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Information Acquisition and Learning from Prices Over the Business Cycle

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Abstract

We study firms’ incentives to acquire costly information in booms and recessions to understand the role of endogenous information in explaining asymmetric business cycles. When the economy has been in a boom in the previous period, and firms enter the current period with an optimistic belief, the incentive to acquire information is weaker than when the economy has been in a recession and firms share a pessimistic belief. However, the price system, in transmitting information from informed to uninformed firms, moderates asymmetric incentives in information acquisition and renders the aggregate learning outcome approximately acyclical. Our results challenge the prevailing view of procyclical learning as the source of asymmetric business cycles.

JEL codes: D51, D82, D83, D84, E39.

Keywords: information choice, rational expectations, asymmetric information, Bayesian learning, strategic substitutability, business cycle asymmetries.

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1 Introduction

A perennial question in macroeconomics is why business cycles exhibit asymmetry. Slow and gradual expansions, contrasted by prompt and sharp contractions have been considered a key stylized fact of the business cycle by such early contributors as Pigou (1926), Mitchell (1927), and Keynes (1936). One strand of the theoretical literature argues that procyclical learning, in the form of more or better information in booms than in recessions, is the main force behind the observed asymmetric dynamics.\(^1\) The idea is that in an environment where agents hold only imperfect information about the current state of the economy, upon a state change, procyclical learning induces only small upward revisions in agents’ beliefs during recessions, but large downward revisions during booms. This pattern of learning triggers a quick response on part of the agents when the state switches from a boom to a recession, but only a slow response when the economy transits from a recession to a boom. Despite that explanation’s intuitive appeal, two, so far unanswered, questions remain. First, is procyclical learning optimal when firms are allowed to choose their information? Second, how does information contained in equilibrium prices affect learning? To address these questions, we develop an equilibrium model of costly information acquisition and study optimal learning in booms and recessions.\(^2\)

Our paper is most closely related to the literature concerned with procyclical learning as the source of asymmetric business cycles. Chalkley and Lee (1998) study a binary state, binary action model of capital utilization with imperfect information about the economy’s state. In their model, due to risk aversion, investors require more precise information to choose the high than the low action, the latter constructed to be the safer choice. Hence, noise investors, whose actions are independent of their belief about the economy’s state, constitute a larger fraction of the economy in recessions than in booms. This, in turn, renders signals about the economy’s state more noisy in recessions than in booms. As a consequence, the dynamics of beliefs and aggregate


\(^2\)Methodologically, the idea of information choice is not novel. See Veldkamp (2011) for a coverage of early and more recent models of information choice in macroeconomics and finance.
activity are characterized by fast declines and slow recoveries. In Veldkamp (2005) asymmetric movements in lending rates are the result of more investment projects being undertaken in good than in bad times which generates a procyclical number of public signals about the unknown probability of a positive return. Slow booms and sudden crashes in asset markets occur as investors face a small sample problem in bad times while learning quickly about a change in the economy’s state during a boom. Similar to the idea of a larger number of signals in good than in bad times in Veldkamp (2005), the explanation for asymmetric movements in macroeconomic aggregates in Van Nieuwerburgh and Veldkamp (2006) relies on procyclical learning as a consequence of higher precision signals in booms than in recessions. In their model, an additional additive shock to aggregate technology ensures that the signal-to-noise ratio and thus learning is procyclical. All aforementioned papers, advocating procyclical learning as the source of asymmetric booms and recessions, share three model features that separate them from our analysis. First, agents in their models are assumed to be passive learners whereas we allow them to choose whether to become informed, i.e. they are active learners. Second, we allow for an informational role of prices, that arises naturally in equilibrium with asymmetrically informed agents, a channel that is however absent in the three papers. Third, public signals about aggregate activity are constructed to be more informative in booms than in recession in their models. In Chalkley and Lee (1998) only the high action, which firms choose when being sufficiently confident that the economy is in the good state, generates publicly observed information. Similarly, in Veldkamp (2005) the precision of the public signal moves procyclically as the number of investment projects is, by construction, greater in booms than in recessions. In Van Nieuwerburgh and Veldkamp (2006), in turn, the variance of the aggregate statistic is smaller in booms than in recessions due to the combination of an additive and a multiplicative shock to aggregate technology. In contrast, our model does not rely on the construction of an asymmetrically informative aggregate statistic. Rather, we analyze an economic environment

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3Although the model in Veldkamp (2005) is primarily about asset markets instead of business cycles, gradual booms and prompt crashes due to procyclical learning make it an interesting and relevant reference for our analysis.

4It is an important and well known result that with asymmetric information at least some agents will wish to reoptimize on their plans if learning from equilibrium prices is suppressed, see e.g. the discussions in Grossman (1981) and Laffont (1989), chapter 9.
where all exogenous, public information is acyclical. This combined with active learning ensures that any variation in the precision of firms’ information about the unknown state over the business cycle is the result of their information choices. Our contribution to this literature is to show that learning is weakly countercyclical rather than procyclical, once one allows for information choice and the price system to transmit information. The learning asymmetry found in our paper challenges the prevailing view of procyclical learning as the source of asymmetric business cycles.

In our model, firms initially hold only imperfect information about the aggregate technology level that varies as the economy moves between booms and recessions. Prior to hiring labor in a perfectly competitive market, firms choose whether to acquire a fully revealing signal about the economy’s true state at some fixed cost, and thus to learn the true technology level. An additional endogenous signal is provided by the labor market clearing wage. As the rational expectations equilibrium wage reflects firms’ employment decisions, and ultimately the information they hold, it transmits information from firms that have bought the fully revealing signal to those that have not. In our model information acquisition is a strategic substitute. An individual firm’s expected gain from acquiring the costly signal decreases as the fraction of informed firms increases. Demand for information and hence the fraction of informed firms differ across booms and recessions, albeit only slightly. When the economy has been in a boom in the previous period, and firms enter the current period with an optimistic belief the fraction of informed firms is smaller than when the economy has been in a recession and firms share a pessimistic belief. However, learning is approximately acyclical, as is the informational content of equilibrium wages. The approximate symmetry of optimal learning arises from two interacting effects. First, firms do not have substantially different incentives to acquire costly information in booms and recessions. Second, learning from equilibrium wages lowers the incentive to become informed equally in booms and in recessions. This is due to uninformed firms being able to refine their information about the unknown state by observing the equilibrium wage. Due to this information transmission, learning from equilibrium wages dampens any asymmetric incentives for costly information acquisition. Thus, firms’ incentives to acquire information are more symmetric than without learning from wages.

The rest of the paper is organized as follows. In the next section, we lay
out the model environment and describe the information structure together with firms’ learning rule and the ordering of events. Section 3 defines and analyzes equilibrium of the model. Here we show existence and uniqueness of rational expectations equilibrium with costly information acquisition. In Section 4 we present our main results: approximate acyclicity of both demand for information and the informativeness of the price system. Section 5 discusses the role of learning from equilibrium wages in our model, and illustrates attainability of rational expectations equilibrium beliefs. Section 6 concludes.

2 The model environment

Time is discrete and indexed by \( t \geq 0 \). In each period the state of the economy is described by \( z_t \in Z = \{z, \bar{z}\} \), \( 0 < z < \bar{z} \), where \( z \) and \( \bar{z} \) indicate a recession, and a boom respectively. The evolution of the state \( z_t \) is governed by a Markov chain with time invariant and symmetric transition probabilities. The persistence of the process is denoted with \( \rho \in (\frac{1}{2}, 1) \), where 

\[
\rho = \mathbb{P}(z_{t+1} = \bar{z} | z_t = z) = \mathbb{P}(z_{t+1} = z | z_t = \bar{z}).
\]

There is a measure-one continuum of ex ante identical firms, indexed by \( i \in [0, 1] \). Firm \( i \) produces output \( y_{it} \) employing labor \( h_{it} \), taking as given the wage rate \( w_t \). The firm’s real profits in period \( t \) are given by

\[
\Pi_{it} = y_{it} - w_{t} h_{it}.
\]

The production technology of the firm exhibits diminishing returns to labor and is hit by an aggregate technology shock that depends on the state of the economy\(^5\)

\[
y_{it} = z_{t} \log(1 + h_{it}).
\]

We close the model by introducing a representative household with preferences represented by the following period utility function defined over consumption and leisure

\[
U(c_t, \ell_t) = c_{t} + \phi_t \log(\ell_{t}),
\]

\(^5\)The log-specification of firms’ production technology in (2) and of the representative household’s utility from leisure in (3) greatly simplifies equilibrium analysis, in that it allows us to derive a unique equilibrium wage functional that is linear in the exogenous shocks.
where \( \phi_t \in \Phi = \left[ \phi, \overline{\phi} \right] \), \( 0 < \phi < \overline{\phi} \), features a uniform i.i.d. taste shock that is independent of the state \( z_t \).\(^6\) The role of this aggregate supply shock, whose realization is known to the household but unknown to firms, is to introduce noise in the information revealed by the labor market clearing wage.\(^7\) As is well known from Grossman and Stiglitz (1976), in the absence of unobservable noise in labor supply, a competitive rational expectations equilibrium with costly information acquisition would fail to exist. Moreover, since our model addresses the information acquisition decision of competitive firms rather than of the representative household, we assume that consumption enters linearly in (3). Under that assumption, the household’s labor supply schedule varies with the shock \( \phi_t \) but remains unaffected by its belief about the state. The household’s endowment of time is normalized to unity, that is \( \ell_t + h_t \leq 1 \). Finally, the representative household owns all firms and finances its consumption expenditures from labor income and aggregate profits. The budget constraint therefore reads

\[
C_t \leq w_t h_t + \int_0^1 \Pi_{it} \, di .
\]

This concludes the description of the physical environment of the model. We now lay out the information structure of the economy and describe firms’ learning rule together with the ordering of events.

**Information structure, learning, and ordering of events**

In our model, the true state is a priori unknown to all firms by assumption. However, firms are allowed to acquire a costly signal about the state prior to choosing their profit maximizing employment level. In addition to this costly and exogenous signal, the labor market clearing wage will provide firms with another costless and endogenous signal about the current state. Whenever firms learn a new piece of information about the state, they update their

\(^6\)As we seek to analyze the microfoundations of learning over the business cycle, our choice of a state independent, uniformly distributed taste shock, together with the symmetric transition probabilities in the binary Markov chain, ensures that the source of any asymmetry does not hinge on the specification of the model’s stochastic environment.

\(^7\)The introduction of unobservable noise in labor supply in our model corresponds to the random asset supply assumption in Grossman and Stiglitz (1980) and many closely related papers, for instance Hellwig (1980), Diamond and Verrecchia (1981), Verrecchia (1982), Admati (1985), and more recently in Ganguli and Yang (2009) and Van Nieuwerburgh and Veldkamp (2009).
belief in a Bayesian fashion. Since firms will hold different beliefs about
the state within a single period, we distinguish between the following three
stages.

Stage 1: Costly information acquisition. At the beginning of each period
the state $z_t \in \mathcal{Z}$ is drawn according to the Markov chain. Firms do not
learn the true state. Instead, they enter the period with a common prior
belief $\mu_t$ about the economy being in a boom, where $\mathbb{P}(z_t = 0 | z_{t-1}) =
\mu_t$ derives from the Markov chain. All firms choose simultaneously
whether to acquire a perfectly revealing signal $\mathcal{S}_t \in \mathcal{S} = \{S, \mathcal{F}\}$ about
the state at a fixed cost $\kappa > 0$ that is equal across all firms and periods.
Reselling purchased information is not permissible. Firms that pay $\kappa$
to observe signal $\mathcal{S}_t$ update their belief to

$$\hat{\mu}_I^t = \begin{cases} 
1 & \text{if } \mathcal{S}_t = \mathcal{F}, \\
0 & \text{if } \mathcal{S}_t = \mathcal{L}, 
\end{cases}$$

where the superscript $I$ identifies firms that become informed. Throughout
the model $\lambda_t \in [0, 1]$ denotes the fraction of firms that ac-
cquire the costly signal in stage 1 and hold the updated belief $\hat{\mu}_I^t$. Ac-
cordingly, in each period there is a fraction $1 - \lambda_t$ of firms that choose
not to observe signal $\mathcal{S}_t$ and keep their initial prior belief $\mu_t$.

Stage 2: Learning from the equilibrium wage. Firms enter stage 2 with
their endogenous belief about the state from stage 1. They maximize
expected profits by choosing the optimal level of employment $h_{it}$. Firms
take as given the real wage rate $w_t$ and account for any information
contained in the equilibrium wage about the state in their optimal labor
demand. In particular, uninformed firms revise their stage 1 belief $\mu_t$
about the state to $\hat{\mu}_U^t$ upon observing the equilibrium real wage $w_t$. To

\footnote{The fact that firms share a common prior is not an assumption. At the end of each period
they learn the true state perfectly by observing their own output in (2) and exploiting their
knowledge of the symmetric transition probability. This yields a common prior belief at the
beginning of each period $t > 0$.}

\footnote{In the following, we will repeatedly refer to firms that acquire the costly signal as informed firms, and those firms refraining from costly information acquisition as uninformed firms. This is not entirely correct however, since the equilibrium wage contains noisy information about the state and thus allows firms that do not acquire the costly signal to become informed to some extent. However, no confusion should arise from our slight abuse of terminology.}
the contrary, informed firms do not revise their belief $\bar{\mu}_t I$ from stage 1, as the equilibrium wage contains information about the state only in the form of the exogenous signal $S_t$ that informed firms already know. The representative household privately learns the realization of the taste shock $\phi_t$ and forms its labor supply $h^S_t$ to maximize expected period utility. The labor market clears.

**Stage 3: End-of-period learning.** Informed and uninformed firms produce outputs $y_t I$ and $y_t U$ according to their employment decisions from stage 2, and given the realized technology level from stage 1. The representative household chooses consumption, and the goods market clears. From observing their own output, uninformed firms can infer the true $z_t$ perfectly. Next period’s common prior belief $\bar{\mu}_{t+1}$ obtains from perfect knowledge of $z_t$ and the transition probabilities of the Markov chain

$$\bar{\mu}_{t+1} = \begin{cases} 1 - \rho & \text{if } z_t = \underline{z}, \\ \rho & \text{if } z_t = \bar{z}. \end{cases}$$

For notational convenience we define the set of possible prior beliefs as $\mathcal{M} = \{1 - \rho, \rho\}$. As a consequence of perfect end-of-period learning, information in the form of the costly signal has value only in the current period. The information acquisition problem in stage 1 is therefore purely static, as are the household’s and firms’ optimization problems in stages 2 and 3. This allows us to drop the time subscript from the next section on.

### 3 Equilibrium

We solve the model backwards, starting from equilibrium in the labor market in stage 2, for a given fraction of informed firms.\textsuperscript{10} Then, we solve the stage 1 information acquisition problem taking as given the distribution of equilibrium outcomes in the labor market.

We solve for the labor market equilibrium using rational expectations equilibrium (REE) under asymmetric information, based on the pioneering

\textsuperscript{10}Given that the household does not have access to a storage technology, goods market equilibrium in stage 3 is given by $\int y_i \, di = c$. 

8
work of Lucas (1972) and Green (1973).\textsuperscript{11} This equilibrium concept accounts for learning from prices by imposing a consistency requirement on equilibrium beliefs. Namely, beliefs are required to be in line with any information contained in the observed equilibrium wage. We show that in our model rational expectations equilibrium à la Lucas and Green exists and is unique.

3.1 Labor market equilibrium

Labor demand and supply schedules are found by solving the household’s and firms’ maximization problems. The household solves its static utility maximization in two steps. First, in stage 2, it chooses how much labor to supply for a given wage and realization of taste shock, $h^S(w, \phi)$. Then, in stage 3, when labor income and profits are realized, it chooses consumption.

For $\lambda > 0$, the equilibrium wage can reveal the signal $S$ the informed firms acquired. Hence, uninformed firms update their belief using any information that may be contained in the equilibrium wage they observe. Letting $\hat{\mu}^U(w, \mu)$ to stand for this updated belief, an uninformed firm’s profit maximization problem reads

$$\max_{h^U \geq 0} \left\{ \hat{\mu}^U(w, \mu) \Pi(w, \bar{z}, h^U) + (1 - \hat{\mu}^U(w, \mu)) \Pi(w, \bar{z}, h^U) \right\}.$$ \hfill (7)

The resulting labor demand of an uninformed firm is denoted by $h^U(w, \hat{\mu}^U)$.

Informed firms maximize expected profits for a given wage, forming expectations with belief $\hat{\mu}^I(w, \mu, S)\textsuperscript{12}$. That is, they solve

$$\max_{h^I \geq 0} \left\{ \hat{\mu}^I(w, \mu, S) \Pi(w, \bar{z}, h^I) + (1 - \hat{\mu}^I(w, \mu, S)) \Pi(w, \bar{z}, h^I) \right\},$$ \hfill (8)

yielding $h^I(w, \hat{\mu}^I)$, the labor demand of an informed firm. Having laid out the maximization problems of the agents, we can now define a rational expectations equilibrium in the labor market.

**Definition 1** (Rational expectations equilibrium in the labor market). \textit{Given a fraction of informed firms, $\lambda \in [0, 1]$, rational expectations equilibrium in

\textsuperscript{11}For surveys on extensions of rational expectations equilibrium to asymmetric information see Radner (1979) and Grossman (1981).

\textsuperscript{12}Informed firms do not learn anything new from the equilibrium wage, but we still write their belief as a function of the wage to indicate that their belief is equally required to be consistent with the equilibrium wage as formalized in (10). Similarly, the prior belief is redundant as an argument due to the fully revealing nature of the costly signal.
the labor market is a pair of demand schedules \( h^U(w, \hat{\mu}^U) \) and \( h^I(w, \hat{\mu}^I) \), a supply schedule \( h^S(w, \phi) \) and a wage functional \( W_\lambda(\phi, \mu, \mathcal{S}) \) such that for all \( (\phi, \mu, \mathcal{S}) \in \Phi \times \mathcal{M} \times \mathcal{S} \) and \( w = W_\lambda(\phi, \mu, \mathcal{S}) \)

1. \( h^U(w, \hat{\mu}^U) \) and \( h^I(w, \hat{\mu}^I) \) solve the uninformed and informed firm’s profit maximization problem in (7) and (8), respectively;
2. beliefs are consistent with the realized wage \( w \)
   \[ \hat{\mu}^U(w, \mu) = \mathbb{P}(z = z_0 | w = W_\lambda(\phi, \mu, \mathcal{S}), \mu) \]  
   \[ \hat{\mu}^I(w, \mu, \mathcal{S}) = \mathbb{P}(z = z_0 | w = W_\lambda(\phi, \mu, \mathcal{S}), \mu, \mathcal{S}) \]  
3. \( h^S(w, \phi) \) solves the household’s stage 2 problem;
4. labor market clears
   \[ (1 - \lambda)h^U(w, \hat{\mu}^U) + \lambda h^I(w, \hat{\mu}^I) = h^S(w, \phi). \]

The following proposition proves to be helpful in establishing existence and uniqueness of the labor market equilibrium. In particular, we show that the combination of uniform taste shocks and binary signals induces “all-or-nothing” learning on part of the uninformed firms.

**Proposition 2** (“All-or-nothing” learning from REE wages). The set of rational expectations wages in the labor market can be partitioned into a set of wages which perfectly reveal the signal \( \mathcal{S} \) of the informed firms and a set of wages which are perfectly uninformative about \( \mathcal{S} \).

**Proof.** Solving the representative household’s labor supply problem yields

\[ h^S(w, \phi) = \begin{cases} 1 - \frac{\phi}{w} & \text{if } w > \phi \\ 0 & \text{otherwise.} \end{cases} \]  

Firm \( i \)'s labor demand, which solves its profit maximization problem is

\[ h_i(w, \mu) = \begin{cases} \frac{\mathbb{E}_i[z | w]}{w} - 1 & \text{if } w < \mathbb{E}_i[z | w] \\ 0 & \text{otherwise,} \end{cases} \]  

where \( \mathbb{E}_i[z | w] \) denotes the expectation with respect to the equilibrium belief \( \hat{\mu}_i(\cdot) \). For strictly positive equilibrium demands and supply\(^\text{13} \), market clearing

\(^{13}\text{A sufficient condition for strictly positive equilibrium quantities is } \bar{z} - \bar{z} < \bar{z} - \bar{\phi}.\)
in the labor market requires

\[(1 - \lambda)E[z \mid w] + \lambda E[z \mid w, \mathcal{F}] = 2w - \phi. \quad (14)\]

First, note that the equilibrium wage can only contain information about state \(z\) through the signal of the informed firms, \(\mathcal{F}\). This follows from the informed and uninformed having identical information about \(z\) prior to information acquisition and \(\phi\) being distributed independently of \(z\). Thus, \(E[z \mid w, \mathcal{F}] = E[z \mid \mathcal{F}]\).

To characterize the informativeness of wages, first suppose \(\mathcal{F} = \mathcal{F}_0\) and \(\phi = \phi' \in \Phi\). Equilibrium wage \(w = \mathcal{W}_\lambda(\phi', \mu, \mathcal{F})\) is determined by

\[(1 - \lambda)E[z \mid w] + \lambda z = 2w - \phi'. \quad (15)\]

Note that if there does not exist \(\phi'' \in \Phi\) such that

\[(1 - \lambda)E[z \mid w] + \lambda z = 2w - \phi'', \quad (16)\]

then \(w\) can only obtain when \(\mathcal{F} = \mathcal{F}_0\), hence perfectly revealing \(\mathcal{F}\). Otherwise, \(w\) does not fully reveal \(\mathcal{F}\). Hence, for \(w\) not to fully reveal \(\mathcal{F}\), taste shock \(\phi'\) and \(\lambda\) have to be such that \(\phi' - \lambda(z - \bar{z}) \geq \phi\). Similarly, for \(\mathcal{F} = \overline{\mathcal{F}}\) and \(\phi = \phi'' \in \Phi\), if \(\phi'' + \lambda(z - \bar{z}) \leq \bar{\phi}\), the resulting equilibrium wage does not fully reveal \(\mathcal{F}\). Otherwise, the uninformed can infer from the equilibrium wage that \(\mathcal{F} = \overline{\mathcal{F}}\).

To show that not-fully-revealing wages are perfectly uninformative about \(\mathcal{F}\), let us derive the probability density function of \(w\), conditional on \(\mathcal{F}\). One obtains

\[f(w \mid \mathcal{F}) = \frac{1}{\phi - \phi'} \left| \frac{\partial \phi'}{\partial w} \right|. \quad (17)\]

Note from (15) and (16) that \(\left| \frac{\partial \phi'}{\partial w} \right|\) is not a function of \(\mathcal{F}\) for not-fully-revealing wages. Hence, from Bayes’ rule we obtain the following ratio of posterior beliefs for all \(\mu \in (0, 1)\)

\[
\frac{\mu \mathcal{U}(w, \mu)}{1 - \mu \mathcal{U}(w, \mu)} = \frac{f(w \mid \mathcal{F})}{f(w \mid \overline{\mathcal{F}})} \frac{\mu}{1 - \mu} \quad (18)
\]

\[
= \frac{\mu}{1 - \mu} \quad (19)
\]

for \(w\) which does not fully reveal \(\mathcal{F}\). Thus, not-fully-revealing wages are perfectly uninformative about \(\mathcal{F}\). \(\Box\)

\[14\] Here and in the rest of this proof, we have suppressed the dependence of the expectation of \(z\) on the prior belief \(\mu\) for conciseness as none of the results depend on the prior belief.
The above characterization allows us to construct the equilibrium wage functional, in contrast to Grossman-Stiglitz type models, which rely on guess-and-verify. Moreover, our approach permits us not only to establish the existence of equilibrium but also its uniqueness.

**Proposition 3** (Existence and uniqueness of REE). *Rational expectations equilibrium in the labor market exists and is unique.*

*Proof.* Note from the previous proof that whether an equilibrium wage \( w \) perfectly reveals \( S \) or is perfectly uninformative does not depend on the equilibrium belief of the uninformed, \( \hat{\mu}_U(w, \mu) \). Thus, for each \((\phi, \mu, S)\) triplet: (1) the resulting equilibrium wage is either perfectly informative or completely uninformative and (2) the induced equilibrium belief of the uninformed is either the prior belief or the belief of the informed. Consequently, for each \((\phi, \mu, S)\) triplet there exists a unique rational expectations equilibrium wage, given by

\[
W_\lambda(\phi, \mu, S) = \begin{cases} 
\frac{1}{2} \left[ \phi + z \right] & \text{if } \phi < \phi + \lambda (z - \bar{z}) \\
\frac{1}{2} \left[ \phi + (1 - \lambda)E[z | \mu] + \lambda \bar{z} \right] & \text{otherwise},
\end{cases}
\]

(20)

\[
W_\lambda^*(\phi, \mu, S) = \begin{cases} 
\frac{1}{2} \left[ \phi + (1 - \lambda)E[z | \mu] + \lambda \bar{z} \right] & \text{if } \phi \leq \phi - \lambda (z - \bar{z}) \\
\frac{1}{2} \left[ \phi + \bar{z} \right] & \text{otherwise}.
\end{cases}
\]

(21)

3.2 Information acquisition equilibrium

Equipped with the unique REE wage functional, we can solve a firm’s information acquisition problem in stage 1. A firm will acquire information at cost \( \kappa \) if the expected profit of an informed firm exceeds that of an uninformed firm by more than \( \kappa \). Letting \( G(\lambda) = E[\Pi^I(w, \lambda) | \mu] - \kappa - E[\Pi^U(w, \lambda) | \mu] \)\(^{15}\) to denote the expected, prior-to-information-acquisition gain from becoming informed, we define stage 1 equilibrium as follows.

**Definition 4** (Information acquisition equilibrium). *Given a rational expectations wage functional \( W_\lambda(\cdot) \), information acquisition equilibrium is a fraction *

\(^{15}\Pi(.) \) represents equilibrium profit.
of informed firms $\lambda^*$ such that

$$
\lambda^* = \begin{cases} 
0 & \text{if } G(0) < 0 \\
1 & \text{if } G(1) > 0 \\
\lambda^* \in [0, 1] & \text{if } G(\lambda^*) = 0.
\end{cases}
$$

(22)

A sufficient condition for the equilibrium fraction of informed firms to be unique is that the expected gain from becoming informed, $G(\lambda)$, is strictly decreasing in $\lambda$. We show below that information acquisition is a strategic substitute, i.e. the expected gain from becoming informed is indeed strictly decreasing in the fraction of informed firms.

4 Results

We first establish strategic substitutability in information acquisition for all prior beliefs. Then, we turn to our main results, the approximate acyclicity of demand for information and of the informativeness of the price system. We first show that firms have a stronger incentive to acquire information when the economy has been in a recession in the previous period, and firms share a pessimistic belief about the economy being in a boom than when the economy has been in a boom and an optimistic belief prevails. As a consequence, the equilibrium fraction of informed firms is higher and the price system more informative when firms have a pessimistic belief than for an optimistic belief. However, we then show that demand for information is approximately acyclic, yielding no significant variation in the fraction of informed firms and the informativeness of the price system over the business cycle.

Proposition 5 (Strategic substitutability in information acquisition). Given that for all $\lambda \in [0, 1]$ there exists a non-degenerate interval of uninformative wages, the expected gain from becoming informed is strictly decreasing in the fraction of informed firms.

Proof. We want to show that the expected gain function satisfies $G'(\lambda) < 0$ for all $(\lambda, \mu) \in [0, 1] \times (0, 1)$. Given that for wages that fully reveal the signal of the informed, the uninformed and informed firms make identical choices, the gain from becoming informed prior to opening of the labor market pertains to realizations of the signal and the taste shock which support uninformative
wages. From (20) and (21) it follows that the lowest and highest uninformative wages, denoting them $w$ and $\overline{w}$, respectively, are given by

$$w = \frac{1}{2} \phi + \frac{1}{2} \left[ (1 - \lambda) \mathbb{E} [z | \mu] + \lambda \overline{z} \right]$$  \hspace{1cm} (23)

$$\overline{w} = \frac{1}{2} \phi + \frac{1}{2} \left[ (1 - \lambda) \mathbb{E} [z | \mu] + \lambda \overline{z} \right].$$  \hspace{1cm} (24)

Moreover, as $\mathbb{E} [z | \mu] = \mathbb{E} [z | \mu]$ for uninformative wages, we have from above that

$$f(w \mid \mathcal{S}) = \frac{2}{\phi - \overline{\phi}} \text{ for } w \in [w, \overline{w}].$$  \hspace{1cm} (25)

Then, the prior-to-information-acquisition probability of observing an uninformative wage is

$$G(\lambda) = \mu \int_{w}^{\overline{w}} \left( \Pi^I(w, z) - \Pi^U(w, \overline{z}) \right) f(w \mid \mathcal{S}) dw$$

$$+ (1 - \mu) \int_{w}^{\overline{w}} \left( \Pi^I(w, z) - \Pi^U(w, z) \right) f(w \mid \mathcal{S}) dw - \kappa$$

$$= \left[ 1 - \frac{\lambda (\overline{z} - z)}{\phi - \overline{\phi}} \right] \left[ \mu \mathbb{E} \log \overline{z} + (1 - \mu) z \log \overline{z} - \mathbb{E} [z | \mu] \log \mathbb{E} [z | \mu] \right] \kappa,$$  \hspace{1cm} (30)

for $\overline{\phi} - \phi > \lambda (\overline{z} - z)$ and 0 otherwise. Note that the expected gain is equal to the probability of observing an uninformative wage multiplied by the difference
in expected profits for a given wage, which is independent of $\lambda$ and strictly positive for $\mu \in (0, 1)$ by Jensen’s inequality. Under the parameter restriction $\overline{\phi} - \phi > \overline{z} - z$ the existence of a non-degenerate interval of uninformative wages is guaranteed for all $\lambda \in [0, 1]$ and we have $G'(\lambda) < 0$ as was to be shown. □

In our model, strategic substitutability in information acquisition arises from an information externality due to rational expectations equilibrium wages transmitting information, similar to Grossman and Stiglitz (1980). As more firms acquire the costly signal and become informed about the economy’s state, the price system becomes more informative in terms of the probability of observing an informative wage in (27). An individual firm’s incentive to acquire the costly signal is reduced. Hence, the expected gain of becoming informed decreases in the fraction of informed firms.

We now turn to our two main results, approximate acyclicity of information demand and of the informativeness of the price system, both of which constitute novel findings in the literature on learning and asymmetric business cycles. These results are stated in the following proposition and its corollary.

**Proposition 6** (Approximately acyclical information demand). *Given that the probability of observing an uninformative wage is strictly positive, the expected gain from becoming informed is higher for the low prior belief $1 - \rho$ than for the high prior belief $\rho$. However, up to a second-order approximation the expected gain is the same for the low and the high prior belief.*

**Proof.** The probability of observing an uninformative wage is independent of $\mu$. Hence, to show the countercyclicity of expected gain with respect to the prior belief $\mu$, it suffices to show that

$$g(\mu) := \mu \overline{z} \log \overline{z} + (1 - \mu) \overline{z} \log \overline{z} - \mathbb{E}[z | \mu] \log(\mathbb{E}[z | \mu])$$

(31)

is such that $g(1 - \rho) > g(\rho)$ for $\rho \in \left(\frac{1}{2}, 1\right)$. Defining $f(\rho) := g(1 - \rho) - g(\rho)$, we have $f\left(\frac{1}{2}\right) = f(1) = 0$. Moreover,

$$f''(\rho) = (\overline{z} - z)^2 \left(\frac{1}{\rho \overline{z} + (1 - \rho) \overline{z}} - \frac{1}{(1 - \rho) \overline{z} + \rho \overline{z}}\right) < 0$$

(32)

for $\rho \in \left(\frac{1}{2}, 1\right)$. Hence, $g(1 - \rho) > g(\rho)$ for all $\rho \in \left(\frac{1}{2}, 1\right)$.

Taking a second-order Taylor approximation of $G(\cdot)$ around $\overline{z} = \overline{z}$ yields

$$G(\lambda) = \frac{(1 - \mu) \mu (\overline{z} - z)^2}{\overline{z}} - \kappa.$$
Thus, the approximate expected gain from becoming informed is the same for the low prior belief $1 - \rho$ and the high prior belief $\rho$.

To grasp the intuition behind this result consider a highly persistent process for the aggregate technology, i.e. $\rho$ close to 1. Then, as the prior belief is either close to 0 or 1, we can ignore the gain from information which confirms the prior view. This is because confirmatory information does not change the optimal quantity of labor significantly. Hence, we can focus on the gain from receiving information contradicting the prior belief. First, consider the case when the economy has been in a boom and firms hold an optimistic prior belief. Now, the gain from contradictory information, which reveals the state to be low, comes from the combination of two effects. On the one hand, it enables the firm to reduce its production costs by hiring less labor than dictated by the prior belief. On the other hand, this decrease in the quantity of labor hired decreases the informed firm’s output relative to that of the uninformed firm. Since the aggregate technology level is low, the output effect is dominated by the cost reduction effect. In the alternative case, when the economy has been in a recession and firms hold a pessimistic belief, these effects operate in the opposite directions while the net effect is of similar magnitude. That is, although the informed firm has higher labor input costs than the uninformed firm, the output effect more than compensates for this as the aggregate technology level is high. In other words, perfectly revealing information allows the informed firm to cut its losses when the prior belief is high and increase its profits when the prior belief is low.

Figure 1 illustrates how the expected gain from becoming informed varies with the prior belief and Figure 2 shows that the higher-order terms of the gain function only give rise to moderately countercyclical information demand.

Weakly countercyclical information demand implies that, given $\kappa$ is such that we have an interior solution for $\lambda^*$, the fraction of informed firms is higher for the pessimistic belief than for the optimistic belief. This, in turn, implies that the probability of observing a perfectly revealing wage is higher for the low prior belief than for the high prior belief, although only slightly as demand for information is approximately acyclical. As equilibrium wages are either perfectly revealing or perfectly uninformative, a straightforward measure of the informativeness of the price system is the probability of observing
Figure 1: The solid line represents the expected gain from becoming informed at $\lambda = 0$, $G(0)$, for $z = 3$, $\bar{z} = 4$ and $\kappa = 0$ and the dashed lines corresponds to a symmetric approximation of the expected gain.
Figure 2: Fraction of informed firms $\lambda^*$ as a function of $\bar{z}$ for $\underline{z} = 1$, $\phi = 0.1$, $\bar{\phi} = 0.5$, $\rho = 0.9$ and $\kappa = 0.001$. 
an informative wage. Hence, from the preceding proposition we obtain the following.

**Corollary 7** (Approximately acyclical informativeness of the price system). Given that the cost of the signal is such that an interior solution for the equilibrium fraction of informed firms obtains, the price system is more informative when the low prior belief $1 - \rho$ prevails than when the high prior belief $\rho$ prevails. However, up to a second-order approximation of the expected gain function, the informativeness of the price system exhibits acyclicity.

**Proof.** From $g(1 - \rho) > g(\rho)$ as shown in the proof of Proposition 6 and from Definition 4, for $\kappa$ such that $\lambda^* \in (0, 1)$, the equilibrium fraction of informed firms, $\lambda^*$ is higher for the pessimistic belief than for the optimistic belief. Then, by equation (27), the probability of observing an informative wage is higher when the prior belief is $1 - \rho$ than for prior belief of $\rho$.

From the proof of Proposition 6, up to a second-order approximation, the expected gain from becoming informed is the same for the prior beliefs of $1 - \rho$ and $\rho$. Hence, from Definition 4 and equation (27), the fraction of informed firms and the probability of observing an informative wage are equal for these two prior beliefs.

\[\square\]

### 5 Discussion

In this section we first delve deeper into the role of learning from equilibrium wages and examine how it relates to the approximate learning symmetry established in Proposition 6. To study how firms' incentives to acquire information will change if learning from wages is suppressed, we consider Walrasian equilibrium in the labor market, which does not require firms' beliefs to be consistent with the observed wage\(^{16}\). We find that suppressing the informational role of wages results in a firm's incentives for acquiring information being no more affected by other firms' information acquisition decisions. Consequently, without learning from wages either all firms are informed or no firm is informed. Therefore, learning from wages dampens any asymmetric incentives to acquire information.

After having examined Walrasian equilibrium, a solution concept with off-equilibrium beliefs, we turn to the question of how firms may come to

\(^{16}\)We follow Grossman (1981) in referring to equilibrium which does not require beliefs to be in line with the observed wage as Walrasian equilibrium.
hold rational expectations equilibrium beliefs. To shed light on attainability of equilibrium beliefs, we consider an equilibrium concept, introduced by Kobayashi (1977) and Jordan (1982, 1985), where a sequence of wages is observed and used to update beliefs in a Bayesian fashion. We find that beliefs and wages under this alternative equilibrium concept converge to their counterparts in REE after observing a single Walrasian equilibrium wage.

5.1 The role of learning from wages

Let us begin by defining an equilibrium concept which disregards learning from wages, namely Walrasian equilibrium.

**Definition 8** (Walrasian equilibrium in the labor market). Given a fraction of informed firms, \( \lambda \in [0, 1] \), Walrasian equilibrium in the labor market is a pair of demand schedules \( h_U(w, \bar{\mu}^U) \) and \( h_I(w, \bar{\mu}^I) \), a supply schedule \( h_S(w, \phi) \) and a wage functional \( \bar{W}_\lambda(\phi, \bar{\mu}^U, \bar{\mu}^I) \) such that for all \( (\phi, \bar{\mu}^U, \bar{\mu}^I) \in \Psi \times [0, 1] \) and \( w = \bar{W}_\lambda(\phi, \bar{\mu}^U, \bar{\mu}^I) \)

1. \( h^U(w, \bar{\mu}^U) \) and \( h^U(w, \bar{\mu}^I) \) solve
   \[
   \max_{h^U \geq 0} \left\{ \bar{\mu}^U \Pi(w, z, h^U) + (1 - \bar{\mu}^U) \Pi(w, z, h^U) \right\}, \tag{34}
   \]
   \[
   \max_{h^I \geq 0} \left\{ \bar{\mu}^I \Pi(w, z, h^I) + (1 - \bar{\mu}^I) \Pi(w, z, h^I) \right\}, \tag{35}
   \]
   respectively;

2. \( h^S(w, \phi) \) solves the household’s stage 2 problem;

3. labor market clears for all realizations of \( \phi \)
   \[
   \lambda h^I(w, \bar{\mu}^I) + (1 - \lambda) h^U(w, \bar{\mu}^U) = h^S(w, \phi). \tag{36}
   \]

To find the prior-to-information-acquisition gain from becoming informed when the stage 2 labor market equilibrium is Walrasian, note from (28) and (29) that the gain from becoming informed at any wage \( w \) is not a function of \( w \). Moreover, as there is no learning from wages, the expected gain from becoming informed is found by integrating over all possible equilibrium wages and accounting for the cost of the signal

\[
\bar{G} = \mu \bar{E} \log \bar{E} + (1 - \mu) \bar{E} \log \bar{E} - \bar{E}[z] \log(\bar{E}[z]) - \kappa. \tag{37}
\]
Comparison of (30) and (37) reveals that the expected gain from becoming informed with learning from wages is equal to the gain when learning from wages is suppressed, scaled by the probability of observing an uninformative wage. Therefore, for $\kappa$ such that $\lambda^*$ is strictly positive, learning from equilibrium wages moderates asymmetric incentives to acquire costly information. Moreover, without learning from wages, depending on whether $\tilde{G}$ is negative or positive, the fraction of informed firms is either 0 or 1. Thus, when equilibrium in the labor market is Walrasian, there exists $\kappa > 0$ such that all firms are informed when the pessimistic belief prevails and no firm informed when the public belief is optimistic, although the demand for information is only weakly countercyclical. On the other hand, when learning from wages is allowed, the fraction of informed firms differs only slightly across the two possible public beliefs.

5.2 Attainability of REE beliefs

Our sequence of markets equilibrium, adopted from Jordan (1985), formalizes what Vives (2008) refers to as information tâtonnement\textsuperscript{17}. Firms express their demand for labor, based only on their private information at the time. Their demand schedules are aggregated and the notional\textsuperscript{18} market clearing wage announced. Firms then update their beliefs using any information that may be contained in the announced notional market clearing wage and adjust their demand schedules to reflect their updated beliefs. Updated demand schedules are collected and a new notional market clearing wage announced. This process is allowed to continue until all firms no more wish to adjust their demand schedule, which requires that firms’ beliefs are not altered by the last market clearing wage. Our sequence of markets equilibrium builds on Walrasian equilibrium as follows.

**Definition 9** (Sequence of markets equilibrium). Given a fraction of informed firms, $\lambda \in [0, 1]$, sequence of markets equilibrium is a sequence of pairs of demand schedules $\{h^U_n(w, \tilde{\mu}_n^U), h^I_n(w, \tilde{\mu}_n^I)\}$, a supply schedule $h^S(w, \phi)$ and a wage functional $\tilde{W}_\lambda(\phi, \tilde{\mu}_n^U, \tilde{\mu}_n^I)$ such that $\forall n \in \mathbb{Z}_+$

1. $h^U_n(w, \tilde{\mu}_n^U)$, $h^I_n(w, \tilde{\mu}_n^I)$, $h^S(w, \phi)$ and $\tilde{W}_\lambda(\phi, \tilde{\mu}_n^U, \tilde{\mu}_n^I)$ constitute a Walrasian equilibrium;

\textsuperscript{17}See Vives (2008), pp. 334–335.

\textsuperscript{18}Notional refers to the wage that would clear the market for the given supply and the current demands. However, trades are not yet executed.
2. \( \bar{\mu}_0^U = \bar{\mu}_0^I = \bar{\mu}(\mu, \mathcal{F}) \) and

\[
\begin{align*}
\bar{\mu}_{n+1}^U(w_n, \mu) &= \mathbb{P}(z = \bar{z} | \bar{w}_n) = \bar{W}_n(\phi, \bar{\mu}_n^U, \bar{\mu}_n^I, \mu), \\
\bar{\mu}_{n+1}^I(w_n, \mu, \mathcal{F}) &= \mathbb{P}(z = \bar{z} | \bar{w}_n) = \bar{W}_n(\phi, \bar{\mu}_n^U, \bar{\mu}_n^I, \mu, \mathcal{F}).
\end{align*}
\]

3. \( \lim_{n \to \infty} \bar{\mu}_n^U = \bar{\mu}_I^U \) and \( \lim_{n \to \infty} \bar{\mu}_n^I = \bar{\mu}_I^I \).

In our model, rational expectations equilibrium beliefs are attainable via observing and processing the information contained in a sequence of notional Walrasian market clearing wages. This is formalized in the following proposition.

**Proposition 10** (Attainability of REE beliefs). Sequence of Walrasian markets equilibrium beliefs and wages converge to their rational expectations equilibrium counterparts after observing a single Walrasian equilibrium wage.

**Proof.** Note from Proposition 2 that whether a REE wage is perfectly informative or perfectly uninformative does not depend on the belief of the uninformed, \( \bar{\mu}_U \), but only on \( \phi \) and \( \mathcal{F} \). Hence, in the first step of the sequence of Walrasian markets, when the uninformed use only their prior belief to formulate their demands, the informativeness of \( \bar{w}_0 \) is determined by exactly the same conditions on \( \phi \) as the informativeness of REE wages. For the same reason, the uninformed cannot learn anything more from \( \bar{w}_1 \). Hence, the sequence of Walrasian markets wage functional satisfies \( \bar{W}_n(\phi, \bar{\mu}_n^U, \bar{\mu}_n^I) = \bar{W}(\phi, \bar{\mu}_n^U, \bar{\mu}_n^I) = W(\phi, \mu, \mathcal{F}) \). That is, Walrasian equilibrium wages in the first step contain the same information as rational expectations equilibrium wages.

\[\square\]

6 Conclusion

We have investigated two so far unanswered questions in the literature on procyclical learning as an explanation for business cycle asymmetries. Namely, whether firms have a stronger incentive to acquire information in booms or in recessions and how learning from prices contributes to aggregate learning outcomes. We find that in a model environment featuring no exogenous source of asymmetry firms’ information demand is approximately acyclical rather than procyclical. We also establish a crucial role for learning
from equilibrium prices. The price system, in transmitting information from the informed to the uninformed, moderates any asymmetric incentives in information acquisition. That is, irrespective of whether firms would have pro- or countercyclical demand for information, the aggregate learning outcome is less cyclical when firms exploit information contained in equilibrium wages than when learning from wages is not taken into consideration. These two novel findings suggest that models relying on procyclical learning to explain slow expansions and sharp contractions require strongly procyclical, exogenous information to survive the moderating effect of information acquisition and learning from prices.
References


