a range for which the entrant does not invest. If the negotiated access fee is close to $\alpha$ prices are higher and total coverage lower compared to a situation with partial facility-based competition. These results are somehow in line with the findings of Bourreau & Doğan (2004, 2005), who showed that an incumbent might provide attractive access conditions to delay entrant’s investments and to avoid facility-based competition. Hence, it might be superior to implement access regulation with a high access fee as this might yield lower retail prices and higher coverage. Thereby, a price-cap regulation might be insufficient to ensure facility-based competition. As a side note, given this results entrants as well as incumbents might argue against higher access fees.

**A note on cost-based access fees**

Figure 14 illustrates the average costs of both firms. This provides a reference point for the access fee and enlighten two aspects. First, a cost-based access fee $\alpha = AC_I$ always yields pure service-based competition. Second, an access fee above costs might yield a superior outcome due to the facility-based entry and the fiercer retail price competition. Hence, the promotion of facility-based competition with higher access fees might yield a welfare enhancing outcome. This insight confirms the findings of other papers, e.g. Sarmento & Brandão (2007), that cost-based access fees, like the forward looking long-run incremental costs, yield inferior results regarding the investment incentives. note that the necessary access fee for an outcome with facility-based competition might be very high in our model. In our simulations an access fee approximately more than twice the average costs of the incumbent is necessary to trigger investment by the entrant.

![Figure 14: Average costs subject to the access fee](image)

Figure 14: Average costs subject to the access fee (for $\nu = 100$, $\lambda = 1$, $\delta = 100$, $\omega = 40$)
Both illustrations show that the initial investment of the entrant is determined by its average costs. As long as the entrant does not invest and only enters via service-based competition, its average costs equal the access fee, i.e. its marginal costs. If the entrant starts to invest, the average costs include the access fee in regions without investment as well the investment costs in regions with own infrastructure facilities. Moreover, the average costs include the competition effect of investment as entrant’s demand increases due to the lower prices.

The incumbent’s average costs increase with the access fee as prices and coverage increase. If the entrant starts to invest and provides access via its own infrastructure in some regions, the incumbent loses consumers in these regions completely. Thus, the incumbent’s average costs increases more steep as soon as the entrant enters facility-based competition in the dense regions.

5 Extensions

In this last section, we want to discuss two extensions of the model and the robustness of our results. First, we will discuss the effects of geographically differentiated prices and the differences to uniform pricing. Second, we will discuss a setup with a lower share of unattached consumers by relaxing the assumption of a myopic monopolist in the previous period. Allowing for a lower price of the incumbent in the previous period yields a situation with a greater installed base with partially locked-in consumers and less consumers without switching costs.

5.1 Geographically differentiated prices

In contrast to the closely related models of Valletti et al. (2002) and Götz (2009), we did not take the opportunity of geographically differentiated prices into account. Allowing for such a price policy reveals that the incentives to implement such a pricing regime differ significantly from those in the above mentioned models. In the model of Valletti et al. (2002) as well as in the model of Götz (2009), the incumbent has an incentive to charge different prices due to its monopoly position in the less densely populated regions in which the entrant does not invest. In our model, the incumbent has no incentive to charge different retail prices at first as there is competition in all covered regions based on an access obligation. Let us now assume that the regulatory authority permits geographical price differentiation. If the incumbent increases its price in regions with only service-based competition, consumers would switch to the entrant and the incumbent would earn less per consumer as long as its price is above the access fee. Hence, a price differentiation would only be profit increasing if its equilibrium price is below the access fee.\footnote{Even if the entrant has an obligation to provide services if the access fee is above equilibrium prices, it is questionable that the regulatory authority would raise the access fee that high, especially if we consider...} In our setup, it is the entrant who has the initial
incentive to provide geographically differentiated prices. Subject to the kind of competition, i.e. service- or facility-based competition, the entrant faces different costs and might therefore increase its profits if it sets the prices accordingly. Hence, the incumbent has an incentive for geographically differentiated prices as reaction to the entrant’s pricing behavior.

To analyze the outcome of this pricing regime, we adjust the profit functions of the firms such that both maximize their profits with consideration of the differentiated prices $p_{j}^{sbc}$ and $p_{j}^{fbc}$. At the pricing stage, the maximization problems of both firms are then given by

$$\max_{p_{I}^{fbc}, p_{I}^{sbc}} \pi_{I} = \int_{\delta_{I}}^{\delta_{E}} p_{I}^{fbc} y_{I} - F_{I} d\delta + \int_{\delta_{I}}^{\delta_{E}} p_{I}^{sbc} y_{I} + \alpha y_{E} - F_{I} d\delta$$

$$\max_{p_{E}^{fbc}, p_{E}^{sbc}} \pi_{E} = \int_{\delta_{E}}^{\delta_{I}} p_{E}^{fbc} y_{E} - F_{E} d\delta + \int_{\delta_{I}}^{\delta_{E}} (p_{E}^{sbc} - \alpha) y_{E} d\delta.$$  \hfill (21)

Differentiating the profit functions with respect to prices and solving the first order conditions yields the equilibrium prices in regions with facility-based competition, i.e.

$$p_{I}^{fbc} = \frac{\omega((2\lambda + 1)\nu + 4\varpi)}{3\nu + 8\varpi} \quad (23)$$

$$p_{E}^{fbc} = \frac{\nu \omega (4\lambda - 1)}{3\nu + 8\varpi}$$  \hfill (24)

which are only subject to exogenously given parameters, i.e. to the reservation utility parameters $\nu$ and $\lambda$ and to the maximum switching costs $\varpi$.

In regions with service-based competition, the equilibrium prices are

$$p_{I}^{sbc} = \frac{\omega((2\lambda + 1)\nu + 4\varpi)}{3\nu + 8\varpi} + \frac{3(\nu + 2\varpi)}{3\nu + 8\varpi} \alpha \quad (25)$$

$$p_{E}^{sbc} = \frac{\nu \omega (4\lambda - 1)}{3\nu + 8\varpi} + \frac{3\nu + 4\varpi}{3\nu + 8\varpi} \alpha$$  \hfill (26)

and include an additional term which reflects the entrant’s additional marginal costs, i.e. the access fee $\alpha$. These equilibrium prices equal the equilibrium prices in our benchmark case without investments of the entrant, i.e. Equation (6) and (7). The equilibrium prices in regions with service-based competition are strictly increasing with the access fee and the same characteristics as in the benchmark case apply. The extent of facility-based competition has no effect on equilibrium prices in the different regions at all and consequently investment of the entrant does not yield fiercer retail competition anymore.

At the investment stage, the firms maximize their investment levels as in the original model given the equilibrium prices of the subsequent pricing stage. Differentiating the reduced profits with respect to the investments and solving the first order conditions of both firms yield the likely complaints about a margin squeeze.
optimal investment decisions of both firms, i.e.

$$\delta_E^* = \frac{F_E (3 \nu + 8 \overline{w})^2}{4 \alpha \overline{w} ((4 \lambda - 1) \nu - 2 \alpha)(\nu + 2 \overline{w})}$$

and

$$\delta_I^* = \frac{2 F_I (3 \nu + 8 \overline{w})^2}{\nu \overline{w} ((2 \lambda + 1) \nu + 4 \overline{w})^2 + 2 \alpha (\nu \overline{w} + \lambda (9 \nu + 32 \nu \overline{w} + 32 \overline{w}^2)) - \alpha (\nu + 2 \overline{w})(9 \nu + 16 \overline{w})}.$$ 

Figure 15 illustrates the investment decisions of both firms with geographically differentiated prices and uniform pricing. Both investment decisions yield qualitatively similar results as before but differ quantitatively.

Compared to the case with uniform pricing, the incumbent invests less if facility-based entry occurs. The intuition of this result is straightforward: In regions with service-based competition, the equilibrium price of the entrant is strictly increasing with the access fee. As a consequence, the incumbent’s profit in the last covered regions does not increase as the positive effect of increasing prices on the revenues per consumer is compensated by the negative effect on demand.

The entrant’s optimal investment differs from the original result in two ways. First, the entrant starts to invest earlier, i.e. for a lower access fee $\alpha^{DP} < \alpha^{UP}$. The incentive to invest is higher as investment only lowers costs without any effect on competition in regions without own infrastructures. Consequently, the initial investment is not delayed due to the strategic effect and the entrant invests for lower access fees. Second, the entrant invests less for high access fees compared to the case with uniform pricing. With differentiated prices, the entrant always realizes a (constant) positive mark-up on the access fee. In the case with uniform pricing, the entrant’s revenue per consumer decreases with increasing access fees as the prices decrease with the investment.\(^{27}\) Hence, there is a stronger incentive to invest in order to avoid the access fee.

\(^{27}\)In some cases, the revenue per consumer might even become negative.
To summarize, if we allow for geographically differentiated prices in our setup, the incentives to provide such a price regime are reversed in comparison to related models of Valletti et al. (2002) and Götz (2009), i.e. the entrant gives the impetus for differentiated equilibrium prices. In this setup, prices are independent from the access fee in regions with facility-based competition and strictly increasing with the access fee in regions with service-based competition. As the investments of both firms are positively related with the access fee, the typical trade-off between static and dynamic efficiency applies, i.e. there might be higher investment with higher prices or lower retail prices with lower coverage and less facility-based competition. Total coverage decreases compared to a uniform pricing regime. This result is based on the entrant’s lower demand for access in the last covered region as the extent of facility-based competition has no effect on the prices in regions with service-based competition.

5.2 Forward-looking incumbent in the previous period

So far, we assumed a myopic monopolist in the previous period and an equal share of partially locked-in and unattached consumers. In our setup, a main driver of the incumbent’s increasing incentives to invest despite - or because of - the intensified retail competition is the entrant’s increasing demand from unattached consumers in the last covered region. This positive investment effect might be lowered or possibly disappear if the share of unattached consumers decreases, i.e. if we lower the incumbent’s price in the previous period. As the investment stage does not provide a closed-form solution of the investment decisions, it is not possible to solve the game for the previous period analytically and to derive the profit maximizing price $p_{I0}^\ast$. Nevertheless, simulating the equilibrium outcome for prices in the previous period below the monopoly price, i.e. for $p_{I0} < \nu_2$, gives some insights regarding the robustness of our results.

Lowering the share of unattached consumer by setting a lower price $p_{I0}$ yields lower equilibrium prices of both firms in average. This is straightforward as the entrant has to set a lower price in order to get a positive demand. Consequently, the incumbent has to react and sets a lower price, too, to avoid the switch of too many of its consumers.

At the investment stage, the entrant’s investment incentives are weakened and facility-based entry is delayed. The positive effect of fiercer retail competition occurs later, i.e. for higher $\alpha$. The incumbent’s optimal investment increases if the price in the previous period and the share of unattached consumers is lowered. This is intuitive as the incumbent’s installed base with consumers who are partially locked-in due to switching costs is greater. Hence, the region with zero profits is characterized by a lower density for a given access fee. If the share is of unattached consumers without switching costs is sufficiently low, i.e. if the price of the previous period $p_{I0}$ is below some threshold, the investment decision of the entrant becomes discontinuous. This situation is comparable to the situation with high investment costs $F_E > \hat{F}_E$ as discussed above. The entrant’s first order condition becomes zero for high access fees only and the entrant’s initial
investment covers a range of regions.

If we compare total profits of both firms with lower initial prices $p_{I0}$, we observe that the profits of the incumbent increase with a decreasing price $p_{I0}$ while the profits of the entrant decrease. Subject to the access fee $\alpha$, this might lead to a situation in which the investment inducing access fee is too high, i.e. the entrant’s access costs are so high, that the entrant realizes negative profits. Hence, if the share of unattached consumers is very low, the entrant will not invest anymore or might even be foreclosed. This finding is in line with the standard results in models with switching costs.\(^{28}\) There is an incentive to lower the price in the previous period in order to weaken competition in subsequent periods. In our model, there is no competition in the previous period and the incumbent might weaken future competition to an extent that entry, at least in facility-based competition, is impeded.

To summarize, as long as the share of unattached consumers without switching costs, i.e. the initial price $p_{I0}$, is not too low, the results do not change qualitatively. If the installed base and the amount of consumers with switching costs is relatively great, the entrant is not able to invest and only service-based competition is realized.

6 Conclusion

In this paper, we showed that an entrant who faces a make-or-buy decision in regions with different population densities might invest in own infrastructure facilities, at least in densely populated areas. The decision between investment and reselling is determined by possible cost savings and by the – strategic – effect of investments on competition and prices. On the one hand, the entrant has an incentive to invest because investment lowers costs in regions with own facilities. This allows the entrant to lower its price in order to attract more consumers, in particular an increasing demand from unattached consumers. On the other hand, investments yield lower equilibrium retail prices in all covered regions. If the entrant lowers its price, the incumbent becomes more aggressive to avoid the loss of too much of its previous consumers. Hence, the entrant faces a dilemma due to the opposite effects of investment on its profits.

This trade-off provides a possible explanation why entrants do not climb the ladder of investment, especially in the presence of cost-based access pricing. In our simulations, the access fee inducing investment is at least twice the average costs of the incumbent. We showed that the access fee which yields lower prices compared to the situation with a cost-based access fee is above the average costs. Therefore, high non-cost-based, access fees might yield a welfare enhancing outcome by fostering the investments of entrant and increasing intensity of competition. The latter point explains why the entrant and even the incumbent might argue against such a policy. The profits of the firms decrease with (partial) facility-based competition whereas

consumers benefits due to lower retail prices. Hence, the effect on total surplus appears to be ambiguous and a detailed welfare analysis is a topic for further research.

Given a sufficiently high access fee and induced facility-based entry, competition on the retail market is spurred. Thereby, the usual trade-off between static and dynamic efficiency, i.e. higher retail prices with lower penetration and higher coverage versus lower retail prices with higher penetration and lower coverage, does not apply. As long as the entrant provides broadband access via the network of the incumbent in regions without own infrastructures, the effect of fiercer retail competition, decreasing retail prices and increasing demand from unattached consumers dominates the negative effect of more intense competition on investments. Hence, the investment incentives of the incumbent increase even though or, to be more precise, because retail competition becomes fiercer.

Another feature of our model is the inclusion of consumer switching costs. We provide a possible explanation for lower prices of entrants in broadband access markets even if they resell the incumbent’s access product, e.g. via bit stream access. The effect of switching costs on prices and coverage includes the expected trade-off between static and dynamic efficiency: Higher maximum switching costs weaken competition, yield higher retail prices and increases investment incentives of both firms. In contrast to many other models with consumer switching costs, total demand is not fixed but increases with more intense retail competition. A more detailed analysis of the effect of previous customers on the equilibrium outcome might be another topic for further research.

In our model, the entrant only competes via facility-based competition if the investment costs are not too high. Given the magnitude of investments in Next Generation Access Networks, it might be interesting to provide an extension of our model which allows for an analysis of currently debated interventions, for instance a setup in which the entrant and the incumbent cooperate, i.e. jointly invest in some regions, or a setup which includes subsidization.
A Appendix

A.1 Proof of proposition 2

The optimal investment decision of the incumbent was given by

$$\delta^*_{sbc} = \frac{2F_I(3\nu + 8\bar{\omega})^2}{2\alpha(\nu\bar{\omega} + \lambda(9\nu^2 + 32\nu\bar{\omega} + 32\bar{\omega}^2)) - \alpha(9\nu^2 + 34\nu\bar{\omega} + 32\bar{\omega}^2) + \nu\bar{\omega}((2\lambda + 1)\nu + 4\bar{\omega})^2}$$

Differentiation with respect to the access fee yields

$$\frac{\partial \delta^*_{sbc}}{\partial \alpha} = \frac{-4F_I(3\nu + 8\bar{\omega})^2(\nu(\nu(\nu^2 + 32\nu^2 + 32\nu\bar{\omega})) - 2\alpha(9\nu^2 + 32\nu^2 + 34\nu\bar{\omega}) + \nu\bar{\omega}((2\lambda + 1)\nu + 4\bar{\omega})^2)}{(2\alpha(\nu(\nu^2 + 32\nu^2 + 32\nu\bar{\omega})) - \alpha(9\nu^2 + 34\nu\bar{\omega} + 32\bar{\omega}^2) + \nu\bar{\omega}((2\lambda + 1)\nu + 4\bar{\omega})^2)^2}$$

Since the denominator is strictly positive, the equation is negative if

$$\nu(\nu(\nu^2 + 32\nu^2 + 32\nu\bar{\omega})) - 2\alpha(9\nu^2 + 32\nu^2 + 34\nu\bar{\omega}) + \nu\bar{\omega}((2\lambda + 1)\nu + 4\bar{\omega})^2 > 0$$

and therefore if the access fee satisfies

$$\alpha < \frac{\nu\bar{\omega} + \lambda(9\nu^2 + 32\nu^2 + 32\nu\bar{\omega})}{2(9\nu^2 + 32\nu^2 + 34\nu\bar{\omega})}.$$  ■

A.2 Proof of proposition 4

The first order condition of entrant’s reduced profit function is given by

$$\frac{\partial \pi_E}{\partial \delta_E} = F_E - 4\alpha\delta_E \bar{\omega} \left((4\lambda - 1)\nu - 4\alpha\frac{\delta^2_E - \delta^2_I}{\nu^2 - \delta^2_I}\right) \frac{\nu + 2\bar{\omega}}{(3\nu + 8\bar{\omega})^2} = 0 \quad (A.1)$$

Solving the first order condition (A.1) for the investment costs, yields

$$F_E = 4\alpha\delta_E \bar{\omega} \left((4\lambda - 1)\nu - 4\alpha\frac{\delta^2_E - \delta^2_I}{\nu^2 - \delta^2_I}\right) \frac{\nu + 2\bar{\omega}}{(3\nu + 8\bar{\omega})^2} \quad (A.2)$$

In the lower limit where the entrant will invest in all covered regions, i.e. $\delta_E = \delta_I$, the first order condition becomes zero if the investment costs equal

$$F_E = 4(4\lambda - 1)\nu\alpha\delta_I \bar{\omega} \frac{\nu + 2\bar{\omega}}{(3\nu + 8\bar{\omega})^2}$$

For lower investment costs, the entrant would like to invest in more regions than the incumbent. Given our assumption $\delta_E > \delta_I$, this is not possible. For higher investment costs, the first order condition is only satisfied for $\delta_E > \delta_I$. Hence, an optimal investment decision $\delta^*_E \geq \delta_I$ exists.
for $F_E \geq 0$.

In the upper limit where the entrant is indifferent between investment and only service-based entry in region $\delta$, there might exist multiple solutions satisfying the first order condition due to the polynomial of degree three. Hence, we have to take the second order condition into account which is given by

$$\frac{\partial^2 \pi_E}{\partial \delta^2_E} = \left( \alpha^2 \frac{16\nu(3\delta^2_E - \delta^2_f)}{\delta^2 - \delta^2_f} - 4(4\lambda - 1)\alpha \nu \bar{\omega} \right) \frac{\nu + 2\nu}{(3\nu + 8\bar{\omega})^2} \tag{A.3}$$

As the first order condition becomes convex for high $\alpha$, the first order condition is tangent to zero in the limit and the second order condition equals zero. Thus, to maximize entrant’s profits, the second order condition has to be strictly non-positive. As the last term is strictly positive, this holds if

$$\alpha^2 \frac{16\nu(3\delta^2_E - \delta^2_f)}{\delta^2 - \delta^2_f} - 4(4\lambda - 1)\alpha \nu \bar{\omega} \leq 0 \tag{A.4}$$

Beside the trivial solution with $\alpha = 0$, the second order condition is strictly non-positive for an access fee

$$\alpha \leq \frac{(4\lambda - 1)\nu(\delta^2 - \delta^2_f)}{4(3\delta^2_E - \delta^2_f)} \tag{A.5}$$

The corner solution for which the entrant is indifferent between investment and reselling in region $\delta$ is optimal if the first order condition is zero and the second order condition is negative at the point $\delta_E = \bar{\delta}$. Substituting in equation (A.2) and (A.5) yields two conditions. For low access fees, i.e. $\alpha < \frac{(4\lambda - 1)\nu(\delta^2 - \delta^2_f)}{4(3\delta^2_E - \delta^2_f)}$, the entrant is indifferent between investment and reselling in region $\delta$ for investment costs

$$F_E = 4\alpha \bar{\omega}((4\lambda - 1)\nu - 4\alpha) \frac{\nu + 2\nu}{(3\nu + 8\bar{\omega})^2} \tag{A.6}$$

For lower investment costs, the first order condition only holds if the entrant invests, i.e. $\delta_E < \bar{\delta}$. Hence, an optimal investment decision $\delta^*_E \leq \bar{\delta}$ exists for $F_E \leq 4\alpha \bar{\omega}((4\lambda - 1)\nu - 4\alpha) \frac{\nu + 2\nu}{(3\nu + 8\bar{\omega})^2}$.

For higher access fees, i.e. $\alpha \geq \frac{(4\lambda - 1)\nu(\delta^2 - \delta^2_f)}{4(3\delta^2_E - \delta^2_f)}$, the second order condition becomes positive at $\delta_E = \bar{\delta}$. This implies that it is never optimal for the entrant not to invest as long as the investment costs are not too high, i.e. investment costs less or equal (A.2). Above condition
(A.4) for a negative slope of the second order condition holds for

\[-\frac{1}{2} \sqrt{\frac{4\delta_I}{3} + \frac{(4\lambda - 1)\nu(\delta^2 - \delta_f^2)}{3\alpha}} \leq \delta_E \leq \frac{1}{2} \sqrt{\frac{4\delta_I}{3} + \frac{(4\lambda - 1)\nu(\delta^2 - \delta_f^2)}{3\alpha}} \]  

(A.7)

As \(-\frac{1}{2} \sqrt{\frac{4\delta_I}{3} + \frac{(4\lambda - 1)\nu(\delta^2 - \delta_f^2)}{3\alpha}} < 0\) this constraint is not binding. Moreover, for high investment costs, the first order condition becomes positive at \(\delta_E = \delta^\star\). Hence, it is never optimal to resell in region \(\overline{\delta}\) and it must apply that

\[
\frac{1}{2} \sqrt{\frac{4\delta_I}{3} + \frac{(4\lambda - 1)\nu(\delta^2 - \delta_f^2)}{3\alpha}} < \delta^\star
\]

which is satisfied for

\[
\alpha > \frac{(4\lambda - 1)\nu(\delta^2 - \delta_f^2)}{4(3\delta^2 - \delta_f^2)}
\]

To summarize, for high investment costs \(F_E > 4\delta^\star\delta(3\nu - 4\alpha)\frac{\nu + 2\nu}{(3\nu + 8\nu^2)^2}\), the entrant will (i) only invest if the access fees is sufficiently high and (ii) initial investment takes place in all regions with higher density as \(\delta_E = \frac{1}{2} \sqrt{\frac{4\delta_I}{3} + \frac{(4\lambda - 1)\nu(\delta^2 - \delta_f^2)}{3\alpha}} < \delta^\star\). Note that the entrant’s optimal investment function subject to investment costs and access fee has a discontinuity at \(\alpha = \frac{(4\lambda - 1)\nu(\delta^2 - \delta_f^2)}{4(3\delta^2 - \delta_f^2)}\).

Finally, we can determine the maximum investment costs for which the entrant invests. Substituting \(\delta_E = \frac{1}{2} \sqrt{\frac{4\delta_I}{3} + \frac{(4\lambda - 1)\nu(\delta^2 - \delta_f^2)}{3\alpha}}\) in equation (A.2) yields

\[
F_E^{\max} = 4\delta^\star\delta(4\lambda - 1)\nu(\delta^2 - \delta_f^2) + 4\alpha\delta_f^2 \frac{\sqrt{\frac{4\delta_I}{3} + \frac{(4\lambda - 1)\nu(\delta^2 - \delta_f^2)}{3\alpha}}}{3\delta^\star} \frac{\nu + 2\nu}{(3\nu + 8\nu^2)^2},
\]

(A.8)

i.e. the investment costs for which the first order condition becomes zero at minimum. For higher investment costs, the first order condition is strictly positive and the entrant will always choose to resell as long as profits are non-negative.

Additionally, we can substitute \(\alpha = \frac{(4\lambda - 1)\nu(\delta^2 - \delta_f^2)}{4(3\delta^2 - \delta_f^2)}\) in equation (A.6) to obtain the maximum investment costs for which initial investment in region \(\overline{\delta}\) might be optimal, i.e.

\[
\hat{F}_E = 2\delta^3((4\lambda - 1)\nu)^2 \frac{2\nu}{(3\delta^2 - \delta_f^2)} \frac{\nu + 2\nu}{(3\nu + 8\nu^2)^2}
\]

(A.9)

Thus, the optimal investment decision of the entrant \(\delta_E^\star\) satisfying the first order condition (A.1) and maximizing entrant’s profits, i.e. ensuring a negative second order condition, exists.
Subject to the access fee, an interior solution might exist for investment costs $4(4\lambda - 1)\nu\delta_E^2 + \frac{\nu + 2\omega}{(3\nu + 8\omega)^2} < F_E < F_E^{\text{max}}$ for which the entrant invests in dense regions and buys access in the others for some access fees. Given this case, it applies

$$\delta_E^* \in \begin{cases} [\delta_I, \delta] & \text{for } 0 \leq F_E \leq \hat{F}_E \\ [\delta_I, \frac{1}{2} \sqrt{\frac{4\lambda - 1}{\alpha} + \frac{(4\lambda - 1)\nu(\delta^2 - \delta_I^2)}{3\alpha}}] & \text{for } \hat{F}_E < F_E \leq F_E^{\text{max}} \end{cases}$$

For investment costs above the maximum switching costs or low access fees, the entrant choose the corner solutions, i.e. pure reselling. ■

### A.3 Proof of Proposition 5

The first order condition of entrant’s reduced profit function is given by

$$\frac{\partial \pi_E}{\partial \delta_E} = F_E - 4\alpha\delta_E\omega \left((4\lambda - 1)\nu - 4\alpha \frac{\delta_E^2 - \delta_I^2}{\delta^2 - \delta_I^2}\right) \frac{\nu + 2\omega}{(3\nu + 8\omega)^2} = 0 \quad (A.10)$$

and the simplified investment decision with only consideration of costs by

$$\frac{F_E}{y_E(p^*, \delta)} = \alpha \iff 2F_E - \alpha\delta_E \left((4\lambda - 1)\nu - 4\alpha \frac{\delta_E^2 - \delta_I^2}{\delta^2 - \delta_I^2}\right) \frac{\nu + 2\omega}{3\nu + 8\omega} = 0 \quad (A.11)$$

Solving (A.11) for the investment costs, substituting in (A.10), and rearranging yield

$$3\alpha\delta_E \nu \left((4\lambda - 1)\nu - 2\alpha \frac{\delta_E^2 - \delta_I^2}{\delta^2 - \delta_I^2}\right) \frac{\nu + 2\omega}{(3\nu + 8\omega)^2} = 0 \quad (A.12)$$

If both approaches, i.e. the first order condition and the simplified decision yield the same investment level, above equation (A.12) has to equal zero. Beside the trivial case with $\alpha = 0$, this is satisfied for

$$\tilde{\alpha} = \frac{(4\lambda - 1)\nu(\delta_E^2 - \delta_I^2)}{4(\delta_E^2 - \delta_I^2)} \quad (A.13)$$

Substituting $\tilde{\alpha}$ in our initial equations (A.10) and (A.11) yields for both $F_E = 0$. Hence, for positive and identical investment costs, both conditions are never satisfied for the same investment level $\delta_E$. 

45
Solving both conditions for the fixed costs yields

\[ F_{E}^{FOC} = 4\alpha \delta_{E} \nu \left( 4\lambda - 1 \right) \left( \nu - 4\alpha \frac{\delta_{E}^{2} - \delta_{I}^{2}}{\delta^{2} - \delta_{I}^{2}} \right) \frac{\nu + 2\varpi}{\left( 3\nu + 8\varpi \right)^{2}} \]

\[ F_{E}^{SID} = \alpha \delta_{E} \left( 4\lambda - 1 \right) \left( \nu - 4\alpha \frac{\delta_{E}^{2} - \delta_{I}^{2}}{\delta^{2} - \delta_{I}^{2}} \right) \frac{\nu + 2\varpi}{2\left( 3\nu + 8\varpi \right)} \]

Subtracting the investment costs and rearranging yield

\[ F_{E}^{SID} - F_{E}^{FOC} = 3\alpha \delta_{E} \nu \left( 4\lambda - 1 \right) \left( \nu - 4\alpha \frac{\delta_{E}^{2} - \delta_{I}^{2}}{\delta^{2} - \delta_{I}^{2}} \right) \frac{\nu + 2\varpi}{2\left( 3\nu + 8\varpi \right)^{2}} \]

which is positive for all access fees \( 0 < \alpha < \tilde{\alpha} \). Therefore, both conditions are only satisfied for the same investment level \( \delta_{E} \) if the investment costs are lower in the case with consideration of the price effect. If the investment costs are the same in both cases, the optimal investment level \( \delta_{E} \) has to be greater to satisfy the first order condition, i.e. investment is lower. Hence, the investment incentives are lower in the case with consideration of the price effect and consequently the entrant will invest less compared to the case in which the investment decision is only based on costs. ■

A.4 Proof of proposition 8 (i)

Let us assume that the entrant will invest in all regions covered by the incumbent. Substituting \( \delta_{E}^{*} = \delta_{I} \) in the first order condition of the incumbent (17) yields

\[ \frac{\partial \pi_{I}}{\partial \delta_{I}} = F_{I} - \delta_{I} \left( \frac{\nu \varpi (2\lambda + 1) \nu + 4\varpi)^{2}}{2\left( 3\nu + 8\varpi \right)^{2}} + \alpha \frac{\nu \varpi + \lambda (9\nu^{2} + 32\varpi + 32\nu^{2})}{(3\nu + 8\varpi)^{2}} \right) \neq 0 \]

and solving for the access fee

\[ \alpha = \frac{F_{I}(3\nu + 8\varpi)^{2}}{\delta_{I} \nu (\nu \varpi + \lambda (9\nu + 32\nu \varpi + 32\nu^{2}))} - \frac{\varpi (3\nu + 4\varpi)^{2}}{2(\nu \varpi + \lambda (9\nu + 32\nu \varpi + 32\nu^{2}))} \] \hspace{1cm} (A.14)

The first order condition of the entrant for \( \delta_{E}^{*} = \delta_{I} \) is given by

\[ \frac{\partial \pi_{E}}{\partial \delta_{E}} = F_{E} - 4(4\lambda - 1)\alpha \delta_{I} \nu \varpi \frac{\nu + 2\varpi}{(3\nu + 8\varpi)^{2}} \neq 0 \]
Substituting the access fee from equation (A.14) yields

\[
\frac{\partial \pi_E}{\partial E} = F_E - 4(4\lambda - 1)\nu \sigma \frac{F_I(3\nu + 8\sigma)^2}{\delta_I \nu \sigma + \lambda (9\nu + 32\nu \sigma + 32\sigma^2)}
- \frac{\sigma (3\nu + 4\sigma)^2}{2(\nu \sigma + \lambda (9\nu + 32\nu \sigma + 32\sigma^2))} \frac{\nu + 2\sigma}{(3\nu + 8\sigma)^2} = 0 \quad (A.15)
\]

Solving for the last region covered by both firms \( \delta_I \) leads to

\[
\delta_I = \frac{(3\nu + 8\sigma)^2 (4(4\lambda - 1)F_I \sigma \nu + 2\sigma) - F_E (\nu \sigma + \lambda (9\nu + 32\nu \sigma + 32\sigma^2)))}{2(4\lambda - 1)\nu \sigma^2 (\nu + 2\sigma)((2\lambda + 1)\nu + 4\sigma)^2} \quad (A.16)
\]

Hence, both first order conditions might be zero if both firms choose this region \( \delta_I \) which is only non-negative if

\[
4(4\lambda - 1)F_I \sigma (\nu + 2\sigma) - F_E (\nu \sigma + \lambda (9\nu + 32\nu \sigma + 32\sigma^2)) \geq 0
\]

On the opposite, if the difference between both investment costs is sufficiently low, i.e. both investment costs are sufficiently correlated, i.e.

\[
F_E > \frac{4(4\lambda - 1)\sigma (\nu + 2\sigma)}{\nu \sigma + \lambda (9\nu + 32\nu \sigma + 32\sigma^2)}
\]

with \( 0 < \frac{4(4\lambda - 1)\sigma (\nu + 2\sigma)}{\nu \sigma + \lambda (9\nu + 32\nu \sigma + 32\sigma^2)} \leq 1 \), the optimal region \( \delta_I = \delta_E \) as defined in equation (A.16) is negative and consequently no feasible solution exists. Hence, for sufficient correlated investment costs, there exists no access fee \( \sigma \) which yields to an optimal investment decision \( \delta_E^* = \delta_I^* \). \( \blacksquare \)
References


Commission of the European Communities (2009), ‘Community Guidelines for the application of State aid rules in relation to rapid deployment of broadband networks ’.


48


