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Should day care be subsidized?

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Abstract

In an economy with distortionary taxes on labor, can subsidies on day care, financed by an increase in taxes, raise welfare by encouraging women with small children to work? We show, within a heterogeneous-agent life-cycle framework, that the Ramsey optimal policy consists in equalizing consumption/leisure wedges over the life cycle and across agents. A simple way to implement this is to make day care expenses tax deductable. Calibrating our model to Germany, we find that tax deductibility for day care expenses leads to an approximate doubling of labor supply for both married and single mothers with small children. The overall welfare gain from optimal reform corresponds to a 1.0 percent increase in consumption.

Keywords: Female labor force participation, Germany, day care subsidies

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1 Introduction

In both Europe and North America, public support for preschool day care is on the rise. At its Barcelona meeting in 2002, the European Union encouraged member countries to supply subsidized full-day places for one-third of 0- to 3-year-olds, and for over 90% of all 3- to 6-year-olds by the year 2010. In the United States from 2002 to 2007, the fraction of four-year-olds attending state-funded preschool education rose from 14 to 22 percent. At the same time, U.S. state funding for preschool education rose from 2.4 to 3.7 billion dollars.\(^1\) In Canada, partly inspired by the $5-a-day day care policy introduced in Québec in 1997, all major parties in the 2006 federal election campaign put forward proposals for a national day care programme.

Is public funding for preschool day care a good idea? We approach this question from an optimal taxation perspective, essentially in the spirit of Ramsey (1927). This means that we ask whether day care subsidies can contribute to a more efficient allocation of resources.

There are of course several alternative justifications for government funding of day care whose validity may be worth exploring. One is that subsidized day care might raise fertility and thereby contribute to solving the problems generated by an aging population. Another is that it might promote early learning, especially among disadvantaged children. Meanwhile, day care subsidies tend to encourage female labor force participation, which might be desirable either because it promotes equality within the household\(^2\) or because it leads to a more efficient allocation of resources. Here we focus entirely on the consequences of day care subsidies for the allocation of resources.

In this paper, we make an optimal-taxation based case for subsidized day care. To do

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\(^2\) Knowles (2006) shows that rising women’s wages relative to men’s wages has strengthened the relative bargaining position of women within U.S. households. Since the introduction of day care subsidies raises the effective wage for women, it is likely to have the same effect.
this, we develop a heterogeneous-agent life-cycle framework where the demand for day care arises from the assumption that every hour of work makes it necessary for the agent to purchase an hour of day care per young child. A non-trivial optimal taxation problem arises from the assumption that the government has an exogenous spending requirement and that lump-sum taxes are ruled out but linear taxes and subsidies are allowed. Thus the consumption/leisure choice is distorted and the problem is to minimize the total welfare cost of this distortion.

Our main finding is that the optimal policy consists in equalizing the consumption/leisure wedge over the life cycle and across agents, and that the most straightforward way to accomplish this is to make day care expenses tax deductible.\textsuperscript{3} The intuitive reason for that is the following. If day care is not subsidized or tax deductible and marginal taxes rates are not conditional on the number of small children, then the presence of small children reduces the ratio of marginal private to marginal social returns to working net of day care expenses. This means that labor supply is more distorted the more small children are present, and this is not consistent with minimizing the total distortion. What is optimal is to even out the distortion. This is essentially a tax smoothing argument.

Having established this theoretical result, we then examine the quantitative significance of day care finance reform by calibrating our model to Germany. There are several strong reasons for studying Germany in the context of day care policy. One, as emphasized in a recent OECD survey of Germany (OECD, 2008), is that “fewer mothers with small children are employed in Germany than in other countries and those who work do so for fewer hours”. As we show in Figure 1, women with small children supply on average 60 percent fewer hours than other women. There is no corresponding difference between fathers with small children and other men.\textsuperscript{4}

\textsuperscript{3} Interestingly, Israel will be making day care expenses tax deductible starting in the fiscal year 2010, following a decision by their Supreme Court. See Segal (2009).

\textsuperscript{4} Merz (2004) studies female labor supply trends in western Germany and finds that although participation has risen somewhat, hours per female employee actually fell between 1980 and 2000, the net
Another reason to consider Germany is that, until recently, availability of subsidized day care is very limited there, especially in western Germany; see Kreyenfeld and Hank (2000), Wrohlich (2005) and OECD (2006). On average, across the OECD countries for which data are available, 23 percent of 0-3 year-olds are in formal day care. In Germany, that number is just 9 percent, and in western Germany it is less than 3 percent.5

A further reason to study Germany is that its day care policy is currently in transition. In 2004, the SPD-Green federal government enacted the *Tagesbetreuungsausbaugesetz*; this legislation requires that, each year, 1.5 billion euros have to be invested in day care, and local authorities are responsible for supplying sufficient day care for children under the age of three. The CDU/CSU-SPD government that took office in 2005 continued in the same spirit. In 2007 the German government announced that it plans by 2013 to triple the number of subsidized day care spots for young children to 750000.6 In 2008, the *Kinderförderungsgesetz* was passed in the *Bundestag* (federal parliament), establishing a right to a day care spot for every child from the age of one.7 Recent reforms in Germany have also included moves towards making day care expenses tax deductible.8 Our work contributes to an evaluation of this and other day care finance reforms in Germany.

When calibrating our model to Germany, it is important to capture the way that labor supply varies with the presence (or not) of small children in the household. In the data, the extent of this variation is strongly associated with gender and marital status.9 Con-

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6 Source: Deutsche Welle (2007).
7 For more details on these reforms and also appropriate references to German legislation, see the website of the German federal ministry of families, seniors, women, and youth, [http://www.bmfsfj.de/](http://www.bmfsfj.de/).
8 See Bundesregierung (2006).
9 When talking about *marital status* we are not interested in whether anyone is legally married, only whether they are living in a relationship that is economically equivalent to marriage. Therefore we will use the terms “married” and “cohabiting” interchangeably throughout the paper.
sequently, we introduce heterogeneity with respect to these features. The introduction of married couples into the model requires us to allow for households with two working members, noting that tax deductibility of day care expenses is the Ramsey optimal policy in this case as well.

Our quantitative findings can be summarized as follows. Optimal day care reform leads to substantial increases in the labor supply of mothers with small children. Specifically, tax deductibility for day care expenses leads to an approximate doubling of labor supply for both married and single mothers with small children. Meanwhile, the labor tax rate increases by 0.4 percentage points. Optimal reform also has substantial welfare effects. Weighting households in such a way that the government has no redistributitional motive for subsidizing day care (setting Pareto weights inversely proportional to the marginal utility of consumption), the overall welfare gains correspond to a 1.0 percent increase in consumption. Among couples, the gains correspond to a 0.9 percent increase; for single men this number is 0.8 percent, and for single women it is 2.0 percent.

Our main results are derived in an environment with linear taxes and no transfer payments. Meanwhile, the current German tax system is very far from linear and there is a generous system of means-tested social assistance payments. In order to investigate the robustness of our results to this policy context, we modify the model environment to be broadly consistent with the existing framework of German fiscal policy. To focus sharply on the effects of day care finance reform, the policy changes that we consider keep other aspects of the tax-transfer system unchanged, subject only to government solvency.

Keeping these other aspects unchanged, we find that making day care expenses tax deductible gives rise to an overall welfare gain corresponding to a 0.7 percent increase in consumption. Interestingly, the reform pays for itself by encouraging people to work rather than live on social assistance. This means that everyone gains from this reform.

\[10\] The effects of reform are compared to an initial situation where day care is neither subsidized nor tax deductible.
Since in this context we don’t have a theoretical result establishing deductibility as an optimal policy, we also consider a set of alternative policies, specifically linear subsidies on day care at various rates. The welfare gain for society as a whole turns out to be maximized at a subsidy rate of about 85 percent. At this rate, the overall welfare gain is 0.8 percent, but because this policy involves a tax increase, only a bare majority of 50.1 percent prefer it to no reform at all. If voters were faced with the options of making day care expenses tax deductible and introducing an 85 percent subsidy on day care, a clear 60 percent majority would vote for deductibility.

Our work is related to several strands of the literature. First, there is an empirical literature documenting that subsidized day care may encourage labor supply of mothers. The seminal work of Heckman (1974) presents strong evidence from the United States that day care subsidies do indeed increase female labor supply; more recently, in a case more directly relevant to this paper, Wrohlich (2006) finds, using an econometric approach, that expanding availability of subsidized day care increases maternal employment in Germany. Bick (2010) draws the same qualitative conclusion in the context of a calibrated life-cycle model, though he finds a smaller effect than Wrohlich (2006).  

Another strand considers the role of public policy in determining the choice between home and market production. Lindbeck (1982) argues that subsidies to day care in many countries has contributed to the increased labor supply of women, counteracting the rise in tax rates that have been required to finance these subsidies.  

The reason for Lindbeck’s conclusion is that he thinks, as we do, of market provided day care as a close substitute for day care at home. Rosen (1997), on the other hand, considers, in the

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11 The contrasting conclusions drawn by Wrohlich (2006) and Bick (2010) is that the latter assumes that day care is not necessary even when both parents are working. Instead he assumes that the mother suffers a utility cost from being away from her children and that this utility cost can be avoided by purchasing day care. Our quantitative results lie between those of Wrohlich (2006) and those of Bick (2010).

12 Bergstrom and Blomquist (1996) argue that the effect on labor supply may be so large that subsidizing day care is self-financing; a similar result is found in Lundholm and Ohlsson (1998).
context of a model conceptually similar to Lindbeck’s, whether high day care subsidies can be justified and concludes that they cannot. This conclusion hinges on imposing a rather low upper limit on the degree of substitutability between market and home produced day care. Another important reason why our conclusions differ from Rosen’s is that the tax smoothing considerations central to our analysis are absent from his static model.

A third strand of the literature is concerned, as we are, with understanding female labor supply. Much of that literature has focused on the remarkable rise of female labor force participation since the 1950s in the United States. Prominent contributions include Jones et al. (2003), Greenwood et al. (2005), and Olivetti (2006). More recently, Attanasio et al. (2008) have emphasized the importance of reduced day care costs in explaining observed changes in U.S. female participation rates.

Other related papers consider the role of fiscal policy in explaining differences in labor supply across time and across countries, starting with Prescott (2004). Of particular relevance to the present paper is the idea that differences in labor-supply-promoting public spending may be an important factor in accounting for differences in labor supply across countries, especially the difference between Scandinavia and (the rest of) continental Europe; this idea is explored in Olovsson (2004), Ragan (2005), and Rogerson (2006).

Our work is also closely related to the growing literature on quantitative evaluation of fiscal policy reforms in dynamic models with heterogeneity. A particularly relevant part of the literature are those papers that deal with multi-member households, such as Chade and Ventura (2002) and Güner et al. (2008) who look at the effects of income tax reforms on labor supply and Erosa et al. (2008) who look at the effects of parental leave policies.

The paper is organized as follows. Section 2 presents the environment, proves the optimality of equalizing consumption/leisure wedges and discusses how that can be implemented by making day care expenses tax deductible. Section 3 provides a quantitative assessment of optimal day care reform in Germany. Section 4 concludes.
2 Theory

Consider a $T$-period life-cycle model with $I$ agents where at age $s$ an agent $i$ has $b_s^i$ small children. The demand for day care arises from the following assumption: for every unit of time that the agent works, it needs to purchase day care for each small child. Beyond that, agents in the model are indifferent to the presence of children; time spent with children is like leisure spent without children, no better and no worse. The only significance of children in the model is that they require constant supervision and so day care is required when the parent is working.

The resource cost of day care is $d$ per unit of time and child. The government levies age- and agent-dependent linear taxes on labor income (at rates $\tau_s^i$) to finance exogenous government purchases $G$ and age- and agent-dependent linear subsidies on day care (at rates $\theta_s^i$). There is a world capital market that enables agents to transform one unit of the age $s$ good into $1 + r$ units of the age $s + 1$ good and vice versa without any non-negativity restrictions on the amount saved. Each agent’s initial endowment of capital is zero. Factor prices are exogenous.

Formally, agent $i$ solves

$$\max \sum_{s=1}^{T} \beta^s \left[ u_s^i(c_s^i) + v_s^i(h_s^i) \right],$$

where $u_s^i: \mathbb{R}^+ \rightarrow \mathbb{R}$ is an increasing, differentiable and concave function and $v_s^i: \mathbb{R}^+ \rightarrow \mathbb{R}$ is a decreasing, differentiable and concave function for each $i$ and $s$, subject to the life-time budget constraint (associated with the Lagrange multiplier $\lambda^i$)

$$\sum_{s=1}^{T} \prod_{t=1}^{s-1} (R_{t+1}^i)^{-1} \left[ c_s^i + (1 - \theta_s^i)db_s^ih_s^i \right] = \sum_{s=1}^{T} \prod_{t=1}^{s-1} (R_{t+1}^i)^{-1} \left[ (1 - \tau_s^i)w_s^ih_s^i + x_s^i \right],$$

where $c_s^i$ denotes consumption, $h_s^i$ denotes hours worked, $w_s^i$ denotes age-specific produc-
activity, $x^i_s$ is a transfer payment that has to sum to zero across agents, i.e.

$$\sum_{i=1}^{I} x^i_s = 0$$

(3)

for all $s$. These transfer payments are introduced only in order to simplify the proofs, and in the equilibria we consider we will make assumptions to ensure that they are all zero. $R^i_{s+1}$ is the after-tax gross rate of return between $s$ and $s + 1$. Denoting the agent- and age-specific capital income tax rate by $\xi^i_s$, we have

$$R^i_s = 1 + (1 - \xi^i_s)r.$$

Notice that we allow the disutility of labor, $v^i_s(h)$, to depend on age and on the agent’s identity; this enables us to establish a slightly more general result than otherwise.

Assuming an interior solution, labor supply and consumption are characterized by the agent’s first order conditions which are

$$\beta^s u^i_{c^i_s} - \lambda^i \prod_{t=1}^{s-1} (R^i_{t+1})^{-1} = 0$$

(4)

and

$$\beta^s v^i_{h^i_s} + \lambda^i \prod_{t=1}^{s-1} (R^i_{t+1})^{-1} [(1 - \tau^i_s)w^i_s - (1 - \theta^i_s)db^i_s] = 0.$$  

(5)

where we define

$$u^i_{c^i_s} = \frac{\partial u^i_s(c^i_s)}{\partial c^i_s}$$

and

$$v^i_{h^i_s} = \frac{\partial v^i_s(h^i_s)}{\partial h^i_s},$$

thus suppressing the $i$ and $s$ arguments of $u$ and $v$.

The economy as a whole faces the following constraint:

$$\sum_{i=1}^{I} \sum_{s=1}^{T} (1 + r)^{-s} [c^i_s + db^i_s h^i_s + G] = \sum_{i=1}^{I} \sum_{s=1}^{T} (1 + r)^{-s} w^i_s h^i_s,$$

(6)

where $G$ denotes government consumption.
2.1 Ramsey government

The Ramsey government, assigning a weight $\pi^i$ to each agent $i$, maximizes

$$\sum_{i=1}^{I} \pi^i \sum_{s=1}^{T} \beta^s \left[ u^i_s(c^i_s) + v^i_s(h^i_s) \right]$$

subject to (3), associated with the Lagrange multipliers $\nu_s$, (6), associated with the Lagrange multiplier $\mu$, and the implementability constraints

$$\sum_{s=1}^{T} \beta^s \left[ u^i_s(c^i_s - x^i_s) + v^i_s(h^i_s) \right] = 0$$

for $i = 1, 2, \ldots, I$, associated with Lagrange multipliers $\phi^i$.

The first order conditions are

$$\beta^s \varphi^i u^i_{c^i} = \nu_s,$$  \hspace{1cm} (9)

$$\beta^s u^i_{c^i} \left[ \pi^i + \varphi^i \left(1 - \sigma^i_s(c^i_s)\right) \right] - \mu(1 + r)^{-s} = 0,$$  \hspace{1cm} (10)

and

$$\beta^s v^i_{h^i} \left[ \pi^i + \varphi^i \left(1 + \frac{1}{\varepsilon^i_s(h^i_s)}\right) \right] + \mu(1 + r)^{-s} \left[w^i_s - dB^i_s]\right] = 0,$$  \hspace{1cm} (11)

for $i = 1, 2, \ldots, I$ and $s = 1, 2, \ldots, T$, where $\varepsilon^i_s$ is the Frisch (1959) elasticity of labor supply defined via

$$\varepsilon^i_s(h) = \frac{\partial v^i_s(h)}{\partial h} \cdot \frac{1}{h}$$  \hspace{1cm} (12)

and $\sigma^i_s$ is defined via

$$\sigma^i_s(c) = \frac{\partial^2 u^i_s(c)}{\partial c^2} \cdot c.$$  \hspace{1cm} (13)

2.2 Ramsey optimum

We now characterize the Ramsey optimal allocation. We will establish that, under certain conditions, the Ramsey optimal allocation equalizes the consumption/leisure wedge over
the life cycle and across agents. The precise definition of this wedge is as follows:\textsuperscript{13}

$$W^i_s = \frac{v_{h^i_s}/u_{c^i_s}}{w^i_s - db^i_s}.$$ \hfill (14)

To see under what conditions it is optimal to equalize \(W^i_s\) over the life cycle, combine the Ramsey government’s first order conditions with respect to labor supply (Equation 11) for agent \(i\) at ages \(s\) and \(t\) to obtain

$$W^i_t = W^i_s \cdot \frac{\pi^i + \varphi^i (1 + 1/\varepsilon^i_s(h^i_s))}{\pi^i + \varphi^i (1 + 1/\varepsilon^i_t(h^i_t))} \cdot \frac{\pi^i + \varphi^i (1 - \sigma^i_s(c^i_s) \cdot (1 - x^i_s/c^i_s))}{\pi^i + \varphi^i (1 - \sigma^i_t(c^i_t) \cdot (1 - x^i_t/c^i_t))}.$$ \hfill (15)

This establishes the following Proposition.

\textbf{Proposition 1} If (1) \(\varepsilon^i_s(h) = \varepsilon^i\) for all \(i, s\) and \(h\), (2) \(\sigma^i_s(c) = \sigma^i\) for all \(i, s\) and \(c\), and (3) \(x^i_s = 0\) for all \(i\) and \(s\), then the Ramsey allocation satisfies

$$W^i_t = W^i_s$$ \hfill (16)

for all \(s\) and \(t\).

As is always the case, the Ramsey optimal allocation can be implemented in many different ways. Generically, in order to implement the Ramsey optimal policy, we need one policy instrument per decision. If more instruments are available, the optimal policy is not unique. In the present setting, we have two decisions \((c^i_s\) and \(h^i_s))\) and three policy instruments \((\tau^i_s, \theta^i_s\) and \(\xi^i_s))\) per agent and age. Since \(\theta^i_s\) and \(\tau^i_s\) both affect the same wedge \(W^i_s\), they are not uniquely determined. Nevertheless, we can characterize the set of optimal policies in the following way. The government should either subsidize day care or give agents more favorable tax treatment in periods when more small children are present. To give a more precise description of the set of optimal policies, it is useful to

\textsuperscript{13} Notice that this wedge is only well-defined for those \(i\) and \(s\) such that \((w^i_s - db^i_s) > 0\), i.e. that the wage exceeds the cost of day care. This is the rationale for the production of day care outside the family; for those \(i\) and \(s\) where it does not hold, the Ramsey optimum dictates that no day care should be consumed.
let \( \theta^i_s = \theta^i \). Because of the indeterminacy, this involves no loss of generality. Then, as we will demonstrate below, there are only three possibilities, all consistent with Ramsey optimality. One is for the subsidy rate to be low, \( \theta^i < \tau^i_s \) for all \( s \). In this case, the tax rate \( \tau^i_s \) should be lower in those periods where a larger number of small children are present. Another possibility is the converse, \( \theta^i > \tau^i_s \) for all \( s \), and a higher tax rate in those periods where a larger number of small children are present.

A final possibility is to set \( \tau^i_s = \theta^i \) for all \( s \). This means that the tax rate is constant over the life cycle—surely an advantage from the point of view of simplicity. One way to describe this policy is that it involves making day care expenses tax deductible and equalizing tax rates over the life cycle. If taxes cannot vary over the life cycle, then this is the uniquely optimal policy. This is what we mean when we say that it is optimal to make day care expenses tax deductible.

To see that these are the only three possibilities consistent with Ramsey optimality, combine the agent’s first order condition (5) and (11) with the assumptions of Proposition 1 to obtain

\[
\tau^i_s = \chi^i_s - (\chi^i_s - \theta^i) \cdot \frac{db^i_s}{w^i_s} \tag{17}
\]

where

\[
\chi^i_s = \frac{\varphi^i(1/\varepsilon^i_s(h^i_s)) + \sigma^i_s(c^i_s) (1 - x^i_s/c^i_s))}{\pi^i + \varphi^i(1 + 1/\varepsilon^i_s(h^i_s))}. \tag{18}
\]

Notice that under the assumptions of Proposition 1 we have \( \chi^i_s = \chi^i > 0 \) for all \( s \). Given our assumption that \( w^i_s > db^i_s \) for all \( i \) and \( s \), this means that Equation (17) only allows three possibilities: either \( \theta^i_s < \tau^i_s < \chi^i \) for all \( s \), \( \theta^i_s > \tau^i_s > \chi^i \) for all \( s \) or \( \theta^i_s = \tau^i_s = \chi^i \) for all \( s \). Meanwhile, Equation (17) implies that

\[
\tau^i_t - \tau^i_s = - (\chi^i_t - \theta^i) \left( \frac{db^i_t}{w^i_t} - \frac{db^i_s}{w^i_s} \right). \tag{19}
\]

It follows that, as described above, there are only three possibilities that are consistent with a Ramsey optimum. They are

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1. \( \tau^i_t > \theta^i \) for all \( r \). Then \( \tau^i_t < \tau^i_s \) whenever \( b^i_t / w^i_t > b^i_s / w^i_s \).

2. \( \tau^i_t < \theta^i \) for all \( r \). Then \( \tau^i_t > \tau^i_s \) whenever \( b^i_t / w^i_t > b^i_s / w^i_s \).

3. \( \tau^i_t = \theta^i \) for all \( r \).

We will now discuss the significance of the assumptions of Proposition 1. Suppose Assumption (1) does not hold and that the Frisch elasticity of labor supply varies over the life cycle. Then it follows from Equation (17) that for any fixed \( \theta^i_s = \theta^i \), taxes should be set so that when the Frisch elasticity is lower (all else equal), the agent should face a higher tax rate, i.e. that \( \tau^i_s \geq \tau^i_t \) if \( \varepsilon^i_s \geq \varepsilon^i_t \), a natural feature of Ramsey optimal taxation.\(^{14}\)

Suppose on the other hand that Assumption (2) does not hold and that \( \sigma^i_s \) varies over the life cycle; then Equation (17) implies that \( \tau^i_s \geq \tau^i_t \) if \( \sigma^i_s \geq \sigma^i_t \), all else equal. Finally, note that Assumption (3), that \( \pi^i_s = 0 \) for all \( i \) and \( s \) holds automatically (by Equation 3) when \( I = 1 \). We discuss below some conditions under which it holds more generally.

Having established conditions under which it is optimal to equalize the consumption/leisure wedge over the life cycle, we now strengthen the conditions to guarantee that it should be equalized across agents as well. Combining the Ramsey government’s first order conditions (9-11), we get

\[
W^i_s = W^j_s, \\
\pi^j u_{\varepsilon^j_s} + \frac{\nu^i_s}{\beta^i} (1 + 1/\varepsilon^j_s) \\
\pi^j u_{\varepsilon^j_s} + \frac{\nu^i_s}{\beta^i} (1 + 1/\varepsilon^j_s) + \frac{\nu^i_s}{\beta^i} \left( \sigma^i_s (1 - x^i_s/c^i_s) - \sigma^j_s (1 - x^j_s/c^j_s) \right) \tag{20}
\]

for all \( i, j, \) and \( s \). This establishes the following Proposition.

**Proposition 2** If, in addition to the assumptions of Proposition 1 we have \( (1) \varepsilon^i = \varepsilon \) for all \( i \) and \( (2) \sigma^i = \sigma \) for all \( i \), then the Ramsey allocation satisfies

\[
W^i_t = W^j_s \tag{21}
\]

for all \( i, j, s \) and \( t \).

\(^{14}\) This has previously been discussed in a life-cycle setting by Erosa and Gervais (2002).
Just as in the case of Proposition 1, the equalization of wedges can be accomplished in three ways. The simplest one involves making day care expenses tax deductible and equalizing tax rates not just over the life cycle but also across agents, thus setting \( \tau^i_s = \theta^i_s = \tau \) for all \( i \) and \( s \).

The conclusions of both Propositions 1 and 2 rely on \( x^i_s = 0 \). What if transfers are non-zero for some \( i \) and \( s \)? Then Equation (17) implies that if \( x^i_s/c^i_s \geq x^i_j/c^i_j \) (for some \( s \), and hence for all \( s \)) then \( \tau^i \geq \tau^j \). In other words, if the Ramsey policy maker would like to redistribute from agent \( i \) to agent \( j \) using transfers \( x \), then the Ramsey policy maker also wants to impose a higher labor tax on agent \( i \) than on \( j \).

Under what conditions are zero transfers optimal? Intuitively, the desire of the government to redistribute purchasing power should depend on the Pareto weights. Does there exist a vector of Pareto weights \( \pi = (\pi^1, \pi^2, \ldots, \pi^I) \) such that \( x^i_s = 0 \) for all \( s \) and \( i \) is optimal, assuming of course the other premises of Propositions 1 and 2? If so, how can we go about constructing such weights? What we can say is the following. First, note that what matters are the relative Pareto weights so that we may assume without loss of generality that \( \sum_{i=1}^I \pi^i = 1 \). This leaves us with \( I - 1 \) degrees of freedom when choosing Pareto weights. Suppose now that there exists a vector \( \pi \) of Pareto weights and an associated allocation that satisfies the Ramsey optimality conditions and \( x^i_1 = 0 \) for all \( i \). This is a reasonable assumption; we can use the \( I - 1 \) degrees of freedom in choosing the Pareto weights to ensure that \( x^i_1 = 0 \) for \( i = 1, 2, \ldots, I - 1 \); Equation (3) ensures that \( x^I_1 = 0 \) also. Then, using Equations (3), (9) and (10), it is straightforward to show that this allocation is such that \( x^i_s = 0 \) for all \( s \) and \( i \). Moreover, it follows that the weights must be inversely proportional to marginal utilities of consumption, a natural result given that maximization of \( (7) \) implies a redistributitional motive whenever agents differ in their weighted marginal utility of consumption.

In the quantitative exercises of Section 3, we will confine our attention to situations where the policy maker has no redistributive motive. This means that we will calibrate
the Pareto weights to be inversely proportional to the marginal utility of consumption and that we will only consider reforms that involve age- and agent-independent tax and subsidy rates.

So far, we have only discussed labor taxes and day care subsidies. It turns out that, under the assumptions of Proposition 2, optimal capital income taxes are zero. To see this, combine Equations (4) and (10) for $s = 1$ and $s$ to obtain

$$\xi_s = \left( \frac{\pi^i + \varphi^i \sigma_s^i(c_s^i)(1 - c_s^i/x_s^i)}{\pi^i + \varphi^i \sigma_{s-1}^i(c_{s-1}^i)(1 - c_{s-1}^i/x_{s-1}^i)} - 1 \right) \cdot \frac{1 - r}{r}. $$

Hence if we have $\sigma_s^i(c_s^i) = \sigma^i$ and $x_s^i = 0$ for all $i$ and $s$ then $\xi_s^i = 0$ for all $i$ and $s$ so that capital income taxes are zero in the Ramsey optimum.

We end this section with a discussion of the factors that determine the magnitude of the effects on allocations and welfare from optimal reform. What that magnitude depends on is the degree to which the consumption/leisure wedge $W_s^i$ is not equal across different values of $s$ in the initial situation. When this wedge is nearly equal over the life cycle, there is not much for day care subsidies to accomplish; when it varies a lot over the life cycle, optimal day care subsidies have large effects on labor supply and on welfare. We now consider what conditions might lead the wedge to vary over the life cycle. Note that, by the agent’s first order condition, we can express it as the ratio of the marginal private to the marginal social return to working, i.e.

$$W_s^i = \frac{(1 - \tau_s^i)w_s^i - (1 - \theta_s^i)db_s^i}{w_s^i - db_s^i}. $$

Important factors that lead to $W_s^i$ being different for different values of $s$ include the unit day care cost, the spacing of children and the initial level of taxation. To see this in more detail, consider an initial situation where $\theta_s^i = 0$ and $\tau_s^i = \tau$. Then the wedge is given by

$$W_s^i = \frac{(1 - \tau)w_s^i - db_s^i}{w_s^i - db_s^i}. $$

Evidently the wedge is constant as a function of $s$ if $\tau = 0$ so there is no rationale for day care subsidy reform in an economy with zero labor taxes. If $\tau > 0$ then the wedge
is a decreasing function of $db_i^s$ which means that the higher the hourly day care cost per child and the greater the number of children, the more severe is the consumption/leisure distortion. This in turn means that the effects of optimal reform will be greater the greater the extent to which having children is confined to a small part of the life cycle.

### 2.3 Discussion

The main optimal taxation result that we have established in this paper says that consumption/leisure wedges should be equalized across ages and across agents. This means that our work is related to the uniform commodity taxation result of Sandmo (1974). What he shows is that if utility is separable in labor and commodities and homogeneous (of arbitrary degree) in the commodities, then commodities should be taxed uniformly. We prove a somewhat stronger result in an environment where labor is supplied in several periods. Under assumptions that imply Sandmo’s and also assuming that the Frisch (1959) elasticity of labor supply is constant and age-independent, then labor supply at different ages should be taxed uniformly as well. Another dimension in which we go beyond Sandmo is that we allow for heterogeneous agents and establish conditions under which all agents should face the same tax rates. Finally, and perhaps this is the most important novel feature of our work, our approach allows us to interpret uniform consumption/leisure wedges as giving favorable tax treatment to households with small children.

The connection to the efficiency-in-production result of Diamond and Mirrlees (1971) is looser. The result of that paper is that technical rates of substitution should be equalized across sectors. That result does not apply to our environment, since labor inputs from different individuals are perfect substitutes. Technical rates of substitution are exogenously fixed and thus either happen to be equal or cannot be equalized.

Meanwhile, our work is obviously related to that of Erosa and Gervais (2002). In that
context it is worth noting that the optimality of age-independent tax rates and subsidies hinges on the Frisch elasticity being constant. If it is not, taxes and subsidies should typically be age-dependent, a feature of Ramsey optimal policy that is emphasized in their work.

3 A quantitative assessment of optimal day care reform

In this section we use the model laid out in Section 2 to provide a quantitative assessment of optimal day care reform. We calibrate the model to match some key features of the German economy. In particular, we want to capture the way that labor supply varies with the presence (or not) of small children in the household. In the data, the extent of this variation is strongly associated with gender and marital status. Consequently, we introduce heterogeneity with respect to these features. Formally, introducing married couples requires us to extend the model to incorporate households with two adult members, but this extension does not undermine any of the theoretical results established in Section 2; a proof of this is conceptually straightforward but heavy in notation and therefore omitted here. Moreover, the calibrated model features overlapping generations as opposed to just a single life cycle; the theoretical results survive this extension as well, provided that the Pareto weights of the generations decline geometrically at a rate given by the households’ subjective discount factor $\beta$.

We now describe how married couples are incorporated into the model economy. The economy is populated by $I$ households, some of which are lifelong couples and some of which are lifelong singles. Singles still solve (1) subject to (2) provided that we suppress
the dependence on birth cohort. A couple consists of a man and a woman and it solves

$$\max \sum_{s=1}^{T} \beta^s \left[ u^i_s(c^i_s) + v^{i,m}_s(h^{i,m}_s) + v^{i,f}_s(h^{i,f}_s) \right],$$

subject to the life-time budget constraint (associated with the Lagrange multiplier $\lambda^i$)

$$\sum_{s=1}^{T} \prod_{t=1}^{s-1} (R_{t+1}^i)^{-1} \left[ c^i_s + (1 - \theta^i_s)db^i_s \min \{ h^{i,m}_s, h^{i,f}_s \} \right] =$$

$$= \sum_{s=1}^{T} \prod_{t=1}^{s-1} (R_{t+1}^i)^{-1} \left[ (1 - \tau^{i,m}_s)w^{i,m}_s h^{i,m}_s + (1 - \tau^{i,f}_s)w^{i,f}_s h^{i,f}_s + x^i_s \right],$$

where the superscript $i, m$ represents the man and $i, f$ represents the woman of household $i$. (Again, we suppress dependence on birth cohort.) The idea behind the $\min \{ h^{i,m}_s, h^{i,f}_s \}$ expression in the budget constraint is that day care has to be purchased for every hour that both parents work in the market and that shift work is not allowed.

3.1 Calibration

As discussed in Section 2.2, the optimal policy consists in equalizing across households and age a certain wedge, defined in Equation (22). This wedge depends on the number of small children, the wage, the pre-reform tax rate and the per-unit day care cost. Therefore the calibration should replicate some of the main facts about how these variables are distributed. We now discuss these facts in turn.

3.1.1 Population

The model population is designed to reflect the German population in terms of the number of singles and married couples and the distribution of children. The data that we use for this purpose are from the database (G)SOEP, see http://www.diw.de.\footnote{For a more detailed description of this dataset, see Appendix A.} A household is
defined in this context as a set of people living at the same address. Two adults living in
the same household are counted as a couple if (1) they are the two eldest in the household,
(2) they are of the opposite sex, and (3) the age difference is strictly less than 20 years.
In GSOEP data from 1984 to 2004, about 72 percent of people between the ages of 20
and 62 are cohabiters in this sense; this is what we assume for the model population as
well. Half of the single people in the model are assumed to be male, and the other half
female.

Children are distributed among households in such a way as to replicate the key features
of the data. We assume that the length of a time period is six years. Adults are assumed
to live for $T = 10$ periods, the last three periods being spent in retirement and we think
of the first period of adult life as 20-25 years of age. Children live with their parents for
three periods, and we assume that they require constant supervision only during their first
period of life. Newborn children may arrive when the parents are aged 1-4 (corresponding
to 20-44 years). At age 1, the maximum number of newborns is 2, reflecting the fact
that young parents very rarely have more than two children. For ages 2-4, the maximum
number of newborns is 3. In addition, we set an upper limit on completed fertility at 3
children. This gives rise to 34 different possible cases and hence our model population
has 34 different child profile types for each of the groups single men, single women and
couples. To determine population shares of these types, we proceed as follows. First we
use GSOEP data to compute the probabilities of children arriving as a function of family
characteristics such as age, number of existing children, marital status and, for singles,
gender. Then, for each of the groups single men, single women and couples, we simulate
a large population on the basis of these probabilities. Based on these populations, we can
determine the population shares of each of the 34 child profiles.

By construction, these population shares ensure that we match the following features of
our GSOEP sample; 3.6 percent of single men, 6.9 percent of single women, and 13.2
percent of couples have small (0-5 years old) children at any given point in time. These
population shares also imply that 18.5 percent of the model population is between 0 and 17 years old. Reassuringly, according to Eurostat, the corresponding number for the entire population of Germany was 18.2 percent in 2004.

### 3.1.2 Wages and taxes

To generate a wage distribution that reflects the key features of its empirical counterpart, we let wages depend on age, marital status, gender and (permanent) productivity type. Wage profiles are calibrated on the basis of a regression of the log wage\(^{16}\) on time dummies, age in years, gender and marital status. Specifically, the regression coefficients are 0.0405 (the age premium), −0.0004 (the coefficient on age squared), 0.174 (the male wage premium), −0.0399 (the gender-independent cohabitation effect) and, finally, 0.0125 (the male-specific cohabitation effect). The residuals of this regression are then modelled as the sum of a purely transitory measurement error and an individual fixed effect. The variance of the fixed effect can then be backed out from the autocovariance of the residual. This autocovariance provides the basis for our modelling of productivity types. In the model we assume that there are two productivity types (high and low). To replicate the empirical autocovariance, we give the high type a wage that is 92.4 percent above that of the low type. Among singles, there are as many high types as there are low types. Among couples, there are four possible combinations; the population shares of each are set so as to match the correlation between spouses’ permanent productivity types. This correlation is 0.27 (with a standard error of 0.0045). Imposing symmetry, this corresponds to 28 percent of couples being high-high/low-low and 22 percent high-low/low-high.

The pre-reform labor tax rate \(\tau\) is set to 46 percent, which is the combined average effective labor and consumption taxes 1991-97 as reported in Carey and Tchilinguirian (2000).

\(^{16}\) In the data, the wage is defined as the ratio of labor earnings to hours worked.


3.1.3 Preferences and the cost of day care

We assume that the utility of consumption takes the form

\[ u_s(c) = \eta_s \frac{c^{1-\sigma}}{1-\sigma}. \]

Meanwhile, the disutility of labor is given by

\[ v_s^{i,g}(h) = \psi_s^{i,g} \frac{h^{1+1/\varepsilon}}{1 + 1/\varepsilon} \]

where \( g \in \{m, f\} \).

The consumption equivalents, \( \eta \), are calculated using the OECD consumption equivalence scale. According to this scale, the first adult counts as one unit, the second adult as 0.7 and each child as 0.5.

The parameter \( \varepsilon \), representing the Frisch elasticity of labor supply, is set to 0.5; see Domeij and Flodén (2006) and Pistaferri (2003). The reciprocal of the intertemporal elasticity of substitution for consumption, \( \sigma \), is set to 2. The parameter \( \beta \) is set to 0.97\(^6\) and the interest rate \( r \) is set so that the subjective and market discount rates are equal; \( r = 1/\beta - 1 \).

The disutility of labor parameters \( \psi^{i,g} \) are set so as to match average labor supply by marital status (cohabiting or single) and age group (26-43 and 44-61) for those without small children.\(^{17}\) Parameter values are given in Table 1. The day care cost parameter \( d \) is set so as to match the labor supply of single mothers with small children and is discussed further below. The results of the calibration are shown in Table 2.

\(^{17}\) We exclude the hours worked by 18-25-year-olds from the calibration target since many people in this age group are students; this fact has a big impact on labor supply in this age range (see Figure 1) in a way that our model is not designed to capture.
## Table 1: Disutility of labor parameter values

<table>
<thead>
<tr>
<th></th>
<th>Cohabiting Females</th>
<th>Cohabiting Males</th>
<th>Single Females</th>
<th>Single Males</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age 20-43</td>
<td>142</td>
<td>80</td>
<td>260</td>
<td>90</td>
</tr>
<tr>
<td>Age 44-61</td>
<td>142</td>
<td>90</td>
<td>272</td>
<td>104</td>
</tr>
</tbody>
</table>

## Table 2: Hours per year

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ages</td>
<td>26-43</td>
<td>44-61</td>
</tr>
<tr>
<td>Cohabiting men with children</td>
<td>2189</td>
<td>2284*</td>
</tr>
<tr>
<td>Cohabiting men without children</td>
<td>2223</td>
<td>2219</td>
</tr>
<tr>
<td>Cohabiting women with children</td>
<td>596</td>
<td>511*</td>
</tr>
<tr>
<td>Cohabiting women without children</td>
<td>1529</td>
<td>1625</td>
</tr>
<tr>
<td>Single men with children</td>
<td>2159</td>
<td>1225*</td>
</tr>
<tr>
<td>Single men without children</td>
<td>1969</td>
<td>1974</td>
</tr>
<tr>
<td>Single women with children</td>
<td>657</td>
<td>655</td>
</tr>
<tr>
<td>Single women without children</td>
<td>1663</td>
<td>1765</td>
</tr>
</tbody>
</table>

*Not a calibration target.

Note: A child in this context is one below the age of 6.

Data source: GSOEP

Although we do not target the labor supplies of married men and women with small children, we nevertheless match them quite well. However, the model does not capture the behavior of single men with small children, who in the data actually work *more* than single men without small children at home. The model as specified can only replicate this fact if we assume that single men with children have a much smaller disutility of working than single men without children. This is not only a questionable assumption as
such, but would tend to push the results in the direction of larger welfare gains from day
care subsidies. In this context it is worth recalling that only a rather small fraction, 3.6
percent, of single men have small children at home. Figures 2 and 3 show that the model
also captures the complete life-cycle profile of hours worked, with the exception of single
men with children where we miss the level but capture the slope.

The numbers we target are average hours worked rather than participation and hours per
worker separately. Nevertheless, it is interesting to note that the model’s implications
for participation are broadly in line with the data with the exception of single men with
children. Our findings are summarized in Table 3. Since the model has full employment
among those without children, we focus on the employment rate of those with small
children relative to those without.

Table 3: Participation rates for those with
small children relative to those without

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ages</td>
<td>26-43</td>
<td>26-43</td>
</tr>
<tr>
<td>Cohabiting men</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>Cohabiting women</td>
<td>0.56</td>
<td>0.61</td>
</tr>
<tr>
<td>Single men</td>
<td>1.07</td>
<td>0.89</td>
</tr>
<tr>
<td>Single women</td>
<td>0.60</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Data source: GSOEP

The parameter $d$ determining the real cost of day care corresponds to €5.15 per hour (at
2004 prices); recall that this number is chosen so that single mothers with small children
work as much on average in the model as in the data. This cost corresponds to 43% of the
wage of an average young woman (age 20-25). It is also worth noting that this number is
very close to the rate recommended by the Bundesverband für Kindertagespflege (German
National Day Care Association), which is €5.50.\footnote{See http://www.tagesmutter.net/themen/infos-fuer-tagesmuetter/finanzen-teil-2.html.} More direct evidence on hourly day
care costs in Germany is hard to come by because of the small size of the day care sector in Germany.\textsuperscript{19} On the other hand, direct evidence on hourly costs of day care is available for the United States and Sweden. According to U.S. Census Bureau (2005), the average ratio of day care costs to earnings for full-time working mothers is about 20 percent in the United States. In Sweden, the cost of a day care spot varies quite a bit across municipalities. In the municipality of Täby, where day care costs are the lowest in the nation, the costs correspond to about 38 percent of the wage of a young woman (age 18-24). Meanwhile, the Swedish national average cost of day care is about 53 percent of young women’s wages.\textsuperscript{20} It is also worth noting that in the steady-state equilibrium, about 0.88 percent of GNP is spent on day care; the corresponding number in Germany is 0.59 percent.\textsuperscript{21}

3.2 Effects of day care reform

We now consider the effects of adopting the Ramsey optimal policy, i.e. making day care expenses tax deductible. We design the reform in such a way that those already born at the moment of reform are not affected; they pay their taxes according to the old system. We adopt this approach in order to avoid any issues of intergenerational redistribution. In particular, we want to avoid the result that the initial old and middle-aged lose from the reform simply because they pay for it but get nothing in return. Such a result would not be particularly interesting. Because of the grandfather clause of our reform, no agent experiences any transition. Nevertheless, we solve for the transition at the aggregate level; this is necessary in order to ensure intertemporal government budget balance. When evaluating welfare gains for a group of households, we use weights that are

\textsuperscript{19} Wrohlich (2005) documents the extent to which demand for subsidized day care falls short of the rather small supply of it.

\textsuperscript{20} Source: SCB (2007).

\textsuperscript{21} Source: OECD (2006).
inversely proportional to the marginal utility of consumption as discussed in Section 2.2.

The effects of day care expense deductibility on labor supply are summarized in Table 4. Hours worked by mothers with small children approximately double. Because the reform involves a 0.4 percentage point tax increase, labor supply is reduced slightly for some groups. Nevertheless, aggregate labor supply increases by 1.7 percent. Meanwhile, GNP increases by 1.1 percent and resources allocated to day care go up from 0.88 percent of GNP to 1.88 percent.

<table>
<thead>
<tr>
<th>Table 4: Hours per year</th>
<th>Pre-reform</th>
<th>Post-reform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ages</td>
<td>26-43 44-61</td>
<td>26-43 44-61</td>
</tr>
<tr>
<td>Cohabiting men with children</td>
<td>2284</td>
<td>2251</td>
</tr>
<tr>
<td>Cohabiting men without children</td>
<td>2225</td>
<td>2207 2197</td>
</tr>
<tr>
<td>Cohabiting women with children</td>
<td>511</td>
<td>1040</td>
</tr>
<tr>
<td>Cohabiting women without children</td>
<td>1533</td>
<td>1520 1611</td>
</tr>
<tr>
<td>Single men with children</td>
<td>1225</td>
<td>1749</td>
</tr>
<tr>
<td>Single women with children</td>
<td>655</td>
<td>1260</td>
</tr>
<tr>
<td>Single women without children</td>
<td>1667</td>
<td>1644 1734</td>
</tr>
</tbody>
</table>

Note: A child in this context is one below the age of 6.

The overall welfare welfare gains from introducing tax deductibility of day care expenses correspond to a 1.0 percent increase in consumption. For couples it corresponds to a 0.9 percent increase in consumption; for single men the number is 0.8 percent, and for single women it is 2.0 percent.

Underlying the aggregated welfare gains, there is considerable heterogeneity, the details of which are depicted in Figure 4. A 59 percent majority supports making day care expenses
tax deductible. Breaking down this figure by demographics, we find that 67 percent of cohabiting couples, 48 percent of single women and 26 percent of single men support moving from not subsidizing day care at all to making it fully tax deductible. Who are the opponents? Because the reform involves a tax increase, those who never have children have nothing to gain from the reform. In addition to the childless, there is a category of people who do not benefit from the reform because in those periods when they have children, their net marginal product of labor \( w - db \) is negative (for couples, what matters is the lowest net marginal product). This means that, in those periods, it does not pay to work even when day care expenses are tax deductible.

### 3.3 Sensitivity analysis

The results above are computed in an environment with linear taxes and zero transfer payments. Meanwhile, the current German tax system is very far from linear and there is a generous system of means-tested social assistance. In order to investigate the robustness of our results to this policy context, we modify the model environment to be broadly consistent with the existing framework of German fiscal policy. In this context, there are many policy changes one might want to consider. However, this paper is about day care finance reform and so the policy changes that we consider in this section all keep the rest of the tax and transfer system unchanged, subject only to government solvency. With this restriction on the set of reforms, tax deductibility of day care expenses in not necessarily optimal anymore. We still consider such a policy, where what we mean by deductibility is that both taxes and transfers are computed on the basis of earnings less day care expenses. In addition, we also consider subsidizing day care at various rates.

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22 15 percent of couples in the model never have children; the corresponding number for single men is 73 percent and for single women it is 48 percent.

23 The reforms are financed by a vertical shift of the marginal tax schedule for those income levels where taxes are paid. See Figure 5 for a depiction of the German marginal tax schedule.
We model the German labor income tax schedule following the description in OECD (2005). For example, the average and marginal tax rates for a single individual with one child are displayed in Figure 5. Consumption is taxed separately from labor income at the rate 15.8 percent, taken from Carey and Tchilinguirian (2000). We model transfers following the description of German social assistance policies in Adema and Kahl (2003). For example, a single woman with one small child receives €884 per month and a married couple with one child 7 years or younger and one child older than 7 receives €1263 per month. Transfers are generally clawed back at a rate of 85 percent as household income rises. See Appendix B for details.

Having introduced this tax and transfer system into the model, we calibrate the disutility of labor parameters $\psi^i_s$ in order to match hours worked for those without children and we set the day care cost parameter $d$ so as to match hours worked for single women with small children, as described in Section 3.1. Recall that in the context of the model with linear taxes and no transfers, such a calibration strategy implied that we also approximately matched the labor of supply of married women with small children, though that was not an explicit calibration target. However, in this context, the means-tested transfer system significantly weakens the incentives for single mothers to work. In order to match their labor supply, the cost of day care needs to be about €2.57 per hour, which is lower than in the model with linear taxes. At this lower cost, married women with small children, very few of whom enjoy any transfer payments on account of their husbands’ earnings, face stronger incentives to work. The model therefore needs to be modified in order to avoid implying counterfactually high hours worked for married mothers with small children. The modification that we adopt, following Güner et al. (2008), is to introduce a utility cost of participation for married couples. This cost is incurred if and only if both spouses work. We set this parameter so as to match average hours worked by married mothers with small children. With this extension, we are able to match our calibration targets

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24 The value added tax in Germany is 19 percent on most goods and services and 7 percent for food. Day care is not subject to value added tax.
very closely.

Our calibration is further validated by the following observations. Before the reform, 28 percent of single mothers in the model receive social assistance. The corresponding number in the data, according to Adema and Kahl (2003), was 26 percent in the year 2000. According to the same source, 1.9 percent of married couples with children received social assistance in 2000; in the model, that number is 1.5 percent.

In this context, making day care expenses tax deductible gives rise to an overall welfare gain corresponding to a 0.7 percent increase in consumption. The reform pays for itself by encouraging people to work rather than live on social assistance; rather than increasing taxes to pay for it, the reform involves a tax cut. (The marginal tax schedule shifts down by 0.7 percentage points.) The fraction of single mothers for whom social assistance is the chief source of income falls from 24 percent to 12 percent. For married couples with children, the corresponding numbers are 0.5 and 0.4 percent. Aggregate social assistance payments fall from 0.6 to 0.3 percent of GNP. Because of the tax reduction, there is unanimous support for this reform when compared to the initial situation where day care expenses are not deductible and not subsidized.25

Since in this context we don’t have a theoretical result establishing deductibility as the optimal policy, we now consider a set of alternative policies, specifically linear subsidies on day care at various rates. The benefits of such policies are depicted in Figure 6 which shows the welfare gains for each of the major groups (single men, single women and married couples) as a function of the subsidy rate from zero up to 100 percent. The welfare gain for society as a whole (using Pareto weights as described in Section 2.2) is maximized at a subsidy rate of about 85 percent. At this rate, the overall welfare gain is 0.8 percent, but because this policy involves a tax increase, only a bare majority of 50.1

25 Given that the presence of means-tested social assistance provides a further argument in favor of day care finance reform, it is perhaps surprising that the welfare gains are not even larger in this new context than they were in Section 3.2. The reason they are not larger is that the unit resource cost of day care \(d\) is lower here, as discussed on page 27.
percent prefer it to the initial situation.

What policy enjoys the most public support? Suppose we are in the initial situation and the options for reform are (1) retain the status quo, (2) let day care expenses be tax deductible and (3) subsidize day care at a rate of 85 percent. If voters choose the option they like best (and in this context there is no tactical reason why they wouldn’t), then about 60 percent would vote for deductibility and about 40 percent would support an 85 percent subsidy.

4 Concluding remarks

In this paper we have shown, within a heterogeneous-agent life-cycle framework, that the Ramsey optimal policy is to make day care expenses tax deductible. Calibrating our model to Germany, we found that implementing tax deductibility for day care would make German mothers work significantly more, leading to a sizeable welfare gain by moving the economy toward a more efficient trade-off between consumption and leisure.

In our analysis we have not considered the possible effects of day care on child welfare and development. It is an open question whether taking these effects into account would weaken or strengthen the case for deductibility of day care. There is some evidence that day care has a positive effect on child development and parental welfare; see OECD (2006). If we trust this evidence, then our assessment of the benefits of day care subsidies are conservative. On the other hand, Baker et al. (2005) find some evidence of negative effects from the province of Québec. More recently, Gruber et al. (2010) have found statistically insignificant effects of day care on people born in Sweden 1974-88, a time and place where subsidized day care expanded rapidly. Thus the issue remains unsettled. Either way, there is a strong efficiency case to be made for day care subsidies that must be weighed against any possible negative effects.
Appendix A  Data

Our main source of data is the German Socio-Economic Panel (SOEP) for the years 1984-2004. More information about SOEP can be found at http://www.diw.de/english/soep/29012.html.

A.1 Measurement of marital status

Conceptually, we are not interested in whether anyone is legally married, only whether they are living with one other person in a relationship that is economically similar to marriage. The approach we adopt is an imperfect attempt to capture that notion. What we do is to group people who belong to the same household and try to pick out among the adult members of a given household a pair of individuals who appear to be in a marriage-like relationship. If the household has just one adult member, the situation is clear: we then consider the sole household member to be single. On the other hand, if the household has more than one adult member, then we order the household members by age and consider the two eldest. Occasionally it happens that there are more than two eldest members; we then randomly choose two of them. If these two individuals turn out to be of the opposite sex and if the age gap is strictly less than 20 years, we consider them to be married. The exclusion of same-sex couples is there because we are interested in using marital status information to draw inferences about the probability of the arrival of children, and the exclusion of couples 20 years apart or more in age is there to exclude single parents living with their children from being assigned as married.

Evidently our approach excludes some couples that, conceptually speaking, are married in the economic sense of that term. Nevertheless, we take the view that our approach yields an acceptable approximation.
A.2 Probabilities of having children

As described in the main text, the distribution of children among households in the model is based on simulating large populations of single men, single women and couples on the basis of probabilities of newborn children arrivals that are taken from the data. Based on these populations, we can then determine the population weights of each of the 34 child profiles.

The probabilities of having (acquiring\textsuperscript{26}) 0, 1, 2 and 3 children (more than that is not allowed in any given six-year period) for single men, single women and couples are set so as to match the number of young (less than six years old) children that these categories of household have (possess) as a function of the age of the parents and the number of children already present in the household, i.e. those children that are between the ages of six and seventeen. Since the fraction of young parents who have more than two children is tiny, we force this fraction to be zero.

In the GSOEP files Xkind (where X is a letter representing the year), there is an entry corresponding to each child in the sample with information on birthyear and a number identifying the household. We then merge this data with the information on marital status, age, and, if unmarried, the gender of each apparent parent (any adult in the same household as the child), and remove those adult-children pairs that are such that the child is less than 18 years younger than the parent. We then consider those children that are between 0 and 5 years old; these are considered “newborn” for the purpose of the calibration. The probabilities of having 0, 1, 2 and 3 new children for potential parents categorized by marital status, age, gender and number of children aged 6-17 are then simply given by the corresponding fractions in the data, e.g. the fraction of 26-30-year-old single women with two children aged 6-17 who have exactly one child between 0 and 5.

\textsuperscript{26} It is of course not idiomatic in English to speak of a household “acquiring” a child, but in this context it is crucial to distinguish between possession and acquisition, and here we are talking about acquisition and not possession.
A.3 Life-cycle hours profiles

The data on hours are based on the GSOEP variable “average hours worked per week”, called, for example, BP41 in 1985 and NP47 in 1997. What we would like to do is to match the life-cycle profile of hours worked for the first cohort to be affected in a major way by day care reform, i.e. those who are young today. For obvious reasons, there is no data on the entire life-cycle profile of hours for this cohort. Therefore, we use the entire GSOEP panel from 1984 to 2004 and regress hours on age and cohort dummies representing the decade of birth. The data presented in Figure 1 are the predicted values for the cohort born in the 1960s.

A.4 Wages

Wages are defined as individual annual labor earnings divided by annual hours worked. The names of these GSOEP variables are e11101XX and i11110XX, where the XX stands for the year. These wages are used to run the regressions described in Section 3.1.2.

Appendix B Tax and transfer system

B.1 Tax system

The German tax system is modelled following the description in OECD (2005). Spouses are assessed jointly using the income splitting method. We define taxable income, \( x \), as earnings less a basic allowance which consists of three parts. First, there is a allowance of \( \€1308 \) for single parents. Second, there is a work-related allowance of \( \€920 \) per employed person. Third, there is lump-sum allowance of \( \€36 \) for singles and \( \€72 \) for couples. The tax liability, \( T \), is then calculated as follows. Let \( y = (x - 7664)/1000 \) and \( z = \)
(x – 12739)/10000. The tax liability is then

\[
T = \begin{cases} 
0 & \text{if } x \leq 7664, \\
(793.10y + 1600) y & \text{if } 7664 < x \leq 12739, \\
(265.78z + 2045) z + 1016 & \text{if } 12739 < x \leq 52152, \\
(0.48x - 10410) & \text{if } 52152 < x.
\end{cases}
\]

These formulae are used directly to calculate the tax liability for a single individual. For couples, we apply these formulae on half the taxable income and then double the resulting amount to arrive at the tax liability. A “solidarity surcharge” (Solidaritätszuschlag) is then levied at 5.5 percent of the tax liability subject to an exemption limit of €972 for singles and €1944 for couples. Total tax payments are equal to the tax liability plus the solidarity surcharge. Note finally that we do not model social security contributions or benefits. The reason is that a large part of benefits are tied to contributions in a more or less actuarially fair way which means that the system of social insurance contributions and benefits is not distortive in the way that income taxes and public purchases are.

### B.2 Social assistance

German social assistance policies are modelled following the description in Adema and Kahl (2003). First, there is a universal child benefit (Kindergeld) of €154 per child and month.\(^{27}\) Second, there is a gross income tested child rearing benefit of €307 per month for children below 2 years of age. Third, households with little earnings are entitled to several additional benefits. For example, a single parent receives a standard monthly payment of €286 plus €395 for housing, €67 for heating, €121 for larger purchases such as clothing and furniture, and child payments of €169 per child not yet 7 years of age and €232 for per child 7 years and older. The sum of these social assistance payments

\(^{27}\) Legally, the child benefit is treated as a tax credit, but in cases where the tax liability is less than the tax credit, the difference is paid out as a cash transfer. The child benefit is equivalent to a direct cash transfer, and we have therefore chosen to model it as such.
are reduced by 85% of earnings net of taxes. Table B1 summarizes the German social assistance benefits.

Table B1: German social assistance policies per month in euros

<table>
<thead>
<tr>
<th>Child benefit per child</th>
<th>Single without children</th>
<th>Couple with children</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>154</td>
<td>154</td>
</tr>
</tbody>
</table>

Net-earnings based benefits<sup>a</sup>

<table>
<thead>
<tr>
<th></th>
<th>Single</th>
<th>Couple</th>
<th>Single</th>
<th>Couple</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard payment</td>
<td>286</td>
<td>515</td>
<td>286</td>
<td>515</td>
</tr>
<tr>
<td>Average housing</td>
<td>357</td>
<td>335</td>
<td>395</td>
<td>444</td>
</tr>
<tr>
<td>Average heating</td>
<td>44</td>
<td>60</td>
<td>67</td>
<td>67</td>
</tr>
<tr>
<td>Large purchases</td>
<td>46</td>
<td>85</td>
<td>121</td>
<td>159</td>
</tr>
<tr>
<td>Children not yet 7 years of age (per child)</td>
<td>15</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Children 7 years and older (per child)</td>
<td>78</td>
<td>78</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Child rearing per child not yet 2 years of age<sup>a</sup> 307 307

<sup>a</sup>These benefits are clawed back at the rate 85 percent as after-tax earnings rise.
References


Figure 1: Average hours worked over the life cycle in Germany. Solid lines refer to people without small children, dashed lines to people with small children. For details, see Appendix A.
Figure 2: Average hours worked over the life cycle in the data and in the model for cohabiting men and women. Solid lines refer to the data, dashed lines to model predictions.
Figure 3: Average hours worked over the life cycle in the data and in the model for single men and women. Solid lines refer to the data, dashed lines to model predictions.
Figure 4: The distribution of welfare gains.
Figure 5: Average and marginal tax rates for a single individual with one child.
Figure 6: Welfare gains associated with day care subsidies between 0 and 100 percent in the economy with progressive taxes and means-tested social assistance.