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Elites or masses? A structural model of policy divergence, voter sorting and apparent polarization in U.S. presidential elections, 1972-2008

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Abstract

One of the most widely discussed phenomena in American politics today is the perceived increasing partisan divide that splits the U.S. electorate. A central contested question is whether this diagnosis is actually true, and if so, what is the underlying cause. We develop a model that relates the parties’ positions on economic and “cultural” issues, the voters’ ideal positions and the electorate’s voting behavior, and apply the model to U.S. presidential elections between 1972 and 2008. The model allows us to recover candidates’ positions from voter behavior; to decompose changes in the overall political polarization of the electorate into changes in the distribution of voter ideal positions and consequences of elite polarization; and to determine the characteristics of voters who changed their party allegiance.

JEL-Code: D720.

Keywords: polarization, differentiated candidates, policy divergence, ideology, voter migration.

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1 Introduction

Polarization in Congress has increased substantially over the last 30 years, from a historic low achieved between roughly 1940 and 1980 (Poole and Rosenthal 1984, 1985, 2000; Groseclose, Levitt, and Snyder 1999; Theriault 2008). Today, Congressional polarization, as measured by the difference between the median Republican and Democratic first-dimension Nominant-score is higher than ever in the last 100 years, and this is true of House and Senate alike. Elite polarization also appears to be prevalent among party activists (Abramowitz and Saunders 2008, Harbridge and Malhotra 2011).

In contrast, conclusions about mass polarization vary substantially in the literature. On the one hand, many political commentators diagnose a sharp and increasing partisan divide that splits the U.S. electorate. For example, the Economist writes that “the 50-50 nation appears to be made up of two big, separate voting blocks, with only a small number of swing voters in the middle”, and that “America is more bitterly divided than it has been for a generation”.\(^1\) Hetherington (2001) demonstrates that voters behave in an increasingly partisan way (e.g. perceive important policy differences between the parties, and are less inclined to vote split-ticket), and links this change in voter behavior to elite polarization. Similarly, Abramowitz and Saunders (1998, 2008) provide evidence that Democratic and Republican party members have become more liberal among Democrats and more conservative among Republicans.

On the other hand, not everybody shares the diagnosis of electoral polarization. In fact, research that analyzes voter preferences on different policy issues directly rather than voter behavior finds strong evidence that the preferences of the American electorate on a number of policy issues are similar to what they were a generation ago (Fiorina, Abrams, and Pope 2006; Bartels 2006; Fiorina and Abrams 2008; Levendusky 2009). From this perspective, there does not appear to be a polarization in the sense that voters have moved from moderate positions to more extreme ones (DiMaggio, Evans, and Bryson 1996; Baker 2005).

The seeming tension between the observation of more partisan voter behavior on the one hand and no fundamental change in voter preferences on the other is puzzling: If voters’ fundamental preferences on issues did not change, why do they now act in more partisan ways? What is necessary to answer this fundamental question is a theoretical framework that provides for an explicit mechanism linking elite actions and mass voting behavior. In this paper, we develop such a model. It distinguishes precisely among the concepts of elite polarization, voter preference polarization, and voter behavior, and also allows us to estimate quan-

---

\(^{1}\)“On His High Horse,” November 9, 2002 issue and “America’s Angry Election,” January 3, 2004 issue, respectively.
titative measures of their development over the course of several elections. This model allows us to answer
the following important questions: First, have the masses in fact become more polarized, or is what has been
perceived and identified as polarization really just a reflection of changes in elite behavior? Second, to what
extent have elites and masses contributed, if at all, to changes in polarization? Third, is polarization driven
primarily by economic or by cultural issues, and which types of voters are most affected by it?

In analyzing the third question, our model also contributes to an ongoing debate in the literature about
what type of issues – economic or cultural – drive vote choice today, and how their relative effects might
have changed over time. A common impression is that moral issues have become more important in re-
cent years. For example, in the popular bestseller “What’s the matter with Kansas?”, Frank (2005) argues
that poor people often vote for Republicans because of cultural issues such as abortion or gay marriage,
while their economic interests would be more closely aligned with the Democratic party. Hunter (1992),
Shogan (2002) and Greenberg (2005) present similar “culture-war” arguments. Others challenge this thesis,
however, and emphasize the importance of economic issues in explaining voter preferences for candidates
(e.g., Bartels 2006; McCarty, Poole, and Rosenthal 2006; Gelman, Shor, Bafumi, and Park 2008; Bartels
2010). Ansolabehere, Rodden, and Snyder (2006) provide some mixed evidence, and show a substantially
increased importance of moral issues for vote choices in the 1990s relative to the 1970s and 80s, but also
find that economic factors are still more important for voters than purely moral ones.

To gain an intuitive understanding of the effects captured by the model, consider a society in which the
parties’ policy platforms are virtually indistinguishable. In this case, whether Democrats or Republicans
win hardly makes a difference for the implemented policy, so that voters may not base their vote choices on
their ideological preferences, but rather on their personal and idiosyncratic perceptions of the candidates.
When empirical researchers analyze the ideological determinants of voting behavior in this society, it looks
as if voters do not care about issues. However, if party elites become more polarized over time, creating
a more meaningful choice, then voters will expose previously buried ideological divisions among them,
even if their preferences remain constant: In short, elite polarization can beget voter behavior that appears
more polarized, but in reality is not. Moreover, whether voters appear to be more strongly polarized on
economic issues or on cultural ones depends crucially on whether the distance between the parties is larger
on economic or cultural issues.

That voters’ issue preferences more strongly affect their vote choices, the more distant party positions
are from each other, assumes only rational behavior by voters and not changes in their underlying policy
preferences. In other words, we do not assume that elite polarization on an issue “makes people think more about that issue” and that they consequently develop more radical preferences on the issues. Rather, voters always have preferences on the issues, and they are always aware of them – however, rational voters will not to condition their vote choices on their issue preferences, if both candidates take the same policy stances.

This core intuition behind our structural model is present as a qualitative idea (and sometimes even as reduced form model) in earlier work. Fiorina et al. (2006) point out that, in a multidimensional setting, the direction of elite polarization has effects for the direction of the fault line through the electorate, and Hetherington (2001) shows that the intensity of elite polarization influences the intensity of voter polarization (see also Coleman (1996), Wattenberg (1998) and Bartels (2000) for earlier manifestations of similar arguments).\(^2\)

In particular, Fiorina et al. (2006, p. 183) point out that the effect of elite polarization on voter behavior constitutes a severe challenge for interpreting empirical studies that analyze the determinants of voter behavior: “The findings of scores if not hundreds of electoral studies are ambiguous. The problem most deeply afflicts attempts to study electoral change by conducting successive cross-sectional analyses and comparing the results.” For example, consider a hypothetical study that analyzes how voters’ preferred positions on abortion affect their voting behavior, and compares voters in the 1970s to voters today. Suppose that the researcher finds a significant increase in the probit regression coefficient of the abortion variable, which means that a voter’s position on abortion today provides a more informative signal about his vote choice than in the 1970s. It is tempting to interpret such a result as “abortion has become a more important issue for voters.” However, the result does not necessarily mean that voters in the 1970s had less intense preferences on the issue of abortion; it might rather be a consequence of stronger distinctions between Democratic and Republican elites on this issue.

Our model provides a formal framework for analyzing the connection between elite actions (i.e., parties’ policy proposals), the distribution of voters’ policy preferences, and their voting behavior. The example above shows that elite actions will influence how voters’ ideal positions on policy issues translate into vote choices. Conversely, observing voters’ behavior provides information about the underlying actions of elites. Using NES data from the U.S. presidential elections between 1972 and 2008, we show how we can use observations of voter preferences on different policy issues and voters’ choices of which candidate to vote

\(^2\)Specifically, Hetherington (2001) shows that the difference between Republican and Democratic DW-Nominate scores in the House as a measure of elite polarization is a highly significant variable in regressions that use several different measures of mass polarization as dependent variables.
for, to draw inferences about the differences between Democratic and Republican presidential candidates’ positions on different issues during this time period.

The model also provides us with a better understanding of the underlying causes of electoral polarization: Does the electorate look more politically polarized today than a generation ago, and if so, is elite polarization, a change in the voters’ preferences, or both responsible for this? To analyze these questions, we first define a measure of the electorate’s polarization on political issues. It quantifies the degree to which voters’ candidate choices depend on their preferred issue positions. Our estimation procedure provides a distribution of voters’ ideal points and the positions of candidates, in different elections. We can therefore synthetically separate and quantitatively estimate the importance of the two potential reasons for changes in the overall polarization measure. In a first thought-experiment, we fix the candidates at their positions in a previous election, and look at only those changes that arise from changes in the distribution of voter ideal points alone. We call this effect “voter polarization.” Second, we fix the electorate of an earlier election year and see how this constant set of voters reacts to the observed change in the parties’ positions. We call this effect “sorting.”

We further separate the two potential causes of sorting, policy divergence on cultural and on economic issues, and use this to infer the marginal importance of cultural and economic preferences for voters’ choices. For example, if most of the policy divergence between parties is on cultural issues, then the impact of voters’ preferences on cultural issues for their vote choices will increase, relative to the impact of economic issues. Again, however, this change reflects elite actions rather than represents genuine changes in voters’ preferences and in how they balance different issues.

We also show that a turn in the direction of political conflict (e.g., an increased importance of cultural issues relative to economic ones) changes the sets of voters who are likely to support Democrats and Republicans, respectively. Specifically, our results shows how socially-conservative, but economically-liberal voters have migrated from mostly supporting the Democrats in the 1970s to mostly supporting the Republicans today, and that there is a group of socially-liberal but economically-conservative voters who migrate in the opposite direction. Analyzing the ideological and social characteristics of these party switchers contributes to the discussion among political practitioners, journalists and political scientists about the impacts of cultural and economic preferences on partisanship, and about the secular shift of white working class voters (“Reagan Democrats”) to the Republican party.

In the next section, we provide an intuitive description of our model, before turning to the formal model and estimation.
2 An intuitive description of our model and procedure

In our model, we assume that voters’ policy preferences are characterized by two parameters $\delta$ and $\theta$ that measure their preferred positions on cultural and economic issues. Apart from a policy payoff determined by the candidates’ economic and cultural policy positions relative to a voter’s own position, each voter also receives a non-policy payoff from each candidate, which captures, like in a probabilistic voting model, both a systematic component (e.g., the candidate’s competence) and an idiosyncratic component (e.g., how likable a voter finds each candidate).

In Figure 1, points D and R indicate the two party platforms (i.e., the voter types for whom the parties’ positions correspond to their ideal policy). Higher values of $\delta$ indicate a more “socially conservative” cultural position, and higher values of $\theta$ indicate more “economically liberal” preferences ($\delta$ and $\theta$ are parameters in the voters’ utility function (1) on page 8). Culturally conservative voters who also prefer a low level of spending support the Republican position on both issues, while culturally liberal voters who also prefer a high level of spending support the Democratic position on both issues. These core supporters are unlikely to have an idiosyncratic candidate-specific preference shock that outweighs their policy preferences. In contrast, economically-conservative, but socially-liberal voters and economically-liberal, but socially-conservative voters are less firm in their support, and the boundary line between the set of (likely) Democratic and Republican supporters goes through these regions of the type space.

![Figure 1: Platform differences and voter intensities](image)

Due to the voters’ idiosyncratic preferences, the separating line in the economic-cultural space does not perfectly separate actual Democratic from Republican voters, but separates only those who are more likely to vote Democratic from those who are more likely to vote Republican. The intensity of a voter’s policy
preferences determines how likely he is to “cross over” to the policy-wise less preferred candidate. The locus of the separating line is the same in both panels of Figure 1. However, in the left panel, candidate positions are quite close, and the importance of idiosyncratic non-policy preferences is high: The transition in terms of the implied probability of voting Republican is rather gradual, with voters above the dividing line slightly more likely to vote Democrat, and those below the line slightly more likely to vote Republican.

In contrast, policy differences between candidates in the right panel are large. While voters who are located exactly on the dividing line are still equally likely to prefer the Democrat and the Republican, those voters who are located slightly off that line are now much more likely to prefer the candidate on their side of the dividing line to his opponent, as policy differences have become more important relative to idiosyncratic non-policy preferences.

For each voter whose ideal positions we know, we can calculate a probability that this voter will vote Republican in an election. This probability can be used to make a prediction (i.e., if this probability is greater/smaller than 1/2, we predict that the voter votes Republican/Democrat). Of course, the prediction will sometimes be wrong, and we can form a measure of how useful knowledge of voters’ preferred positions is, on average, for making such predictions based on how often our prediction is correct. We call this measure predictiveness, and compare changes in it over time.

The argument above discussing Figure 1 suggests one possible reason for an intertemporal increase in predictiveness: Elite polarization (i.e., the movement of candidate platforms) generates the appearance of a more “politically divided” electorate, in the sense that ideal points of voters who are not exactly on the dividing line become better predictors of voting behavior. Alternatively, predictiveness can increase because voters (on average) move away from the dividing line, and this means that an external observer who learns the voters’ ideal issue positions can, on average, make better predictions about voting behavior, even if the candidates’ positions are unchanged. We call the first effect “sorting” and the second one (electoral) “polarization.” Of course, between two actual elections, both effects arise simultaneously, but our model can synthetically separate these effects, so that we learn which one is quantitatively more important for the observed increase in political divisions.

What does voter behavior tell us about candidate platforms? First, from the discussion above, it is clear that the extent to which the two voter blocs can be neatly divided by a line enables us to infer how far apart the two candidates’ positions are. Second, the data also reveal information about the relative difference between party platforms in the economic and cultural dimension. To see this, consider Figure 2.
left panel, the platforms of the two candidates differ primarily along the economic dimension, while their cultural positions are fairly close. As a result, the separation line is fairly flat: Most Republicans have low values of $\theta$, while most Democrats have high values of $\theta$. In the intermediate range, cultural preferences do play a role, but the “marginal rate of substitution between economic and cultural issues” is low: Suppose we start with a voter who is just indifferent between the Democratic and Republican position. If this voter becomes more socially conservative (i.e., if $\delta$ increases by one unit), how much does $\theta$ have to increase in order to keep this voter on the separating line, i.e. just stochastically indifferent between candidates? We call this marginal rate of substitution the importance of cultural relative to economic issues.

![Voter separation lines](image)

Figure 2: Voter separation lines

Changing from the left to the right panel, the importance of cultural relative to economic issues increases, i.e., the dividing line becomes steeper. The reason for the pivot of the separating line is the change of the Democratic and Republican positions – cultural differences between the candidates have become more pronounced relative to the left panel, and economic ones less so. Consequently, the polity has become more divided along cultural lines. Note that the slope of the dividing line depends solely on the difference between the candidates’ economic and social positions. An increased importance of cultural relative to economic issues is not driven by voters “becoming more concerned” with cultural issues and/or less concerned with economic issues. If the distribution of voter preferences changes, e.g. probability mass shifting from the middle of the distribution to more extreme positions, this is a completely separate effect that leaves the dividing line unchanged (though it affects how many voters “cross-over” to the other party).

Finally, we can analyze the types of voters who switched their party allegiance as a result of changing party platforms. Think of a graph that superimposes the two panels of Figure 2. A pivot of the dividing line has the effect that socially-liberal and economically-conservative voters move to the Democratic party, while
socially-conservative and economically-liberal voters move to the Republican party. From our estimation, we can identify those ideological voter types who are most likely to switch their allegiance from Democrats to Republicans and vice versa, and we can then look in the data for the demographic characteristics of these voters. For example, do those socially-conservative and economically-liberal voter types who we identify as most likely to have shifted from Democrats to Republicans indeed look like the “Reagan Democrats” or the “angry white religious fundamentalists” that have been described in the popular literature?

3 Model

Two candidates, labeled $D$ and $R$, are endowed with a cultural-ideological position $\delta_P \in [0, 1]$, $P \in \{D, R\}$, an economic position $g_P$ that denotes the quantity of a public good that the candidate provides if elected, and an associated cost of public good provision $c_P$. Each voter is characterized by his cultural ideology $\delta \in [0, 1]$; a parameter $\theta \in [0, 1]$ that determines his preferences for public goods, and a parameter $\xi_P \in \mathbb{R}$ that measures the impact of the personal charisma of the candidate $P = D, R$ on the voter. Specifically, a voter’s utility from candidate $P$ is given by

$$u(\delta, \theta, \xi_P) = \theta v(g_P) - c_P - (\delta - \delta_P)^2 + \xi_P.$$  (1)

Note that $v(\cdot)$ is an increasing and strictly concave function that is the same for all voters. Since a voter’s gross utility from public goods is $\theta \cdot v(g)$, high $\theta$-types receive a higher payoff from public goods and thus, their preferred public good provision level, accounting for the cost of provision, is higher than for low $\theta$-types. We assume that there is a continuous distribution of $(\delta, \theta, \xi_D, \xi_R)$ in the electorate, that $\theta \in [0, 1]$, and that $\xi \equiv \xi_R - \xi_D$ is independent of $\theta$ and $\delta$. A voter is indifferent between the two candidates if and only

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3 Note that the two-dimensional nature of policy in our model is essential for this part of the analysis because it allows for movement (by different groups) in both directions.

4 For simplicity and in order to focus on the effects of cultural and economic policy divergence on voter behavior, we use a model in which policy choices are exogenous. However, one could, for example, modify a differentiated candidate framework to obtain policy divergence as an equilibrium result (Krasa and Polborn 2009, 2010a, 2010b). In particular, Krasa and Polborn (2011) show this for voter utility functions very similar to the ones used here. Alternatively, Schnidman and Schofield (2011) present an alternative model of non-convergence of party positions in a two-dimensional policy space. Their main driving force is the presence of policy-motivated party activists who support the candidates conditional on their policy choices.

5 We could generalize the utility function to $u(P, g) = \theta v(g) - c_P - s(\delta - \delta_P)^2 + \xi_P$, where $s > 0$. The case $s = 1$ corresponds to (1), and higher $s$ means that voters put more emphasis on cultural issues. By setting $\chi = \sqrt{s(\delta - \delta_P)^2}$, for arbitrary $\delta$ we can write the new utility function as $u(P, g) = \theta v(g) - c_P - (\chi - \chi_P)^2 + \xi_P$, which is exactly the same form (1) (just with $\chi$ replacing $\delta$). Thus, our assumption that the parameter multiplying the ideological loss $(\delta - \delta_P)^2$ is one is without loss of generality.

6 This is just a normalization because $v(\cdot)$ can take arbitrary values.

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if \( \theta v(g_D) - c_D - (\delta - \delta_D)^2 + \xi_D = \theta v(g_R) - c_R - (\delta - \delta_R)^2 + \xi_R \), which implies

\[
-2\delta(\delta_R - \delta_D) + (v(g_D) - v(g_R))\theta = c_D - c_R - (\delta_R^2 - \delta_D^2) + \xi.
\]

(2)

We assume that the Democrat provides (weakly) more of the public good (i.e., \( g_D \geq g_R \)) for a higher tax cost (i.e., \( c_D \geq c_R \)), and that the Republican is (weakly) to the right of the Democrat on cultural issues (i.e., \( \delta_R \geq \delta_D \)).

For any given value of \( \xi \), if \( g_D = g_R \), the line of indifferent or cutoff voters in a \((\delta, \theta)\)-space is vertical. Intuitively, if Democrat and Republican provide the same amount of public goods, then only the voters’ ideological preferences (\( \delta \)) matter for their voting choice, while the voters’ economic preference (\( \theta \)) is immaterial. If, instead, \( g_D > g_R \), the cutoff value for \( \theta \) is given by

\[
\theta(\delta, \xi, g_D, g_R) = \frac{2\delta(\delta_R - \delta_D) + c_D - c_R - (\delta_R^2 - \delta_D^2) + \xi}{v(g_D) - v(g_R)}.
\]

(3)

Equation (3) is a straight line in the \( \delta - \theta \) space, and has a positive slope. Intuitively, if the Democrat provides more public goods than the Republican, then a voter is indifferent between the candidates either if he is socially relatively liberal, but wants lower spending on public goods (i.e., low \( \delta \) and low \( \theta \)), or if he is socially conservative, but likes substantial government spending on public goods (i.e., high \( \delta \) and high \( \theta \)). Higher types of \( \theta \) are more likely to vote for the Democrat, and for any given economic preference type \( \theta \), higher \( \delta \)-types are more likely to vote for the Republican.

4 Estimating the Model

4.1 Overview

Our objective is to determine how a respondent’s answers to the survey questions translates into a position in the \( \delta - \theta \)-space, and a probability of voting Republican. Our model shows that the position of the separating line is determined by the candidates’ positions and may therefore change from one election to the next. Thus, we estimate voters’ preference parameters \( \delta \) and \( \theta \) simultaneously with \( \xi \) and the position of the separating

\[\text{7} \text{From a theoretical point of view, these are mere normalizations: We can simply call the candidate who provides more public good the “Democrat,” and measure \( \delta \) in a way that the Democrat’s position is weakly to the left of the Republican’s. As we will see below, these normalizations make sense empirically in the U.S. context.}\]
line. Specifically, we proceed as follows: Equation (3) implies that the slope, \( k \), and the intercept, \( a \), of the
separating line are given by
\[
k = \frac{2(\delta_R - \delta_D)}{v(g_D) - v(g_R)}, \quad a = \frac{c_D(g_D) - c_R(g_R) - (\delta_R^2 - \delta_D^2) + \bar{\xi}}{v(g_D) - v(g_R)},
\]
where \( \bar{\xi} = E[\xi] \). Define
\[
\varepsilon = \frac{\xi - \bar{\xi}}{v(g_D) - v(g_R)}.
\]
We assume that \( \varepsilon \) is normally distributed with standard deviation \( \sigma \) (given the normalization in (5), the mean
of \( \varepsilon \) is 0). Equations (3), (4) and (5) imply that a citizen votes Republican if and only if
\[
\theta - k\delta - a - \varepsilon < 0.
\]
Let \( X_i, i = 1, \ldots, n \) and \( Y_i, i = 1, \ldots, m \) be random variables that describe the answers to survey questions on
cultural and economic issues, respectively. From these data, we construct an index of cultural and economic
preferences. Specifically, we assume that \( \delta = \sum_{i=1}^{n} \lambda_i X_i \) and \( \theta = \sum_{i=1}^{m} \mu_i Y_i \), where, of course, the \( \lambda_i \) and \( \mu_i \)
are parameters to be estimated.

We normalize \( X_i \) and \( Y_i \) such that (i) the lowest and highest realizations for each question are 0 and 1;
(ii) high values on \( X_i \) and \( Y_i \) increase the estimated value of \( \delta \) and \( \theta \), respectively (i.e., we code answers such
that all \( \lambda_i \) and \( \mu_i \) are non-negative).\(^8\) Finally, we normalize \( \sum_{i=1}^{n} \lambda_i = 1 \) and \( \sum_{i=1}^{m} \mu_i = 1 \) so that \( \theta, \delta \in [0, 1] \),
to keep the distribution of \( \theta \) and \( \delta \) comparable over time. This normalization is without loss of generality
because multiplying all variables in (6) by a positive constant will not change whether (6) is satisfied.\(^9\)

Let \( \Phi(\cdot) \) denote the cdf of a normal distribution with mean 0 and standard deviation 1. Then (6) implies
that the probability that a voter votes Republican is given by
\[
\Phi \left( \frac{1}{\sigma} \left[ k \sum_{i=1}^{n} \lambda_i X_i - \sum_{i=1}^{m} \mu_i Y_i + a \right] \right).
\]
In principle, this objective function is similar to a probit model. However, the challenge is that, unlike in a
standard probit model, the argument of \( \Phi \) is not a linear function of the model parameters to be estimated.

\(^8\)We can do (ii) without loss of generality by redefining a new variable \( \bar{X}_i = 1 - X_i \) (or \( \bar{Y}_i = 1 - Y_i \)) if the corresponding coefficient \( \lambda_i \) (or \( \mu_i \)) in a regression using the original answers \( X_i \) or \( Y_i \) is negative.

\(^9\)In the estimation, multiplying all variables in (6) by the same constant leaves the parameter estimate for \( k \) unchanged and multiplies the estimate of the standard deviation of \( \varepsilon \) accordingly.
We now describe how the model can be used to identify changes in the distance between the candidates’ platforms. Taking the standard deviation on both sides of (5) we get

\[ \sigma_j = \frac{\sigma_{\xi}}{\nu(g_{D,j}) - \nu(g_{R,j})} \]  

(8)

where \( \sigma_{\xi} \) is the standard deviation of \( \xi \).\(^{10}\) Using (4) implies

\[ \delta_{D,j} - \delta_{R,j} = \frac{\sigma_{\xi} k_j}{2 \sigma_j}, \text{ and } \nu(g_{D,j}) - \nu(g_{R,j}) = \frac{\sigma_{\xi}}{\sigma_j} \]  

(9)

We can use (16) and (17) to estimate the values \( \sigma_j \) and \( k_j \) from the data of the election in year \( j \) (as described in more detail in Theorem 1 below). This allows us to identify both the cultural and economic difference in the candidates’ platforms, if we normalize the policy difference \( \nu(g_D) - \nu(g_R) \) in a base year.

In Section 4.2, which the more substantively interested reader can skip, we provide more technical details on the estimation procedure.

\subsection*{4.2 Estimation Procedure}

In order to get the best estimate of voters’ values of \( \delta \) and \( \theta \), we estimate \( \lambda \) and \( \mu \) using pooled data from several elections. Because candidate platforms change from one election to the next, this means that we must allow that \( k \) and \( \sigma \) change over time and thus index them by the year of the election. Let \( D_t, t = 1, \ldots, s \) be the year dummy for year \( t = 1, \ldots s \) (i.e., \( D_t = 1 \) if the observation occurred in year \( t \), and 0 otherwise). Then (7) generalizes to

\[ \Phi \left( \sum_{t=1}^{s} \frac{D_t}{\sigma_t} \left[ \sum_{i=1}^{n} \lambda_i X_i \right] - \sum_{i=1}^{m} \mu_i Y_i + \sum_{t=1}^{s} D_t a_t \right) \].

(10)

In order to determine \( k_t, a_t, \sigma_t, t = 1, \ldots, s, \lambda_i, i = 1, \ldots, n, \) and \( \mu_i, i = 1, \ldots, m, \) we first estimate the model in which the probability of voting Republican is given by

\[ \Phi \left( \left[ 1 + \sum_{t=2}^{s} \alpha_t D_t \right] \left[ \sum_{i=1}^{n} \lambda_i \tilde{X}_i \right] - \left[ 1 + \sum_{t=2}^{s} \rho_t D_t \right] \left[ \sum_{i=1}^{m} \tilde{\mu}_i \tilde{Y}_i \right] + \sum_{t=1}^{s} \tilde{a}_t D_t \right), \]  

(11)

\(^{10}\)Here, \( \sigma_{\xi} \) is assumed constant over time. We discuss this assumption in Section 7.3.
where there are no restrictions on the $\tilde{\lambda}_i$, and $\tilde{\mu}_i$, i.e., they could be negative or greater than 1. $\tilde{X}_i$ and $\tilde{Y}_i$ are the responses to the survey questions, solely normalized to be between 0 and 1, but absent the additional requirement that higher realizations of the response to each question increase $\delta$ and $\theta$.

Denote by $d_{t,j}$, $\tilde{x}_{i,j}$, and $\tilde{y}_{i,j}$ observation $j$ of random variables $D_t$, $\tilde{X}_i$, and $\tilde{Y}_i$, respectively. Let

$$z_j = \left[1 + \sum_{t=2}^{s} \alpha_t d_{t,j}\right]\left[\sum_{j=1}^{n} \tilde{\lambda}_i \tilde{x}_{i,j}\right] - \left[1 + \sum_{t=2}^{s} \rho_t d_{t,j}\right]\left[\sum_{j=1}^{m} \tilde{\mu}_i \tilde{y}_{i,j}\right] + \sum_{t=1}^{s} \tilde{a}_t d_{t,j}, \tag{12}$$

and let $v_j = 1$ if the voter in observation $j$ votes Republican, and $v_j = 0$ if he votes Democrat. To estimate $\alpha_i, \beta_i, \tilde{\lambda}_i, \tilde{\mu}_i$, and $\tilde{a}_i$, we maximize the log-likelihood function, i.e., solve

$$\max_{\{\alpha_i, \beta_i, \tilde{\lambda}_i, \tilde{\mu}_i, \tilde{a}_i\}} \sum_{j=1}^{J} v_j \ln \Phi(z_j) + (1 - v_j) \ln (1 - \Phi(z_j)). \tag{13}$$

We use Newton’s method to determine a zero of the first order condition of this maximization problem. Note that, in contrast to a standard probit model, $z_j$ is not a linear function of the model parameters. This generates some numerical challenges, as the region of convergence is relatively small, thus requiring a good start value. The computer code for performing the estimation can be obtained from the authors. Theorem 1 shows how the parameter estimates of (13) translate into parameters of the original model.

**Theorem 1** Define $\rho_1 = \alpha_1 = 1$. Let $\alpha_t, \rho_t$ and $\tilde{a}_t$ for $t \in \{1, \ldots, s\}; \tilde{\lambda}_i, i \in \{1, \ldots, n\}; \tilde{\mu}_i, i \in \{1, \ldots, m\}$, be the parameters of the modified model in (11). Then the parameters of the original model (10) are determined as follows:

1. $\delta$ and $\theta$ are given by

$$\delta = \frac{\sum_{j=1}^{n} [\tilde{\lambda}_i \tilde{X}_i - \min(\tilde{\lambda}_i, 0)]}{\sum_{j=1}^{n} |\tilde{\lambda}_i|}, \quad \theta = \frac{\sum_{j=1}^{n} [\tilde{\mu}_i \tilde{Y}_i - \min(\tilde{\mu}_i, 0)]}{\sum_{i=1}^{m} |\tilde{\mu}_i|}. \tag{14}$$

2. The weights of cultural and economic issues are given by

$$\lambda_i = \frac{|\tilde{\lambda}_i|}{\sum_{j=1}^{n} |\tilde{\lambda}_i|}, \quad \mu_i = \frac{|\tilde{\mu}_i|}{\sum_{i=1}^{m} |\tilde{\mu}_i|} \tag{15}$$

11We obtain such a start value by first optimizing over $\tilde{\lambda}_i$, $\tilde{\mu}_i$, and $\tilde{a}_i$, use the resulting solution as a start value for optimizing over $\alpha_t$, $\rho_t$, and $\tilde{a}_t$. Starting from this value, convergence can be obtained for the complete optimization problem.
3. The standard deviation of the individual preference shock $\epsilon_t$ in period $t$ is given by

$$\sigma_t = \frac{1}{(1 + \rho_t) \sum_{i=1}^{m} |\tilde{\mu}_i|} \sum_{i=1}^{m} |\tilde{\mu}_i|$$

(16)

4. The slope of the separating line in the ($\delta, \theta$) space in period $t$ is

$$k_t = \frac{(1 + \alpha_t) \sum_{i=1}^{n} |\tilde{\lambda}_i|}{(1 + \rho_t) \sum_{i=1}^{m} |\tilde{\mu}_i|}$$

(17)

5. The vertical intercept of the separating line in the ($\delta, \theta$) space in period $t$ is

$$a_t = \tilde{a}_t - (1 + \rho_t) \sum_{i=1}^{m} \min(\tilde{\mu}_i, 0) + (1 + \alpha_t) \sum_{i=1}^{n} \min(\tilde{\lambda}_i, 0)$$

$$= \frac{(1 + \rho_t) \sum_{i=1}^{m} |\tilde{\mu}_i|}{(1 + \rho_t) \sum_{i=1}^{m} |\tilde{\mu}_i|}.$$  

(18)

After determining weights ($\lambda, \mu$) for a set of base years, we can determine $\delta$ and $\theta$ by using (14). Different base years give slightly different results because which preferred policy positions are economically and culturally “conservative” (i.e., leaning towards the Republican position) may change over time. Different approaches have different advantages. Pooling all years gives us the largest data set and compares all years against a common benchmark. In contrast, focusing on late base years has the advantage of measuring people’s preferences in a way that is more consistent with what is considered economically and culturally liberal or conservative today as opposed to an average over the last generation, and this the approach that we will choose for the main part of the paper. However, it should be noted that our main results are not sensitive with respect to the choice of the base period.

For given values of ($\delta, \theta$) (i.e., obtained for ($\lambda, \mu$) from fixed base years), estimating $k$, $a$, and $\sigma$ for a single election year is much easier. In particular, the probability of voting Republican is given by

$$\Phi \left( \frac{1}{\sigma} [k\delta_i - \theta_i + a] \right).$$

(19)

We can estimate this model by first estimating

$$\Phi (\beta_\delta \delta_i - \beta_\theta \theta_i + \beta_a),$$

(20)
which is a standard probit model, and by then using the identities

$$\sigma = \frac{1}{\beta_0}, \quad k = \frac{\beta_\delta}{\beta_0}, \quad a = \frac{\beta_a}{\beta_0}. \quad (21)$$

Of course, if we compare the values that we get from a direct estimation of (13), and the values obtained by this simplified method in a year that is one of the base years used in the estimation of (13) (i.e., using the values of ($\delta, \theta$) obtained from the direct estimation), then we get identical values for $k, \sigma$ and $a$.

5 Concepts and Data

Economic and cultural issues. It is useful to start by defining what we mean by economic and cultural issues. We think of economic policies as those policies that affect net personal income or consumption of public goods directly for a significant number of people. For example, this policy area would contain the level of taxation and of public good provision, legislation affecting the power of unions in wage bargaining, and general business regulation affecting profits and capital incomes.

In contrast, the notion of cultural issues is somewhat more amorphous. In our view, policies in this area have to do with the government regulating or influencing behavior, and most people care about these policies even if they are not personally affected one way or the other. For example, most heterosexual voters have a view on gay marriage, even though the legality of gay marriage does not affect their effective personal choice set (i.e., marrying someone from the opposite gender, or not marrying) at all. Clearly, those policies labeled “moral issues” by Ansolabehere, Rodden, and Snyder (2006) fall into this category. However, there are other policy issues that do, too, but are not “moral issues” in a narrow sense.12 For example, people differ widely in whether they see the U.S. as a force for good in the world that should impose its policy preferences on other countries, often by using military means.13 Related to this specific example is the whole complex of patriotism/jingoism which is also broached by Frank (2005) as an important cultural wedge issue.

We use data from the post-election survey of the American National Election Survey for Presidential election years during the time period from 1972 to 2008. We considered all questions that were continu-

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12We discuss the quantitative robustness to the definition of cultural/moral issues in Section 7.1, where we restrict the cultural factors to purely moral ones.

13Of course, one could expand the definition of the “moral” category to cover these cases. It is not immediately obvious why the legality of abortion for U.S. residents is a “moral” issue, but the consequences of U.S. military occupation in foreign countries (whether killing children as collateral damage in drone strikes, or enabling girls to go to school) are not a “moral” issue.
ously available between 1972 and 2008 and could be identified as either cultural or economic. As a result, we use the following questions in order to determine the cultural ideology index 𝛿 of a voter: Questions VCF0837 (1980 and before) and VCF0838 (1984 and after) about whether abortion should be always legal, mostly legal, mostly illegal or always illegal; Question VCF0834 about the role of women, with answers ranging from “Women and men should have an equal role” to “Women’s place is in the home”; Question VCF0206, about the respondent’s thermometer score feeling towards blacks; Question VCF0830, about affirmative action and the government’s responsibility to help minorities, with answers ranging from “Government should help minority groups/blacks” to “Minority groups/blacks should help themselves”; Question VCF0213 about the respondent’s thermometer score towards the U.S. military; Question VCF0130 about church attendance, which we use as a dummy with 1 for respondents who go to church weekly or almost every week.

For economic preferences, we use the following questions: Question VCF0809 on the role of the government in the economy, with answers ranging from “Government should see to job and good standard of living” to “Government should let each person get ahead on his own”; Questions VCF0209 and VCF0210 about the respondent’s thermometer scores towards unions and “big business”, respectively; Question VCF0114 about family income. Here, respondents are put into 5 groups according to how their income compares with the percentiles of the U.S. income distribution.

The thermometer issues we include measure the respondents’ affinity to certain groups, which we interpret as proxies for policy issues. For example, we believe that the attitude towards unions and big business should be a good proxy for right-to-work legislation or business regulation in general. We interpret the question about aid to minorities as primarily about affirmative action and hence more cultural than economic. With respect to the patriotism/chauvinism complex, there is unfortunately no direct question about nationalism; but since chauvinism (i.e., extreme nationalism characterized especially by a belligerent foreign policy) requires power projection by means of armed forces, the attitude towards the military is a useful proxy.

No demographic variables. We do not include demographic measures such as gender, race or age because we believe it is more useful to take the voter’s preferences on policy issues as a measure of his ideological position. It is certainly true that a voter’s demographic characteristics influence his preferred positions. For example, women have on average a more liberal position on abortion rights than men, so if one did not know

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14 Because we need continuously available questions, we start our analysis in 1972, as moving to the 1960s would have meant losing a substantial number of questions.
a voter’s preferences on abortion, including information on the voter’s gender is a useful proxy for preferred positions. However, since the NES has information on policy preferences, we prefer to use this information directly. The idea is that, controlling for the respondent’s opinion about abortion and the role of women, the respondent’s gender does not provide much additional information about the voter’s preferences.\footnote{In fact, we have run our regression including a number of demographic controls, and with some exceptions, they have turned out to be small and often insignificant. Also, dummies for the major religious groups (Protestants, Catholics) turned out to be very close to zero and statistically insignificant.}

We do not include any measure of partisan affiliation or self-placement on a one-dimensional liberal-to-conservative scale. Including such a measure would defy the purpose of our analysis. We want to know which policy-preferences (on both the economic and the cultural dimension) translate into a preference for the candidate of one of the parties. Regressing individuals’ vote choices for Democrats or Republicans on whether the individuals feel attached to either party is not very helpful.

Similarly, the liberal-conservative scale is not helpful because it collapses the two dimensions of our interest into one: For example, if a voter claims to be moderate, is that because he is a social liberal but a fiscal conservative, or a social conservative but a fiscal liberal, or a moderate in both dimensions? Also, when comparing the distribution of political self-identification over a long time period, it is unclear whether the social constructs of “liberal”, “moderate” and “conservative” mean the same to voters in 2004 as they meant in 1972. Attitudes on particular political issues that made an individual reasonably classified as “liberal” or “conservative” some decades ago may today lead to a different classification, and a different voting behavior. Our model allows us to analyze how this reclassification has played out over the last 35 years with respect to economic and cultural positions.

\textbf{Missing issues and bias.} Finally, it is useful to discuss the impact of data limitations on our results. Suppose that the true relationship for year \( t \) has the same structure as the model we estimate, but has \( \delta \) and \( \theta \) influenced by more issues than we have data for: \( \delta = \sum_{i=1}^{N} \lambda_i X_i \) and \( \theta = \sum_{i=1}^{M} \mu_i Y_i \), where \( N > n \) and \( M > m \) (i.e., we have data only on the first \( n \) and \( m \) issues, respectively, but the true model is determined by all \( N \) and \( M \) issues). This problem may arise particularly acutely because we have to restrict ourselves to questions that were asked in the NES in every year from 1972 to 2008. Clearly, missing some issues on the cultural dimension will lead to an underestimate of the importance of cultural issues relative to economic issues, and vice versa. Moreover, missing questions implies that we will ascribe more variation to the idiosyncratic shock \( \varepsilon \) than justified in the true model. Thus, the absolute value of \( k \) should not be over-interpreted in the
sense that $k < 1$ ($k > 1$) implies that “cultural issues are less (more) important than economic issues.” The value of $k$ depends, among other things, on which questions we use for our measurement of economic and cultural preferences and therefore, how well measured preferences reflect “true” preferences on economic and cultural issues. If, for example, we measure cultural preferences much better than economic ones, then $k$ is higher than it would be if we measured economic preferences in a better way. However, the interpretation of the development of variables over time is not systematically affected by this problem as long as the true issue weights (of the included and omitted issues) do not change systematically over time.

Which type of systematic change of the issue weights of included and omitted variables over time can we expect? Presumably, the committee deciding on which questions to ask in the NES has some notion of the importance of different issues that guides their decision – when a new issue becomes sufficiently important in political discourse, a new question will be included, and if the importance of an existing issue falls below some threshold, its usage will be discontinued. However, since continuity of questions is a very important feature for many studies, the importance threshold for inclusion is presumably higher than the threshold for exclusion. Thus, if a question remains in the NES for the whole period between 1972 and 2008, the NES committee must have felt in 1972 that its importance warranted inclusion, and its importance remained sufficiently high over the entire period to prevent exclusion. Issues that became important within this time period, but were not yet sufficiently important to be included in 1972 are not in our data set so that we would expect that the sum of the true normalized weights of the questions included in our analysis may have been higher in 1972 than in 2008. If this is the case, then our estimate of the degree to which policy positions influence voting behavior is biased downward late in our sample period (relative to the estimate in early years), as too much variation is attributed to idiosyncratic shocks rather than unmeasured variations in a voter’s position. Fortunately, in the empirical results, the influence of policy positions on voting behavior increases significantly even without taking into account this bias, which strengthens our results.

16For example, a respondent’s attitude towards gay people is now probably a good predictor of social conservatism, but in the 1970s, the NES did not contain any questions on this complex. Similarly, we would suspect that a question about the respondent’s confidence in scientific results (say, in evolution or global warming) would be more informative about a respondent’s social conservatism today than it was 40 years ago.
6 Empirical Results

6.1 Probit regression for $\delta$ and $\theta$

Table 1 reports the values of $\lambda$, $\mu$, $\lambda$, and $\mu$ for two different pooled base periods, the first five elections (1972-1988) and the last five presidential elections (1992-2008). Below the point estimates for each parameter, we report the corresponding 95 percent confidence interval (obtained by using bootstrap resampling).

<table>
<thead>
<tr>
<th></th>
<th>$(\lambda, \mu)_{1972-1988}$</th>
<th>$(\lambda, \mu)_{1992-2008}$</th>
<th>$(\lambda, \mu)_{1972-1988}$</th>
<th>$(\lambda, \mu)_{1992-2008}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>military (thermometer)</td>
<td>1.183</td>
<td>1.254</td>
<td>0.331</td>
<td>0.308</td>
</tr>
<tr>
<td></td>
<td>[0.819,1.601]</td>
<td>[0.911,1.672]</td>
<td>[0.269,0.393]</td>
<td>[0.248,0.369]</td>
</tr>
<tr>
<td>aid to minorities (high answers = against aid to minorities)</td>
<td>0.543</td>
<td>0.618</td>
<td>0.152</td>
<td>0.152</td>
</tr>
<tr>
<td></td>
<td>[0.335,0.760]</td>
<td>[0.401,0.852]</td>
<td>[0.093,0.209]</td>
<td>[0.099,0.205]</td>
</tr>
<tr>
<td>black (thermometer)</td>
<td>-1.294</td>
<td>-0.912</td>
<td>0.362</td>
<td>0.224</td>
</tr>
<tr>
<td></td>
<td>[-1.728,-0.924]</td>
<td>[-1.303,-0.589]</td>
<td>[0.297,0.426]</td>
<td>[0.161,0.283]</td>
</tr>
<tr>
<td>role of women (high answers = women’s place is in the house)</td>
<td>0.147</td>
<td>0.454</td>
<td>0.041</td>
<td>0.112</td>
</tr>
<tr>
<td></td>
<td>[-0.014,0.320]</td>
<td>[0.250,0.691]</td>
<td>[0.003,0.088]</td>
<td>[0.063,0.162]</td>
</tr>
<tr>
<td>abortion (high answers = should be legal)</td>
<td>-0.304</td>
<td>-0.746</td>
<td>0.085</td>
<td>0.183</td>
</tr>
<tr>
<td></td>
<td>[-0.479,-0.142]</td>
<td>[-0.977,-0.559]</td>
<td>[0.041,0.128]</td>
<td>[0.143,0.227]</td>
</tr>
<tr>
<td>attends church</td>
<td>0.106</td>
<td>0.082</td>
<td>0.030</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>[0.004,0.214]</td>
<td>[0.027,0.187]</td>
<td>[0.004,0.057]</td>
<td>[0.001,0.045]</td>
</tr>
<tr>
<td>income</td>
<td>-0.541</td>
<td>-0.665</td>
<td>0.122</td>
<td>0.159</td>
</tr>
<tr>
<td></td>
<td>[-0.722,-0.363]</td>
<td>[-0.887,-0.469]</td>
<td>[0.084,0.159]</td>
<td>[0.119,0.199]</td>
</tr>
<tr>
<td>big business (thermometer)</td>
<td>-1.458</td>
<td>-0.818</td>
<td>0.330</td>
<td>0.196</td>
</tr>
<tr>
<td></td>
<td>[-1.815,-1.155]</td>
<td>[-1.154,-0.537]</td>
<td>[0.289,0.371]</td>
<td>[0.140,0.249]</td>
</tr>
<tr>
<td>union (thermometer)</td>
<td>1.478</td>
<td>1.739</td>
<td>0.334</td>
<td>0.416</td>
</tr>
<tr>
<td></td>
<td>[1.194,1.813]</td>
<td>[1.421,2.124]</td>
<td>[0.295,0.374]</td>
<td>[0.367,0.468]</td>
</tr>
<tr>
<td>government standard of living (high answer = no gov. welfare)</td>
<td>-0.947</td>
<td>-0.962</td>
<td>0.214</td>
<td>0.230</td>
</tr>
<tr>
<td></td>
<td>[-1.184,-0.735]</td>
<td>[-1.226,-0.740]</td>
<td>[0.178,0.252]</td>
<td>[0.187,0.274]</td>
</tr>
</tbody>
</table>

Table 1: Estimation of Parameters; 95 percent confidence interval

The first two columns of Table 1 show the expected effects of political positions on voting behavior. Remember that our model is normalized in a way that a high value of the cultural index $\delta$ and a low value of the preference for public goods, $\theta$, increase a voter’s likelihood of voting Republican. Consequently, Table 1 indicates that a person is more culturally conservative (i.e., high $\delta$) if he likes the military; is against special government support for minorities; feels “less warm” towards blacks, believes that caring for the family is better for women than working outside the home; believes that abortion should be illegal; and attends church weekly or almost every week. A person is more economically conservative (i.e., low $\theta$) if he has a high income; likes big business; dislikes unions; and does not feel that government should provide
guaranteed jobs and a standard of living for everyone.

The third and fourth column report the implied values for the $\lambda_i$ and $\mu_i$. Remember that these are normalized so that they are positive and sum to 1, respectively. The values can be interpreted as the relative weight of different issues in determining whether a person is culturally or economically conservative. Since answers are normalized such that they go from 0 to 1, the value of $\lambda_i$ is the effect on the value of $\delta$ that arises when a respondent changes from the most liberal answer in question $i$ to the most conservative one.

Overall, the importance of different issues for the determination of the cultural and economic scores are relatively stable when comparing the earlier and the later period. In terms of cultural issues, the gender-specific questions (role of women, abortion) increase in importance, while the importance of racial relations decreases. For the measurement of economic preferences, the opinion about unions gains some importance relative to the opinion on big business.

We choose the 1992-2008 weights as the standard base because they reflect best what identifies voters’ cultural and economic conservatism today (rather than more than 20 years ago). We report only results based on the 1992-2008 weights but figures for the two alternative bases (1972-1988, and 1972-2008; available on request) show that the choice of base period does not matter qualitatively for our results.

We now analyze how much the distribution of voter ideal points on these two dimensions changed over time. Note that this is a logically independent concept – the preference distribution may change significantly even if the determining factors of conservatism remain constant. Table 2 reports the average values of $\delta$ and $\theta$ (based on $(\lambda, \mu)_{1992-2008}$) for all years between 1972 and 2008, as well as the corresponding standard deviations for both voters and non-voters.

<table>
<thead>
<tr>
<th>Year</th>
<th>av. $\delta$</th>
<th>std $\delta$</th>
<th>av. $\theta$</th>
<th>std $\theta$</th>
<th>corr.</th>
<th>Year</th>
<th>av. $\delta$</th>
<th>std $\delta$</th>
<th>av. $\theta$</th>
<th>std $\theta$</th>
<th>corr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>0.500</td>
<td>0.149</td>
<td>0.496</td>
<td>0.149</td>
<td>-0.190</td>
<td>1972</td>
<td>0.526</td>
<td>0.151</td>
<td>0.521</td>
<td>0.156</td>
<td>-0.141</td>
</tr>
<tr>
<td>1976</td>
<td>0.502</td>
<td>0.142</td>
<td>0.451</td>
<td>0.154</td>
<td>-0.121</td>
<td>1976</td>
<td>0.489</td>
<td>0.135</td>
<td>0.493</td>
<td>0.145</td>
<td>-0.089</td>
</tr>
<tr>
<td>1980</td>
<td>0.485</td>
<td>0.135</td>
<td>0.486</td>
<td>0.151</td>
<td>-0.190</td>
<td>1980</td>
<td>0.478</td>
<td>0.140</td>
<td>0.532</td>
<td>0.162</td>
<td>-0.105</td>
</tr>
<tr>
<td>1984</td>
<td>0.470</td>
<td>0.140</td>
<td>0.495</td>
<td>0.156</td>
<td>-0.193</td>
<td>1984</td>
<td>0.486</td>
<td>0.132</td>
<td>0.537</td>
<td>0.134</td>
<td>0.002</td>
</tr>
<tr>
<td>1988</td>
<td>0.491</td>
<td>0.133</td>
<td>0.478</td>
<td>0.159</td>
<td>-0.208</td>
<td>1988</td>
<td>0.491</td>
<td>0.131</td>
<td>0.534</td>
<td>0.153</td>
<td>-0.062</td>
</tr>
<tr>
<td>1992</td>
<td>0.466</td>
<td>0.137</td>
<td>0.477</td>
<td>0.153</td>
<td>-0.242</td>
<td>1992</td>
<td>0.493</td>
<td>0.119</td>
<td>0.536</td>
<td>0.152</td>
<td>-0.135</td>
</tr>
<tr>
<td>1996</td>
<td>0.489</td>
<td>0.130</td>
<td>0.474</td>
<td>0.150</td>
<td>-0.249</td>
<td>1996</td>
<td>0.477</td>
<td>0.127</td>
<td>0.537</td>
<td>0.138</td>
<td>-0.115</td>
</tr>
<tr>
<td>2000</td>
<td>0.486</td>
<td>0.125</td>
<td>0.485</td>
<td>0.150</td>
<td>-0.293</td>
<td>2000</td>
<td>0.482</td>
<td>0.143</td>
<td>0.526</td>
<td>0.151</td>
<td>-0.057</td>
</tr>
<tr>
<td>2004</td>
<td>0.490</td>
<td>0.143</td>
<td>0.502</td>
<td>0.159</td>
<td>-0.372</td>
<td>2004</td>
<td>0.492</td>
<td>0.131</td>
<td>0.545</td>
<td>0.156</td>
<td>-0.215</td>
</tr>
<tr>
<td>2008</td>
<td>0.483</td>
<td>0.139</td>
<td>0.534</td>
<td>0.171</td>
<td>-0.427</td>
<td>2008</td>
<td>0.493</td>
<td>0.128</td>
<td>0.547</td>
<td>0.145</td>
<td>-0.159</td>
</tr>
</tbody>
</table>

Table 2: Cultural and economic indices: Average and standard deviation
The average $\delta$ and $\theta$ move around in a relatively unsystematic way over time. Looking at the development of the standard deviations, it is also quite apparent that there is no clear time trend. The distribution of economic or cultural issue preferences certainly does not become a lot more polarized over time, as this would require a substantial increase in the standard deviations. This confirms the results of DiMaggio, Evans, and Bryson (1996), Fiorina, Abrams, and Pope (2006) and Fiorina and Abrams (2008) who all find that overall issue preferences of American voters have remained mostly stable over time.

Table 2 reports the ideological distribution of voters and nonvoters in the corresponding years. The correlation between economic and cultural conservatism among voters (the left table) has increased from a low of $-0.12$ in 1976 to $-0.43$ in 2008. Since high values of $\delta$ and low values of $\theta$ correspond to cultural and economic conservatism, this means that the two types of conservatism are today more closely related among voters, although that correlation is still not perfect. While it is often claimed that voting behavior of members of Congress has become essentially one-dimensional in recent years, it will be quite clear from the figures in the next subsection that such a claim cannot be made for the American electorate at-large.

The right part of Table 2 reports the ideological distribution of nonvoters in the corresponding years. There is not much of a systematic difference between voter and nonvoters on cultural issues, but nonvoters tend to be on average more liberal than voters on economic issues. A plausible interpretation is that (some) liberals face substantially higher costs of voting, for example, because of poor organization and therefore longer voting lines in inner cities, so that they are more likely to abstain. It is also interesting that, among non-voters, the correlation between the two types of conservatism is weaker and does not follow a clear trend over time.

### 6.2 Platform Differentiation

We now turn to the changes in the distance between the candidates’ platforms. Recall from equation (9) in Section 4.1 that the model identifies changes in the policy distance, relative to the distance in the base year. We chose 1976 as base year since divergence on both policies is lowest in that year. Figure 3 displays the results for cultural and economic positions.

The difference between the two parties’ cultural positions, $\delta_R - \delta_D$ increases by more than 200 percent in all years after 1992, and by about 300 percent in the last decade. For economic positions, the change in the distance between positions is considerably smaller; the maximum increase is about 50 percent in
1976. However, we should stress that our method only allows us to identify changes of the distance in cultural positions relative to the same distance in 1976, and many researchers have argued that the parties’ positions on “moral issues” (a subset of our cultural issues here) were quite close to each other in the 1970s (e.g. Fiorina, Abrams, and Pope 2006; Ansolabehere, Rodden, and Snyder 2006), while the distance on economic issues may have been more substantial already in the base year.

It is useful to contrast our model and its implications about the polarization of candidate platforms in presidential elections with Poole and Rosenthal’s DW-nominate score that measures polarization in Congress. The DW-nominate score is based on legislators’ votes in Congress. Party polarization is commonly operationalized by considering the difference between the average Democratic and the average Republican score. The positions of presidential candidates usually cannot be compared using the DW-nominate measures because very rarely, both candidates serve in Congress during the same time period and thus voting on the same laws (Obama vs. McCain in 2008 was the only exception to this in the recent past, and clearly, a single data point does not tell us anything about the development of polarization over time).

In contrast, our method is based on comparing the behavior of voters, and thus on their understanding of what the differences between candidates are. Crucially, our data have a measure of the voters’ preferred positions, as well as their vote choices. This allows us to reconstruct both the importance of economic and cultural positions for vote choices and a measure of the distance between policy platforms on both the cultural and the economic dimension.
6.3 Polarization and Sorting of the Electorate

Figure 4 provides some insights into the changing voting behavior of the electorate. It displays the values of $\delta$ and $\theta$ for all voters, together with the voter’s choice (red for Republican, blue for Democrat). The left panel is for the 1976 election, the right one for the 2004 election. In both panels, we have drawn the 50 percent separating line, i.e., voters on this line have an implied probability of voting Republican or Democrat that is exactly $1/2$. Voters below and to the right of the separating line are more likely to vote for the Republican, while voters above and to the left of the line are more likely to vote for the Democrat.

Remember that the electorate’s voting behavior becomes more determined by policy preferences if the two party platforms are farther apart from each other. Thus, we expect that the increase in policy divergence is reflected in a cleaner separation of the voting blocks in 2004, and this is exactly what we see in Figure 4.

It is useful to provide a more formal measure for this separation effect intuitively captured by the visual comparison of elections. A natural way to measure the importance of policy positions for voter choices is the following measure of position predictiveness, $\Psi$. Suppose that we have to predict the voting behavior of a large group of voters in a tight election. If we did not have any information about these voters, we could not do better than flipping a coin, and this would give us a 50 percent “success quota.” Using information about the preferred political positions of a voter enables us to make better predictions: We predict that a voter
votes Republican (Democrat) whenever his position is below (above) the separating line, and the probability of being correct for voter $i$ with this prediction is simply $\Phi\left(\frac{1}{\sigma_t}(k_t\delta_i - \theta_i + a_t)\right)$, where $(k_t, a_t, \sigma_t)$ denote the parameters for a separating line for year $t$. When we average this measure over all voters, we have a measure of how important political issue preferences are for predicting voting behavior.

Note that a problem could arise in lopsided elections. For example suppose that 70 percent of voters vote for the Republican candidate in an election because that candidate has a large expected valence $\bar{\xi}$. Then even a completely uninformed guesser could achieve a 70 percent success quota (by guessing that each voter votes Republican). To avoid this problem, we adjust the valence such that the election would have ended in a tie. More formally, we find a new intercept $a_t'$ such that the weighted vote share of the Democrat (and Republican) is exactly $1/2$, i.e. $\sum_{i=1}^{I_i} \frac{\Phi\left(\frac{1}{\sigma_t}(k_t\delta_i - \theta_i + a_t')\right)w_i}{\sum_{i=1}^{I_i} w_i} = 0.5$, where $w_i$ is the sample weight of voter $i$.\textsuperscript{17} We then measure the quality of information about political positions by how much the success quota of our forecasting system lies above the success quota of a pure coin flip:

$$
\Psi_t = 2 \sum_{i=1}^{I_i} \left| \Phi\left(\frac{1}{\sigma_t}(k_t\delta_i - \theta_i + a_t')\right) - 0.5 \right| w_i
$$

(22)

Note that $\left| \Phi\left(\frac{1}{\sigma_t}(k_t\delta_i - \theta_i + a_t')\right) - 0.5 \right|$ is the increase in the success probability relative to a pure coin flip, and the factor 2 in front normalizes $\Psi$ such that it lies between 0 and 1. For example, if knowledge of political preferences allows to correctly forecast 80 percent of voters, then this is $2(0.8-0.5) = 60\%$ better than a pure coin flip.

If $\Psi = 1$, society is extremely divided along ideological lines: Every conservative votes Republican, and every liberal votes Democratic. This means that most voters would know which party they will vote for before they know who are the actual candidates of each party – they are not going to give the other party’s candidate a chance to convince them to switch parties in this election, so they are not “swing voters.” In contrast, if $\Psi = 0$, knowledge of a voter’s issue preferences does not help to predict voting behavior – all voters are ex-ante open to both candidates.

Figure 5 shows the development of $\Psi$ over the last 10 presidential elections, and the parallels to cultural polarization in Figure 3 are quite obvious. $\Psi$ decreases from 1972 to 1976 (to around 0.35), and then increases substantially throughout our observation period to end at a level of about 0.58. In other words,\textsuperscript{17}

\textsuperscript{17}In most years in the NES, all observations have the same sample weight, in which case $w_i = 1$ and the term in the denominator is simply the number of voters.
voters’ political issue preferences are a substantially better predictor of their voting behavior in the 2000s than in the 1970s – knowing them allows about 65 percent better predictions in 2004 than it did in 1976.

Figure 5: Position predictiveness, 1972-2008, with 95% confidence intervals

To put Figure 5 into historical context, note first that the lowest value of $\Psi$ occurs in the 1976 election between Ford and Carter. From today’s perspective, Ford was clearly a moderate Republican. While Jimmy Carter today is probably considered the most liberal president in the last 50 years, this reputation to a large extent derives from his foreign policy positions (say, not bombing Iran in 1979, or being unusually critical of Israel for a U.S. politician), and therefore may have been surprising to voters who probably expected Carter (an evangelical Christian and Southern governor) to be a relatively conservative Democrat. It is therefore plausible that the predictiveness of policy positions for vote choices is quite low in 1976.

A particularly large increase in $\Psi$ occurs with Ronald Reagan’s first presidential election in 1980. Reagan’s success as a conservative is generally considered a key turning point in American politics and initiates a process of ideological realignment of the parties, with liberal Republicans and conservative Democrats switching party affiliations throughout the 1980s and 1990s. There is no significant change in $\Psi$ from 1980 to 1988, but the values in the next decade are somewhat higher. The next big increase in $\Psi$ occurs in 2004. While George W. Bush had campaigned as a “compassionate conservative” (i.e., a relatively moderate Republican), his first term showed that he was much more conservative than expected; moreover, in 2004, he
ran against John Kerry, a very liberal Democrat. Thus, the resulting stark difference between candidates that gave rise to the quote in the first paragraph of the introduction (that “the 50-50 nation appears to be made up of two big, separate voting blocks, with only a small number of swing voters in the middle”) is reflected by the large increase in $\Psi$. Interestingly, $\Psi$ remained at almost the same level in the 2008 election.

It is instructive to compare the development of predictiveness as shown in Figure 5 with different measures of mass polarization in the literature. For example, the percentage of voters casting a straight ticket for President and House (Hetherington 2001, Figure 3), and the percentage of respondents who perceive important differences between the parties (ibid., Figure 5) show a secular increase from the 1970s on, just like $\Psi$. The same is true of the percentage of strong partisans (Bartels 2000, Figure 1) and the estimated impact of party identification on presidential voting (ibid., Figure 4). Overall, this external validation confirms that $\Psi$ measures something that has been interpreted as mass polarization in the existing literature. The main advantage of $\Psi$ is, though, that we can decompose it to disentangle the effects of elite polarization and changes in the voters’ preference distribution. Evidently, this would be impossible for any of the measures cited above, and we turn to this task next.

Our model suggests two distinct possible reasons for the deeper political divisions between voters reflected in the increase of predictiveness: First, for a fixed distribution of political preferences in society, divergence of the two candidates’ policy platforms implies that every voter who is not exactly on the dividing line moves more firmly into his ideologically preferred camp, in the sense that his probability of voting for his ideologically-preferred party increases, while the likelihood of crossing-over because of idiosyncratic preferences for the other candidate diminishes. Second, if we keep the candidates’ policy platforms fixed, but move voters’ ideal positions away from the dividing line, then the average preference of voters for their ideologically closer party also increases. The first effect captures what we call sorting, while the second one relates to the notion of voter polarization. Since we estimate the voter preference distribution and the dividing line for each year, we can isolate these effects: We can fix the electorate in the previous election and just focus on the change in sorting, or we can fix the dividing line in the previous election and thus isolate

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18The only substantial qualitative difference is for the 1972 election, which has no particularly remarkable feature in these 4 measures (and is often measured as less polarizing than 1976), but is identified as a considerably more polarizing election than 1976 by $\Psi$.

19Levendusky (2009) explains sorting and polarization using an example with three voter types, liberals, conservatives and moderates. Suppose that in year $t_0$ half of the liberals vote Republican and half of the conservatives vote Democratic, while in year $t_1$ all liberals vote Democratic and all conservative vote Republican. Then the electorate is more sorted in $t_1$ than in $t_0$, but polarization has not changed since the number of voters of each type remained the same. In contrast, suppose that in year $t_1$ there are more conservatives and liberals, and fewer moderates than in $t_0$. Then the electorate is more polarized in $t_1$ than in $t_0$. 

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changes in voter polarization.

Formally, let $\Psi(t_e, t_p)$ denote the predictiveness for the electorate of year $t_e$ if the politicians’ positions are as in year $t_p$. The total change in predictiveness in year $t$ from the previous election in year $t - 4$ is $\Delta \Psi = \Psi(t, t) - \Psi(t - 4, t - 4)$. We call $\Delta S(t) \equiv \Psi(t - 4, t) - \Psi(t - 4, t - 4)$ the level of sorting in year $t$, taking as given the base electorate of the last election. The remaining change in $\Psi$ in comparison to the last year is due to polarization, measured as $\Delta P(t) \equiv \Psi(t, t) - \Psi(t - 4, t)$.

![Figure 6: Sorting and polarization contributions to position predictiveness, 1972-2008, by election (left) and cumulative relative to 1976 (right).](image)

The left panel of Figure 6 plots $\Delta S(t)$ and $\Delta P(t)$ to decompose the change in predictiveness into polarization and sorting. Since we do not have data for 1968 (i.e., the election before 1972), the left panel starts from 1976. Note that, in those years where both polarization and sorting increase (1984, 1992, 2004), we draw the effects stacked above each other so that the height of the column in these years is equal to the whole effect (i.e., $\Delta \Psi_t$). In the other years, both polarization and sorting are drawn starting from zero, and the total change in $\Delta \Psi_t$ is equal to the difference between the positive and the negative column. The right panel presents the same information in a different way, plotting the aggregate change of polarization and sorting, relative to 1976: The values in 1976 are zero by definition, and in all following years are $\sum_{t=1976}^t \Delta S(i)$ and $\sum_{t=1976}^t \Delta P(i)$; the values for 1972 are just the negative of the values for 1972 in the left panel.

It is apparent from the left panel that sorting is more volatile than polarization: Sorting increases in five elections, and decreases in four elections, while polarization increases in most elections, though usually by a small amount. Also, the average absolute change in sorting is considerably larger than the average
absolute change in polarization. This is intuitive because changes in sorting are caused by changes in the
distance between the candidates’ positions from year to year, and since the candidates change from election
to election, while the electorate remains mostly the same as in the previous election, it is very plausible that
there are much larger swings possible in sorting than in polarization.

Sorting decreases sharply in 1976 (i.e., going from Nixon vs. McGovern to Ford vs. Carter) and re-
bounds sharply in 1980 (Reagan vs. Carter). Somewhat surprisingly, the second largest increase in sorting
is in 1996 (Dole vs. Clinton) relative to 1992 (G.H.W. Bush vs Clinton). As a person, Dole does not nec-
essarily appear to be that much more polarizing than G.H.W. Bush. However, it could well be the case that
the first term of Bill Clinton with the attempted health care overhaul, and the following take-over of the
House of Representatives by the Republicans lead by Newt Gingrich generated the perception among voters
of increased policy divergence between Democrats and Republicans, independent of the specific candidates
for the presidency. The values for the next two elections are consistent with the general perception among
political pundits: In 2000, the electorate was not perceived as polarized (and, in part, rather frustrated with
the small perceived policy difference between Bush and Gore), while 2004 was perceived as an election with
a stark policy contrast between Bush and Kerry.

Finally, it is useful to compare the aggregate contributions of sorting and polarization to predictiveness.
Of course, since polarization changes usually by a much smaller amount from election to election, and
the sign of the change in sorting is sometimes positive and sometimes negative, the aggregate numbers
depend a lot on the time period considered. Relative to 1976 as the base year, the aggregate sorting effect is
considerably larger than the aggregate polarization effect in all years, and sorting accounts for about three
quarters of the total change by 2008. However, the aggregate polarization effect in the last three elections
(i.e., in 2008 relative to 1996) is considerably larger than the aggregate change in sorting during the same
time period. This may indicate that the elite polarization that started around 1980, apart and in addition to
its effect on voter behavior, is eventually also having an effect on the fundamental preferred policy positions
of the electorate. In this context, remember that the correlation between δ and θ takes its highest value in
2004 and 2008 (see Table 2 in Section 6.1), so voters who are socially conservative are also becoming more
fiscally conservative (and vice versa).

Because the 1972 election was also very characterized by deep policy divisions, these numbers change significantly when we
measure changes relative to 1972 instead of 1976. Predictiveness in 2008 is about one third higher than in 1972, with the increase
coming from polarization and sorting to about equal parts.
6.4 Relative importance of cultural and economic issues

We now turn to another interesting question: How did the relative importance of cultural and economic issues for the determination of individuals’ voting behavior change from 1972 to 2008, and how did this change affect the behavior of different voter types? Remember that we can interpret the slope of the dividing line as a “marginal rate of substitution” between cultural and economic positions. If an individual on the dividing line becomes one unit more culturally conservative, his economic liberalism needs to increase by $k$ units in order for him to remain stochastically indifferent between the candidates.

Returning to Figure 4 in the preceding subsection with the 1976 and 2004 election, we see that the slope of the dividing line, $k$, is low 1976: Voters split primarily along economic issues (with high $\theta$ types mostly voting for Carter, and low $\theta$ types mostly voting for Ford). In contrast, in 2004, the separating line is considerably steeper and thus, to a higher degree along cultural lines, with social liberals primarily voting for Kerry, social conservatives for Bush.

Figure 7 displays the development of the slope $k$ for all years. Given the values of $\delta_i$ and $\theta_i$ for each voter in each year, we estimate the model given in equation (20), and use (21) to determine $k$ and $\sigma$. After the initial decrease in $k$ from 1972 to 1976, the relative importance of cultural issues starts to increase to reach high points in 2000 and 2008. The confidence intervals in Figure 7 clearly indicate that, while election-to-election changes are often not statistically significant, the long-term trend definitely is statistically significant.

![Figure 7: The development of $k$ from 1972 to 2008, with 95% confidence intervals](image-url)
Since the standard deviation of $\theta$ is about 10-20 percent larger than that of $\delta$, a value of $k$ around 1.1 or 1.2 would indicate that an increase of cultural conservatism by one standard deviation can approximately be compensated by an increase in economic liberalism by one standard deviation, to leave the voter’s probability of voting Republican unaffected. In this sense, cultural and economic factors are roughly equally important in the last elections, while in the 1970s, economic factors were substantially more important than cultural ones.\(^{21}\)

### 6.5 Voter Migration

The secular increase in $k$ is accompanied by a downward trend in the intercept $a$, so that the dividing line pivots around some center of gravity in the general neighborhood of the center of the voter preference distribution. Thus, the change of the separating line affects some voter types much more than others: Voters whose ideological types are close to the pivot point remain more-or-less evenly split between both parties. In contrast, an increase in $k$ means that, to the right of the pivot point, voters become more likely to vote Republican, and to the left of the pivot point, they become more likely to vote Democratic.

Specifically, the voters who move in the Republican direction are those with socially conservative, but economically liberal views. It is tempting to identify these voters with what has been called “Reagan Democrats” (we will discuss this in more detail in Section 6.6 below). Those voters who move in the opposite direction, towards the Democrats, are voters with socially liberal and economically conservative views. For example, it is often argued that a substantial number of professionals or financial sector executives fall into this group.

Figure 8 illustrates this effect for the election with the largest increase in $k$, in 1980. Consider first the left panel. We draw the separating line of the 1976 election as a dashed line, and the 1980 one as a solid line. The area between the dashed and solid lines shows the area of voter types that are most affected. To the Northeast of the intersection between the two separating lines, there are 18.6 percent of the electorate who change from being more likely to vote for the Democrat in 1976 to being more likely to vote for the Republican in 1980. Of course, due to the stochastic nature of voter preferences, not all voters in this area change their actual vote (i.e., some may already have voted Republican in 1976, and others may continue

\(^{21}\)Remember that this result is not a consequence of voters directly putting more weight on cultural issues now than they did in the past, but rather follows from increased party divergence. Changes in the distribution of preferences are independent of these changes in $k$.\)
to vote Democratic in 1980). However, in terms of their probability of switching allegiance, these are the most affected voters. They have a high $\theta$, for example because of lower income or positive attitude towards unions, but they are culturally conservative and many found Reagan’s cultural conservatism appealing. In exchange for these voters, some fiscally-conservative but socially liberal voters might have migrated to the Democrats, but in the NES sample, there was not a single voter in this region, resulting in a landslide victory for Reagan.

Of course, focusing on the actual separating lines may be a bit misleading in that it conflates the effects of increased policy divergence on cultural issues (i.e., higher $k$) and changes in net valence: In addition to emphasizing cultural conservatism, Reagan may also have been “better” than Carter; by itself, such a pure valence difference results in a parallel upward shift of the separating line, and thus, in an increase of the “Reagan Democrat” area (and a corresponding decrease of the “Wall Street Liberals” area). To correct for this valence effect, the right panel of Figure 8 shifts in parallel the separating lines such that they correspond to a tied election, respectively.$^{22}$ The Reagan Democrat area in the right panel is therefore the area of voters who would (stochastically) switch from Democrats to Republicans just based on the new platforms, even if valence is corrected in a way that keeps the parties’ vote shares constant. Again, it is evident that there are

$^{22}$That is, we calculate a value of the intercept $\alpha$ such, if we substitute this value in (7) and sum over all voters, the expected number of Republican voters equals that of Democratic voters.
significantly more voters in the Reagan Democrat area.

Finally, it is instructive to discuss why having a model with (at least) two policy dimensions is very helpful for the analysis of voter migration. By contrast, consider what happens in a one-dimensional model where voters have an ideal position in a liberal-conservative spectrum, as well as an idiosyncratic preference shock like in our model. When comparing two tied elections (i.e., when adjusting the net valence just as we did above), there is always the same cutoff voter type such that voters who are more liberal are more likely to vote for the Democrat, and those who are more conservative are more likely to vote for the Republican. The only difference that can arise between overall tied elections is whether the percentage of liberals and conservatives that vote for Democrats and Republicans, respectively, is very high (this will happen if there are large policy differences between candidates) or barely higher than 1/2 (this will happen if the policy differences are small). However, a type that votes predominantly Republican in a tied election can never vote predominantly Democratic in another tied election – in contrast to our framework, such an event is excluded by construction in a one-dimensional framework. Furthermore, systematic electoral shifts that increase or decrease the republican vote share (induced by net changes in valence) must affect all voter types in the same direction, making all types either more or less likely to vote Republican.

6.6 Who is changing party preferences?

The question of the changing fault line through the electorate and the resulting voter migration is a highly controversial issue in the study of American voting behavior. The term “Reagan Democrats” was originally coined by Democratic pollster Stan Greenberg for the culturally conservative voters of Macomb County, Michigan (largely white, unionized auto-workers). More recently, Journalist Thomas Frank (2004) has written the bestseller “What’s the matter with Kansas?” in which he argues that white working class voters in relatively poor states often vote for Republicans because Democrats became more similar to Republicans on economic issues, and because their preferences on cultural issues such as abortion or gay marriage are more closely aligned with Republicans: “The Democratic Leadership Council has long been pushing the party to forget blue-collar workers and concentrate instead on recruiting affluent white-collar professionals, who are liberal on social issues. [...] They] stand rock solid on, say, the pro-choice position while making endless concessions on economic issues, on welfare, NAFTA, social security, labor law, privatization, deregulation and the rest of it” (p. 243). “By dropping the class language that once distinguished them sharply from Republicans they have left themselves vulnerable to cultural wedge issues like guns and abortion and the
rest whose hallucinatory appeal would ordinarily be far overshadowed by material concerns.” (p.245)

Interpreted in terms of our model framework, we read Frank’s claim as follows: (1) Over time, the policy difference between Democrats and Republicans has diminished on economic issues, and increased on cultural “wedge” issues; (2) this change in the economic and cultural party platforms has induced some “natural Democrats” (which Frank identifies as the white working class) to switch to the Republicans.

Regarding the first claim, remember that Figure 3 indicates that economic policy divergence was lowest in the 1970s and thus has increased rather than decreased over this time period. However, this divergence has been overshadowed by the relatively much larger increase in divergence on cultural issues. In relative terms, Frank’s first claim is correct: Policy differences on cultural issues have become more salient for the distinction between Democrats and Republicans, and while economic issues are still very important for vote choices, they have become relatively less important.

The analysis of the last subsection identifies people who were most likely to switch from Democrats to Republicans as those with culturally conservative, but economically liberal policy preferences. To evaluate whether these party switchers are in fact predominantly the “Reagan Democrats” or “white working class voters” that Greenberg and Frank talk about, some additional analysis is needed.

For each voter type (i.e., a $(\delta, \theta)$ combination), we can calculate the probability that the type “moved” from the Democrats in year $t_0$ to the Republicans in year $t_1$ as $M_{D \rightarrow R} = (1 - \Phi_{t_0})\Phi_{t_1}$, where $\Phi_t$ is the probability of voting Republican in year $t$. We can then rank all different voter types that are present in our sample of year $t_1$ according to their value of $M_{D \rightarrow R}$ (or the analogously defined $M_{R \rightarrow D}$), and we will consider the 5 or 10 percent of types that rank highest according to these measures as the “most likely party switchers” (either in the Republican or in the Democratic direction). We will then analyze how these voter types look demographically and in terms of issue preferences.

Table 3 shows the 5 and 10 percent of most likely party switchers in the each direction, by income category, race, education, occupation and social class. Voter types most likely to move in the Republican direction ((first two lines, our notion of “Reagan Democrats”) are considerably more likely to be lower

\footnote{Clearly, the extent and direction of economic policy divergence depends on what one considers as the base period. For example, in the 2000 election (the last one before Frank wrote his book), economic divergence is in fact rather low and quite a bit lower than in the 1980s.}

\footnote{It is useful to think of $\Phi_t$ as the proportion of all those voters in the overall population (i.e., not just in the NES sample) with a specific preference type who vote Republican in year $t$. However, one should not necessarily think of $M$ as the transition probability of a specific individual, especially from one election to the next, because the error terms $\varepsilon$ of an individual voter may be intertemporally correlated (nothing in our estimation procedure needs to be adapted if they are, because the NES data are anyway just sequential cross sections rather than panel data).}
middle class than the general population, with about 30 percent located between the 16th and the 33rd percentile of the income distribution, and another 40 percent between the 33rd and the 67th percentile; in contrast, fewer of them are either very poor or very rich. They are also significantly whiter and less educated than the population at-large: about 50 percent have high school or less as their highest degree, and they are less likely to be in professional and managerial positions. They are also significantly more likely to identify as “working class” and less likely to identify as upper middle class (or upper class).\(^{25}\)

In contrast, the voter types most likely to switch from Republicans to Democrats (i.e., “country club liberals”, in the second two lines) are considerably more likely to be high income (54 or 60 percent from the upper third of the income distribution), and considerably less likely to be low education than the population at-large. They are also considerably more likely to be in professional and managerial positions.

Table 4 continues to provide some information about the behavior and the political points of view of the two groups. Reagan Democrats are extremely unlikely to be agnostic or non-Christian, and they attend church at least almost weekly substantially more often than the population at large (or even Republicans).

In contrast, the voters who are most likely to switch from Republicans to Democrats are more than twice as likely than the average voter to be non-Christian or agnostic, and only about 15 percent of them attend

\(^{25}\)We use question VCF0148 and take answers 1 and 2 (“working class” and “average working class”) as the dummy variable in the penultimate column, and answers 6 and 7 (“upper middle class” and “upper class” as the dummy variable in the last column of Table 3.
church often. Reagan Democrats are also significantly more likely to “feel cold” about Jews (thermometer score of 49 or less) than the general population; the 5 percent of the electorate most likely to switch from Democrats to Republicans accounts for a quarter of all voters who “feel cold” about Jews.

The difference in religiosity between the two switching groups is also reflected in attitudes. None of the 10 percent of the electorate who we identified as most likely to switch to the Democrats believes that abortion should be always illegal, while around 40 percent of Reagan Democrats do (by comparison, only every sixth Republican voter holds this point of view). Similarly glaring differences obtain for the question whether the whole Bible is literally true, and whether gay couples should be allowed to adopt children. Overall, the voters most likely to have switched from the Democrats to the Republicans contain a large percentage of extremely religiously and socially conservative voters, and the voters who are most likely to have switched from Republicans to Democrats are mostly on the extreme liberal end of the cultural spectrum.\(^{26}\)

**Previous empirical analysis of Frank’s hypotheses.** Thomas Frank’s book has stimulated substantial research that critically examines some of Frank’s diagnosis and that we now discuss. Bartels (2006) uses NES data to empirically analyze whether white working class voters attach more weight to social issues than to economic issues, either absolutely or relative to other voters. Specifically, Bartels runs two regressions

\(^{26}\)This is quite consistent anecdotal evidence about party switchers. For example, in 1988, Rick Perry was the Texas chairman of the Al Gore campaign, while Jon Stewart of the Daily Show recently admitted to having voted for George H.W. Bush for President (see http://articles.latimes.com/2011/jun/22/entertainment/la-et-onthemedia-20110622).
similar to those in our Table 1, one for college educated whites and one for non-college educated whites, and then compares the regression coefficients of the two regressions. In the terminology of our framework, this approach tests whether different subsets of voters are characterized by different separating lines (specifically, different slopes $k$).\textsuperscript{27}

Bartels finds that the regression coefficients for most cultural issues are smaller among non-college educated whites than among college-educated whites. On the other hand, non-college educated whites also have generally smaller regression coefficients for economic issues. Interpreted in our model framework, this says that the vote-choice of college educated voters is determined by their preferred policy positions to a larger extent than the vote-choice of non-college educated voters, and conversely, less by their idiosyncratic preferences for the candidates. This is a very interesting and plausible finding, especially considering that college educated voters may be better in figuring out what the positions of the parties actually are. (We have run a similar test on our data set and receive the same result).

However, whether the separating line has a different slope among two groups only tells us whether two individuals who have identical policy preferences but belong to different groups would vote differently. Independent of whether the two groups have the same separating line, the voting behavior of the average group member can be affected differentially by changes in the parties’ positions (and the resulting changes in the separating line) if the distribution of preferences differs between the two groups.

Figure 9 provides an intuition for why this is the case. There are two sets of voters that differ in observable characteristics — think “working class voters” and the complement set of “non-working class voters” — but also in the distribution of voter preferences in each group (the color of the shading indicates the density of voters, with darker shades denoting the areas with most voters). In each set, some voters vote Democratic and some vote Republican, and this can be used to estimate a separating line, separately for each group. This provides us with $k_1$ and $k_2$, the relative importance of cultural and economic issues in groups 1 and 2, respectively.

To keep things simple, suppose that we find that the separating line between Democrats and Republicans in each group is actually the same (so, $k_1 = k_2$). Does this mean that the behavior of voters is necessarily affected in the same way if $k$ changes over time (again, remaining the same across the two groups)? Clearly, given the way we have drawn the preference distributions for the two groups in Figure 9, the answer is neg-

\textsuperscript{27}Bartels does not aggregate economic or cultural issues, and he does not argue based on relative importance of cultural and economic issues, but rather focuses on absolute differences between the coefficients of the same exogenous variable in both regressions. However, it is clear that one can translate his findings into our framework.
Figure 9: Differential impact of increased importance of cultural issues in groups 1 (left) and 2 (right)

In group 1, some voters change from Democrats to Republicans, and some change from Republicans to Democrats. The gains and losses of each party in this group are approximately equal. Thus, if we just look at the average propensity to vote republican in group 1, it would appear that it did not change at all (although, of course, there are also migrations of group 1 voters, in both directions). In contrast, most of the voter migration in group 2 is from Democrats to Republicans.

Thus, the following two statements are both true in Figure 9: (1) “Voters from group 2 have become more supportive of Republicans because of an increased cultural polarization (or a decreased economic difference) between parties” and (2) “The vote choice of voters in group 1 is determined by their cultural and economic positions in the same way as the vote choice of voters in group 2.”

7 Robustness

7.1 Robustness to the selection of policy issues

Our measure of “cultural issues” is rather broad: It includes essentially all “non-economic” policy questions that have been asked in the National Election Survey in all presidential election years from 1972 to 2008. Much of the existing literature has focused instead on purely “moral” questions, i.e. excluding both racial issues and those related to patriotism/militarism. For example, Ansolabehere, Rodden, and Snyder (2006)
find that economic issues are at least twice as important as moral issues, but that the importance of moral issues for vote choice has increased from close to zero in the 1970s and 80s to a nontrivial size in the latest elections.

We include more than just “moral” issues in our cultural category, and this necessarily increases the weight of that category and the overall explanatory power of the analysis. However, we will now show that the main results of our analysis remain qualitatively unchanged if we reduce the variables in our “cultural” category to the narrowly “moral” ones (abortion, role of women and church attendance). Figures 10, 11 and 12 are the analogues of Figures 3, 5, 7 and 6. The results display the same qualitative behavior as in

![Figure 10: “Moral” and economic policy divergence of candidates, 1972 to 2008](image)

the corresponding figures in the main text, but there are also some interesting differences. Figure 10 shows that “moral” policy divergence increases even more dramatically than cultural policy divergence, reaching, between 1996 and 2004, more than 600 percent of the difference in 1976. Moreover, there is a substantial decline of moral policy divergence in 2008.

The values of $\Psi$ display the same secular increase as with cultural issues, but, as expected, the values are somewhat lower. Also, $k$ increases steadily from a low in 1976, but the numerical value of $k$ is substantially lower than in the basic model – clearly, when only “moral issues” are included, the importance of this category relative to the unchanged economic category is lower. Finally, decomposing the increase in $\Psi$ into changes in sorting and polarization in Figure 12 shows essentially the same picture as for general cultural issues in Figure 6.
Figure 11: Position predictiveness and relative importance of moral and economic issues

Figure 12: Contributions of sorting and polarization to position predictiveness, by election (left) and cumulative (right) (cultural issues restricted to purely moral ones)

7.2 Restricting the set of voters

Several papers in the existing literature look at the question whether different subsets of voters such as working class voters and those with college education differ in what determines their vote choice. That is, do the estimated coefficients for the same variables in probit regressions on different subsets of voters differ in size? There are different interesting partitions that one could look at: For example, does the separating line differ by income, race or gender? How does the standard error of $\xi$ differ by the voters’ educational level? In principle, we can analyze these questions in our framework. A practical problem is, however, that
focusing on subsets of voters reduces the respective sample size substantially and thus, the size of confidence intervals increases substantially so that it is often impossible to know whether estimated differences in $\Psi$ or $k$ in different subsets are due to real differences in the underlying true parameters, or due to random variations.

For example, one of the secular changes in the U.S. political landscape is the partisan realignment of in the former confederate states. After the Civil War and Reconstruction periods, most Southern whites felt an animosity against the Republican party, and in the 1930s, Roosevelt managed to include Southern whites in his New Deal coalition. As a consequence, the Deep South remained one of the most Democratic regions of the country for the next generation. Thus, during this time, both parties had culturally conservative wings. Following the civil rights legislation of the 1960s, a large block of social conservatives (white, southern evangelicals) migrated to Republican party. This partisan realignment of the South proceeds throughout the period we consider in our paper. The reader may therefore wonder whether our results pick up this realignment of voters, rather than a change in the position of parties. However, for all years, there is no $k$ for any subset of voters (either Southerners or Northerners) that is significantly different from the $k$ for the whole United States. Details are available from the authors upon request.

7.3 Changes in the Variance of Valence $\xi$

In our analysis in Section 6, we assume that the standard deviation of $\xi$ does not change over the sample years. In a probit model that analyzes data from only one year, the assumption that the residual error is drawn from a standard normal distribution is a mere normalization – if we write the minimization problem of a probit regression, but assume that the probability of voting Republican is $\Phi_\sigma(\alpha + \beta x)$ (where $\Phi_\sigma$ is the cdf of a $N(0,\sigma)$ distributed random variable), then the objective function is homogeneous of degree zero in $(\alpha,\beta,\sigma)$. Thus, $\sigma$ is not determined and can be normalized to one, without loss of generality. In contrast, we look at a sequence of years. Of course, the argument above holds for each year, but since we want to interpret the change of regression coefficients (or functions of regression coefficients) over time, we effectively have assumed that the standard deviation of idiosyncratic preference shocks is constant over time. This is a standard assumption when the analysis is based on a comparison of regression coefficients over time (e.g. Bartels 2006, McCarty, Poole, and Rosenthal 2006) and usually not even discussed.

If the standard deviation of idiosyncratic preference shocks is constant over time, we can interpret our

---

28We do not need to make any assumption about the average value of $\xi$ in the population, i.e. the average net valence of candidates is allowed to vary over time.
empirical results as evidence of policy divergence. If, instead, one allows for $\sigma_\xi$ to vary over time, the interpretation of the policy divergence results can change; for example, if one were to assume that $\sigma_\xi$ decreased considerably over time (i.e., the size of the average idiosyncratic preference shock decreased), then one would have to think of overall policy divergence between parties as relatively constant (though there still would have to be an increase of cultural divergence relative to economic divergence). If, instead, $\sigma_\xi$ increases over time, the divergence effects would be magnified relative to the basic model. The mathematical logic behind our model (and, more generally, intertemporal probit models) does not allow us to isolate one of these interpretations as the “true” one any more than a relativist physicist can determine an absolute coordinate system.\footnote{I.e., if the physicist pushes the gas pedal in a car, does the car accelerate in the direction it is pointing, or does the car stand still, but the trees move faster in the opposite direction? Modern physics is built on the notion that there is no absolute coordinate system, so we cannot say which of the two statements is in any absolute sense “true”, but it is still the case that certain interpretations are more natural than others in certain applications.}

This said, what is a natural way of thinking about the temporal development of $\sigma_\xi$ in our context? The net-valence term $\xi$ is determined by the voters’ interpretation of candidate traits that are not directly linked to the candidate’s economic or cultural platform, and the NES contains several question about such characteristics that go back sufficiently many years to enable a comparison across different elections: VCF0354 – VCF0356 and VCF0366 – VCF0366 ask, respectively, whether the Democrat and Republican presidential candidates are knowledgeable, moral and provide strong leadership. Each of these variables is measured on a 4-point scale, and if we denote the responses of voter $j$ to the questions about the Democratic and Republican candidate at time $t$ by $X_{i,t}^j$, $Y_{i,t}^j$, $i = 1, 2, 3$, respectively, then $Z_{t}^j = \sum_{i=1}^{3} (X_{i,t}^j - Y_{i,t}^j)$ is a useful proxy that is proportional to the net valence of the Democratic candidate that voter $j$ perceives. We can then compute the standard deviations $\sigma(Z_t) = \sqrt{E[(Z_t - E(Z_t))^2]}$ for the presidential election years from 1980 (the first year for which these data are available) to 2008, which gives the following values: 3.10, 3.00, 2.62, 3.08, 3.13, 3.20, 4.21, and 4.05.

The solid line in Figure 13 recalculates the time series from Figure 3 in Section 6, using these standard deviations for $\sigma_\xi$. For comparison, we plot the values derived from assuming that $\sigma(\xi)$ is constant (i.e., the values of Section 6) as a dashed line. Note that the two curves are very close to each other until 2000, and thus, the overall picture of the development until this time is qualitatively unchanged. However, for 2004 and 2008, the adjusted curve displays even more policy divergence than in the basic model where $\sigma_\xi$ is assumed to be constant.
Figure 13: Cultural and economic policy divergence of candidates, 1972 to 2008, when $\sigma_\xi$ changes. Dashed line: constant $\sigma_\xi$.

8 Conclusion

In this paper, we analyze some of the central questions in American political behavior. Do voters increasingly split along ideological lines, and if so, what does their behavior tell us about the underlying fundamental causes? Are voters today more polarized than they were a generation ago, or do they just appear more polarized because they face more polarizing choices? And, if the types of voters that support the two parties, respectively, have changed over the long run, who has become more Democratic and who has become more Republican?

When candidate positions are very similar, then voters choose candidates primarily based on non-policy attributes. This is true even if voters would, in principle, care a lot about policy — if there is no meaningful policy difference between candidates, the voters cannot express the direction or intensity of their policy preferences through the act of voting for one of the candidates. In contrast, more divergence between party positions translates into a starker choice for voters, and one that is influenced more by the voters’ ideal positions relative to the candidates.

Our formal model shows that we can recover the extent and the direction of policy divergence from the voting behavior of voters whose ideal positions we know from their answers to policy question in the National Election Survey. Our empirical results show that Democrats and Republicans have diverged substantially since the mid-1970s, in particular on cultural, but also on economic issues. As a consequence,
policy positions have become significantly and substantially more important for the determination of voting behavior. Most of this effect is due to increased “sorting” because of increased policy divergence between the parties, but our model can also identify some “electoral polarization,” mostly due to the fact that the voters’ ideal economic and cultural positions have become more strongly correlated recently.

Finally, our results resolve a seeming paradox in the existing literature. Political pundits often claim that, over the last generation, the Republican party gained substantial support by their increased emphasis on cultural issues which appealed to “Reagan Democrats” — socially conservative voters who previously voted Democratic because of their economically liberal preferences. Yet, convincing political science research appears to show that working class voters do not put more emphasis on social issues than other groups of voters. Our model shows why these claims can be logically consistent with each other. Statements about the weights of different issues for different groups of voters deal with the question how voters trade-off policy differences between their own preferred positions and those of the candidates, and the marginal trade-off may very well be the same for two very distinct groups. The reason why certain segments of the electorate move in the Republican direction is that they are socially conservative and the social policy difference between parties has increased, and not because they “care more” about these issues than other voters (in the sense of having a higher weight on these issues when deciding whom to vote for).

We show empirically that the voters whose behavior is most likely to be affected by the change in the fault line brought about by the parties’ changing positions in fact display many of the characteristics stipulated by the informal literature. The voters who our model predicts to have (probabilistically) shifted from Republicans to Democrats are significantly better educated, likely to be in professional or managerial positions, are more agnostic, and more liberal in terms of gay rights and abortion than the population at large; and those voters who have moved in the opposite direction, from Democrats to Republicans, display the opposite characteristics: They are less educated, lower-middle class (but not poor), and are more likely to be religious fundamentalists.

Our methods are, of course, applicable to other data sets and the question of “polarization” in other countries. In particular, it would be very interesting to analyze whether the developments that we identified for the US in the last generation — policy divergence between parties, and stronger divergence on cultural issues than on economic ones — are also reflected in other countries (and in other voting systems such as proportional representation), or whether the experience in the United States is unique in this respect. Such a cross-country comparison will be instrumental for finding out the root cause for the development — why
is it that parties have diverged over the last generation? And, is this a bad development that should be corrected (and, if so, how?), or is the increased extent of choice between parties actually a desirable feature. Evidently, these are some very fundamental questions that will require a lot more work, but we hope that the instruments that we have developed in this paper will prove useful in this long-term project.
Appendix

**Proof of Theorem 1.** Let $N_A$ be the set of all $i$ with $\bar{\lambda}_i < 0$. Then let $X_i = 1 - \tilde{X}_i$ if $i \in N_A$, and $X_i = \tilde{X}_i$, otherwise.

Similarly, let $N_M$ be the set of all $i$ with $\bar{\mu}_i < 0$. Then let $Y_i = 1 - \tilde{Y}_i$ if $i \in N_M$, and $Y_i = \tilde{Y}_i$, otherwise.

Note that $\bar{\lambda}_i \tilde{X}_i = \bar{\lambda}_i (1 - \tilde{X}_i) + \bar{\lambda}_i$. Thus, for $i \in N_A$ we get $\bar{\lambda}_i \tilde{X}_i = \bar{\lambda}_i X_i \sum_{i=1}^n |\tilde{\lambda}_i| + \bar{\lambda}_i$. For $i \notin N_A$ it follows that $\bar{\lambda}_i \tilde{X}_i = \bar{\lambda}_i X_i \sum_{i=1}^n |\tilde{\lambda}_i|$. Similarly, $\bar{\mu}_i \tilde{Y}_i = \bar{\mu}_i Y_i \sum_{i=1}^m |\tilde{\mu}_i| + \bar{\mu}_i$ for $i \in N_M$ and $\bar{\mu}_i \tilde{Y}_i = \bar{\mu}_i Y_i \sum_{i=1}^m |\tilde{\mu}_i|$ for $i \notin N_M$. Thus,

$$
\sum_{i=1}^n \bar{\lambda}_i \tilde{X}_i = \sum_{i=1}^n \bar{\lambda}_i X_i \sum_{i=1}^n |\tilde{\lambda}_i| + \sum_{i=1}^n \min(\tilde{\lambda}_i, 0), \text{ and } \sum_{i=1}^m \bar{\mu}_i \tilde{Y}_i = \sum_{i=1}^m \bar{\mu}_i Y_i \sum_{i=1}^m |\tilde{\mu}_i| + \sum_{i=1}^m \min(\tilde{\mu}_i, 0) \tag{23}
$$

Since $\delta = \sum_{i=1}^n A_i X_i$, and $\theta = \sum_{i=1}^m B_i Y_i$ equation (23) immediately implies (14).

It remains to prove that the modified model corresponds to the original model.

Note that (18) and (16) imply

$$
\frac{\alpha_t}{\sigma_t} = \bar{\alpha}_t - (1 + \rho_t) \sum_{i=1}^m \min(\tilde{\mu}_i, 0) + (1 + \alpha_t) \sum_{i=1}^n \min(\tilde{\lambda}_i, 0). \tag{24}
$$

(23) implies

$$
(1 + \alpha_t) \sum_{i=1}^n \bar{\lambda}_i \tilde{X}_i = (1 + \alpha_t) \sum_{i=1}^n \bar{\lambda}_i X_i \sum_{i=1}^n |\tilde{\lambda}_i| + (1 + \alpha_t) \sum_{i \in N_A} \bar{\lambda}_i = \frac{k_t}{\sigma_t} \sum_{i=1}^n \bar{\lambda}_i X_i + (1 + \alpha_t) \sum_{i=1}^n \min(\tilde{\lambda}_i, 0) \tag{25}
$$

$$
(1 + \rho_t) \sum_{i=1}^m \bar{\mu}_i \tilde{Y}_i = (1 + \rho_t) \sum_{i=1}^m \bar{\mu}_i Y_i \sum_{i=1}^m |\tilde{\mu}_i| + (1 + \rho_t) \sum_{i \in N_M} \bar{\mu}_i = \frac{1}{\sigma_t} \sum_{i=1}^m \bar{\mu}_i Y_i + (1 + \rho_t) \sum_{i=1}^m \min(\tilde{\mu}_i, 0). \tag{26}
$$

Next, note that $\sum_{i=1}^s D_i = 1$, since the year dummy for exactly one of the years is 1, and all other ones are zero. Thus, $(1 + \sum_{i=1}^s D_i \alpha_t) = \sum_{i=1}^s D_i (1 + \alpha_t)$. Similarly, it follows that $\sum_{i=1}^s D_i \frac{\kappa_t}{\sigma_t} = \sum_{i=1}^s \frac{D_i}{\sigma_t} \sum_{i=1}^s D_i \kappa_t$. Let $\alpha_0 = \rho_0 = 0$. Then, (25), and (26) imply

$$
\left(1 + \sum_{i=2}^s \alpha_i D_i \right) \sum_{i=1}^n \bar{\lambda}_i \tilde{X}_i - \left(1 + \sum_{i=2}^s \rho_i D_i \right) \sum_{i=1}^m \bar{\mu}_i \tilde{Y}_i + \sum_{i=1}^s \bar{\alpha}_i D_i
\right.

= \sum_{i=1}^s D_i (1 + \alpha_t) \sum_{i=1}^n \bar{\lambda}_i \tilde{X}_i - \sum_{i=1}^s D_i (1 + \rho_t) \sum_{i=1}^m \bar{\mu}_i \tilde{Y}_i + \sum_{i=1}^s \bar{\alpha}_i D_i.
\[
= \sum_{t=1}^{s} D_t \frac{k_t}{\sigma_t} \sum_{i=1}^{n} \lambda_i X_i - \sum_{t=1}^{s} \frac{D_t}{\sigma_t} \sum_{i=1}^{m} \mu_i Y_i \\
+ \sum_{t=1}^{s} D_t \left[ \tilde{a}_t + (1 + \alpha_t) \sum_{i=1}^{n} \min\{\tilde{\lambda}_i, 0\} - (1 + \rho_t) \sum_{i=1}^{m} \min\{\tilde{\mu}_i, 0\} \right] \\
= \sum_{t=1}^{s} \frac{D_t}{\sigma_t} \left[ \sum_{i=1}^{s} k_t D_t \sum_{i=1}^{n} \lambda_i X_i - \sum_{i=1}^{m} \mu_i Y_i + \sum_{i=1}^{s} D_t a_t \right],
\]

where the last equality follows from (24). The two models are therefore equivalent. ■
References


