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Influence Costs and Hierarchy

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Abstract

In an internal capital market, individual departments may compete for a share of the firm’s budget by engaging in wasteful influence activities. We show that firms with more levels of hierarchy may experience lower influence costs than less hierarchical firms, even though the former provide more opportunities for exerting influence. We further argue that the widely discussed change from the U-form to the M-form organization in the 1920’s may be related to attempts to limit divisional lobbying. In particular, we show that influence costs under the U-form organization are lower than under the M-form organization if and only if the firm’s operations are sufficiently small.

Keywords: Hierarchies, influence activities, internal capital markets, U-form vs. M-form organization.

JEL Classification Numbers: D74, G31, G34.

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1 Introduction

It is a widely held view in the literature on organizations that firms with more levels of hierarchy also experience greater influence costs, since multi-tier firms have more levels of executives upon which influence can be exerted. Informal arguments along these lines can be found in, e.g., Milgrom and Roberts (1990) or Meyer, Milgrom, and Roberts (1992). In this paper, we formally investigate the relationship between hierarchical structure and influence activities. In contrast with the received literature, we find that firms with more levels of hierarchy may experience lower influence costs than less hierarchical firms.

Influence costs inevitably arise in any organization where individuals have discretion over decisions which have distributional implications. Other individuals affected by the decision will then spend time and energy attempting to influence the decision maker in their favor. Influence activities come in many guises. For instance, individuals may engage in lobbying, or providing information which makes them look good relative to others. In extreme cases, this may involve sabotage or overt conflict where individuals or departments present evidence with the sole purpose of damaging each other’s position. For instance, Rotemberg and Saloner (1995) cite an article by Morrill (1991) describing in detail the clashes between a firm’s marketing department and its operations department where the two sides engage in open war over a proposal to expand production.

On a priori grounds, influence activities need not be harmful. As they involve the presentation of evidence, influence activities may provide decision makers with information that helps them make better decisions. As the objectives of the organization and the individuals taking part in the conflict typically diverge, however, it is likely that there will be too much influence activity (Rotemberg and Saloner 1995). In this paper, like most existing work in this area, we focus directly on this dark side by assuming that any influence activity is purely wasteful. According to Milgrom and Roberts (1988), there are then three options the organization can adopt. First, it can close communication channels for certain decisions. Second, it can reduce the return to influence activities by limiting decision makers’ discretion and restricting their ability to respond to information supplied by others. Finally, it can adjust compensation, promotion, investment, and other criteria from what would otherwise be optimal to align individual goals with those of the organization. As the first method is rather crude, existing research has mainly focused on the second and third option.

Examples of organizational responses to influence activities are the use of non-discretionary (e.g., seniority-based) promotion schemes and the narrowing of wage differentials (Milgrom 1988; Milgrom and Roberts 1988), the divestiture of poorly performing divisions (Meyer, Milgrom, and Roberts 1992), and the design of a firm’s capital structure (Bagwell and Zechner 1993). What these papers have in common is that they
emphasize the benefits of committing to a certain course of action, thereby limiting discretionary power within the organization. While our paper is also concerned with ways to reduce influence activities, we adopt a different approach. Instead of considering commitment technologies allowing the organization to limit discretionary power, we explore ways of channelling discretionary power by distributing authority optimally across the organization’s members. To the extent that the distribution of authority defines an organization’s hierarchy, we thus address the question of optimal hierarchy.

Our model is set in the context of a capital budgeting problem, but the results apply equally to other problems of intrafirm resource allocation or rent distribution. Suppose a firm has $N$ departments or projects— to be called units—that compete on the firm’s internal capital market for a share of the corporate budget by engaging in wasteful rent-seeking or influence activities.\(^1\) In the literature on organizational behavior, the notion that resources are distributed on the basis of power struggles rather than efficiency is well known. Cherrington (1989, p.727) notes:

> Political models of organizational behavior view organizations as collections of people and subgroups competing for scarce resources and the right to determine the organizations’ strategies and objectives. Political models of decision making focus on allocating resources according to the relative power of individuals and subunits rather than an objective assessment of organizational effectiveness.

The exact nature of the competition between units depends on the firm’s type of hierarchy. If the firm has a one-level hierarchy, the budget allocation decision resides with a single executive, e.g., the chief financial officer (CFO). Consequently, there is only a single round of conflict where the $N$ units directly compete with each other in trying to win the CFO’s favor.

If the firm has a two-tier hierarchy, the $N$ units are grouped in $M \leq N$ divisions. In this case, the budget allocation is performed by $M + 1$ executives: the CFO, who is responsible for the allocation of the budget to the $M$ divisions, and $M$ lower-level financial managers (one for each division), each of whom is responsible for the allocation of the division budget to the units in the respective division. Viewing this as a two-step procedure captures the idea that top-level managers usually deal with strategic issues while lower-level managers deal with operational issues.

Under a two-tier hierarchy, there are two rounds of competition. In the first round, each of the $N$ units tries to persuade the CFO to allocate as large a budget share as

\(^1\)To motivate why the units engage in rent-seeking at all, we assume that they derive private benefits of control from the output produced with the acquired capital.
possible to the unit’s division. The second round consists of $M$ local conflicts where the units in each division compete with each other for a share of the division budget. One of the key insights of this paper is that introducing a second level of hierarchy (and thus a second round of conflict) may reduce total influence costs although the number of executives upon which influence can be exerted increases from one to $M + 1$.\footnote{Wärneryd (1998) studies a related effect in the context of jurisdictional organization.} More precisely, we show that the unique influence-cost-minimizing hierarchy within the class of all one- and two-tier hierarchies has two levels.

An unappealing feature of the influence-cost-minimizing hierarchy is that it involves a capital distribution which is highly asymmetric. One would therefore expect that in certain situations hierarchies other than the influence-cost-minimizing hierarchy involve higher output. Indeed, we show that if the production technology exhibits sufficiently decreasing returns to scale, total output under a symmetric two-tier hierarchy may be greater than under the influence-cost-minimizing hierarchy.

A phenomenon that has received considerable attention in the literature is the change from the U-form to the M-form organization by many US corporations in the 1920’s (see, e.g., Chandler 1962). In this connection, Williamson (1975) conjectures that, besides other reasons, the change may have been related to attempts to influence the firms’ internal resource allocation process as operations expanded. We show that a change from the U-form to the M-form structure may indeed be optimal (in terms of minimizing influence costs) if the firm’s products or regions grow beyond a certain level. Our paper thus provides an explanation for the change from the U-form to the M-form structure that is complementary to existing explanations based on the superiority of the M-form structure in auditing division managers and assigning cash flows to high yield uses (Williamson 1970, 1975), in coping with “control loss effects” (Williamson 1975; Sen 1993), in using implicit incentives (Aghion and Tirole 1995), and in promoting relative performance evaluation (Maskin, Qian, and Xu 1999).

To isolate the effect of influence activities on intrafirm resource allocation, we assume that, as suggested in the Cherrington passage quoted earlier, resources are allocated solely on the basis of the units’ relative rent-seeking efforts. In particular, this implies that the CFO or lower-level financial managers have no intrinsic motivation in allocating capital efficiently. Moreover, it implies that the firm’s owner(s) cannot commit financial managers to a particular resource allocation by resorting to penalties or other incentive devices. While these are clearly strong limitations, they allow us to focus on the question we are interested in:\footnote{While we believe that incentive contracts are useful in channeling resources to their most effective use in an organization, the earlier quotation from Cherrington and similar evidence suggests that in-} if, for whatever reason, it is optimal to equip decision makers with
discretion, how should discretionary power be distributed among decision makers to limit the costs arising from distributional conflict over scarce resources?

In much of the theoretical literature on capital budgeting, the resource allocation process is impeded by private information on the part of division or project managers (e.g., Harris, Kriebel, and Raviv 1982; Antle and Eppen 1985; Harris and Raviv 1996, 1998). In the context of our model, this means that if units have private information about their productivities, influence activities could serve to make this information public. As argued elsewhere, however, even if influence activities serve to convey information, it is likely that there will be too much influence activity as the interests of the individual units and the organization typically diverge. Hence organizational measures such as the ones discussed in this paper are likely to remain relevant if units have private information about their productivities.

Let us briefly relate our results to the literature on optimal hierarchies. While there is by now a vast literature on optimal hierarchies, most of these papers focus on issues that are not, or only remotely, related to our model, e.g., information processing and team theory (Keren and Levhari 1979; Radner 1992; Bolton and Dewatripont 1994), supervision and task assignment (Williamson 1967, 1975; Rosen 1982), or incentives (Calvo and Wellisz 1979). The paper that is perhaps closest to ours is Hart and Moore (1999). Like our paper, Hart and Moore develop a theory of hierarchy based on the allocation of authority. Unlike our model, however, interaction between agents in the Hart-Moore model is solely “top down”, i.e., lower-level agents have no means to influence the decisions of agents higher up in the hierarchy.

Our paper is also related to recent work by Stein and Scharfstein (1998), who study the implications of divisional rent-seeking on the internal allocation of capital in firms. The authors show that, in an effort to stop rent-seeking, corporate headquarters may optimally bribe the division managers in the form of preferential capital allocation. The basic, but crucial, difference between their and our model is that in our model there is conflict between units or divisions over scarce input, while in their model there is conflict between headquarters and divisions over produced output.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 derives the influence cost-minimizing hierarchy within the class of all one- and two-tier hierarchies. Section 4 introduces heterogeneous productivities and decreasing returns to scale. Among other things, it shows that for sufficiently concave production functions the influence-cost-minimizing hierarchy produces less output than the optimal symmetric influence activities still play an important role, implying that incentive contracts are unable to solve the resource allocation problem completely. For the same reason as in our paper, incentive contracts are also ruled out in the capital budgeting models of Harris and Raviv (1996, 1998).
hierarchy. Section 5 examines the change from the U-form to the M-form organization from the viewpoint of minimizing influence costs. Section 6 provides a numerical example illustrating the main results of the paper. Section 7 concludes.

2 The Model

Consider a firm with $N$ departments or projects (henceforth units) that possibly differ in their productivities. Eventually, we are interested in organizing these units along $M \leq N$ divisions so as to maximize firm output. Divisions are denoted by $z_k$, where $k = 1, ..., M$. Formally, a division is a subset of the set of units, i.e., we have $z_k \subseteq \{1, ..., N\}$ for all $k$. We frequently refer to divisions with only one unit as stand-alone units and divisions with more than one unit as proper divisions. A partitioning of the set of units into divisions is called a hierarchy.

Implicit in our notion of hierarchy is a restriction to one- and two-level hierarchies. While allowing for three- or four-level hierarchies is straightforward, it considerably complicates the analysis without generating additional insights. Real-world corporate hierarchies frequently consist of only two levels.\footnote{What we have in mind here are not personnel hierarchies, which typically consist of more than two levels, but organizational charts.} At the division level, firms are typically organized along functional, geographic, product, or brand lines. At the next lower level, divisions are subdivided into functional, geographic, product, or brand subdivisions. In most cases, the second divisionalization follows different criteria than the first. For instance, if a firm is organized along product lines, the product divisions are typically subdivided into geographic or functional units (i.e., manufacturing, sales, finance, etc.), but not into (sub)product units.

For the units to be productive, they need capital. Depending on the particular form of hierarchy, the capital allocation is performed by one or more managers whose only role is to distribute resources. Managers operate on different levels. At the top level, the firm’s CFO decides on the fraction of the budget to be allocated to each division. At the division level, a lower-level financial manager then divides the budget allocated to a particular division among the units in that division. There are thus $M + 1$ managers in total.

For convenience, we assume that the budget that is to be distributed is fixed. This may be due to, e.g., agency problems between owners and managers (Holmström and Ricart i Costa 1986) or capital market imperfections in conjunction with agency conflicts between existing and future shareholders (Thakor 1990). In practice, budgets in most firms are set at fixed dates (e.g., annually or quarterly) and kept constant in between.
Empirically, there is strong evidence suggesting that intrafirm capital rationing is pervasive. For instance, Gitman and Forrester (1977) report that 52 per cent of the firms in their survey allocated a fixed annual budget among competing projects. In Ross’s (1986) study, half of the firms employed capital rationing. According to Ross (p.15), an immediate implication of such rationing is that “projects compete against each other, not against a profitability standard”. This is precisely what we model in this paper.

In particular, we assume that the budget assigned to a particular division or unit is determined by the division’s or unit’s rent-seeking expenditures relative to the rent-seeking expenditures of all other divisions or units. Implicitly, this assumes that it is optimal for the firm’s owner(s) to grant the managers discretion over the allocation of the budget.

Except for the hierarchy where all units are located on the same level, the capital allocation process outlined above gives rise to two levels of distributional conflict. On both levels, we assume that the units make their rent-seeking expenditures noncooperatively and independently of each other.

On the first level, individual units try to influence the CFO regarding the allocation of the firm budget to the different divisions. We shall assume that the budget share awarded to division \( k \) is

\[
\beta_k := \begin{cases} 
\frac{\sum_{i \in k} t_i}{\sum_j t_j} & \text{if } \sum_j t_j > 0 \\
1/M & \text{otherwise,}
\end{cases}
\]

where \( t_j \) denotes the rent-seeking expenditure of unit \( j \).

That is, the budget share awarded to a division equals the sum of rent-seeking expenditures by the units in that division divided by total rent-seeking expenditures.\(^5\) In case the division consists of more than one unit, fighting for a larger division budget is therefore a public good. As investments in rent-seeking are made noncooperatively, this means that units which belong to proper divisions will necessarily underinvest in the inter-division conflict. Technically, the fact that fighting for a larger division budget is a public good means that the first-order conditions in the inter-division conflict only determine total expenditures by the units in each division. In this case, we focus on within-group symmetric equilibria, i.e., equilibria where all units in a given division make the same expenditure.

\(^5\)This particular contest success function was introduced by Tullock (1975, 1980) for the analysis of court proceedings and rent-seeking contests. More recently, Fullerton and McAfee (1990) use the same contest success function, derived from more primitive assumptions, to analyze research contests, and Baye, Kovenock, and de Vries (1993) use the related success function where the highest bidder wins with certainty to study lobbying. For a more general discussion of conflicts of this nature, see, e.g., Skaperdas (1992, 1996) and Dixit (1987).
On the second, subordinate conflict level, the units in each division try to influence the lower-level manager in charge of the division regarding the allocation of the division budget. In analogy with the inter-division conflict, we shall assume that the share of division $k$'s budget awarded to unit $i$ is

$$\alpha_{i,k} := \begin{cases} \frac{r_i}{\sum_{j \in z_k} r_j} & \text{if } \sum_{j \in z_k} r_j > 0 \\ 1/n_k & \text{otherwise,} \end{cases}$$

where $r_i$ is the rent-seeking expenditure of an individual unit in division $k$, and $n_k$ is the total number of units in division $k$.

To capture the notion that rent-seeking is costly to the firm, we assume that it diminishes the amount of resources available for productive activities. For instance, if individual units try to influence managers via cash bribes, we assume that the cash bribe reduces the amount of capital available to the respective unit.\(^6\) Alternatively, if individual units try to influence managers via lobbying or manipulating information, we assume that the respective activities use up time and effort that could be spend more productively elsewhere.

The simplest way of formalizing the notion that rent-seeking uses up productive input is to reduce the input in the production function by the amount of rent-seeking investments. Hence we shall assume that the output of unit $i$ in division $k$ is

$$\pi_i := (1 + \theta_i) \left( \delta_i B - r_i - t_i \right)\eta,$$

where $\theta_i \geq 0$ is a productivity parameter, $\eta \in (0,1]$ is a concavity parameter, $B$ is the total firm budget, and $\delta_i := \alpha_{i,k} \beta_k$ is the budget share awarded to unit $i$.\(^7\) For convenience, we normalize the budget to $B = 1$.

Finally, we need to elaborate on why individual units should engage in influence activities at all. The easiest way of motivating this is to assume that units can divert a (possibly small) fraction $\lambda$ of the produced output as private benefit.\(^8\) The notion that

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\(^6\)We implicitly assume that managers cannot directly appropriate a fraction of the budget for which they are responsible, e.g., by transferring cash into their private accounts. The only way for managers to improve their salary is thus to accept cash bribes.

\(^7\)An alternative way of modelling production is as $\pi_i = (1 + \theta_i) \left( K_i + L_i \right)^\eta$, where $K_i := \delta_i B$ denotes capital input and $L_i$ denotes labor input. Assuming that a unit can spend time either on productive activities or on rent-seeking, we obtain the resource constraint $1 = L_i + r_i + t_i$, where $1$ is the (normalized) time available to the unit. Inserting this constraint in the production function yields $\pi_i := (1 + \theta_i) \left( \delta_i B + 1 - r_i - t_i \right)^\eta$. Except for the additional constant $1$, the production function is identical to (1).

\(^8\)An alternative way of modelling taste for capital would be to assume that managerial utility is directly increasing in the amount of capital invested (Harris and Raviv 1996, 1998) or that capital and managerial effort are substitutes in the production of output (Harris, Kriebel, and Raviv 1982).
there are private benefits of control is standard by now. It captures the idea that the accounting technology is imperfect, which allows individual units to spend a fraction of the produced surplus on perks. The objective function of unit \( i \) is then
\[
u_i := \lambda \pi_i.
\]

We proceed by deriving the optimal, i.e., output-maximizing hierarchy for the case of constant returns to scale (\( \eta = 1 \)) and homogeneous productivities (\( \theta_i = \theta \)). We thus implicitly take the viewpoint of the firm’s owner(s) who design the firm’s hierarchy to maximize shareholder value. Decreasing returns to scale and heterogeneous productivities are considered in Section 4.

3 Influence-Cost-Minimizing Hierarchies

With constant returns to scale and homogeneous productivities, the optimal hierarchy corresponds to the hierarchy which minimizes influence costs.

By (2), the objective function of unit \( i \) in division \( k \) is
\[
u_i := \lambda (1 + \theta) \left( \delta_i - r_i - t_i \right).
\]

We find subgame perfect equilibria through backward induction. We begin with the intra-division conflict where the \( n_k \) units in division \( k \) compete for a share of division \( k \)’s budget. In that conflict, the division budget and the expenditures in the inter-division conflict are already given. It thus remains to determine the equilibrium levels of \( r_i \) and \( \alpha_{i,k} \).

Clearly, there is no equilibrium such that no unit makes a positive expenditure, since then an individual unit could get the entire division budget by expending an arbitrarily small amount. The optimal rent-seeking expenditure of unit \( i \), given the expenditures of all other units in division \( k \), is therefore given by the first-order condition
\[
\frac{\partial u_i}{\partial r_i} = \lambda (1 + \theta) \left( \frac{\sum_{j \in z_k} r_{j,i} \beta_k - 1}{\left( \sum_{j \in z_k} r_{j} \right)^2} \beta_k - 1 \right) = 0.
\]

Since this condition implies that equilibrium expenditures are the same for all units, there is a unique equilibrium where each unit invests
\[
r := \frac{n_k - 1}{n_k} \beta_k.
\]

Moreover, each unit receives an equal share of the division budget, i.e., we have \( \alpha_{i,k} = 1/n_k \).
In the preceding inter-division conflict, the objective function of unit $i$ in division $k$ is then
\[ u_i := \lambda (1 + \theta) \left( \frac{1}{n_k^2} \beta_k - t_i \right). \] (5)

The term $1/n_k^2$ may be said to represent a unit’s valuation for the corporate budget. The fact that the sum of valuations is strictly less than unity reflects the fact that a dollar going to division $k$ is worth strictly less than one dollar to the division’s units as it induces wasteful rent-seeking in the subsequent intra-division conflict. By contrast, the valuation of a stand-alone unit is always equal to unity.

The fact that valuations are possibly asymmetric simplifies the analysis greatly. In particular, if the asymmetry is sufficiently large, only the units with the highest valuation make positive investments in the inter-division conflict. The following lemma concerns one such example.

**Lemma 1.** In an inter-division conflict with two or more stand-alone units, only the stand-alone units make positive expenditures in equilibrium.

**Proof.** Let $m \geq 2$ denote the number of stand-alone units. Maximizing (5) with respect to $t_i$ and dividing the resulting first-order condition through by $\lambda (1 + \theta)$ yields
\[ \frac{1}{n_k^2} \left( \sum_{j \not\in z_k} t_j \right) - \left( \sum_{j \in z_k} t_j \right)^2 \leq 0. \] (6)

We claim that (6) is slack for all divisions with $n_k > 1$. Suppose to the contrary that there exists a proper division $z_3$ with $n_3 > 1$ for which (6) is binding, and consider two arbitrary stand-alone divisions $z_1$ and $z_2$. Adding up (6) for $z_1$ and $z_2$ and rearranging gives
\[ \frac{\sum_j t_j - t_1 - t_2 + \sum_{j \in z_3} t_j}{\left( \sum_j t_j \right)^2} + \frac{\sum_j t_j - \sum_{j \in z_3} t_j}{\left( \sum_j t_j \right)^2} \leq 2. \] (7)

However, inserting the binding first-order condition for $z_3$ in (7), we have
\[ \frac{\sum_j t_j - t_1 - t_2 + \sum_{j \in z_3} t_j}{\left( \sum_j t_j \right)^2} + n_3^2 \leq 2, \]
a contradiction as the leftmost term must be nonnegative.

As there cannot be an equilibrium where no unit makes a positive expenditure, it follows from (3) that there exists a unique equilibrium where each of the $m$ stand-alone units invests a positive amount. □

Hence, if the asymmetry in the inter-division conflict is sufficiently large, divisions with low valuations (i.e., those with a large number of units) may opt not to participate.
in the competition. Formally, this means that in hierarchies with two or more stand-alone units, we can safely ignore all proper divisions when computing total rent-seeking costs. By \((4)\), there then exists a unique equilibrium where each stand-alone unit invests the common amount
\[
t = \frac{m - 1}{m^2}. \tag{8}
\]

If there is only a single stand-alone unit, Lemma 1 does not apply since for a profile of expenditures to be an equilibrium, there must at least be two units expending a positive amount. The following result is the analogue of Lemma 1 for conflicts with a single stand-alone unit.

**Lemma 2.** In an inter-division conflict with a single stand-alone unit, only the stand-alone unit and the units in the division(s) with the smallest number of units make positive expenditures in equilibrium.

**Proof.** The proof is analogous to that of Lemma 1. Let \(n\) denote the smallest number of units in any proper division. Adding up the first-order conditions of the stand-alone unit and some arbitrary division \(k\) with \(n\) units, we have
\[
\frac{\sum_j t_j - t_1 - \sum_{j \in z_k} t_j + \sum_{j \in z_2} t_j}{(\sum_j t_j)^2} + \frac{\sum_j t_j - \sum_{j \in z_2} t_j}{(\sum_j t_j)^2} \leq 1 + n^2, \tag{9}
\]
where \(t_1\) is the investment by the stand-alone unit and \(z_2\) is some arbitrary division with \(n_2 > n\) units.

Inserting the binding first-order condition for \(z_2\) in \((9)\) yields
\[
\frac{\sum_j t_j - t_1 - \sum_{j \in z_k} t_j + \sum_{j \in z_2} t_j}{(\sum_j t_j)^2} + n^2_2 \leq 1 + n^2,
\]
contradiction as the leftmost term must be nonnegative.

Recall that we focus on within-group symmetric equilibria, i.e., equilibria where all units within a division make the same expenditure. As there cannot be an equilibrium where no unit makes a positive expenditure, this then implies that the expenditures of both the stand-alone unit and the proper division(s) with \(n\) units must be positive. \(\square\)

By Lemma 2, the first-order conditions of both the stand-alone unit and the proper division(s) with \(n\) units must be binding. Suppose there are \(m \geq 1\) divisions with \(n\) units. Solving the resulting system of \(mn + 1\) first-order conditions, we obtain
\[
t_1 = \frac{m (1 + m (n^2 - 1))}{(n^2 m + 1)^2} \tag{10}
\]
for the stand-alone unit and
t = \frac{m}{n(n^2m + 1)^2} \tag{11}
for each unit in a division with \( n \) units.

We now come to our main result. It shows that, although the number of possible hierarchies grows exponentially as the number of units becomes large, there is always a unique influence-cost-minimizing hierarchy. Of particular interest is that the influence-cost-minimizing hierarchy is independent of the number of units \( N \).

**Proposition 1.** With homogeneous productivities and constant returns to scale, the unique influence-cost-minimizing hierarchy is the hierarchy where a stand-alone unit competes against a division comprising all other units.

**Proof.** See Appendix.

Proposition 1 offers several insights into the optimal allocation of resources in multi-project firms. First and foremost, it shows that if the objective is to minimize influence costs, two levels of hierarchy are always better than one. To some extent, this is counterintuitive as it suggests that influence costs can be reduced by adding additional levels of (wasteful) conflict. To see why the statement is true, recall from (4) that total rent-seeking expenditures under the one-level hierarchy are

\[ Nr = \frac{N - 1}{N}, \tag{12} \]

which implies that the dissipation rate, i.e., the fraction of the budget that is wasted on rent-seeking, is bounded from below by 2/3 for \( N \geq 3 \).

Consider now a symmetric two-tier hierarchy where the \( N \) units are organized in \( N/2 \) divisions (assuming that \( N \) is an even number). By (4), total rent-seeking expenditures in the \( N/2 \) intra-division conflicts are

\[ Nr = \sum_k \frac{1}{2} \beta_k = \frac{1}{2}. \tag{13} \]

To this must be added the extra rent-seeking expenditures incurred in the inter-division conflict. However, since the division budget is a public good, the units in the respective divisions underinvest, with the result that rent-seeking expenditures in the inter-division conflict remain moderate. As is shown in the Appendix, total rent-seeking expenditures in the inter-division conflict amount to

\[ Nt = \frac{1}{4} \frac{N - 2}{N} \tag{14} \]
It is easily seen that for $N \geq 3$ the sum of (13) and (14) is strictly less than (12), implying that the symmetric two-tier hierarchy with $N/2$ divisions entails strictly lower influence costs than the one-level hierarchy. Thus, adding an additional level of conflict may reduce influence costs.

While the symmetric two-tier hierarchy involves less rent-seeking than the one-level hierarchy, Proposition 1 suggests that one can do even better with a hierarchy where a stand-alone unit competes against a division comprising all other units. This way, the free-rider problem in the division with the $N - 1$ units is as large as possible. As a result, the units in this division heavily underinvest in the conflict with the stand-alone unit. As for the stand-alone unit, this implies that in order to obtain the bulk of the corporate budget, it must also invest only little. Overall, rent-seeking expenditures in the inter-division conflict are therefore low. With the bulk of the budget going to the stand-alone unit, however, rent-seeking expenditures in the subsequent internal conflict among the $N - 1$ units remain modest as well.

The influence-cost-minimizing hierarchy thus exploits the following properties:

1. By grouping $N - 1$ units in a common division, these units face a severe free-rider problem in the provision of influence activities in the inter-division conflict.

2. As in equilibrium the bulk of the capital goes to the stand-alone unit, only little capital is dissipated in the subsequent internal conflict among the $N - 1$ units.

Up to this point, we have assumed that productivities are homogeneous and the production technology exhibits constant returns to scale. If productivities are heterogeneous, Proposition 1 continues to hold as long as the production technology is linear, for one can then assign the position of the stand-alone unit to the most productive unit, thereby reinforcing the result. If the production function is concave, however, the result may no longer hold. The following section elaborates on this point.

4 Symmetric vs. Asymmetric Capital Distribution

In this section, we show that with heterogeneous productivities and decreasing returns to scale, there may be a tradeoff between minimizing influence costs and allocating capital optimally. As a consequence, it may no longer be true that the influence-cost-minimizing hierarchy, i.e., the hierarchy where a stand-alone unit competes against a division comprising all other units, is also the hierarchy which maximizes output. In particular, it may then be true that a fully symmetric two-tier hierarchy is superior to the influence-cost-minimizing hierarchy. To illustrate this, we compare the output under
the influence-cost-minimizing hierarchy with the output under the optimal symmetric hierarchy for arbitrary productivity distributions \((\theta_1, \ldots, \theta_N)\) and concavity parameters \(\eta\). To simplify the analysis, we assume that \(N\) is an even number.

Using results derived in the Proof of Proposition 1, it is easy to show that total output under the hierarchy where a stand-alone unit competes against the \(N - 1\) other units is

\[
(1 + \theta_1)^\eta \left(\frac{(N - 1)^4}{((N - 1)^2 + 1)^2}\right) + \sum_{i=2}^{N} (1 + \theta_i)^\eta \left(\frac{(N - 1)^2 + 2 - N}{(N - 1)^2 \left(\frac{(N - 1)^2 + 1}{2}\right)^2}\right),
\]

where \(\theta_1\) denotes the productivity parameter of the stand-alone unit, and \(\theta_1 \geq \theta_i\) for all \(i\) by optimality.

To determine the optimal symmetric hierarchy, consider some arbitrary symmetric hierarchy with \(m \leq N/2\) divisions and \(N/m\) units per division. By symmetry, all \(N\) units must expend positive resources in equilibrium both in the intra- and inter-division conflict. By (4), the common expenditure in the intra-division conflict in division \(k\) is then

\[
r = \frac{m(N - m)}{N^2} \beta_k,
\]

implying that \(\alpha_{i,k} = m/N\) for all \(i\) and \(k\).

Maximizing (5) with respect to \(t_i\) and solving the resulting system of first-order conditions, we have\(^9\)

\[
t = \frac{m(m - 1)}{N^3},
\]

which, among other things, implies that \(\beta_k = 1/m\) for all \(k\).

From (16) and (17), it then follows that the total output under the symmetric hierarchy with \(m\) divisions is

\[
\sum_{i=1}^{N} (1 + \theta_i)^\eta \left(\frac{m(N - m + 1)}{N^3}\right).
\]

Differentiating (18) with respect to \(m\) shows that the output under the symmetric hierarchy is increasing in the number of divisions, which implies that the constraint \(m \leq N/2\) must be binding. For this reason, we shall henceforth refer to the symmetric hierarchy with \(m = N/2\) divisions as optimal symmetric hierarchy.

We are now in the position to compare the output under the influence-cost-minimizing hierarchy with that under the optimal symmetric hierarchy for arbitrary productivity distributions and concavity parameters. Denote the collection of actual productivities

\(^9\)Observe that the first-order conditions for \(\eta = 1\) and \(\eta < 1\) are identical.
by the $N$ units by $\Theta := \{1 + \theta_1, \ldots, 1 + \theta_N\}$. In the analysis that follows, it is useful to order productivity distributions along the one-dimensional index $\omega \in [1, \infty)$ defined by

$$\omega := \frac{(\max \Theta)(N - 1)}{\sum_i (1 + \theta_i) - \max \Theta}. \quad (19)$$

In words, (19) represents the ratio of the productivity of the most productive unit divided by the average productivity of the $N - 1$ remaining units. If $\omega = 1$, we are back to the case of homogeneous productivities analyzed in Section 3.

We then have the following result.

**Proposition 2.** For each $N$, there exists a unique index value $\overline{\omega} > 1$ such that

i) for all $\omega > \overline{\omega}$, the total output under the influence-cost-minimizing hierarchy is greater than the total output under the optimal symmetric hierarchy for all values of $\eta$, and

ii) for all $\omega \leq \overline{\omega}$, there exists a unique concavity parameter $\pi(\omega) \in (0, 1)$ such that the total output under the influence-cost-minimizing hierarchy is greater than the total output under the optimal symmetric hierarchy if and only if $\eta > \pi(\omega)$.

**Proof.** Comparing (15) with (18) and rearranging, we have that the total output under the influence-cost-minimizing hierarchy exceeds the total output under the optimal symmetric hierarchy if and only if

$$\omega > (N - 1) \frac{\left(\frac{N - \frac{\omega}{N^2} + 1}{N^2}\right)^{\eta} - \left(\frac{(N - 1)^2 + 2 - N}{(N - 1)^2 (N - 1)^2 + 1}\right)^{\eta}}{\left(\frac{\omega}{N^2}\right)^{\eta} - \left(\frac{N - \frac{\omega}{N^2} + 1}{N^2}\right)^{\eta}} =: \xi(N, \eta).$$

Define $\overline{\omega} := \lim_{\eta \to 0} \xi(N, \eta)$. First, we show that $\overline{\omega}$ exists. By L’Hôpital’s rule we have that

$$\overline{\omega} = (N - 1) \frac{\ln \frac{1}{2N^2} \left(\frac{1}{2} N + 1\right) - \ln \frac{(N - 1)^2 + 2 - N}{(N - 1)^2 (N - 1)^2 + 1}}{\ln \frac{(N - 1)^2}{(N - 1)^2 + 1} - \ln \frac{1}{2N^2} \left(\frac{1}{2} N + 1\right)},$$

which is finite for all $N$. Furthermore, it is true that $\overline{\omega} > 1$ for all $N \geq 3$ (recall that if $N = 2$, the result is trivial as there is only a single possible hierarchy).

Differentiating $\xi(N, \eta)$ with respect to $\eta$ shows that $\xi(N, \eta)$ is strictly decreasing in $\eta$ for all $N$. Thus, to establish the proof it only remains to show that $\xi(N, 1) < 1$. As is easy to show, $\xi(N, 1)$ is bounded from above by $9/11$ for all $N \geq 2$. \qed

Proposition 2 confirms our earlier intuition that with heterogeneous productivities and decreasing returns to scale there may be a tradeoff between influence cost-minimization and optimal capital allocation. In particular, if $\omega \leq \overline{\omega}$ and $\eta > \pi(\omega)$, i.e.,
if the productivity of the most productive unit is not too high and returns to scale are sufficiently decreasing, distributing capital evenly among the units is more important than reducing influence costs. In this case, the optimal symmetric hierarchy is superior to the influence-cost-minimizing hierarchy. On the other hand, if the productivity of the most productive unit is sufficiently high, optimality may require that the bulk of the corporate budget be allocated to the most productive unit even if the production function exhibits decreasing returns to scale. In this case, the influence-cost-minimizing hierarchy is superior to the optimal symmetric hierarchy.

5 U-Form vs. M-Form Organization

The early 1920’s witnessed a radical change in the way many large US corporations such as DuPont or General Motors were organized (for a discussion, see Chandler 1962). While most of these firms were previously organized along functional lines (e.g., manufacturing, sales, finance, etc.), with the functional divisions possibly subdivided into brand, regional, or product subdivisions, they were now organized along brand, regional, or product lines, with each of the brand, regional, or product divisions subdivided into functional subdivisions. Chandler calls the former organizational form U-form organization and the latter M-form organization.\(^\text{10}\)

The merits of the M-form organization vis-à-vis the U-form organization are discussed in Chandler (1962) and Williamson (1975). To paraphrase Williamson (1975, p.133), “what is involved is that radical expansion of the U-form enterprise (1) experiences cumulative ‘control loss’ effects, which have internal efficiency consequences, and (2) eventually alters the character of the strategic decision-making process in ways that favor attending to other-than-profit objectives.” In other words, the U-form structure is no longer suited if the firm’s operations expand. In this case, the M-form structure proves superior as it serves “both to economize on bounded rationality and attenuate opportunism” and “reduced partisan political input into the resource allocation process” (Williamson 1975, p.137-138).

In what follows, we shall examine the second argument in greater detail. We will not, however, examine under what conditions, if any, the U-form or the M-form organization

\(^{10}\)Note that the U-form organization need not have fewer levels of hierarchy than the M-form organization, for the functional divisions in the U-form organization may be subdivided into brand, product, or geographic subdivisions (Williamson 1975, p.137). An important distinction between the two structures, however, is that the operating divisions in the M-form organization are quasi-autonomous as each division contains the full range of operating functions (i.e., manufacturing, sales, etc.) whereas the divisions in the U-form organization naturally depend on each other.
are optimal. Suppose the firm has $m$ functions and $n$ products (or regions). In terms of our model, an expansion of operations corresponds to an increase of $n$ vis-à-vis $m$ (for simplicity, we assume that $m$ is fixed).\textsuperscript{11} Under the U-form structure the firm has then $m$ divisions with $n$ units each, while under the M-form structure it has $n$ divisions with $m$ units each.\textsuperscript{12} To simplify the analysis, we abstract from heterogeneous productivities and decreasing returns to scale and set $\eta = 1$ and $\theta_i = \theta$. Similar to Section 2, the problem of comparing the U-form and M-form organization then boils down to determining which of the two organizations involves lower influence costs.

Consider first the U-form organization. From (4), we have that total rent-seeking expenditures in the $m$ intra-division conflicts are

$$mnr = \frac{n - 1}{n}.$$  

In the preceding inter-division conflict, the objective function of each of the $mn$ units (or product subdivisions) is given by (5), with $n_k = n$ and $k = 1, \ldots, m$. Solving the resulting system of first-order conditions, we get

$$mnt = \frac{m - 1}{n^2m}.$$  

Thus, total rent-seeking expenditures under the U-form organization are

$$mn (r + t) = \frac{m - 1 + mn (n - 1)}{n^2m}. \quad (20)$$

Consider next the M-form organization. By analogy with (20), total rent-seeking expenditures under the M-form organization are

$$mn (r + t) = \frac{n - 1 + mn (m - 1)}{m^2n}. \quad (21)$$

Comparing (20) with (21), we have the following proposition.

**Proposition 3.** If $n < m$, influence costs under the U-form organization are strictly lower than under the M-form organization. Conversely, if $n > m$, influence costs under the M-form organization are strictly lower than under the U-form organization. If $m = n$ influence costs are the same under both organizational forms.

Proposition 3 supports Williamson’s conjecture that, as operations expand, the M-form structure may be superior at reducing influence costs arising from conflict over

\textsuperscript{11} An expansion of operations may also involve an increase in the firm’s budget. As our results are independent of $B$, however, we may as well set $B = 1$.

\textsuperscript{12} Recall the definition of the two organizational forms at the beginning of this section.
resources than the U-form structure. More precisely, if the firm has fewer products (or brands or regions) than functions, Proposition 3 implies that the U-form structure is superior to the M-form structure. If, however, the firm has more products (or brands or regions) than functions, the M-form structure is superior.

6 An Example

We conclude by illustrating the possible tradeoff between optimal capital allocation and influence cost-minimization with an example. Unlike Proposition 2, where we compared the influence-cost-minimizing hierarchy with the optimal symmetric hierarchy, we now endogenously derive the optimal, i.e., output-maximizing hierarchy for arbitrary productivity distributions and concavity parameters. The result is similar to Proposition 2, i.e., for high values of $\eta$ the influence-cost-minimizing hierarchy is optimal, whereas for low values of $\eta$ the symmetric hierarchy is optimal.

Suppose the firm has four units. As before, we restrict attention to hierarchies with at most two levels. Ignoring for a moment the issue of where individual units should be positioned within the hierarchy, this gives rise to four possible hierarchies:

$$\mathcal{H}_1 := \{1, 2, 3, 4\},$$
$$\mathcal{H}_2 := \{1, 2, \{3, 4\}\},$$
$$\mathcal{H}_3 := \{1, \{2, 3, 4\}\},$$ and
$$\mathcal{H}_4 := \{\{1, 2\}, \{3, 4\}\}.$$

Hierarchies $\mathcal{H}_1 - \mathcal{H}_4$ are depicted in Figure 1. Managers (i.e., the CFO and lower-level financial managers) are marked by squares, and units are marked by circles.

In what follows, we shall consider all four hierarchies in turn. Let $\theta_1$ denote the productivity parameter of the most productive unit, $\theta_2$ the productivity parameter of the second-most productive unit, and so on. For convenience, we normalize the sum of productivity parameters to $\sum_i \theta_i = 1$.

Hierarchy $\mathcal{H}_1$

Under $\mathcal{H}_1$, the game reduces to a symmetric single-stage conflict. By (4), we then have $t = 3/16$, implying that total firm output is

$$\sum_i \pi_i = \sum_i (1 + \theta_i) \left(\frac{1}{16}\right)^\eta.$$

Since net inputs are the same for all units, the position of an individual unit within the hierarchy is irrelevant.
Figure 1: The four different four-unit hierarchies.

Hierarchy $\mathcal{H}_2$

By Lemma 1, only the stand-alone units make positive investments in the inter-division conflict, which implies that $\beta_3$ (the budget share awarded to the proper division) must be zero. Accordingly, equilibrium rent-seeking expenditures by the stand-alone units are $t = .25$ each. Total firm output is then

$$\sum_i \pi_i = \sum_{i=1}^{2} (1 + \theta_i) \left( \frac{1}{4} \right)^\eta.$$

Clearly, output is maximized if and only if the stand-alone positions are filled with the two units with the highest productivity.

Hierarchy $\mathcal{H}_3$

By (10)-(11), rent-seeking expenditures in the inter-division conflict are $t_1 = .09$ for the stand-alone unit and $t = 1/300$ for each of the three units in the proper division. Consequently, the budget share awarded to the stand-alone unit is $\beta_1 = .9$. Moreover, we have from (4) that each of the three units in the proper division invests $r_2 = 2/90$ in
the intra-division conflict. Total firm output is thus
\[ \sum_i \pi_i = (1 + \theta_1) \left( \frac{81}{100} \right)^{\eta} + \sum_{i=2}^4 (1 + \theta_i) \left( \frac{7}{900} \right)^{\eta}. \]

Clearly, output is maximized if and only if the position of the stand-alone unit is filled with the most productive unit.

**Hierarchy \( \mathcal{H}_4 \)**

By (4), each unit in \( z_1 \) and \( z_2 \) invests \( r_1 = .25\beta_1 \) and \( r_2 = .25\beta_2 \), respectively, in the intra-division conflict. In the inter-division conflict, the units’ objective function is thus given by (5) with \( n_k = 2 \), implying that in the unique within-group symmetric equilibrium, each of the four units invests \( t = 1/32 \). Total firm output is then
\[ \sum_i \pi_i = \sum_i (1 + \theta_i) \left( \frac{3}{32} \right)^{\eta}. \]

As in the case of \( \mathcal{H}_1 \), the position of an individual unit within the hierarchy is irrelevant.

It remains to compare the four hierarchies. It is easy to show that for all \( \eta \) and all distributions of productivities \( \mathcal{H}_1 \) is dominated by \( \mathcal{H}_4 \), and \( \mathcal{H}_2 \) is dominated by \( \mathcal{H}_3 \). Comparing \( \mathcal{H}_3 \) with \( \mathcal{H}_4 \) yields the following result, which is illustrated in Figure 2.

If \( \eta \) is close to one (region I), hierarchy \( \mathcal{H}_3 \) dominates hierarchy \( \mathcal{H}_4 \). In the light of Proposition 1, this is indeed what we would expect. There, we showed that the unique optimal hierarchy under homogeneous productivities and constant returns to scale is the one where a single stand-alone unit competes against the remaining \( N - 1 \) units. Allowing for asymmetric productivities reinforces this result since, by making the most
productive unit the stand-alone unit, the bulk of the capital can be allocated to the unit with the highest productivity. By continuity, we would then expect that this hierarchy is also optimal for values of $\eta$ sufficiently close to one.

On the other hand, for values of $\eta$ close to zero (region III), hierarchy $H_4$ dominates hierarchy $H_3$. Again, this is not surprising as decreasing returns to scale tend to discriminate against asymmetric hierarchies.

Finally, for intermediate values of $\eta$ (region II), the result is mixed. In particular, for each such $\eta$ there exists a unique productivity parameter $\theta_1(\eta)$ such that for all $\theta_1 \geq \theta_1(\eta)$, hierarchy $H_3$ is optimal, while for all $\theta_1 < \theta_1(\eta)$, hierarchy $H_4$ is optimal (note the equivalence to Case ii) in Proposition 2). Moreover, the function $\theta_1(\eta)$ is strictly decreasing, implying that hierarchy $H_4$ is dominated by $H_3$ for a greater range of productivity distributions as returns to scale become less important.

7 Concluding Remarks

This paper shows that firms with more levels of hierarchy may experience lower influence costs than less hierarchical firms even though the former have more executives upon which influence can be exerted. The result is primarily due to a free-rider problem in the provision of influence activities. Suppose a firm has two levels of hierarchy. If individual units (i.e., projects or departments) are grouped in two or more common divisions, they must exert influence on two levels: on the top level, with the intention of maximizing the budget allocated to their respective division, and on the division level, with the intention of securing as large a share as possible of the division budget for themselves. On the top level, however, the units face a free-rider problem as all other units in their division also benefit from their influence activity. Consequently, influence activities at the top level remain fairly low. In the paper, we show that the influence-cost-minimizing hierarchy heavily exploits this property.

We also address the change from the U-form to the M-form organization by many large US corporations in the 1920’s. In particular, we show that influence costs are lower under the M-form organization than under the U-form organization if and only if the number of products or regions in which the firm is active exceeds a certain threshold. Our results thus confirm an early conjecture by Williamson (1975) that the change may, at least partly, be motivated by attempts to influence the firms’ internal resource allocation process while their operations expanded.

Although we have not pursued this issue here, our model may also be used to shed light on the question of whether firms should merge or remain separate. As is easy to show, if the merged firm is reorganized optimally after the merger (in the sense of
Proposition 1), integration always reduces total influence costs, the reason being that the merger opens up better ways of reorganizing the firm’s hierarchy in an influence-cost-minimizing manner. This accords with a claim by Bolton and Scharfstein (1998, p.111) that “integration fundamentally changes the resource allocation process by increasing centralized decision making under corporate headquarters.” Observe that our model explicitly applies to large, multidepartmental firms. By contrast, the theory of integration advanced by Grossman and Hart (1986) and Hart and Moore (1990) applies mainly to entrepreneurial firms, which makes it difficult to address issues like postmerger reorganization or hierarchies. Or, to paraphrase Bolton and Scharfstein again (p.111), “the Grossman-Hart-Moore paradigm ... does not see integration as leading to greater centralization; only as reallocating bargaining power.”

Appendix: Proof of Proposition 1

If we have $N = 2$, the result is trivial as there is only one possible hierarchy. We therefore assume that we have $N \geq 3$. The proof proceeds in a series of steps.

Step 1. We first compute total influence costs for the hierarchy where a stand-alone unit competes against a proper division consisting of the remaining units. Denote the stand-alone unit by $z_1$ and the proper division by $z_2$. In the intra-division conflict in $z_2$, equilibrium rent-seeking expenditures are given by (4), implying that there exists a unique equilibrium where each unit in $z_2$ expends

$$r_2 = \frac{N - 2}{(N - 1)^2} \beta_2,$$

where $\beta_2$ is the budget share awarded to $z_2$. It follows that $\alpha_{i,2} = 1/(N - 1)$.

In the inter-division conflict, the objective functions of $z_1$ and unit $i$ in $z_2$ are given by (5) with $n_1 = 1$ and $n_2 = N - 1$, respectively. As there cannot be an equilibrium where no unit makes a positive expenditure, the expenditures of both the stand-alone unit and the $N - 1$ units in $z_2$ must be strictly positive in any within-group symmetric equilibrium. Solving the resulting system of first-order conditions, we have

$$t_1 = \frac{(N - 1)^2}{((N - 1)^2 + 1)^2}$$

and

$$t_2 = \frac{1}{(N - 1)((N - 1)^2 + 1)}.$$

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where $t_2$ is the common expenditure in $z_2$. The budget share awarded to $z_1$ is thus

$$
\beta_1 = \frac{(N - 1)^2}{(N - 1)^2 + 1}
$$

whereas the budget share awarded to $z_2$ is $\beta_2 = 1 - \beta_1$.

It follows that total influence costs under the hierarchy where a stand-alone unit competes against the remaining units are

$$
t_1 + (N - 1)(t_2 + r_2) = \frac{2N - 3}{(N - 1)\left((N - 1)^2 + 1\right)}, \quad (22)
$$

Observe that for $N \geq 3$, this number is bounded from above by $.3$.

**Step 2**: Next, we show that the hierarchy where a stand-alone unit competes against the $N - 1$ other units involves lower influence costs than the optimal hierarchy within the class of hierarchies having at least two stand-alone units. By Lemma 1, only the stand-alone units make positive expenditures in the inter-division conflict. Suppose there are $m$ stand-alone units. From (8), it then follows that total influence costs in the inter-division conflict are

$$
mr = \frac{m - 1}{m}.
$$

Hence, influence costs are minimized by having exactly two stand-alone units and putting the remaining $N - 2$ units in proper divisions where rent-seeking expenditures are zero. Total influence costs under the optimal hierarchy within the class of hierarchies having at least two stand-alone units are then $.5$ (if $N = 3$, the optimal hierarchy within this class necessarily has three stand-alone units, implying that total influence costs are even $2/3$). By contrast, in (22) it was shown that for $N \geq 3$ total influence costs under the hierarchy with a single stand-alone unit and a proper division consisting of the remaining units are bounded from above by $.3$.

**Step 3**: We proceed by showing that the hierarchy where a stand-alone unit competes against a division consisting of the remaining units is optimal within the class of hierarchies having a single stand-alone unit. By Lemma 2, only the stand-alone unit and the proper division(s) with the smallest number of units make positive expenditures in the inter-division conflict. We can thus safely restrict attention to hierarchies with a single stand-alone unit and $m$ proper divisions having $\underline{a}$ units each, where $m\underline{a} \leq N - 1$.

Given the restriction, it follows from (4) that total rent-seeking expenditures in the $m$ intra-division conflicts are

$$
nmr = m\frac{\underline{a} - 1}{\underline{a}}(1 - \beta_1),
$$

where $\beta_1$ is the budget share awarded to the stand-alone unit, and $1 - \beta_1 = \sum_{k=2}^{m+1} \beta_k$. 

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Equilibrium rent-seeking expenditures in the inter-division conflict are given by (10)-(11). Accordingly, the budget share awarded to the stand-alone unit is

$$\beta_1 = \frac{n^2 m + 1 - m}{n^2 m + 1},$$

while the budget share awarded to each of the $m$ proper divisions is

$$\beta_k = \frac{1}{n^2 m + 1}.$$ 

Total influence costs in the hierarchy with one stand-alone unit and $m$ proper divisions are therefore

$$t_1 + m n (t + r) = \frac{m(2n - 1)}{mn^2 + n}. \quad (23)$$

The problem of finding the optimal hierarchy within the class of hierarchies having a single stand-alone unit hence boils down to minimizing (23) subject to the constraint that $m n + 1 \leq N$. Differentiating (23) with respect to $m$ shows that total influence costs are strictly increasing in $m$. As it is always possible to reduce $m$ while leaving $n$ constant (the remaining units can be put in a division with $n \geq \underline{n}$ units where equilibrium rent-seeking expenditures are zero), the optimal number of proper divisions with $\underline{n}$ units is one. Inserting $m = 1$ in (23) and differentiating with respect to $\underline{n}$ shows that influence costs are strictly decreasing in $\underline{n}$, implying that the unique solution is $m = 1$ and $\underline{n} = N - 1$, i.e., the hierarchy where a single stand-alone unit competes against all other units is optimal within the class of hierarchies having just one stand-alone unit.

**Step 4.** It remains to show that total influence costs under the hierarchy with one stand-alone unit and one proper division are less than under any hierarchy with no stand-alone unit. A hierarchy with no stand-alone unit is feasible if and only if $N \geq 4$. Suppose there are $M$ divisions. By (4), we have that total rent-seeking expenditures in the $M$ intra-division conflicts are

$$\sum_{k=1}^{M} \frac{n_k - 1}{n_k} \beta_k,$$

which is bounded from below by .5 since $n_k \geq 2$ and $\sum_{k=1}^{M} \beta_k = 1$. By contrast, in (22) it was shown that for $N \geq 3$, total rent-seeking expenditures under the hierarchy where a single stand-alone unit competes against a division comprising all other units are bounded from above by .3.

\[ \square \]

**References**


