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Contracts with Social Multipliers
Mary A. Burke and Kislaya Prasad

Abstract:
We develop a model of contracting in which individual effort choices are subject to social pressure to conform to the average effort level of others in the same risk-sharing group. As in related models of social interactions, a change in exogenous variables or contract terms generates a social multiplier. In this environment, small differences in fundamentals such as skill or effort cost can lead to large differences in group productivity. We characterize the optimal contract for this environment and describe the properties of equilibria, properties that agree with stylized facts on effort compression in revenue-sharing settings. The model also implies potential sorting into groups on the basis of idiosyncratic effort costs. We estimate a significant social multiplier on physician productivity, using data on medical partnerships.

Keywords: agency theory, peer pressure, social interactions, contracts, social multipliers
JEL Codes: D2, D8, C7

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1 Introduction

Agency theory provides us with a framework for the study of contracts and their effect on behavior in organizations. Starting from the assumption that agents are rational and narrowly pursue self-interest, the theory shows how appropriately designed contracts can resolve conflicts between principals and agents, and between partners. However, in the extensive literature on behavior within partnerships, teams, and organizations, a neglect of the social components of group interaction is conspicuous (exceptions include Kandel and Lazear 1992, Rob and Zemsky 2002, and Encinosa, Gaynor, and Rebitzer 2000). In contrast, the noneconomic literature on organizational behavior has long stressed the importance of social-psychological factors such as peer pressure, need for affiliation, group identity, social norms, and organizational culture (for example, Pfeffer 1997 and Granovetter 1985). Recent work in economics (Lazear 2001, Brock and Durlauf 2001a and 2001b, Postlewaite 2001, Bernheim 1994, Young 1993, Ichino and Maggi 2000, Akerlof and Kranton 2000, Glaeser and Scheinkman 2002, Becker and Murphy 2000) makes it feasible to develop models in which social interactions play a central role. This research has yielded an abundance of innovation that has expanded the empirical reach of economics. In this paper, we examine the effects of one type of non-market social interaction—a desire for conformity among workers. We deduce implications for the design of optimal contracts, outline features of data generated by such a model, and argue that our results are broadly consistent with stylized facts about incentives and performance.

The assertion that workers care about how their effort levels compare with those of their co-workers sounds on its face uncontroversial, and yet standard models of labor economics and industrial organization systematically omit any such concerns. The reasons to expect such invidious comparisons are many. A worker might experience direct pressure from peers not to shirk, especially in a revenue-sharing setting, or self-imposed pressure to live up to an established group standard. Even within groups in which outputs are not explicitly shared, such as among academic colleagues who are not co-authors, members are expected to do their
fair share to maintain the prestige of the institution. Who among us has never felt either guilty for leaving earlier than our colleagues or righteous (or perhaps resentful) for staying later?

Formally, we can capture these concerns as a cost of deviating from the average effort of one’s contracting partners. In such a model, average effort is an endogenously emerging work norm towards which people are drawn. The cost of nonconformity is symmetric: exceeding the norm is just as costly as falling short of it. We can view this as capturing an individual’s desire not to be taken advantage of or as pressure from others not to outdo them. When preferences embody this desire to conform, the group’s behavior becomes subject to a social multiplier (see Becker and Murphy 2000). Any change in contract terms or exogenous parameters sets off cascading effects: each individual responds directly to the parameter change and makes a further adjustment as peers’ efforts are also adjusted, both to the parameter change and to their peers’ responses. Small changes in fundamentals therefore lead to relatively large changes in average performance. The efficacy of incentives is altered by the social multiplier, as are the optimal contract terms. In the presence of such interactions, then, the standard relationships between incentives, risk, and performance may be distorted or overridden. The results imply, for example, that organizational success or failure may depend as much on the nature of social interactions as on objective factors, giving the latter less predictive power over organizational performance.

In the standard moral hazard model, the optimal contract strikes the proper balance between spreading risk and encouraging work effort. Group members choose efforts to maximize expected utility, which depends on an individual’s effort, on the income process implied by the contract, and on exogenous factors. While risk-sharing contracts have been studied exhaustively in many contexts (see, for instance, the review by Prendergast 1999), social interactions among group members have, until very recently, been largely absent from this literature. To study this, we use the contracting framework of Gaynor and Gertler (1995). Specifically, we consider the group contracting problem when each member of the group cares

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1This framework has been extended previously by Encinosa, Gaynor, and Rebitzer (2000) and Dutta and Prasad (2002) to incorporate social interactions
about his or her own effort relative to the average effort of his or her partners, as well as
his or her absolute effort and his or her (uncertain) income under the contract. In Encinosa,
Gaynor, and Rebitzer (2000), a version of the model includes an effort norm to which indi-
viduals aspire, but the effect is linear in the difference between individual and group effort.
The group mean does not enter the first-order condition for individual effort, and a social
multiplier cannot arise. Kandel and Lazear (2001) examine the effect of peer pressure on
worker effort under profit-sharing contracts, but they focus on the case in which the cost
associated with peer pressure is monotonically decreasing in a worker’s own effort, rather
than symmetric about average effort. While their model admits a version in which the cost
of peer pressure basically mimics our nonconformity cost, no social multiplier arises, because
workers are homogeneous. They do, however, argue that the symmetric cost structure, re-
flecting the desire to conform to a group norm, seems apt in the case of partnerships. The
Kandel-Lazear model treats contract structure as exogenous and focuses on the case of pure
profit-sharing, whereas we admit a broader class of contracts and derive optimal contract
terms.

Following Glaeser and Scheinkman (2002) and Brock and Durlauf (2001a)\(^2\), we model
the cost of deviation from the average action of a peer group as quadratic in the difference
between individual and mean peer efforts. Optimal individual effort depends directly on
mean peer effort, and the coefficient of mean peer effort captures the degree of preference for
conformity. A change in the contract’s parameters (or in an exogenous variable) sets off two
effects—the direct adjustment of effort to the changed variable, as well as an indirect effect
that arises as a response to the adjustment of others. The workings of the multiplier are
seen explicitly in analyzing the effects on behavior of a change in the distribution of effort
costs and changes in contract parameters. The optimal contract, after taking such effects
into account, is quite different than it would be in the absence of social interactions and also
responds differently to changes in environmental parameters. Depending on the parametric
specification, introducing a preference for conformity can either increase or decrease the

\(^2\)These papers model preference for conformity and study social multipliers but do not consider the con-
tracting problem.
incentive intensity in the optimal contract, depending on how average effort is altered by the conformity effects. When average effort increases due to conformity, the optimal contract is “high-powered,” using higher incentive intensity. When average effort decreases, optimal incentives are low-powered.

While we work with an explicit parametric form, the principal qualitative features that drive our results are conformity, strategic complementarity, and a property introduced in Glaeser and Scheinkman (2002) called “moderate social influence.” Following Gaynor and Gertler (1995), we focus on linear, budget-balancing contracts. We find that, for a given contract, a unique equilibrium profile of effort levels exists. In addition, an optimal contract—maximizing the sum of utilities for the group, subject to enforceability constraints—exists quite broadly. Given the social interaction effects, a small increase in a given individual’s cost of effort diminishes his or her own effort as well as that of group members, although the own effect is more significant. We find that individuals with low cost of effort prefer to be in a group of similar individuals. Consequently, it is possible to separate individuals into a “high action” group and a “low action” group through an appropriate choice of group-specific fees. Preference for conformity then leads to the use of low-powered incentives in the high-cost group, while low-cost groups use high-powered incentives.

We complement the theoretical analysis of this paper with an empirical investigation aimed at testing for the presence of social interaction effects in the data and recovering parameters that allow us to assess the magnitude of the social multiplier. We reanalyze the medical partnership data of Gaynor and Gertler (1995). This data set has been studied extensively and is somewhat dated, but our principal objectives here are to illustrate the feasibility of estimating social interaction effects and to highlight some of the methodological issues that arise. We find that tests support the presence of interactions but suggest values of social multipliers that are small.

The remainder of the paper is organized as follows. The basic model is described in Section 2. For a given contract, we study the equilibrium and assess the effects of the social

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3Linear contracts are widely used in the real world, even though theory often predicts very complicated contracts. For our purposes, and given our data on medical partnerships, this restriction seems reasonable.
multiplier in Section 3.1. In Section 3.2, we study existence and properties of the optimal contract. We consider sorting by individual characteristics when individuals are free to choose among different groups in Section 4. In addition, we report in Section 5 the results from estimating the magnitude of social interaction effects using data on group medical practices from Gaynor and Gertler (1995). A conclusion follows.

2 Model

We look at a very simple model with mean-variance utility in which effort affects only the mean of the distribution of output. To begin, we fix the number of workers at \( n \). Each worker \( i \) chooses a level of effort \( e_i \) from \( \mathbb{R} \). This results in a random contribution to output of \( y_i(e_i) \). We let \( y \equiv (y_1, y_2, \ldots, y_n) \). The total output of the group is additively separable and written as

\[
\bar{y}(e) \equiv \sum_{i=1}^{n} y_i(e_i),
\]

where \( e = (e_1, e_2, \ldots, e_n) \). In addition,

\[
y_i(e_i) = e_i + \varepsilon_i,
\]

where \( \varepsilon_i \) has zero mean and finite variance \( \nu \) (independent of the level of effort). Thus,

\[
\bar{y}(e) = \sum_{i=1}^{n} e_i + \sum_{i=1}^{n} \varepsilon_i.
\]

For a given profile of effort choices, the expected output is \( E(\bar{y}|e) = \sum_{i=1}^{n} e_i \). Effort is costly, and we represent the cost function by \( C^i(e_i) \):

\[
C^i(e_i) = \theta_i e_i^\beta.
\]

The only difference between workers is in the cost parameter \( \theta_i \). We assume \( \beta \geq 1 \) to guarantee nondecreasing marginal effort cost.
We assume preference for conformity. Individuals get disutility if their actions depart from those of their peer group (taken to be everyone else in the group). The cost takes the following form:

\[ G(e_i) = \frac{J}{2}(e_i - \hat{e}_i)^2, \]

where \( \hat{e}_i = \sum_{j \neq i} e_j / (n - 1) \).\(^4\) The non-negative parameter \( J \) indexes the strength of conformity pressure: the greater is \( J \), the greater the cost of a given deviation from average effort.

If each worker’s share of output is given by \( s_i(y) \), payoffs may be written as

\[ V_i(e) = \mathbb{E}[s_i(y(e)) - C_i(e_i) - G(e) - \frac{1}{2}r \text{Var}(s_i(y(e)))], \tag{1} \]

expectations being taken with respect to the distribution of the \( \epsilon_i \).\(^5\) The parameter \( r \) is the coefficient of absolute risk aversion.

When each of the \( y_i \) is observed, we have essentially the environment studied by Mirrlees (1974), so that his nonexistence result applies. Like numerous other papers, we restrict contracting possibilities. Lang and Gordon (1995) and Gaynor and Gertler (1995) have studied this environment, assuming budget-balancing linear contracts. Specifically, they examine contracts of the form:

\[ s_i(y_1, \ldots, y_n) = \xi y_i + \frac{(1 - \xi)}{(n - 1)} \sum_{j \neq i} y_j. \tag{2} \]

One gets to keep a fraction \( \xi \) of one’s own contribution and the rest goes into a pool that is shared equally among the others. When \( n = 1 \), there is only one possible value of \( \xi \), that is, \( \xi = 1 \). Other groups must choose an optimal value for \( \xi \). Equal sharing corresponds to \( \xi = 1/n \).

\(^4\)We assume that efforts are observable to co-workers, but not contractible. It is possible to specify the conformity effect as \( J(y_i - \hat{y}_i)^2 \). We expect this to lead to identical results.

\(^5\)This is the approximate certainty equivalent wealth of \( i \). The approximation is exact for exponential utility when \( y(e) \) has a normal distribution.
3 Contracts in the presence of social interactions

We examine the problem in two parts. First, we take a fixed contract $\xi$ and characterize equilibrium action levels chosen in the presence of social interactions, explicating the role of social multipliers. Next, we examine the choice of $\xi$ that maximizes the sum of utilities of group members. Agents are said to display preference for conformity when $J > 0$.

3.1 Equilibrium effort

The next proposition follows Glaeser and Scheinkman (2002) (hereafter GS). In particular, we verify their Moderate Social Influence (MSI) condition.

Definition 1 (MSI). The MSI condition holds if the marginal utility of an agent’s own action is less affected (in absolute value) by a change in the average action of his or her peers than by a change in his or her own action.

Proposition 1. For any given $\theta = (\theta_1, \ldots, \theta_n) \in \mathbb{R}_+^n$, $\xi \in (0,1]$, $\beta > 1$ and $J > 0$, there exists a unique equilibrium $e(\theta, \xi, \beta, J)$.

Existence is established using a simple fixed-point argument. For uniqueness, we verify the MSI condition and invoke GS.

The next corollary examines the effect on efforts when some $\theta_i$ is changed. The corollary takes into account both the direct effect of an increase in $\theta_i$, as well as the change that results when the direct effect on $e_i$ causes a response in the $e_j$. What is ultimately of interest is the response to a change in $\theta_i$ when the system has finally settled into a new equilibrium. The proof relies on a framework that we elaborate upon in the discussion following the corollary.

Corollary 1. (1) An increase in $\theta_i$ decreases the effort level of each player in the group, with a greater decrease in $e_i$ than in $e_j$ ($j \neq i$). (2) In equilibrium, players with a smaller value of $\theta_i$ exert greater effort.

The corollary examines how equilibrium efforts change in response to changes in $\theta_i$. We may, similarly, obtain comparative statics results for changes in other parameters, and we
are particularly interested in changes in $\xi$.

We proceed using familiar methods based on the implicit function theorem. The interesting features of our results stem from the associated social multiplier. Since strategic complementarity holds (between a worker’s action and the average action of his or her peer group), anything that increases the action of any one worker will end up increasing the actions of the other workers in the peer group. This will feed back and increase the workers’ actions even further. This means, for instance, that two “slightly different” realizations of parameters could have fairly large effects. This has great relevance for the optimal contracting problem, in which small increases in $\xi$ could have a significant effect on the workers’ actions.

The analysis starts with the system of first-order equations that define equilibrium,

$$
\xi - \theta_i e_i^{\beta - 1} - J(e_i - \hat{e}_i) = 0 \quad \text{for } i = 1, 2, \ldots, n.
$$

(3)

Proposition 1 assures us that there is an equilibrium $e(\xi, \theta, \beta, J)$, assumed to be a smooth function of the parameters. We are interested in evaluating the response of $e_i$ to a change in $\xi$. Changes in other parameters can be analyzed similarly. We adopt the notation that $\partial e_i / \partial \xi$ denotes the response of $e_i$ when all other effort levels are held constant, and $de_i / d\xi$ denotes the equilibrium response. Taking the total differential of the equations (3) we obtain,

$$
\frac{de}{d\xi} = F_1^{-1} \frac{\partial e}{\partial \xi},
$$

(4)

where $de / d\xi$ and $\partial e / \partial \xi$ are vectors and $F_1$ is the matrix where all diagonal elements are one, and off-diagonal elements in the $i$-th row equal

$$
F_1(i, j) = \left( \frac{J}{n - 1} \right) \frac{-1}{J + \theta_i (\beta - 1)e_i^{\beta - 2}}.
$$

It will be convenient to denote $x_i \equiv (J + \theta_i (\beta - 1)e_i^{\beta - 2})^{-1}$, and $z_i \equiv Jx_i / (n - 1)$. Observe that $x_i = \partial e_i / \partial \xi$. 9
We use the representation of $F_1^{-1}$ given in GS (the Neumann expansion):

$$(F_1)^{-1} = I + (I - F_1) + (I - F_1)^2 + \cdots,$$

where all diagonal elements of $(I - F_1)$ are zero, and off-diagonal elements of row $i$ equal $z_i$.

This allows us to write

$$\frac{de}{d\xi} = (I + H) \frac{\partial e}{\partial \xi},$$

where $H$ is a non-negative matrix. This leads to the following expression for the derivative of interest:

$$\frac{de_i}{d\xi} = \frac{\partial e_i}{\partial \xi} + \sum_j H_{ij} \frac{\partial e_j}{\partial \xi}. \quad (6)$$

The first term on the right side of this equation, $\frac{\partial e_j}{\partial \xi}$, gives us the direct effect of a change in $\xi$. A change in $\xi$ affects $e_i$ through its effect on $e_j (j \neq i)$ as well, and any change in $e_i$ causes further changes in the $e_j$. The second term in equation (6) gives the additional response from the social multiplier. The size of the social multiplier depends on

$$\frac{\partial e_i}{\partial \hat{e}_i} \equiv (n - 1)z_i,$$

and becomes larger as all the $\frac{\partial e_i}{\partial \hat{e}_i}$ approach one.

**Computational Experiment**

To assess the role of the social multiplier, we conduct some computational experiments. For any given $(\xi, \theta, J)$, we can compute the equilibrium, $e$, by solving the system of equations (3). This allows us to compute changes in equilibrium efforts as the parameters are changed. In our computations, we assume quadratic effort costs and compare the results for the following two models:

$$C^i(e_i) = \theta_i \frac{e_i^2}{2},$$

$$C^i(e_i) = (1 - J + \theta_i) \frac{e_i^2}{2}.$$
The former is the model from Proposition 1, with $\beta = 2$. The latter is a form similar to Example 2 of GS. $J$ now appears as the relative weight of effort costs and the costs from departing from the norm. This form is convenient: we have

$$e_i = \frac{\xi + J\hat{e}_i}{1 + \theta_i},$$

so that $\partial e_i/\partial \xi = 1/(1 + \theta_i)$, which is also the total response to a change in $\xi$ when $J = 0$. We have

$$\frac{\partial e_i}{\partial \hat{e}_i} = \frac{J}{1 + \theta_i},$$

with $J \leq 1$ to ensure MSI. The two models yield qualitatively similar results regarding the multiplier, but very different results for optimal contracts. So we present results only for the latter model, except when we compute optimal contracts (in which case we present results for both).

Figure 1 illustrates the effort distributions for the $J = 0$ and $J = 1$ case. We let $N = 10$ and fix the value of $\xi$ at 0.3 and that of $\theta$ at $(0.1, 0.2, \ldots, 1.0)$. In other words, each individual has a distinct value of $\theta_i$, and these values are spaced 0.1 units apart. This allows us to plot the effort, $e_i$, corresponding to each $\theta_i$, and we observe that efforts are much larger when $J = 1$. This comparison of outcomes tells us little, since we are comparing two models with different cost functions. Our interest is in the qualitative properties of data generated according to the two models. The rest of the figures, and especially Tables 1–2, address this.

Of greater interest is Figure 2, which shows the response of efforts when $\xi$ is increased from 0.3 to 0.35. The lower two lines, identified by + and $\Box$, are drawn with $J = 0$, so that their difference ($\equiv \Delta \xi \cdot (\partial e_i/\partial \xi)$) identifies the direct effect. The difference between the two upper lines, identified by $\circ$ and $\triangledown$, gives the total effect (for $J = 0.75$). We see this is larger than the direct effect, the difference being attributable to the social multiplier.

In Figure 3, we illustrate the effect of the social multiplier when costs are scaled down (with $\xi$ fixed). We multiply the cost vector, $\theta$, by a scalar: first 0.6, and then 0.5. The
effort distribution is graphed for $J = 0$ (denoted by + for both scale values), and for $J = 1$ (denoted by ◦). The direct effect is very small, virtually indistinguishable in the graph. The total effect and the social multiplier are both quite large. Relative to Figure 2, the parameter values are chosen to make the multiplier much larger ($\partial e_i/\partial \hat{e}_i$ is closer to one if $\theta_i$ is smaller and $J$ is closer to one).

3.2 Optimal contracts

From Proposition 1, for any $\xi$, we have a unique equilibrium $e$. We find $\xi^*$, the value of $\xi$ that maximizes group utility, subject to the constraint that individuals in the group choose equilibrium effort levels. In other words, $\xi^*$ solves the following problem:

$$
\max_{\xi} V \equiv \sum_i (e_i - \theta_i \frac{e_i^\beta}{\beta} - \frac{nr\nu}{2}[\xi^2 + \frac{(1 - \xi)^2}{n - 1}] - \frac{J}{2} \sum_i (e_i - \hat{e}_i)^2
$$

subject to

$$
\xi - \theta_i e_i^{\beta - 1} - J(e_i - \hat{e}_i) = 0 \quad (\forall i).
$$

(Recall that the parameter $\nu$ above represents the variance of the deviation of output from effort.) In Proposition 2, we show that an optimal contract, $\xi^*$, exists.

**Proposition 2.** Given any $\theta = (\theta_1, \ldots, \theta_n) \in \mathbb{R}^n$, $\beta \geq 2$ and $J > 0$, an optimum $\xi^* \in (0, 1)$ exists.

We now examine the properties of the optimal contract using a computational approach, extending the example from section 3.1.

**Computational Experiment (continued)**

Figure 4 illustrates the determination of the optimal share. From the first-order condition, we look for a zero of the marginal social utility. When $J = 0$, the optimal share is $\xi^* = 0.17$. When $J = 1$, the optimal share is $\xi^* = 0.34$. When the social multiplier is taken into account, it is optimal to use high-powered incentives. Figure 5 considers the effects of a decrease in
costs. When $J = 0$, the marginal social utility for Scale = 0.4 is virtually indistinguishable from that for Scale = 0.5. However, with $J = 1$, the two are noticeably different—leading to a significant increase in $\xi^*$.

In Table 1, we consider the effects of changing costs (using ten different values of the scale, ranging from 0.05 to 0.50). In each case, we compute the optimal contract and the distribution of effort at this contract. We observe, once again, the effect of the social multiplier. When $J = 0.9$, average efforts are noticeably higher. The range of outcomes is also much greater. When we compute the coefficient of variation of the average group outcomes, the values are 0.0957 (for $J = 0$) and 0.5544 (for $J = 0.9$). This is accompanied by smaller values of the coefficient of variation within groups (columns 5 and 9 of Table 1). The optimal share is much greater when $J = 0.9$, and rises when costs are scaled down. A striking feature of Table 1 is the much greater variation in the optimal shares when social interactions are present (the coefficient of variation is 0.1757 as opposed to 0.0301) than when they are absent.

Finally, in Table 2, we consider the effects of changing the risk premium term—either the risk aversion coefficient, $r$, or the variance, $\nu$. We use 10 different values, ranging from 0 (risk-neutrality) to 1.8. In each case, we compute the optimal contract and the distribution of effort at this contract. We note first that $\xi^*$ decreases. This is actually easy to show analytically—$\xi^*$ decreases as $r \nu$ increases, converging to $1/n$. We notice that average efforts are significantly higher when $J = 0.9$. The standard deviation is also higher, but the coefficient of variation is a little smaller. What is particularly interesting about these results is that changes in $r \nu$ have no direct effect on efforts. All of the effect here comes from the change in $\xi^*$, which then has both a direct and a multiplied effect on actions. There is a much broader range of outcomes when $J = 0.9$, but this is associated with smaller variation within groups. The across-group coefficient of variation (of average effort) is actually smaller when $J = 0.9$, but this is reversed as soon as we drop the risk-neutral outcome. Similarly, the range of contracts is greater for $J = 0.9$ when we drop the risk-neutral outcome.

Tables 3 and 4 present results for the case where $C_i(e_i) = \theta_i e_i^2/2$. Now, average efforts
are higher for $J = 0$. The coefficient of variation of effort continues to be lower for the $J = 1$ groups. The variation across groups is higher when $J = 1$ (COV = 1.3032, as opposed to COV = 1.2480 when $J = 0$). Now, the values of $\xi^*$ are larger when $J = 0$, and greater conformity leads to lower values of $\xi^*$. The $\xi^*$ are more variable when $J = 0$ (COV = 0.3107 as opposed to COV = 0.2548 when $J = 1$). There is greater within-group variation in efforts for $J = 0$, but smaller variation in average efforts across groups (COV = 0.3973 when $J = 0$, but COV = 0.4590 when $J = 1$).

A preference for conformity can lead to either higher or lower incentive intensity, $\xi^*$, depending on how the intensity of conformity preference affects average effort. In the model of Tables 1 and 2, all effort levels go up when $J$ increases (although the higher effort levels go up by less). In the model of Tables 3 and 4, low efforts increase, high efforts decrease, and the average effort decreases. In general, the high-effort (low-cost) people respond most to stronger incentives, but in the latter model they are behaving more like the high-cost people, and in this case the high incentives are not worth the cost (in terms of sacrificed risk-sharing). The reverse is true when all efforts increase with $J$.

To summarize: (1) a striking feature of the generated data in the presence of preference for conformity is that small differences across groups, whether in individual characteristics or environmental conditions, are reflected as large differences in outcomes and endogenously chosen organizational variables (for example, contracts). At the same time, because of preference for conformity, we have greater uniformity within groups. (2) Optimal incentive intensity, $\xi^*$, can be either greater or smaller, and this depends on the nature of effort costs. When average efforts increase with conformity, high-powered incentives are optimal. If average efforts decrease, low-powered incentives are warranted.

The simulation results are consistent with the empirical findings of Hansen (1997), and Weiss (1987), both of whom report a convergence of individual output levels toward a standard, following the introduction of group incentive schemes. Observing telephone operators at American Express, Hansen found that those workers who were least productive before introduction of the plan increased their output significantly under the plan, but the more
productive workers did not alter output significantly. The net result was an increase in average productivity for the group, and a decrease in the variance of individual output. For manufacturing workers, Weiss observed that most of the least productive workers increased their output levels under the group incentive plan, but nearly all of the most productive workers actually reduced their outputs. We can think of such plans as introducing a concern for conformity by linking individual output to group output, where prior to the plan no such link existed.

4 Choice of groups

Suppose there are $\ell = 2$ groups, and $2n$ people. There are $n$ “slots” in each group. Suppose $\theta = (\theta_1, \ldots, \theta_{2n})$, with $\theta_1 < \theta_2 < \ldots < \theta_{2n}$. After the realization of $\theta$, the two “groups” choose shares, labelled $\xi_1$ and $\xi_2$ ($\xi_\ell \in [0, 1]$). Then, agents choose groups based on the their realization of $\theta_i$. We ask if segregation by $\theta_i$ could arise, that is, can we have group 1 with $\theta^1 = (\theta_1, \ldots, \theta_n)$ and group 2 with $\theta^2 = (\theta_{n+1}, \ldots, \theta_{2n})$ in equilibrium?

The next proposition shows two things. First, agents with lower values of $\theta_i$ will gain more from belonging to a group where the others also have lower values of $\theta_i$ (this is the “high-action” group $\theta^1$, where average actions are higher). This result can be used to show that group-specific prices, $q_\ell$, can separate players by $\theta_i$ in the following sense: There is an assignment of players to groups $\ell \in \{1, 2\}$ such that, if agent $i$ is assigned to group $\ell$, there is no $\ell'$ such that

$$\sup_{e_i} U(e_i, \theta_i, \xi_{\ell'}, A_{\ell'}^I) - q_{\ell'} > \sup_{e_i} U(e_i, \theta_i, \xi_{\ell}, A_{\ell}^I) - q_{\ell},$$

where $A_{\ell}^I$ is used to denote both the peer group of $i$ and the average action $\hat{e}_i$ for this peer group. Share parameters can be chosen such that $\xi_1$ is optimal for group 1 ($\theta^1$) and $\xi_2$ is optimal for group 2 ($\theta^2$). We denote $\sup_{e_i} U(e_i, \theta_i, \xi_{\ell}, A_{\ell}^I)$ by $U^\ell(\theta_i)$ for $\ell \in \{1, 2\}$. As before, group 1 has $\theta^1$ as the cost parameters of its members.

**Proposition 3.** Suppose there are two peer groups, $(A^1_i, A^2_i)$, such that $\theta_k < \theta_j$ whenever $k$
is in $A^1_i$ and $j$ is in $A^2_i$.

1. For any fixed $\xi$, $U^1(\theta_i) - U^2(\theta_i)$ decreases with $\theta_i$.

2. There exists a set of prices $(q_1, q_2)$ that separate players by $\theta_i$.

An examination of the proof demonstrates that $\xi_1 > \xi_2$, so that low-cost groups use high-powered incentives. This is further evident from Table 1.

5 Empirical tests of the model

A central prediction of our theoretical model is that individual effort under a group contract depends not only on contract terms and effort costs, but also on the average effort level of other group members. We reanalyze the data of Gaynor and Gertler (1995) (hereafter GG) and test for peer effects, using data on group medical practices under revenue-sharing contracts, and adopting the standard approach in the literature of regressing individual outcomes against group outcomes (Arcidiacono and Nicholson 2002; Epple and Romano 1998; Hoxby 2000; Encinosa, Gaynor, and Rebitzer 2000; Glaeser and Scheinkman 2001 and 2002, among others). The approach constitutes an estimation of our equation (3), after the equation is modified to suit the context of the medical practice data. Using office visits as a proxy for physician effort, we obtain an estimate of the theory’s social interaction term, $J$, that supports the hypothesis of positive peer effects.

The remainder of this section is organized as follows: In Section 5.1 we derive a testable empirical model based on our theory. Section 5.2 describes the empirical methodology. A data appendix describes the data sources and the measurement of key variables. In Section 5.3 we discuss the empirical results, which are presented in full in Table 3.

5.1 The empirical model

In the data, each physician gets a percentage, call it $\zeta$, of her self-generated revenues, together with the fraction $1/n$ of the pooled income, where $n$ is the total group size (the pooled income
is \((1 - \zeta)\) times total group revenues. In our theoretical model, the physician gets the average of the pooled income of the others, not including herself. Under the observed contracts, then, the effective percentage of an individual’s revenues retained is \(\zeta + \frac{(1 - \zeta)}{n}\), denoted by \(\hat{\zeta}\). In the empirical model, we use \(\hat{\zeta}\) where appropriate, computing it using the data’s measures of \(\zeta\) and \(n\). We retain the theory’s assumption that the social multiplier acts on the difference between a physician’s own effort and the mean effort of his or her peers, not including himself or herself.

For each physician in the sample, we observe the self-reported typical quantity of office visits per week, \(q_i\), and use this as a proxy for physician effort. Given this substitution, our theory predicts a positive relationship between the quantity of office visits to an individual physician and the average number of visits to that physician’s practice partners, a relationship analogous to that between individual and peer effort described in equation 3. Assuming quadratic costs of the form \(\theta_i e_i^2\), we can express this relationship as

\[
q_i = \left(\frac{1}{J + \theta_i}\right)\hat{\zeta} P + \left(\frac{J}{J + \theta_i}\right)\bar{q}_{-i} + \epsilon_i. \tag{7}
\]

Here we have substituted individual office visits \(q_i\) for effort \(e_i\), mean peer visits \(\bar{q}_{-i}\) for mean peer effort \(\bar{e}_i\), and the adjusted incentive parameter \(\hat{\zeta}\) for \(\zeta\). We multiply the latter by \(P\), the office visit fee (common to all group members), which we had normalized to 1 in the theoretical model. We have also added a stochastic component \(\epsilon_i\), assumed i.i.d. across physicians, with mean zero and variance \(\sigma_\epsilon\).

To test this relationship we must take into account factors other than effort that might affect the demand for office visits to a given physician. The data contain several likely demand factors, such as physician experience and board certification status; market factors, such as income per capita and education levels; and group-practice factors, such as the number of exam rooms, the quantity of non-physician labor, and the price per visit. Assuming demand is linear and additively separable in each of the included factors, we specify the empirical model as follows:
\[ q_i = \left( \frac{1}{J + \theta_i} \right) \hat{\zeta} P + \left( \frac{J}{J + \theta_i} \right) \hat{q}_{-i} + g(N_i, X, P, H, K) + \epsilon_i, \]  

(8)

in which \( g(.) \) is a linear function of its arguments. \( N_i \) represents a vector of physician characteristics, and \( X \) is a vector of market-area statistics. \( P, H, K \) denote three variables specific to the physician’s group practice: respectively, the office visit fee, hours of non-physician labor, and number of examination rooms. Using the group practice data, we estimate the above equation in order to test for the presence and magnitude of social interactions. The estimated coefficient on group mean output indicates the strength of such interactions, and can be used to obtain an estimate of the average social multiplier across practice groups, as described below.

5.2 Empirical methodology

We face a number of well-known problems identifying peer effects in this manner. First, individual and peer-group outputs may be correlated for reasons other than genuine, social influence (Manski 1993; Arcidiacono and Nicholson 2002). Such correlated effects, to use Manski’s terminology, would occur when physicians sort themselves into groups on the basis of unobserved productivity factors, or when physicians in the same group practice experience similar unobserved environmental influences on output such as local epidemiological factors. In either case, mean peer output proxies for the unobserved group-level influences to produce spurious peer effects. However, if the peer effects are genuine, average practice-partner output is simultaneous with individual output, and therefore endogenous. Yet another problem concerns the separation of endogenous peer effects (operating through effort) and any so-called “contextual” social interactions deriving from exogenous group characteristics. In some group settings, such as school classrooms, peer gender composition has been found to exert independent effects on individual achievement separate from the simultaneous choice of effort (Hoxby 2000). In such cases, it may be impossible to separately identify the effects of exogenous and endogenous peer variables because the latter may constitute linear functions.
of the former (Manski 1993). In this setting we assume there are no independent contextual interactions, so we don’t face the problem of separating the two types of effects.

To identify the peer effects, we use an instrumental variable for mean peer output, employing instruments that predict average group output but not individual output. For example, we use the group average of years of experience and the percentage of group members with board certification. Assuming production is non-joint across physicians, neither of these aggregate factors should cause variation in individual output. While an individual’s board certification status may affect his or her productivity, we do not expect the certification status of an individual’s peers to be a source of direct social influence on individual productivity apart from its influence on the productivity of peers. Of course, we include the individual’s own certification status and years of experience (and squared experience) as explanatory variables, together with the physician’s specialty field and an indicator for whether he or she also has a subspecialty. Including these variables controls for selection into groups on the basis of these factors and on any unobserved factors correlated with them. Further control for unobserved physician type obtains from including the price per visit and contract incentive parameters. If, as in the theoretical framework, physicians can select into groups on the basis of these contract terms, these parameters proxy for unobserved effort costs. We include other group-level factors, such as quantity of exam rooms and hours of non-physician labor, and the market-level factors income per capita and average education level, in order to control for additional correlated influences on members of a given practice. Although the instruments could be correlated with the average values of unobserved factors in the group, the exogeneity requirement is that they need only be uncorrelated with the residual unobserved factors not absorbed by the individual and group-level controls.

Another econometric issue (raised first in GG) we must address is the endogeneity of the variables $P$ (the office visit fee), $\hat{\zeta}$ (the incentive term), $H$ (hours nonphysician labor), and $K$.

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6In binary choice models, however, Brock and Durlauf (2001) obtain separate identification of endogenous and exogenous effects based on the non-linearity of outcomes in the exogenous variables.

7Peer effects do not amount to joint production: under joint production the marginal product of an individual’s effort depends on the quantity and/or quality of other physicians’ inputs, a condition wholly independent of the presence or absence of peer effects as we model them.
(exam rooms). Because these values are chosen (jointly) by the group members themselves, presumably to optimize group welfare, they may be correlated with the errors on individual output. While this is a virtue for the purpose of preventing bias on the coefficient for group mean output (or its instrumental variable), if we want to observe the effect of exogenous price and incentive changes we should remove this endogeneity. Following the results of Hausman specification tests conducted in GG, which do not reject exogeneity for $H$ and $K$, but do reject it for $P$ and $\zeta$, we employ instrumental variables for $P\zeta$ and $P$. Following GG we use the wages for non-physician staff (including physician’s assistants, lab technicians, and nurses) to identify the effects of $P$ and $P\zeta$, because these wages affect a group’s costs, and therefore its price and incentive choices, but not individual demand. Risk preferences and income variability within the group also play a role in optimal contract determination, but not in the choice of individual effort. Thus, it is appropriate to include as additional instruments the group-average values of risk-preference measures and of variables that proxy for income variability, such as physician specialty. We adopt a two-stage least-squares estimation with three endogenous variables: group mean office visits, price per visit, and incentive intensity. The results of the estimation are reported in Table 5. However, we also report results from OLS estimation, because including the endogeneous contract parameters offers a better defense against correlated effects bias on the estimation of the social interactions.

Notice that the first two coefficients in equation 8 depend on the idiosyncratic variable $\theta_i$, which represents subjective effort cost. Viewing $\theta_i$ as a random variable implies random coefficients on $P\zeta$ and $\bar{q}_{-i}$. With random coefficients, the standard OLS estimates will be biased because of the likely correlations between the effort costs, mean group output, and the incentive parameter. Standard instrumental-variables (that is, two-stage least-squares) regression yields inefficient estimates, and may or may not produce inconsistent estimates. Consistency requires that the variable $\theta_i$ be independent of the instrumental variables for $P\zeta$ and $\bar{q}_{-i}$, and

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8We conduct our own specification tests for the endogeneity of $P$, $H$, $K$, $\zeta$, and mean peer output $\bar{q}_{-i}$, which do not reject exogeneity for any of them. The discrepancies between our test results and GG’s most likely owe to two factors: first, GG use the logarithms of these variables while we use the values directly; and second, the data set that survives omission of missing values is in our case smaller than in GG’s. We are able to nearly replicate GG’s results by replicating their specification.
we proceed assuming such independence. This assumption simply reiterates the condition that idiosyncratic effort cost cannot be predicted on the basis of a group’s non-physician wage payments, nor on the basis of the average observed group-member characteristics (such as average years experience), controlling for the other included variables.

5.3 Interpretation of results

The empirical results are presented in Table 5. The relevant data description and variable definitions are in an appendix. Here, we focus only on the variables of theoretical interest. The model implies that the social multiplier varies inversely across groups with the average effort cost in the group. We can estimate the average value of the social multiplier across groups as a simple transformation of the estimated coefficient on GRPMEAN. Denoting this coefficient as \( \gamma \) and the average multiplier as \( M \), we get \( M = \frac{1}{1-\gamma} \) (Becker and Murphy 2000). Given the IV estimate of \( \gamma \) of 0.376, we get an average multiplier value of 1.6 that is significant at the .01 level. Under standard OLS regression, the estimated mean multiplier is 2.07, significant at the .01 level. The IV estimate suggests that, on average, a change in an exogenous variable (for example an exogenous shock to price) will have on average a 60 percent greater impact on group productivity than it would have in the absence of social interactions. The estimated coefficient on GRPMEAN lies between zero and one, and so satisfies the moderate social influence condition required for unique equilibrium in the theoretical model. The signs of other key coefficients take the predicted values: under the IV specification, the coefficient of the incentive term (INCENTIVE) is positive and significant at the .10 level, and the coefficient on the visit fee (PRICE) is negative and significant at the .10 level. Under OLS, the incentive effect is small and insignificant while the price effect is negative, significant at the .05 level, and yet smaller in absolute value than under the IV estimation.

\(^9\)A single asterisk (*) denotes significance at the .10 level, (**) denotes significance at the .05 level, and (***) denotes significance at the .01 level.
6 Conclusion

A criticism often levelled at the economics literature on organizations is that it neglects cultural and social processes in the workplace. People involved in economic transactions are also socially related to each other, and to properly understand the functioning of organizations we must understand the nature of social interactions within them. Addressing a conspicuous gap in the formal theory of organizations, we have shown that social interactions can have profound effects on individual behavior and optimal contractual incentives. Further, the model’s predictions can be tested empirically, and we find support for the presence of a significant social multiplier among partners in a group medical practice. The limitations of the data notwithstanding, the test is valuable, given the relative scarcity of empirical work on social interactions, and it contributes to the discussion of how best to approach such work.

Our computational results explore properties of data generated from our model. One of these properties, the compression of individual effort levels in the presence of a social multiplier, has been observed empirically as an effect of introducing group profit-sharing. We know of no other model that can explain this observation.

The model teaches us that a given organization’s success may well depend upon details of the social interaction among the members of the organization. Replicating such success, therefore, is difficult because such interactions cannot be reproduced at will. This lesson applies quite generally at the societal level, despite our formal focus on organizations. In response to the puzzling observation that societies in apparently similar situations, given similar economic prescriptions, experience vastly different results, there has been a push towards understanding the role of norms and social capital in development. (See, for example, http://www.worldbank.org/poverty/scapital/.) The consensus emerging from this literature is that such divergent paths can be understood only in light of differences in institutions. Institutions, in turn, can be understood only in light of both market and non-market interactions.
7 References


8 Proofs

Proof of Proposition 1

Proof. We begin by noting that

\[ V_i^1 = \xi - \theta_i e_i^{\beta - 1} - J(e_i - \hat{e}_i). \]

Now let \( \theta_m = \min_i \{\theta_i\} \) and define the interval

\[ I \equiv [0, (\xi/\theta_m)^{\frac{1}{\beta - 1}}]. \]

If \( e_i = 0, V_i^1 = \xi + J\hat{e}_i > 0 \) whenever \( \xi > 0 \). If \( e_i = (\xi/\theta_m)^{\frac{1}{\beta - 1}} \), then

\[ V_i^1 = \xi - \theta_i \frac{\xi}{\theta_m} - J((\xi/\theta_m)^{\frac{1}{\beta - 1}} - \hat{e}_i). \]

Since \( \theta_i > \theta_m \), for any \( \hat{e}_i \in I \) we must have \( V_i^1 < 0 \). Recalling that \( V_{i1} < 0 \), and \( V_i^1 \) is continuous, we have \( e_i \in I \) whenever \( \hat{e}_i \in I \). A fixed point argument, as in Proposition 1 of GS, now shows that there exists at least one equilibrium \( e(\theta, \xi, \beta, J) \).

To show that there is at most one equilibrium we verify the MSI condition. Note that

\[ \left| \frac{V_{i2}}{V_{i1}} \right| = \left| -\frac{J}{J + \theta_i(\beta - 1)e_i^{\beta - 2}} \right| < 1 \]

for all \( \theta_i > 0, \beta > 1 \) and \( e_i > 0, \xi > 0 \) ensures that \( e_i > 0 \) for all \( i \). Now Proposition 3 of GS establishes the desired result. \( \Box \)

Proof of Proposition 2

Proof. We find \( \xi^* \), the share parameter that maximizes group utility subject to the constraint that individuals in the group choose equilibrium effort levels:

\[
\max_{\xi} V \equiv \sum_i (e_i - \theta_i e_i^{\beta}) - \frac{nr\nu}{2} \left[ \frac{(1 - \xi)^2}{n - 1} \right] - \frac{J}{2} \sum_i (e_i - \hat{e}_i)^2
\]

subject to

\[
\xi - \theta_i e_i^{\beta - 1} - J(e_i - \hat{e}_i) = 0 \quad (\forall i).
\]

From Proposition 1 we know there is a unique \( e(\theta, \xi, \beta, J) \) that satisfies the constraints. The first order condition from the maximization problem can be simplified to

\[
\sum_i (1 - \xi) \frac{de_i}{d\xi} - nr\nu \left( \frac{n\xi - 1}{n - 1} \right) + J \sum_i (e_i - \hat{e}_i) \frac{d\hat{e}_i}{d\xi} = 0.
\]

Upon expanding the sum, we see that

\[
\sum_i (e_i - \hat{e}_i) \frac{d\hat{e}_i}{d\xi} = \sum_i \sum_{j \neq i} (e_j - \hat{e}_j) \frac{de_i}{d\xi}.
\]
Since $\sum_i(e_i - \hat{e}_i) = 0$, we have
\[ \sum_{j \neq i}(e_j - \hat{e}_j) = -(e_i - \hat{e}_i), \]
which allows us to write
\[ \sum_i(e_i - \hat{e}_i) \frac{de_i}{d\xi} = - \sum_i(e_i - \hat{e}_i) \frac{de_i}{d\xi}. \]

The first order condition can be rewritten as
\[ V_{\xi}(\xi) \equiv \sum_i(1 - \xi) \frac{de_i}{d\xi} - r n_v \left( \frac{n\xi - 1}{n - 1} \right) - J \sum_i(e_i - \hat{e}_i) \frac{de_i}{d\xi} = 0. \]

Observe that
\[ \frac{de_i}{d\xi} = \frac{\partial e_i}{\partial \xi} + \sum_j H_{ij} \frac{\partial e_j}{\partial \xi}, \]
with $H \equiv F_1^{-1}$ as defined in the discussion following Proposition 1. When $\xi = 0$, we have $e_i = 0$ and (with $\beta \geq 2$) $\partial e_i/\partial \xi > 0$ for each $i$ so that, together with $(n\xi - 1) = -1$ we have $V_{\xi}(0) > 0$. In case $J = 0$,
\[ V_{\xi}(1) = \sum_i(1 - \xi) \frac{de_i}{d\xi} - r n_v \left( \frac{n\xi - 1}{n - 1} \right) = -r n_v < 0. \]

Continuity of $V_{\xi}$ shows that $\xi^* \in (0, 1)$ exists.

For $J > 0$, we show that $J \sum_i(e_i - \hat{e}_i) \frac{de_i}{d\xi}$ is positive for all $\xi$. This follows from two claims: (1) $\frac{de_i}{d\xi}$ is smaller when $\theta_i$ is larger, and (2) $e_i > \hat{e}_i$ when $\theta_i$ is smaller than average, while $e_i < \hat{e}_i$ when $\theta_i$ is greater than average. Since $\sum_i(e_i - \hat{e}_i) = 0$, weighting positive terms by larger (positive) numbers yields the required result. Claim (2) is immediate from Corollary 1. We prove claim (1).

(1) We denote $x_i \equiv \frac{\partial e_i}{\partial \xi}$ and $z_i \equiv (\frac{\partial e_i}{\partial \hat{e}_i})/(n - 1)$, observing that $z_i = J x_i/(n - 1)$. Suppose $\theta_i < \theta_j$. From the equilibrium conditions it is possible to show that $\theta_i(\beta - 1)e_i^{\beta - 2} < \theta_j(\beta - 1)e_j^{\beta - 2}$. This implies $x_i > x_j$. Now we examine $\frac{de_i}{d\xi}$ when the effect of changes in other players’ actions is taken into account. We first enumerate some properties of the matrix $H$ that appears in
\[ \frac{de_i}{d\xi} = \frac{\partial e_i}{\partial \xi} + \sum_j H_{ij} \frac{\partial e_j}{\partial \xi}, \]
where $F_1^{-1} = I + H$. Now $F_1$ has 1 as each diagonal element, and every off-diagonal element of row $i$ equals $-z_i$. From the Neumann expansion (see GS),
\[ H = (I - F_1) + (I - F_1)^2 + (I - F_1)^3 + \ldots. \]

Denote a term in the expansion $(I - F_1)^m$ by $A^m$ for $m \geq 1$. For each $m$ we have
1. $A^m(i, k) > A^m(j, k)$ when $k \notin \{i, j\}$,
2. $A^m(i, i) > A^m(j, j)$, and
3. $x_i A^m(j, i) = x_j A^m(i, j)$.

Consequently each property holds for $H$ as well. Now write
\[
\frac{de_i}{d\xi} = (1 + H_{ii})x_i + H_{ij}x_j + \sum_{k \neq i, j} H_{ik}x_k,
\]
\[
\frac{de_j}{d\xi} = (1 + H_{jj})x_j + H_{ji}x_i + \sum_{k \neq i, j} H_{jk}x_k.
\]

A comparison shows that $de_i/d\xi > de_j/d\xi$ when $\theta_i < \theta_j$.

We have $V_\xi(1) < 0$ and $V_\xi(\cdot)$ continuous so that a solution $\xi^* \in (0, 1)$ exists. \hfill \qed

**Proof of Corollary 1**

**Proof.** (1) Following the approach explained after Proposition 1 we see that, for each $i$,
\[
\frac{\partial e_i}{\partial \theta_i} = \frac{-e_i^{\beta-1}}{J + \theta_i(\beta - 1)e_i^{\beta-2}} < 0.
\]

Incorporating the social multiplier we have
\[
\frac{de_i}{d\theta_i} = (1 + H_{ii}) \frac{\partial e_i}{\partial \theta_i} < 0.
\]

An increase in $\theta_i$ will also increase $e_j$, by increasing $e_i$:
\[
\frac{de_j}{d\theta_i} = H_{ji} \frac{\partial e_i}{\partial \theta_i} < 0.
\]

An increase in $\theta_i$ will decrease all action levels. Since $(1 + H_{ii}) > H_{ji}$, part (1) is now proved.

(2) The equilibrium condition, for each $i$, can be written as
\[
\xi + J\hat{e}_i = \theta_i e_i^{\beta-1} + Je_i.
\]

This can, in turn, be written as
\[
\xi + \frac{J}{n-1} \sum_j e_j = \theta_i e_i^{\beta-1} + Je_i \frac{n}{n-1}.
\]

For every $e$,
\[
\theta_i e_i^{\beta-1} + Je \frac{n}{n-1} > \theta_j e_j^{\beta-1} + Je \frac{n}{n-1}
\]

whenever $\theta_i > \theta_j$. Together with the previous condition this implies that $e_i < e_j$ whenever $\theta_i > \theta_j$. \hfill \qed

**Proof of Proposition 3**

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Proof. (1) Let $U^\ell(\theta_i)$ denote
\[
\sup_{e_i} U(e_i, \theta_i, \xi, A_i^\ell).
\]
We follow GS (Section 2.5) and show that an agent with low $\theta_i$ gains more from belonging to the high action group. Consider the utility $V$ of an agent with share parameter $\xi$ when the average action of her peers is $A$.
\[
dV\,dA = \xi \frac{de_i}{dA} + (1 - \xi) - \theta_i e_i \beta - 1 \frac{de_i}{dA} - J(e_i - A) \frac{d}{dA} (e_i - A).
\]
From equilibrium conditions, and since $d(e_i - A)/dA = de_i/dA - 1$, this equals:
\[
J(e_i - A) + (1 - \xi).
\]
Differentiating with respect to $\theta_i$ we get
\[
\frac{d^2V}{dAd\theta_i} = J \frac{d}{d\theta_i} (e_i - A),
\]
which must be less than zero in light of the arguments in the Corollary and at the end of Proposition 2 (the decrease in $e_i$ when $\theta_i$ increases is greater than the decrease in any other $e_j$, hence $A$). Since the low cost group $\theta^1$ will have higher actions in equilibrium, the first part of the proposition follows.

(2) For a fixed $\xi$, in a segregated equilibrium, every agent would like to join the high action group. We choose a price for group 1 that is just higher than the increase in utility for the agent in group 2 who has the highest gain from switching.

Now suppose that for each group $\theta^\ell$, $\xi^\ell$ is optimal for $\ell$. The value of $\xi$ which maximizes a player’s utility is higher when $\theta_i$ is lower. In addition, $\xi^\ell$ is found by averaging the first order conditions that define each individual’s best $\xi$. Hence, each individual in group 1 must have a higher gain from belonging to group 1 than a member of group 2. As before, we can find a price for group 1 that separates the players by $\theta_i$.

9 Data

9.1 Data sources

The physician level and group-practice data are the results of a 1978 survey conducted by Mathematica Policy Research Group. These are the same data used by GG to test their model of moral hazard under revenue-sharing contracts. Controlling for missing values, we observe 755 physicians in 236 groups. Prices are reported in 1978 dollars. All the groups operated under a fee-for-service system and provided primarily ambulatory care. Specialties among the observed physicians consisted of general practice, internal medicine, pediatrics, and obstetrics/gynecology. We also use data describing characteristics of the market area served by each group. These data were collected by GG from a list of sources including the American Medical Association, The County and City Data Book, and the American Hospital Association Guide. The sources are described at length in Boldin et al. (1979).

\[\text{The reported sample sizes are after omission of records with missing values. Average actual group size for the 236 observed groups is about 16 physicians. When computing group-average values for given variables, we use all available data, including data reported by physicians ultimately excluded based on their not reporting for other variables.}\]
9.2 Variable names and measurement

The dependent variable in equation (8) is physician output, measured by the number of office visits per week (VISITS). The key independent variables are the incentive term and the mean peer output, both of which are represented by instrumental variables. The incentive term (INCENTIVE) is the product of the group’s office visit fee and its ‘compensation scale’ variable. The compensation scale corresponds roughly to the term $\zeta$ from our theoretical model and captures the extent of revenue sharing in the group. To measure this variable, the survey asked respondents to rank, on a scale of one to ten, the relationship between individual productivity and compensation.$^{11}$ Mean peer output (GRPMEAN) represents, for any given physician, the average office visits over all other physicians in the same group practice. These values were computed prior to omitting physician records with missing values for other variables of interest in order to use all available information.

The remaining independent variables for the second stage are as follows: hours of non-physician labor per week (HRSNON), number of exam rooms at the group facility (EXAM-RMS), office visit fee (PRICE--given by an instrumental variable), individual years experience and experience squared (EXPER and EXPERSQ), foreign medical graduate dummy (FMG), subspecialty dummy (SUBSPEC), dummies for specialty in pediatrics (PDS), internal medicine (IM) and obstetrics/gynecology (OBS), board certification dummy (BOARD), multispecialty dummy (MULTSPEC) indicating whether the group employs physicians of more than one specialty, hospital beds per capita in the relevant market region (BEDSPOP), physicians per capita in the region (MDPOP), average regional apartment rental rate (RENT), percent of regional population receiving AFDC payments (AFDC), regional population per square mile (POPDENS), regional per capita income (PCAPINC), and average years education among the regional population (EDUC).

As explained above we use instruments for the endogenous firm-level variables and for mean peer output. The list of instruments includes non-physician wages for each of four classes of non-physician labor (WAGELPN, WAGECLT, WAGEULT, WAGEBA), and the group averages of variables measuring risk preferences, income variance, preferred group size, years experience, medical training location (foreign vs. domestic), and board certification. To measure risk aversion, the survey asked physicians to rank the importance of income regularity on a scale of one to ten. The group average of this ranking is denoted AIMPR-EGY. The average value for preferred group size is denoted APREFSIZ. To measure income variance, we use the specialty and subspecialty dummies mentioned above. For group-level measures of these variables, we use the percentage composition of each specialty (PCTGS for general surgery, PCTOB for obstetrics/gynecology, PCTIM for internal medicine, and PCTPD for pediatrics), and the average value of the subspecialty dummy (ASUBSPEC). We also include the average of the foreign medical graduate dummy (AFMG), and percent board certified (PCTBOARD), average years experience (AEXPER) and average experience squared (AEXPERSQ).

$^{11}$The reported values are highly correlated with alternate measures of the compensation structure from the same survey (see GG p. 599).
Table 1: Statistics on equilibrium effort and optimal share for groups with small cost differences

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<thead>
<tr>
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<th>Scale</th>
<th>Average Effort</th>
<th>Stdev</th>
<th>COV</th>
<th>ξ*</th>
<th>Average Effort</th>
<th>Stdev</th>
<th>COV</th>
<th>ξ*</th>
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Parameters: \( N = 10, \beta = 2, \gamma = 1. \)

Table 2: Statistics on equilibrium effort and optimal share for groups with differences in risk premium

<table>
<thead>
<tr>
<th>Group</th>
<th>Risk Premium</th>
<th>Average Effort</th>
<th>Stdev</th>
<th>COV</th>
<th>ξ*</th>
<th>Average Effort</th>
<th>Stdev</th>
<th>COV</th>
<th>ξ*</th>
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Parameters: \( N = 10, \beta = 2, \gamma = 1. \)
Table 3: Statistics on equilibrium effort and optimal share for groups with small cost differences (Case 2)

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<tr>
<th>Group</th>
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<th>Stdev</th>
<th>COV</th>
<th>( \xi^* )</th>
<th>Average Effort</th>
<th>Stdev</th>
<th>COV</th>
<th>( \xi^* )</th>
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Parameters: \( N = 10, \beta = 2, r \nu = 1. \)

Table 4: Statistics on equilibrium effort and optimal share for groups with differences in risk premium (Case 2)

<table>
<thead>
<tr>
<th>Group</th>
<th>Risk Premium</th>
<th>Average Effort</th>
<th>Stdev</th>
<th>COV</th>
<th>( \xi^* )</th>
<th>Average Effort</th>
<th>Stdev</th>
<th>COV</th>
<th>( \xi^* )</th>
</tr>
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Parameters: \( N = 10, \beta = 2, \text{Scale} = 0.5. \)
Table 5: Instrumental variable and OLS estimates of the individual physician demand function. Dependent variable: office visits.

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† Instrumental variable
Figure 1. Effort distribution for $J = 0$ and $J = 1$

Figure 2. Effort distribution when share increases from 0.3 to 0.35
Figure 3. Effort distribution when costs are scaled down

Figure 4. Optimal shares when $J = 0$ and $J = 1$
Figure 5. The effect on optimal share of a decrease in costs

Marginal social utility (with equilibrium effort)

Scale = 0.4
(J = 1)

Scale = 0.5
(J = 1)

J = 0