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Working Paper
Reciprocal dumping with Bertrand competition


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Supplement (Friberg and Ganslandt, 2005)

Cournot duopoly

A representative consumer in the home market has the following utility function

\[ U(q_H, q_F) = a(q_H + q_F) - \frac{1}{2} b(q_H^2 + 2q_Hq_F + q_F^2) + m. \] (1)

Utility maximization subject to the budget constraint gives the following inverse demand functions for variety H and F:

\[ p_H = a - bq_H - \theta bq_F \] (2)
\[ p_F = a - bq_F - \theta bq_H \] (3)

Firms simultaneously solve the first order conditions

\[ \frac{d}{dq_H} ((p_H - c)q_H) = 0 \] (4)
\[ \frac{d}{dq_F} ((p_F - c - t)q_F) = 0. \] (5)

The unique solution is

\[ q_H = \frac{(2 - \theta)(a - c) + \theta t}{b(2 - \theta)(2 + \theta)} \] (6)
\[ q_F = \frac{(2 - \theta)(a - c) - 2t}{b(2 - \theta)(2 + \theta)}. \] (7)

The prohibitive trade cost is given by \(q_F(t) \equiv 0\)

\[ t = \frac{(a - c)(2 - \theta)}{2}. \] (8)

In autarchy \(q_F = 0\) and

\[ q_H = \frac{a - c}{2b}. \] (9)

Welfare is

\[ W = a(q_H + q_F) - \frac{1}{2} b(q_H^2 + 2\theta q_Hq_F + q_F^2) - c(q_H + q_F) - t q_F. \] (10)

In the trade equilibrium:
\[ W = \frac{4\theta^2ac + 32a\theta c + 2\theta^2at - 2\theta^2ct + 16a\theta t - 16\theta tc - 4\theta^3ac - 2\theta^3at + 2\theta^3ct}{2b(\theta - 2)^2(2 + \theta)^2} \]
\[ + \frac{24a^2 - 16\theta a^2 - 48ac - 2\theta^2 a^2 - 2\theta^2 c^2 - 16\theta c^2 - \theta^2 t^2}{2b(\theta - 2)^2(2 + \theta)^2} \]
\[ + \frac{24c^2 - 24at + 24ct + 2\theta^3 a^2 + 2\theta^3 c^2 + 12t^2}{2b(\theta - 2)^2(2 + \theta)^2}. \] (11)

The derivative is
\[ \frac{dW}{dt} = \frac{(a - c)(\theta^2 + 8\theta - \theta^3 - 12) - (\theta^2 - 12)t}{b(2 - \theta)^2(2 + \theta)^2}. \] (12)

Welfare is convex in \( t \). The second order derivative is
\[ \frac{d^2W}{dt^2} = \frac{12 - \theta^2}{b(2 - \theta)^2(2 + \theta)^2} > 0. \] (13)

The minimum is an interior trade equilibrium since the minimum is at a trade cost strictly lower than the prohibitive level. The minimum can be found by setting \( \frac{dW}{dt} = 0 \), with a unique solution
\[ t = \frac{(a - c)(3 + \theta)(2 - \theta)^2}{12 - \theta^2}. \] (14)

Welfare at this trade cost is
\[ W_{\text{min}} = \frac{(a - c)^2 (9 - \theta^2)}{2b(12 - \theta^2)}. \] (15)

Trade can reduce welfare for any degree of product differentiation. At the prohibitive trade cost the derivative is
\[ \frac{dW}{dt} = \frac{(a - c)(\theta)}{2b(2 - \theta)(2 + \theta)} > 0 \] (16)

This is confirmed by a comparison of welfare at the minimum and in autarchy. Using the autarchy quantity we obtain welfare in autarchy
\[ W^M = \frac{3(a - c)^2}{8b}. \] (17)

The quote between the minimum welfare with trade and welfare under autarchy is
\[ \frac{W_{\text{min}}}{W^M} = \frac{4}{3} \left( \frac{9 - \theta^2}{12 - \theta^2} \right) < 1. \] (18)
Bertrand duopoly

Re-arrange inverse demand equations to obtain the demand functions for H and F:

\[ q_H = \frac{a - p_H + \theta p_F - a\theta}{b (1 - \theta^2)} \quad (19) \]
\[ q_F = \frac{a - p_F + \theta p_H - a\theta}{b (1 - \theta^2)} \quad (20) \]

First order conditions (reaction functions):

\[ \frac{d}{dp_H} ((p_H - c) q_H) = 0 \quad (21) \]
\[ \frac{d}{dp_F} ((p_F - c - t) q_F) = 0 \quad (22) \]

The unique Bertrand equilibrium is:

\[ p_H = \frac{2a - a\theta - a\theta^2 + 2c + c\theta + t\theta}{4 - \theta^2} \quad (23) \]
\[ p_F = \frac{2a - a\theta - a\theta^2 + 2c + c\theta + 2t}{4 - \theta^2} \quad (24) \]

and we insert the equilibrium prices in demand functions to obtain the equilibrium quantities.

\[ q_H = \frac{(2 - \theta^2 - \theta) (a - c) + \theta t}{b (4 - \theta^2) (1 - \theta^2)} \quad (25) \]
\[ q_F = \frac{(2 - \theta^2 - \theta) (a - c) - 2t + \theta^2 t}{b (4 - \theta^2) (1 - \theta^2)} \quad (26) \]

Welfare in a trade equilibrium is

\[ W = \frac{36ac\theta^2 - 16\theta tc - 12a\theta^3 c - 6a\theta^3 t - 8a\theta^4 c + 18a\theta^2 t - 18c\theta^2 t + 6\theta^3 tc}{2b (1 - \theta^2) (4 - \theta^2)^2} \]
\[ + \frac{32ac\theta + 16a\theta t - 4a\theta^4 t + 4c\theta^4 t + 24a^2 - 18a^2 \theta^2 - 16a^2 \theta - 48ac + 4a^2 \theta^4}{2b (1 - \theta^2) (4 - \theta^2)^2} \]
\[ + \frac{6a^2 \theta^3 - 18c^2 \theta^2 - 16c^2 \theta + 6c^2 \theta^3 - 9\theta^2 t^2}{2b (1 - \theta^2) (4 - \theta^2)^2} \]
\[ + \frac{24c^2 + 4c^2 \theta^4 - 24at + 24ct + 2\theta^4 t^2 + 12t^2}{2b (1 - \theta^2) (4 - \theta^2)^2} \quad (27) \]
Welfare has a unique minimum \( \frac{dW}{dt} = 0 \) at
\[
t = \frac{(3 - 2\theta)(a - c)(1 - \theta)(2 + \theta)^2}{2\theta^4 - 9\theta^2 + 12}
\] (28)
Welfare is strictly convex in \( t \). The second order derivative is
\[
\frac{d^2W}{dt^2} = \frac{12 + 2\theta^4 - 9\theta^2}{b(1 - \theta^2)(4 - \theta^2)^2} > 0
\]
The corresponding welfare at minimum is
\[
W_{\text{min}} = \frac{(9 - 4\theta^2)(a - c)^2}{2b(12 + 2\theta^4 - 9\theta^2)}
\] (29)
The prohibitive trade cost is given by \( q^*_F(t) \equiv 0 \), i.e.
\[
t = \frac{(a - c)(2 - \theta - \theta^2)}{2 - \theta^2}.
\] (30)
Welfare at the prohibitive level is
\[
W|_{t=\tilde{t}} = \frac{ac - a^2}{b\theta^2 - 2b} + \frac{ac - c^2}{b\theta^2 - 2b} + \frac{2ac - a^2 - c^2}{8b - 8b\theta^2 + 2b\theta^4}.
\] (31)
The interior minimum is a strict minimum. The derivative at the prohibitive level shows that a higher trade barrier and less trade can increase welfare at the margin:
\[
\left. \frac{dW}{dt} \right|_{t=\tilde{t}^+} = \frac{(a - c)\theta}{(2 - \theta^2)(2 - \theta)(2 + \theta)b} > 0
\] (32)
Nevertheless, welfare in autarchy is
\[
W^{\text{M}} = \frac{3}{8} \frac{(a - c)^2}{b}.
\] (33)
The quote between the minimum and autarchy welfare is
\[
\frac{W_{\text{min}}}{W^{\text{M}}} = \frac{4(3 - 2\theta)(2\theta + 3)}{3(12 + 2\theta^4 - 9\theta^2)} > 1.
\] (34)

Cournot oligopoly (duopoly in H and duopoly in F)
Inverse demand functions with four firms are
\[
p_1 = a - bq_1 - \theta bq_2 - \theta bq_3 - \theta bq_4
\] (35)
\[
p_2 = a - \theta bq_1 - bq_2 - \theta bq_3 - \theta bq_4
\] (36)
\[
p_3 = a - \theta bq_1 - \theta bq_2 - bq_3 - \theta bq_4
\] (37)
\[
p_4 = a - \theta bq_1 - \theta bq_2 - \theta bq_3 - bq_4
\] (38)
Firms simultaneously solve

\[
\frac{d}{dq_1} ( (p_1 - c) q_1 ) = 0 \quad (39)
\]
\[
\frac{d}{dq_2} ( (p_2 - c) q_2 ) = 0 \quad (40)
\]
\[
\frac{d}{dq_3} ( (p_3 - c - t) q_3 ) = 0 \quad (41)
\]
\[
\frac{d}{dq_4} ( (p_4 - c - t) q_4 ) = 0 \quad (42)
\]

The equilibrium is

\[
q_1 = q_2 = \frac{(a - c)(2 - \theta) + 2\theta t}{b(3\theta + 2)(2 - \theta)} \quad (43)
\]
\[
q_3 = q_4 = \frac{(a - c)(2 - \theta) - (2 + \theta) t}{b(3\theta + 2)(2 - \theta)} \quad (44)
\]

Welfare is

\[
W = \left( (a - c)(q_1 + q_2 + q_3 + q_4) - \frac{1}{2} b(q_1^2 + q_2^2 + q_3^2 + q_4^2) \right) - \theta b(q_1 q_2 + q_1 q_3 + q_1 q_4 + q_2 q_3 + q_2 q_4 + q_3 q_4) - t(q_3 + q_4) \quad (45)
\]

Explicitly

\[
W = \frac{6\theta t c - 12a\theta^3 c - 6\theta^3 t a - 3\theta^3 t^2 + 24a^2 - 48ac + 18\theta^2 t a}{b(3\theta + 2)^2 (\theta - 2)^2} + \frac{3\theta^2 t^2 - 24at + 12t^2 + 24ct + 16\theta t^2 - 18a^2 \theta^2 - 18c^2 \theta^2}{b(3\theta + 2)^2 (\theta - 2)^2} + \frac{24c^2 + 6c^2 \theta^3 + 6a^2 \theta^3 + 36a\theta^2 c - 18\theta^2 t c}{b(3\theta + 2)^2 (\theta - 2)^2} \quad (46)
\]

The derivative is

\[
\frac{dW}{dt} = -\frac{2}{b} \frac{12a - 12c - 12t - 9a\theta^2 + 3a\theta^3 + 9c\theta^2 + 3\theta^3 t - 3c\theta^3 - 16\theta t - 3\theta^2 t}{(3\theta + 2)^2 (\theta - 2)^2} \quad (47)
\]

Welfare is a strictly convex function in \( t \). The second order derivative is

\[
\frac{d^2 W}{dt^2} = \frac{2}{b(3\theta + 2)^2 (2 - \theta)^2} > 0 \quad (48)
\]
Welfare has a unique minimum, $\frac{dW}{dt} = 0$, at

$$t = \frac{3(a - c)(1 + \theta)(2 - \theta)^2}{12 + 16\theta + 3\theta^2 - 3\theta^3} \quad (49)$$

Welfare at this point is

$$W_{\text{min}} = \frac{3 \left(3 - \theta^2 + 2\theta\right)(a - c)^2}{b \left(12 + 16\theta + 3\theta^2 - 3\theta^3\right)} \quad (50)$$

The prohibitive trade cost, $q_3(t) = q_4(t) = 0$, is

$$t = \frac{(2 - \theta)(a - c)}{(2 + \theta)} \quad (51)$$

Autarchy is

$$q_1 = q_2 = \frac{a - c}{b(2 + \theta)} \quad (52)$$
$$q_3 = q_4 = 0 \quad (53)$$

Welfare under autarchy is

$$W^{\text{D}} = \frac{(3 + \theta)(a - c)^2}{b(\theta + 2)^2} \quad (54)$$

The quote between the minimum welfare with trade and welfare under autarchy is

$$\frac{W_{\text{min}}}{W^{\text{D}}} = \frac{3(\theta + 1)(3 - \theta)(\theta + 2)^2}{(12 + 16\theta + 3\theta^2 - 3\theta^3)(\theta + 3)} < 1$$

**Bertrand oligopoly (duopoly in H and duopoly in F)**

Demand functions are

$$q_1 = \frac{a + \theta p_2 - \theta a + p_4\theta + p_3\theta - 2\theta p_1 - p_1}{(1 - \theta)(3\theta + 1)b} \quad (55)$$
$$q_2 = \frac{a + \theta p_1 - \theta a - 2\theta p_2 + p_4\theta + p_3\theta - p_2}{(1 - \theta)(3\theta + 1)b} \quad (56)$$
$$q_3 = \frac{a + \theta p_2 - \theta a + p_3\theta - 2p_3\theta + \theta p_1 - p_3}{(1 - \theta)(3\theta + 1)b} \quad (57)$$
$$q_4 = \frac{a + \theta p_2 - \theta a + p_3\theta - 2p_4\theta + \theta p_1 - p_4}{(1 - \theta)(3\theta + 1)b} \quad (58)$$

Firms simultaneously solve
\[
\frac{d}{dp_1} (q_1 (p_1 - c)) = 0 \quad (59)
\]
\[
\frac{d}{dp_2} (q_2 (p_2 - c)) = 0 \quad (60)
\]
\[
\frac{d}{dp_3} (q_3 (p_3 - c - t)) = 0 \quad (61)
\]
\[
\frac{d}{dp_4} (q_4 (p_4 - c - t)) = 0 \quad (62)
\]

The unique trade equilibrium is

\[
p_1 = p_2 = \frac{(5\theta + 2) (a + 2c\theta - a\theta + c) + 4\theta^2 t + 2\theta t}{(5\theta + 2) (2 + \theta)} \quad (63)
\]
\[
p_3 = p_4 = \frac{(5\theta + 2) (a + 2c\theta - a\theta + c) + 6\theta^2 t + 7\theta t + 2t}{(5\theta + 2) (2 + \theta)} \quad (64)
\]

Welfare is

\[
W = \frac{1}{b (3\theta + 1) (1 - \theta) (2 + \theta)^3 (5\theta + 2)^2} \left[ -90\theta^3 tc + 180a\theta^3 c + 90\theta^3 ta + 201\theta^3 t^2 + 24a^2 - 48ac \\
+144a^2\theta - 222a^2 ta + 219\theta^2 t^2 - 24at + 12t^2 + 24ct \\
+88\theta t^2 + 222a^2 \theta^2 + 222c^2 \theta^2 + 24c^2 + 144c^2 \theta - 300a^2 \theta^4 \\
-90c^2 \theta^3 - 90a^2 \theta^3 - 444a^2 c - 144\theta ta + 144\theta tc - 288ac\theta \\
+222a^2 t c + 50a^4 t^2 + 24\theta^5 t^2 - 300c\theta^4 t - 300c^2 \theta^4 + 600a\theta^4 c + 300a \theta t^3 \right] \\
\]

The derivative is

\[
\frac{dW}{dt} = 2 \frac{b (3\theta + 1) (1 - \theta) (2 + \theta)^2 (5\theta + 2)^2}{[24\theta^5 t - 150c\theta^4 + 50\theta^4 t + 150a\theta^4 - 45c\theta^3 + 201\theta^3 t + 45a\theta^3 \\
+219\theta^2 t - 111a\theta^2 + 111c\theta^2 - 72a\theta + 72c\theta + 88\theta t - 12a + 12c \theta t^3]} \\
\]

Welfare is strictly convex in \( t \). The second order derivative is

\[
\frac{d^2W}{dt^2} = 2 \frac{24\theta^5 + 50\theta^4 + 201\theta^3 + 219\theta^2 + 88\theta + 12}{b (3\theta + 1) (1 - \theta) (2 + \theta)^2 (5\theta + 2)^2} > 0 \quad (67)
\]

There is a unique minimum, \( \frac{dW}{dt} = 0 \), at

\[
t = \frac{3 (5\theta + 2)^2 (1 - \theta) (a - c)}{12\theta^4 + 19\theta^3 + 91\theta^2 + 64\theta + 12} \quad (68)
\]
Welfare at this point is

\[
W_{\text{min}} = \frac{3 \, 16a^2\theta^2 - 32a\theta^2c + 16c^2\theta^2 - 28ac\theta + 14a^2\theta + 14c^2\theta - 6ac + 3c^2 + 3a^2}{12\theta^4 + 19\theta^3 + 91\theta^2 + 64\theta + 12}
\]  
(69)

Welfare in autarchy, \(q_3 = q_4 = 0\), is

\[
W^{D} = \frac{(a - c)^2 (3 - 2\theta)}{b (\theta + 1) (\theta - 2)^2}
\]  
(70)

The quote between the minimum welfare with trade and welfare under autarchy is

\[
\frac{W_{\text{min}}}{W^{D}} = \frac{3 (2 - \theta)^2 (\theta + 1) (16\theta^2 + 14\theta + 3)}{(3 - 2\theta) (12\theta^3 + 19\theta^2 + 91\theta^2 + 64\theta + 12)}
\]  
(71)