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Lie, Denny

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FEDERAL RESERVE BANK OF BOSTON

No. 09-20

State-Dependent Pricing and Optimal Monetary Policy

Denny Lie

Abstract:
This paper analyzes optimal monetary policy under precommitment in a state-dependent pricing (SDP) environment. Under SDP, monopolistically competitive firms are allowed to endogenously change the timing of price adjustments. I show that this endogenous timing of price adjustment alters the tradeoff and the cost of inflation variation faced by the monetary authority in comparison to the standard time-dependent pricing (TDP) assumption. In particular, it is desirable to let inflation vary more under SDP. Despite the change in the policy tradeoff, however, the optimal response under SDP to either a productivity shock or a government purchase shock under the timeless perspective (long-run) policy can still be characterized as an approximate price stability rule. In addition to a standard first-order approximation to the equilibrium solution, this paper also computes a second-order solution where the effect of state-dependence can play a central role. The policy response under SDP exhibits some degree of nonlinearity, especially in the presence of larger shocks and when the state of the economy is farther away from the steady state. Finally, this paper also studies the optimal policy start-up problem related to the cost of adopting the timeless perspective policy instead of the true Ramsey policy. The SDP assumption leads to different start-up dynamics compared to the dynamics under the TDP assumption in several interesting ways. In particular, the change in the policy tradeoff gives rise to much higher start-up inflation under SDP.

JEL Classifications: E31, E52, E61

Denny Lie is a research associate at the Federal Reserve Bank of Boston and is affiliated with Boston University. His e-mail address is dlie@bu.edu.

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1 Introduction

In modern macroeconomic theory, monetary policy is thought to have real effects because of a tradeoff between nominal and real activity, such as that implied by predetermined prices. The task of the monetary authority is to optimally exploit this tradeoff in response to various shocks to the economy. Currently there are a large and growing number of studies in the literature that analyze the nature of optimal monetary policy. A common denominator of all these studies is that the nature of the pricing friction responsible for the presence of predetermined prices is time-dependent. That is, firms have no choice about the timing of their price adjustments, as in the familiar work of Calvo (1983) and Taylor (1980). One reason why time-dependent pricing (henceforth, TDP) models are popular in optimal monetary policy analysis is because there is no need to track the price distribution of firms and the frequency of price adjustment. For example, in Calvo (1983), a single parameter (the probability of price adjustment) summarizes the whole distribution. In state-dependent pricing (henceforth, SDP) models, one needs to track many state variables involving predetermined prices and the distribution of firms according to the time since the last price adjustment.

There are at least two reasons why it is desirable to move away from the realm of TDP in optimal monetary policy analysis. First, the endogenous timing of price adjustment under SDP may alter the inflation-output tradeoff faced by the monetary authority. This modification to the tradeoff in turn may lead to a different prescription as to the optimal conduct of monetary policy. In such an environment, the monetary authority needs to take into account the effect of its policy on the frequency of price adjustment, since this affects the course of real economic activity. Second, there is increasing evidence documenting state dependence in firms' price adjustment activities. For example Nakamura and Steinsson (2008), among others, show that it is the frequency, rather than the size of price adjustment, that has a strong positive correlation with inflation.

In this paper, I study optimal monetary policy under precommitment in a state-dependent pricing environment. Specifically, I assume that firms' price adjustment process follows the model of Dotsey, King, and Wolman (1999), in which firms are allowed to endogenously change their timing of price adjustments by paying a small fixed menu cost. In terms of the optimal monetary policy approach, this paper follows the common practice in the public finance literature, for instance, as in Chamley (1986). Other examples of optimal monetary policy studies in the literature using this approach include Erceg, Anderson, and Levin (2000); Khan, King, and Wolman (2003); Levin et. al.

(2005); Schmitt-Grohe and Uribe (2005, 2007), all of whom analyze a closed-economy setting, while Benigno and Benigno (2003) and Faia and Monacelli (2004) analyze an open-economy setting. This paper thus departs from the widespread use of the linear-quadratic (LQ) approach, as surveyed in Clarida, Galí, and Gertler (1999). In such an approach, one needs an explicit reduced-form equation involving inflation dynamics (the Phillips curve) and the literature has not provided a convenient and thorough representation of state-dependent pricing for that purpose.[1] Furthermore, the public finance approach uses a direct and natural criterion for the evaluation of welfare: households' lifetime utility. In this approach, market distortions (inefficiencies) are identified and monetary policy affects welfare through its influence on the variations in these distortions.

Specifically, there are four set of distortions present in the model: (i) the markup distortion that arises from firms' monopoly power, which causes market-generated output level to be inefficient; (ii) the relative-price distortion arising from firms' asynchronous price-adjustment process; (iii) the monetary (exchange) distortions due to the use of money and credit to purchase final consumption goods; and (iv) the menu cost distortion due to the fixed cost of price adjustment. There are generally tradeoffs between these distortions requiring the monetary authority to balance the extent of these distortions in the process of achieving the socially optimum allocation. The monetary authority is assumed to solve a precommitment Ramsey problem in a decentralized economy setting where the private sector's efficiency conditions must be respected at all times.

Another contribution of this paper is in terms of the method used to solve for the optimal policy problem. Starting from the work of King, Plosser, and Rebelo (1988), most dynamic stochastic general equilibrium models are solved using a first-order (linear) approximation method. Many have argued that linear approximations to SDP models miss the state-dependence nature and the nonlinearity property of such models. In light of recent development in the literature on second-order approximation solution methods—for instance, Kim et. al. (2008), Lombardo and Sutherland (2007), and Schmitt-Grohe and Uribe (2004)—I also compute a second-order approximate equilibrium solution to the optimal policy problem, in addition to the standard linear approximate solution. In particular, I use the recent perturbation approach of Johnston, King, and Lie (2009). This approach is convenient because the second-order approximate solution can be directly cast in a state-space form, which permits a ready comparison to the first-order approximate solution.

The first finding of this paper involves the timeless perspective (Woodford, 2003) responses to

[1] Studies that attempt to derive a state-dependent pricing Phillips curve include Bakhshi, Khan, and Rudolf (2007), Gertler and Leahy (2008), and Hernandez (2004).

a temporary productivity shock and a temporary government purchase shock. I show that the optimal response to either shock can be characterized as an approximate price stability rule, in a sense that the price level is still largely stabilized around its deterministic trend. Hence, the optimal policy under SDP is to closely replicate the dynamics under the TDP assumption previously found in studies such as Khan, King, and Wolman (2003) and Schmitt-Grohe and Uribe (2007). Despite the close association to optimal response under TDP, the presence of endogenous timing of price adjustment under SDP alters the policy tradeoff faced by the monetary authority. In particular, I show that it is optimal for the monetary authority to let inflation vary *more* under SDP. As we shall see, this somewhat smaller focus on inflation stabilization is precisely due the changing nature of the policy tradeoff under SDP.

Within the long-run timeless perspective policy, I also show that the optimal response based on a second-order approximation to the policy problem is virtually identical to the response computed using a standard linear approximation method. This property holds even under the SDP assumption. This finding suggests that there are neglible second-order components (state-dependence and nonlinearity) in the optimal policy response under SDP. However, as is standard, this conclusion is achieved when the state of the economy prior to a shock is at the deterministic steady-state. When this assumption on the starting state is relaxed, there are indeed some differences between the first- and second-order approximate dynamics under the assumption of SDP. The degree of nonlinearity is shown to depend on the interaction between the state of the economy, the size of the shock, and the assumed policy rule.

Finally, this paper also studies the optimal monetary policy start-up problem. As described in Woodford (2003), a timeless perspective policy is a policy that is assumed to have been long implemented. Most studies on optimal precommitment monetary policy have focused exclusively on optimal timeless-perspective policies. However, even though a precommitment policy is optimal from the timeless perspective, it is not the true Ramsey solution that maximizes the welfare of the representative agent. The true Ramsey solution specifies that the monetary authority should treat the early period of precommitment policy implementation differently than subsequent periods. This is because in the starting period, there is no past commitment that the monetary authority must follow. With monopolistically competitive firms and nominal price rigidity as in the model in this paper, this so-called *start-up problem* is manifested in the optimal decision of the monetary authority to temporarily stimulate the economy by generating surprise inflation in the starting

period. It is optimal to do so since the economy operates inefficiently due to the presence of firms' monopoly power. Given the suboptimality of the timeless perspective policy, it is of interest to investigate the cost of adopting this policy instead of the true Ramsey policy. The start-up problem essentially measures the welfare loss of ignoring this starting period of policy implementation.

I find that incorporating SDP in the model leads to different start-up dynamics compared to the dynamics under the standard TDP assumption along several interesting dimensions. In particular, it is optimal to generate much *higher* start-up inflation despite the fact that the monetary authority is shown to have *less* leverage over real activity in the presence of SDP. This result is once again due to the subtle modification to the policy tradeoff involving the lower cost of inflation variation on the relative-price distortion. However, the welfare improvement from generating this surprise inflation is shown to be relatively small. Thus, the timeless-perspective policy may be a good approximation to the true Ramsey policy.

The rest of this paper is organized as follows. Section 2 presents the model used in the analysis, and lays out the optimal monetary policy approach and the solution methodology. I also present the calibration of the model and the steady-state property under the optimal precommitment policy. Section 3 analyzes the timeless perspective (long-run) responses to the two shocks, for both first- and second-order approximate solutions. Section 4 studies the nonlinearity issue further, showing how impulse responses depend on the state of the economy and the shock size. Section 5 investigates the start-up problem. Section 6 concludes.

2 The model

This section starts by discussing the private sector's efficiency conditions of the model economy. The description of the optimal precommitment policy problem, in which all the private sector's efficiency conditions must be respected at all times, follows next.

2.1 The private sector's constraints

The private sector consists of two sets of agents, a representative household and a continuum of intermediate-good firms on a unit measure. These private sector agents solve dynamic optimization problems given the state of the economy and the knowledge of the optimal policy rule employed by

a committed monetary policy authority.[2] The economy has two sets of state variables. The first set, s_t^1, concerns those state variables involving the private sector. Further, s_t^1 can be decomposed into the endogenous state vector, k_t and the exogenous state vector, ς_t. Hence, $s_t^1 = (k_t, \varsigma_t)$. Further, I assume that the exogenous state vector evolves according to the process $\varsigma_{t+1} = v(\varsigma_t, \eta_{t+1})$, where η_t is a vector of serially independent shocks. Lagged optimal policy multipliers, collected in a vector s_t^2, are the additional set of state variables. These multipliers summarize the past policy plans of the monetary authority that must be respected under the precommitment policy. The entire set of state variables is s_t, with $s_t = (s_t^1, s_t^2)$. Later in the section the individual elements of these state vectors will be clarified. In the model, aggregate fluctuations are driven by two exogenous shocks, a productivity shock and a government purchase shock, which affect s_t.

2.1.1 Intermediate-good firms

There is a continuum of monopolistically competitive firms in the economy, indexed by $i \in [0, 1]$, where each firm produces a differentiated intermediate good. These varieties of intermediate goods are bundled together into a final consumption good according to the constant elasticity of substitution (CES) aggregator,

$$c_t = \left[\int_0^1 c_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε is both the constant elasticity of substitution and the relative demand elasticity for any variety i. There is also a government that purchases final consumption goods (g_t) aggregated according to the same process.[3] Hence, if we let $p_t(i)$ be the relative price of variety i, the demand for variety i is then given by $p_t(i)^{-\varepsilon} y_t = p_t(i)^{-\varepsilon}(c_t + g_t)$, where $y_t = c_t + g_t$ is the aggregate economy-wide output.

Each firm i is subject to a fixed cost that needs to be paid every time it adjusts its (nominal) price. As in Dotsey, King, and Wolman (1999), the fixed adjustment costs (in labor units) are heterogenous across firms and are drawn each period independently from a time-invariant continuous distribution with cumulative distribution function $G(\cdot)$. Firms that choose not to adjust prices, after observing the fixed cost draw and the state of the economy, do not have to pay these fixed costs but must keep the nominal price from the previous period. This specification means that, at any given period, firms will be distributed according to "the time since the last price adjustment"

[2] This paper abstracts from deriving such an optimal policy rule. Instead, I search for the allocation that is consistent with this optimal policy rule.

[3] Government purchases are assumed to be financed by lump-sum taxes.

j, with $j = 0$ indicating those firms that adjust in the current period. Let $\omega_{j,t}$ be the end-of-period fraction of firms that last adjusted their prices j periods ago. By end-of-period, I mean that these fractions are observed in the current period t after firms' production and pricing decisions are made. These variables summarize the distribution of firms according to the time since the last price adjustment and can be expressed as

$$\omega_{j,t} = (1 - \alpha_{jt})\omega_{j-1,t-1} \tag{1}$$

for $j = 1, ..., J-1$, where J is the longest period of price fixity that has to be endogenously determined in the steady state.[4] For each j, the variable $\alpha_{j,t}$ is the fraction of firms j—firms that last adjusted prices j periods ago—that decide to adjust at the beginning of period t right after the fixed cost draw. Since the fraction of all firms sums to one, the fraction of adjusting firms in the current period t ($\omega_{0,t}$) is then given by[5]

$$\omega_{0,t} = 1 - \sum\nolimits_{j=1}^{J-1} \omega_{j,t} \ . \tag{2}$$

Note that in the current SDP specification, both $\{\alpha_{jt}\}_{j=1}^{J-1}$ and $\{\omega_{j,t}\}_{j=0}^{J-1}$ are determined endogenously and depend on aggregate variables and firms' prices. Under the TDP assumption as in Levin (1991) and Khan, King, and Wolman (2003), these variables are exogenous and become parameters of the model.[6]

Next, I describe the dynamic maximization problem for adjusting firms. Firms are monopolistically competitive and use labor as the only production input. Specifically, the production function for any firm i is given by $y_t(i) = a_t n_t(i)$ where a_t is the exogenous aggregate productivity level and $n_t(i)$ is the production labor used by firm i. The labor market is global so that all firms faced the same real marginal cost w_t/a_t, where w_t is the aggregate real wage. Let $p_{j,t}$ be the relative price of firms that have not adjusted for j period(s) at time t. Given the form of the CES consumption aggregator above, it follows that the real profit of a firm with price $p_{j,t}$ is given by

$$z(p_{j,t}, s_t) = [p_{j,t} - \frac{w_t}{a_t}] \cdot p_{j,t}^{-\varepsilon}(c_t + g_t) \ .$$

[4] As long as inflation is not zero and the support of the fixed cost distribution is finite, J will be finite as well.

[5] One can also pin down $\omega_{0,t}$ by defining beginning-of-period fractions $\{\theta_{j,t}\}_{j=1}^{J}$. The fraction of adjusting firms can now be written as $\omega_{0,t} = \sum_{j=1}^{J} \alpha_{j,t}\theta_{j,t}$, with the understanding that $\alpha_{J,t} = 1$. The end-of-period and beginning-of period fractions are related by the relationship $\theta_{j,t} = \omega_{j-1,t-1}$, for $j = 1, ..., J$.

[6] The TDP specification is a generalization of two popular price/wage rigidity specifications. When $J \to \infty$ and α_j are identical across j, the specification collapses to Calvo (1983). For any finite J and $\{\alpha_j\}_{j=1}^{J-1} = 0$ with $\alpha_J = 1$, we have the contracting model of Taylor (1980).

Let's denote $v_0(s_t)$ as the value function of a typical adjusting firm, gross of the adjustment cost. Given the state vector s_t, the maximization problem of a typical adjusting firm is

$$\begin{aligned} v_0(s_t) &= \max_{p_{0,t}} \{z(p_{0,t}, s_t) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_{1,t+1})v_1(p_{1,t+1}, s_{t+1}) \\ &\quad +\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\alpha_{1,t+1}v_0(s_{t+1}) - \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\alpha_{1,t+1}w_{t+1}\Xi_{1,t+1}\} \ , \end{aligned} \tag{3}$$

where the choice variable $p_{0,t}$ is the optimal relative price. The above expression says that an adjusting firm chooses the optimal price to maximize its expected present discounted values of current and future profits. In doing so, it has to take into account the possibility of price adjustment in future periods. There is an expected probability of $(1-\alpha_{1,t+1})$ that it will choose not to adjust so that its expected value in the next period is $v_1(p_{1,t+1}, s_{t+1})$. With expected probability $\alpha_{1,t+1}$, it will optimally choose to adjust so that its value becomes $v_0(s_{t+1})$. The last term in the second line of (3) reflects the fact that if the firm decides to adjust in the next period, it must also pay the fixed adjustment cost. In the Bellman equation above, $\Xi_{1,t+1}$ is the expected fixed adjustment cost in the future period, conditional on adjustment. Since the fixed costs are in terms of labor unit, the expected cost in consumption units is then given by $w_{t+1}\Xi_{1,t+1}$. Note that since households own the firms, future periods are discounted by the effective discount factor $\beta E_t \frac{\lambda_{t+1}}{\lambda_t}$, where λ_t is the shadow value of households' income.

For firms that do not adjust ($j = 1, ..., J-1$) and hence simply apply prices from the previous period, the value functions can be expressed as[7]

$$\begin{aligned} v_j(p_{j,t}, s_t) &= z(p_{j,t}, s_t) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_{j+1,t+1})v_{j+1}(p_{j+1,t+1}, s_{t+1}) \\ &\quad +\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\alpha_{j+1,t+1}v_0(s_{t+1}) - \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\alpha_{j+1,t+1}w_{t+1}\Xi_{j+1,t+1} \ . \end{aligned} \tag{4}$$

The first order necessary condition of the the dynamic problem (3) and recursive differentiation of (4) lead to a formula for the optimal nominal price,

$$P_{0,t} = p_{0,t} \cdot P_t = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t \frac{\omega_{j,t+j}}{\omega_{0,t}} \frac{\lambda_{t+j}}{\lambda_t} \frac{W_{t+j}}{a_{t+1}} (P_{t+j})^{\varepsilon} y_{t+j}}{\sum_{j=0}^{J-1} \beta^j E_t \frac{\omega_{j,t+j}}{\omega_{0,t}} \frac{\lambda_{t+j}}{\lambda_t} (P_{t+j})^{\varepsilon-1} y_{t+j}} \ , \tag{5}$$

where P_t and W_t are the aggregate price level (index) and nominal wage at time t, respectively. The CES aggregator implies the aggregate price index is given by

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \ .$$

[7] There is no max operator in this expression since the only decision made by non-adjusting firms is the input decision for production to meet demand.

The expression $(\omega_{j,t+j}/\omega_{0,t}) = (1-\alpha_{j,t+j}) \cdot (1-\alpha_{j-1,t+j-1}) \cdot \ ... \cdot \ (1-\alpha_{1,t+1})$ in the optimal price equation is the expected probability of keeping the price constant (non adjustment) from time t until time $t+j$. Equation (5) is similar in many respects to the optimal nominal price expression under the familiar time-dependent Calvo pricing. In such a setup, $(\omega_{j,t+j}/\omega_{0,t})$ is constant but it still reflects the probability of non adjustment for j periods. In the current SDP specification, this term is time-varying and thus serves as a time-varying effect on the discount factor. The optimal price thus depends on current and expected future demands, price levels, marginal costs, and probabilities of non adjustment. In a special case where the price level, the adjustment probability, and the marginal cost are expected to be constant, the optimal nominal price is simply a constant markup over current nominal marginal cost $(P_{0,t} = (\varepsilon/(\varepsilon-1)) \cdot (W_t/a_t))$, as in the flexible-price solution.

On the equilibrium determination of the probability of adjustment, firms only adjust if there is a positive benefit of doing so; that is, if the value of adjustment outweighs the fixed cost associated with adjustment. Given the continuous distribution of the fixed adjustment cost, there will be a mass of firms at the margin for each j that are indifferent between adjusting or keeping the price from the previous period. For these firms, there is a zero benefit to adjust, so that $(v_{0,t} - v_{j,t}) = w_t \bar{\epsilon}_{j,t}$, where $\bar{\epsilon}_{j,t}$ is the fixed cost at the margin for bin j.[8] Hence, the proportion or probability of firms adjusting for each j is given by

$$\alpha_{j,t} = G\left(\frac{v_{0,t} - v_{j,t}}{w_t}\right) \tag{6}$$

for $j = 1, ..., J-1$. For $j = J$, we have $\alpha_{J,t} = 1$ since all firms will find it optimal to adjust after J periods. Finally, the average or expected adjustment cost conditional on adjustment is given by

$$\Xi_{j,t} = \frac{1}{\alpha_{j,t}} \int_0^{G^{-1}(\alpha_{j,t})} x dG(x) \tag{7}$$

for $j = 1, ..., J$. This concludes the description of the decisions faced by intermediate-good firms.

As a preview of the optimal monetary approach used in this paper, all the efficiency conditions of the firms' problem above become relevant constraints that must be respected by the committed monetary authority. Specifically, the monetary authority must respect the optimal price decisions by adjusting firms given by equation (5).[9] It must also respect the firms' adjustment decision,

[8] This fixed cost at the margin is the largest cost that is actually paid by adjusting firms.

[9] In terms of actual computation of the optimal policy, equation (5) must be written in a recursive form. This can be done by reformulating this constraint using the marginal value recursions, meaning the derivatives of the value functions (3) and (4) with respect to the optimal relative price $p_{0,t}$.

which are summarized by (3) and (6). The evolution of the distribution of firms in (1) and (2) must also be respected. In particular, note that some of these constraints are forward-looking, which is why there needs to be a commitment mechanism in the optimal policy problem. As we will see later in the section, this commitment mechanism is summarized by the lagged policy multipliers in a recursive Lagrangian problem.

2.1.2 Households

I turn next to the households' optimization problem. Since most elements of the households' optimization problem are identical to those in Khan, King, and Wolman (2003), I simply mention the core assumptions and proceed directly to the resulting efficiency conditions. Appendix A provides additional detail on the households' optimization problem.

Households choose the amount of final goods consumption (c_t) and leisure (l_t) to maximize a lifetime utility function subject to a budget constraint. The instantaneous utility function is assumed to be given by $u(c_t, l_t) = \frac{1}{1-\sigma} c_t^{1-\sigma} + \chi \frac{1}{1-\phi} l_t^{1-\phi}$. Here, σ is the inverse elasticity of intertemporal substitution and the parameter ϕ governs the labor supply elasticity. Final consumption goods can be purchased using credit or money (cash). Specifically, households choose to purchase a fraction of goods ξ_t with credit and the balance $1 - \xi_t$ with money. There is a cost associated in using credit, as in Baumol (1954) and Tobin (1956) in a partial equilibrium setting and Prescott (1987) and Dotsey and Ireland (1996) in a general equilibrium setting. These transaction time costs of using credit (in terms of labor time units) are heterogenous across goods and are randomly drawn from a continuous distribution with cumulative distribution function (CDF) $F(.)$. Since using credit is costly, money can be used to facilitate the purchase of final consumption goods. However, there is an opportunity cost to using money since households can purchase one-period discount bonds and obtain a return of R_t (the nominal interest rate). Households thus must balance the costs associated with holding cash and using credit in purchasing final consumption goods. In the absence of costly credit (transaction time), households would purchase all goods using credit, as in the cashless model of Woodford (2003). At the other extreme where no credit is allowed, we have a familiar cash-in-advance model as in Lucas (1980). Finally, households also derive income from their labor effort in the amount of $w_t n_t$.

Proceeding to households' efficiency conditions, the labor-leisure optimal choice implies the equality between the utility cost of foregone leisure and the value of income gained by working;

thus,

$$u_l(c_t, l_t) = w_t \lambda_t \ . \tag{8}$$

The holdings of one-period nominal discount bonds require that

$$\frac{\lambda_t}{1 + R_t} = \beta E_t \frac{\lambda_{t+1}}{1 + \pi_{t+1}} \ , \tag{9}$$

where π_t is the net inflation rate. Next, the marginal utility of consumption must equal to the full price of a unit of final consumption good,

$$u_c(c_t, l_t) = \lambda_t \left(1 + R_t \left(1 - \xi_t\right)\right) \ . \tag{10}$$

Note that since there is an opportunity cost of using money, the full consumption price above involves the nominal interest rate R_t, multiplied by the fraction of goods bought using money, $(1 - \xi_t)$. In the cashless version of the model, the condition is simply $u_c(c_t, l_t) = \lambda_t$.

The largest credit time cost that the representative household will choose to pay is $R_t c_t / w_t$ so that the fraction of goods purchased using credit is given by

$$\xi_t = F\left(\frac{R_t c_t}{w_t}\right) \ , \tag{11}$$

where $F(\cdot)$ is the CDF of the distribution of time costs of credit use. Finally, the time use constraint for the economy is given by

$$l_t + n_t + \int_0^{(R_t c_t / w_t)} x dF(x) = 1 \ , \tag{12}$$

where $\displaystyle\int_0^{(R_t c_t / w_t)} x dF(x)$ is the aggregate time costs of credit. Note that given ξ_t, the real money demand in the economy at any period t is then $m_t = (1 - \xi_t) c_t$. Appendix B provides more details on the money demand function, along with the credit cost distribution.[10]

[10] In addition to being more realistic, the inclusion of a monetary sector in the model is required from a technical standpoint. Note that without a monetary sector, the optimal steady-state inflation is zero. But the SDP model of Dotsey, King, and Wolman (1999) used here requires a non-zero steady state inflation rate. If inflation rate is zero, all firms would then optimally choose not to adjust forever in the steady state, so that $J \to \infty$. Hence, computing optimal policy under SDP here is not possible in a cashless model such as Woodford (2003).

2.1.3 Other equilibrium conditions and the state vectors

To complete the model, there are several aggregate equilibrium conditions that need to be satisfied. First, the aggregate production of goods must be equal to the aggregate demand:

$$a_t n_t^y = \left[\sum\nolimits_{j=0}^{J-1} \omega_{j,t} p_{j,t}^{-\varepsilon}\right](c_t + g_t) \ , \tag{13}$$

where n_t^y represents the total labor used in production. Since the fixed adjustment costs are in term of labor costs, there is also the aggregate labor used for this price adjustment process given by

$$n_t^p = \sum\nolimits_{j=0}^{J-1} \omega_{j,t-1} \Xi_{j+1,t} \ . \tag{14}$$

The sum of the aggregate production labor and pricing labor must be equal to the total labor supply so that we have

$$n_t^y + n_t^p = n_t \ . \tag{15}$$

Next, the relative price aggregation implied by the CES aggregator is given by

$$\sum\nolimits_{j=0}^{J-1} \omega_{j,t} p_{j,t}^{1-\varepsilon} = 1 \ . \tag{16}$$

Given the gross inflation $(1+\pi_t)$, the evolution of predetermined relative prices (for $j = 1, ..., J-1$) can be expressed by[11]

$$p_{jt} = p_{j-1,t-1} \frac{1}{1+\pi_t} \ . \tag{17}$$

Together with the lagged fractions of firms, $\{\omega_{j,t-1}\}_{j=0}^{J-1}$, the lagged relative prices, $\{p_{j,t-1}\}_{j=0}^{J-2}$, are elements of the endogenous state vector k_t in s_t^1.

Finally, I assume simple driving processes for the exogenous variables (productivity and government purchases),

$$\log(a_t) = \rho_a \log(a_{t-1}) + \sigma_a e_{z,t} \tag{18}$$

$$\log(g_t) = \rho_g \log(g_{t-1}) + \sigma_g e_{g,t} \ , \tag{19}$$

where $[e_{z,t}, e_{g,t}] \in \eta_t$ are i.i.d. shocks, normalized to have unit variance. Here, σ_a and σ_g are the standard deviations of productivity and government purchase shocks, respectively. The exogenous state vector ς_t thus consists of a_t and g_t. This completes the definition for the state vector s_t^1.

[11] Specifically, suppose that $P_{j-1,t-1}$ is the nominal price of the intermediate-goods for firms that have not adjusted their prices for $j-1$ periods at time $t-1$, with $j \leq J-1$. If a firm in this bin chooses not to adjust at time t, then $P_{j,t} = P_{j-1,t-1}$, so that $P_{j,t}/P_t = P_{j-1,t-1}/P_t \Rightarrow p_{j,t} = (P_{j-1,t-1}/P_{t-1})(P_{t-1}/P_t)$ $\Rightarrow p_{j,t} = p_{j-1,t-1}\frac{1}{1+\pi_t}$.

2.2 Identifying the four sets of distortions

As mentioned previously, monetary policy influences various distortions in the economy; hence it is useful to explicitly identify these distortions. There are four sets of distortions present in the model economy. One can view these distortions as inefficiencies that exist in the economy. Although I choose not to pursue it here, it is possible to eliminate these distortions through the use of fiscal subsidies, as in Woodford (2002) and Levin et.al. (2005).

First, there is a distortion due to firms' monopoly power that is frequently termed the *markup* distortion and is captured by the reciprocal of the real marginal cost,

$$\psi_t = \frac{a_t}{w_t} \ . \tag{20}$$

If firms are perfectly competitive as in a standard real business cycle (henceforth, RBC) model, $\psi_t = 1$ since the real wage is always equal to productivity. This distortion influences, for example, the firms' value functions (3), (4) and the optimal price chosen by adjusting firms (5).

Second, there is a distortion arising from infrequent price adjustments (sticky prices), frequently termed the *relative-price* distortion[12]

$$\Delta_t = \frac{a_t n_t^y}{(c_t + g_t)} = \sum\nolimits_{j=0}^{J-1} \omega_{j,t} p_{j,t}^{-\varepsilon} \ . \tag{21}$$

This distortion measures the extent of lost aggregate output due to sticky prices—if Δ_t is greater than unity, then it takes more labor to produce a given volume of output as suggested by (13). Both the markup and relative-price distortions are eliminated at zero inflation where both ψ_t and Δ_t are equal to unity. The model then collapses to an RBC framework if monetary frictions are absent.

The third set of distortions involve the use of money and credit in purchasing consumption goods. Frequently termed the monetary distortions, these distortions can be decomposed further into the *monetary wedge* and *shopping-time* distortions. The monetary wedge distortion is the inefficiency that arises because the full cost of consumption is augmented by the requirement that money must be held to finance a portion of consumption purchase. It can be expressed as

$$\Phi_t = R_t(1 - \xi_t) \ . \tag{22}$$

[12] This relative-price distortion is already widely identified in various studies in the literature, such as Damjanovic and Nolan (2006) King and Wolman (1999), and Yun (2008).

As suggested by (10), this distortion drives a wedge between the marginal utility of consumption and the shadow value of wealth.[13] The shopping-time distortion arises because the use of credit in purchasing consumption goods requires some use of labor; this distortion affects the total available resources in (12). It is governed by the expression

$$h_t = \int_0^{(R_t c_t / w_t)} x dF(x) \ . \tag{23}$$

Both the monetary wedge and the shopping-time distortions are minimized at zero nominal interest rate as prescribed by the Friedman rule, so that the inflation rate must be negative (deflation).

In the time-dependent-pricing version of the model, all three distortions mentioned above are present. Under SDP, there exists another distortion involving the costs associated with price adjustments, represented by (14):

$$n_t^p = \sum\nolimits_{j=0}^{J-1} \omega_{j,t-1} \Xi_{j+1,t} \ .$$

Like the shopping-time distortion, this *menu cost* distortion can be viewed as a drain on the economy's resources.

To gain more insight into the distortions, Figure 1 looks at the steady-state implications of these various inefficiencies in the model economy. First, we look at the relationship between inflation and the relative price distortion in panels A and B. For simplicity and comparability to the literature, these two panels are produced under the assumptions that there are no monetary distortions and firms' pricing decisions are time-dependent.[14] Looking first at panel A, the optimal relative price (p_0) is equal to 1 (one) under zero inflation and is increasing with inflation. When inflation is positive, price-adjusting firms will charge a higher price since they anticipate that the relative price will be eroded by future inflation. On the contrary, under negative inflation (deflationary steady state), the optimal relative price is lower since the relative price will increase if firms do not adjust in future periods. That is, those firms which do not adjust will be faced with a decreasing relative price under positive inflation and an increasing relative price under deflation—as a consequence, price dispersion is increasing with both non zero inflation and deflation (panel B). Only under zero inflation is the relative price distortion is eliminated and a unit of labor will exactly produce a

[13] This monetary-wedge distortion is highlighted in cash-in-advance models, for instance, Cooley and Hansen (1991) and Stockman (1981).

[14] Specifically, we assume that $J = 6$, with $\{\omega_j\}_{j=0}^{J-1}$ and $\{\alpha_j\}_{j=1}^{J-1}$ equal to their steady-state values under the optimal policy presented in figure 2. For any given inflation rate, we can compute the optimal reset price and the corresponding relative price distortion at the steady state. Assuming state dependence does not change the figures in panels A and B.

unit of output. Panel C looks at the implication of varying degrees of average markup on market activity (production labor). As the average markup increases, there is a corresponding decrease in the real wage paid by firms so that households substitute away from market activity. As we will see in a later section, the monetary authority can temporarily erode this average markup by generating surprise inflation.

Moving on to the monetary distortions in the right-side panels of Figure 1, we see that households subsitute away from cash transactions into costly credit since the opportunity cost of holding money becomes higher as the nominal interest rate increases.[15] As a consequence, the fraction of goods bought using credit increases with the nominal rate (panel D). Higher nominal rates are also associated with higher monetary wedge distortions (panel E), which in turn create a substitution away from consumption since this wedge affects the full cost of consumption as shown in equation (10). Finally, as displayed in panel E, the higher fraction of credit goods associated with higher nominal rates will in turn increase the transaction (shopping) time associated with costly credit.

Figure 1 also shows that there is a tradeoff among the distortions, even in this simple static illustration. As an illustration, we know from panel B that the relative price distortion is minimized at zero steady-state inflation. However, at zero inflation, the nominal interest rate must be positive for the Fisher equation (9) to be satisfied—hence, the monetary wedge distortion (panel E) is not minimized. On the contrary, if the nominal interest rate is zero so that there is a small deflation at the steady state, the relative price distortion will not be minimized. The task of the monetary authority then is to balance the costs of these various distortions, both in the steady state and in the near-steady-state dynamics in response to any exogenous shock.

2.3 Optimal monetary policy: approach and methodology

2.3.1 The optimal policy approach

Several private sector efficiency conditions described above involve forward-looking constraints and are defined recursively. As such, I use the recursive optimal policy design approach along the lines of Kydland and Prescott (1980) and Marcet and Marimon (1999). I present this approach briefly below, where a complete and formal treatment can be found in Johnston, King, and Lie (2008). This

[15] To generate panels D, E, and F, we assume that the credit cost distribution is beta with the parameters as in table 1. We also assume that the production labor is $n^y = 0.2$, and the real wage $w = a = 1$, with consumption $c = a \cdot n^y = 0.2$. For any given nominal interest R, we can then compute the credit good fraction ξ and the two monetary distortions.

optimal policy approach is similar to that used for example in Khan, King, and Wolman (2003) and Schmitt-Grohe and Uribe (2007). Yet, as shown in Rotemberg and Woodford (1999) and Woodford (2002) in the case of undistorted steady state and Benigno and Woodford (2004, 2008) in the more general case of distorted steady state, the optimal policy problem described below can be also presented in a linear-quadratic (LQ) framework. That is, under some regularity conditions, one can derive a quadratic loss function using the second-order approximation to the lifetime utility function. The problem of the monetary authority authority is then to minimize this loss function with the constraints being the linear approximations to the private sector efficiency conditions, which usually result in an aggregate supply equation (the Phillips curve) and an aggregate demand equation (the IS curve). In addition to having to derive such a loss function, which is not a trivial task itself, a drawback of this approach is that the resulting policy reaction function is linear. The optimal policy problem presented below allows an implied policy reaction function that is nonlinear if the equilibrium equations of the model are approximated to the second order. Hence, it allows for a richer analysis, especially if one is interested in analyzing the degree of nonlinearity and state-dependence in the model and in the policy rule.[16]

The benevolent policy authority is concerned with the maximization of the present value expression

$$E_t \left\{ \sum_{j=0}^{\infty} \beta^j U(b_{t+j}, k_{t+j}, x(\varsigma_{t+j})) \right\}$$

via suitable choices of control variables, b_t, and is subject to constraint sets

$$\psi(b_t, k_t, x(\varsigma_t)) \geq 0 \tag{24}$$

$$g^1(b_t, k_t, x(\varsigma_t)) + \beta E_t g^2(b_{t+1}, k_{t+1}, x(\varsigma_{t+1})) \geq 0 \ , \tag{25}$$

where k_t and ς_t are vectors of endogenous and exogenous state variables, respectively. The processes for these state vectors are given by

$$\varsigma_{t+1} = \nu(\varsigma_t, \eta_{t+1}) \tag{26}$$

$$k_{t+1} = \kappa(b_t, k_t, x(\varsigma_t), x(\varsigma_{t+1})) \ . \tag{27}$$

Note that I only present the case where the forward-looking constraints involve one-period ahead in time, but it is possible to generalize this set to include forward-looking constraints that stretch

[16] I note that this section on the optimal policy approach has a separate notation than the rest of the paper.

to infinity as shown in Marcet and Marimon (1999). One can then form a standard Lagrangian problem with (24), (25), (26), and (27) as the full constraint sets.

Following Marcet and Marimon (1999), the set of forward-looking constraints can be handled with a generalization of Bellman equation of the form

$$W(\mu_t, k_t, \varsigma_t) = \min_{\gamma_t} \max_{b_t} \{h(b_t, \gamma_t, \mu_t, k_t, x(\varsigma_t)) + \beta E W(\mu_{t+1}, k_{t+1}, \varsigma_{t+1}) | (\mu_t, k_t, \varsigma_t)\}$$

where the evolution of the pseudo state vector, μ_t, is given by

$$\mu_{t+1} = \gamma_t \ . \tag{28}$$

Here, γ_t is the vector of current (time t) multipliers associated with the forward-looking constraints (25) and μ_t is the vector of the lags of these multipliers. One can view these lagged multipliers as explicit commitment technologies. Note that in the Bellman equation above the modified objective function takes the form

$$\begin{aligned} h(b_t, \gamma_t, \mu_t, k_t, x(\varsigma_t)) &= u(b_t, k_t, x(\varsigma_t)) \\ &\quad + \gamma_t g^1(b_t, k_t, x(\varsigma_t)) \\ &\quad + \mu_t g^2(b_t, k_t, x(\varsigma_t)) \ . \end{aligned}$$

Further, the modified objective function above can be augmented with the Lagrangian terms involving the point-in-time constraint sets in (24) yielding $p_t \psi(b_t, k_t, x(\varsigma_t))$, where p_t is the vector of Lagrange multipliers associated with these constraints. We can thus focus our attention on the composite expression

$$\begin{aligned} W(\mu_t, k_t, \varsigma_t) &= \min_{p_t, \gamma_t} \max_{b_t} \{w(b_t, p_t, \gamma_t, \mu_t, k_t, x(\varsigma_t)) \\ &\quad + \beta E W(\mu_{t+1}, k_{t+1}, \varsigma_{t+1}) | (\mu_t, k_t, \varsigma_t)\} \ , \end{aligned} \tag{29}$$

where the maximization-minimization problem is also subject to the state evolution equations in (26), (27), and (28) and the initial conditions on μ_t, k_t, and ς_t. In the above expression the function $w(.)$ is $h(b_t, \gamma_t, \mu_t, k_t, x(\varsigma_t)) + p_t \psi(b_t, k_t, x(\varsigma_t))$. The results of solving this problem are the decision rules

$$\begin{aligned} b_t &= b(k_t, \varsigma_t, \mu_t) \\ p_t &= p(k_t, \varsigma_t, \mu_t) \\ \gamma_t &= \gamma(k_t, \varsigma_t, \mu_t) \ , \end{aligned}$$

so that the state and pseudo-state vectors evolve according to

$$\begin{aligned} \varsigma_{t+1} &= \nu(\varsigma_t, \eta_{t+1}) \\ k_{t+1} &= \kappa(k_t, b(k_t, \varsigma_t, \mu_t), x(\varsigma_t), x(\varsigma_{t+1})) \\ \mu_{t+1} &= \gamma(k_t, \varsigma_t, \mu_t) \ . \end{aligned}$$

2.3.2 Application to the model economy

The optimal policy approach described above can be easily applied to solve for the optimal precommitment policy that is the subject of this paper. The objective function of the monetary policy authority is simply the households' lifetime utility function in our model economy. Each of the private sector constraints described previously fits into either the constraint set in (24) or (25) and the evolution of the state vectors s_t^1 fall into the restrictions described in (26) and (27). For example, (6), (8), (10), and (13) are parts of the point-in-time constraint sets (24), while (3), (4), and (9) are forward-looking constraints as identified in (25). The evolution of the fractions of firms in (1) and predetermined prices in (17) would fit into the evolution equation described in (27). In terms of vectors of variables, consumption (c_t) and labor (n_t) are parts of the control vector b_t, for example. The endogenous state vector k_t would consist of the lagged predetermined relative prices, $\{p_{j,t-1}\}_{j=0}^{J-2}$ and the lagged fractions of firms, $\{\omega_{j,t-1}\}_{j=0}^{J-2}$. The vector p_t includes the Lagrange multipliers attached to all of the point-in-time private sector constraints in the model. On the other hand, the multipliers associated with forward-looking constraints such as (9) are elements of the vector γ_t. The vector μ_t consists of lagged multipliers associated with forward-looking constraints—this vector is thus the model's second set of state variables, s_t^2, described earlier. Recall that this vector of lagged multipliers summarizes the past policy plans or commitment that must be followed by the monetary authority. To solve for the optimal policy, we can form a *saddle point functional equation* as in (29) and take all the private-sector efficiency conditions as the policy constraints. The first-order conditions of this optimization problem would then represent the allocations under the optimal policy.

Recursive equilibrium The recursive equilibrium is defined as follows: given the state vector s_t, the evolution of the exogenous state variables, and the optimal precommitment policy rule employed by the monetary authority, both households and intermediate-good firms solve their optimization problems as described above. The market-clearing conditions for the goods and the labor markets,

(13) and (15), are also satisfied in equilibrium. Finally, the optimal policy rule is consistent with and respects households' and firms' (the private sector's) efficiency conditions.

2.3.3 Approximate solutions

The model is solved using the perturbation approximation solution method of Johnston, King, and Lie (2009). Specifically, the first order necessary conditions of the optimal policy problem above can be written as a system of nonlinear equations in the expectational form

$$E_t F(z_{t+1}, z_t, \eta_{t+1}) = 0 \ ,$$

where z_t is the vector of all variables (including both endogenous and exogenous variables) at time t and η_{t+1} is the vector of exogenous shocks at time $t+1$. Johnston, King, and Lie (2009) show that the solution to a second-order Taylor-series approximation to the above system can be cast into a nearly linear state-space representation of the form

$$z_t \simeq \underbrace{(z + \Pi_1 ds_t)}_{1st} + \underbrace{\frac{1}{2} Ed^2 z_t + \frac{1}{2} \left[\begin{array}{cc} \Pi_1 & \Pi_2 \end{array} \right] \left[\begin{array}{c} d^2 s_t - Ed^2 s_t \\ \xi_t \end{array} \right]}_{2nd} \tag{30}$$

$$\left[\begin{array}{c} ds_{t+1} \\ d^2 s_{t+1} - Ed^2 s_t \\ \xi_{t+1} \end{array} \right] = \left[\begin{array}{ccc} \delta & 0 & 0 \\ 0 & \delta & \lambda \\ 0 & 0 & \rho \end{array} \right] \left[\begin{array}{c} ds_t \\ d^2 s_t - Ed^2 s_t \\ \xi_t \end{array} \right] + \left[\begin{array}{cc} \alpha & 0 \\ 0 & \gamma_\eta \\ 0 & \gamma_e \end{array} \right] \left[\begin{array}{c} \eta_{t+1} \\ e_{t+1} \end{array} \right] \tag{31}$$

where the first equation is the observational output equation and the second equation represents the state evolution equation. Here $d^j x_t$ is the $j-th$ differential of the vector of variables x_t, $\xi_t = ds_t ds_t^T - E(ds_t ds_t^T)$, and $e_{t+1} \equiv \left[\begin{array}{cc} vech(\eta_{t+1}\eta_{t+1}^T - E(\eta_{t+1}\eta_{t+1}^T))^T & vec(ds_t \eta_{t+1}^T)^T \end{array} \right]^T$.[17] Π_1, Π_2, δ, λ, ρ, α, γ_η, and γ_e are matrices of coefficients. Also, $Ed^2 z_t$ and $Ed^2 s_t$ are the unconditional expectations of the second differential of the vector of endogenous and state variables, respectively. Note that this second-order state space form also encompasses the linear (first-order) approximate solution. That is, under a linear approximation, the output equation only contains the first bracketed term (*1st*) and the state evolution system in the second equation above only contains the first line ($ds_{t+1} = \delta ds_t + \alpha \eta_{t+1}$).

[17] The supercript ^{T} is the transpose of a vector or a matrix. Here, $vec(.)$ is the standard vectorization operation. For any symmetric matrix A, some of the elements of $vec(A)$ are redundant. If we define D_m as the unique duplication matrix for A (see for instance, Magnus and Neudecker, 2002) that selects the unique (upper triangular) elements of A, so we can then define $vec(A) = D_m vech(A)$, where $vech(A)$ now contains only unique elements of $vec(A)$.

Defining impulse response as the revision in forecast errors, $E_t z_{t+j} - E_{t-1} z_{t+j}$, one can apply the state-space form above recursively to calculate the response of the economy to any given shock for a given starting state.[18] The analyses in the next section use this state-space form to generate various impulse responses. For later references, I will call the response based on the first-order approximate solution as the *first-order impulse response*, while the *second-order impulse response* refers to the response based on the second-order approximate solution.

Several comments on the second-order state-space form are in order. Compared to the first-order, there are some additional terms in the exogenous shock vector in the second-order state-space form within (31). First, there is a $vech(\eta_{t+1}\eta_{t+1}^T - \Omega_{\eta\eta})$ term, where $\Omega_{\eta\eta} = E(\eta_{t+1}\eta_{t+1}^T)$ is the covariance matrix of the exogenous shocks–in the context of my model, $\Omega_{\eta\eta} = \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_g^2 \end{bmatrix}$. This term in general captures the nonlinearity of the exogenous shocks if the shocks themselves enter nonlinearly in the model. Such a nonlinearity is not present in the first-order approximate solution. Even when shocks enter linearly in the model as in the current model, the effect of this term may not be zero. This is because this term is also related to the uncertainty of the shock process itself, meaning it is related to the "uncertainty correction" term in the jargon of the literature on second-order approximation.[19] This term then leads to asymmetric responses to different shock sizes. For example, in the first-order impulse response when shocks enter linearly, the response to a two-standard-deviation shock has the same property as the response to a one-standard-deviation shock. That is, if output increases by 1 percent in response to a one-standard-deviation productivity shock, a two-standard-deviation shock will increase output by 2 percent. A shock of a given magnitude is also symmetric irrespective of whether the shock is positive or negative: to the first-order, the response of a variable to an m percent negative shock is just the opposite of the response to an m percent positive shock. This symmetric property is unappealing for various reasons. For example, one might expect that the monetary authority or an optimizing agent might behave differently in response to an unfavorable shock compared to a favorable shock. Only in the second-order impulse response is this asymmetric property captured.

The second additional term is the state-dependent response to shocks. This *state-dependence*

[18] The method here is related to several recent contributions on second-order approximations such as Schmitt-Grohe and Uribe (2004), Lombardo and Sutherland (2007), and Kim, et. al. (2008). A main insight of Johnston, King, and Lie (2009) is that if one derives restrictions directly on the equilibrium equations of the model, rather than on the assumed policy function, the approximate solution will always be in a state-space form. Hence, no additional modification such as "pruning" in Kim, et. al. (2008) is necessary. Moreover, the state-space form is always achieved for any n-th order approximation.

[19] See for example, Schmitt-Grohe and Uribe (2004).

property is captured in the second-order approximation through the term $vec(ds_t\eta_{t+1}^T)$. That is, the economy's response to a shock depends nonlinearly on the state of the economy, which is an especially appealing property for state-dependent pricing models. Once again, the first-order impulse response does not capture this *state-dependence* effect. As we see in section 4, these two additional nonlinear terms can be particularly important, especially when the state of the economy is farther away than the certainty steady state.

2.4 Calibration and the steady-state inflation under optimal policy

Table 1 presents the calibration of the parameters in the model. The time unit is meant to be a quarter. The choice of σ and ϕ equal unity means that the utility function involves log function in consumption and leisure. The parameter χ in the utility function is chosen such that the steady-state total labor is equal to 0.2, which is equal to the value observed in the U.S. postwar period reported in various real business cycle studies, such as King, Plosser, and Rebelo (1988). The choice of demand elasticity ($\varepsilon = 10$) is a compromise among various studies that report various values for this parameter, for instance Altig et. al. (2005), Basu and Fernald (1997), and Rotemberg and Woodford (1999). This value implies that the markup is equal to about 11.11 percent in the flexible-price equilibrium.

In terms of the fixed adjustment cost distribution, I use a beta distribution with left and the right parameters equal to $a_l = 2.1$ and $a_r = 1$, respectively. This implies a probability density function (PDF) that is strictly increasing and strictly convex.[20] Appendix C provides more details on the specification of this adjustment cost distribution. I calibrate the largest possible fixed cost paid by firms, B, so that the maximum number of quarters of price fixity (J) is 6 quarters. Figure 2 displays the steady-state probability (α_j) and firms' distribution (ω_j) of price adjustment based on these calibrated parameters under the optimal precommitment policy. This figure shows that the price adjustment hazard is increasing with the time since last price adjustment, unlike the constant hazard implied by the standard Calvo model. In addition, the figure implies that the mean and median durations of price fixity are 4.02 quarters and 3.00 quarters, respectively, and the average frequency of price adjustment is about 24 percent per quarter. This degree of

[20] I also experiment with various shape parameters of the beta distribution that yield various shapes of the PDF: linear, strictly increasing, strictly convex, or combinations of these. Each time, similar results are found for both the steady-state and near steady-state dynamics analyzed in subsequent sections. Changing the distribution to the one used in Dotsey and King (2005) also preserves the paper's results.

price rigidity is in the ballpark of available empirical estimates using macroeconomic data such as Christiano, Eichenbaum, and Evans (2005). It is higher than the evidence based on microeconomic data reported in Bils and Klenow (2004), but still close to the finding in Nakamura and Steinsson (2008).

The parameters of the credit cost distribution, which governs the demand for money, are set according to the estimation in Khan, King, and Wolman (2003). Appendix B describes the implied money demand function and elasticity based on their estimates. The rest of the parameters involve the exogenous driving processes. Following King and Rebelo (1999), the aggregate productivity is very persistent ($\rho_a = 0.95$), with the standard deviation (σ_a) equal to 0.0072. Finally, the choices of ρ_g and σ_g are in line with the estimates in Ravn (2007).

Optimal steady-state inflation I next consider the steady-state inflation under the optimal policy. Recall that the task of the benevolent monetary authority is to balance the costs of various distortions in the economy. As shown by various studies, e.g. King and Wolman (1999) and Woodford (2002), the optimal steady-state inflation is zero when only relative-price and markup distortions are present. That is, both of these distortions are eliminated under zero steady-state inflation and there is no long-run tradeoff between them. The presence of the menu cost distortion in the current SDP model does not alter this conclusion since this distortion is also eliminated under zero inflation. (Firms would find no need to change prices under zero inflation and hence would never have to pay the adjustment costs.) However, as shown by Khan, King, and Wolman (2003) and Schmitt-Grohe and Uribe (2007) in the TDP case, the presence of monetary distortions alters the tradeoff faced by the monetary authority. As prescribed by the Friedman rule, the distortions due to money demand are eliminated when the opportunity cost of holding money—the nominal interest rate—is zero. This requirement implies that inflation must be negative (deflation). It follows that the optimal steady-state inflation when all distortions are present (henceforth, "full" distortions) should also be negative, but not as negative as implied by the Friedman rule. Put another way, the monetary authority compromises between maintaining price stability and following the Friedman rule.

Table 2 shows that the optimal steady-state inflation rate in the SDP model reflects the above discussion. As expected, in the benchmark case with all the distortions present, the steady-state (net) inflation is negative (-0.99 percent at the annualized rate under SDP) but not as negative

as the Friedman rule (-2.88 percent with the calibrated discount rate β). Under a lower elasticity of demand ($\varepsilon = 7$), the steady state deflation increases to -1.50 percent per annum. This higher deflation is associated with the lower cost of relative price distortion implied by the lower elasticity of demand. Next, when the variations in either the monetary wedge or the shopping-time distortion are eliminated, the steady state deflation is closer to zero since the relative-price, menu cost, and markup distortions, which are minimized at zero inflation, become relatively more important.[21] Eliminating variations in the markup distortion leads to a larger steady-state deflation due to greater relative importance of the two monetary distortions.[22]

I next compare the optimal steady-state inflation under SDP and TDP. Note that the TDP model counterpart is constructed under the assumption that the parameters $\{\alpha_j\}_{j=1}^{J}$ and $\{\omega_j\}_{j=0}^{J-1}$ are equal to the steady-state values under the SDP model, with all other structural parameters remain the same as their counterparts under SDP. In the "full" distortions case, there is a slightly larger deflation rate (-0.988 percent) under TDP than under SDP (-0.992 percent). This pattern is also apparent under lower ε or when one or more sets of distortions are eliminated. This can be understood as follows. First, even though in the TDP model we have the same adjustment patterns (α_j and ω_j) as in the steady state under SDP, the steady state allocation under SDP is not the equilibrium allocation under TDP. This is because there are "freed" resources in terms of labor/leisure time under TDP since firms do not have to pay the adjustment costs. These freed resources would be optimally dispersed as additional leisure or labor time. Since this additional labor time can be used to produce more output, the economy under TDP would be producing more efficiently (closer to potential) with lower average markup distortions than the economy under SDP. Since a lower steady state markup is associated with smaller deflation (inflation is closer to zero), it follows that the optimal steady-state deflation rate should be smaller under TDP than under SDP. However, the difference is quite small since these extra resources from price adjustment activity under SDP are also small.[23] This result also means that if firms under TDP were also required to

[21] By *eliminating variations in distortions*, we mean that we make it impossible for the monetary authority to influence or manipulate the variations in the distortions. This can be thought using the idea of fiscal subsidy: government would provide subsidy when there are movements in distortions so that these distortions would always be equal to a certain level. In terms of the model specific, for example when we want to remove variations in the monetary-wedge distortion, we then set $R(1-\xi)$ in (10) to a certain constant level and resolve the model. At least one of the distortions have to be present in the economy—otherwise, there is no scope for stabilization policy.

[22] Here, the (constant) markup is set to equal to the steady-state markup under optimal policy with all the distortions present.

[23] Specifically, the aggregate labor resources used in price adjustment activities in the steady state is about

pay the same amount of fixed costs when they adjust as in the SDP case, the optimal steady-state inflation should be identical under the two cases. This is as expected since both cases are calibrated under identical steady-state frequency of price adjustment, meaning α_j and ω_j are identical in the steady state for all j.

3 Optimal timeless perspective (long-run) responses

3.1 Productivity shock

This section considers the first- and second-order optimal responses to a temporary productivity shock under the assumption of timeless perspective (Woodford, 2003) where the optimal policy has been long implemented since some distant past. That is, although the true Ramsey solution involves a monetary authority that treats initial periods differently due to the absence of past commitments, we assume that the monetary authority ignores this "start-up problem." Put another way, the monetary authority is assumed to have committed to the state-contingent policy rule since time immemorial. The cost of of ignoring these initial conditions is analyzed in section 5.

Throughout the analysis below the economy is assumed to be always at the steady state prior to the shock and the shock size is 2 (two) standard deviations, normalized so that productivity increases by 1 percent on impact relative to its steady state value. This starting state assumption means that the term $vec(ds_{t-1}\eta_t)$ in the second-order approximate solution is zero since $ds_{t-1} = 0$. That is, I temporarily ignore this state-dependence effect when I analyze the second-order impulse response.

3.1.1 First-order approximate solution

In the first-order impulse response analysis below, I begin by reviewing some familiar results from the literature under the time-dependent pricing (TDP) assumption before comparing them to the dynamics in the state-dependent pricing (SDP) assumption.

RBC and TDP without monetary distortions I begin the analysis with a review of the optimal monetary policy response in the time-dependent pricing (TDP) model with only relative-price and markup distortions, as in Benigno and Woodford (2005), Goodfriend and King (1997),

0.016 percent of aggregate steady-state total labor.

King and Wolman (1999), and Woodford (2002). The response in a familiar simple real business cycle (RBC) model is calculated and displayed as well to serve as a benchmark.[24] Both the RBC model and the "cashless" TDP model share the same zero steady-state inflation.

The optimal response to a productivity shock in this cashless TDP model is to exactly replicate the response in the RBC environment, albeit in a monopolistically competitive setting (Figure 3). Here, it is optimal to keep inflation at its zero steady-state level and to fully accomodate the productivity shock. That is, price stability in response to a productivity shock is optimal when monetary distortions are absent. Consumption increases one-to-one (1 percent on impact) with the shock throughout the whole period of higher productivity. Labor is essentially constant: there is an exact offset of wealth and substitution effects reflecting the households' preference specification.[25] Since inflation remains equal to its zero steady-state level, the average markup and relative price distortions continue to be minimized and hence there are zero variations in each of these distortions (not shown). Regarding the interest rate's movements, initially there is a decrease in the real rate when consumption is high relative to the steady state, but it grows over time to its steady-state level as consumption is expected to decrease to its steady-state level. Since inflation is zero, the nominal rate and real rate responses exactly coincide.

This complete price stability result is in accord with that obtained in a linear-quadratic (LQ) framework such as Clarida, Galí, and Gertler (1999) and Benigno and Woodford (2005). In an LQ framework, the loss function in this two-distortion case is a quadratic function of inflation and an output gap measure. As stressed by Benigno and Woodford (2005) among others, in the isoelastic utility case under zero steady-state government purchase, there is no tradeoff between inflation and output gap stabilizations in response to a productivity shock. Minimizing inflation variation also minimizes the variation in the output gap.

TDP with and without monetary distortions Figure 4 compares the optimal responses under TDP with and without monetary distortions.[26] The presence of additional frictions from money

[24] To permit direct comparison with the TDP model, the firms in the RBC model also use labor as the only input in production, hence we abstract from capital accumulation. Prices are flexible and firms are assumed to be monopolistically competitive in our RBC model.

[25] Labor in this figure and subsequent ones only include the dynamics of labor used in production. The dynamics of production labor are virtually identical to the dynamics of total labor since labor used in costly credit transaction and in price adjustment activity (under SDP) are small, both in the steady state and in the near steady-state dynamics.

[26] The case considered here with monetary distortions is identical to the model in Khan, King, and Wolman (2003), with the exception of the difference in calibration of the firms' distribution parameters $\{\omega_j\}_{j=0}^{J-1}$.

demand alters the tradeoff faced by the monetary authority. In addition to the costs of variations in the relative-price and markup distortions, the monetary authority also needs to manage the cost of variations in the nominal interest rate that affects the opportunity cost of holding money. As shown in Figure 4, complete inflation stabilization is no longer optimal, though feasible, in such an environment. The monetary authority in this environment has to compromise between complete price stability and the Friedman rule.

One can see this compromise from the responses of inflation and the nominal interest rate in Figure 4. First, it is no longer optimal for inflation to stay constant since we know from Figure 3 that the nominal rate has to decrease by a bit more to be consistent with constant inflation. Hence, in such a scenario, the monetary authority completely ignores the welfare cost of nominal interest rate movements. On the other hand, if the nominal rate is completely stabilized as in the Friedman rule, inflation has to vary by more to accomodate movements in the real interest rate associated with consumption movements. Under the optimal policy, the compromise means that the nominal rate responses are smoothed out relative to the case where monetary distortions are absent. Inflation is no longer constant: there is a slight deflation on impact but inflation is higher relative to the steady state in subsequent periods. The inflation and nominal rate responses imply that consumption increases by a bit less than 1 percent before subsequently tracking productivity.

Despite the differences in the dynamics between the TDP case with and without monetary frictions, the optimal policy here can still be considered as an "approximate price stability" rule in a sense the price level remains largely stabilized. For example, the largest deviation of inflation is only about 5 basis points deviation (0.05 percent) at the annual rate. Using a medium-scale macroeconomic model that accords well with postwar U.S. business cycles, Schmitt-Grohe and Uribe (2007) report the same conclusion regarding this approximate price stability result.

TDP and SDP with "full" distortions Finally, Figure 5 compares the long-run timeless perspective responses under the TDP and SDP models when all four sets of distortions are present. The optimal response under SDP is to closely replicate the dynamics under the TDP model counterpart, indicating that approximate price stability is also optimal. It is again optimal to have a slight deflation on impact in response to a productivity shock, although the inflation variation is larger under SDP. However, inflation response is again muted: the largest deviation of inflation is

Johnston, King, and Lie (2008) instead consider Calvo (1983) price setting but with the same four distortions. Both studies display largely similar dynamics as those presented here.

less than an 8-basis-point deviation from the steady-state value (in quarter three). The monetary authority also smooths out the nominal interest rate movements due to the presence of the monetary distortions. As in the TDP case, consumption also increases by a bit less than productivity, although it is closer to the 1-percent increase in productivity. One subtle difference between the TDP and SDP cases is that there are now movements in the fraction of firms adjusting ($\omega_{0,t}$). Under SDP, on impact, there is an increase in the fraction of firms adjusting in response to higher deflation since firms' relative prices increase more quickly than before. There is a high correlation between the responses of inflation and the fraction of firms adjusting, which reflects the fact that firms' adjustment decision crucially depends on the expected path of the price level.

Despite the conclusion that the long-run response to a productivity shock under SDP is largely similar to that under TDP, I note that there are some important quantitative differences in the dynamics. These differences are at the core of the analysis since they reflect the presence of endogenous timing and frequency of price adjustment under SDP. Recall that the task of the monetary authority is to optimally balance the costs of variations in various distortions. Since the variation in the additional adjustment cost (menu cost) distortion under SDP is minimized when inflation stays constant, one might think that the optimal response should involve a smaller inflation variation under SDP. Yet, as shown in Figure 5, inflation is actually more variable under SDP than under TDP. This conjecture misses the changing nature of the relative costs of various distortions and the policy tradeoff between various stabilization goals faced by the monetary authority under SDP.

The optimal policy response can be understood as follows. First, note that for a given change in inflation, the variation in the relative-price distortion is smaller under SDP than under TDP. This smaller price dispersion is due to the fact that firms are able to optimally respond to changing inflation. For example, if inflation deviates farther away from zero or a trend, additional firms will choose to adjust so that prices are more closely synchronized. Under TDP, firms do not have this opportunity and will only adjust if they are given the exogenous signal to adjust, so that there is higher price dispersion. This lower relative-price-distortion cost of inflation variation under SDP means that the monetary authority can "afford" to let inflation vary more and to optimally put more weight on the stabilization of other distortions or variables. For example, this is apparent in the response of the nominal rate, which affect the variations in the two monetary distortions, depicted in Figure 5. The nominal rate varies by less under SDP, indicating that the optimal policy

is closer to the Friedman rule. The monetary authority can also afford to stabilize the average markup distortion by more, which is apparent by looking at the consumption response relative to productivity. Compared to under TDP, consumption tracks productivity much closer under SDP, indicating a smaller decrease in the output gap.[27] Overall, however, this modification to the policy tradeoff in response to a productivity shock is relatively small. Hence, from the timeless perspective, the optimal response under TDP is a good approximation to the optimal response under SDP.

3.1.2 Second-order approximate solution

Figures 6 and 7 present both the first- and second-order impulse responses together under the TDP and SDP models, respectively, when all four sets of distortions are present. Specifically, the second-order responses are calculated using the state-space form presented in (30) and (31). The figures show that the second-order optimal responses to a productivity shock under the timeless perspective are virtually identical to the first-order responses under both TDP and SDP. This observation suggests that there are negligible second-order shifts and nonlinearity under the timeless perspective policy and when the economy starts from the certainty steady state point. Johnston, King, and Lie (2008) show that this result also holds under the familar Calvo time-dependent pricing assumption. As we see in the next section, this close association between the first- and second-order responses does not always hold when various starting states and shock sizes are considered.

3.2 Government purchase shock

So far I have exclusively discussed the optimal response to a productivity shock. Figure 8 now displays the dynamics when the economy is hit by a positive two standard-deviation government purchase shock.[28] A temporary, but persistent, government purchase shock can be thought as a temporary drain on economy's resources from the perspective of the representative household. Households thus choose to reduce consumption (top left panel of Figure 8) and increase work effort in response to this negative wealth effect. Since the increase in government purchase, and hence, the decrease in consumption, are temporary, the real interest rate is temporarily higher. Inflation movements are generally small so that the price level is largely stabilized around its steady state.

[27] Government purchase is assumed to be at its zero steady-state level throughout this higher-productivity period, so that consumption is always equal to output.

[28] As in case of productivity shock, the response is normalized so that government purchase increases by 1% on impact.

These dynamic patterns occur under both the TDP and SDP assumptions. As noted by Schmitt-Grohe and Uribe (2007), the monetary authority chooses an allocation that resembles the allocation in the flexible-price case, but with due consideration regarding the presence of predetermined prices and monetary frictions.[29]

The conclusion regarding the optimal long-run dynamics in the SDP model versus the TDP model in the productivity shock case carries over to the case of a government purchase shock. In particular, inflation tends to be more variable and the optimal policy is closer to the Friedman rule under SDP. The latter point is apparent by a more muted nominal interest rate response under SDP as depicted in the top right panel of Figure 8. Once again, the lower cost of inflation variation on the relative-price distortion under SDP is responsible for this result. As in the productivity shock case, the response under SDP still closely tracks the response under TDP.

Finally, as in the case of a productivity shock, the second-order response (not shown) is virtually identical to the first-order response under both model assumptions for the particular shock size and when the economy is at the steady state prior to the shock.

4 Nonlinearity and the effects of shock size and state-dependence

The result on the optimal timeless perspective (long-run) policy responses to both productivity and government purchase shocks in section 3 shows that the first- and second-order responses are nearly identical even under the SDP model specification, meaning the policy responses are highly linear and a first-order approximation seems to be good enough for characterizing the equilibrium solution. However, the previous exercise assumes that the economy starts from the deterministic steady-state point for a given shock size, but the linearity result may not hold when one moves away from this assumption. In this section, I consider further this nonlinearity issue and examine how responses depend on the size of the shock and the state of the economy under the SDP model specification.[30] Throughout the analysis below, I focus on the responses to a productivity shock.

Size of shock I first look at the optimal response to a productivity shock for various shock sizes (3, 2, ..., -2, and -3 standard deviations), presented in Figure 9. The starting state of the

[29] Schmitt-Grohe and Uribe (2007) consider a *permanent* increase in the growth rate of government purchase. Yet, the same intuition holds in the case of a temporary shock.

[30] There is also an issue of how accurate is the approximate solution. Like any other approximation method, the method used in this paper is only accurate in the neighbourhood of the point of the approximation. Future research should consider this accuracy issue.

economy prior to this shock is again the certainty steady-state point. Hence, this is essentially the same experiment as in Figure 7 when I assume a given shock size.[31] Looking first at the top two panels, the evolutions of the productivity shock are identical in the first- and second-order approximations since the productivity process itself (18) is assumed to be (log) linear. A quick glance at the inflation responses in the bottom two panels may indicate that the responses are identical under the first- and second-order approximations. However, there are indeed some small differences across the two panels. For example, in the impact period (quarter 0), in response to a negative three (-3) standard deviation shock, inflation increases by exactly 10 basis point in the first order, while it jumps slightly more than 10 basis point in the second-order response. There is also an asymmetry between negative and positive shocks of a given size that is only captured in the second-order approximate solution. On the contrary, the first-order responses exhibit symmetric responses as shown in the bottom left panel, which is a particular implication of the general nature of responses coming from a first-order (linear) approximation. I will revisit this asymmetry issue below. Overall, however, the first- and second-order responses are largely identical for a range of reasonable shock sizes when the starting state is the deterministic steady-state point.

State-dependence Figure 10 displays the state-dependence effect of the optimal responses under SDP. In particular, I assume that the economy is perturbed by a repeated sequence of productivity shocks of equal magnitude (3 standard deviations) at periods 0, 1, and 2. As previously, I assume that prior to the shock at period 0, the economy is at its steady state. Note that the state of the economy in period 1 (period 2) when the shock occurs would not be the steady-state point since the economy is also hit by a productivity shock in the previous period, period 0 (period 1). Hence, this exercise captures the *state-dependence* effect. In particular, the term $vec(ds_{t-1}\eta_t^T)$ in the second-order state-space form displayed in (31) would not be zero in periods 1 and 2 since $ds_{t-1} \neq 0$. Such a state-dependence effect is not captured in the first-order response as apparent from the first-order state-space form.

Looking at Figure 10, there are indeed some effects of state-dependence which make the optimal responses quite nonlinear: although they move in the same directions, the second-order responses now exhibit somewhat different patterns quantitatively compared to the first-order responses. This

[31] Note that in figure 7, the shock size is two standard deviations, but with the normalized response so that productivity is 1 percent higher compared to the steady state value. Here in figure 9, I instead normalize the response so that an m standard deviation shock corresponds to an m percent increase in productivity. The scales for the same two standard deviations shock in figures 7 and 9 are therefore different.

is apparent from the response of inflation in the bottom-left panel of Figure 10 for example. The SDP model under the optimal precommiment policy thus exhibits an important degree of nonlinearity if one assumes that the state of the economy is farther away from the steady state. Under the TDP model specification (not shown), there is a smaller degree of nonlinearity in the sense that the second-order responses still track the first-order responses more closely.

Shock size and state-dependence The previous examination on state-dependence effect assumes a given size of the shock. In Figure 11, I repeat the same experiment of a repeated sequence of productivity shocks but with a range of different shock sizes (3, 2, and 1 standard deviations). It is apparent from the figure that the degree of nonlinearity in the SDP model depends on the interaction between the state of the economy and the size of the shock. That is, the degree of nonlinearity increases with the shock size. For example, the first- and second-order inflation responses (middle panels) are nearly identical when the economy is hit by a one standard deviation shock. When the shock size is two standard deviations, we start to see some differences between the first- and second-order responses. Under the three standard-deviation shock (as in Figure 10), the inflation response now becomes much more muted in the second-order approximation. For example, inflation deviates by about 70 basis points in quarter four in the first-order case, while it only increases by slightly more than 40 basis points in the same quarter in the second-order case.

Asymmetry of responses I next revisit the issue of asymmetric responses between the positive and negative shocks under the same exercise of repeated productivity shocks discussed above. Figure 12 displays the result for the inflation response. Looking at the first-order response in the top panel, there is a perfect symmetry between negative and positive shocks of equal magnitude as expected. There is, however, a different story when one considers the second-order response in the bottom panel: the asymmetric property is apparent, reflecting the fact that the nonlinearity is only captured in the second-order approximate solution. And once again, the degree of asymmetry or nonlinearity of the responses depends positively on the magnitude of the shock. To be more concrete, consider the second-order inflation responses in the 3 and -3 standard-deviation shock cases. Comparing these two cases, inflation is more variable when the shock is negative. An interpretation of this observation is that since a negative productivity shock is more harmful to the economy due to a decreasing marginal utility, the monetary authority moves inflation more (compared to the opposite magnitude when the shock is positive) to cushion the negative impact

of this negative shock on consumption and the welfare of the representative household.

Exogenous linear policy rule Although the experiments above show that there is some degree of nonlinearity in the SDP model, it could be the case that this nonlinearity actually comes from the implied optimal precommitment policy rule. To investigate this, Figure 13 displays the optimal responses when we assume that the policy rule is linear and in the form of a simple Taylor rule,

$$R_t = \bar{R} + \rho_\pi(\pi_t - \bar{\pi}) + \rho_y \left(\frac{y_t - \bar{y}}{\bar{y}} \right) ,$$

where R_t, π_t, and y_t are the nominal rate (the monetary policy instrument), the four-quarter inflation rate, and the aggregate output, respectively. The inflation target $\bar{\pi}$ and potential output $\bar{y}$ are set to equal to the steady state values under the benchmark optimal precommitment policy environment. We also assume conventional values for the policy coefficients: $\rho_\pi = 1.5$ and $\rho_y = 0.5$.

Looking first at the responses under the TDP environment in the left-side panels of Figure 13, we see that in response to a sequence of repeated positive productivity shocks, the Taylor rule prescribes that both nominal rates and inflation must decrease. The important main conclusion is that in the TDP environment under this simple linear Taylor rule, the first- and second-order impulse responses are nearly indistinguisable, which is an indication of a high degree of linearity in the TDP model. On the contrary, the right-side panels of Figure 13 show that there is quite a degree of nonlinearity in the SDP model even though the policy rule itself is linear.

Summary From various exercises above we conclude that unlike TDP models, SDP models generally exhibit some degree of nonlinearity, both under the optimal monetary policy and when the policy rule itself is linear. And this nonlinearity property depends on the interaction between the state of the economy and the size of the shock. Although this nonlinearity does not seem to change the qualitative property of the responses, there are indeed some important quantitative differences. It follows that a conventional first-order approximation to the equilibrium solution may be good enough for some purposes, such as when an analyst is only interested in looking at the qualitative dynamics of an SDP model. However, a second- or higher-order approximation may be warranted for some other purposes such as forecasting, estimation, and so on.

5 The start-up problem

In this section, I consider a departure from the timeless perspective policy. One should note that the optimal policy from the timeless perspective is not the true Ramsey solution, which generally involves treating the starting period of optimal policy implementation differently than the subsequent periods. That is, a Ramsey planner takes into account that there is no past commitment that she must abide by in the starting period during which the optimal state-contingent plan is set and announced. The timeless perspective optimal policy is thus suboptimal in a sense that it is welfare-dominated by the true Ramsey solution, although it welfare-dominates other time-consistent rules.[32] This so-called "start-up problem" means that the policy rule in the starting period is different than that in the subsequent periods. One reason to analyze this start-up problem is that it essentially measures the cost of adopting the timeless perspective policy instead of the true Ramsey policy, which I undertake in this section.

In the context of monetary models with monopolistic competition like the model in this paper, this start-up problem takes the form of a monetary authority that generates surprise inflation in the starting period of optimal policy implementation. It is optimal to do so since the economy operates inefficiently due to firms' monopoly power. Generating temporary higher inflation would erode the markup of firms when prices are sticky, and hence, would temporarily bring the economy closer to the first-best allocation.

One question of interest that one can ask is whether the start-up inflation would be more muted under the SDP assumption. An implication of SDP is that inflation is more variable since firms can change their prices more freely. This property of SDP means that the monetary authority has less leverage over real activity—or from a standpoint of the New Keynesian Phillips curve, the slope of the Phillips curve is steeper under SDP than under TDP. Hence, the conjecture that the start-up inflation would be lower under SDP is a reasonable one to make. As we see below, this conjecture is incorrect for the very reason that the policy tradeoff is altered under SDP.

We can analyze the dynamics of the start-up problem in the context of the present model by setting the lagged policy multipliers to zero since these multipliers essentially represent the monetary authority's past commitments.[33] Furthermore, I assume that the economy was at the steady state

[32] One can view the optimal policy from the timeless perspective as time consistent since if the policy itself is assumed to have been implemented since time immemorial, there is no reason for the monetary authority to deviate from this behavior now or in the future.

[33] Specifically, I assume that the optimal policy's starting period is at $t = 0$ and set the vector of state

prior to $t = 0$ and there is zero stochastic disturbance or shock. These last two assumptions make sure that the dynamics coming from this exercise are entirely due to the start-up problem and not to the transition across steady states or to the response to an exogenous shock. After the starting period, the monetary authority is assumed to follow the timeless-perspective policy rule. We can then look at the response of the economy based on these assumptions.[34] For interested readers, appendix D provides further background on the analogous start-up problem in a simple linear-quadractic (LQ) framework under the time-dependent Calvo pricing as in Clarida, Galí, and Gertler (1999).

The start-up problem under TDP Let's first look at the start-up problem under TDP, with the dynamics depicted in Figure 14. As in the discussion above, the monetary authority optimally chooses to generate higher inflation in the starting period of the precommitment policy. Inflation goes up initially by more than 250 basis points from its steady state and this higher-than-steady-state inflation lasts for about five to six quarters. The transition speed of inflation back to its steady-state level mainly depends on the probabilities of price adjustment (α_j), which are assumed to be constant under TDP.[35] The top right panel of Figure 14 displays the motivation behind this surprise inflation: the temporary increase in inflation temporarily erodes firms' average markup, which stimulates consumption and output. (Since there is no government purchase, output and consumption dynamics are identical.) Cumulatively over the entire stimulation periods, consumption is higher relative to its steady state by about 6 percent. Both the real and nominal interest rates decrease during these early periods, but the nominal rate decreases less on average due to expected inflation.

Note that the size of the start-up inflation depends on several factors. First, it depends on the size of the firms' monopoly power and hence, the size of the average markup distortion, in the economy. The higher is the markup distortion, the higher the start-up inflation is.[36] Second, it depends on the shorty-run inflation-output tradeoff, provided by the presence of predetermined prices. Other things held constant, the start-up inflation is lower if prices are more flexible as the ability of the monetary authority to stimulate real activity in the short run is more limited. Third,

variables s_0^2 to zero, which reflect the fact that there is no past commitment in this starting period.

[34] Here, I only consider the first-order response.

[35] This also holds in the analysis of the start-up problem in a simple LQ framework in appendix D.

[36] If there were no markup distortion so that market-generated output level is efficient, there would be no need for the monetary authority to generate this surprise inflation.

it depends on the relative cost of inflation variation since the optimal decision of the monetary authority to generate this surprise inflation and to erode the average markup is *not* a free lunch. In this case, higher inflation increases the size of the relative-price distortion (price dispersion), which affects the welfare of the representative agent. In determining the size of the start-up inflation, the monetary authority has to take into account these various factors. The last two factors are especially important when we consider the start-up problem under the corresponding SDP specification, which I discuss below.

The start-up problem under SDP Figure 15 compares the start-up problem in the SDP and TDP models. We can see that it is still optimal for the monetary authority to generate surprise inflation in the first period under the SDP assumption. Yet, there are two important differences regarding the dynamics. The first one involves the *size* of the start-up inflation at quarter zero. Under SDP, the surprise inflation is much higher: inflation jumps by more than a 500 basis point deviation from the steady state. This result can be understood as follows. As noted above, an economy under SDP can be thought as one with less nominal rigidity due to firms' ability to change their timing of price adjustment. An implication of this lower degree of nominal rigidity is that the monetary authority has less leverage over real activity. This is apparent if we look at the relative consumption (output) and average markup movements in the top right panel of Figure 15. Under SDP, the average markup is eroded by *less* and consumption increases by *less* than in the TDP case in spite of the higher increase in inflation. It is in this sense that the presence of endogenous timing of adjustment under SDP alters the short-run inflation-output tradeoff faced by the monetary authority. Yet, despite this lower leverage over real activity, it is optimal to generate a *higher* start-up inflation under SDP. This is precisely because the cost of inflation variation on the relative-price distortion is *lower* under SDP. That is, even though the decrease in the monetary authority's leverage over real activity means that there is less incentive to generate surprise inflation, the changing nature of the policy tradeoff involving the relative-price distortion makes it optimal to increase inflation by more.

The second difference between the start-up dynamics under the TDP and SDP assumptions is in the transition of inflation back to its steady-state level. Under SDP, inflation is *less* persistent than under TDP. This is intuitive since more firms choose to adjust their prices in response to higher inflation in the SDP case, as can be seen in the bottom left panel of Figure 15.

To summarize, under the SDP environment, the lower cost of inflation variation in terms of relative price distortion gives rise to much *higher* start-up inflation under SDP. This conclusion is achieved even though the monetary authority has less leverage over real activity under SDP. As shown in appendix D, a similar result can also be observed in a simple LQ framework with Calvo pricing when we vary the parameter governing the probability of price adjustment. To a certain degree, the incorporation of SDP in this context can be thought as a Calvo model with a higher probability of price adjustment, provided that stabilization (welfare) weights are adjusted appropriately.

Welfare cost The start-up problem involves the monetary authority choosing to inflate (the true Ramsey policy) rather than adopting the timeless perspective policy. There is a cost associated with ignoring this start-up period. One way to quantify this cost is to compute the unconditional welfare cost associated with *not* taking this opportunity to generate surprise inflation in the starting period of policy implementation. Following Schmitt-Grohe and Uribe (2005, 2007), I define this welfare cost as the fraction of consumption that the representative household would be willing to give up in the benchmark policy environment without past commitment (the true Ramsey policy), so that it would be as well off to be in the alternative policy environment with past commitment (timeless perspective). This welfare cost, w^c, is implicit in the expression

$$E_0 \sum_{t=0}^{\infty} \beta^t u((1-w^c)c_t^c, l_t^c) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \ ,$$

where the superscripts c in consumption and leisure variables in the left-hand-side of the above expression indicate the contingent plans under the benchmark policy without past commitment. The expectation operator above makes clear that the starting implementation period of the optimal policy is at period $t = 0$. Hence, we can compute the welfare cost w^c using the second-order approximation method employed here.[37]

I find that under SDP, w^c is equal to about 0.0088 percent. Using the 2008 nominal U.S. per capita personal consumption expenditures figure of $33,264, this amounts to a welfare cost of about $2.92 per capita per annum. This number means that the cost of adopting the timeless perspective policy is quite small, although it is not negligible. Hence, even though the timeless perspective policy is suboptimal, it is still a good enough representation of the true Ramsey solution.

[37] A second- or higher- order approximation is generally needed for an accurate welfare computation. Kim and Kim (2003) show that using linear approximation to compute welfare can lead to spurious welfare reversals, which is due to the inaccuracy of the linear approximate solution.

6 Conclusion

This paper provides an analysis of optimal monetary policy under precommitment with the assumption that firms' pricing decisions are state dependent. The main lesson from the analysis is that the presence of endogenous timing of price adjustment alters the tradeoff faced by the monetary authority. I also show that the cost of inflation variation on the relative-price distortion is lower under state-dependent pricing than under the standard time-dependent pricing assumption. An implication of this is that it is desirable for the monetary authority to put less weight on inflation stabilization, relative to other stabilization goals.

Several additional findings come from the analysis in the paper. First, the optimal timeless perspective responses to both productivity and government purchase shocks under state-dependent pricing are shown to involve greater inflation variations. Yet the optimal response can still be characterized as an approximate price stability rule. Second, I show that even though the monetary authority has less leverage over real activity under state-dependent pricing, the optimal policy start-up problem involves a higher surprise inflation. Once again, the changing nature of the policy tradeoff is responsible for this result. I also show that the welfare cost associated with adopting the timeless perspective policy instead of the true Ramsey policy is quite small, which is an indication that the optimal policy from the timeless perspective is a good approximation to the true Ramsey solution.

Another finding of this paper comes from the use of a second-order approximation to the equilibrium solution. I show that the policy response under state-dependent pricing exhibits some degree of nonlinearity and state-dependence. This result is especially true for large shocks and when the economy is farther away from the steady state. In a broader sense, this result shows that the aforementioned property of state-dependent pricing models may not be appropriately captured if one only uses a standard linear approximation method.

In terms of future research, the analysis of the paper can be extended in several ways. The central focus of the present study is the characterization, and not the implementation, of optimal monetary policy under state-dependent pricing. However, to be useful for policymakers, one should consider the implementability issue in the future. Another important extension involves the driving process in the model. In this paper, aggregate fluctuations are driven by two standard shocks in neoclassical models: a productivity shock and a government purchase shock. It would be desirable to extend the analysis to a setup where there exists a cost-push shock as is common in

the New Keynesian literature (e.g. Clarida, Galí, and Gertler, 1999). Since the presence of such a shock means that prices tend to move beyond the movement in excess demand, there will be a further nontrivial modification to the policy tradeoff in combination with the state-dependent pricing assumption. Yet another promising research avenue is in terms of the use of a second-order approximation method in characterizing the dynamics of the optimal response. This paper has taken a first step in that direction.

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Appendix

Appendix A: Details of households' intertemporal optimization problem

Households choose the amount of final goods consumption (c_t) and leisure (l_t) to maximize the lifetime utility function,

$$U = \sum\nolimits_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} c_t^{1-\sigma} + \chi \frac{1}{1-\phi} l_t^{1-\phi} \right] ,$$

subject a budget constraint which I describe next. In each period households receive income from their total labor labor effort (n_t) with nominal wage W_t, nominal one-period bond (B_t) from the last period, and dividends from their ownership of the intermediate-goods firms (Z_t). Households also inherit their previous period's portfolio of intermediate-goods firms (x_t). The (pre-dividend) value of the portfolio of firms is V_t. They then pick the amount of consumption for that period, buy current period bonds (B_{t+1}), and may buy more claims on the ownership of the intermediate goods firms. As mentioned in the main text, households optimally choose to finance some portions of consumption goods using money and the other portions using credit. Money is used since credit is costly, as in the transaction cost models of Baumol (1954) and Tobin (1956). Some goods are bought using credit because there is an opportunity cost of holding money (the nominal interest rate). Households thus accumulate debt from the credit use. This debt is assumed to be paid in the next period without any interest. Let ξ_t be the proportion of credit goods at time t. If we let $\bar{P}_t$ as the nominal price of final consumption goods at time t, then the amound of nominal money holding is $M_t = (1- \xi_t)\bar{P}_t c_t$. The amount of nominal debt that must be paid in the next period is then $D_{t+1} = \xi_t \bar{P}_t c_t$. There is also a lump-sum tax T_t that must be paid to the government. Hence, the budget constraint at time t is given by

$$M_t + \frac{1}{1+R_t} B_{t+1} + x_{t+1}(V_t - Z_t) \leq x_t V_t + B_t + W_t n_t - D_t + T_t \ .$$

Note that one can make the nominal budget constraint into a real one by dividing it by the aggregate price level P_t - in the case of the CES aggregator,

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} ,$$

where $P_t(i)$ is the nominal price of intermediate good i. On the price of final consumption goods, $\bar{P}_t$, the assumption is that the final-goods producers (retailers or households) must borrow to finance

the production with interest rate R_t. Since the final-goods producers are perfectly competitive and receive zero profits, it follows that $\bar{P}_t = (1 + R_t)P_t$.

As in Khan, King, and Wolman (2003), I assume that the each of the (mass) of final consumption goods has a random fixed time cost in terms of labor unit. (Although it is common to assume the representative household is assumed to purchase a single final consumption good, one can think of the final good as a continuum of final goods in a unit interval). Let $F(.)$ be the cumulative distribution function of the time cost. The largest time cost that households are willing to pay is then $F^{-1}(\xi_t)$. A good would be purchased with credit if its time cost is below this largest time cost, while money would be used to purchase a good if its time cost is above this value. The total transaction time at time t in labor unit is then given by

$$h_t = \int_0^{F^{-1}(\xi_t)} v dF(v) \ .$$

Since labor must also be used in production (n_t^y) and in price adjustment activity (n_t^p), the total labor time is then

$$n_t = n_t^y + n_t^p + h_t = 1 - l_t \ .$$

We can then form a Lagrangian to solve for the optimization problem described above. Let λ_t be the multiplier attached to the budget constraint. Solving for the Lagrangian and rearranging the first order necessary conditions lead to equations (8), (9), (10), (11), and (12) in the main text.

Appendix B: Credit cost distribution and money demand function

This appendix describes the credit cost distribution and the implied money demand function used in the main text. In this shopping-time assumption, the implied optimal choice between using money or credit based on households' efficiency conditions is given by

$$F^{-1}(\xi) = \frac{Rc}{w}$$

where $F(\cdot)$ is the CDF of the credit cost distribution and ξ is the proportion of goods bought using credit. If we let P as the nominal price of final goods (so that Pc is the nominal consumption expenditure), it follows that the nominal demand for money is $M = (1 - \xi)Pc$. Combining this with the efficient credit used above, we have

$$\frac{M}{Pc} = 1 - F(\frac{Rc}{w})$$

Note that when all goods are bought using money ($\xi = F(\frac{Rc}{w}) = 0$), the money demand function then collapses to $\frac{M}{P} = c$, as in cash-in-advance models.

Khan, King, and Wolman (2003) uses the above expression to estimate the money demand function given a particular distribution $F(\cdot)$. Specifically, they assume that the distribution is a generalized beta distribution of the form

$$F(x) = \check{\xi} + \hat{\xi} \cdot F^{beta}(\frac{x}{\Omega}; a_1, a_2)$$

where a_1 and a_2 are the two parameters the beta distribution and where the support has to be normalized by Ω, the largest credit cost. Here, $\check{\xi}$ is the proportion of goods that have zero credit cost with the property $0 < \check{\xi} < 1$, so that there are always some goods with zero credit cost. $\hat{\xi} \leq 1 - \check{\xi}$ is then the proportion of goods that have non-zero cost distributed according to beta distribution. Given this specified distribution, Khan, King, and Wolman (2003) estimate the five parameters $(\check{\xi}, \hat{\xi}, a_1, a_2, \Omega)$ by minimizing the distance (sum of squared errors) between $1 - \frac{M}{Pc}$ and $F(\frac{Rc}{w})$ based on the money demand function above and given data for M (nominal money), Pc (nominal consumption expenditure), R (nominal interest rate), and w (real labor compensation). The estimation used in this paper is for the sample period 1959:Q1 through 1989:Q4. The implied money demand elasticity based on the estimation is less than 0.3 in absolute value thoughout the sample period used.

Appendix C: Adjustment cost distribution

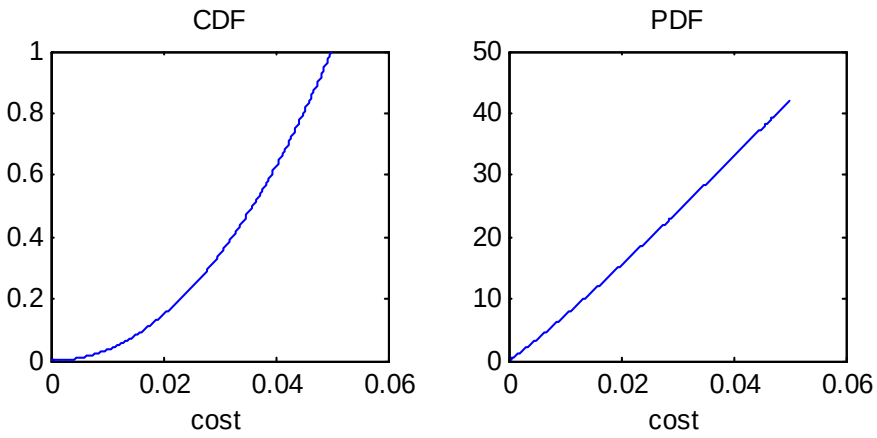
The paper uses beta distribution for the distribution of the price adjustment costs. Specifically, the PDF of the beta distribution is

$$g^{beta}(x; a_l, a_r) = \frac{1}{\beta(a_l, a_r)} \left[x^{a_l - 1}(1 - x)^{a_r - 1} \right] ,$$

where $\beta(a_l, a_r) = \frac{\Gamma(a_l + a_r)}{\Gamma(a_l)\Gamma(a_r)}$ is the beta function which is a normalization of the gamma function $\Gamma(\cdot)$. This beta function can also be written as $\beta(a_l, a_r) = \int_0^1 u^{a_l - 1}(1-u)^{a_r - 1} du$. Note that the beta distribution is known to be a flexible distribution in which various shapes of the PDF (increasing, decreasing, convex, concave, etc) can be attained for suitable choices of parameters a_l and a_r. The CDF is given by

$$G^{beta}(x; a_l, a_r) = \frac{\beta_x(a_l, a_r)}{\beta(a_l, a_r)} ,$$

where $\beta_x(a_l, a_r)$ is the incomplete beta function.



Since the support of the beta distribution lies on a unit interval, $x \in [0,1]$, we need to normalize the support by the largest price adjustment cost B, so that the PDF of the distribution of adjustment cost is given by

$$g(\epsilon; a_l, a_r; B) = g^{beta}(\frac{\epsilon}{B}; a_l, a_r)$$

for a given fixed adjustment cost ϵ. Hence, the support for the distribution above lies on the interval $0 \leq \epsilon \leq B$. The same procedure goes for the CDF as well. Next, using the specified distribution above, we can compute the expected level of cost of adjustment, Ξ, for a given proportion of firms adjusting, α. Since in the model, α is the determined by the cutoff rule $v_0 - v_j = w\bar{\epsilon}$, we can express Ξ as a function of the fixed cost cutoff $\bar{\epsilon}$. This expression can be cast as

$$\Xi(\bar{\epsilon}) = B \int_0^{\bar{\epsilon}/B} x g^{beta}(x; a_l, a_r) dx \ .$$

For the parameters of the distribution, I choose $a_l = 2.1$ and $a_r = 1$. This leads to the following shapes for the CDF and PDF

Appendix D: The start-up problem under linear-quadratic (LQ) framework

This appendix serves to put forth the start-up problem, present in all optimal monetary policy models under precommitment with imperfect competition. I do this by considering optimal policy problem in a familiar linear-quadratic (LQ) framework, for example as in Clarida, et. al. (1999) with Calvo staggered pricing. I note some notational overlaps with the main text in the paper.

Here, the familiar model has two distortions present in the economy: the relative price distortion and the markup distortion due to monopolistically competitive firms that set prices according to

Calvo's price setting. As in Woodford (2003) and Benigno and Woodford (2004, 2005), we can derive a quadratic loss function using the second order approximation to the utility function. I focus on the deterministic case and the case in which the size of the markup distortion is small in deriving the loss function. The more realistic stochastic case is not relevant for our purpose here since we can just consider the start-up problem assuming that the stochastic disturbances are zero. It is true that the size of the markup distortion will matter for how large the start-up effect is, but since our purpose is just to collect idea on the monetary authority's incentive to inflate, we can just assume a small markup distortion without any loss of generality—as long as there is a markup distortion, there would an incentive to inflate in the first period.

Given the discussion above, the monetary authority's objective function is thus given by

$$L = -\frac{1}{2}E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\lambda\left(x_t - x^*\right)^2 + \pi_t^2\right]\right\}, \tag{32}$$

where x_t is the output gap, π_t is the inflation rate (percentage change of price level from time $t-1$ to time t), and x^* is the target output gap. The form of the loss function indicates that the monetary authority should seek to stabilize inflation around zero and the output gap around the target level x^*. To derive the loss function above, I assume that the momentary utility function is given by $u(c_t, n_t) = \log(c_t) - \varepsilon\frac{n_t^{1+v}}{1+v}$, with c_t represents the Dixit-Stiglitz aggregate consumption. The production function for the monopolistically competitive firms is linear in labor and these firms hire labor competitively in a global labor market. Given these assumptions, it follows that $x^* = v\Phi = v(1 - \frac{\theta-1}{\theta})$, where θ is the elasticity the substitution across goods varieties. In this expression, Φ measures the inefficiency of steady state output level due to the markup distortion. Also, the relative weight on output gap stabilization is given by $\lambda = \frac{\kappa}{\theta}$, where κ is the coefficient on the output gap in the Phillips curve

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}. \tag{33}$$

With Calvo's probability of non-adjustment denoted by α and assuming a unit output elasticity of real marginal cost, it follows that $\kappa = (1-\alpha)(1-\alpha\beta)/\alpha$.

The optimal policy under commitment then involves a benevolent monetary authority that selects a state-contingent sequence of output gap x_t and inflation π_t for all periods $t \geq 0$ by

maximizing (32) subject to (33).[38] The Lagrangian for the maximization problem is

$$L_0 = -\frac{1}{2}E_0\left\{\sum_{t=0}^{\infty}\beta^t\left\{\left[\alpha\left(x_t - x^*\right)^2 + \pi_t^2\right] + \phi_t\left[\pi_t - \lambda x_t + \beta E_t\pi_{t+1}\right]\right\}\right\}$$

where ϕ_t is the multiplier of the constraint. The optimality conditions therefore are

$$\alpha(x_t - x^*) - \frac{\lambda}{2}\phi_t = 0 \tag{34}$$

$$\pi_t + \frac{1}{2}\phi_t - \frac{1}{2}\phi_{t-1} = 0 \tag{35}$$

$$\pi_0 + \frac{1}{2}\phi_0 = 0\ , \tag{36}$$

where the first condition holds for all $t \geq 0$ and the second condition holds for all $t > 0$. Equations (35) and (36) show that the monetary authority should treat the starting period ($t = 0$) differently than the subsequent periods. That is, the lagged multipler associated with past commitments prior to the start-up period is zero, highlighting that that there is no past commiment in the start-up period.

Following Marcet and Marimon (1999), we can cast the maximization problem into a recursive form. Formally, let's consider the following augmented Lagrangian:

$$\begin{gathered} V(\phi_{-1}, s_0) = min_{\{\phi_t\}_{t=0}^{\infty}} max_{\{\pi_t, x_t\}_{t=0}^{\infty}} \\ -\tfrac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{[\lambda\left(x_t - x^*\right)^2 + \pi_t^2] \\ -\phi_t(\kappa x_t) + (\phi_t - \phi_{t-1})\}\ . \end{gathered}$$

Recursively, the above Lagrangian can be written as

$$\begin{gathered} V(\phi_{t-1}, s_t) = min_{\{\phi_t\}} max_{\{\pi_t, x_t\}} \\ -\tfrac{1}{2}\{[\lambda\left(x_t - x^*\right)^2 + \pi_t^2] + \beta E_t V(\phi_t, s_{t+1}) \\ -\phi_t(\kappa x_t) + (\phi_t - \phi_{t-1})\}\ . \end{gathered}$$

Hence, by adding a lagged multiplier to the the policy problem, we get the same optimality conditions, given by (34) and (35), for all periods $t \geq 0$. One can interpret this lagged multiplier as

[38] There is another constraint involving an aggregate demand condition (the IS curve equation), but this condition is not needed to solve for the optimal sequences of inflation and the output gap.

the commitment that must be followed by the monetary authority. Note that equations (33), (34), and (35) are a system of difference equations, of which the stable stationary solutions are given by

$$x_t = \delta x_{t-1}$$

$$\phi_t = \delta \phi_{t-1} + \frac{2(\delta - 1)\lambda}{\kappa} x^*$$

$$\pi_t = \delta \pi_{t-1} \ ,$$

where $a \equiv \frac{\lambda}{\lambda(1+\beta)+\kappa^2}$ and $\delta = \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0,1)$. The unique solution for optimal π_t that is consistent with the initial condition $\phi_{-1} = 0$ is given by[39]

$$\pi_t = (1-\delta)\frac{\lambda}{\kappa}\delta^t x^* \ . \tag{37}$$

Equation (37) shows that inflation in the start-up period $t = 0$ is higher than those in subsequent periods, illustrating that surprise inflation is optimal in the start-up period. Several comments on (37) are in order. First, the size of the start-up problem depends on several features of the model economy, with the size of the target output gap x^* playing a prominent role. In this model setup, we can interpret the size of the target output gap x^* as the size of the steady-state markup distortion in the economy—that is, the inefficiency in the economy due to firms' monopoly power. If there is no monopoly power, there would be no incentive to inflate since $x^* = 0$ in this case. Equation (37) tells us that the monetary authority has an incentive to inflate only if the steady-state output is inefficient ($x^* > 0$). Put another way, the start-up problem is present because there is a markup distortion that still exists even under zero inflation, and the monetary authority can erode this markup distortion because there is no past commitment that must be respected in the start-up period 0 ($\phi_{-1} = 0$). The higher is the degree of inefficiency due to markup distortion (higher Φ and x^*), the higher would the start-up inflation be. The start-up effect will eventually die down with the rate δ per period.

What is the effect of the probability of non-adjustment α on the start-up problem? We can investigate this by decomposing (37) into period $t = 0$ and the subsequent periods:

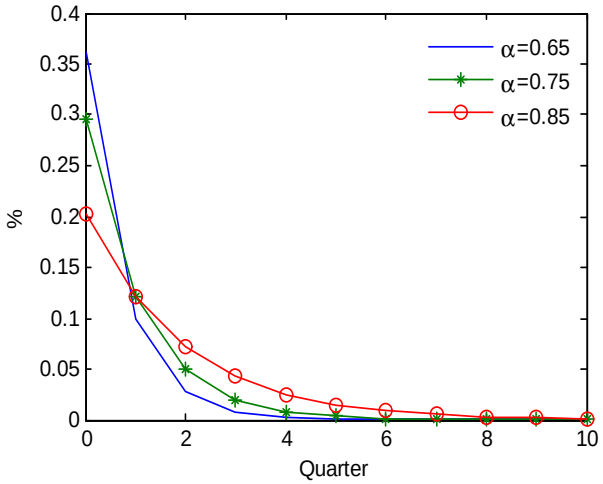
$$\pi_0 = (1-\delta)\frac{1}{\theta}x^*, \tag{38}$$

[39] Using the same technique, we can also find the evolution of the output gap that is consistent with $\phi_{-1} = 0$. This is given by $x_t = \delta^{t+1} x^*$.

$$\pi_t = \delta \pi_{t-1} \ , \forall \ t > 0. \tag{39}$$

Note that I have used the fact that $\lambda = \kappa/\theta$ in deriving (38) from (37). One can show that the probability of price fixity α positively affects δ for given values of β, v, and θ. Hence, based on (38), a higher α (more price stickiness) translates into lower start-up inflation in period 0, for a given size of the steady-state markup distortion. This feature is related to the cost of higher inflation on the relative-price distortion: since a higher degree of price stickiness leads to a higher relative-price distortion for a given inflation rate, the optimal level of surprise inflation in the first period should be lower when α is higher. That is, the start-up inflation is lower if prices are stickier since the welfare cost of inflation is higher. Equation (39) shows that the speed of the transition of inflation back to its steady state level depends entirely on δ. The higher is the probability of price fixity, the higher is δ—hence, the slower the transition is. The figure below shows the evolution of inflation as in (37) for several parametrizations of α , with all other parameters fixed at particular values.[40]

Figure D.1



Note that we can draw a parallel between Figure 3.1 above and the result on the start-up problem under SDP in the main text. Within this context, the economy under SDP can be approximately thought of as a Calvo model with a lower α (less price stickiness), provided that the relative stabilization weight (λ) is appropriately adjusted.

[40] Specifically, I assume the following parameter values: $\beta = 0.99$, $\theta = 10$, and $v = 0.5$.

Tables and Figures

Table 1: Calibrated Parameters

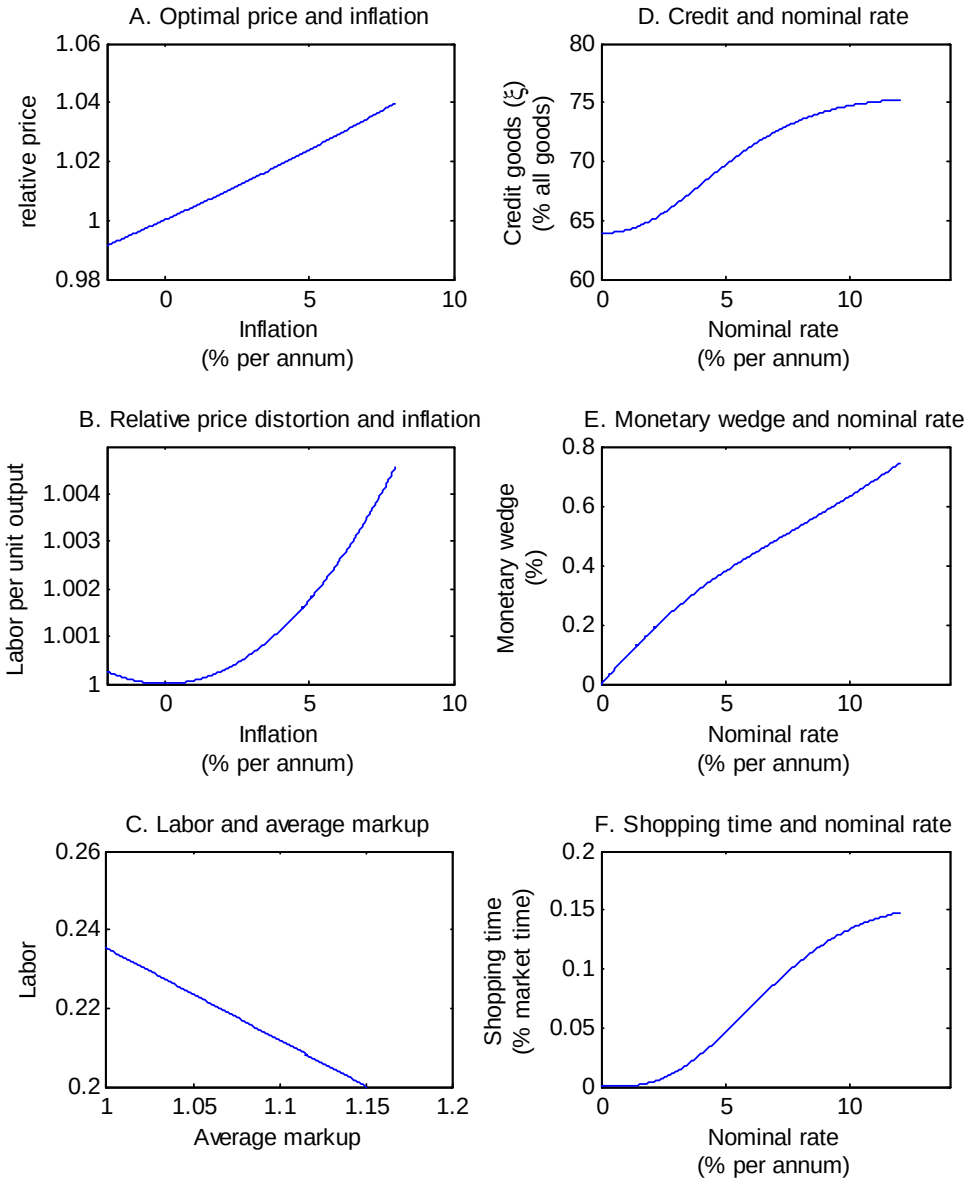
Rate of time preference (β)	0.99
Labor supply elasticity (ϕ)	1
Elasticity of intertemporal substitution (σ)	1
Scale parameter in the utility (χ)	3.59
Demand elasticity (ε)	10
Fixed adjustment cost distribution (beta)	
Beta left parameter (a_l)	2.10
Beta right parameter (a_r)	1
The largest fixed cost (B)	0.00057
Credit cost distribution (generalized beta)	
Mass of goods with zero credit cost ($\check{\xi}$)	0.639
Beta left parameter (a_1)	2.806
Beta right parameter (a_2)	10.446
The largest credit cost (Ω_c)	0.013
Productivity shock	
Persistence parameter (ρ_a)	0.95
Standard deviation of shock (σ_a)	0.0072
Government purchase shock	
Persistence parameter (ρ_g)	0.90
Standard deviation of shock (σ_g)	0.008

Table 2: Steady-state annualized inflation under optimal policy

	SDP	TDP
Benchmark ("full" distortions)	-0.993%	-0.988%
Lower ε ($\varepsilon = 7$)	-1.502%	-1.489%
Eliminating variations in		
Shopping-time	-0.471%	-0.466%
Monetary wedge	-0.661%	-0.657%
Shopping-time and monetary wedge	——	0%
Markup	-1.048%	-1.048%

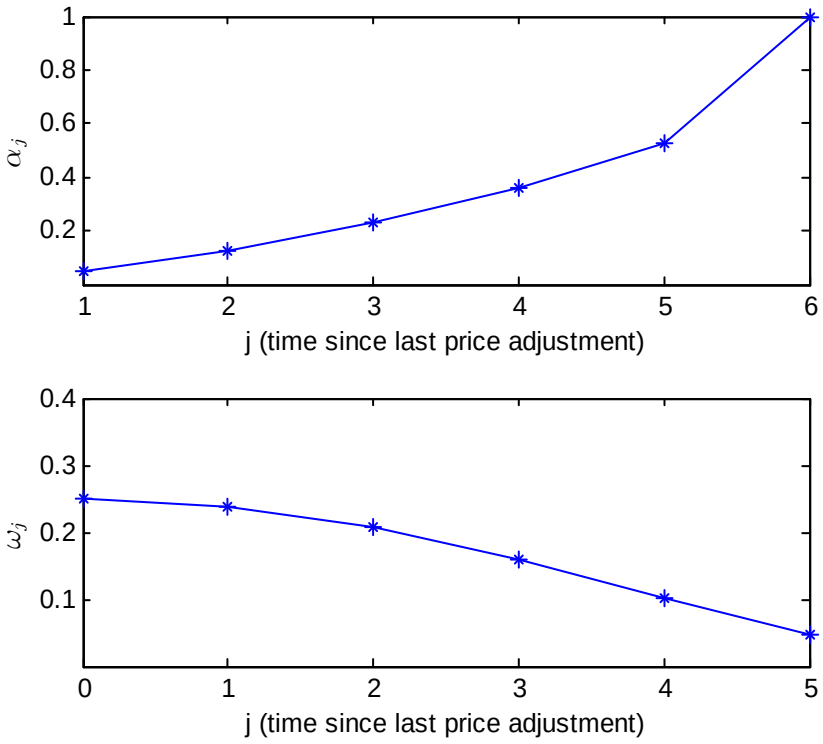
Notes: (1) the TDP model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$) are equal to the steady-state values under TDP, with the same $J = 6$; (2) the shopping-time distortion is eliminated by setting $\int_0^{(R_t c_t/w_t)} x dF(x)$ in equation (12) to a constant and resolving the optimal policy problem; (3) the monetary wedge distrotion is eliminated by setting $R_t(1-\xi_t)$ in equation (10) to a constant and resolving the optimal policy problem; (4) the markup distortion is eliminated by setting w_t/a_t to a constant and resolving the optimal policy problem.

Figure 1: Steady state implications of various distortions



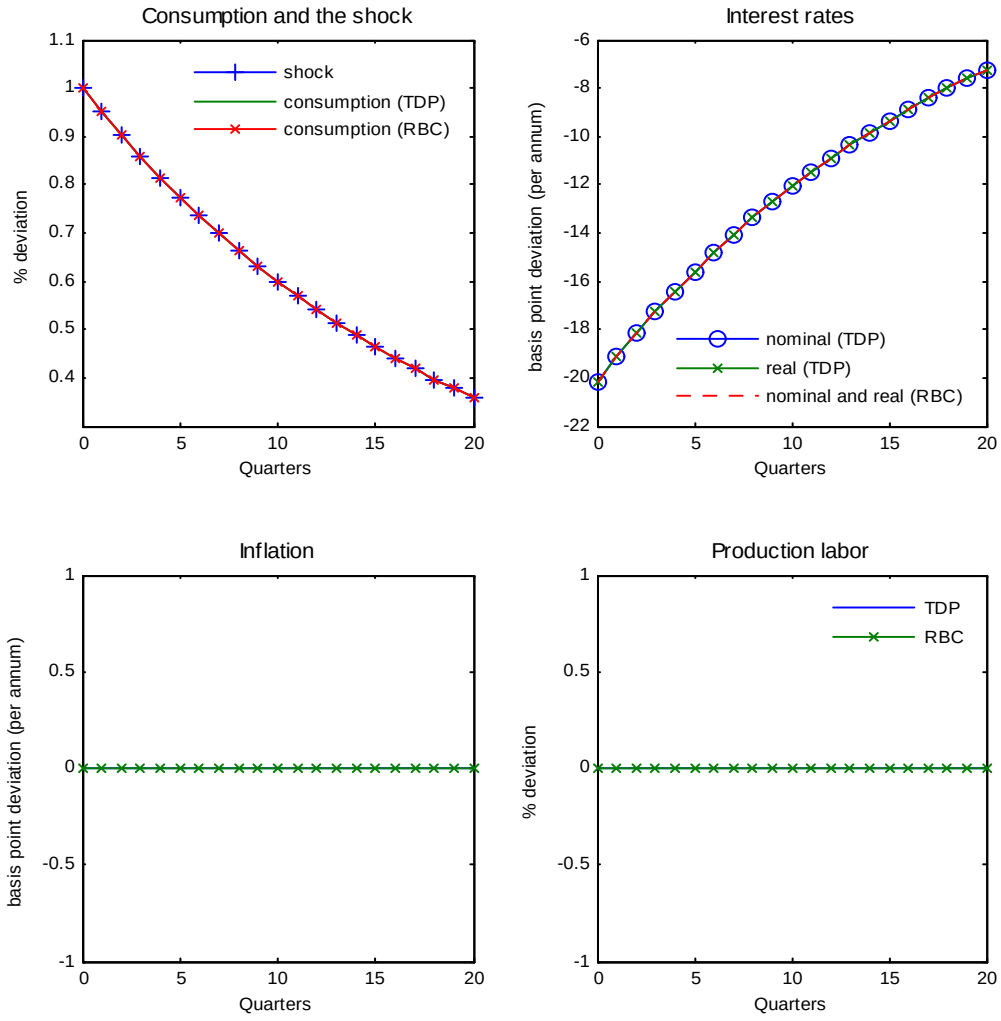
Notes: (1) panels A, B, and C are generated using $J = 6$ with $\{\omega_j\}_{j=0}^{J-1}$ and $\{\alpha_j\}_{j=1}^{J-1}$ equal to their steady-state values under the optimal policy presented in Figure 2, with monetary and menu cost distortions are assumed to be absent; (2) panels D, E, and F are constructed using a beta distribution for the credit cost distribution, with parameter values as in table 1—further, I assume constant $n^y = 0.2$ and $w = a = 1$ so that $c = a \cdot ny = 0.2$, as the nominal interest rate (R) varies.

Figure 2: Steady-state probability (α_j) and distribution (ω_j) of price adjustment
Optimal policy under SDP



Notes: (1) the longest period of price fixity is $J = 6$; (2) all market distortions are present (full distortions); (3) the horizontal (j) axis is in quarters.

Figure 3: Optimal timeless perspective responses to a productivity shock
RBC vs. TDP cashless
First-order approximation

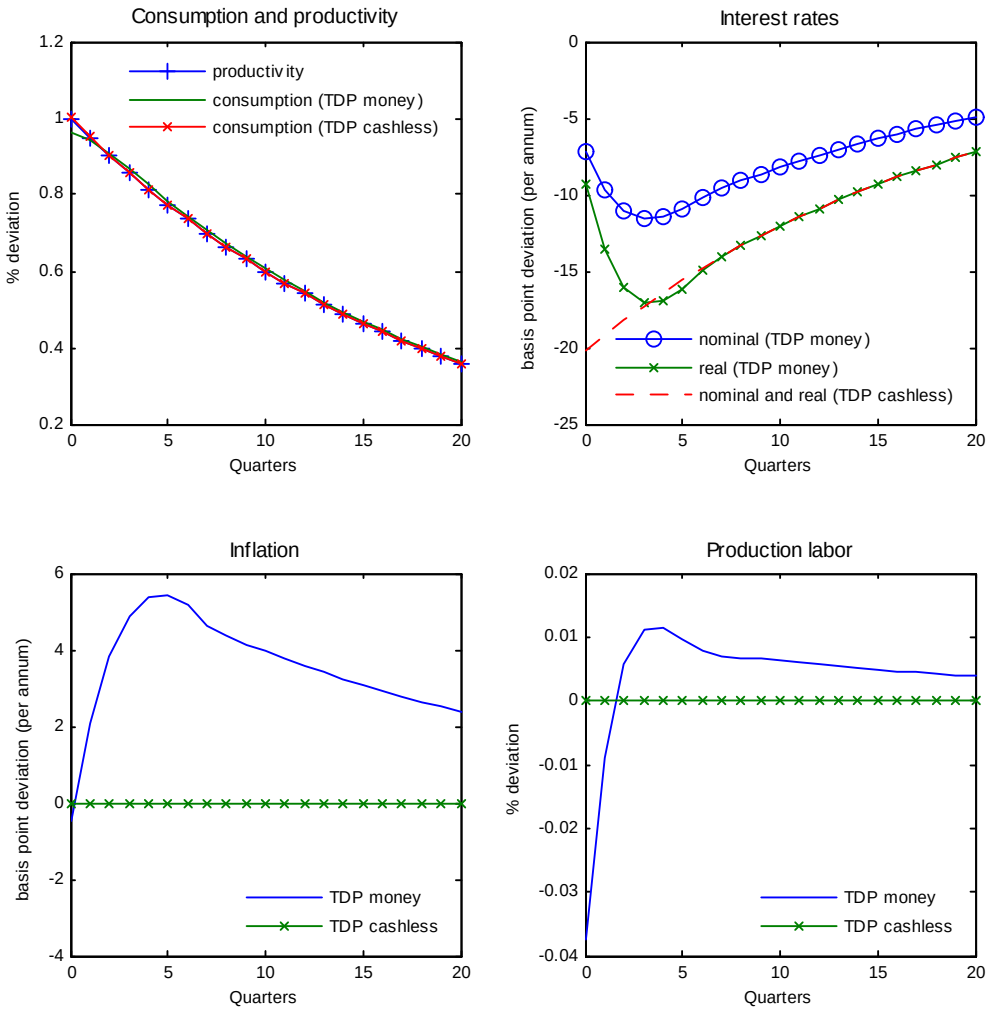


Notes: (1) *RBC* refers to the dynamics of a real business cycle model with flexible price and where firms are monopolistically competitive; (2) *TDP cashless* refers to the dynamics under TDP model when monetary distortions (shopping-time and monetary wedge) are absent; (3) the shock size is 2 standard deviation, normalized so that productivity increases by 1 percent on impact; (4) the TDP model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$ are equal to the steady-state values under TDP, with the same $J = 6$.

Figure 4: Optimal timeless perspective responses to a productivity shock

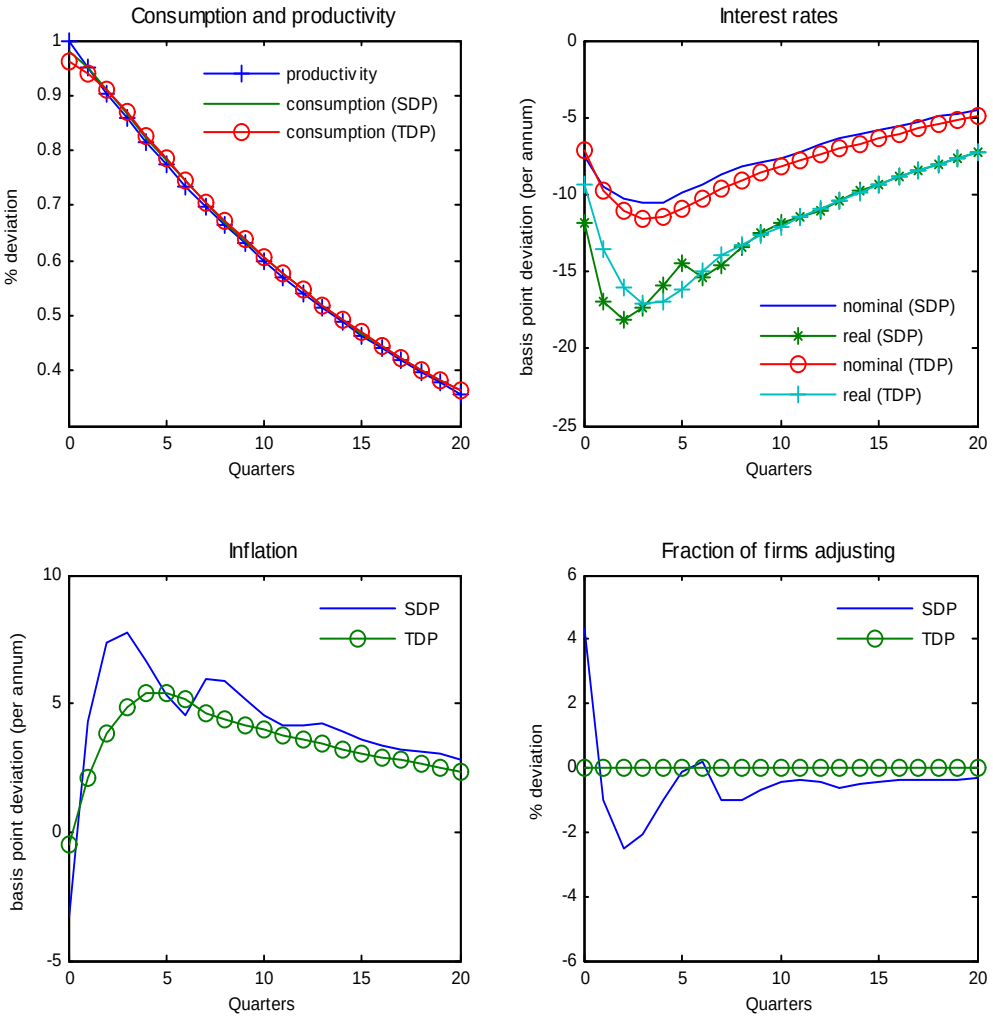
TDP cashless vs. TDP money

First-order approximation



Notes: (1) *TDP cashless* refers to dynamics under TDP model when monetary distortions (shopping-time and monetary wedge) are absent; (2) *TDP money* refers to the dynamics under TDP model when all distortions, including monetary distortions, are present; (3) the shock size is 2 standard deviation, normalized so that productivity increases by 1 percent on impact; (4) the TDP model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$ are equal to the steady-state values under TDP, with the same $J = 6$.

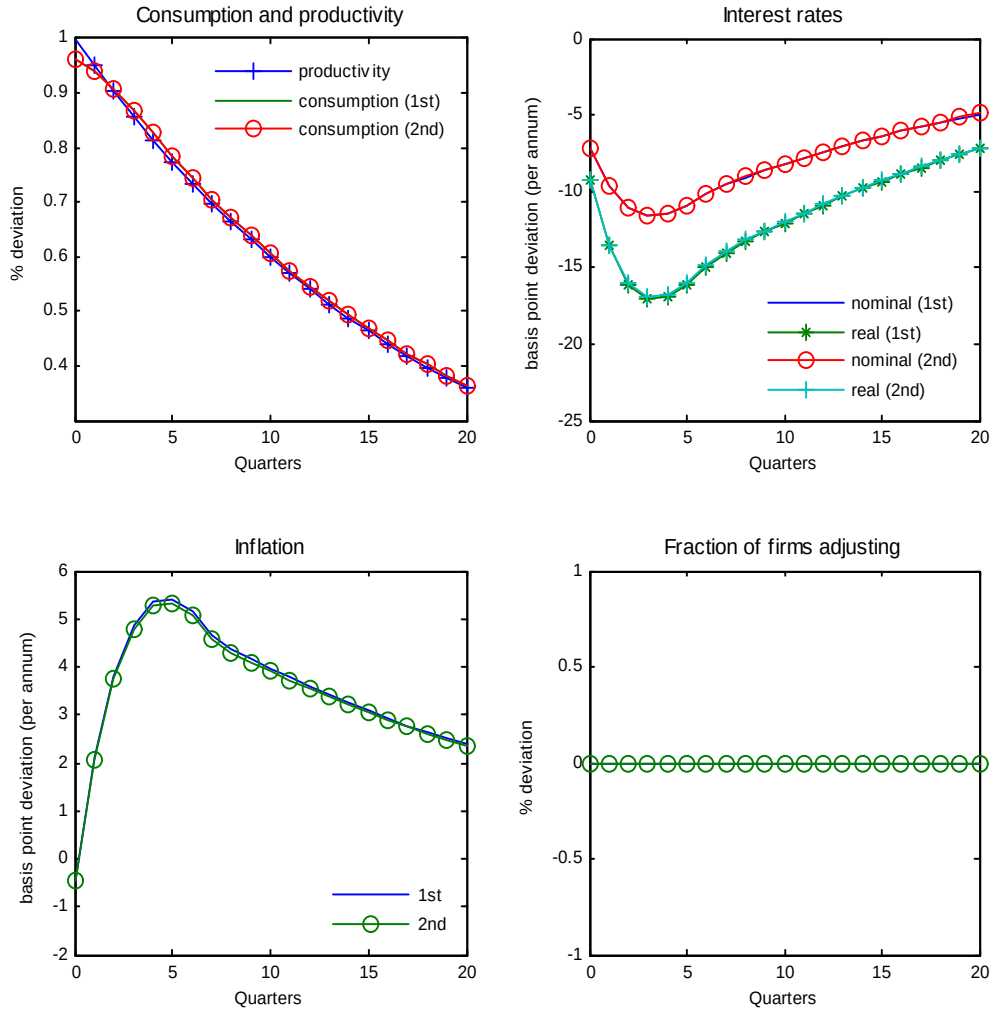
Figure 5: Optimal timeless perspective responses to a productivity shock
TDP vs SDP
First-order approximation



Notes: (1) the figure is constructed under the assumption that all distortions are present; (2) the shock size is 2 standard deviation, normalized so that productivity increases by 1 percent on impact; (3) the TDP model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$ are equal to the steady-state values under TDP, with the same $J = 6$.

Figure 6: Optimal timeless perspective responses to a productivity shock under TDP

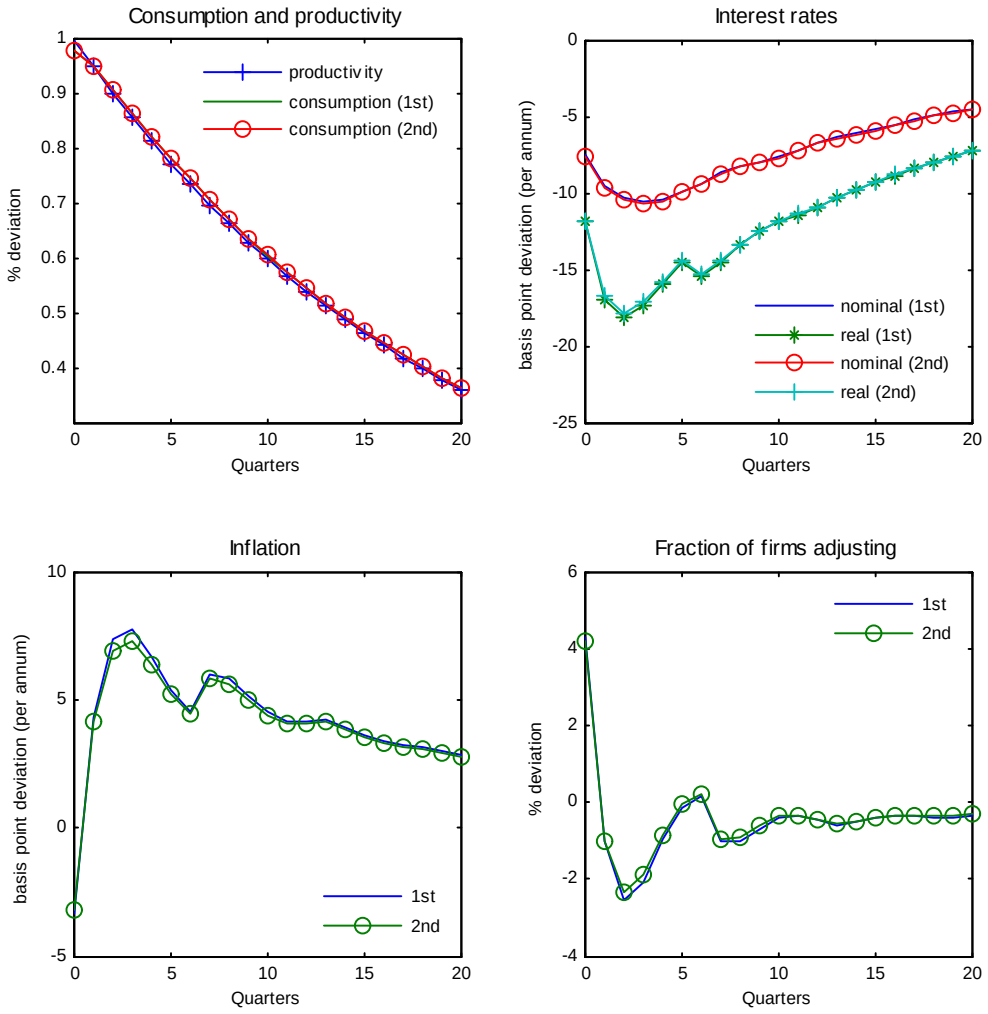
First- vs. second-order approximations



Notes: (1) the figure is constructed under the assumption that all distortions are present; (2) *1st* and *2nd* refer to the dynamics under the first- and second- order approximation solutions, respectively; (3) the shock size is 2 standard deviation, normalized so that productivity increases by 1 percent on impact; (4) the TDP model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$ are equal to the steady-state values under TDP, with the same $J = 6$.

Figure 7: Optimal timeless perspective responses to a productivity shock under SDP

First- vs. second-order approximations

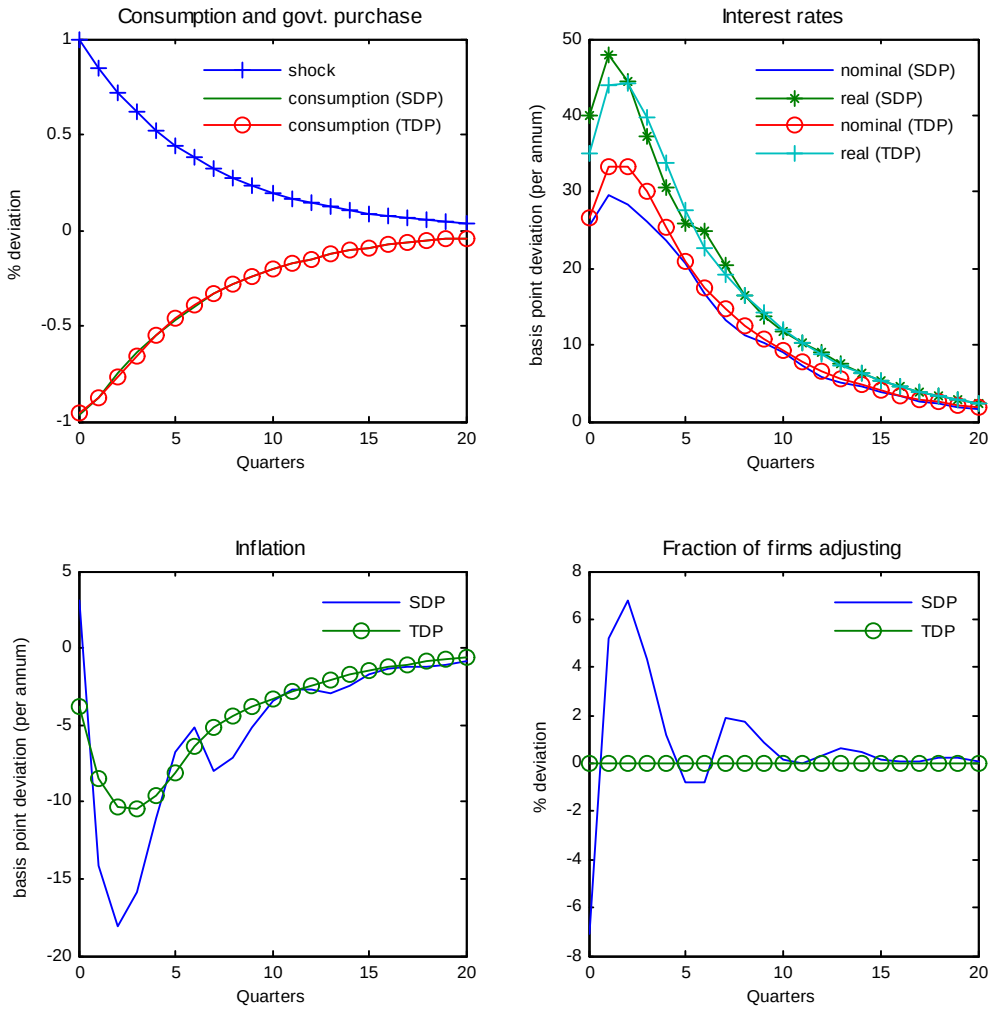


Notes: (1) the figure is constructed under the assumption that all distortions are present; (2) *1st* and *2nd* refer to the dynamics under the first- and second- order approximation solutions, respectively; (3) the shock size is 2 standard deviation, normalized so that productivity increases by 1 percent on impact.

Figure 8: Optimal timeless perspective responses to a government purchase shock

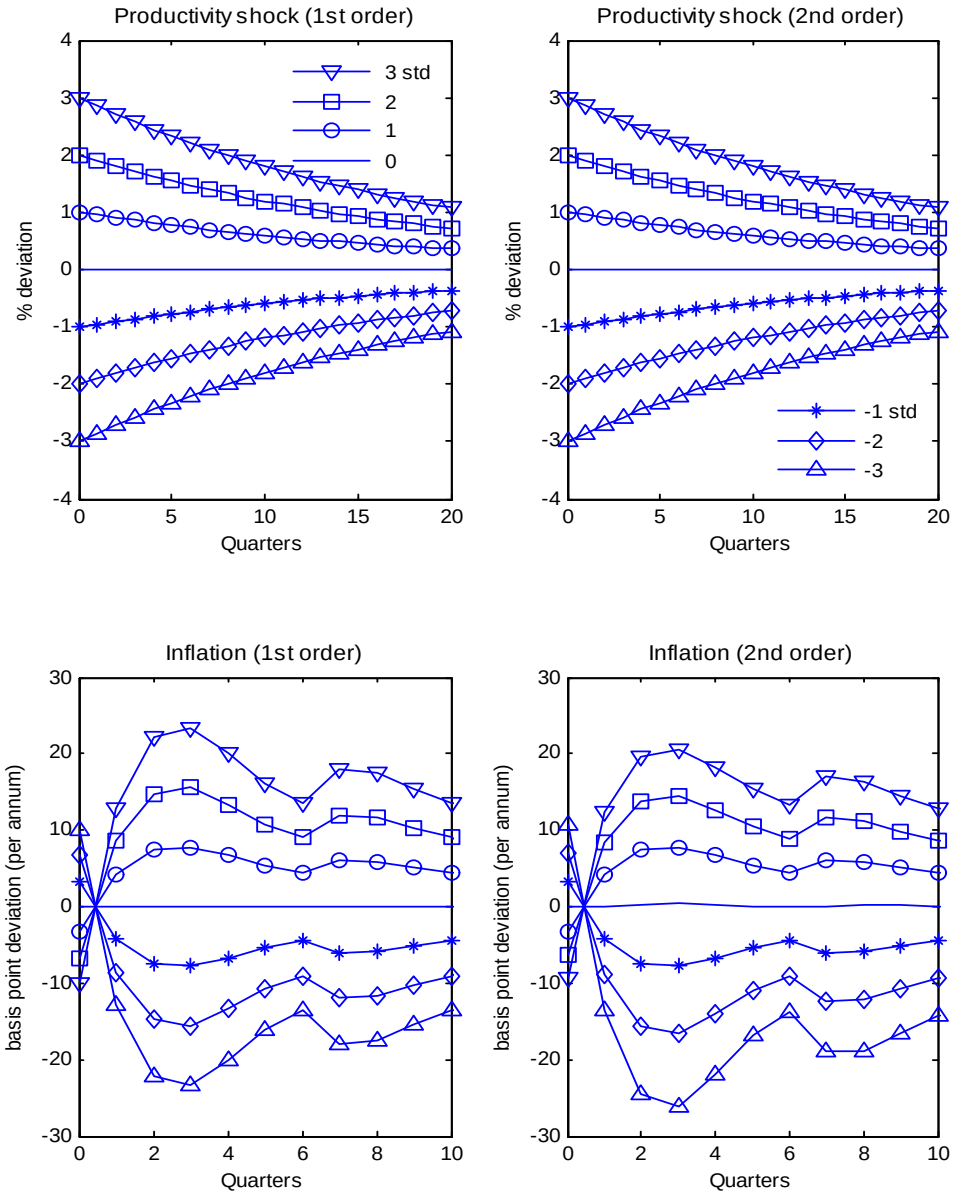
TDP vs SDP

First-order approximation



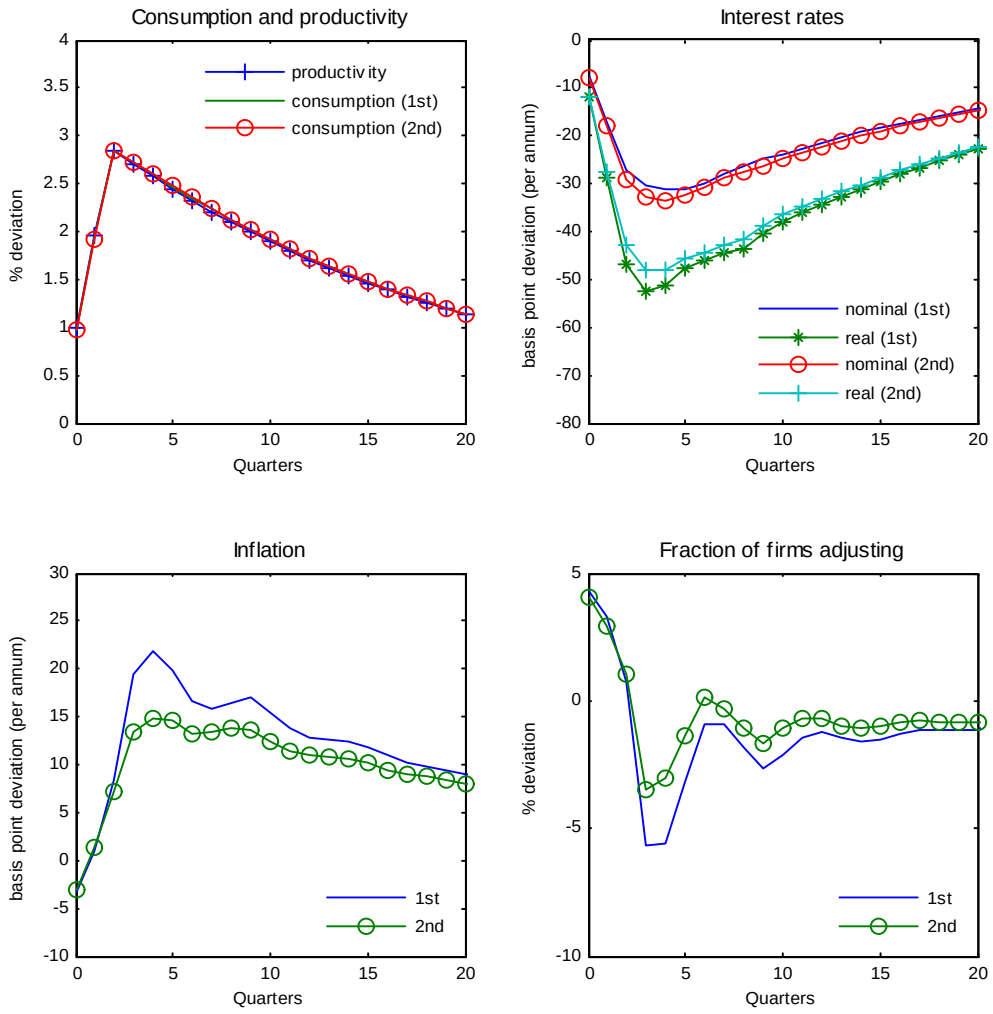
Notes: (1) the figure is constructed under the assumption that all distortions are present; (2) the shock size is 2 standard deviation, normalized so that productivity increases by 1 percent on impact; (3) the TDP model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$ are equal to the steady-state values under TDP, with the same $J = 6$.

Figure 9: Inflation response to a productivity shock for various shock sizes
Optimal timeless perspective policy under SDP



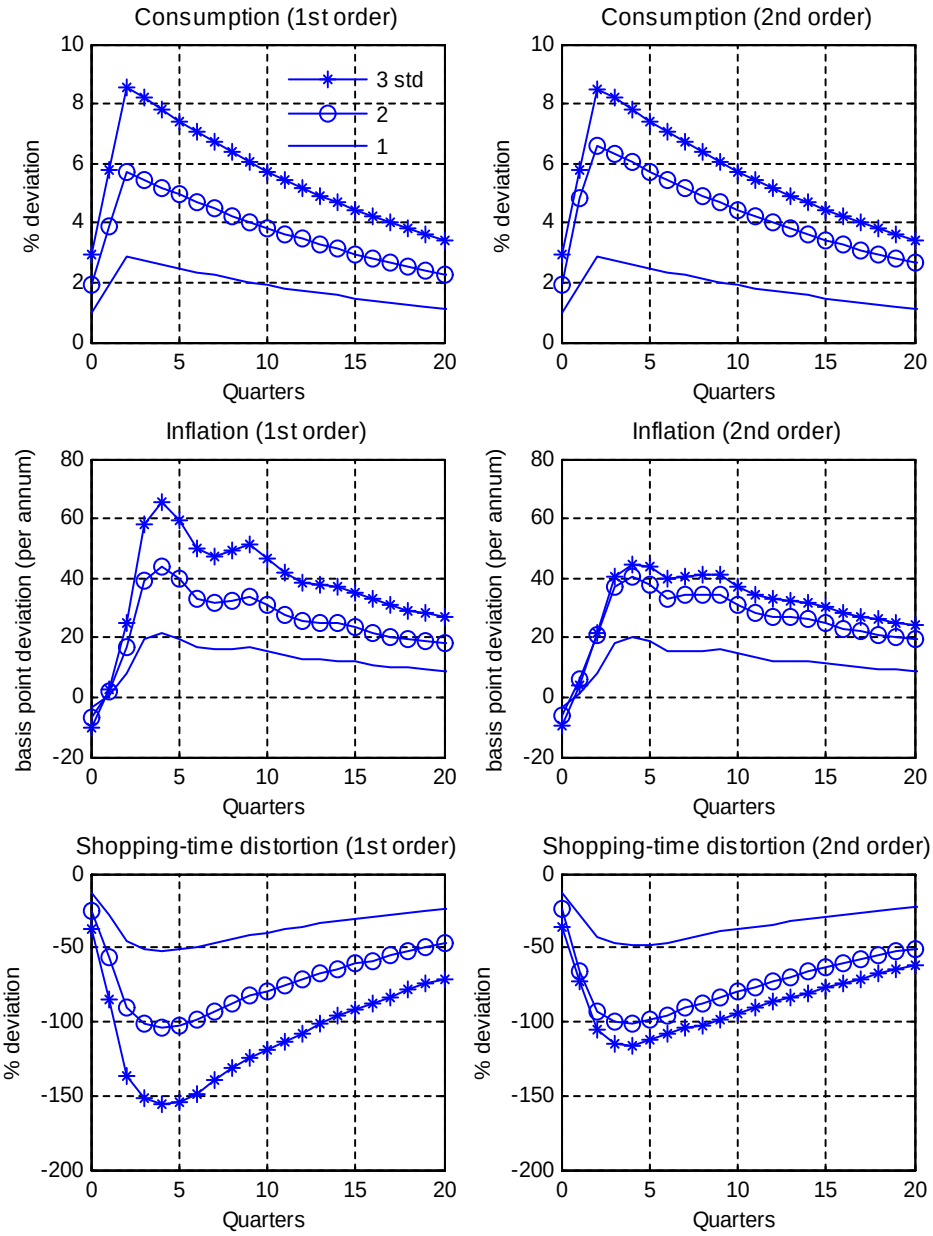
Notes: (1) these are responses to a productivity shock at period 0, starting from steady state, for various shock sizes (3, 2, 1, -1, -2, and -3 standard deviation shocks; (2) responses are normalized such that an m standard deviation shock corresponds to an m percent increase or decrease in productivity on impact relative to the steady-state value; (3) all market distortions are present.

Figure 10: State-dependence effect and nonlinearity
Optimal timeless perpective policy under SDP, productivity shocks



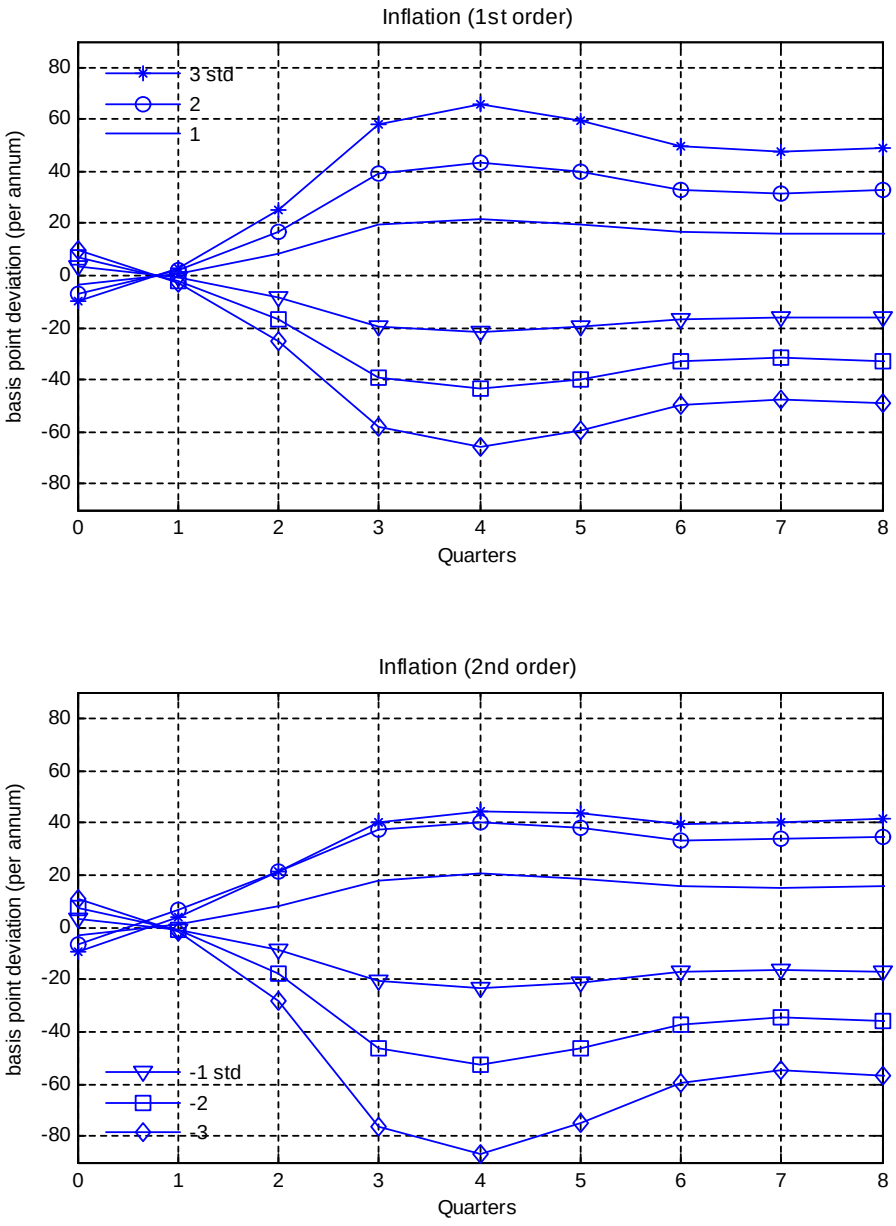
Notes: (1) these are responses to a repeated sequence of productivity shocks at periods 0, 1, and 2; (2) each shock is a 3 standard deviation shock, normalized so that on impact it translates to a 1 percent increase in productivity relative to its steady-state value; (3) all market distortions are present.

Figure 11: Interaction between state-dependence and shock size
Optimal timeless perspective policy under SDP, productivity shocks



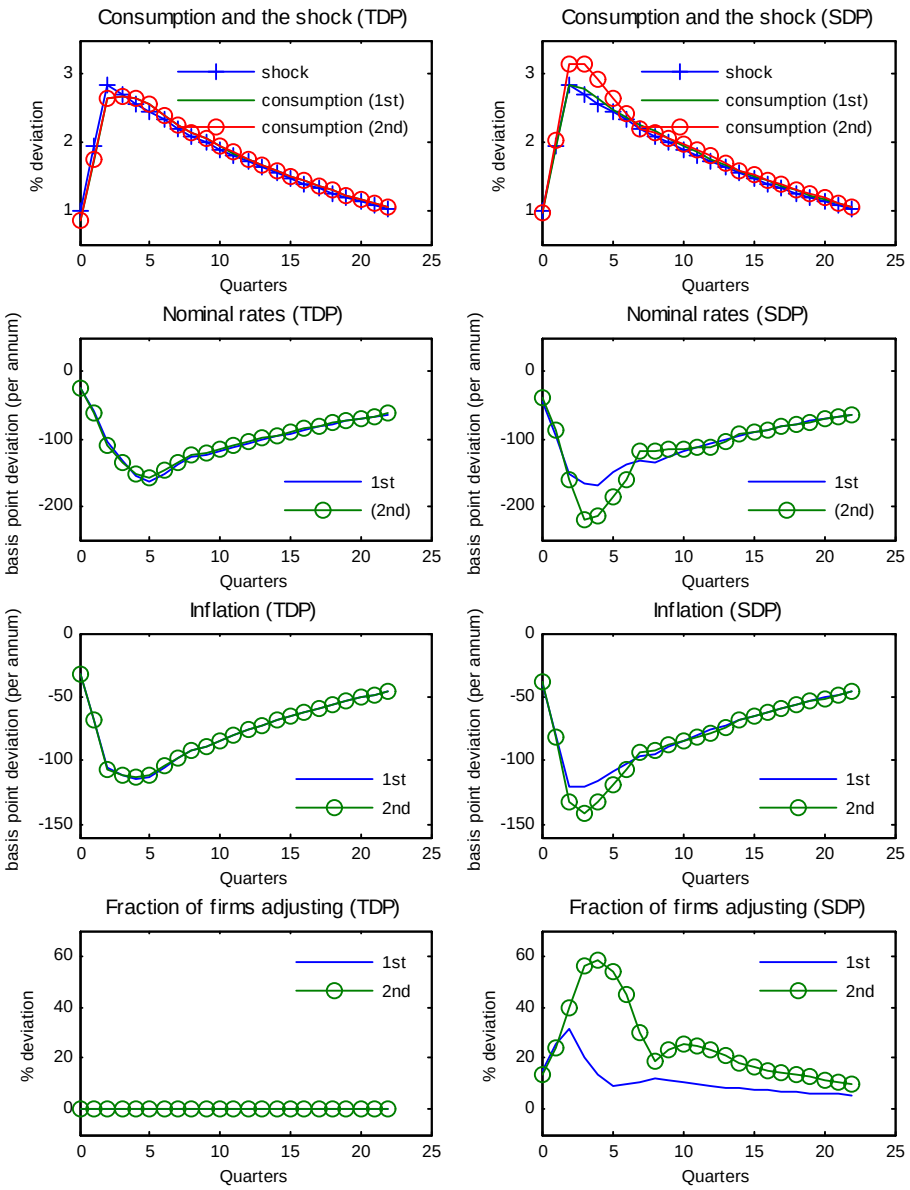
Notes: (1) these are responses to a repeated sequence of productivity shocks at periods 0, 1, and 2 for various shock sizes (3, 2, and 1 standard deviation); (2) responses are normalized so that an m standard deviation shock corresponds to an m percent increase in productivity on impact, relative to the steady-state value; (3) all market distortions are present.

Figure 12: Assymetry of inflation responses to shocks
Optimal timeless perspective policy under SDP, productivity shocks



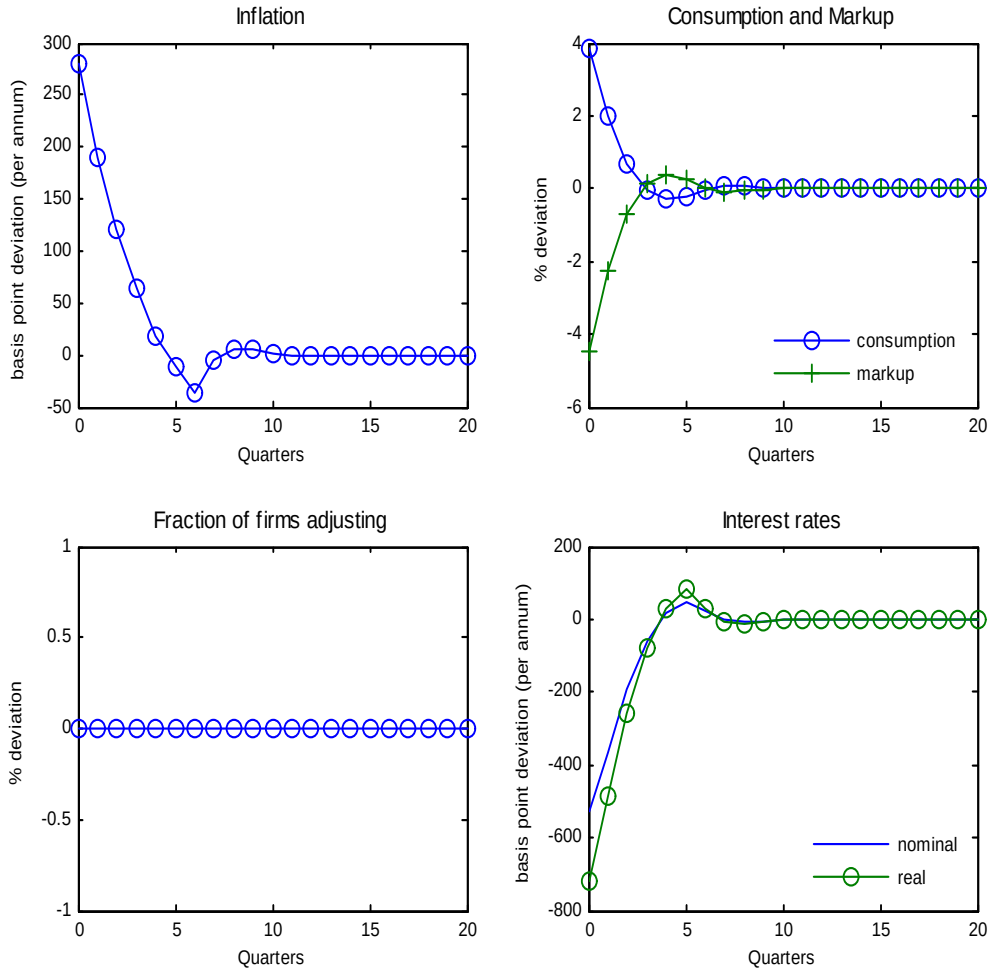
Notes: (1) these are inflation responses to a repeated sequence of productivity shocks at periods 0, 1, and 2 for various shock sizes (3, 2, 1, -1, -2, and -3 standard deviation shocks); (2) responses are normalized so that an m standard deviation shock corresponds to an m percent increase in productivity on impact, relative to the steady-state value; (3) all market distortions are present.

Figure 13: State-dependence effect and nonlinearity under Taylor rule
TDP vs. SDP, first- vs. second-order approximations, productivity shocks



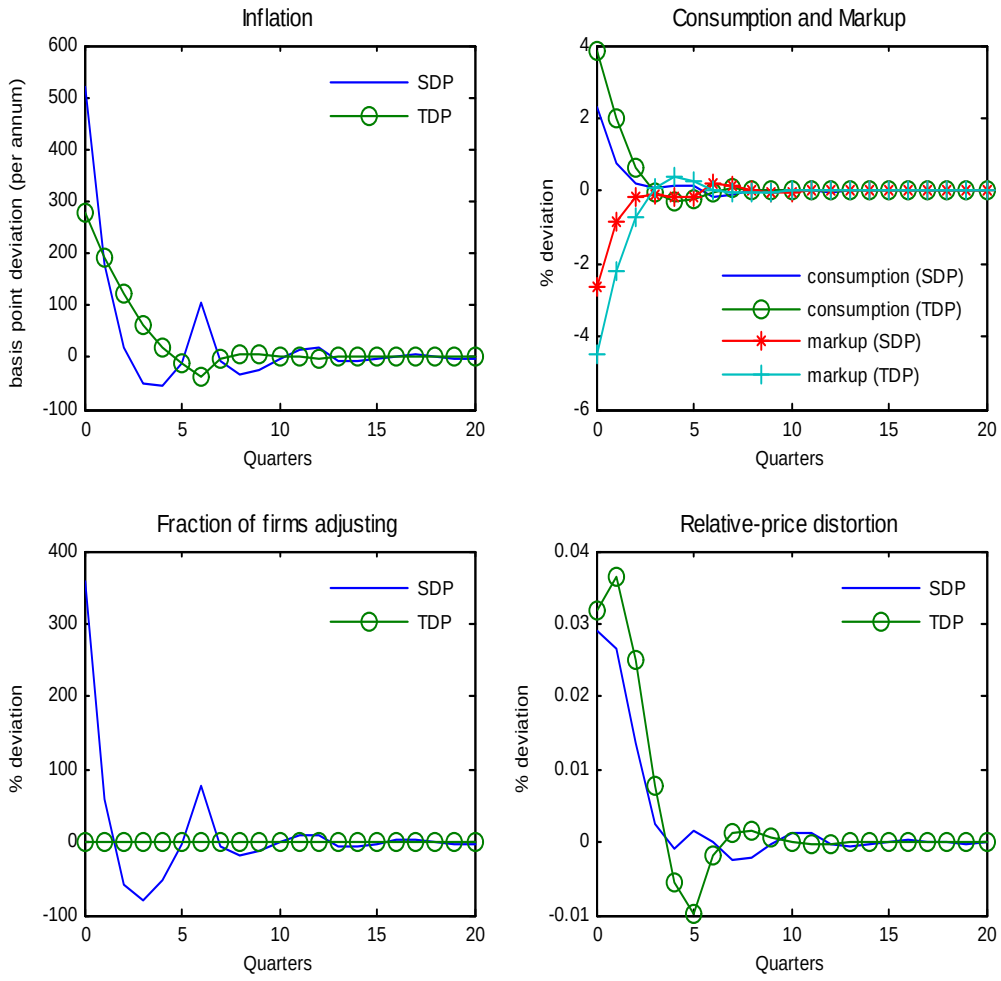
Notes: (1) these are responses to a repeated sequence of productivity shocks at periods 0, 1, and 2 under a simple linear Taylor rule; (2) *1st* and *2nd* refer to the dynamics under the first- and second- order approximate solutions, respectively; (3) each shock is a 3 standard deviation shock, normalized so that on impact it translates to a 1 percent increase in productivity from its steady-state value; (4) the TDP model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$ are equal to the steady-state values under TDP, with the same $J = 6$; (5) all market distortions are present.

Figure 14: The start-up problem
Optimal policy under TDP



Notes: (1) the figure is constructed under the assumption of zero exogenous shock; (2) the starting point prior to the optimal policy implementation is the deterministic steady state under optimal policy; (3) all market distortions are assumed to be present.

Figure 15: The start-up problem
Optimal policy under SDP vs. TDP



Notes: (1) the figure is constructed under the assumption of zero exogenous shock; (2) the starting point prior to the optimal policy implementation is the deterministic steady state under optimal policy; (3) all market distortions are assumed to be present.