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A tale of tails: An empirical analysis of loss distribution models for estimating operational risk capital

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An Appendix to
A Tale of Tails:
An Empirical Analysis of Loss Distribution Models for
Estimating Operational Risk Capital

Kabir Dutta and Jason Perry

The public release of *A Tale of Tails* generated many emails to the authors with questions about the research results not included in the paper. This appendix contains tables that summarize the unpublished results relevant to those emails, and is intended to enhance the reader's understanding of the research.

These tables are similar to tables included in the paper. They differ in the exposure indicators used to scale the data (gross income and total assets) and include some business lines and event types originally excluded. As in the paper, the designs of the tables are such that they anonymize the participant institutions.

The authors produced these results while they were employees of the Federal Reserve Bank of Boston.

Table 1: *Power Law Capital Estimates at the Business Line Level as a Percentage of Total Assets*

	5% Threshold	10% Threshold
Business Line 2: Trading & Sales		
Panel A: ξ Values by Institution		
	-	1.16
	-	0.92
	-	1.75
Panel B: Summary Statistics of Capital Estimates as a Percentage of Assets		
25th	-	15.75
Med	-	30.84
75th	-	1654.53
Business Line 3: Retail Banking		
Panel A: ξ Values by Institution		
	0.87	0.87
	1.04	0.94
	1.18	1.14
	1.25	1.18
	-	0.76
Panel B: Summary Statistics of Capital Estimates as a Percentage of Assets		
25th	3.70	1.31
Med	7.74	1.67
75th	29.28	8.31
Business Line 5: Payment & Settlement		
Panel A: ξ Values by Institution		
	-	0.78
	-	0.81
	-	0.77
Panel B: Summary Statistics of Capital Estimates as a Percentage of Assets		
25th	-	0.03
Med	-	0.03
75th	-	0.37

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Business Line 6: Agency Services		
Panel A: ξ Values by Institution		
	0.86	0.90
	1.00	1.17
	1.02	1.02
	-	0.80
	-	0.85
	-	0.93
Panel B: Summary Statistics of Capital Estimates as a Percentage of Assets		
25th	9.84	0.30
Med	15.71	3.43
75th	21.17	13.23
Business Line 9: Other		
Panel A: ξ Values by Institution		
	-	0.90
	-	1.41
	-	1.98
Panel B: Summary Statistics of Capital Estimates as a Percentage of Assets		
25th	-	79.53
Med	-	158.60
75th	-	10484.09

Panel A presents ξ for each business line, computed using the Hill estimator, with 5 and 10 percent of the data in the tail. Panel B presents summary statistics for capital estimates at the 99.9% level for each institution as a percentage of total assets. These capital estimates were not simulated, but instead calculated using the power law approximation method described in the paper. No goodness-of-fit tests are presented in this table.

Table 2: *Power Law Capital Estimates at the Event Type Level as a Percentage of Total Assets*

	5% Threshold	10% Threshold
Event Type 2: External Fraud		
Panel A: ξ Values by Institution		
	0.71	0.76
	0.74	0.74
	0.80	0.73
	0.95	1.00
Panel B: Summary Statistics of Capital Estimates as a Percentage of Assets		
25th	0.10	0.10
Med	0.11	0.11
75th	0.15	0.19
Event Type 3: Employment Practices & Workplace Safety		
Panel A: ξ Values by Institution		
	0.73	0.80
	0.86	0.97
	0.89	0.96
Panel B: Summary Statistics of Capital Estimates as a Percentage of Assets		
25th	0.20	0.39
Med	0.30	0.58
75th	0.37	0.89
Event Type 4: Clients, Products & Business Practices		
Panel A: ξ Values by Institution		
	0.78	0.81
	1.56	1.21
	1.62	1.62
	-	2.98
Panel B: Summary Statistics of Capital Estimates as a Percentage of Assets		
25th	734.25	27.11
Med	1468.13	5305.10
75th	6193.56	145333968.31

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**Event Type 7: Execution, Delivery & Process
Management**

Panel A: ξ Values by Institution

0.80	0.90
0.83	0.99
0.86	1.00
0.89	0.98
0.90	0.83
1.09	0.93
1.09	1.08

**Panel B: Summary Statistics of Capital Estimates as a
Percentage of Assets**

25th	2.10	7.59
Med	4.53	8.87
75th	6.06	15.62

Panel A presents ξ for each event type, computed using the Hill estimator, with 5 and 10 percent of the data in the tail. Panel B presents summary statistics for capital estimates at the 99.9% level for each institution as a percentage of total assets. These capital estimates were not simulated, but instead calculated using the power law approximation method described in the paper. No goodness-of-fit tests are presented in this table.

Table 3: *Standard Errors for g-and-h Parameter Estimates at the Business Line and Event Type Level*

	g	Std. error for g	h/η_0	Std. error for h/η_0	η_1	Std. error for η_1	η_2	Std. error for η_2	η_3	Std. error for η_3
Panel A: Business Line Level										
2	1.6554	0.0007	0.4107	0.0004	-	-	-	-	-	-
	1.9613	0.0004	0.2095	0.0006	-	-	-	-	-	-
	2.6413	0.0009	0.2102	0.0010	-	-	-	-	-	-
3	1.5473	0.0004	0.3250	0.0002	-	-	-	-	-	-
	1.5611	0.0005	0.5420	0.0004	-	-	-	-	-	-
	1.7750	0.0006	0.4326	0.0004	-	-	-	-	-	-
	1.7941	0.0001	0.2663	0.0009	-0.1199	0.0004	0.0224	0.0001	-0.0010	0.0000
	1.8615	0.0010	0.1684	0.0007	-	-	-	-	-	-
	1.9410	0.0010	-0.0403	0.0006	-	-	-	-	-	-
4	1.8568	0.0020	0.4831	0.0012	-	-	-	-	-	-
	2.0952	0.0006	0.3298	0.0005	-	-	-	-	-	-
	2.2063	0.0012	-0.1076	0.0004	-	-	-	-	-	-
5	1.2487	0.0004	0.2690	0.0004	-	-	-	-	-	-
	1.5550	0.0005	0.0615	0.0005	-	-	-	-	-	-
	1.6361	0.0006	-0.0173	0.0005	-	-	-	-	-	-
	1.6523	0.0022	0.5079	0.0012	-	-	-	-	-	-
6	1.7197	0.0007	0.4113	0.0005	-	-	-	-	-	-
	1.7604	0.0009	0.0978	0.0005	-	-	-	-	-	-
	1.7665	0.0011	0.2331	0.0006	-	-	-	-	-	-
	1.8576	0.0004	0.0868	0.0002	-	-	-	-	-	-
	1.8861	0.0005	0.2663	0.0004	-	-	-	-	-	-
7	1.3902	0.0011	0.1108	0.0015	-	-	-	-	-	-
	1.7967	0.0018	0.0331	0.0011	-	-	-	-	-	-
	1.8526	0.0011	-0.0518	0.0006	-	-	-	-	-	-
	1.9809	0.0014	0.0340	0.0007	-	-	-	-	-	-
8	1.4874	0.0004	-0.0825	0.0002	-	-	-	-	-	-
	1.5582	0.0003	0.1530	0.0002	-	-	-	-	-	-
	1.5896	0.0011	-0.0637	0.0009	-	-	-	-	-	-
9	1.3932	0.0006	0.1254	0.0004	-	-	-	-	-	-
	1.4855	0.0006	0.0480	0.0004	-	-	-	-	-	-
	2.2056	0.0010	0.2418	0.0010	-	-	-	-	-	-

TABLE CONTINUED ON NEXT PAGE

	g	Std. error for g	h/η_0	Std. error for h/η_0	η_1	Std. error for η_1	η_2	Std. error for η_2	η_3	Std. error for η_3
Panel B: Event Type Level										
1	1.5363	0.0010	0.3349	0.0005	-	-	-	-	-	-
	1.8990	0.0006	0.2250	0.0005	-	-	-	-	-	-
2	1.4264	0.0002	0.2471	0.0003	-	-	-	-	-	-
	1.5319	0.0009	0.1418	0.0008	-	-	-	-	-	-
	1.5446	0.0004	0.1341	0.0003	-	-	-	-	-	-
	1.6084	0.0006	0.2445	0.0007	-	-	-	-	-	-
	1.7683	0.0003	-0.1583	0.0016	0.0192	0.0008	0.0012	0.0001	-0.0001	0.0000
3	1.1909	0.0004	0.1943	0.0003	-	-	-	-	-	-
	1.4207	0.0006	0.2031	0.0010	-	-	-	-	-	-
	1.4672	0.0004	0.3239	0.0002	-	-	-	-	-	-
4	1.5876	0.0003	0.5658	0.0006	-	-	-	-	-	-
	1.6956	0.0005	0.0026	0.0002	-	-	-	-	-	-
	2.1618	0.0005	0.2905	0.0003	-	-	-	-	-	-
	2.4195	0.0017	0.2066	0.0021	-	-	-	-	-	-
6	1.9810	0.0016	0.1702	0.0010	-	-	-	-	-	-
7	0.9910	0.0005	0.9800	0.0004	-	-	-	-	-	-
	0.9920	0.0003	0.9909	0.0002	-	-	-	-	-	-
	0.9942	0.0003	0.9933	0.0002	-	-	-	-	-	-
	0.9970	0.0006	0.9910	0.0003	-	-	-	-	-	-
	0.9989	0.0003	0.9972	0.0003	-	-	-	-	-	-
	0.9992	0.0005	0.9900	0.0003	-	-	-	-	-	-
	1.6600	0.0002	-0.0314	0.0017	0.1149	0.0009	-0.0068	0.0001	0.0000	0.0000
8	1.9534	0.0014	0.4146	0.0006	-	-	-	-	-	-

This table presents the g and h parameters and the bootstrap standard error estimates for the g-and-h distribution. Panel A shows the enterprise level, Panel B shows the business line Level, and Panel C shows the event type level. Standard errors were estimated using 50,000 samples bootstrapped from the original data for each institution. Some institutions were fitted using only one h parameter. For the other institutions, four h parameters were used and $h(Z^2) = \eta_0 + \eta_1 * Z^2 + \eta_2 * Z^4 + \eta_3 * Z^6$.

Table 4: Capital Estimates at the Business Line Level as a Percentage of Total Assets

	Reasonable Results		Rarely Fit the Data			Generally Yielded Unreasonable Capital Estimates				
	g-and-h	Emp	Exp	Gamma	Weibull	EVT 5%	EVT 10%	GPD	Log-logistic	Truncated Lognormal
Business Line 2: Trading & Sales										
Panel A: Summary Stats of Capital Estimates as a Percentage of Total Assets for All Models										
# Modeled	3	6	6	6	6	3	3	6	6	5
# that Fit	3	6	1	1	2	2	2	6	6	5
Mean	0.27	0.05	0.02	0.02	0.01	12.75	2.32	225.72	2.85	5.83
Med	0.26	0.01	0.01	0.01	0.01	2.46	1.28	7.57	0.62	0.19
SD	0.09	0.07	0.02	0.03	0.01	19.34	2.65	532.98	3.91	12.77
25th	0.22	0.00	0.00	0.00	0.00	1.60	0.82	0.11	0.17	0.00
75th	0.31	0.10	0.03	0.03	0.01	18.77	3.31	22.98	6.00	0.26
Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)										
0-1.5%	3	6	1	1	2	0	1	3	4	4
1.5-3%	0	0	0	0	0	1	0	0	0	0
3-20%	0	0	0	0	0	0	1	1	2	0
20-100%	0	0	0	0	0	1	0	1	0	1
100-500%	0	0	0	0	0	0	0	0	0	0
500+%	0	0	0	0	0	0	0	1	0	0
Business Line 3: Retail Banking										
Panel A: Summary Stats of Capital Estimates as a Percentage of Total Assets for All Models										
# Modeled	6	6	6	6	6	4	4	6	6	6
# that Fit	6	6	0	0	0	4	4	3	4	5
Mean	0.30	0.04	0.02	0.02	0.01	41.51	4.32	1.14	0.96	1.40
Med	0.11	0.05	0.02	0.02	0.01	13.76	0.67	0.63	0.79	0.36
SD	0.42	0.03	0.01	0.01	0.01	65.14	7.71	0.99	0.48	2.73
25th	0.07	0.02	0.01	0.01	0.01	2.98	0.20	0.53	0.75	0.12
75th	0.29	0.06	0.03	0.03	0.02	52.29	4.79	1.42	0.94	0.59
Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)										
0-1.5%	6	6	0	0	0	1	3	1	3	4
1.5-3%	0	0	0	0	0	0	0	2	1	0
3-20%	0	0	0	0	0	1	1	0	0	1
20-100%	0	0	0	0	0	1	0	0	0	0
100-500%	0	0	0	0	0	1	0	0	0	0
500+%	0	0	0	0	0	0	0	0	0	0

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	Reasonable Results		Rarely Fit the Data			Generally Yielded Unreasonable Capital Estimates				
	g-and-h	Emp	Exp	Gamma	Weibull	EVT 5%	EVT 10%	GPD	Log-logistic	Truncated Lognormal
Business Line 4: Commercial Banking										
Panel A: Summary Stats of Capital Estimates as a Percentage of Total Assets for All Models										
# Modeled	3	5	5	5	5	-	-	5	5	5
# that Fit	3	5	0	0	1	-	-	5	4	5
Mean	0.16	0.01	0.00	0.00	0.00	-	-	9.09	0.64	0.22
Med	0.20	0.01	0.00	0.00	0.00	-	-	5.50	0.73	0.11
SD	0.15	0.01	0.00	0.00	0.00	-	-	13.54	0.54	0.29
25th	0.10	0.00	0.00	0.00	0.00	-	-	0.31	0.14	0.07
75th	0.24	0.01	0.00	0.00	0.00	-	-	6.83	0.94	0.18
Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)										
0-1.5%	3	5	0	0	1	-	-	2	4	5
1.5-3%	0	0	0	0	0	-	-	0	0	0
3-20%	0	0	0	0	0	-	-	2	0	0
20-100%	0	0	0	0	0	-	-	1	0	0
100-500%	0	0	0	0	0	-	-	0	0	0
500+%	0	0	0	0	0	-	-	0	0	0
Business Line 6: Agency Services										
Panel A: Summary Stats of Capital Estimates as a Percentage of Total Assets for All Models										
# Modeled	5	6	6	6	6	4	4	6	6	6
# that Fit	5	6	0	1	1	4	4	6	6	6
Mean	0.59	0.04	0.02	0.02	0.02	1.03	0.17	2.85	2.00	0.35
Med	0.14	0.02	0.01	0.01	0.01	0.90	0.12	2.92	1.60	0.20
SD	0.97	0.05	0.02	0.02	0.02	1.00	0.19	2.57	1.98	0.39
25th	0.04	0.00	0.00	0.00	0.00	0.44	0.05	0.52	0.28	0.04
75th	0.48	0.05	0.02	0.03	0.02	1.50	0.24	5.06	3.55	0.67
Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)										
0-1.5%	4	6	0	1	1	3	4	2	3	6
1.5-3%	1	0	0	0	0	1	0	1	1	0
3-20%	0	0	0	0	0	0	0	3	2	0
20-100%	0	0	0	0	0	0	0	0	0	0
100-500%	0	0	0	0	0	0	0	0	0	0
500+%	0	0	0	0	0	0	0	0	0	0

This table presents a summary of the 99.9% capital estimates for each Basel business line as a percentage of total assets. The capital estimates were simulated from one million trials. Panel A presents the first, second and third quartiles, which were calculated across each model. These statistics include both capital estimates that statistically fit and do not fit. Panel B presents a frequency distribution of banks whose capital estimates fit according to one or more of the following goodness-of-fit tests: Kolmogorov-Smirnoff, Chi-Square, and Anderson-Darling. The fit for the g-and-h distribution was only tested using Q-Q plots. No goodness-of-fit tests were performed for the empirical distribution. By construction, the empirical distribution would fit the data. All empirical estimates are included in these counts. The total number of business lines modeled and the number that fit are also presented in Panel A.

Table 5: *Capital Estimates at the Business Line Level as a Percentage of Total Assets*
(Additional Business Lines Not Originally Included in Paper)

	Reasonable Results		Rarely Fit the Data			Generally Yielded Unreasonable Capital Estimates				
	g-and-h	Emp	Exp	Gamma	Weibull	EVT 5%	EVT 10%	GPD	Log-logistic	Truncated Lognormal
Business Line 5: Payment & Settlement										
Panel A: Summary Stats of Capital Estimates as a Percentage of Total Assets for All Models										
# Modeled	4	5	5	5	5	3	3	5	5	5
# that Fit	4	5	0	0	0	3	3	5	5	5
25th	0.00	0.00	0.00	0.00	0.00	0.09	0.01	0.02	0.05	0.00
Med	0.01	0.00	0.00	0.00	0.00	0.17	0.01	0.12	0.08	0.02
75th	0.04	0.01	0.00	0.01	0.01	0.19	0.05	0.16	0.22	0.04
Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)										
0-50%	4	5	-	-	-	3	3	5	5	5
50-100%	-	-	-	-	-	-	-	-	-	-
100-200%	-	-	-	-	-	-	-	-	-	-
200-1000%	-	-	-	-	-	-	-	-	-	-
1000%+	-	-	-	-	-	-	-	-	-	-
Business Line 7: Asset Management										
Panel A: Summary Stats of Capital Estimates as a Percentage of Total Assets for All Models										
# Modeled	4	7	5	5	5	0	0	5	5	5
# that Fit	4	7	0	0	2	0	0	5	5	4
25th	0.01	0.00	0.00	0.00	0.00	-	-	1.33	0.56	0.01
Med	0.01	0.01	0.01	0.01	0.01	-	-	1.65	1.08	0.03
75th	0.05	0.02	0.02	0.02	0.02	-	-	3.62	1.49	0.08
Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)										
0-50%	4	7	-	-	2	-	-	4	5	4
50-100%	-	-	-	-	-	-	-	-	-	-
100-200%	-	-	-	-	-	-	-	1	-	-
200-1000%	-	-	-	-	-	-	-	-	-	-
1000%+	-	-	-	-	-	-	-	-	-	-
Business Line 8: Retail Brokerage										
Panel A: Summary Stats of Capital Estimates as a Percentage of Total Assets for All Models										
# Modeled	3	5	4	4	4	2	2	4	4	4
# that Fit	3	5	1	1	1	2	2	3	3	3
25th	0.01	0.00	0.00	0.00	0.00	0.27	0.09	0.01	0.18	0.00
Med	0.02	0.00	0.00	0.00	0.00	0.47	0.10	0.06	0.35	0.01
75th	0.02	0.01	0.01	0.01	0.01	0.68	0.10	0.16	1.01	0.01
Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)										
0-50%	3	5	1	1	1	2	2	3	3	3
50-100%	-	-	-	-	-	-	-	-	-	-
100-200%	-	-	-	-	-	-	-	-	-	-
200-1000%	-	-	-	-	-	-	-	-	-	-
1000%+	-	-	-	-	-	-	-	-	-	-

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Business Line 9: Other

Panel A: Summary Stats of Capital Estimates as a Percentage of Total Assets for All Models

# Modeled	3	7	0	0	0	3	3	5	5	0
# that Fit	3	7	0	0	0	0	0	3	4	0
25th	0.01	0.04	-	-	-	1648.21	1.33	0.10	0.26	-
Med	0.01	0.11	-	-	-	3295.69	2.19	0.56	0.70	-
75th	0.10	1.34	-	-	-	67851981.14	816.36	251.26	3.97	-

Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)

0-50%	3	7	-	-	-	-	-	1	3	-
50-100%	-	-	-	-	-	-	-	-	-	-
100-200%	-	-	-	-	-	-	-	-	-	-
200-1000%	-	-	-	-	-	-	-	1	-	-
1000%+	-	-	-	-	-	-	-	1	1	-

This table presents a summary of the 99.9% capital estimates for each Basel business line as a percentage of total assets. The capital estimates were simulated from one million trials. Panel A presents the first, second and third quartiles, which were calculated across each model. These statistics include both capital estimates that statistically fit and do not fit. Panel B presents a frequency distribution of banks whose capital estimates fit according to one or more of the following goodness-of-fit tests: Kolmogorov-Smirnoff, Chi-Square, and Anderson-Darling. The fit for the g-and-h distribution was only tested using Q-Q plots. No goodness-of-fit tests were performed for the empirical distribution. By construction, the empirical distribution would fit the data. All empirical estimates are included in these counts. The total number of business lines modeled and the number that fit are also presented in Panel A.

Table 6: *Capital Estimates at the Business Line Level as a Percentage of BL Gross Income*
(Additional Business Lines Not Originally Included in Paper)

	Reasonable Results		Rarely Fit the Data			Generally Yielded Unreasonable Capital Estimates				
	g-and-h	Emp	Exp	Gamma	Weibull	EVT 5%	EVT 10%	GPD	Log-logistic	Truncated Lognormal
Business Line 5: Payment & Settlement										
Panel A: Summary Stats of Capital Estimates as a Percentage of Business Line Gross Income for All Models										
# Modeled	4	5	5	5	5	3	3	5	5	5
# that Fit	4	5	0	0	0	3	3	5	5	5
25th	1.28	0.93	0.53	0.60	0.57	12.65	3.30	10.44	28.09	1.81
Med	1.81	1.12	0.60	0.65	0.60	14.50	4.80	11.57	35.63	3.17
75th	16.52	2.67	0.99	1.04	0.70	64.65	5.54	54.38	38.61	8.58
Panel B: Capital Estimates as a Percentage of Business Line Gross Income for Models that Fit (Frequency)										
0-50%	3	5	-	-	-	2	3	3	4	5
50-100%	1	-	-	-	-	-	-	2	1	-
100-200%	-	-	-	-	-	1	-	-	-	-
200-1000%	-	-	-	-	-	-	-	-	-	-
1000%+	-	-	-	-	-	-	-	-	-	-
Business Line 7: Asset Management										
Panel A: Summary Stats of Capital Estimates as a Percentage of Business Line Gross Income for All Models										
# Modeled	4	7	5	5	5	0	0	5	5	5
# that Fit	4	7	0	0	2	0	0	5	5	4
25th	2.46	1.10	0.79	0.94	0.95	-	-	178.84	145.22	6.92
Med	4.03	1.61	1.17	1.39	0.96	-	-	275.18	156.90	11.39
75th	6.71	3.14	1.17	1.50	1.42	-	-	1064.43	362.04	18.95
Panel B: Capital Estimates as a Percentage of Business Line Gross Income for Models that Fit (Frequency)										
0-50%	4	6	-	-	2	-	-	1	1	4
50-100%	-	-	-	-	-	-	-	-	-	-
100-200%	-	1	-	-	-	-	-	1	2	-
200-1000%	-	-	-	-	-	-	-	1	1	-
1000%+	-	-	-	-	-	-	-	2	1	-
Business Line 8: Retail Brokerage										
Panel A: Summary Stats of Capital Estimates as a Percentage of Business Line Gross Income for All Models										
# Modeled	3	5	4	4	4	2	2	4	4	4
# that Fit	3	5	1	1	1	2	2	3	3	3
25th	1.15	1.54	1.17	1.24	1.13	24.83	10.51	6.80	85.09	1.96
Med	1.24	1.76	1.38	1.44	1.27	36.45	15.32	15.01	130.02	2.04
75th	2.68	1.91	1.49	1.57	1.44	48.08	20.14	22.05	159.82	2.31
Panel B: Capital Estimates as a Percentage of Business Line Gross Income for Models that Fit (Frequency)										
0-50%	3	5	1	1	1	1	2	3	1	3
50-100%	-	-	-	-	-	1	-	-	-	-
100-200%	-	-	-	-	-	-	-	-	2	-
200-1000%	-	-	-	-	-	-	-	-	-	-
1000%+	-	-	-	-	-	-	-	-	-	-

This table presents a summary of the 99.9% capital estimates for each Basel business line as a percentage of business line gross income. The capital estimates were simulated from one million trials. Panel A presents the first, second and third quartiles, which were calculated across each model. These statistics include both capital estimates that statistically fit and do not fit. Panel B presents a frequency distribution of banks whose capital estimates fit according to one or more of the following goodness-of-fit tests: Kolmogorov-Smirnoff, Chi-Square, and Anderson-Darling. The fit for the g-and-h distribution was only tested using Q-Q plots. No goodness-of-fit tests were performed for the empirical distribution. By construction, the empirical distribution would fit the data. All empirical estimates are included in these counts. The total number of business lines modeled and the number that fit are also presented in Panel A.

Table 7: Capital Estimates at the Event Type Level as a Percentage of Total Assets
(Additional Event Type Not Originally Included in Paper)

	Reasonable Results		Rarely Fit the Data			Generally Yielded Unreasonable Capital Estimates				
	g-and-h	Emp	Exp	Gamma	Weibull	EVT 5%	EVT 10%	GPD	Log-logistic	Truncated Lognormal
Event Type 1: Internal Fraud										
Panel A: Summary Statistics of Capital Estimates as a Percentage of Total Assets for All Models										
# Modeled	2	7	4	4	4	-	-	4	4	4
# that Fit	2	7	0	2	2	-	-	4	4	4
25th	0.022	0.001	0.001	0.001	0.003	-	-	0.146	0.623	0.014
Med	0.028	0.004	0.001	0.001	0.004	-	-	0.379	0.886	0.019
75th	0.035	0.005	0.002	0.003	0.004	-	-	0.619	1.900	0.117
Panel B: Capital Estimates as a Percentage of Total Assets for Models that Fit (Frequency)										
0-1.5%	2	7	-	2	2	-	-	4	3	4
1.5-3%	-	-	-	-	-	-	-	-	-	-
3-20%	-	-	-	-	-	-	-	-	1	-
20-100%	-	-	-	-	-	-	-	-	-	-
100%+	-	-	-	-	-	-	-	-	-	-

This table presents a summary of the 99.9% capital estimates for each Basel event type as a percentage of total assets. The capital estimates were simulated from one million trials. Panel A presents the first, second and third quartiles, which were calculated across each model. These statistics include both capital estimates that statistically fit and do not fit. Panel B presents a frequency distribution of banks whose capital estimates fit according to one or more of the following goodness-of-fit tests: Kolmogorov-Smirnoff, Chi-Square, and Anderson-Darling. The fit for the g-and-h distribution was only tested using Q-Q plots. No goodness-of-fit tests were performed for the empirical distribution. By construction, the empirical distribution would fit the data. All empirical estimates are included in these counts. The total number of event types modeled and the number that fit are also presented in Panel A.

Table 8: *Summary Statistics for Allocation of g-and-h Capital Estimates at the Business Line Level*

	BL1	BL2	BL3	BL4	BL5	BL6	BL7	BL8	BL9
Mean	2.07%	12.91%	20.88%	12.88%	2.13%	32.98%	6.70%	1.14%	21.22%
SD	3.01%	14.83%	26.23%	15.55%	2.23%	38.10%	7.01%	1.21%	29.89%
25	0.03%	3.23%	3.23%	0.53%	0.20%	3.57%	0.06%	0.10%	1.38%
Med	0.06%	10.58%	10.04%	5.17%	0.91%	14.79%	5.27%	1.11%	4.54%
75	2.19%	12.32%	23.66%	26.00%	3.78%	65.82%	12.87%	1.18%	33.40%

This table gives the allocation of the 99.9% capital estimates across Basel event types (as a percentage of the total capital) for the g-and-h distribution. For the g-and-h distribution, event types with 100 or more observations were estimated. Some event types were combined in order to estimate using g-and-h. In these cases, the total capital for the combined event types was then reallocated among those event types using the empirical model results. For all other event types that could not be estimated due to limited data, estimates from the empirical model were substituted.

Table 9: *Summary Statistics for Allocation of g-and-h Capital Estimates at the Event Type Level*

	ET1	ET2	ET3	ET4	ET5	ET6	ET7	ET8
Mean	1.11%	1.43%	8.72%	40.03%	9.70%	2.91%	34.44%	3.91%
SD	1.49%	1.45%	20.52%	44.43%	24.35%	5.51%	40.64%	2.58%
25	0.32%	0.82%	0.52%	3.92%	0.07%	0.34%	4.98%	2.64%
Med	0.62%	0.99%	0.97%	10.14%	0.44%	0.70%	17.94%	4.06%
75	1.02%	1.21%	1.83%	85.13%	1.18%	1.79%	58.06%	5.25%

This table gives the allocation of the 99.9% capital estimates across Basel event types (as a percentage of the total capital) for the g-and-h distribution. For the g-and-h distribution, event types with 100 or more observations were estimated. Some event types were combined in order to estimate using g-and-h. In these cases, the total capital for the combined event types was then reallocated among those event types using the empirical model results. For all other event types that could not be estimated due to limited data, estimates from the empirical model were substituted.

Table 10: *Summary Statistics for Allocation of g-and-h Capital Estimates at the Business Line Level as a Percentage of Total Assets*

	BL1	BL2	BL3	BL4	BL5	BL6	BL7	BL8	BL9
Mean	0.01%	0.07%	0.19%	0.06%	0.02%	0.20%	0.02%	0.01%	0.21%
SD	0.01%	0.07%	0.30%	0.06%	0.02%	0.28%	0.03%	0.01%	0.31%
25	0.00%	0.02%	0.03%	0.01%	0.00%	0.02%	0.00%	0.00%	0.00%
Med	0.00%	0.07%	0.07%	0.04%	0.01%	0.08%	0.02%	0.01%	0.04%
75	0.01%	0.12%	0.12%	0.11%	0.04%	0.22%	0.05%	0.01%	0.29%

This table gives the allocation of the 99.9% capital estimates across Basel business lines (as a percentage of total assets) for the g-and-h distribution. For the g-and-h distribution, business lines with 100 or more observations were estimated. Some business lines were combined in order to estimate using g-and-h. In these cases, the total capital for the combined business lines was then reallocated among those business lines using the empirical model results. For all other business lines that could not be estimated due to limited data, estimates from the empirical model were substituted.

Table 11: *Summary Statistics for Allocation of g-and-h Capital Estimates at the Event Type Level as a Percentage of Total Assets*

	ET1	ET2	ET3	ET4	ET5	ET6	ET7	ET8
Mean	0.01%	0.01%	0.04%	0.30%	0.11%	0.03%	0.20%	0.04%
SD	0.01%	0.01%	0.07%	0.39%	0.27%	0.06%	0.30%	0.03%
25	0.00%	0.01%	0.00%	0.02%	0.00%	0.00%	0.04%	0.02%
Med	0.00%	0.01%	0.01%	0.06%	0.00%	0.00%	0.10%	0.03%
75	0.01%	0.01%	0.02%	0.49%	0.01%	0.01%	0.18%	0.05%

This table gives the allocation of the 99.9% capital estimates across Basel event types (as a percentage of total assets) for the g-and-h distribution. For the g-and-h distribution, event types with 100 or more observations were estimated. Some event types were combined in order to estimate using g-and-h. In these cases, the total capital for the combined event types was then reallocated among those event types using the empirical model results. For all other event types that could not be estimated due to limited data, estimates from the empirical model were substituted.

Table 12: *Difference Between Aggregate Capital Estimates (as a Percentage of Enterprise Assets) Under Comonotonic and Independence Structures at the Business Line Level*

	Emp	EVT 5%	EVT 10%	Exp	Gamma	g-and-h	GPD	Log- logistic	Weib
A	0.05%	0.04%	0.03%	0.01%	0.01%	0.41%	-6.44%	-0.59%	0.01%
B	0.05%	-324.43%	-0.37%	0.01%	0.01%	0.10%	0.22%	0.26%	0.01%
C	0.24%	-36.65%	-629.08%	0.02%	0.05%	0.25%	-80661.37%	-29.44%	0.04%
D	0.10%	-0.10%	-0.97%	0.01%	0.02%	0.49%	32.61%	-15.15%	0.01%
E	0.07%	-7.32%	-296.59%	0.01%	0.01%	0.05%	-120.04%	-1.02%	0.01%
F	0.07%	0.00%	0.00%	0.02%	0.03%	0.49%	-7.19%	-1.31%	0.03%
G	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	-5.07%	-0.56%	0.00%
Mean	0.08%	-52.64%	-132.43%	0.01%	0.02%	0.26%	-11538.18%	-6.83%	0.02%
Std Dev	0.07%	120.59%	245.27%	0.01%	0.02%	0.21%	30480.50%	11.35%	0.01%
25th	0.05%	-21.98%	-148.78%	0.01%	0.01%	0.08%	-63.62%	-8.23%	0.01%
Med	0.07%	-0.10%	-0.37%	0.01%	0.01%	0.25%	-6.44%	-1.02%	0.01%
75th	0.09%	0.00%	0.00%	0.01%	0.02%	0.45%	-2.43%	-0.58%	0.02%

This table shows the difference between the 99.9% capital estimates calculated under two dependence assumptions: comonotonicity (simple addition) and independence (zero correlation), summed from the Basel business line level to the enterprise level and given as a percentage of the total assets. The mean, median, standard deviation, and interquartile statistics were calculated across all institutions. If certain business lines were not large enough to estimate under the given distribution, those business lines were left off of these calculations.

Table 13: *Difference Between Aggregate Capital Estimates (as a Percentage of Enterprise Assets) Under Comonotonic and Independence Structures at the Event Type Level*

	Emp	EVT 5%	EVT 10%	Exp	Gamma	g-and-h	GPD	Log- logistic	Weib
A	0.56%	0.00%	0.00%	0.01%	0.02%	0.01%	-4.56%	-0.31%	0.02%
B	0.05%	-82.26%	-2.40%	0.01%	0.01%	0.10%	-0.56%	0.06%	0.01%
C	0.09%	1.13%	-52.17%	0.00%	0.01%	0.07%	-18.06%	-0.56%	0.00%
D	0.04%	0.05%	0.94%	0.01%	0.01%	0.11%	-850.49%	-8.60%	0.01%
E	0.03%	-0.39%	-34.48%	0.00%	0.01%	0.11%	-0.44%	0.14%	0.01%
F	0.08%	0.00%	0.00%	0.02%	0.03%	0.10%	-215.42%	-6.53%	0.04%
G	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	-1.36%	-0.80%	0.00%
Mean	0.12%	-11.64%	-12.59%	0.01%	0.01%	0.07%	-155.84%	-2.37%	0.01%
Std Dev	0.20%	31.14%	21.63%	0.01%	0.01%	0.05%	316.25%	3.61%	0.01%
25th	0.03%	-0.20%	-18.44%	0.00%	0.01%	0.04%	-116.74%	-3.66%	0.01%
Med	0.05%	0.00%	0.00%	0.01%	0.01%	0.10%	-4.56%	-0.56%	0.01%
75th	0.08%	0.02%	0.00%	0.01%	0.01%	0.11%	-0.96%	-0.13%	0.01%

This table shows the difference between the 99.9% capital estimates calculated under two dependence assumptions: comonotonicity (simple addition) and independence (zero correlation), summed from the Basel event type level to the enterprise level and given as a percentage of the independence estimates. The mean, median, standard deviation, and interquartile statistics were calculated across all institutions. If certain event types were not large enough to estimate under the given distribution, those event types were left off of these calculations.

Table 14: *g-and-h Loss Multiplier at the Enterprise Level*

	Largest Loss	Second Largest	Third Largest	Fourth Largest	Fifth Largest	Median Loss
Mean	12.8	20.5	47.4	61.5	111.7	155,510.6
Std Dev	12.2	15.1	30.0	32.1	76.7	204,049.2
25th	2.2	9.4	24.2	46.7	66.0	31,233.5
Med	8.1	17.1	49.3	57.4	91.6	50,157.5
75th	21.7	28.1	66.4	80.6	154.3	194,616.3

This table presents sample statistics for the g-and-h distribution loss multipliers, where the loss multiplier is defined as the 99.9% g-and-h capital estimate divided by the given loss. These statistics are calculated across all institutions for the median and five largest losses.

Table 15: *g-and-h Loss Multiplier at the Business Line Level*

	Largest Loss	Second Largest	Third Largest	Fourth Largest	Fifth Largest	Median Loss
Business Line 2: Trading & Sales						
Mean	24.7	57.1	102.3	141.2	244.3	97,441.6
Std Dev	29.4	56.9	118.9	138.8	109.7	53,720.3
25th	7.7	25.0	34.5	61.0	181.3	78,628.1
Med	7.9	37.3	52.0	65.1	192.4	121,336.8
75th	33.2	79.3	145.1	183.3	281.4	128,202.6
Business Line 3: Retail Banking						
Mean	49.8	119.3	188.4	218.2	258.4	105,407.4
Std Dev	49.8	129.4	218.6	252.7	308.0	191,654.6
25th	7.7	29.4	39.3	45.6	48.8	7,405.0
Med	40.6	60.9	84.4	105.0	121.1	20,548.4
75th	79.4	196.3	306.0	333.8	375.9	77,960.4
Business Line 4: Commercial Banking						
Mean	67.7	321.5	355.0	419.3	715.5	67,036.7
Std Dev	81.5	423.9	428.2	505.8	998.7	74,976.9
25th	21.8	80.0	113.7	134.4	142.5	26,264.7
Med	39.2	149.0	206.8	244.9	253.5	51,453.4
75th	99.4	476.7	522.2	617.1	1,057.5	100,017.1
Business Line 5: Payment & Settlement						
Mean	153.5	185.9	276.5	316.6	343.5	7,654.4
Std Dev	298.4	341.7	496.2	555.3	582.9	12,488.6
25th	3.7	11.4	23.4	28.9	32.6	906.4
Med	5.0	19.6	32.5	45.1	62.9	1,736.1
75th	154.8	194.1	285.5	332.8	373.8	8,484.1
Business Line 6: Agency Services						
Mean	45.7	55.3	80.3	124.9	166.2	14,809.1
Std Dev	27.4	30.8	50.2	68.1	84.3	13,145.6
25th	26.4	38.1	38.2	102.2	110.6	5,406.2
Med	34.6	45.2	81.5	121.4	216.6	10,205.4
75th	73.4	81.4	127.6	147.6	225.1	20,396.3
Business Line 7: Asset Management						
Mean	16.0	30.8	37.0	54.3	72.5	1,311.6
Std Dev	15.0	9.0	9.5	22.1	31.8	752.0
25th	4.6	25.7	33.5	41.9	59.5	752.9
Med	15.7	28.7	37.5	58.5	81.0	1,176.8
75th	27.2	33.8	41.0	70.9	94.0	1,735.4
Business Line 8: Retail Brokerage						
Mean	14.9	19.8	25.1	39.3	45.7	3,014.1
Std Dev	16.8	15.1	22.0	21.1	20.6	3,918.8
25th	5.5	11.1	12.4	32.7	39.7	769.0
Med	9.1	11.5	13.9	50.5	57.6	1,254.5
75th	21.5	24.4	32.2	51.5	57.6	4,379.4
Business Line 9: Other						
Mean	0.4	1.9	8.1	35.6	73.8	31,501.4
Std Dev	0.2	1.3	5.8	32.3	84.6	51,313.1
25th	0.3	1.2	5.3	18.1	26.0	1,876.7
Med	0.4	1.8	8.6	29.2	43.7	2,277.8
75th	0.5	2.5	11.1	49.9	106.5	46,514.3

This table presents sample statistics for the g-and-h distribution loss multipliers, where the loss multiplier is defined as the 99.9% g-and-h capital estimate divided by the given loss. These statistics are calculated across all institutions for the median and five largest losses.

Table 16: *g-and-h Loss Multiplier at the Event Type Level*

	Largest Loss	Second Largest	Third Largest	Fourth Largest	Fifth Largest	Median Loss
Event Type 2: External Fraud						
Mean	7.3	21.6	54.1	71.4	83.9	4,572.6
Std Dev	6.9	14.1	33.9	36.5	44.7	3,106.3
25th	2.2	14.0	24.5	50.4	54.5	3,110.6
Med	4.4	18.8	50.2	56.5	73.0	3,807.6
75th	13.4	24.4	78.8	103.3	119.2	6,552.9
Event Type 3: Employment Practices & Workplace Safety						
Mean	18.8	37.9	52.4	87.8	156.3	167,607.7
Std Dev	27.0	51.4	50.9	41.7	97.2	182,269.3
25th	2.8	13.7	24.3	61.4	96.0	12,761.2
Med	4.1	18.3	24.7	85.5	148.9	118,821.0
75th	18.9	20.3	67.3	101.6	153.1	274,325.5
Event Type 4: Clients, Products & Business Practices						
Mean	10.2	18.1	35.0	53.0	126.0	206,165.6
Std Dev	8.7	3.1	21.6	26.5	131.0	185,442.2
25th	2.8	17.1	24.1	36.6	55.7	89,814.2
Med	9.6	19.2	24.5	51.1	81.1	196,573.3
75th	17.0	20.1	35.4	67.4	151.3	312,924.7
Event Type 7: Execution, Delivery & Process Management						
Mean	14.5	20.9	39.7	61.7	68.1	12,715.2
Std Dev	7.4	7.7	21.7	34.9	39.4	4,889.0
25th	9.2	15.8	24.1	31.8	35.4	10,404.0
Med	12.1	18.5	38.0	74.7	80.2	13,862.7
75th	19.0	28.4	50.6	83.3	87.5	15,228.2

This table presents sample statistics for the g-and-h distribution loss multipliers, where the loss multiplier is defined as the 99.9% g-and-h capital estimate divided by the given loss. These statistics are calculated across all institutions for the median and five largest losses.

Table 17: Average *b* Parameter
by Business Line, Event Type and Overall

	b
Business Line 2: Trading & Sales	0.0043
Business Line 3: Retail Banking	0.0019
Business Line 4: Commercial Banking	0.0039
Business Line 5: Payment & Settlement	0.0023
Business Line 6: Agency Services	0.0028
Business Line 7: Asset Management	0.0044
Business Line 8: Retail Brokerage	0.0039
Event Type 1: Internal Fraud	0.0031
Event Type 2: External Fraud	0.0011
Event Type 3: Employment Practices & Workplace Safety	0.0036
Event Type 4: Clients, Products & Business Practices	0.0033
Event Type 6: Business Disruption & System Failures	0.0031
Event Type 7: Execution, Delivery & Process Management	0.0023
Overall	0.0024